

Optimal Urban Transportation Policy: Evidence from Chicago

Milena Almagro, Juan Camilo Castillo, Nathaniel Hickok, Felipe Kup, and Tobias Salz

Motivation

Some Facts about Public Transit in the US

The current state of public transit:

- Customers only pay about 24% of the trip cost directly through fares.

Source: newgeography.com

Motivation

Some Facts about Public Transit in the US

The current state of public transit:

- Customers only pay about 24% of the trip cost directly through fares.

Source: newgeography.com

- The average bus utilization rate is very low.

Source: DOT

Motivation

Some Facts about Public Transit in the US

The current state of public transit:

- Customers only pay about 24% of the trip cost directly through fares.

Source: newgeography.com

- The average bus utilization rate is very low.

Source: DOT

- Only 5% of commuting trips use public transit.

Source: Census.gov

Motivation

Some Facts about Public Transit in the US

The current state of public transit:

- Customers only pay about 24% of the trip cost directly through fares.

Source: newgeography.com

- The average bus utilization rate is very low.

Source: DOT

- Only 5% of commuting trips use public transit.

Source: Census.gov

- A private car emits about twice the amount of CO₂ per passenger mile as public transit.

Source: DOT

Motivation

Some Facts about Public Transit in the US

The current state of public transit:

- Customers only pay about 24% of the trip cost directly through fares.

Source: newgeography.com

- The average bus utilization rate is very low.

Source: DOT

- Only 5% of commuting trips use public transit.

Source: Census.gov

- A private car emits about twice the amount of CO₂ per passenger mile as public transit.

Source: DOT

→ Is current urban transit running **efficiently**?

Challenges in urban transportation policy

For a city government that operates and regulates urban transportation

People travel using different transportation modes

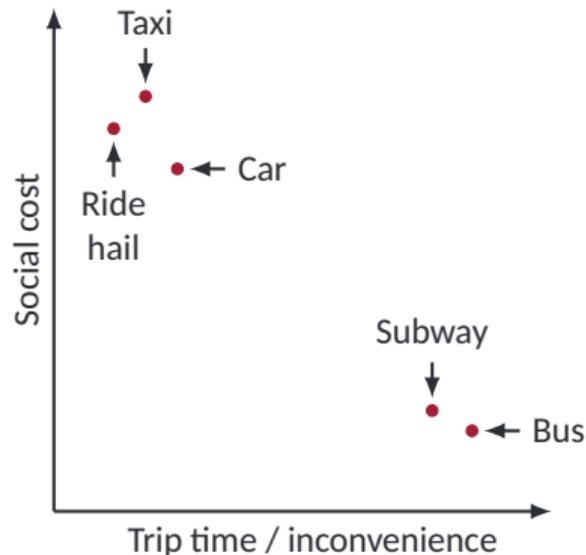
- Heterogeneous: car, bus, subway, taxi, ride-hailing

Challenges:

- Externalities: congestion, environmental, network
- Limited budget
- Distributive role of transportation

Policy levers:

- Prices and capacities of public transit
- Congestion pricing for private transit



Urban Transportation

Main focus of this project

City planner's problem:

- Maximize:

$$\text{Consumer Surplus} - \text{Costs} - \text{Externalities}$$

- Setting prices and capacities (frequencies) across modes → decentralized equilibrium

- Subject to budget constraint:

$$\text{Revenue} - \text{Costs} \leq \text{City Budget}$$

Accounting for:

- Heterogeneity across consumers and locations
- Response of private providers: taxi/ride-hailing drivers, (later: optimal response of ride-hailing platforms)

Scope: conditional on current infrastructure (short-run!)

This Project

Research Questions:

1. What are the optimal **prices** and **capacities/frequencies** of an urban transit system?
2. What are the welfare gains and distributional effects relative to the current state?

This Project

Research Questions:

1. What are the optimal **prices** and **capacities/frequencies** of an urban transit system?
2. What are the welfare gains and distributional effects relative to the current state?

Strategy:

- Model of a city planner
 - Discrete choice **mode demand** (McFadden 1974, Berry et al. 1995)
 - Transportation **technology**: cost, trip times, and congestion/network externalities
- Estimation: construct novel dataset for all relevant modes for **Chicago**
- Counterfactuals to determine optimal policy and measure welfare effects

Preview of Results

Preliminary!

The city should:

1. Lower price of public transit by $\sim 90\%$, decrease capacity by 20%-30%
2. Charge congestion/carbon tax on private cars $\sim \$0.25/\text{km}$

Tax achieves a larger welfare gain ($\sim \$6\text{M}/\text{week}$ vs $\sim \$2\text{M}/\text{week}$)

- Leads more people to switch
- But with huge, regressive decrease in CS ($\sim \$35\text{M}/\text{week}$)

Ride hailing prices only slightly higher than socially optimal

- Market power markup plays the role of a Pigouvian tax, no need for additional tax

Literature Review

1. **Transportation:** McFadden (1974), Small (1982), Small (2005), Parry and Small (2009)
2. **Spatial Equilibrium Models:**
 - Theory: Arnott (1996), Lagos (2003)
 - Empirics: Frechette et al. (2019), Buchholz (2021), Brancaccio et al. (2020), Arora et al. (2020), Castillo (2020), Buchholz et al. (2020), Cairncross et al. (2021), Rosaia (2021), Leccese (2021), Brancaccio et al. (2021), Kreindler (2022), Durrmeyer and Martínez (2022)
3. **Transportation in the long run:** Tsivanidis (2018), Allen and Arkolakis (2020), Barwick et al. (2021)
4. **Geo-location Data and Mobility:** Miyahuchi et al. (2020), Glaeser et al. (2020), Couture et al. (2021)

Literature Review

1. **Transportation:** McFadden (1974), Small (1982), Small (2005), Parry and Small (2009)
2. **Spatial Equilibrium Models:**
 - Theory: Arnott (1996), Lagos (2003)
 - Empirics: Frechette et al. (2019), Buchholz (2021), Brancaccio et al. (2020), Arora et al. (2020), Castillo (2020), Buchholz et al. (2020), Cairncross et al. (2021), Rosaia (2021), Leccese (2021), Brancaccio et al. (2021), Kreindler (2022), Durrmeyer and Martínez (2022)
3. **Transportation in the long run:** Tsivanidis (2018), Allen and Arkolakis (2020), Barwick et al. (2021)
4. **Geo-location Data and Mobility:** Miyahuchi et al. (2020), Glaeser et al. (2020), Couture et al. (2021)

⇒ **Budget constrained city with heterogeneous agents, optimal prices and capacities**

Roadmap

1. Data
2. Model and Estimation
 - 2.1 City Planer
 - 2.2 Demand
 - 2.3 Transportation Technology
3. Counterfactuals

Roadmap

1. Data

2. Model and Estimation

2.1 City Planer

2.2 Demand

2.3 Transportation Technology

3. Counterfactuals

Data Sources

Chicago, June 2019-February 2020.

