Optimal Feedback in Contests

George Georgiadis (Northwestern Kellogg)

with

Jeff Ely  •  Sina Khorasani  •  Luis Rayo
Motivation

- Contests can be an effective way to organize economic activity
  - Labor market (promotion) tournaments
  - Innovation contests
  - All-pay auctions
  - Legal & political battles
  - Athletic tournaments

- Contests are inherently dynamic, and designer may have informational advantage over participants about how they are doing mid-contest

This paper:
Characterizes optimal dynamic contests when the designer chooses \textit{when} the contest ends, \textit{how} a prize is allocated, and a real-time feedback policy
Applications

1. Promotion contests
   - A firm has an open VP slot and wants to promote one of its associates
   - It monitors efforts imperfectly, and is better informed than the associates themselves about their performance
   - How to design a contest to maximize the associates’ efforts?

2. Innovation races
   - 2006 Netflix Prize: $1M prize for an algorithm that predicts user film ratings with at least 10% better accuracy than Netflix’ own algorithm
   - How to design the rules of contest?
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2. Innovation races
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Related Literature

- **Static tournaments / contests:**
  - Lazear & Rosen ('81), Green & Stokey ('83), Nalebuff & Stiglitz ('83)
  - *Optimal prize allocation*: Moldovanu & Sela ('01), Drugov & Ryvkin ('18, '19), Olszewski & Siegel ('20)
  - "*Turning down the heat*": Fang, Noe & Strack ('18), Letina, Liu & Netzer ('20)

- **Dynamic contests:**
  - Taylor ('95), Benkert & Letina ('20)

- **Feedback in contests:**
  - "*Reveal intermediate progress?*": Yildirim ('05), Lizzeri, Meyer & Persico ('05), Aoyagi ('10), Ederer ('10), Goltsman & Mukherjee ('19)
  - *Contests for experimentation*: Halac, Kartik & Liu ('17)
Model (1/4): Players & Timing

- **Players:** A principal and $n \geq 2$ agents

- At $t = 0$, the principal designs a mechanism (contest) comprising
  - i. a rule specifying *when* the mechanism will end,
  - ii. a rule for allocating a $1$ prize, and
  - iii. a real-time feedback policy

- At every $t > 0$, each agent
  - receives a message per the feedback policy, and
  - chooses to *work* or *shirk*; i.e., $a_{i,t} \in \{0, 1\}$

- When mechanism ends, prize is awarded according to allocation rule
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Each agent’s effort generates a binary signal: a Poisson “success”

- Conditional on not having succeeded by \( t \), an agent succeeds during \((t, t+dt)\) with probability \( a_{i,t} dt \); i.e., constant hazard rate of success
- Each agent can succeed at most once (*extend to multiple successes later)

Who observes what:

- Principal observes successes but not efforts
- Agents do not observe their rivals’ successes
- Ea. agent may or may not observe own success or do so probabilistically

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Model (3/4): Principal’s Choice Variables

i. A **termination rule** is a stopping time w.r.t each agent’s success time
   - *e.g.*, mechanism may end at deadline, upon first success, randomly, etc

ii. A **prize allocation rule** specifies each agent’s share of the prize $q_i$ as a function of when each agent succeeds
   - *e.g.*, prize may be awarded to first / second agent to succeed, split, etc

iii. A **feedback policy** specifies the message sent to each agent at every instant as a function of the agents’ success times and past messages
   - *e.g.*, Random feedback, private or public feedback, feedback about one’s own or others’ successes, feedback about feedback, etc
   - $M_{\text{pronto}}$: Keeps agents apprised of own success (but no other feedback)
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Model (4/4): Payoffs

- Given a contest, each agent’s expected payoff is

\[ u_{i,t} = \max_{a_{i,t} \in \{0,1\}} \mathbb{E} \left[ q_i - c \int_0^\tau a_{i,t} dt \right], \]

where \( c \in (1/n, 1) \).

