Delegation under Liquid Democracy
Two Experiments.*

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Abstract

Under Liquid Democracy (LD), decisions are taken by referendum, but voters are allowed to delegate their votes to other voters. Theory shows that in common interest problems where experts are correctly identified, the outcome can be superior to simple majority voting. However, delegation reduces the variety of independent information sources and can be advantageous only if used sparely. We report the results of two experiments, each studying two treatments: in one treatment, participants have the option of delegating to better informed individuals; in the second, participants can choose to abstain. The first experiment follows a tightly controlled design planned for the lab; the second is a perceptual task run online where information about signals' precision is ambiguous. Although the experiments are very different, they reach the same result: in both, LD underperforms relative to both universal voting and abstention. In the lab experiment, we observe systematic overdelegation relative to the best theoretical equilibrium; in the perceptual task, we lack a precise theoretical benchmark, but delegation rates remain significantly higher than abstention rates. Faced with better informed experts, voters neglect the value of their own independent information.

JEL codes: **C92, D70, D72, D91**

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1 Introduction

> I believe that some sort of computerized participation by large numbers of the public in opinion formation and direct policy-making is in the cards in the next ten to twenty years. It may be that we will be able to turn this new technology to the improvement and defense of democratic institutions. I hope so. However, it is by no means evident that this will be the result. (Martin Shubik, 1970, commenting on Miller, 1969)

In Western societies, the sense of living a crisis of traditional political institutions is bringing calls for different, more participatory forms of democracy. Among these, Liquid Democracy has caught the imagination of the young and the tech-savvy. It advocates a voting system where all decisions are submitted to referendum, but voters can delegate their votes freely. Beyond its intellectual roots in the writings of Charles Dogson (in particular, Dogson, 1884), Liquid Democracy was proposed more recently by James Miller in 1969. It has been adopted occasionally for internal decisions by European protest parties–the Swedish and the German Pirate parties being the most famous examples–and now finds vocal support in the tech community, where it aligns both with the emphasis on a non-hierarchical order and with the use of cryptographic tools to maintain confidentiality and reliability.[1] Although the details vary, the common point of different implementations is the ease and specificity of delegation. Supporters herald it as the golden medium between representative and direct democracy: better than the former because representatives can be chosen according to their specific competence on each decision, better than the latter because uninformed or uninterested voters can delegate their votes.

There is a clear, immediate problem: are experts correctly identified? But there is also a second, more subtle, but also more fundamental question: even if the experts are correctly identified, delegation deprives the electorate of the richness of noisy but abundant information distributed among all voters. Unless the extent of delegation is modulated correctly, Condorcet has taught us that a smaller number of independent voters, even if more accurate, may well lead to worse decision-making. This very basic trade-off is the necessary point of departure of Liquid Democracy and is the focus of this paper.

We study a canonical common interest model where voters receive independent signals, conditional on an unknown state ex ante equally likely to take one of two values. The

[1] See for example LiquidFeedback (https://liquidfeedback.com/en/), the Association for Interactive Democracy (https://interaktive-demokratie.org/association.en.html), or Democracy.Earth (https://democracy.earth/). Google ran a 3-year experiment on its internal network, implementing Liquid Democracy for decisions like food menu choices, tee-shirt designs, or logos for charitable events (Hardt and Lopes, 2015). Liquid Democracy is becoming the governance choice for cryptoworld DAO's (Decentralized Autonomous Organizations)–see for example Element Finance (https://medium.com/element-finance).

common objective is to identify the state correctly, aggregating information via majority voting. Signals vary across individuals in the probability of being correct—a variable we denote as *precision*. Experts are publicly identified and the precision of their signals is known; for all other voters, signals' precisions are private information but known to be weakly lower than the experts'. If a voter chooses to delegate, the vote is randomly assigned to one of the experts. We begin by showing theoretically that for any size of the group and any number of experts, there is an equilibrium with positive delegation such that the outcome is superior to majority voting without delegation. However, in such an equilibrium delegation must not be too frequent, given its informational cost. The finding is not surprising, but the equilibrium frequency is counter-intuitively low. For example, consider one of the parametrizations we study: a group of 15 voters of which 3 are experts; the experts' information is correct with probability 70%, while the precision of non-experts' signals can take any value between 50 and 70%, with equal probability. Then a non-expert with information that she knows is only 55% likely to be correct should not delegate to experts whom she knows to be correct with probability 70%. And mistakes are costly: small errors towards over-delegation lead to expected losses that soon become severe. In actual implementations, other factors, for example overconfidence and overweighing of own information, could introduce countervailing forces. It is with these concerns in mind that we test Liquid Democracy with two very different experimental designs.

Before describing the experiments in detail, note that the informational benefit from overweighing voters with more precise signals can be achieved via abstention as well, as long as abstention correlates with more inaccurate signals, as it will in the equilibria of a pure common interest model without idiosyncratic voting costs. Abstention differs from delegation because the increase in voting weight concerns *all* individuals who choose to vote, not only those targeted as delegates. Yet, we know from McMurray (2013) that, under common interest and in the absence of voting costs, it too can lead to improvements over simple majority voting, and for reasons very similar to those favoring delegation. Abstention is a familiar option and does not require any transfer of votes, reducing the appearance of suspicious deals. Its performance relative to delegation is thus an interesting question per se, and our experiments compare the two alternatives.

The first experiment was designed for the lab and follows the theory very closely. We study groups of either 5 voters (of which 1 is an expert) or 15 voters (of which 3 are experts). We observe the frequency of delegation and the fraction of group decisions that yield the correct outcome. We then compare these results to a second treatment, where the option of abstention takes the place of delegation, and we evaluate both relative to simple majority voting with voting by all. We find systematic over-delegation: delegation rates that are between two and three times the rate in the unique strict equilibrium, given the realized experimental precisions. As a result, Liquid Democracy (LD) underperforms, relative to simple majority without delegation. Under Majority Voting with Abstention (MVA), abstention rates are instead very close to the theory, and the fraction of correct decisions is comparable to what majority voting without abstention (or delegation) would deliver. Interestingly, MVA suffers from its own sub-optimal behavior: the environment is symmetric, and voting in line with one's own signal is optimal, but experimental subjects occasionally deviate, and deviate more when abstention is allowed. In the data, voting according to signal correlates positively with the signal's precision, and since more subjects vote under abstention, at lower precisions, we also observe more votes against signal. Although the frequency of such deviations remains low, the result is a decline in correct group decisions that prevents MVA from reaping the gains over universal voting that theory predicts. The conclusion of our first experiment, then, is that even when experts are correctly identified and both LD and MVA have the potential to dominate universal majority voting, both systems in fact fail to do so. LD in particular shows a more clearly detectable negative effect.

The experimental design we implemented is canonical: it follows standard procedures for voting experiments with common values and has been widely and succesfully used in the literature (add refs: Battaglini et al. Morton and Tyran; others?) But could the design itself be biasing results against LD and MVA? There are three reasons to consider the question. First, as mentioned, the theoretical thresholds for equilibrium delegation seem counter-intuitive. But what makes them counter-intuitive, in our view, is not their low value per se but the detailed mathematical manner in which information is conveyed in the experiment, and thus thresholds are chosen. Each participant is told a number for her own precision and is naturally induced to compare such number to the known precision of the experts–in the earlier example, the fact that 55% is transparently lower than 70% makes the difference very salient. In reality, voting decisions take place in an ambiguous world, where individuals do not have explicit numerical knowledge of the reliability of their and others' information. Evaluations are fuzzier. The large over-delegation we find in our first experiment under LD could be less pronounced in a less precise world. A similar argument may also affect voting against signal under MVA. The detailed mathematical design gives us clean theoretical predictions, but could in fact be confusing participants, no matter how much we clarify the instructions. At 55% precision, thinking that one should vote against signal about half the time is a reasonable enough thought. In an actual voting situation, however, lacking an explicit mathematical value for the probability that one's information is correct, it is unlikely that individuals would vote against their best estimate of the right decision.

Finally, there is a third reason for considering a less controlled environment. Could our detailed mathematical design be favoring universal majority voting? In a still current analysis of the Condorcet Jury theorem, suggestively titled "A Note on Incompetence", Margolis (1976) commented on what he saw as the contradiction between the Condorcet theorem and political reality: in common interest problems, "Why is not decision-making trivially easy?". The existence of private interests, correlated signals, asymmetric scenarios, all may lead direct democracy to function less well, but Margolis proposed a different explanation. What if, over some questions and for some voters, information is actually correct with probability *lower* than 1/2? For any individual voter this will not be true when averaging over many decisions, but may well be true over some. And it will affect the probability that the majority decision is correct. By opening some space for improvement over universal majority voting, note also that the possibility of worse than random information allows testing LD in larger electorates, where it is meant to be applied, while maintaining pure common interest and independent signals. With binary choices, conveying information with less than random accuracy requires adding a second level of noise–noise in information about the accuracy of one's noisy signal. It could be introduced in the experiment in precise mathematical form.[2] However, together with the arguments above, we see the desirability of additional for noise as another reason to accept the realistic ambiguity of collective decision-making.

Our second experiment then is meant to capture a voting environment such that voters have "some sense" of how well-informed they are and how likely to be correct, and similarly of how likely experts are to be correct, but such sense is vague and instinctive. There is of course a cost: we lose the precise control granted by Experiment 1. However, even though our aims are different, we can exploit a very rich literature that studies problems with exactly these features: the large literature in psychology and neuroscience that studies perceptual tasks. Our focus is not on measuring accuracy of perception, but on designing the task as a group decision problem.[3]

The Random Dot Kinematogram (RDK) is a classic perceptual task amply used in vision and cognitive research.[4] A number of moving dots are displayed for a very short interval; some move in a coherent direction, either Left or Right in our binary implementation, others

[2]In the absence of noise, if it is known that a signal is more likely to be wrong than right, it is also known that its negative is more likely to be right than wrong..

[3]After having completed this study, we discovered an intriguing paralellism with Margolis' own thinking after the 1976 article. Margolis went on to advocate understanding judgement, including judgement in voting and political reasoning, through the lenses of patterns recognition, starting with perception biases (Margolis, 1987).

[4]It was originally developed to study the perception of motion under noisy conditions in humans and non-humans primates (e.g. van de Grind et al., 1983). In neuroscience, it has been used to study the neuronal correlates of motion perception (Newsome et al., 1989, Britten et al., 1992 and Roitman and Shadlen, 2002)

move at random; subjects report in which direction they think coherent dots are moving. We can label experts ex post as the individuals with performance in the highest quintile, and generate a collective decision by aggregating individual responses, with the additional option of delegation to the experts (in the Liquid Democracy treatments) or abstention (In the MVA treatments). We ran the experiment on Amazon Mechanical Turk with three electorate sizes: $N = 5$, and $N = 15$, as in Experiment 1, and a larger electorate of $N = 125$.

We reach three results. First, given our experimental parameters, it is not rare for individuals' accuracy to be worse than random. And this even over a large number of decisions: aggregating at the subject level over all 120 tasks, between 9 and 12% of subjects have ex post accuracy strictly below randomness; between 11 and 17% do no better than randomness. If we want to study voting and information aggregation when information may be faulty, perceptual tasks can provide a very useful tool.

Second, we do not know individuals' beliefs about their accuracy and the accuracy of others, and cannot compare the extent of delegation or abstention we see in the data to precise theoretical benchmarks. Yet, remarkably, we find the same patterns we saw in Experiment 1. The distributions of voters' accuracy we observe in the two samples–Liquid Democracy and MVA–are effectively identical, but delegation is twice as frequent than abstention when $N = 5$, and more than 50% more frequent when $N = 15$[5], if anything accentuating the disparity observed in the first experiment. Between one fourth and one third of subjects choose to abstain, but about half, in all treatments, choose to delegate.

Third, the high frequency of delegation exacts its expected informational costs. Even with a relatively high fraction of random, or below random subjects, universal majority voting remains the best information aggregator, delivering the highest frequency of correct group decisions in all treatments; MVA is only slightly less efficient, while Liquid Democracy is dominated by both in all treatments.

The robustness of the conclusions across two very different experimental designs is the main contribution of this study. The second design sacrifices experimental control in exchange for a less mathematical formulation; it conveys less information and leaves more space for idiosyncratic responses; it is run on MTurkers rather than students, it is much shorter, and it includes one treatment with a much larger group. And yet, and contrary to our expectations, treatment effects in the second design closely replicate the effects we observe in the first. In a pure common interest setting where experts are correctly identified, individuals over-delegate. The resulting increase in the voting weight of the experts does not lead to an increase in efficiency because the extent of delegation is too high, and thus the net informational effect is negative. Experimental subjects are less prone to abstaining,

[5]It is 75% more frequency when $N = 125$.

and thus the simpler routine option of allowing abstention leads to better outcomes than allowing delegation.

The second contribution of this study is methodological. While the controlled design of our first experiment in the end delivers robust conclusions, we think that it is important to add to our experimental tool-kit designs that recognize the ambiguity present in group decision-making. Experiments on ambiguity at the individual level are common; to our knowledge they are much less so for collective decision-making.[6] In voting problems, in particular, the complexity of many questions and the asymmetry between the cost of acquiring information and the small marginal impact of a single vote make the lack of precise information very likely. Perceptual tasks, with the large and sophisticated literature that accompanies them, can be a particularly usable tool. In this study, it is the combination of a strictly controlled design in the lab with the freer design of the perceptual task that we think teaches us the most. And it is this combination that we recommend.

Our work is related to three separate literatures. First, to the study of voting as information aggregation. The informational costs and benefits of delegating to better informed individuals in pure common interest voting problems were the subject of early studies on the Condorcet Jury theorem (Margolis, 1976, Grofman et al., 1983 and 1983, Shapley and Grofman, 1984), highlighting, as we do, the trade-off between the loss in aggregate information and the more precise information of the individuals actually casting votes. These studies asked important statistical questions but did not focus on rational equilibrium behavior. More recent work (Austen-Smith and Banks, 1996; Feddersen and Pesendorfer 1997; McLennan, 1998; Wit, 1998) put the analysis of the Condorcet Jury theorem on solid equilibrium grounds, but abstracted from the focus on delegation. We did not find in the literature our starting theoretical result, on the existence of an equilibrium with delegation that dominates universal majority voting, but the result builds on the work of McLennan. As we discussed, the trade-off identified in the case of delegation exists also in the case of abstention. Here the best-known work includes partisan voters (Feddersen and Pesendorfer, 1996), but the analysis can also be profitable and rich in a pure common interest setting, as shown by Morton and Tyran (2011) in the case of three voters, and more generally by McMurray (2013). Battaglini et al (2010) and Morton and Tyran (2011) test the theoretical predictions in the lab. The latter model is closer to ours, and, contrary to Battaglini et al., allows for a range of information types and does not rely on the existence of perfectly informed voters. Interestingly, it finds that experimental subjects appear predisposed towards abstaining,

[6]There is an increasing focus on strategic uncertainty. But the question is different from the lack of basic information about the distributions of relevant parameters in the population, and even about own parameters (accuracies, for us).

doing so also even when abstention is dominated. In Morton and Tyran's words, subjects "follow a norm of "letting the experts decide"". According to our results, this tendency is strengthened further when the choice is explicitly phrased as delegation. We are not aware of existing experimental studies in this literature that expressly study delegation in voting. [**To be checked further**].

The second strand of related works are studies of Liquid Democracy. Most of these works belong either in normative political theory or in computer science. Green-Armytage (2015) and Blum and Zuber (2016) discuss what they see as normative advantages of Liquid Democracy, on both epistemic and equalitarian reasons: decisions are taken by better informed voters, and at the same time LD avoids the creation of a detached class of semi-permanent professional representatives. Because the focus is normative, these studies do not analyze strategic incentives. The computer science literature is instead largely concerned with understanding how Liquid Democracy would work in practice. It models behavior via a priori algorithms and studies rich interactions where delegation takes place on networks (Christoff and Grossi, 2017; Kahng, Mackenzie and Procaccia, 2018; Bloembergen, Grossi and Lackner, 2019; Caragiannis and Michas, 2019). These authors connect Liquid Democracy to the social choice tradition, but here too strategic considerations are absent. An exception is Armstrong and Larson (2021) which discusses the informational trade-off involved in delegation and identifies a Nash equilibrium that always strictly improves over universal majority voting. The paper retains the algorithmic flavor of this literature by modelling the delegation choice as sequential; the common interest nature of the problem, together with the added assumptions of complete information and costly voting, then delivers the equilibrium superiority of delegation. Finally, strategic concerns are at the heart of two recent paper in economics, Ravindran (2021) and Dhillon et al. (2021). In Ravindran's model, voters' types are binary and known, with either higher or low information accuracy, and the goal is the characterization of the efficient equilibrium. With a single expert, optimal delegation is defined precisely; with multiple experts, complications can arise although, as in Armstrong and Larson, they can be solved if delegation decisions are sequential. Dhillon et al. study delegation in a model à la Feddersen and Pesendorfer (1996), with partisan voters and perfectly informed experts. As in the papers just discussed, they show that under complete information delegation has desirable properties: the game is dominance-solvable and delegation allows voters to coordinate on the best equilibrium. With incomplete information, multiple equilibria are more difficult to avoid and results are weaker. None of these works is experimental.