Raw data sets:

1. Individual cell phone location records: 40% of all devices.
2. Universe of public transit trips through MIT-CTA partnership.
3. Universe of taxi and ride hailing (pooled + single rides) trips from the city of Chicago.
4. Block level census data.
5. 2019 Chicago transit survey for validation and calibration.

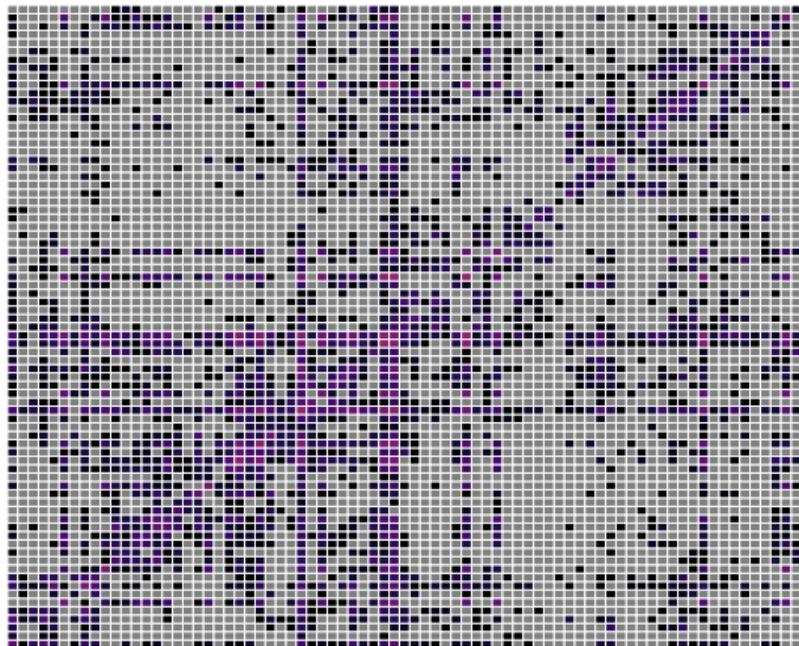
Goal: Hourly flows, prices, and travel times by mode across community areas + micro moments.

Car trips are identified as:

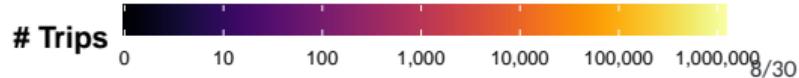
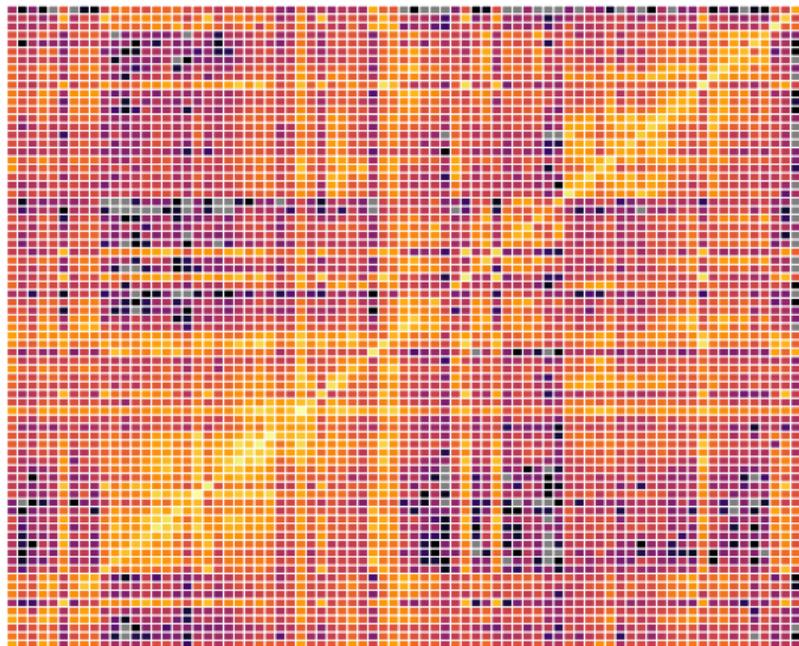
$$\text{Car Trips} = \underbrace{\text{Cell Phone Trips}}_{\text{Cell phone data}} - \underbrace{\text{Public Transit Trips} - \text{Ride-hailing Trips} - \text{Taxi Trips}}_{\text{CTA data}}$$