- Principal designs a mechanism to maximize total effort

\[
\max \mathbb{E} \left[ \sum_{i=1}^n \int_0^\tau a_{i,t} dt \right] \\
\text{s.t. } \{a_{i,t}\} \text{ forms an equilibrium} \\
\sum_{i=1}^n q_i \leq 1. \\
\text{(Budget Constraint)}
\]

* Will argue that effort-maximizing contest also maximizes \( \mathbb{E}[\# \text{successes}] \)
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Roadmap

I. Sufficiency result for a mechanism to maximize total effort

II. Examples of effort-maximizing contests

III. Necessary conditions for optimality

IV. Effort-maximizing contest with shortest expected duration

V. Extensions: Multiple successes & Limited commitment
A Sufficiency Result

Finding an optimal contest is hard because the choice variables are high-dimensional objects and can condition on the entire history.

Lemma 1. A contest is guaranteed to be optimal if in equilibrium:

i. The prize is awarded with probability 1

ii. Each agent earns zero rents

The principal’s objective can be written as

\[
\mathbb{E} \left[ \sum_{i=1}^{n} \int_{0}^{\tau} a_{i,t} dt \right] = \frac{1}{c} \left( \mathbb{E} \sum_{i} [q_i] - \sum_{i} u_{i,0} \right) \leq \frac{1}{c}
\]

Total Surplus \leq 1  \quad Rents \geq 0

If a contest attains those bounds, it must be optimal!
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$$\mathbb{E} \left[ \sum_{i=1}^{n} \int_{0}^{T} a_{i,t} dt \right] = \frac{1}{c} \left( \mathbb{E} \sum_{i} [q_i] - \sum_{i} u_{i,0} \right) \leq \frac{1}{c}$$

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Example 1. Cyclical-Egalitarian Contest

- **Termination** $\tau^\ast$. Runs in cycles of length $T^\ast$ and is terminated at the end of the first cycle in which at least one agent has succeeded.

- **Egalitarian prize allocation**. Prize is shared equally among agents who have succeeded *irrespective of when they did so*.

**Proposition 1.**

- The contest with $\tau^\ast$, EGA, and feedback policy $M^{\text{pronto}}$ is optimal.

- In equilibrium, each agent works until they succeed and earns no rents.

- Contest is optimal because it meets sufficiency conditions of Lemma:
  - $T^\ast$ chosen such that marg. benefit of effort is equal to marg. cost
  - Cyclical structure ensures that at least one agent succeeds
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Cyclical-Egalitarian Contest. Proof Sketch

- **Lemma 1**: Zero rents & prize awarded w.p 1 $\Rightarrow$ Contest is optimal
  - Because contest ends only after an agent succeeds, $2^{nd}$ criterion is met

- Each agent’s flow payoff can be expressed as

  $$(1 - p_t) \times a_t \times \frac{R_t}{\mathbb{E}[\text{prize}|\text{success at } t]} - c \times a_t$$

  - $\Pr\{\text{no success by } t\}$
  - Success rate
  - $\mathbb{E}[\text{prize}|\text{success at } t]$
  - Cost of effort

- $M^{\text{pronto}}$ implies that $p_t = 0$, and it jumps to 1 as soon as he succeeds

- Each agent’s expected reward from success at $t$ is:

  $$R_t = \mathbb{E}\left[\frac{1}{1 + (\#\text{rivals who succeed by } T^*)}\right]$$

  Can choose $T^*$ such that $R_t = c$ so that working is *just* IC for each agent until he succeeds, and he earns zero rents.
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Example 2: Beeps Contest

- **Termination rule.** Conditional on at least one success, at $T^*$ the contest ends w.p $q$, and from then onwards with rate $r$.
- **Prize allocation.** Prize shared equally among agents who succeeded prior to $T^*$. Otherwise, the first agent to succeed wins entire prize.