Finally, a literature in social psychology studies a question that is closely related to our second experiment: if a group of individuals face, individually, a perceptual task but can then aggregate their reactions into a group decision, which decision rule for the group will reach the correct answer most frequently? How does simple majority rule compare to supermajority thresholds? In Sorkin et al.(1998), a small group of subjects are faced with a signal detection task and asked whether the display reflects noise only or signal plus noise. Although the group falls short of normative predictions, simple majority rule leads to the highest accuracy. Individual behavior is modeled as reflecting two main parameters, detection sensitivity and confidence, and the emphasis on confidence shapes the direction this research has since taken. In small groups, decision typically follows discussion, and during discussion individual confidence translates into influence. Communication thus threatens group accuracy, unless confidence correlates positively with individual sensitivity (Sorkin et al., 2001, Bahrami et al. 2012, Silver et al. 2021).[7] Although it seems a natural next step, we are not aware of similar studies that include the possibility of delegation.

In what follows, we begin by describing the theoretical model (Section 2) and its equilibrium properties (Section 3). We then discuss our first experiment: its parametrization and treatments (Section 4); its implementation (Section 5), and its results (Section 6). Section 7 describes the motivation and the design of our second experiment; Section 8 reports its results. Section 9 concludes. The Appendix collects longer proofs and some additional experimental findings.

2 The Model

We study the canonical problem of information aggregation through voting in a pure common interest problem. N odd voters face an uncertain state of the world ω and must take a decision d. There are two possible states of the world, $\omega \in \{\omega_1, \omega_2\}$, and two alternative decisions $d \in \{d_1, d_2\}$. Every voter's payoff equals 1 if the decision matches the state of the world ($d = d_s$when $\omega = \omega_s, s = 1, 2$), and 0 otherwise. Voters share a common prior $\pi = Pr(\omega_1)$ and receive conditionally independent signals $\sigma_i \in \{\sigma_1, \sigma_2\}$. We call q_i the *precision* of individual i's signal, or the probability that i's signal is correct. Precision varies across individuals but is symmetric over the two possible states of the world: $q_i = Pr(\sigma_i = \sigma_1|\omega_1) = Pr(\sigma_i = \sigma_2|\omega_2)$.

The group of N voters is composed of K (odd) experts and M (even) non-experts. All experts receive signals of known precision $q_e = p$. The precision of a non-expert signal is instead private information: q_i ($i \neq e$) is an independent draw from a commonly known distribution $F(q)$ with support $[\underline{q}, \overline{q}]$, with $\underline{q} = 1/2$ and $\overline{q} = p$. The signals themselves are

[7] The earlier experiments in this tradition studied a very large number of tasks, in the hundreds for each subject, but a very small group of subject, as small as 8 or 12.

also private information, for both experts and non-experts, but the type of each voter, as either non-expert or expert, is commonly known. We denote by EU every voter's ex ante expected payoff, before the realizations of precisions and signals. EU equals the ex ante probability that the groups reaches the correct decision.

Each individual holds a single, non-divisible vote. Under Liquid Democracy (LD), after having learnt her precision q_i but before having received the signal, the voter decides whether to keep her vote or to delegate it. If the voter chooses delegation, she specifies whether it is to an expert or to a non expert; the vote is then assigned randomly, with equal probability, to any individual in the indicated category.[8] We denote the probability of delegation by δ_e, for delegation to an expert, and δ_{ne} for delegation to a non-expert. Once delegation is decided, signals are realized and individuals who have not delegated their vote indicate their preferred decision. At the voting stage, the strategic decision is the probability of voting according to one's signal, or against it, a decision we denote by $\{v_\sigma, v_{-\sigma}\}$. When counting votes, each voter who has chosen not to delegate receives a weight equals to the number of votes delegated to her, plus 1. The decision receiving more votes is chosen. Note two implications. First, because delegation is decided before the signal is received, the delegation decision itself cannot convey information about the signal. Second, because voters holding multiple votes communicate only their preferred decision, they are constrained to cast all votes in the same direction. Both implications simplify the equilibrium characterization.

The game is thus a two-stage game, with strategies $\{\delta_e(q_i), \delta_{ne}(q_i)\}$ in the first stage, and, conditional on no delegation, $\{v_\sigma(q_i), v_{-\sigma}(q_i)\}$ in the second stage. With an eye to the experimental implementation, we will select equilibria that require little coordination, and in particular such that experts never delegate, and non-experts only delegate to experts. Hence, multi-step delegation (i delegates to j who delegates to z) will not be observed in equilibrium, and thus neither will circular delegation flows (i delegates to j who delegates to z who delegates to i). To characterize the equilibrium, the model nevertheless needs to specify what would happen in such cases. We allow for multi-step delegation: if delegation targets a voter who has herself chosen delegation, the full packet of votes is delegated according to her instructions. However, if a set of delegation decisions results in a circular delegation flow, we specify that all delegations involved are void.[9]

[8]The random assignment of delegated votes is the natural assumption in the absence of distinguishing characteristics across experts. It also leads to some desirable spreading of delegated votes, as advocated for example by Gőlz et al. (2018).

[9]We include all such delegation decisions: if i and j delegate to z, and z delegates to i, the original votes are returned to all three voters (all three are left with a single vote).

3 Equilibrium

We study an environment that matches the experimental set-up, and where, specifically, $\pi = Pr(\omega_1) = 1/2$. In this symmetric environment, voting according to signal ($v_\sigma = 1$ for all q_i) is an undominated strategy, a result that holds whether delegation is allowed, as in our model, or is not, as in traditional majority voting. We focus on semi-symmetric Perfect Bayesian equilibria in undominated strategies where, when voting, voters follow their signal, and delegation strategies are symmetric for all voters of given type (expert or non-expert). In what follows, "equilibrium" refers to such a notion. We are interested in the welfare properties of delegation, and say that an equilibrium "strictly improves over majority voting" if in equilibrium the ex ante probability of reaching the decision that matches the state of the world is strictly higher than under (sincere) majority voting (MV), or $EU_{LD} > EU_{MV}$. Our most general theoretical result is summarized in the following theorem:[10]

Theorem. *Suppose $\pi = Pr(\omega_1) = 1/2$. Then for any F, and for any N and K odd and finite there exists an equilibrium with delegation that strictly improves over MV.*

The result is of interest because the environment we are studying is particularly favorable to MV. Given the symmetric prior and information structure, the Condorcet Jury theorem applies to rational voting, and thus we know that MV converges to the correct decision with probability 1 asymptotically, as the size of the electorate becomes unbounded. In addition, we are restricting our attention to semi-symmetric equilibria under LD, and thus excluding asymmetric profiles of strategies that we know efficient but that require demanding coordination.[11] And yet, the theorem states that there always exists an equilibrium where the possibility of delegation strictly improves over MV.

We prove the theorem in the appendix, but the intuition is both straightforward and interesting. The essence of the proof is that, when delegation is possible and some voters' information may be barely better than random, there cannot be an equilibrium where delegation is excluded with probability 1: every voter casting their vote with probability 1 (and thus replicating MV) is not an equilibrium. But in this common interest problem, we know from McLennan (1998) that the set of strategies that maximize expected utility must be an equilibrium. Together, the two observations imply that an equilibrium exists,

[10] We grant the result the name of "theorem" because it refers to the default canonical model of information aggregation under common interest, and yet we have not found it in the literature. However, many results with similar flavor do exist. See for example Grofman, Owen and Feld (1982) and (1983).

[11] Building on Nitzan and Parousch (1982), and Shapley and Grofman (1984), Ravindran (2021) characterizes the highest welfare equilibrium in the case of a single expert. It is an asymmetric equilibrium where the total number of votes delegated to the expert mirrors the expert's precision (more precisely, is proportional to $ln(p/(1-p))$).

that it must include a positive probability of delegation, and that it must dominate MV. In addition, again from McLennan (Theorem 2), because the environment is fully symmetric for all voters of given type, the conclusion continues to apply when we restrict attention to semi-symmetric strategies.

To see intuitively why there cannot be an equilibrium where every voter casts her vote with probability 1, consider, for ease of exposition only, the simple case of a single expert: $K = 1$ and $N = M + 1$ Suppose everyone else votes (sincerely) and consider the choices of a voter with precision q_i near the lower bound of the precision distribution $F(q)$. The voter must decide whether to cast her vote and not delegate (an action yielding expected utility $EUND(q_i)$), or delegate the vote to others, and in this case the best choice is to delegate to the expert. Delegation to the expert yields expected utility $EUD(q_i)$, and because upon delegation i will not be voting and non-expert precisions are independent draws from $F(q)$, $EUD(q_i) = EUD$. It is not difficult to verify that:

$$
\begin{aligned}
&lim_{q_i \to \underline{q}}(EUND(q_i) - EUD) = \\
&\quad \underline{q}(1-p)\left[\binom{M-1}{M/2}\mu^{M/2}(1-\mu)^{M/2-1}\right] - p(1-\underline{q})\left[\binom{M-1}{M/2-1}\mu^{M/2-1}(1-\mu)^{M/2}\right] \qquad (1)
\end{aligned}
$$

where $\mu = \int_{\underline{q}}^{\overline{q}} q dF(q)$ and, because signals are independent, $\binom{M-1}{x}\mu^x(1-\mu)^{M-1-x}$ is the probability that x of the other $M - 1$ non-expert votes are correct.

Voter i's decision matters only if i disagrees with the expert and her vote is pivotal. Not delegating is advantageous if $i's$ signal is correct while the expert's signal is not (an event with probability $q_i(1-p)$), and the total number of correct votes cast by the other non-experts falls short of majority by a single vote (with probability $\binom{M-1}{M/2}\mu^{M/2}(1-\mu)^{M/2-1}$)–the first term on the right-hand side of equation (1). Delegating is advantageous if the expert's signal is correct, while i's signal is not (with probability $p(1-q_i)$) and the number of correct votes cast by non-experts falls short of majority by 2 votes (with probability $\binom{M-1}{M/2-1}\mu^{M/2-1}(1-\mu)^{M/2}$)– the second term on the right-hand side of equation (1). Since $\binom{M-1}{M/2} = \binom{M-1}{M/2-1}$,it follows that:

$$Sign[lim_{q_i \to \underline{q}}(EUND(q_i) - EUD)] = Sign\left[\underline{q}(1-p)\mu - p(1-\underline{q})(1-\mu)\right]$$

or:

$$lim_{q_i \to \underline{q}}(EUND(q_i) - EUD) < 0 \Longleftrightarrow [\underline{q}(1-p)\mu - p(1-\underline{q})(1-\mu)] < 0.$$

With $\underline{q} = 1/2$, the condition simplifies to $\mu < p$, which is always satisfied. Hence there exists no equilibrium such that all non-experts prefer voting to delegation for any precision of their signal.

Two observations conclude the argument. First, note that assuming $\underline{q} = 1/2$, while useful for the proof, is a natural assumption: $\underline{q}$ cannot be inferior to 1/2, and thus 1/2 is the natural lower boundary of the support of the precision distribution.[12] The probability of realizations near such lower bound needs to be positive, but can be arbitrarily small. Second, as we show in the appendix, generalizing the model to K experts does not modify the argument, and indeed the same simple condition identified here remains the critical condition when there are multiple experts.

The theorem does not characterize the equilibria with delegation. In Section 4 we will do so when we specialize the model to the parameter values we use in the experiment. Here, to make sure the mechanisms that drive the model are intuitively clear, we describe in detail the **unique** semi-symmetric equilibrium in undominated strategies that exists in the case of a single expert.

3.1 A Single Expert ($K = 1$)

Proposition 1. *Suppose $\pi = Pr(\omega_1) = 1/2$ and $K = 1$. Then for any N odd and finite and for any F atomless and continuous, there exists an equilibrium such that: (i) the expert never delegates her vote and always votes according to signal; (ii) there exists $\widetilde{q}(N) \in (\underline{q}, \overline{q})$ such that non-expert i delegates her vote to the expert if $q_i < \widetilde{q}$ and votes according to signal otherwise. (iii) Such an equilibrium strictly improves over MV and is ex ante maximal among sincere semi-symmetric equilibria where the expert never delegates and non-experts delegate to the expert only.*

The proposition is proved in the appendix. The structure of the equilibrium, however, is intuitive. Note that in all equilibria (in fact, for any number of experts), non-experts must adopt monotone threshold strategies–there must exist a precision threshold $\widetilde{q}$ such that voters with lower precision delegate, and voters with higher precision do not. The reason is immediate: if the voter delegates, expected utility does not depend on the voter's precision. But if the voter does not delegate, there is a non-zero probability that the voter is pivotal, in which case expected utility increases with the voter's precision. The conclusion then follows. Given monotone threshold strategies, the appendix shows that the delegation directions in the proposition–the expert never delegating and non-experts delegating to the expert only–are indeed best responses when all others adopt them too. And since we know,

[12] As noted earlier, with binary states, a signal correct with probability inferior to 1/2 is equivalent to the opposite signal being correct with probability higher than 1/2.

from the earlier theorem, that an equilibrium with partial delegation must exist, it follows that an equilibrium with the strategies characterized in the proposition must exist. Finally, we also find that the condition identifying the equilibrium threshold corresponds to the first order condition from the maximization of ex ante expected utility, over all profiles of semi-symmetric monotone threshold strategies with sincere voting and the specified directions of delegation. Hence, again invoking the theorem, the equilibrium is maximal over such profiles and improves strictly over majority voting.

It is important to note that the equilibrium threshold $\widetilde{q}$ that supports the improvement over MV is strictly interior to the range $(\underline{q}, \overline{q})$. Because $\overline{q} = p$, that means that in equilibrium there are voters who know that their precision is strictly lower than the expert's precision, and yet cast their vote, rather than delegating. Since delegation decreases the aggregate information in the system, and yet the equilibrium with delegation is superior to MV, we expect the threshold $\widetilde{q}$ to be low–only voters with very imprecise information delegate in equilibrium. Indeed, this is what the numerical examples will show. Studying in detail a particularly simple example makes clear why.

Suppose $N = 3$ and $K = 1$, and consider non-expert i's choice of whether or not to delegate to the expert. Note that i conditions on the other non-expert not delegating, because only in such a case does i's decision matter. If i delegates, the expert has 2 votes and chooses unilaterally. Hence $EUD_i = p$. If i does not delegate, the correct decision is reached if i's signal is correct and at least one of the other two signals is correct as well, or if i's signal is incorrect but both of the others are correct. Or:

$$EUND_i = q_i[1 - (1 - \mu_v(\widetilde{q}))(1 - p)] + (1 - q_i)p\mu_v(\widetilde{q})$$

where $\mu_v(\widetilde{q}) = E(q|q > \widetilde{q})$.

Equilibrium $\widetilde{q}$ solves $EUD_{q_i=\widetilde{q}} = EUND_{q_i=\widetilde{q}}$:

$$p = \widetilde{q}[1 - (1 - \mu_v(\widetilde{q}))(1 - p)] + (1 - \widetilde{q})p\mu_v(\widetilde{q})$$

Or

$$p(1 - \widetilde{q})(1 - \mu_v(\widetilde{q})) = (1 - p)\widetilde{q}\mu_v(\widetilde{q})$$

Note that the equilibrium condition thus equalizes the probability that the expert is correct and *both the non-experts* are not, with the probability that the expert is incorrect and *both the non-experts* are correct. Intuitively, the delegation decision only matters: (i) in case of disagreement with the expert (since the choice is otherwise moot), and (ii) if the vote is pivotal, and the other non-expert not only does not delegate (has precision above $\widetilde{q}$),but disagrees with the expert (and hence agrees with the non-expert voter with precision $\widetilde{q}$). The non-expert signal with precision $\widetilde{q}$ receives implicit validation from a second independent and more accurate signal. It is this implicit validation that pushes equilibrium behavior away from delegation, and the equilibrium $\widetilde{q}$ towards low values. For example, with $N = 3$ and $K = 1$, if $p = 0.7$ and $F(q)$ is Uniform over [0.5,0.7], $\widetilde{q} = 0.572$, below the mean non-expert precision of 0.6. A non-expert voter's precision is always lower than the expert's, but the ex ante individual probability of delegation is only 36 percent.

The good properties of the equilibrium with $\widetilde{q} \in (0, 1)$ depend strongly on the optimal, spare use of delegation. But internalizing such reasoning is difficult. Our first experiment tests participants' behavior in an environment that mirrors the model closely.