Combined vs. Survey Data: Flows Across Community Areas

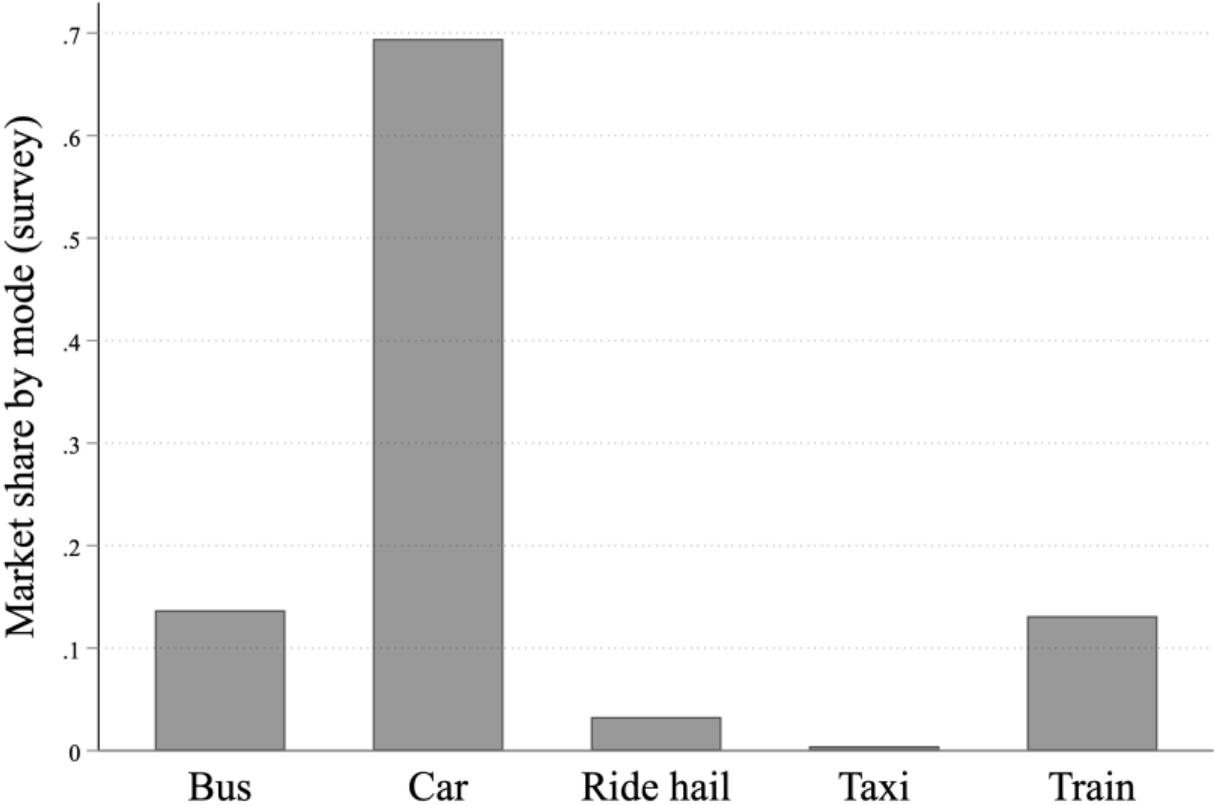
Heatmap, survey data



Heatmap, combined data

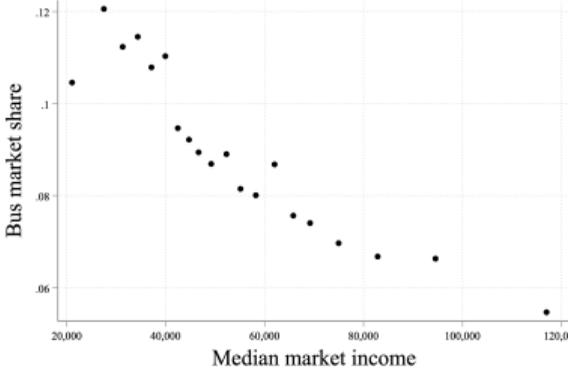
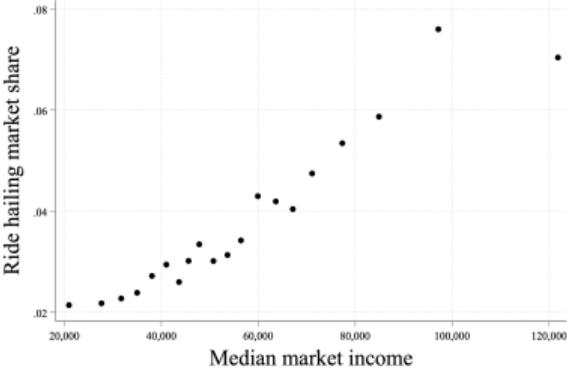
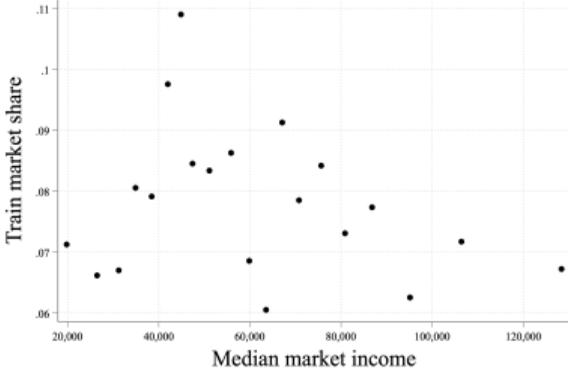
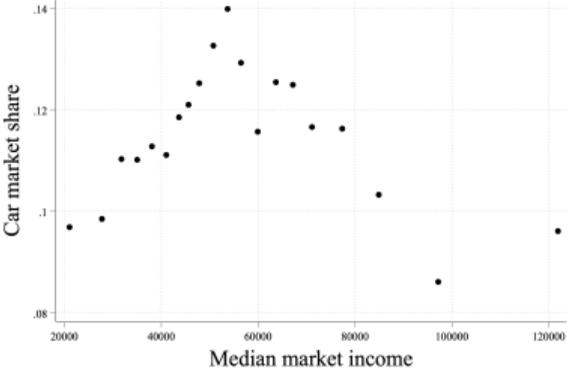


Mode Market Shares



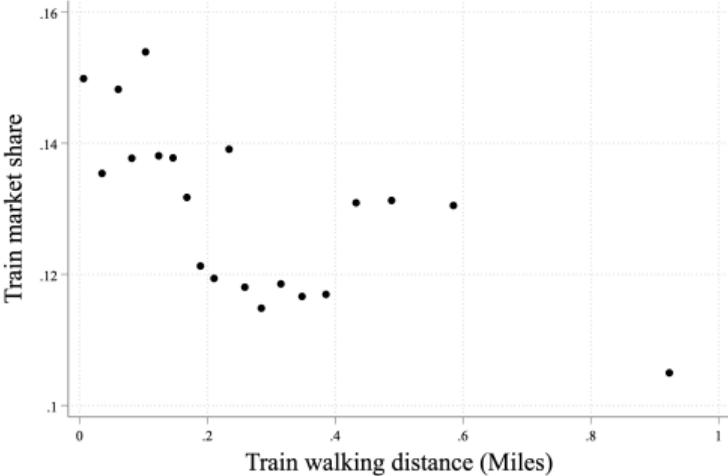
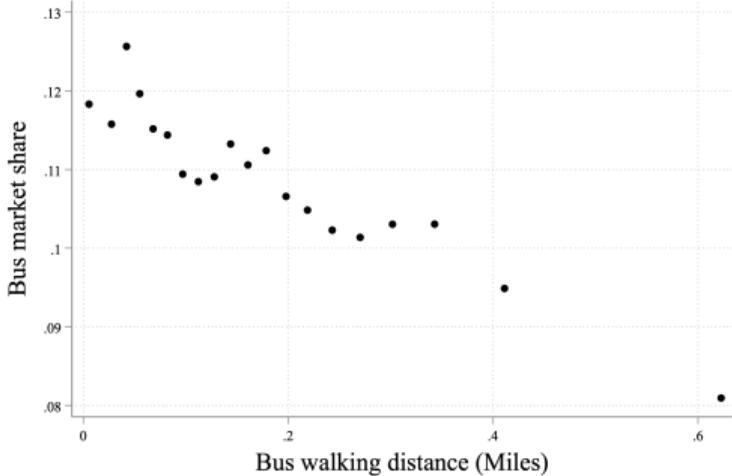
Heterogeneity across locations