**Proposition 2.**
There exist $\{q, r\}$ such that this contest, coupled with $M^{pronto}$ is optimal

- Before $T^*$, resembles a single cycle of the cyclical-egalitarian contest
- After $T^*$, termination rule keeps ea. unsuccessful agent’s belief that nobody has succeeded constant at $c$. Flow payoff from working:

$$\Pr\{\text{no success yet}\} \times (HR \text{ success}) \times \mathbb{E}[\text{prize}] - c = 0$$

so ea. unsuccessful agent is just willing to work and earns no rents
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Example 3: Netflix-Style Contest

- **Termination.** The first agent to succeed triggers countdown $T^c$
- **Prize allocation.** First agent to succeed earns prize $\frac{\alpha}{\alpha + N}$, and each agent who succeeds during countdown earns $\frac{1}{\alpha + N}$

**Proposition 3.**
There exist $\{T^c, \alpha < 1\}$ s.t this contest, coupled with $M^\text{pronto}$ is optimal.

- If the first agent to succeed won the entire prize, he would earn rents
  - Can extract rents by extending contest & giving rivals another chance
- **Aim:** Expected reward from success $R_{i,t} = c$ for all $i, t$
  - During countdown agents know one agent has already succeeded, so must earn a bigger share of the prize than the first agent; hence $\alpha < 1$
- Resembles Netflix prize: first success triggered a 30-day countdown
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**Why a contest instead of individual contracts**

- *Definition*: “Contest” if effort creates a negative externality
  - *i.e.*, if an agent’s payoff decreases in others’ efforts or successes

- Suppose principal splits the prize and offers individual contracts
  - Because prize = $1, the marginal benefits of effort $\sum_i R_{i,t} \leq 1$

- Optimal contests have $R_{i,t} = c$ for all $i, t$, so $\sum_i R_{i,t} = cn > 1$
  - The advantage of a contest is that it allows pooling the agents’ ICs
  - Prize not awarded to one agent can be used to incentivize another
  - This pooling is valuable whenever $c > 1/n$; *i.e.*, when prize is scarce

- *Remark*: If principal can meet $1$ budget constraint *in expectation*, then individual contracts suffice
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Optimal Contests: Examples

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A Necessity Result

- \textit{Obs.} Every optimal contest meets sufficiency conditions of Lemma 1

**Proposition 4.** Every optimal contest features:

i. Termination rule s.t. contest doesn’t end until 1+ agents succeed

ii. $\mathcal{M}^{\text{pronto}}$ feedback

iii. Egalitarian prize structure; \textit{i.e.,} $R_{i,t} = c$ whenever $a_{i,t} = 1$

- $\mathcal{M}^{\text{pronto}}$ ensures there is never asymmetric info btw principal & agent
  - Suppose on the eq’m path, there is an interval in which $p_{i,t} \in (0,1)$
  - IC requires $(1 - p_{i,t})R_{i,t} \geq c$, so $R_{i,t} > c$ during that interval
  - Agent could shirk until that interval so that $p_{i,t} = 0$ and earn rents

- Given $\mathcal{M}^{\text{pronto}}$, full rent extraction requires $R_{i,t} = c$ whenever an agent is supposed to be working
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Optimal Contests: Remarks

In every optimal contest:

i. Owing to $M^{\text{pronto}}$, it is immaterial whether agents observe their successes directly, or do so probabilistically.
   - It may be important however that they don’t observe others’ successes.

ii. Due to $M^{\text{pronto}}$, an optimal contest maximizes total effort conditional on not having succeeded already. So it also maximizes $E[\#\text{successes}]$.

iii. Principal would be no better off with a more precise monitoring tech.
   - To extract all rents, monitoring tech. must generate no type-I errors.