4 Experiment 1: Treatments and Parametrizations

In the lab, we make two simplifications relative to the model. First, we constrain the direction of delegation: experts cannot delegate, and non-experts can only delegate to the experts. Second, we combine the delegation and voting decisions: after being told her own precision and receiving her signal, every participant faces a single ballot with three possible alternatives: delegate, or vote for one or the other of the two options. In the theoretical model, the delegation decision is modeled as preceding the signal to ensure that delegation itself does not convey information about the signal. In the lab, with random group formation and partial feed-back, the coordination required to use delegation informatively with respect to the signal is in practice impossible to achieve. Without such a concern, unifying the decisions in a single ballot streamlines the experiment. Note, and this is important, that it also allows us to observe the signals of the delegators and evaluate the counterfactual outcomes with universal majority voting (MV).

We are interested in three main questions: (1) We consider first the simplest setting, when decisions are taken by a small group and a single expert. How well does LD perform, relative to MV? (2) According to the theorem, LD's potential to improve over MV persists with larger group sizes and multiple experts. In the lab, do results change qualitatively when the size of the group and the number of experts increase? (3) LD makes it possible to shift voting weight away from less informed voters and towards more informed ones. But reducing the weight of less informed voters can also be achieved, more simply, by allowing abstention. How does LD compare to MV with abstention? We denote such a rule by MVA, and as in the case of LD, study it both in a small group with a single expert, and in a larger group with multiple experts.

In all experiments, we set $p = 0.7$, and $F(q)$ Uniform over $[0.5, 0.7]$. We study four

Table 1: $p = 0.7$, $F(q)$ Uniform over $[0.5, 0.7]$

$LD1 : N = 5;\ K = 1$

$\widetilde{q}$	$F(\widetilde{q})$	EU_{LD}	EU_{MV}
0.7	1	0.7	0.717
0.543	0.215	0.731	

$LD3 : N = 15;\ K = 3$

$\widetilde{q}^3$	$F(\widetilde{q}^3)$	EU^3_{LD}	EU^3_{MV}
0.532	0.162	0.843	0.832

treatments. Two treatments concern LD. In LD1, groups consist of 5 voters with a single expert: $N = 5$, $K = 1$. In LD3, each group has 15 voters in all, of which 3 are experts: $N = 15$, $K = 3$. Hence in both treatments one fifth of the group are experts: $K/N = 1/5$. The two treatments with abstention, MVA1 and MVA3, substitute abstention for the possibility of delegation, again either with $N = 5$, $K = 1$ (MVA1), or with $N = 15$, $K = 3$ (MVA3).

4.1 Liquid Democracy

Table 1 reports the theoretical predictions when delegation is possible.[13]

In treatment LD1, we find two semi-symmetric equilibria. For any realization of non-expert precisions, there always exists an equilibrium where every voter delegates to the expert with probability 1: no individual non-expert is ever pivotal, and delegating one's vote is a (weak) best response. The expert then alone controls the outcome. With semi-symmetric strategies, such an equilibrium corresponds to $\widetilde{q} = \overline{q}$ and yields ex ante utility $EU_{LD}(\widetilde{q} = \overline{q}) = p = 0.7$. In addition, there is a unique equilibrium where $\widetilde{q}$ is strictly interior. As argued earlier, the $\widetilde{q}$ threshold is low, and the ex ante probability of delegation is only just above 20 percent. The ex ante probability of reaching the correct decision, equivalent to the expected utility measures, is lowest when the expert decides alone ($\widetilde{q} = \overline{q}$), intermediate under MV, and highest in the equilibrium with delegation and interior $\widetilde{q}$. However, the proportional increase in the probability that the group selects the correct option is small, about 2 percent at each step.[14]

In LD3, full delegation is not an equilibrium any longer. Intuitively, when there are multiple experts and all other non-experts delegate, voter i can be pivotal only if the experts disagree among themselves. The disagreement reduces the attraction of delegation and for sufficiently high q (still smaller than $\overline{q}$) casting a vote is preferable. As we know, the

[13] The details of the derivations are in the Appendix.

[14] With a single expert, the uniqueness of the semi-symmetric equilibrium with interior $\widetilde{q}$ can be proven analytically and holds for arbitrary N. Absent either communication or repetition, asymmetric equilibria are implausible in the lab if they require coordination. But when the expert is dictator and no other voter is pivotal, the choice to delegate or not becomes irrelevant and trivial asymmetric equilibria becomes more realistic. For example, for any realization of non-expert precisions, there are equilibria with 3 non-experts delegating and 1 voting.

equilibrium with interior $\widetilde{q}$ continues to exist. Equilibrium delegation, however, is rare: the expected frequency of individual delegation falls to 16 percent. As theory teaches, the interior equilibrium yields a higher probability of a correct decision than MV. However, with the increase in the size of the group, the Condorcet Jury Theorem effect becomes very pronounced: majority voting works very well and the scope for improvement is small. The percentage gain is only 1.3 percent.

The table conveys two main messages. First, we see concretely what the interior equilibrium entails for the experimental parametrizations. In particular, as expected, equilibrium delegation is not frequent and concerns only voters with precisions not far from randomness. Second, the improvement in the probability of taking the correct decision is small, almost certainly too small to be detectable in the lab. Setting a higher p, and/or setting $\overline{q} < p$ would increase the scope and expected gain from delegation. Increasing $\underline{q}$ or N would have the opposite effect. We have chosen a parametrization that delivers similar efficiencies for the two rules to leave the data free to favor either. The realistic challenge for the experiment will be to see whether indeed in this environment LD and MV's welfare properties are comparable.

4.2 Abstention

Like delegation, abstention can lower the voting weights of less informed voters, with the major advantage of being a simpler and familiar option. However, the two mechanisms are not equivalent: under abstention, voting weight is redistributed towards *all* voters who choose to vote; under LD, delegated votes target the experts only.

We implement the MVA treatments in the identical environment we study under LD. After non-expert voters learn, privately, the precision and the content of their personal signal, they decide, simultaneously and independently, whether to vote or to abstain. Experts are not given the option of abstaining. Everything else remains unchanged. The model of abstention is closely related to McMurray (2013), and its main results– the existence of an equilibrium in monotone cutpoint strategies, and its superiority to MV–carry over to our setting. We report the relevant equations in the Appendix.[15] As in the case of delegation, and for very similar reasons, abstention too is limited to voters with weak information.

[15] McMurray's model and ours differ in two main aspects. First, for comparison to LD, we assume the existence of a known group of experts with higher, known, but not perfect precision. McMurray does not distinguish experts, but widens the support of the distribution of precisions $F(q)$ to cover the full interval $[1/2, 1]$. Second, because of our experimental aim, we assume that the size of the electorate is known and need not be large, deviating from McMurray's large Poisson game set-up. The logic of the two models is otherwise identical. The central intuition comes from the fact that best response strategies are monotone in individual precision and thus abstaining in equilibrium shifts voting weight towards better informed individuals.

Table 2: $p = 0.7$, $F(q)$ Uniform over $[0.5, 0.7]$

$MVA1 : N = 5; K = 1$

$\widetilde{\alpha}$	$F(\widetilde{\alpha})$	EU_{MVA}	EU_{MV}
0.7	1	0.7	
0.580	0.40	0.724	0.717
0.5	0	0.717	

$MVA3 : N = 15; K = 3$

$\widetilde{\alpha}^3$	$F(\widetilde{\alpha}^3)$	EU^3_{MVA}	EU^3_{MV}
0.7	1	0.784	
0.580	0.40	0.849	0.832
0.5	0	0.832	

Table 2 shows the equilibria with abstention, for the experimental parametrizations. We denote by $\widetilde{\alpha}$ the precision threshold below which in equilibrium a non-expert abstains, and above which a non-expert votes.

For both group sizes, there are three semi-symmetric equilibria. Two are boundary equilibria, with either zero ($\widetilde{\alpha} = 0.5$) or full ($\widetilde{\alpha} = 0.7$) abstention; one is an interior equilibrium where, for both group sizes, a non-expert abstains if precision is below 0.58, i.e. with ex ante probability of 40 percent. The boundary equilibrium with zero abstention corresponds to MV; the one with full abstention, where the decision is delegated to the experts, is inferior to MV. As in McMurray's analysis, the interior equilibrium does deliver expected gains over MV, but these remain quantitatively small.[16] The interior equilibrium threshold for abstention is higher than the threshold for delegation, and remains constant in the two group sizes. It implies a larger expected number of abstentions than delegations: for example, and rounding up to integers, when $N = 15$, in equilibrium we expect 2 non-experts to delegate under LD, but 5 non-experts to abstain under MVA.

Under both LD and MVA, the expected improvements over MV are minor. It is natural to ask how sensitive such potential improvements are to strategic mistakes.

4.3 Robustness

We consider here a particularly simple parametrization of strategic mistakes: we suppose that behavior remains symmetric, but the precision threshold for delegation or abstention is chosen incorrectly. In Figure 1, the horizontal axis is the common threshold, and the vertical axis reports expected utilities, or the probability of making the correct choice under each of the three voting rules, LD (in Blue), MVA (in Green) and MV (in Grey). The highest points on the Blue and Green curves coincide with the respective equilibrium thresholds. The first panel corresponds to $N = 5$, $K = 1$; the second to $N = 15$, $K = 3$.

At $\widetilde{q}$ or $\widetilde{\alpha} = 1/2$, no-one delegates their vote or abstains, and all curves equal MV

[16] The existence of the boundary equilibria depends on N and Kodd. Note that because MV without abstention is an equilibrium when abstention is allowed, the simple proof used to establish the earlier theorem cannot be extended from delegation to abstention.

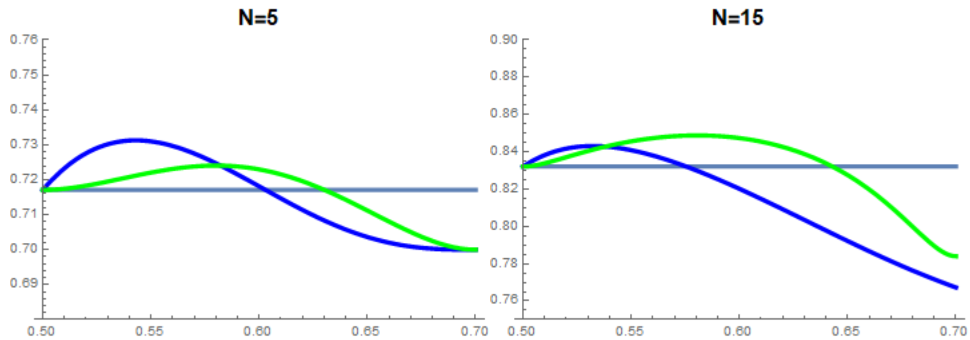


Figure 1: *Robustness to strategic errors.* The horizontal axis is $\widetilde{q}$ ($\widetilde{\alpha}$), the vertical axis is EU. Blue is LD, Green is MVA, and Grey is MV.

and coincide. At $\widetilde{q}$ or $\widetilde{\alpha} = 0.7$, all non-experts delegate or abstain, and only the expert/s decide(s).[17] In the first panel, with a small group, the maximum potential improvement over MV from delegation (from LD) is higher than from abstention (MVA). However, this is not true in the second panel, with the larger group. Both results were already shown in Tables 1 and 2. More interesting is the range of thresholds for which each voting rule dominates MV. Here the message is consistent across the two group sizes: in both cases, the range of thresholds that deliver improvements over MV is limited, and particularly limited for LD. When the group is larger, LD's potential for losses is evident in the figure, as is its increased fragility, relative to MVA: the range of thresholds that improve over MV is half as large under LD3 than under MVA3. With both voting schemes, but with LD in particular, while potential gains are small, there is the real danger of reaching worse decisions: under LD3, maximal potential losses are more than six times maximal potential gains.

5 Experiment 1: Implementation

We ran the experiment online over the Summer of 2021, using the Zoom videoconferencing software. Participants were recruited from the Columbia Experimental Laboratory for the Social Sciences (CELSS)' ORSEE website.[18] They received instructions and communicated with the experimenters via Zoom, and accessed the experiment interface on their personal computer's web browser. The experiment was programmed in oTree and, with the exception of a more visual style for the instructions, developed very similarly to an in-person experiment. Each session lasted about 90 minutes with average earnings of $26, including a

[17]When $K = 3$, the Blue and Green curves do not coincide at $\widetilde{q} = 0.7$ because under MVA3 each expert has the same weight; while under LD3 the number of votes each of them commands is stochastic.

[18]Greiner (2015). CELSS' ORSEE subjects are primarily undergraduate students at Columbia University or Barnard College.

Table 3: Experiment 1: Experimental Design

Sessions	Treatments	Rounds	Subjects	Groups
1a	LD1, LD3	20,20	15	3, 1
1b	LD3, LD1	20,20	15	1, 3
2a	MVA1, MVA3	20,20	15	3, 1
2b	MVA3, MVA1	20,20	15	1, 3
3a, 3a’	LD3, MVA3	20,20	15	1, 1
3b, 3b’	MVA3, LD3	20,20	15	1, 1
4a	LD1, MVA1	20,20	15	3, 3
4b	MVA1, LD1	20,20	15	3, 3

show-up fee of $5.

Participants were asked to vote on the correct selection of a box containing a prize, out of two possible choices, a Green box and a Blue box. The computer selected the winning box putting equal probability on either; conditionally on the computer’s random choice, participants then received a message suggesting a color, and were told the probability that the message was accurate.[19] The same screen also informed them of whether or not they were an expert (for that round). Participants were then asked to vote for one of the two boxes, if experts, or, if non-experts, to either choose one of the boxes or delegate their vote to an expert (in the LD treatments), or abstain (in the MVA treatments). Across rounds, expert/non-expert identities were re-assigned randomly, under the constraint that groups of 5 voters had a single expert, and a group of 15 had three; if the session involved more groups, they were re-formed randomly. A copy of the instructions is reproduced in the Appendix.

We ran 10 sessions, each involving 15 subjects (150 subjects total). Participants played 20 rounds each of two treatments (40 rounds in total), according to the experimental design reproduced in the following table. Hence in total we have data for 240 rounds for LD1 and for MVA1, and 120 rounds for LD3 and for MVA3.

6 Experiment 1: Results

6.1 Frequency of delegation and abstention

Figure 2 reports the aggregate frequencies of delegation and abstention in the data (the dark columns), and according to the predictions of the interior equilibrium, given realized signal

[19]We used examples to instruct participants on what the probability of the message being accurate meant. To limit decimal digits, the precision of the signal was drawn uniformly from a discrete distribution with bins of size 0.01. When comparing the experimental results to the theory, below, we compute equilibria using the corresponding discrete distribution of precisions. The differences are minute.

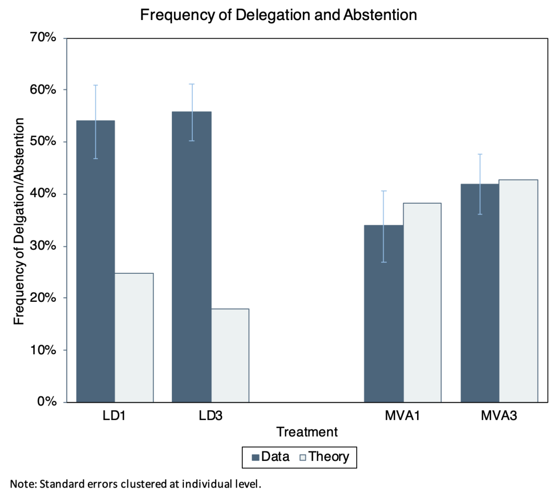


Figure 2: *Aggregate frequency of delegation and abstention.* Confidence intervals are calculated from standard errors clusered at the indivi

precisions in the experiment (the light columns). Columns on the left refer to LD treatments; columns on the right to MV. The light blue lines are 95% confidence intervals, calculated with standard errors clustered at the individual level.

The result is unambiguous: delegation rates in the experiment are between two and three times what theory predicts for the equilibrium that improves over MV. Abstention rates on the other hand are comparable to the predictions. With such high propensity to delegate, the conclusion is robust to all plausible ways of cutting the data: disaggregating by session, considering only the 10 final rounds, clustering standard errors at the session level. Under LD1, there is a second symmetric equilibrium with universal delegation. We do not see it in the data, but, as noted earlier, asymmetric equilibria with very high delegation also exist.[20] The specific claim here then is not that subjects were necessarily playing non-equilibrium strategies, but rather that they were not playing the symmetric interior equilibrium that dominates MV.

Regressions on individual behavior that control for signal quality and for round and treatment order effects lead to the same conclusion. Table 2 report linear probability and probit regressions, with Panel A referring to LD1 and MVA1 ($N = 5$); Panel B to LD3 and MVA3 ($N = 15$). In both panels, the excluded case is MVA played as first treatment in MVA-only sessions. Standard errors are clustered at the individual level. The results are very similar for both group sizes. As expected, the propensity to abstain or delegate is

[20]Out of the 240 LD1 rounds, there are 87 rounds (36.25%) where at least 3 non-experts delegated. In such rounds, the expert is dictator and no individual delegation decision is pivotal. Thus the strategies are an equilibrium.

affected negatively by higher precision of the signal, with no significant difference in impact over the two treatments. Order of treatments and number of rounds have no sustained effects, although there is some weak evidence for increasing abstention/delegation in later rounds in the larger groups (when $N = 15$), or when the treatment is played second in the smaller groups. The strongly significant and surprising result is the higher propensity to delegate, relative to abstaining. If we focus on interior equilibria, recall that the theoretical prediction is in fact in the opposite direction: abstention is predicted to be more frequent than delegation.