Mode choice by income



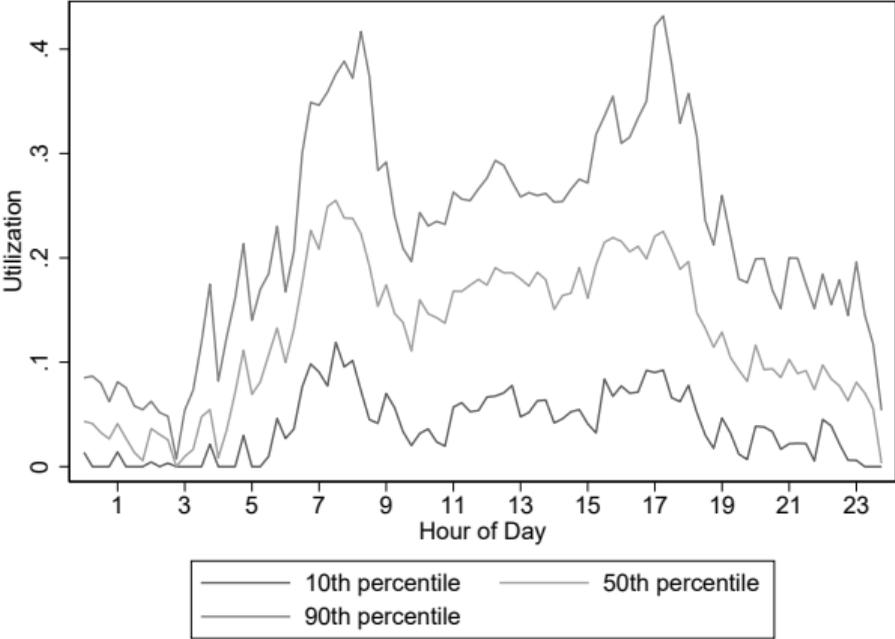
Heterogeneity across locations

Mode choice correlates with accessibility



Utilization of Buses during Weekdays

The city runs 129 bus routes with 1,864 buses.



Roadmap

1. Data

2. Model and Estimation

2.1 City Planer

2.2 Demand

2.3 Transportation Technology

3. Counterfactuals

Model

Outline

There are 3 parts:

1. Commuters make mode choices: (prices and times) \mapsto (# of people choosing each mode)
2. Transportation technology: (# of people choosing each mode) \mapsto (times and social cost)
3. City government: chooses prices and capacity of each mode, trades off welfare and budget (Ramsey)

Next few slides: model with one O-D pair, one time period, government-run modes

- Main empirical model: CA by CA by hour of the week, privately owned modes

Model

Outline

Demand for mode j : $D_j(\mathbf{p}, \mathbf{t})$

- p_j : price
- t_j : Total time (wait + travel time)

Model

Outline

Demand for mode j : $D_j(\mathbf{p}, \mathbf{t})$

- p_j : price
- t_j : Total time (wait + travel time)

Transportation technology j is described by three functions of quantity and capacity (\mathbf{q}, κ) :

1. $\tau_j(q_j, \kappa_j)$: Wait + travel times (congestion, bus/train routing, taxi/ride-hail matching)
2. $C(\mathbf{q}, \kappa)$: Cost (fuel, depreciation, labor)
3. $E(\mathbf{q}, \kappa)$: Environmental externalities

Model

Outline

Demand for mode j : $D_j(\mathbf{p}, \mathbf{t})$

- p_j : price
- t_j : Total time (wait + travel time)

Transportation technology j is described by three functions of quantity and capacity (\mathbf{q}, κ) :

1. $\tau_j(q_j, \kappa_j)$: Wait + travel times (congestion, bus/train routing, taxi/ride-hail matching)
2. $C(\mathbf{q}, \kappa)$: Cost (fuel, depreciation, labor)
3. $E(\mathbf{q}, \kappa)$: Environmental externalities

κ_j : Capacity of mode j , determines waiting times

- Taxis, ride-hailing, shared: Number of drivers working. More idle drivers \rightarrow lower times.
- Buses, trains: Route frequency. More buses \rightarrow lower times.

Model

City Government and Equilibrium

City government chooses prices and capacities (\mathbf{p}, κ) . Equilibrium (\mathbf{q}, \mathbf{t}) such that:

$$\underbrace{\mathbf{q} = \mathbf{D}(\mathbf{t}, \mathbf{p})}_{\text{demand}} \quad \text{and} \quad \underbrace{\mathbf{t} = \boldsymbol{\tau}(\mathbf{q}, \kappa)}_{\text{technology}}$$

- For any (\mathbf{q}, κ) , there is a unique $\mathbf{p}(\mathbf{q}, \kappa)$ that satisfies the equilibrium conditions.

Government's problem (Ramsey):

$$\max_{(\mathbf{q}, \kappa)} \underbrace{U(\mathbf{t}, \mathbf{p}) - \mathbf{p}(\mathbf{q}, \kappa) \cdot \mathbf{q}}_{\text{Consumer Surplus}} + \underbrace{\mathbf{p}(\mathbf{q}, \kappa) \cdot \mathbf{q} - C(\mathbf{q}, \kappa)}_{\text{Gov. revenue \setminus Profits}} - E(\mathbf{q}, \kappa) \quad \text{s.t.} \quad \underbrace{\mathbf{p}(\mathbf{q}, \kappa) \cdot \mathbf{q} - C(\mathbf{q}, \kappa)}_{\text{Budget constraint}} \geq B$$

Welfare

Optimal Pricing

Lagrangian for government's problem:

$$\max_{\mathbf{q}, \boldsymbol{\kappa}} U(\mathbf{q}, \boldsymbol{\kappa}) - C(\mathbf{q}, \boldsymbol{\kappa}) - E(\mathbf{q}, \boldsymbol{\kappa}) + \lambda \cdot \left\{ \sum_j p_j(\mathbf{q}, \boldsymbol{\kappa}) \cdot q_j - C(\mathbf{q}, \boldsymbol{\kappa}) - B \right\}$$

First-order conditions

1. Quantity:

$$p_j = \frac{\partial C}{\partial q_j} - \frac{1}{1+\lambda} \cdot \frac{\partial U}{\partial t_j} \cdot \frac{\partial t_j}{\partial q_j} + \frac{1}{1+\lambda} \cdot \frac{\partial E}{\partial q_j} - \frac{\lambda}{1+\lambda} \cdot \sum_k q_k \cdot \frac{\partial p_k}{\partial q_j}$$

2. Capacity:

$$\frac{\partial C}{\partial \kappa_j} = \frac{1}{1+\lambda} \cdot \frac{\partial U}{\partial t_j} \cdot \frac{\partial t_j}{\partial \kappa_j} - \frac{1}{1+\lambda} \cdot \frac{\partial E}{\partial \kappa_j} + \frac{\lambda}{1+\lambda} \cdot \sum_k q_k \cdot \frac{\partial p_k}{\partial \kappa_j}$$

Optimal Pricing

$$p_j - \underbrace{\tilde{C}_j}_{\text{Mg. cost}} = \underbrace{\tilde{E}_j}_{\text{Mg. environ. externality}} - \sum_k \underbrace{\bar{u}_k^T}_{\text{Utility of time}} \cdot \underbrace{\tilde{T}_{kj}}_{\text{Network effects}} + \frac{\lambda}{1 + \lambda} \cdot \left\{ -\tilde{E}_j - \underbrace{\sum_{k \in J} q_k \cdot \Omega_{kj}}_{\text{Market power distortion}} - \underbrace{\sum_k (\tilde{u}_k^T - \bar{u}_k^T) \cdot \tilde{T}_{kj}}_{\text{Spence distortion}} \right\}$$