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Optimal Contests: Remarks

In every optimal contest:

i. Owing to $M^{pronto}$, it is immaterial whether agents observe their successes directly, or do so probabilistically.
   - It may be important however that they don’t observe others’ successes.

ii. Due to $M^{pronto}$, an optimal contest maximizes total effort conditional on not having succeeded already. So it also maximizes $\mathbb{E}[\#\text{successes}]$.

iii. Principal would be no better off with a more precise monitoring tech.
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Minimum-duration, Effort-maximizing Contest

- Every effort-maximizing contest implements total effort $1/c$
- Here, we characterize the one with the shortest expected duration
  - e.g., suppose the principal incurs a small cost p.u of time contest is on

- Fix an effort-maximizing contest, and define for each $k$,
  $$T_k := \mathbb{E}[^{\text{time when } k \text{ agents are working}}].$$

  - $T_k \leq 1/k$ because when $k$ agents work, next success $\sim \exp(1/k)$
  - Total effort $= \sum_k kT_k = 1/c$
  - Expected duration of contest $= \sum_k T_k$

- Roadmap:
  a. Suppose we can choose $T_1, \ldots, T_n$ directly $\Rightarrow$ Lower bound on duration
  b. Find a contest that achieves this lower bound
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A Lower Bound for Contest Duration

Consider the following relaxed problem:

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\min_{T_1, \ldots, T_n} \sum_{k=1}^{n} T_k \quad \text{s.t.} \quad \sum_{k=1}^{n} kT_k = \frac{1}{c} \quad \text{and} \quad 0 \leq T_k \leq \frac{1}{k}.
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Define \( K^* = \lceil 1/c \rceil \). The following is the unique solution:

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T_k = \begin{cases} 
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Lemma 2. Every effort-maximizing contest has \( \mathbb{E}[\text{duration}] \geq \sum_k T_k \).

- W.p 1, contest must end after \( K^* \) but before \( K^* + 1 \) agents succeed.
- None of the earlier examples satisfy this criterion!
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Second-Chance Contest

- **Termination rule.** After $K^*$ agents succeed, contest is terminated upon the next success or countdown $T^{sc}$ ends, whichever comes first.

- **Prize allocation rule.**
  - If an agent succeeds during the countdown, he earns $c$
  - Remaining prize is shared equally among the first $K^*$ successful agents.

**Proposition 5.**
There exists a $T^{sc}$ such that this contest coupled with $M^{pronto}$ feedback has the smallest duration among effort-maximizing contests.

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Extensions

Extension I: Agents can succeed multiple times

- Assume principal wants to implement max. effort in shortest duration
- A second-chance contest with countdown duration $T^{sc} \in \{0, \infty\}$:
  - Ends w.p. $(1 - 1/c + K^*)$ upon $K^{th}$ success; otherwise upon $K^* + 1^{st}$
  - Prize $c$ for $K^* + 1^{st}$ success; remaining prize split among $K^*$ successes
  - Agents are told when $K^{th}$ success occurs (other feedback immaterial)

Proposition 6.
This contest implements total effort $1/c$ and all agents work until the end
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Extension II: Limited Commitment

- Principal must credibly commit to feedback policy & termination rule
  - Ex-post, she has incentives to keep contest / agents “going”
- Suppose principal cannot credibly provide feedback and termination cannot condition on successes

Proposition 7. Assume agents do not observe their own successes.
- Optimal no-feedback contest ends at a deterministic deadline $T$, and the prize is shared equally among all agents who succeed by deadline.
- In equilibrium, all agents work continuously throughout $[0, T]$
- Over time, each agent believes it is ever more likely they have already succeeded, in which case continuing to exert effort is moot
- Egalitarian prize counteracts this by maximally backloading incentives
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Conclusions

- Contest design with endogenous feedback to maximize total effort

- Many contests are optimal. Every optimal one satisfies two criteria:
  1. Agents are kept fully apprised of their own success
  2. Expected reward from success is constant

- Characterize the minimum-duration, effort-maximizing contest
  - Countdown is triggered once a pre-specified number of agents succeed
  - Contest ends when countdown ends or another agent succeeds
  - Prize is shared (approximately) equally among successful agents

- Broader agenda: Information design in agency models
  - How to use information to provide incentives (under moral hazard)