Participants' choices appear coherent, if not optimal. Delegation and abstention decisions are not only negatively correlated to signals precision, as the regressions show; we find that they are also monotonic in signal precisions (if non-expert i votes at precision $q(i)$, then i votes at all $q'(i) > q(i)$). We report histograms of monotonicity violations for all four treatments in the Appendix. There is weak evidence of fewer violations under MVA, but the two treatments are effectively comparable. Just below 60 percent of subjects have no violations at all under LD; just above 60 percent under MVA, and the results are invariant to the size of the group.[21] In all cases, it is possible to generate perfect monotonicity for at least 80 percent of participants by changing at most 2 of their non-expert choices.[22]

We use monotonicity to estimate individual precision thresholds for delegation and abstention–the thresholds below which each participant delegates or abstains. Figure 3 reports, for each participant, the mean of the range of thresholds that, for each individual, are consistent with minimal monotonicity violations; the size of the dots is proportional to the number of participants at that threshold. The darker diamonds correspond to the average empirical thresholds, and the lighter ones to the theory. The figure confirms the over-delegation that characterizes LD, while again average values for abstention are close to the theoretical predictions. The dispersion in estimated thresholds is typical of similar experiments (for example, Levine and Palfrey, 2007; Morton and Tyran, 2011), but is in clear tension with the focus on symmetric equilibria.

The observation that thresholds tend to be lower for delegation rather than abstention is confirmed in Figure 4, where we plot the cumulative distribution functions of the estimated thresholds.

For both group sizes, the LD distribution first order stochastically dominates the MVA distribution: at any precision, including at the lower boundary of the support, the fraction of subjects estimated to delegate is above the corresponding fraction of abstainers (the fraction

[21]The fraction becomes 80 percent if we limit attention to the last 10 rounds of each treatment.

[22]With type randomly assigned, the expected number of rounds played as non-experts is 16. The maximum possible number of monotonicity violations over 16 rounds is 8.

Frequency of Delegation or Abstention

	(1) Linear Probability	(2) Probit
LD	0.328***	0.938***
	(0.0971)	(0.333)
	[0.00107]	[0.00491]
Round	0.0309	0.0851
	(0.0442)	(0.158)
	[0.486]	[0.590]
Signal Quality	-0.777***	-2.624***
	(0.0759)	(0.283)
	[0.000]	[0.000]
Second	0.154*	0.534*
	(0.0868)	(0.307)
	[0.0801]	[0.0815]
Second * Mixed	-0.129	-0.451
	(0.0967)	(0.337)
	[0.186]	[0.182]
LD * Second	-0.0896	-0.332
	(0.118)	(0.413)
	[0.449]	[0.421]
LD * Second * Mixed	-0.0253	-0.0327
	(0.144)	(0.492)
	[0.861]	[0.947]
LD * Round	-0.112*	-0.345
	(0.0665)	(0.233)
	[0.0956]	[0.138]
LD * Signal	-0.0776	0.0653
	(0.0893)	(0.348)
	[0.387]	[0.851]
Constant	0.675***	0.582***
	(0.0759)	(0.216)
	[0.000]	[0.00705]
Observations	1,920	1,920
R-squared	0.309	

*** p<0.01, ** p<0.05, * p<0.1

Notes: Standard errors in parentheses, clustered at the individual subject level. P-values in brackets. Delegation/abstention is measured as a binary 0-1 subject decision. Experts are dropped because they cannot delegate or abstain. Only LD1 and MVA1 rounds are included. "Second" indicates that the treatment appeared second in the session. "Mixed" indicates that both an LD treatment and an MVA treatment appeared in the session.

Table 4: Determinants of delegation and abstention; N=5

Frequency of Delegation or Abstention

	(1) Linear Probability	(2) Probit
LD	0.208***	0.677**
	(0.0746)	(0.283)
	[0.00609]	[0.0167]
Round	0.0783*	0.274**
	(0.0401)	(0.136)
	[0.0532]	[0.0444]
Signal Quality	-0.861***	-2.691***
	(0.0553)	(0.220)
	[0.000]	[0.000]
Second	-0.0963	-0.341
	(0.0797)	(0.275)
	[0.229]	[0.214]
Second * Mixed	0.0784	0.295
	(0.0865)	(0.298)
	[0.367]	[0.322]
LD * Second	0.0367	0.125
	(0.113)	(0.384)
	[0.745]	[0.745]
LD * Second * Mixed	-0.166	-0.577
	(0.118)	(0.402)
	[0.160]	[0.151]
LD * Round	-0.0506	-0.174
	(0.0600)	(0.205)
	[0.401]	[0.395]
LD * Signal	0.0107	0.102
	(0.0690)	(0.287)
	[0.877]	[0.721]
Constant	0.832***	0.992***
	(0.0614)	(0.199)
	[0.000]	[0.000]
Observations	2,880	2,880
R-squared	0.309	

*** p<0.01, ** p<0.05, * p<0.1

Notes: Standard errors in parentheses, clustered at the individual subject level. P-values in brackets. Delegation/abstention is measured as a binary 0-1 subject decision. Experts are dropped because they cannot delegate or abstain. Only LD3 and MVA3 rounds are included. "Second" indicates that the treatment appeared second in the session. "Mixed" indicates that both an LD treatment and an MVA treatment appeared in the session.

Table 5: Determinants of delegation and abstention; N=15

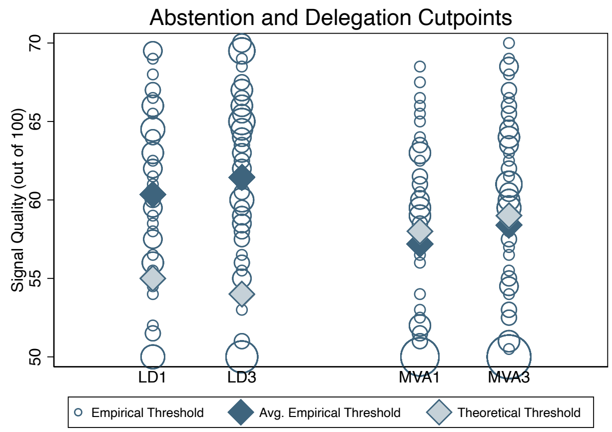


Figure 3: *Delegation and Abstention Thresholds.* All rounds.

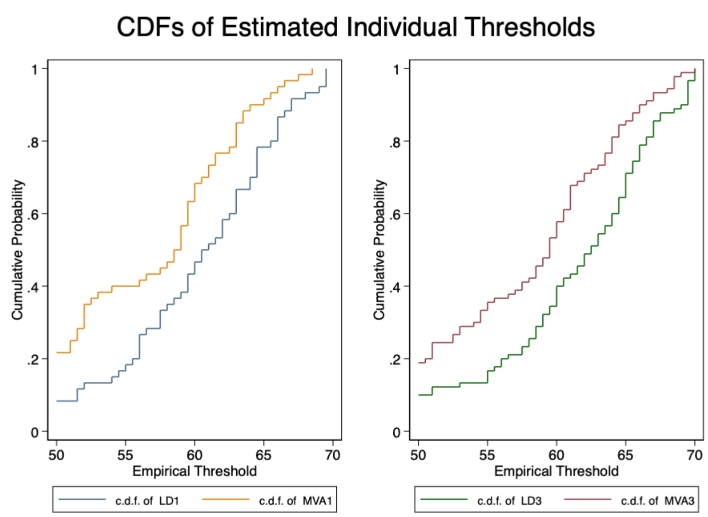


Figure 4: *Individual delegation/ abstention thresholds. CDF's.*

of subjects whose estimated threshold is below the threshold, and thus are voting, is lower). The data do not show substantive differences between the two group sizes.

6.2 Frequency of correct choices

Beyond regularities of delegation and abstention, the real variable of interest is the frequency with which the voting system leads to the correct choice. Figure 5 reports the experimental data and compares them to the theoretical interior equilibrium and to MV.[23] Because we are studying variations of majority voting, a majority of outcomes under both LD and MVA correspond to MV.[24] The figure describes the data; later, we will test the performance of LD and MVA, relative to MV, by conditioning on reaching different outcomes.

We report results grouped by N. The vertical axis is the frequency of correct outcomes over the full data set for the corresponding treatment. The figure holds three main lessons. First, for both group sizes, LD and MVA yield very similar frequencies of correct decisions: given N, the dark blue columns have very similar heights. Second, for both group sizes, both systems fall short of their possible best performance: the light Grey columns are consistently higher than the Blue and dark Grey columns. Third, MVA outcomes are closely comparable to MV for both group sizes; LD outcomes fall short of MV, especially for small groups.

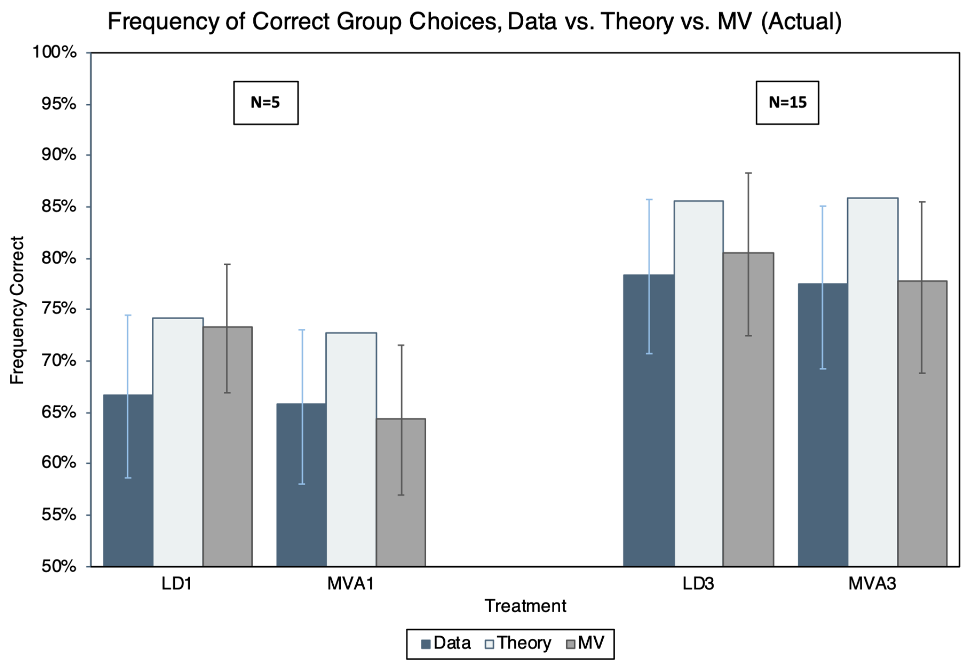
Two main deviations could be responsible for the systems' underperformance relative to the theory.[25] The first is the erroneous choice of delegation/abstention thresholds. Figures 2 and 3 support this interpretation for LD, with its consistent over-delegation, but not for MVA. The second is random voting in the form of voting against signal. As we show in the Appendix, the frequency of voting against signal correlates negatively and significantly with signal precision, and experts vote against signal more rarely than non-experts. Both because experts cast multiple votes under LD and because subjects choose to vote at lower precision under MVA, the share of votes cast against signal is lower in the LD treatments (at about 6%) than in the MVA ones (at about 10%), with little difference across group sizes.[26] MVA

[23] All results are calculated given the experimental realizations of the state and of signals. The 95% confidence intervals are calculated from standard errors clustered at the group level. The results under MV were constructed by imputing votes for those subjects who delegated or abstained in the experiment. Because some of the votes we do observe were cast contrary to signal, we allowed for such a possibility. We calculate the outcome under MV by supposing that unobserved votes follow signals with probability equal to the average frequency observed in the treatment. The randomization is repeated 10,000 times; each time yields a share of correct decisions under MV for the relevant data set, and the figure reports the mean over the 10,000 randomizations. We have verified that calculating the probability of voting sincerely by using the subject's own frequency over the observed votes does not affect the results.

[24] More than 70% of all outcomes under LD, and more than 80% for MVA

[25] A priori, a third possibility would be non-monotonicity in delegation and abstention decisions. But as we described earlier, violations of monotonicity are rare.

[26] The numbers are comparable to those reported, for example, by Guarnaschelli et al. (2000) and Goeree



Notes: MV frequency calculated by randomly assigning delegators/abstainers to vote according to their signal according to the frequency of voting sincerely among non-experts in the treatment; the randomization is repeated 10,000 times, with the MV share calculated each time, and the mean of those 10,000 repetitions is shown.

Figure 5: *Frequency of correct outcomes*

suffers from more random voting. Both systems thus fail to realize their potential gains over MV, but for different reasons.

The comparison to MV shows that the penalty is higher for LD. The better performance of MV in the LD samples reflects the random superiority of the signal draws in those samples: although signal realizations were drawn from the same probability distribution, the frequency of correct signals was higher in the LD treatments. Thus, although LD and MVA have similar shares of correct decision in our data, LD treatments could have performed better, given the superiority of the realized signals.[27]

6.3 Comparing LD and MVA to MV

Evaluating the significance of the disparities observed between LD or MVA on one side, and MV on the other is not immediate. To begin with, calculating correct indices of statistical significance is complicated by several factors: we are interested in group results, with complex correlations across data points: the same individuals are observed over multiple rounds; the frequency with which they are assigned the role of experts is random and variable; the voting group is assembled randomly every round when $N = 5$, but is fixed when $N = 15$; finally, the imputation of missing votes under MV creates randomness in the MV outcomes. But the fundamental difficulty is simpler: as mentioned above, outcomes coincide in a large majority of cases. Restricting the data sample to those elections in which outcomes under delegation, or abstention, differ from outcomes under MV leaves us with little information. To overcome this difficulty, we use bootstrapping methods to simulate a large number of elections in a populations for which our data are representative. By simulating many elections, conditioning on different outcomes becomes feasible.

The procedure we implement allows for correlation across an individual's multiple decisions, and uses randomization to generate the correct balance of experts and non-experts. For each voting system and group size, we generate outcomes by drawing subjects, with replacement, each with their full set of 20-round decisions, and matching them randomly into groups. We then study the outcomes corresponding to 100,000 replications of the experiment for each treatment, using the population of subjects for that voting system and group size. We describe the procedure in more detail in the Appendix. Figure 6 shows the distributions of the differential frequency of correct decisions between the voting systems we are studying and MV, for each group size, conditioning on the decisions being different. Consider for

and Yariv (2011) for juries voting under simple majority and pure common interest, in the absence of communication (6-9%).

[27] If votes were not cast against signals, all three voting systems would be more efficient, but we have verified that, as expected, the difference between LD and MV, and the lack of a difference between MVA and MV, would not be affected.