Optimal Pricing

$$p_j - \underbrace{\tilde{C}_j}_{\text{Mg. cost}} = \underbrace{\tilde{E}_j}_{\text{Mg. environ. externality}} - \sum_k \underbrace{\bar{u}_k^T}_{\text{Utility of time}} \cdot \underbrace{\tilde{T}_{kj}}_{\text{Network effects}} + \frac{\lambda}{1+\lambda} \cdot \left\{ -\tilde{E}_j - \underbrace{\sum_{k \in J} q_k \cdot \Omega_{kj}}_{\text{Market power distortion}} - \underbrace{\sum_k (\tilde{u}_k^T - \bar{u}_k^T) \cdot \tilde{T}_{kj}}_{\text{Spence distortion}} \right\}$$

Notation:

- $\tilde{C}_j = \frac{\partial C_j}{\partial q_j} + \frac{\partial C}{\partial \kappa_j} \frac{\kappa_j}{q_j}$: Mg. cost of a trip, keeping capacity per trip $\frac{\kappa_j}{q_j}$ constant
- $\tilde{E}_j = \frac{\partial E}{\partial q_j} + \frac{\partial E_j}{\partial \kappa_j} \frac{\kappa_j}{q_j}$: Mg. externality of a trip, keeping capacity per trip $\frac{\kappa_j}{q_j}$ constant

⇒ “Standard” **Pigouvian tax/subsidy**

Optimal Pricing

$$p_j - \underbrace{\tilde{C}_j}_{\text{Mg. cost}} = \underbrace{\tilde{E}_j}_{\text{Mg. environ. externality}} - \sum_k \underbrace{\bar{u}_k^T}_{\text{Utility of time}} \cdot \underbrace{\tilde{T}_{kj}}_{\text{Network effects}} + \frac{\lambda}{1+\lambda} \cdot \left\{ -\tilde{E}_j - \underbrace{\sum_{k \in J} q_k \cdot \Omega_{kj}}_{\text{Market power distortion}} - \underbrace{\sum_k (\tilde{u}_k^T - \bar{u}_k^T) \cdot \tilde{T}_{kj}}_{\text{Spence distortion}} \right\}$$

Notation:

- $\bar{u}_j^T = \frac{\partial CS}{\partial t_j}$: How does CS change if the time of mode j increases?
- $\tilde{T}_{kj} = \frac{\partial T_j}{\partial q_k} + \frac{\partial T_j}{\partial \kappa_k} \frac{\kappa_k}{q_k}$: How does mode- j time change if mode- j trips and capacity increase? Two effects:
 1. Congestion: negative externality
 2. Returns to scale: positive externality (Arnott, 1996)

⇒ **Tax** modes that cause congestion, **subsidize** modes with increasing returns to scale

Optimal Pricing

$$p_j - \underbrace{\tilde{C}_j}_{\text{Mg. cost}} = \underbrace{\tilde{E}_j}_{\text{Mg. environ. externality}} - \sum_k \underbrace{\bar{u}_k^T}_{\text{Utility of time}} \cdot \underbrace{\tilde{T}_{kj}}_{\text{Network effects}} + \frac{\lambda}{1+\lambda} \cdot \left\{ -\tilde{E}_j - \underbrace{\sum_{k \in J} q_k \cdot \Omega_{kj}}_{\text{Market power distortion}} - \underbrace{\sum_k (\tilde{u}_k^T - \bar{u}_k^T) \cdot \tilde{T}_{kj}}_{\text{Spence distortion}} \right\}$$

Notation:

- Ω : Inverse Jacobian of demand (\sim inverse price elasticities)
- \tilde{u}_j^T : If t_j increases by 1%, how much does utility of *marginal* commuters change?

Optimal Pricing

$$p_j - \underbrace{\tilde{C}_j}_{\text{Mg. cost}} = \underbrace{\tilde{E}_j}_{\text{Mg. environ. externality}} - \sum_k \underbrace{\bar{u}_k^T}_{\text{Utility of time}} \cdot \underbrace{\tilde{T}_{kj}}_{\text{Network effects}} + \frac{\lambda}{1+\lambda} \cdot \left\{ \underbrace{-\tilde{E}_j - \sum_{k \in J} q_k \cdot \Omega_{kj}}_{\text{Market power distortion}} - \underbrace{\sum_k (\tilde{u}_k^T - \bar{u}_k^T) \cdot \tilde{T}_{kj}}_{\text{Spence distortion}} \right\}$$

Notation:

- Ω : Inverse Jacobian of demand (\sim inverse price elasticities)
- \tilde{u}_j^T : If t_j increases by 1%, how much does utility of *marginal* commuters change on avg.?

Government cares about **budget**, so it behaves like a **monopolist** to some extent:

- Underweights externalities
- Market power markup: higher price for less elastic modes
- Caters to marginal rather than average consumer

Roadmap

1. Data

2. Model and Estimation

2.1 City Planer

2.2 Demand

2.3 Transportation Technology

3. Counterfactuals

Demand

Market Definition:

- m = community area a to a' during hour h
- Exogenous arrival rate λ_m of travelers

Choices:

$$j^* = \operatorname{argmax}_{j \in \mathcal{J}_i \cup \{0\}} u_{mj}^i$$

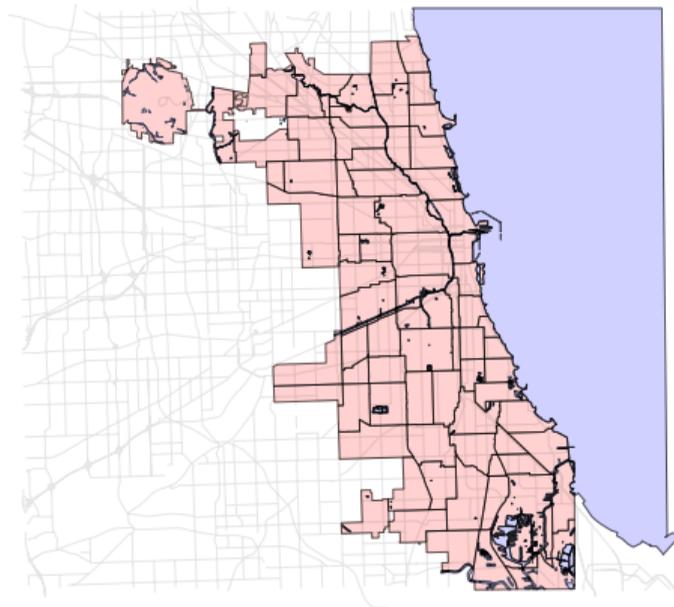
Utility of agent i :

$$u_{mj}^i = \xi_{mj} - \alpha_T^i \cdot T_{ij} - \alpha_p^i \cdot p_{ij} + \epsilon_j^i,$$

Estimation: Instrument prices with inflow of trips

1. Value of time: $\sim \$75/\text{h}$ on avg, higher for high income
2. Taxi/ride hailing elasticities ~ 2

Chicago Community Areas



Roadmap

1. Data

2. Model and Estimation

2.1 City Planer

2.2 Demand

2.3 Transportation Technology

3. Counterfactuals

Transportation technology

Three parts:

1. Congestion
2. Public transit (bus/subway)
3. Private transit (taxis/ride hailing)

Congestion

Given vector q of all trips, what is the travel time from a to a' using mode j ?

Approach: Model city as directed graph of community areas

- Edges e are adjacent community areas
- Agents in market m take a route r_m , a sequence of edges

During hour h , the congestion on edge e is

$$Q_{eh} = \sum_j \sum_{r \text{ s.t. } e \in r} \overbrace{b_j}^{\text{Congestion caused by mode } j} q_{rhj}$$

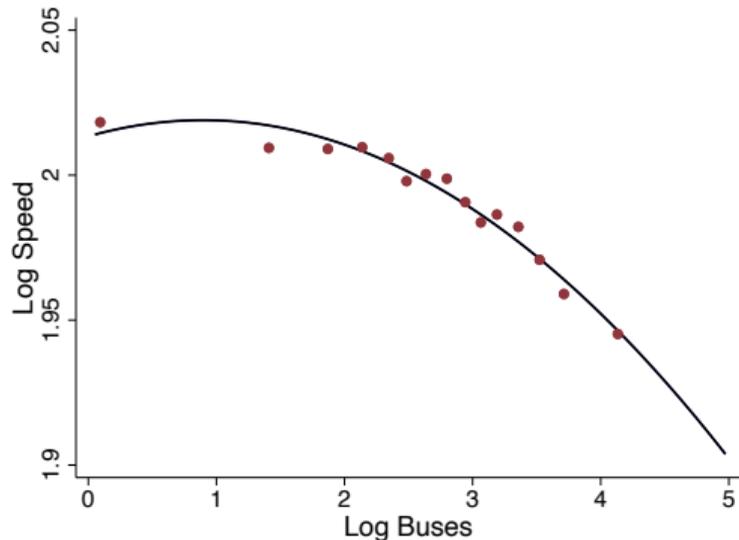
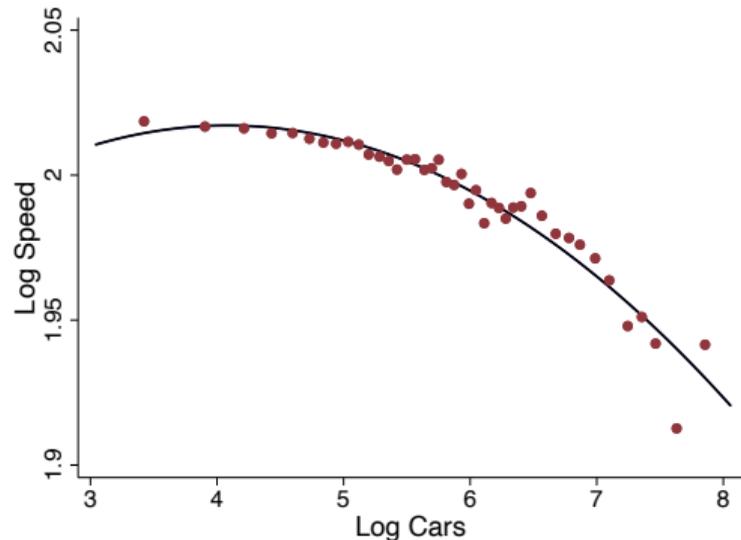
Travel times:

$$\text{By edge: } \tau_{ehj} = \overbrace{A_{ej}}^{\text{"Fixed effect"}} \overbrace{Q_{eh}^{\alpha_j}}^{\text{Mode } j \text{ congestion elast.}}$$
$$\text{By market: } T_{mj}^{\text{travel}} = \sum_{e \in r_{mj}} \tau_{ehj}$$

Congestion estimation

Data: travel times and vehicle flows between adjacent CAs at the hourly level

Binscatters with o-d pair fixed effects:



We calibrate congestion elasticities based on this data

- Later: capture nonlinearity with more flexible functional form

Public transit (bus/subway)

Agents in market $m = (a, a', h)$ take a bus/train route R_m

Total time for agent i in market $m = (a, a', h)$, who takes route R :

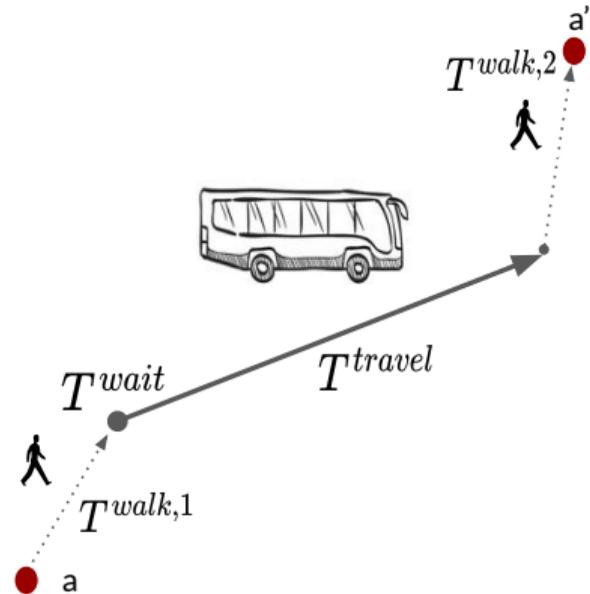
$$T_i = T_i^{\text{walk},1} + \underbrace{T_{Rh}^{\text{wait}}}_{\text{See below}} + \underbrace{T_{Rh}^{\text{travel}}}_{\text{Congestion}} + T_i^{\text{walk},2}$$

Let κ_{Rh} be the frequency of route R . Then the waiting time is

$$T_{Rh}^{\text{wait}} = \frac{1}{\kappa_{Rh}}$$

Cost and externality:

$$C_{Rh} = c_R \kappa_{Rh} \quad E_{Rh} = e_R \kappa_{Rh}$$



Private transit (taxis and ride hailing)

Total time in private mode j in market $m = (a, a', h)$ is $T_{mj}^{\text{wait}} + T_{mj}^{\text{travel}}$.