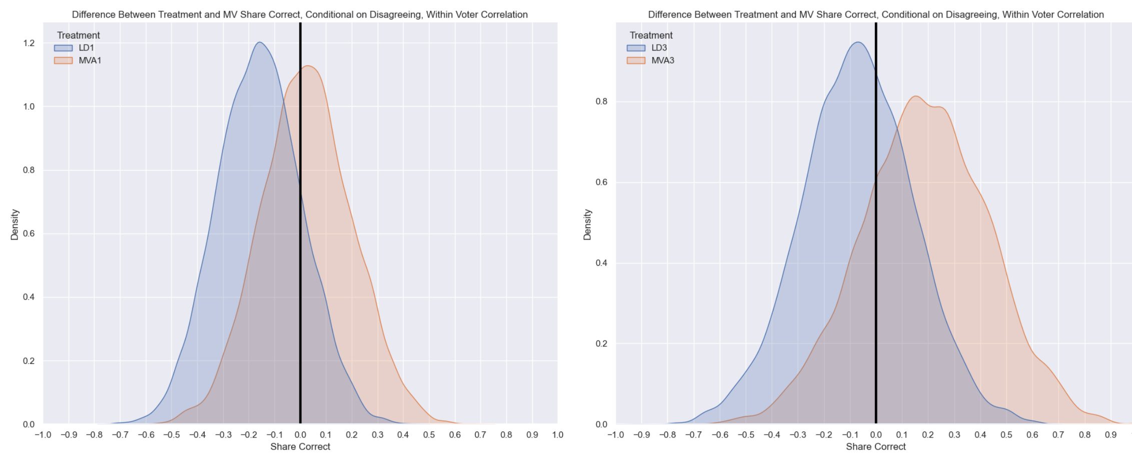


Figure 6: *Differential frequency of correct decisions, relative to MV, conditional on different outcomes.* Distributions over 100,000 bootstrap replications.

example LD1. For each of the 100,000 simulations, we focus on the subset of elections D_{LD1} such that LD1 and MV reach a different outcome. Call $\gamma_{LD1}(D_{LD1})$ ($\gamma_{MV}(D_{LD1})$) the frequency with which LD1 (MV) is correct over subsetD_{LD1}, a variable that ranges from 0 to 1. We are interested in $\gamma_{LD1}(D_{LD1}) - \gamma_{MV}(D_{LD1})$. Because the outcomes differ, by construction $\gamma_{MV}(D_{LD1}) = -\gamma_{LD1}(D_{LD1})$. Hence $\gamma_{LD1}(D_{LD1}) - \gamma_{MV}(D_{LD1}) = 2\gamma_{LD1}(D_{LD1}) - 1$. Our measure then ranges from 1–when, conditional on disagreement, LD1 always reaches the correct outcome, and MV the incorrect outcome–to -1–when the opposite holds; a value of zero indicates that the two rules are correct with equal frequency, conditioning on disagreement. The first panel of Figure 6 plots, in blue, the distribution of such variable over the 100,000 replications. The equivalent distribution for MVA is plotted in the same panel in red. The second panel reports the results for groups of size 15.[28]

For both group sizes, the blue distribution is shifted to the left, relative to the zero point indicated by the vertical black line: when LD and MV differ, the correct decision is more likely to be the one reached by MV. The asymmetry is more pronounced for $N = 5$, where the Blue mass to the left of zero–the probability that MV is superior to LD1, conditional on disagreement–is 85%, versus 67% for LD3. MVA on the other hand, when disagreeing with MV, is more likely to be right than wrong: only barely when $N = 5$, and the probability that MV is superior to MVA1 is just below 50% (48%), but more substantially when $N = 15$ and the probability that MV is correct, conditional on disagreement with MVA3, falls to 26%. The distributions as a whole are informative of the quantitative gap in the probability of being correct, relative to MV. In the panel oon the left, for example, the mode of the Blue

[28]Averaging over all replications, the share of elections in which the outcome differs from MV is 23.4% for LD1, 15.3% for MVA1, 20.1% for LD3, and 15% for MVA3.

distribution at -16% tells us that over the 100,000 replications, conditional on disagreement, the highest probability mass is around a frequency of correct decisions of about 42% for LD1, versus 58% for MV. [29]

In our first experiment then, LD falls short of the hopes if its supporters, even in a streamlined environment where experts are correctly identified. Like delegation, abstention allows voters with weak information not to influence the final choice, but is simpler and performs better. In our data, its efficiency is either comparable or somewhat superior to universal majority voting, contrary to what we see for delegation.

But do these results reflect some core feature of the systems we are studying, or are they artifacts of the lab? We analyze this question in our second experiment.

7 Experiment 2: The Random Dot Kinematogram

[**Should we repeat here the motivations for exp. 2?**] As we discussed in the Introduction, the goal of our second experiment is to evaluate whether the deviations from optimal behavior we see in the lab may stem from the over-mathematization of the environment. Although it is not clear why a detailed mathematical frame should affect the relative performance of delegation and abstantion, it is quite possible that the frame's high precision may distort behavior. With this in mind, we chose for the second experiment a perceptual task–the Random Dot Kinematogram (RDK)–where individual signals correspond to the accuracy of individual perceptions, and neither own or others' accuracy is described or known in precise probabilistic terms. Because the task may be unfamiliar, we describe it in some detail. More information as well as the instructions are reproduced in the Appendix.

We ran the experiment on Amazon Mechanical Turk (with prescreening of subjects by CloudResearch) with three electorate sizes: $N = 5$, and $N = 15$, as in Experiment 1, but also $N = 125$, i.e. with a larger size than we could run in the lab or conveniently on Zoom. In our implementation, 300 moving dots appear in each subject's screen for 1 second; a small fraction of them (dependent on treatment) moves in a coherent direction, either Left or Right, with equal ex ante probability. After 1 second, the image disappears and each participant reports whether the perceived coherent direction was Left or Right. We divide the experiment into two parts; each preceded by a few practice tasks, but with the first part effectively playing the role of extended training. Both parts are divided into six blocs, with each bloc consisting of 20 tasks of equal coherence. We report in the Appendix the precise

[29] We have verified that conclusions remain qualitatively similar if we construct the bootstrap ignoring the possibility of correlation in individual behavior across rounds, and thus draw each individual choice from the full data set for that treatment.

parameters we used for the task (the size and color of the dots, the movements per frame, the random process for the dots moving randomly, etc.), but it should be clear that our experiment does not aim at measuring perception per se–for example, we cannot control the ambient light, screen size, or contrast of the monitors our subjects use. Rather, our focus remains on collective decision-making.[30]

In Part 1, subjects are rewarded on the basis of their individual accuracy. Coherence–the fraction of dots that move in the same direction–ranges from 20% in bloc 1 (one fifth of the 300 dots move synchronously) to 10% in bloc 2, 8% in bloc 3, 6% in bloc 4, and finally the same coherence used in Part 2, and dependent on N, for the final two blocs. At the end of Part 1, each subject is informed of her fraction of correct answers in each bloc. In part 2, each task has both an individual component ("Choose the coherent direction"), and a subsequent group decision with the possibility of delegation (under LD), or abstention (under MVA). ("You said Left. Do you want to Vote or to Delegate/Abstain?"). When delegation is chosen, the vote is assigned randomly to an "expert", that is, one participants whose accuracy is in the top 20% of the group over the last 2 blocs (40 tasks); experts are not allowed to delegate (under LD) or to abstain (under MVA). Thus in line with Experiment 1, groups of 5 have 1 expert, and groups of 15 have 3; the group of 125 has 25 experts, and, following our standard notation, we denote the two treatments on the larger group by LD25 and MVA25. The group decision corresponds to the majority of votes cast, and individuals are rewarded both for their individual accuracy and for the accuracy of the group. As in Part 1, feedback about average individual accuracy in each bloc is provided at the end of Part 2.[31] In Part 2, coherence is kept constant across all blocs. We chose its value according to two main criteria: the task should not be so difficult that subjects are discouraged and act randomly, and should not be so easy that MV accuracy, especially in the large group, leaves effectively no room for possible improvement. Based on the results of two preliminary pilots, we fixed coherence in Part 2 at 5% for electorates of sizes 5 and 15, and at 3% for the electorate of size 125. The task is objectively hard, as the reader can verify at the following link: `https://blogs.cuit.columbia.edu/ac186/files/2022/05/rdk-video.gif`

The experiment used the RDK plugin in jsPsych (Rajananda et al., 2018) and was hosted on cognition.run. For each of LD and MVA, we recruited 60 subjects divided into 12 groups for the $N = 5$ treatment and 90 subjects divided into 6 groups for $N = 15$, (thus replicating the corresponding number of subjects and groups in Experiment 1), and an additional 125 subjects for the largest group. There were then 275 subjects for each voting system, or

[30]Heer and Bostock (2010) and Woods et al. (2015) report on the replication successes and challenges of conducting research on perceptual stimuli online.

[31]Feedback over group accuracy cannot be provided because it depends on choices made by others and is calculated ex post. Recall that participants are online and will come to the experiment at different times.

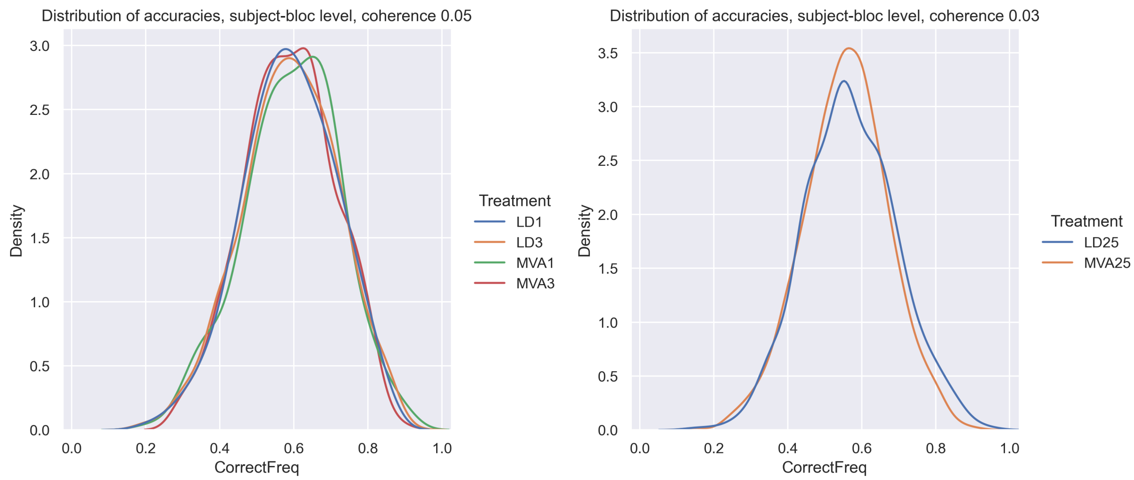


Figure 7: *Accuracies per bloc per subject. Distributions.* The panel on the left corresponds to coherence of 0.05; the panel on the right to coherence of 0.03.

550 in total. The group size and the relevant number of experts were always made public. The experiment lasted about 20 minutes. Subjects earned $1 for participation and a bonus proportional to the number of correct responses, for a total average compensation of $4.92, or just below $15 an hour.

8 Experiment 2: Results

8.1 Accuracy

One motivation for considering a perceptual experiment is the possibility of generating signals, here accuracies, of less than random precision. Figure 8 reports the distributions of correct responses in Part 2, aggregating by level of coherence (0.05 on the left; 0.03 on the right). Accuracy is the fraction of correct responses calculated over each of the 6 blocs for each subject, that is, over 20 tasks.

For given coherence, the distributions are very similar across treatments. In all cases, the spread in the distribution of accuracies is large, ranging from about 25% all the way to 95%. The frequency of blocs with accuracy below 50% is non-negligible and, surprisingly, persists when we aggregate over a larger number of tasks. Aggregating at the subject level, over all 120 tasks, 9% of subjects have accuracy below randomness with coherence 0.05, and 12% with coherence 0.03; 11% and 17% respectively do no better than randomness.[32] If we want to study voting and information aggregation when information may be faulty, perceptual

[32]Note however that individual subjects' accuracies show high variability across blocs, evidence of random noise in perceiving and recording the stimulus in the brain, as formalized in psychophysics research.

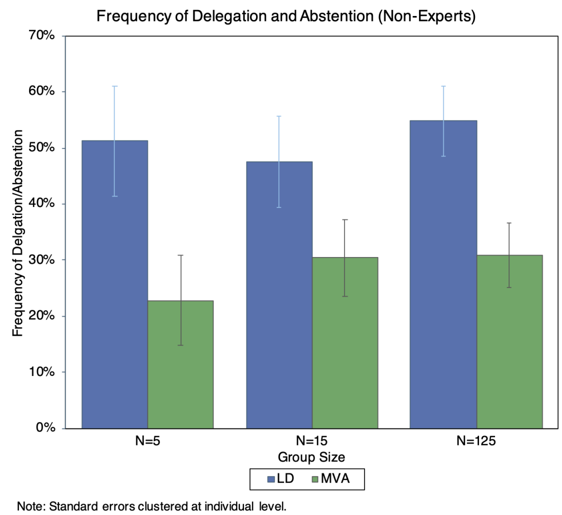


Figure 8: *Frequency of delegation and abstention (non-experts)*

tasks can provide a very useful tool.

8.2 Frequency of delegation and abstention

Absent information about the distributions of subjects' beliefs, we do not have a theoretical reference point for the extent of delegation and abstention we see in the data. We can however compare the two, under the plausible assumption, supported by Figure 7, that accuracies and beliefs about accuracies are comparable across the LD and MVA samples. [**But we do have average beliefs.**] Figure 8 plots the frequencies of delegation and abstention for each group size, calculated over non-experts only for possible comparison to Experiment 1.[33] The 95% confidence intervals are calculated from s.e's clustered at the individual level.

In Experiment 2, delegation remains much more common than abstention, for all group sizes. In groups of 5, where the disparity is largest, delegation is more than twice as frequent; in groups of 15, where we see the least disparity, delegation is still 60% more common. The decline in coherence, from $N = 5$ or 15 to $N = 125$, has smalll effects on the data. Unexpectedly, considering the rather radical change in experimental design, Figure 8 looks quite similar to Figure 2, for the group sizes for which we have data from both experiments.

The higher frequency of delegation is confirmed in the regressions reported in Table 6. The unit of analysis is the bloc at the individual subject level (hence 6 blocs per subject), with data grouped by coherence level.[34] The regressions reported below confirm the results of the figure: in all treatments, delegation is significantly more frequent than abstention. In

[33]The figure is almost identical if frequencies are calculated over the full sample.
[34]The results are unchanged if the data are separated by group size

Experiment 2, accuracy is at best a very weak predictor of participation in voting, never significant at conventional levels and with the wrong sign in the probit estimation, confirming the high uncertainty in subjects' evaluation of their own accuracy. The probit regressions detect a decline in abstention as blocs proceed, which would be consistent with increased familiarity with the task.

8.3 Frequency of correct outcomes

How well did the three voting systems do in Experiment 2? Figure 9 reports the frequency of correct group decisions, aggregated over all groups and tasks for given treatment. As in the case of Figure 5, a large fraction of outcomes are identical.

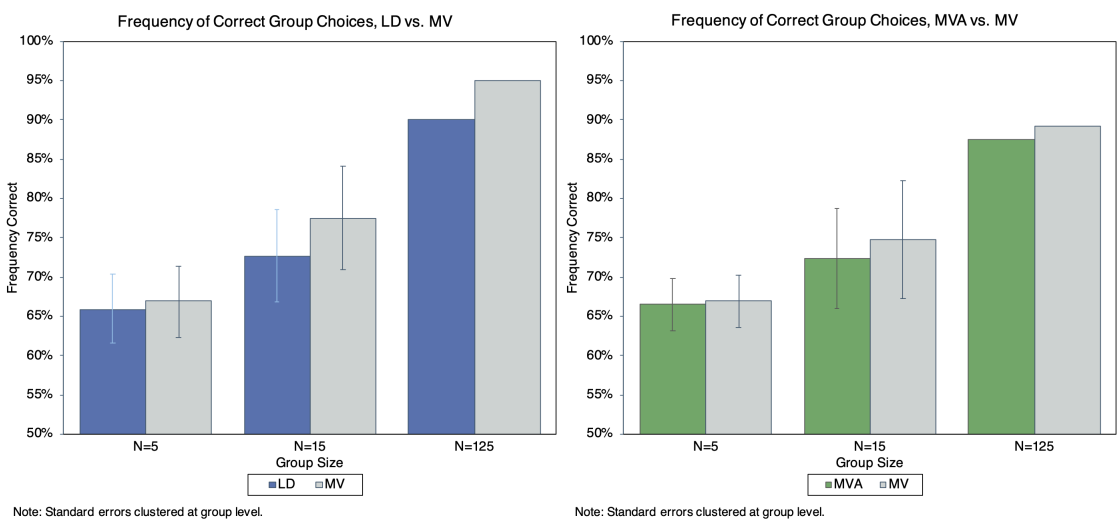


Figure 9: Frequency of correct outcomes

As expected, for all three systems, the fraction of correct decisions increases with the size of the group, ranging from about 65% at $N = 5$ to 90-95% at $N = 125$. In the pure common interest environment, the aggregation of independent signals remains very powerful, even in the presence of weak accuracies. In line with this logic, and with Condorcet, universal voting consistently achieves the best results. Here too, for the group sizes that are common to both experiments, the numbers are surprisingly similar to the numbers in Figure 5. Relative to MV, MVA performs better than LD, but the result in principle could be due to the lower propensity to abstain rather than delegating. Because abstention is less common, the fraction of decisions in which MVA and MV disagree is about the half fraction under LD (between 8 and 12% across the different treatments under MVA, v/s 12 to 24% under LD). And if MVA and MV agree more, their performance overall is more similar. To achieve better estimates of the differences across the three voting systems, we once again bootstrap the data and

Frequency of Delegation/Abstention, N=5 and N=15

	(1) Linear Probability	(2) Linear Probability	(3) Probit	(4) Probit
Accuracy	-0.122	-0.122	0.356	0.354
	(0.0839)	(0.0839)	(0.309)	(0.308)
	[0.146]	[0.146]	[0.249]	[0.251]
LD	0.226***	0.226***	0.547***	0.548***
	(0.0378)	(0.0378)	(0.139)	(0.140)
	[0.000]	[0.000]	[0.000]	[0.000]
N=15	0.00451	0.00446	0.0506	0.0503
	(0.0384)	(0.0384)	(0.139)	(0.140)
	[0.907]	[0.908]	[0.717]	[0.719]
Keys: [E][Y]		-0.00884		-0.0401
		(0.0379)		(0.139)
		[0.816]		[0.772]
Bloc		0.000511		-0.256***
		(0.0115)		(0.0654)
		[0.965]		[0.000]
Constant	0.339***	0.343***	0.163	0.313
	(0.0627)	(0.0656)	(0.222)	(0.234)
	[0.000]	[0.000]	[0.463]	[0.181]
Observations	1,800	1,800	1,800	1,800
R-squared	0.100	0.100		

*** p<0.01, ** p<0.05, * p<0.1

Notes: Standard errors in parentheses, clustered at the individual subject level. P-values in brackets. Delegation/abstention is measured as the share of rounds in a given bloc that a subject chose to delegate/abstain (with a range from 0 to 1). Accuracy is the share of rounds in the bloc that subject answered correctly. Subjects randomly use either keys [E] and [Y] or [V] and [N] to decide whether to delegate; a dummy for being assigned [E][Y] is included. The values for bloc have been scaled to be between 0 and 1; the coeffient for "bloc" thus indicates the effect of going from the first to last bloc.