$\underbrace{T_{mj}^{\text{wait}}}_{\text{See below}} + \underbrace{T_{mj}^{\text{travel}}}_{\text{Congestion}}$

Waiting time depends on L_{ahj} , the number of available drivers at the origin a :

$$T_{mj}^{\text{wait}} = A_{aj} L_{ahj}^{-\phi_j}$$

The distribution of available drivers arises from a model of driver movements (Buchholz, 2021; Rosaia, 2020)

- There is a total number of drivers κ_{hj}
- Travelers who chose mode j matched to drivers, who later become available at drop off
- Available drivers relocate \rightarrow tendency to move towards high earnings neighborhoods

Roadmap

1. Data

2. Model and Estimation

2.1 City Planer

2.2 Demand

2.3 Transportation Technology

3. Counterfactuals

Counterfactuals

Benchmark: Status quo

Counterfactuals: City government maximizes welfare by adjusting:

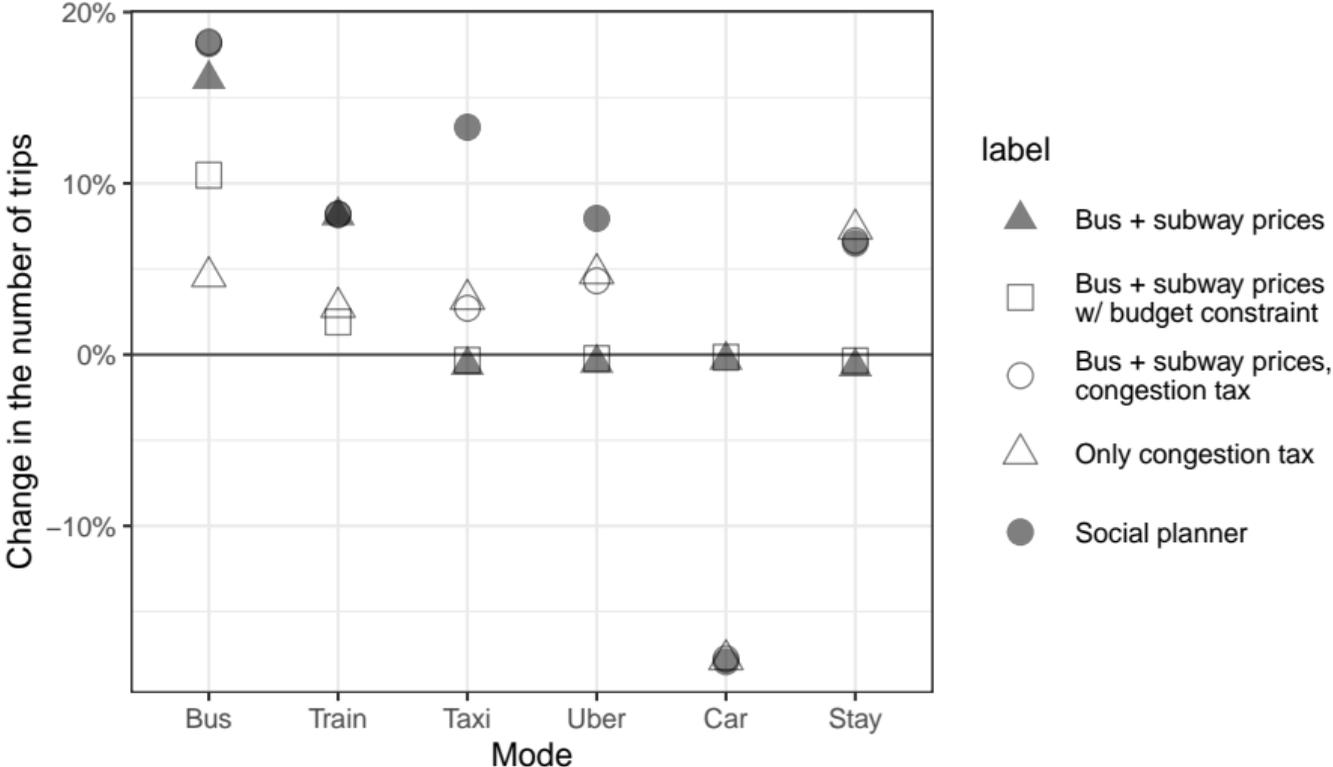
1. *Transit:* Prices and frequencies of public transit
2. *Tax:* Congestion tax
3. *Transit + tax:* Prices and frequencies of public transit + congestion tax
4. *Social planner:* Prices and frequencies of public transit + congestion charge + price of Uber and taxis

We consider alternatives with and without budget constraint

Counterfactual Results

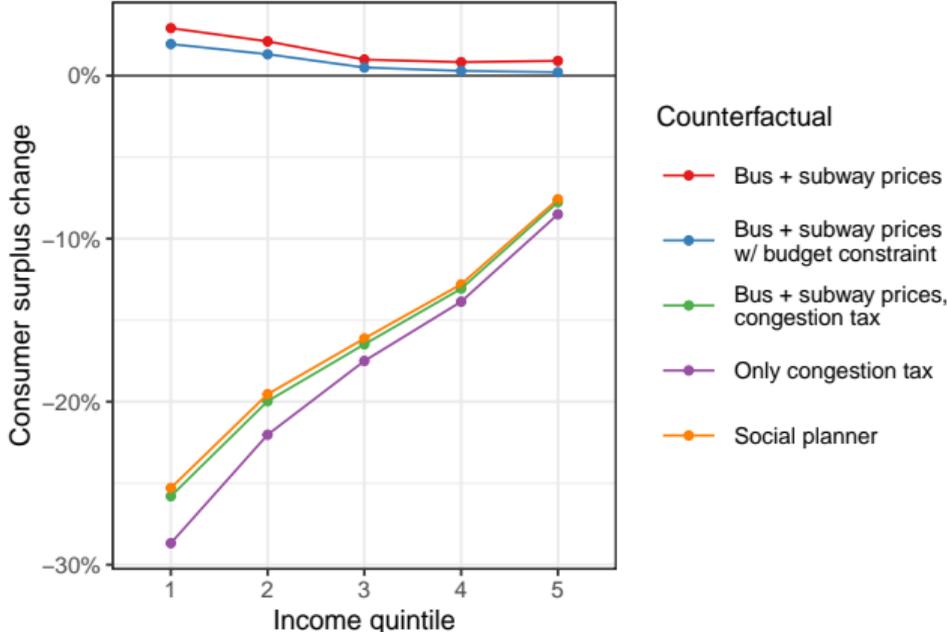
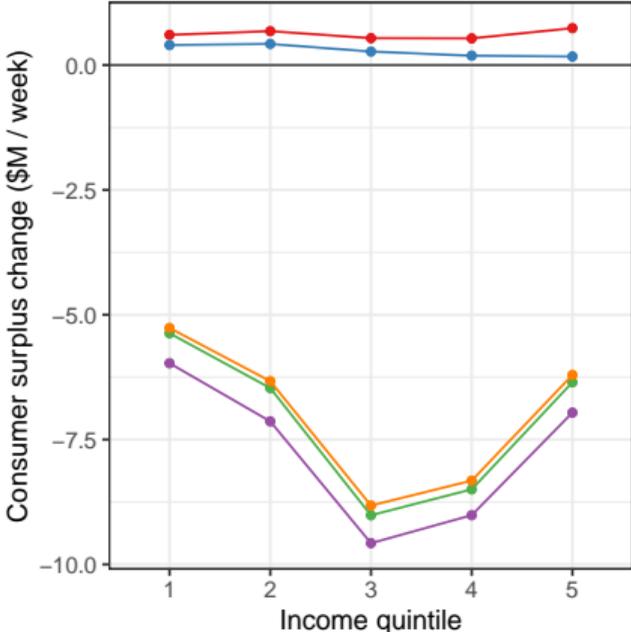
Status quo	Bus + subway	subway prices w/ budget	subway prices, congestion	subway prices, congestion	Only congest	Social planner	variable
0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	-5.2%	uber_price_change
0.0%	-104.5%	-88.4%	-93.4%	-83.1%	0.0%	-93.4%	bus_price_change
0.0%	-107.6%	-91.0%	-96.4%	-86.6%	0.0%	-96.4%	train_price_change
0.0%	0.0%	0.0%	26.0%	26.1%	26.4%	26.0%	car_surcharge
0.0%	-23.7%	-25.8%	-23.3%	-24.3%	0.0%	-23.3%	bus_capacity_change
0.0%	-27.9%	-30.1%	-27.8%	-29.1%	0.0%	-27.8%	train_capacity_change
0.000	2.065	2.035	7.960	7.948	6.085	8.289	welfare_change
0.000	3.105	1.462	-35.701	-36.749	-38.652	-34.940	CS_change
-13.045	-14.616	-13.070	-13.852	-12.924	-12.815	-13.852	profit_city
0.000	0.000	0.000	37.361	37.421	37.842	37.248	taxes
26.080	25.472	25.452	20.567	20.547	21.087	20.646	externalities