Frequency of Delegation/Abstention, N=125

	(1) Linear Probability	(2) Linear Probability	(3) Probit	(4) Probit
Accuracy	0.000304	0.00380	0.479	0.455
	(0.101)	(0.102)	(0.357)	(0.359)
	[0.998]	[0.970]	[0.180]	[0.205]
LD	0.224***	0.223***	0.661***	0.663***
	(0.0415)	(0.0415)	(0.164)	(0.164)
	[0.000]	[0.000]	[0.000]	[0.000]
Keys: [E][Y]		0.0291		-0.0430
		(0.0414)		(0.161)
		[0.484]		[0.790]
Bloc		0.00192		-0.0363**
		(0.00258)		(0.0147)
		[0.459]		[0.0138]
Constant	0.316***	0.282***	0.313	0.694**
	(0.0623)	(0.0721)	(0.225)	(0.288)
	[0.000]	[0.000119]	[0.163]	[0.0162]
Observations	1,500	1,500	1,500	1,500
R-squared	0.095	0.097		

*** p<0.01, ** p<0.05, * p<0.1

Notes: Standard errors in parentheses, clustered at the individual subject level. P-values in brackets. Delegation/abstention is measured as the share of rounds in a given bloc that a subject chose to delegate/abstain (with a range from 0 to 1). Accuracy is the share of rounds in the bloc that subject answered correctly. Subjects randomly use either keys [E] and [Y] or [V] and [N] to decide whether to delegate; a dummy for being assigned [E][Y] is included. The values for bloc have been scaled to be between 0 and 1; the coeffient for "bloc" thus indicates the effect of going from the first to last bloc.

Table 6: *Frequency of delegation/abstention.*

replicate the group decisions a large number of times, generating a large sample of decisions over which the voting results would differ, as we did with the data from Experiment 1.

Figure 10 reports the results of 100,000 simulations, with three panels corresponding, in order, to $N = 5$,$N = 15$, and the single large group at $N = 125$. As in Figure 6, we plot the distributions of the differential frequency of correct decisions under LD (in Blue) or MVA (in Orange), relative to MV. Recall that, if the distribution is skewed to the left of the vertical line at zero, then conditional on disagreement, the correct decision is more likely to be the one reached by MV; and viceversa if the distribution is skewed to the right.

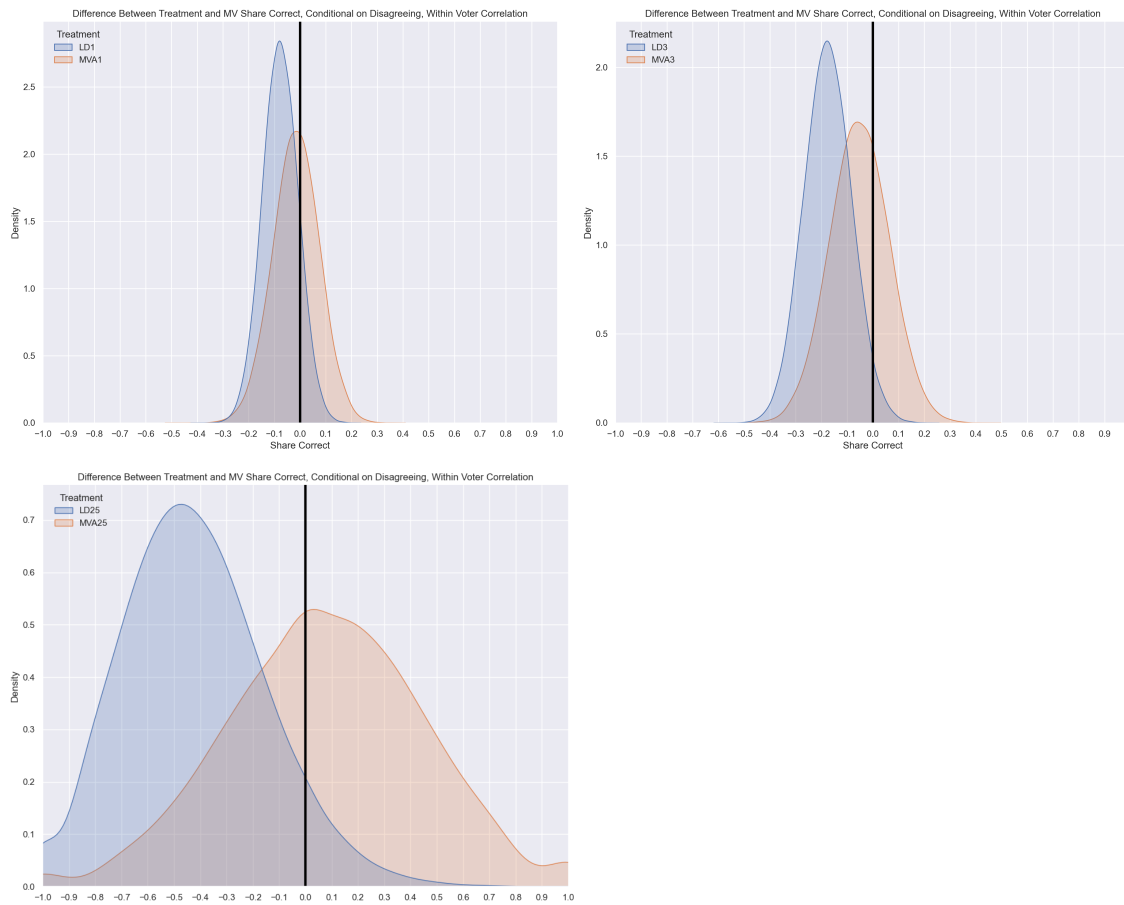


Figure 10: *Differential frequency of correct decisions, relative to MV, conditional on different outcomes*. Distributions over 100,000 bootstrap replications.

In all three panels, the blue mass is shifted to the left. Conditional on disagreement, the share of simulated experiments in which MV is more likely than LD to yield the correct outcome is 87% for LD1, 97% for LD3, and 95% for LD25. MVA (the orange mass in the figure) fares better: the corresponding numbers are 58% for MVA1, 69% for MVA3, and 48% for MVA25, when MVA is just barely more likely to be correct than MV, conditional on disagreement. As in Figure 6, the shapes of the distributions tell us the frequencies with which the two systems are correct, relative to MV. The proximity to zero indicates that even if the voting rule performs less well than MV, the difference need not be large. When $N = 5$ for example, the mode of the Blue distribution at -0.08 says that over the 100,000 simulations the most likely result is a frequency of correct decision of 46% for LD1 versus 54% for MV.

9 Conclusions

LD:

In two very different experiments, delegation is much more frequent than abstention.

LD is dominated by MV, MVA is comparable to MV.

On informational grounds, the value of LD seems at best uncertain.

Other objectives?

A methodological suggestion:

Collective decision-making in an ambiguous world.

Perceptual tasks are a good tool.

If possible, combined with a more controlled lab experiment.

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A Appendix

A.1 Theoretical results

Theorem. *Suppose $\pi = Pr(\omega_1) = 1/2$. Then for any F and for any N and K odd and finite, there exists an equilibrium with delegation that strictly improves over MV.*

Proof. The proof proceeds in four steps. The first three are well-known. First, as we noted in the text, with $\pi = 1/2$, and $Pr(\sigma_i = \sigma_1|\omega_1) = Pr(\sigma_i = \sigma_2|\omega_2) > 1/2$ for all i, voting according to signal is an undominated strategy, and we can focus on sincere equilibria.[35] Second, because this is a voting game of pure common interest, we can apply McLennan (1998)'s first result: a set of strategies that maximizes each voter's expected utility is an equilibrium. Hence an equilibrium exists. Third, the environment is fully symmetric for voters of each given type (experts and non-experts). All voters have identical preferences and are endowed with a single vote. All experts have equal precision and equal probability of receiving any delegated vote, and thus, for any delegation strategy by non-experts, each expert's vote has equal expected weight on the final decision. Non-experts will generally have heterogeneous realized precisions, and the equilibrium action will depend on individual precision, but each precision $q(i)$ is an independent draw from the same distribution F. Hence any permutation of realized precisions to different non-expert voters is assigned equal probability, and each voter holds equal beliefs about the others' precisions. For each type of voters, then, these symmetry conditions satisfy the requirements of McLennan's Theorem 2: if a semi-symmetric strategy profile maximizes each voter's expected utility in the set of semi-symmetric profiles, then it is an equilibrium. We can search for an equilibrium while restricting attention to semi-symmetric profiles.

The only new step is the fourth and last: can there be an equilibrium where no-one delegates? If the answer is negative, then the equilibrium that is maximal for expected utility in the profile of semi-symmetric strategies, and that we know exists, must involve a strictly positive probability of delegation for some realized precisions. But note that if no-one delegates, LD is equivalent to sincere MV. Thus if the profile of strategies where no-one delegates is not an equilibrium, then there must exist an equilibrium of the LD voting game that both involves a strictly positive probability of delegation and strictly dominates MV.

Consider the perspective of non-expert voter i, with q_i in the neighborhood of $\underline{q}$. Suppose no-one else delegates. We show in what follows that i's best response is to delegate his vote. Note first that if no-one delegates, all non-i voters cast a single vote and have equal weight on the group decision. Hence if i delegates, he delegates the vote to an expert, with precision

[35] With $F(q)$ defined over $[1/2, p]$ the event $q_i = 1/2$ is of measure zero.

$p \geq q_j$ for all j.

We need to calculate i's interim expected utility from non-delegating ($EUND(q_i)$) or delegating (EUD). Recall that the expected utility from delegating does not depend on q_i. The expressions are somewhat cumbersome but conceptually straightforward. We find:

$$EUND(q_i) = \sum_{c_n=0}^{M-1} \binom{M-1}{c_n} \mu^{c_n}(1-\mu)^{M-1-c_n} \times \\ \times \left[\sum_{c_e=0}^{K} \binom{K}{c_e} p^{c_e}(1-p)^{K-c_e} \left(q_i I_{c_n+c_e+1>\frac{(M+K)}{2}} + (1-q_i) I_{c_n+c_e>\frac{(M+K)}{2}} \right) \right]$$

$$EUD = \sum_{c_n=0}^{M-1} \binom{M-1}{c_n} \mu^{c_n}(1-\mu)^{M-1-c_n} \times \\ \times \left[\sum_{c_e=0}^{K} \binom{K}{c_e} p^{c_e}(1-p)^{K-c_e} \left(\left(\frac{c_e}{K}\right) I_{c_n+c_e+1>\frac{(M+K)}{2}} + \left(\frac{K-c_e}{K}\right) I_{c_n+c_e>\frac{(M+K)}{2}} \right) \right]$$

where $c_n(c_e)$ indexes the number of non-experts (experts) whose signals are correct, and I_C is an indicator function that takes value 1 if condition C is satisfied and 0 otherwise. As described in the text, μ is the expected precision of non-experts who choose to vote, and thus in this conjectured scenario, $\mu = \int_{\underline{q}}^{\overline{q}} q dF(q)$. For each realized c_n and c_e, i's expected utility always equals 1 if $(c_n + c_e) > (M+K)/2$, i.e. if the other voters with correct signals constitute a majority of the electorate. The choice to delegate or not matters when $(c_n + c_e)$ falls short of the majority by one vote. In such a case, $EUND(q_i)$ equals 1 if i's own signal is correct (with probability q_i) and zero otherwise; EUD equals 1 if i's vote is delegated to an expert with a correct signal (with probability c_e/K) and zero otherwise.

Voter i, with q_i in the neighborhood of $\underline{q}$, strictly prefers delegation if it yields higher expected utility, or:

$$lim_{q_i \to \underline{q}} (EUND(q_i) - EUD) < 0$$

Denote by r the number of additional correct votes required to reach a majority, given the votes of the non-experts, excluding i, or $r \equiv (M+K+1)/2 - c_n$. After some simplifications, we can write:

$$lim_{q_i \to \underline{q}} (EUND(q_i) - EUD) =$$
$$= \sum_{r=1}^{K+1} \binom{M-1}{\frac{M+K+1}{2} - r} \mu^{\frac{M+K+1}{2} - r} (1-\mu)^{\frac{M-(K+3)}{2} + r} \binom{K}{r-1} p^{r-1} (1-p)^{K-(r-1)} \left(\underline{q} - \frac{r-1}{K} \right) \tag{2}$$

Signing this expression is not immediate because the sign depends on the last term. However, the problem is simplified by noticing that:

$$\binom{M-1}{\frac{M+K+1}{2} - r} = \binom{M-1}{\frac{M+K+1}{2} - (K+2-r)}.$$

Equation (2) can then be written as:

$$lim_{q_i \to \underline{q}} (EUND(q_i) - EUD) = \sum_{x=1}^{(K+1)/2} \binom{M-1}{\frac{M+K+1}{2} - x} (\mu(1-\mu))^{\frac{M+K+1}{2} - (K+2-x)} \binom{K}{x} p^x (1-p)^x \times$$
$$\times \left\{ (\mu(1-p))^{K+2-2x} \left[\underline{q} - \frac{x-1}{K} \right] + ((1-\mu)p)^{K+2-2x} \left[\frac{x-1}{K} - (1-\underline{q}) \right] \right\}$$

or, with $\underline{q} = 1 - \underline{q} = 1/2$:

$$lim_{q_i \to \underline{q}} (EUND(q_i) - EUD) = \sum_{x=1}^{(K+1)/2} \binom{M-1}{\frac{M+K+1}{2} - x} (\mu(1-\mu))^{\frac{M+K+1}{2} - (K+2-x)} \binom{K}{x} p^x (1-p)^x \times$$
$$\times \left[\underline{q} - \frac{x-1}{K} \right] \left\{ (\mu(1-p))^{K+2-2x} - ((1-\mu)p)^{K+2-2x} \right\}$$

With $\underline{q} = 1/2$, $\left[\underline{q} - (x-1)/K\right] > 0$ for all $x < (K+2)/2$, and thus for all relevant x values. It follows that:

$$(\mu(1-p))^{K+2-2x} < ((1-\mu)p)^{K+2-2x} \Rightarrow lim_{q_i \to \underline{q}} (EUND(q_i) - EUD) < 0$$

With all $x < (K+2)/2$, the exponent on both sides is positive, and we can compare the roots:

$$\mu(1-p) < (1-\mu)p \Rightarrow lim_{q_i \to \underline{q}} (EUND(q_i) - EUD) < 0,$$

a condition that reduces to:

$$\mu < p$$

and is always satisfied. Hence $lim_{q_i \to \underline{q}} (EUND(q_i) - EUD) < 0$: delegation is the best response. A profile of strategies such that all non-voters cast their vote with probability 1 cannot be an equilibrium. $\square$

Proposition 1. *Suppose $\pi = Pr(\omega_1) = 1/2$ and $K = 1$. Then for any N odd and finite and for any F* ***[properties?]****, there exists an equilibrium such that: (i) the expert never delegates her vote and always votes according to signal; (ii) there exists a unique $\widetilde{q}(N) \in (\underline{q}, \overline{q})$ such that non-expert i delegates her vote to the expert if $q_i < \widetilde{q}$ and votes according to signal otherwise. Such an equilibrium strictly improves over MV and is maximal among sincere semi-symmetric equilibria where the expert never delegates and non experts delegate to the expert only.*

We prove the proposition in three steps. Following the argument in the text, we focus on equilibria with sincere voting. We begin by taking as given the directions of delegation: the expert never delegates, and non-experts only delegate to the expert. The first lemma shows that, if the assumed pattern of delegation holds, then there must exist a $\widetilde{q} \in (\underline{q}, \overline{q})$ such that non-expert i delegates her vote to the expert if $q_i < \widetilde{q}$ and votes according to signal otherwise. Because $\widetilde{q}$ is strictly interior to the interval $[\underline{q}, \overline{q}]$, the conjectured equilibrium must have strictly interior probability of delegation. Lemma 2 shows that given Lemma 1, the assumed directions of delegation are indeed best responses when others follow them. Hence the profile of strategies described in Proposition 1 is an equilibrium. Finally, Lemma 3 proves that such interior equilibrium maximizes ex ante expected utility among sincere, semi-symmetric equilibria with the specified directions of delegation.

Lemma 1. *Suppose $\pi = Pr(\omega_1) = 1/2$ and $K = 1$. Suppose in addition that the expert never delegates her votes, and non-experts only delegate to the expert. Then there exists $\widetilde{q} \in (\underline{q}, \overline{q})$ such that non-expert i delegates her vote to the expert if $q_i < \widetilde{q}$ and votes according to signal otherwise.*

Proof. Under the conjectured pattern of delegation, the expert never delegates, and cycles of delegation are not possible. The only question is the delegation decision by non-experts. As argued earlier, keeping in mind that all q_i's are independent draws from $F(q)$, non-expert i's expected utility conditional on delegation, EUD_i, does not depend on q_i (because i would not be voting). Hence $EUD_i = EUD$ for all i. On the other hand, i's expected utility when not delegating, $EUND_i$, must be weakly increasing in q_i (because the probability of reaching the correct outcome must be weakly increasing in q_i), and strictly increasing if i's probability of being pivotal is positive. Suppose first $\widetilde{q} = \overline{q}$: all non-experts delegate to the expert. In such a case, the expert is dictator, and no individual non-expert can make herself pivotal by deviating: $EUND_i = EUND = EUD$, and delegating is indeed a (weak) best response for any q_i. Hence $\widetilde{q} = \overline{q}$ can be supported as a system of mutual best responses by all non-experts. However, we know from the Condorcet Jury Theorem that in the environment we are studying, MV dominates dictatorship by any voter. When the expert never delegates, MV corresponds to $\widetilde{q} = \underline{q}$: all voters cast their vote. And yet we know from our earlier theorem that MV cannot be an equilibrium when delegation is allowed. By the logic of the theorem, there must be a profile of mutual best responses with positive but partial expected delegation.[36] But if delegation is not full (or $\widetilde{q} \neq \overline{q}$), then any voter who casts a vote has strictly positive probability of being pivotal. Thus for any non-expert i, $(EUND_i(q_i) - EUD)$ is strictly increasing in q_i. It then follows that there must exist a $\widetilde{q} \in (q, \overline{q})$ such that $EUND_i(q_i) = EUD$ at $q_i = \widetilde{q}$, $EUND_i(q_i) > EUD$ for all $q_i > \widetilde{q}$, and $EUND_i(q_i) < EUD$ for all $q_i < \widetilde{q}$: all non-experts i with $q_i > \widetilde{q}$ prefer to vote, and all non-experts i with $q_i < \widetilde{q}$ prefer to delegate to the expert. □

Lemma 2. *Suppose $\pi = Pr(\omega_1) = 1/2$ and $K = 1$. Then: (i) If non-experts adopt the threshold strategies described in Lemma 1 and only delegate to the expert, it is optimal for the expert never to delegate. (ii) Consider non-expert i. If the expert never delegates and all other non-expert $j \neq i$ delegates to the expert if and only if $q_j < \widetilde{q}$, with $\widetilde{q} \in (\underline{q}, \overline{q})$, and never to a non-expert, then it is optimal for i to delegate to the expert if and only if $q_i < \widetilde{q}$, and never to a non-expert. Hence the strategies are mutual best responses.*

Proof. We begin by proving claim (i): delegation from the expert to a non-expert cannot be optimal. First consider expert e delegating to some non-expert j when j does not delegate to e (and thus there is no cycle). Expected utility when all M non-experts use threshold $\widetilde{q}$ is higher than expected utility when $M-1$ non-experts use cutoff $\widetilde{q}$ and one non-expert i delegates to the expert for all q_i (because i delegating when $q_i > \widetilde{q}$ strictly decreases expected utility). In turn, expected utility in this latter case is higher than expected utility from the same actions if the expert's precision were drawn from $[\widetilde{q}, \overline{q}]$ according to distribution F, rather than being p. But the expected utility from this last scenario is identical to the expected utility from e delegating to j: $M-1$ non-experts delegate using cutoff $\widetilde{q}$, but all their delegated votes are turned over to non-expert j, with precision randomly drawn from $[\widetilde{q}, \overline{q}]$, and e, who always delegates regardless of precision, is the analogue of voter i in the constructed scenario. Now suppose that when e delegates to j, j also delegates to e. This creates a cycle, and all delegations are canceled. Note that having one's delegation canceled

[36] We proved the theorem for arbitrary directions of delegation. But notice that the theorem continues to hold if we fix delegation to the pattern conjectured here: the expert never delegates and non-experts only delegate to the expert.

conveys no information about signal realizations. Hence the voting game after delegations are returned is simply majority voting. But we know that majority voting is strictly dominated by the profile of threshold strategies with positive probability of delegation to the expert. In particular, canceling all delegations must strictly decrease expected payoff since j only delegates to e when $q_j < \widetilde{q}$, and thus optimal delegation in this scenario is strictly positive.

Consider now claim (ii). Given threshold $\widetilde{q}$, consider the difference in expected utility for a non-expert i between delegating to expert e or instead delegating to some non-expert j who has not herself delegated her vote to e (the choice would otherwise be irrelevant). Expert e has unconditional expected precision p; non-expert j has unconditional expected precision $\mu(\widetilde{q}) \equiv \mathbb{E}_F[q_j|q_j > \widetilde{q}] < p$. Voter i's expected utility from the two forms of delegation can differ only if i's vote is pivotal and e and j's signals differ ($\sigma_e \neq \sigma_j$): for any number of delegated votes z, the expert agrees with $\frac{M}{2} - (z+1)$ non-experts and disagrees with $\frac{M}{2}$ (j included, i not included). The delegation choice is not trivial because, although $\mu(\widetilde{q}) < p$, when i is pivotal and $\sigma_e \neq \sigma_j$, there must be fewer independent signals agreeing with e than with j. Let$piv_i(z)$ be the event corresponding to the set of signal realizations at which i's vote is pivotal, conditional on the expert being delegated z votes, and $pivj_i(z)$ be the same event additionally conditioning on $\sigma_e \neq \sigma_j$. Note that in both events the expert agrees with $\frac{M}{2} - (z+1)$ non-experts and disagrees with $\frac{M}{2}$: the events contain the same information content and $Pr(\sigma_e = \omega|piv_i(z)) = Pr(\sigma_e = \omega|pivj_i(z))$ for all z. Then, noting that the summation below is to $M-2$ to exclude i and j:

$$EUD(i \text{ delegate to } e) - EUD(i \text{ delegate to } j) =$$

$$= \sum_{z=0}^{M-2} \binom{M-2}{z} F(\widetilde{q})^z (1-F(\widetilde{q}))^{M-2-z} Pr(pivj_i(z)|z)[Pr(\sigma_e = \omega|pivj_i(z)) - Pr(\sigma_j = \omega|pivj_i(z))]$$

Define $r(z) \equiv Pr(\sigma_e = \omega|pivj_i(z)) = Pr(\sigma_e = \omega|piv_i(z))$. Then:[37]

$$r(z) = \frac{Pr(pivj_i(z)|\sigma_e = \omega)Pr(\sigma_e = \omega)}{Pr(pivj_i(z))} =$$

$$= \frac{p(\mu(\widetilde{q}))^{\frac{M}{2}-(z+1)}(1-\mu(\widetilde{q}))^{\frac{M}{2}}}{p(\mu(\widetilde{q}))^{\frac{M}{2}-(z+1)}(1-\mu(\widetilde{q}))^{\frac{M}{2}} + (1-p)(1-\mu(\widetilde{q}))^{\frac{M}{2}-(z+1)}(\mu(\widetilde{q}))^{\frac{M}{2}}}$$

As $Pr(\sigma_j = \omega|pivj_i(z)) = 1 - Pr(\sigma_e = \omega|pivj_i(z))$, we can rewrite:

[37]Note that $r(z)$ is strictly decreasing in z for all z small enough that e is not dictator.

$EUD(i \text{ delegate to } e) - EUD(i \text{ delegate to } j) =$

$$= \sum_{z=0}^{M-2} \binom{M-2}{z} F(\widetilde{q})^z (1 - F(\widetilde{q}))^{M-2-z} Pr(pivj_i(z)|z)[2r(z) - 1]$$

We can sign this expression by exploiting the equilibrium condition for $\widetilde{q}$. Consider the difference in expected utility between i delegating to e and i voting when her precision is $q_i = \widetilde{q}$:

$EUD(i \text{ delegate to } e) - EUND(q_i = \widetilde{q}) =$

$$\sum_{z=0}^{M-2} \binom{M-2}{z} F(\widetilde{q})^z (1 - F(\widetilde{q}))^{M-2-z} Pr(piv_i(z)|z)[Pr(\sigma_e = \omega|piv_i(z)) - \widetilde{q}] =$$

$$= \sum_{z=0}^{M-2} \binom{M-2}{z} F(\widetilde{q})^z (1 - F(\widetilde{q}))^{M-2-z} Pr(piv_i(z)|z)[r(z) - \widetilde{q}]$$

Note that $Pr(pivj_i(z)|z) = \frac{\frac{M}{2}}{M-1} Pr(piv_i(z)|z)$ (i.e. j must be part of the $M/2$ non-experts who disagree with e, out of $M-1$ non-experts, ignoring i). For equilibrium $\widetilde{q}$, $EUD(i \text{ delegate to } e) - EUND(q_i = \widetilde{q}) = 0$ which implies:

$$\frac{M-1}{\frac{M}{2}} \sum_{z=0}^{M-2} \binom{M-2}{z} F(\widetilde{q})^z (1 - F(\widetilde{q})^{M-2-z} Pr(pivj_i(z)|z) r(z) =$$

$$= \frac{M-1}{\frac{M}{2}} \sum_{z=0}^{M-2} \binom{M-2}{z} F(\widetilde{q})^z (1 - F(\widetilde{q}))^{M-2-z} Pr(pivj_i(z)|z) \widetilde{q}$$

or:

$$r(z) = \widetilde{q}.$$

Hence:

$EUD(i \text{ delegate to } e) - EUD(i \text{ delegate to } j) =$

$$= \sum_{z=0}^{M-2} \binom{M-2}{z} F(\widetilde{q})^z (1 - F(\widetilde{q}))^{M-2-z} Pr(pivj_i(z)|z)[2\widetilde{q} - 1]$$

But $(2\widetilde{q} - 1) > 0$ for all $\widetilde{q} \in (\underline{q}, \overline{q})$, and thus $EUD(i \text{ delegate to } e) - EUD(i \text{ delegate to } j) >$

0. $\square$

Lemma 3. *Suppose $\pi = Pr(\omega_1) = 1/2$ and $K = 1$. Then the equilibrium strategies characterized in Lemma 2 maximize ex ante expected utility over sincere, semi-symmetric equilibria where the expert never delegates and non-experts delegate to the expert only.*

Proof. Given threshold $\widetilde{q}$,we can write ex ante expected utility as:

$$EU(\widetilde{q}) = \sum_{z=0}^{M} \binom{M}{z} F(\widetilde{q})^z (1 - F(\widetilde{q}))^{M-z} EU(z, \widetilde{q}),$$

where $EU(z, \widetilde{q})$ is expected utility conditioned on z delegated votes to the expert. The first order condition corresponding to the maximization of $EU(\widetilde{q})$ with respect to $\widetilde{q}$ is given by

$$\frac{dEU(\widetilde{q})}{d\widetilde{q}} = 0$$

where

$$\frac{dEU(\widetilde{q})}{d\widetilde{q}} = \sum_{z=0}^{M} \binom{M}{z} f(\widetilde{q})(z)F(\widetilde{q})^{z-1}(1 - F(\widetilde{q}))^{M-z} EU(z, \widetilde{q}) -$$
$$-\sum_{z=0}^{M} \binom{M}{z} f(\widetilde{q})(M-z)F(\widetilde{q})^z(1-F(\widetilde{q}))^{M-z-1} EU(z, \widetilde{q}) + \sum_{z=0}^{M} \binom{M}{z} F(\widetilde{q})^z(1-F(\widetilde{q}))^{M-z} \frac{d}{d\widetilde{q}} EU(z, \widetilde{q}). \quad (3)$$

We prove the lemma by showing that if there is an equilibrium in strictly interior threshold strategies, then first order condition 3 is satisfied at the equilibrium threshold. But since we know from the Theorem that an interior threshold equilibrium must exist, it follows that such threshold maximizes expected utility, in the class of equilibrium strategies embodied in $EU(\widetilde{q})$ (sincere, semi-symmetric strategies where the expert never delegates and non-experts delegate to the expert only).

Suppose that an interior equilibrium threshold $\widetilde{q}$ exists. Note that:

$$EU(z, \widetilde{q}) = \int_{\widetilde{q}}^{\overline{q}} \dots \int_{\widetilde{q}}^{\overline{q}} U(z, q^1, ..., q^{M-z}) f(q^1 | q_{i_1} > q) ... f(q^{M-z} | q_{i_{M-z}} > q) dq^1 ... dq^{M-z}$$

where the function $U(z, q^1, ..., q^{M-z})$ corresponds to the probability of the group choosing the correct alternative when z votes are delegated to e and $i_1, ..., i_{M-z}$ non-experts, with precisions $q_{i_1}, ..., q_{i_{M-z}}$, have not delegated their votes.

Using Leibnitz' integral rule, we find:

$$\frac{d}{d\widetilde{q}}EU(z,\widetilde{q}) = (M-z)\frac{f(\widetilde{q})}{(1-F(\widetilde{q}))}\left(EU(z,\widetilde{q}) - EU(z,q_i=\widetilde{q},\widetilde{q})\right) \tag{4}$$

where:

$$EU_i(z,\widetilde{q},q_i) = \int_{\widetilde{q}}^{\overline{q}} ... \int_{\widetilde{q}}^{\overline{q}} U_i(z,q_i,q^1,...,q^{M-z}) f(q^1|q_{i_1} > q)...f(q^{M-z}|q_{i_{M-z}} > q)dq^1...dq^{M-z}$$

(The function $U_i(z,q_i,q^1,...,q^{M-z})$ denotes the probability of choosing the correct alternative from the perspective of voter i with precision q_i who has not delegated). Equation 4 tells us that the marginal effect on expected utility of an infinitesimal change in the threshold $\widetilde{q}$ corresponds to the change in utility from one voter switching to delegating, multiplied by the probability of such a switch (the hazard rate), times the number of susceptible voters $(M-z)$.

Consider now the equilibrium condition for the threshold $\widetilde{q}$. A voter who delegates her vote to the expert e has expected utility EUD:

$$EUD(\widetilde{q}) = \sum_{z=1}^{M} \binom{M-1}{z-1} F(\widetilde{q})^{z-1}(1-F(\widetilde{q}))^{M-z}EU(z,\widetilde{q})$$

If instead voter i does not delegate her vote, expected utility $EUND_i$is given by:

$$EUND_i(\widetilde{q},q_i) = \sum_{z=0}^{M} \binom{M-1}{z} F(\widetilde{q})^{z}(1-F(\widetilde{q}))^{M-z-1}EU_i(z,\widetilde{q},q_i)$$

An equilibrium in strictly interior threshold strategies exists if there exists $\widetilde{q} \in (\underline{q},\overline{q})$ such that $EUD(\widetilde{q}) = EUND_{q_i=\widetilde{q}}(\widetilde{q})$.

Returning now to the first order condition 3, note that the first term can be simplified to:

$$Mf(\widetilde{q})\sum_{z=1}^{M} \binom{M-1}{z-1} F(\widetilde{q})^{z-1}(1 - F(\widetilde{q}))^{M-z}EU(z,\widetilde{q}) = Mf(\widetilde{q})EUD(\widetilde{q}). \tag{5}$$

Similarly, using 4, we can simplify the third term in 3:

$$\sum_{z=0}^{M} \binom{M}{z} F(\widetilde{q})^z (1 - F(\widetilde{q}))^{M-z} \frac{d}{d\overline{q}} EU(z, \widetilde{q}) =$$

$$= \sum_{z=0}^{M} \binom{M}{z} f(\widetilde{q})(M-z) F(\widetilde{q})^z (1 - F(\widetilde{q}))^{M-z-1} EU(z, \widetilde{q}) - M f(\widetilde{q}) EUND_i(\widetilde{q}, q_i) \quad (6)$$

Substituting 5 and 6 in 3, we thus obtain:

$$\frac{dEU(\widetilde{q})}{d\widetilde{q}} = Mf(\widetilde{q})EUD(\widetilde{q}) - \sum_{z=0}^{M} \binom{M}{z} f(\widetilde{q})(M-z) F(\widetilde{q})^z (1 - F(\widetilde{q}))^{M-z-1} EU(z, \widetilde{q}) +$$

$$+ \sum_{z=0}^{M} \binom{M}{z} f(\widetilde{q})(M-z) F(\widetilde{q})^z (1 - F(\widetilde{q}))^{M-z-1} EU(z, \widetilde{q}) - Mf(\widetilde{q}) EUND_i(\widetilde{q}, q_i) =$$

$$Mf(\widetilde{q}) \left(EUD(\widetilde{q}) - EUND_i(\widetilde{q}, q_i) \right)$$

or: if there exists $\widetilde{q}$' such that $EUD(\widetilde{q}') - EUND_{q=\widetilde{q}'}(\widetilde{q}')$, then $\left.\frac{dEU(\widetilde{q})}{d\widetilde{q}}\right|_{\widetilde{q}=\widetilde{q}'} = 0$.

Together with the Theorem, this result establishes the Lemma. We know from the Theorem that an equilibrium with $\widetilde{q} \in (\underline{q}, \overline{q})$ exists and must be such that $EU(\widetilde{q}) > EU(\underline{q}) > EU(\overline{q})$,(where the last inequality follows from the Condorcet Jury Theorem). Hence there exists $\widetilde{q}' = argmax EU(\widetilde{q})$, with $\widetilde{q}' \in (\underline{q}, \overline{q})$ and by the result above, such $\widetilde{q}'$ must be an equilibrium. □

A.1.1 LD: K (odd) experts; M (even) non-experts.

We report here, for generic parameter values, the formulas we used to derive the equilibria for the experimental parametrizations. As always, $EUND(q_i)$ is interim expected utility for a voter with realized precision q_i; the equilibrium threshold is denoted $\widetilde{q}$ and solves $EUND(q_i = \widetilde{q}) = EUD$, and $\mu_v(\widetilde{q}) \equiv \mathbb{E}_F[q_j | q_j > \widetilde{q}]$. We find:

$$EUND(q_i,\widetilde{q}) = \sum_{z=0}^{M-1} \binom{M-1}{z} (1-F(\widetilde{q}))^z F(\widetilde{q})^{M-1-z} \sum_{c_n=0}^{z} \binom{z}{c_n} (\mu_v(\widetilde{q}))^{c_n} (1-\mu_v(\widetilde{q}))^{z-c_n} \times$$
$$\times \left\{ \left(\sum_{c_e=0}^{K} \binom{K}{c_e} p^{c_e} (1-p)^{K-c_e} \sum_{x_1=0}^{M-z-1} \sum_{x_2=0}^{M-z-1-x_1} \cdots \sum_{x_{K-1}=0}^{M-z-1-\sum_{k=1}^{K-2} x_k} \frac{(M-z-1)!}{\prod_{k=1}^{K} x_k!} \right) \times \right.$$
$$\times \left(q_i \left((1/K)^{M-z-1} I_{c_n+1+c_e+\sum_{k=1}^{c_e} x_k > (M+K)/2} \right) + \right.$$
$$\left. \left. (1-q_i) \left((1/K)^{M-z-1} I_{c_n+c_e+\sum_{k=1}^{c_e} x_k > (M+K)/2} \right) \right) \right\}$$

,

where $x_K \equiv M - z - 1 - \sum_{k=1}^{K-1} x_k$, and I_C is an indicator function that equals 1 if condition C is realized and 0 otherwise. Similarly:

$$EUD(\widetilde{q}) = \sum_{z=0}^{M-1} \binom{M-1}{z} (1-F(\widetilde{q}))^z F(\widetilde{q})^{M-1-z} \sum_{c_n=0}^{z} \binom{z}{c_n} (\mu_v(\widetilde{q}))^{c_n} (1-\mu_v(\widetilde{q}))^{z-c_n} \times$$
$$\times \left\{ \left(\sum_{c_e=0}^{K} \binom{K}{c_e} p^{c_e} (1-p)^{K-c_e} \sum_{y_1=0}^{M-z} \sum_{y_2=0}^{M-z-y_1} \cdots \sum_{y_{K-1}=0}^{M-z-\sum_{k=1}^{K-2} y_k} \frac{(M-z)!}{\prod_{k=1}^{K} y_k!} \right) \times \right.$$
$$\left. \times \left((1/K)^{M-z} I_{c_n+c_e+\sum_{k=1}^{c_e} y_k > (M+K)/2} \right) \right\}$$

where $y_K \equiv M - z - \sum_{k=1}^{K-1} y_k$.

We use as welfare criterion ex ante expected utility, i.e. expected utility before the realization of q_i (but under the correct expectation of $\widetilde{q}$). Hence:

$$EU(\widetilde{q}) = \int_{\underline{q}}^{\widetilde{q}} EUD f(q) dq + \int_{\widetilde{q}}^{p} EUND(q_i) f(q) dq$$

Under MV, ex ante expected utility is given by:

$$EU_{MV} = \sum_{c_n=0}^{M} \binom{M}{c_n} \mu^{c_n} (1-\mu)^{z-c_n} \sum_{c_e=0}^{K} \binom{K}{c_e} p^{c_e} (1-p)^{K-c_e} I_{c_n+c_e > \frac{(M+K)}{2}}$$

where, as in earlier use, $\mu \equiv \mathbb{E}_F(q_i)$.

A single expert With a single expert, equations simplify:

$$EUD(\widetilde{q}) = \sum_{z=0}^{M-1} \binom{M-1}{z} (1-F(\widetilde{q}))^z \, F(\widetilde{q})^{M-1-z} \sum_{c_n=0}^{z} \binom{z}{c_n} \mu_v(\widetilde{q})^{c_n} (1-\mu_v(\widetilde{q}))^{z-c_n} \times$$
$$\times \left[pI_{c_n+M-z+1>\frac{(M+1)}{2}} + (1-p)I_{c_n>\frac{(M+1)}{2}} \right]$$

$$EUND(q_i, \widetilde{q}) = \sum_{z=0}^{M-1} \binom{M-1}{z} (1-F(\widetilde{q}))^z \, F(\widetilde{q})^{M-1-z} \sum_{c_n=0}^{z} \binom{z}{c_n} \mu_v(\widetilde{q})^{c_n} (1-\mu_v(\widetilde{q}))^{z-c_n} \times$$
$$\times \left[q_i \left(pI_{c_n+M-z+1>\frac{(M+1)}{2}} + (1-p)I_{c_n+1>\frac{(M+1)}{2}} \right) + (1-q_i) \left(pI_{c_n+M-z>\frac{(M+1)}{2}} + (1-p)I_{c_n+>\frac{(M+1)}{2}} \right) \right],$$

A.1.2 MVA: K (odd) experts; M (even) non-experts.

Under the possibility of abstention as well, all equilibria are in monotone threshold strategies. We denote by $\widetilde{\alpha}$ the equilibrium threshold such that all i with $q_i < \widetilde{\alpha}$ choose to abstain, and all i with $q_i > \widetilde{\alpha}$ choose to vote. With a known and finite electorate size, the equilibria are sensitive to whether K and M are odd or even. In particular, an equilibrium with $\widetilde{\alpha} = \underline{q}$ (all voters cast their vote) exists if and only if N is odd. An equilibrium with $\widetilde{\alpha} = p$ (all experts vote, and none of the other voters do) exists if and only if K is odd. This both equilibria exist in our experimental paramterizations. In addition, there are interior equilibria where $\widetilde{\alpha} \in (\underline{q}, p)$. Denoting by $EUV(q_i, \widetilde{\alpha})$ interim expected utility from voting, given q_i, and by $EUA(\widetilde{\alpha})$ interim expected utility from abstaining (which does not depend on q_i), $\widetilde{\alpha}$ must solve $EUV(q_i, \widetilde{\alpha}) = EUA(\widetilde{\alpha})$, where:

$$EUV(q_i, \widetilde{\alpha}) = \sum_{v=0}^{M-1} \binom{M-1}{v} (1-F(\widetilde{\alpha}))^v \, F(\widetilde{\alpha})^{M-1-v} \sum_{c_n=0}^{v} \binom{v}{c_n} (\mu_v(\widetilde{\alpha}))^{c_n} (1-\mu_v(\widetilde{\alpha}))^{v-c_n} \times$$
$$\times \left\{ \left(\sum_{c_e=0}^{K} \binom{K}{c_e} p^{c_e} (1-p)^{K-c_e} \right) \times \right.$$
$$\times \left(q_i \left(I_{c_n+1+c_e>(v+1+K)/2} + (1/2) I_{c_n+1+c_e=(v+1+K)/2} \right) + \right.$$
$$\left. \left. (1-q_i) \left(I_{c_n+c_e>(v+1+K)/2} + (1/2) I_{c_n+c_e=(v+1+K)/2} \right) \right) \right\},$$

and:

$$EUA(\widetilde{\alpha}) = \sum_{v=0}^{M-1} \binom{M-1}{v} (1 - F(\widetilde{\alpha}))^v F(\widetilde{\alpha})^{M-1-v} \sum_{c_n=0}^{v} \binom{v}{c_n} (\mu_v(\widetilde{\alpha}))^{c_n} (1 - \mu_v(\widetilde{\alpha}))^{v-c_n} \times$$
$$\times \left\{ \left(\sum_{c_e=0}^{K} \binom{K}{c_e} p^{c_e} (1-p)^{K-c_e} \right) \left(I_{c_n+c_e>(v+K)/2} + (1/2) I_{c_n+c_e=(v+K)/2} \right) \right. .$$

Ex ante expected utility, before the realization of q_i but under the correct expectation of $\widetilde{\alpha}$, is given by:

$$EU_{MVA}(\widetilde{\alpha}) = \int_{\underline{q}}^{\widetilde{\alpha}} EUA(\widetilde{\alpha}) f(q) dq + \int_{\widetilde{q}}^{p} EUV(q_i, \widetilde{\alpha}) f(q) dq$$

A.2 Monotonicity Violations

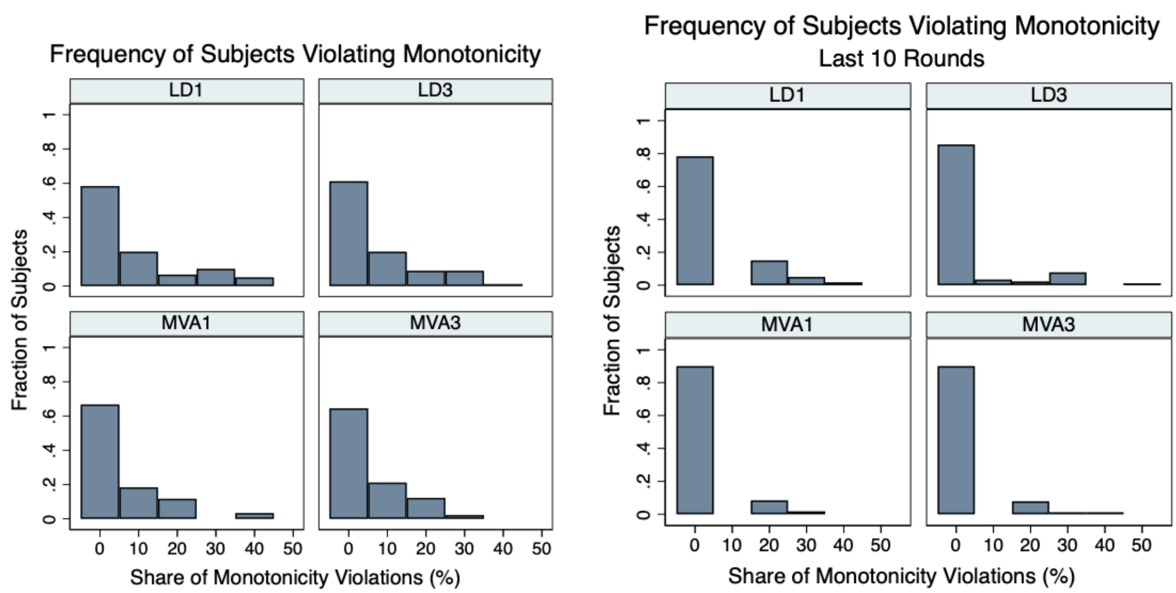

Figure 11: Monotonicity violations - histograms

A.3 Voting against signal and signal quality

Note that the only significant effect is from signal quality (precision). Neither the voting system nor the size of the group matter, but a decrease in signal quality is strongly correlated with an increase in voting against signal. Although voting against signal is an inferior action, the loss from doing so is indeed increasing in signal quality. There could be a small learning

Frequency of Voting Against Signal

	(1) Linear Probability	(2) Probit
Signal Quality	-0.472***	-2.244***
	(0.0560)	(0.235)
	[0.000]	[0.000]
Round	0.00264	0.00454
	(0.0158)	(0.0832)
	[0.868]	[0.956]
LD	-0.00539	-0.0447
	(0.0270)	(0.150)
	[0.842]	[0.765]
N = 15	-0.00637	-0.0728
	(0.0362)	(0.197)
	[0.860]	[0.712]
Second	-0.0115	-0.0700
	(0.0383)	(0.200)
	[0.764]	[0.726]
Second * Mixed	-0.0372	-0.208
	(0.0332)	(0.195)
	[0.264]	[0.286]
N = 15 * Second	0.0288	0.164
	(0.0614)	(0.338)
	[0.640]	[0.626]
N = 15 * Mixed	-0.00469	0.00404
	(0.0398)	(0.242)
	[0.906]	[0.987]
Constant	0.452***	0.221
	(0.0692)	(0.239)
	[0.000]	[0.356]
Observations	2,552	2,552
R-squared	0.154	

*** p<0.01, ** p<0.05, * p<0.1

Notes: Standard errors in parentheses, clustered at the individual subject level. P-values in brackets. The dependent variable is a 0-1 indicator of whether the subject voted against their signal. Only non-experts are included, and only the instances in which they did not delegate or abstain are included. "Second" indicates that a treatment appeared second in the session. "Mixed" inidcates that both an LD treatment and MVA treatment appeared in the session.

effect (voting against signal is less frequent in the treatments run second), but it is very variable and counterbalanced by an equally small and insignificant increase in voting against signal as rounds proceed, possibly from fatigue.

A.4 The bootstrapping procedure: allowing for individual correlation across rounds

Replicating what happens in an individual session, we draw with replacement 15 subjects from the relevant treatment, each with all choices made over the 20 rounds. Among these 15 subjects, we draw, with replacement, 3 subjects, assigning to each of them one choice they made as expert,[38] and 12 subjects, assigning to each one choice made as non-expert. In $N = 15$, that constitute the group and yields one group decision; in $N = 5$ treatments, we divide the 12 subjects randomly into three groups of 4 and assign to each group one of the experts drawn earlier, generating three group decisions. We repeat new draws of 3 experts and 12 experts as above 20 times, generating 20 decisions from the same sample of 15 subjects if the treatment has $N = 15$, and 60 if the treatment has $N = 5$, thus simulating one experimental session. We then draw, always with replacement, a new group of 15 subjects, and repeat the procedure, each time generating 20 (60) decisions from the same group of 15 subjects, depending on the size of the group in the treatment. We repeat the whole procedure 4 times ($N = 5$), or 6 times ($N = 15$) generating 240 (120) decisions, as in our data from each of the $N = 5$ ($N = 15$) treatments. We then calculate the frequency of correct decisions, and consider that one data point for that treatment. We repeat the whole process 100,000 times and generate a distribution of the frequency with which the correct decision was reached.

A.5 The Random Dot Kinematogram

In a Random Dot Kinematogram (RDK), the perceptual stimulus consists of a number of dots being displayed on a screen. A proportion of these dots are determined to be signal dots, while the remaining are noise dots. Signal dots all move in a determined direction, while noise dots move at random according to an algorithm. The tasks consists in reporting the direction in which the signal dots are moving. This direction is called the coherent direction and the proportion of signal dots, the coherence, is the main factor in determining the difficulty of the task.

The task can be programmed in various ways, using a variety of parameters (e.g., color,

[38] If the subject was never an expert, the subject is dropped and another one is drawn.

Duration:	1 second
Directions:	Left/Right
Number of dots:	300
Background color:	Black
Color of dots:	White
Dot radius:	2 pixels
Dot movement per frame:	1 pixel
Aperture width:	600 pixels
Aperture height:	400 pixels
Signal selection:	Same
Noise type:	Random direction
Aperture shape:	Ellipse
Reinsertion:	Dots reappear randomly when hitting edge
Fixation cross:	No
Aperture border:	No
Coherence:	20% to 3% (according to treatment)

Table 8: Experiment 2: RDK parameters

duration, algorithm, number of dots). Research has been done to study the various effects of using different combinations of these parameters (Pilly and Seitz, 2009, Schütz et al., 2010). We take advantage of the recent development of a customizable version of the RDK (Rajananda et al., 2018) which can be implemented as a plugin in jsPsych. This version allows for the configuration of various parameters in order to adjust the task as desired by the researchers. We report in the table below the parameters that we used in our experiment. The reader can find details about how they affect the task in Rajananda et al., 2018 and in the following link: https://www.jspsych.org/6.3/plugins/jspsych-rdk/. It is important to emphasize again that our objective in using the RDK was not to study perception in itself, but rather to create a common task that is reasonably well controlled and calibrated and where, nevertheless, the information about the accuracy of the signals remains ambiguous.