Substitution patterns



Distributional effects

Changes in consumer surplus relative to status quo



- Counterfactual
- Bus + subway prices
 - Bus + subway prices w/ budget constraint
 - Bus + subway prices, congestion tax
 - Only congestion tax
 - Social planner

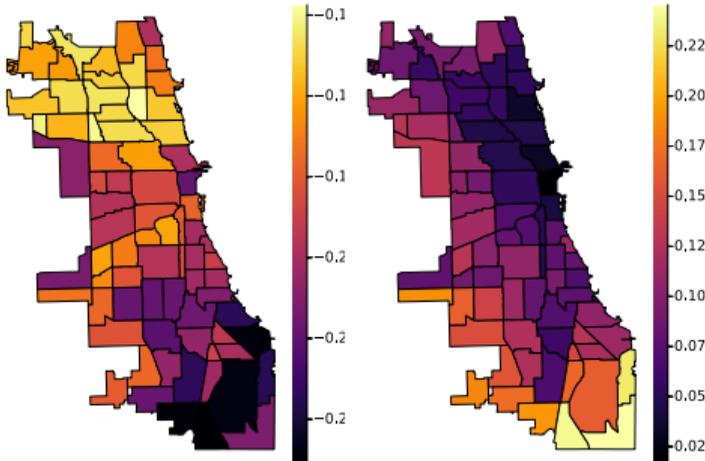
Distributional effects

Changes in consumer surplus relative to status quo

Congestion price

Δ car trips

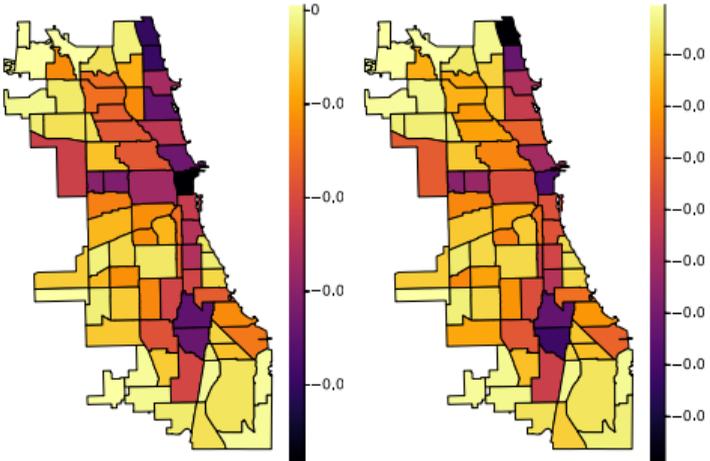
Δ stay



Public transit prices + capacity

Δ car trips

Δ stay



Future Steps

Much Work Ahead!

Refine estimation:

- Add demand heterogeneity
 - Car ownership as choice set variation
 - Include micro variation in access
- Flexible congestion and driver supply models

Exploit fine spatial resolution

- More sophisticated policy levers
- More results on heterogeneity

Additional counterfactuals

- Endogenous response by ride hailing platforms

Conclusions

- Construct new dataset of all trips across all modes for a major urban area.
- Compute optimal transit **prices**, **capacities**, and **subsidies** with government budget constraints.
- Preliminary results:
 - Public transit should be further subsidized and its frequency should be reduced
 - Congestion/carbon taxes → large welfare gains, large CS decrease
 - Markup of ride hailing platforms serves as a Pigouvian tax

Thank you!

Supply

Parametrizing Pooling Technology

Total trip time:

$$T = \underbrace{w(l)}_{\text{Waiting time}} + \underbrace{0.5 \cdot \tau}_{\text{Batch Length}} + \underbrace{T^0}_{\text{Direct time}} + \underbrace{P(q)}_{\text{Match prob.}} \cdot \underbrace{d(q)}_{\text{Expected detour}},$$

where l is the number of idle drivers, $w(l)$ and $d(q)$ are decreasing, $P(q)$ is increasing.

Supply

Parametrizing Pooling Technology

Total trip time:

$$T = \underbrace{w(l)}_{\text{Waiting time}} + \underbrace{0.5 \cdot \tau}_{\text{Batch Length}} + \underbrace{T^0}_{\text{Direct time}} + \underbrace{P(q)}_{\text{Match prob.}} \cdot \underbrace{d(q)}_{\text{Expected detour}},$$

where l is the number of idle drivers, $w(l)$ and $d(q)$ are decreasing, $P(q)$ is increasing.

Suppose that density of **trips with detour distance** x is proportional to

$$\underbrace{\alpha}_{\text{Efficiency}} \cdot \underbrace{q}_{\text{Requests}} \cdot \underbrace{\tau}_{\text{Batch length}} \cdot \frac{1}{\eta} \cdot x^{\underbrace{\eta-1}_{\text{Skewness}}},$$

then $x \sim$ **Weibull** (also shortest path btn. vertices on random graph; Bauckhage et al., 2013).

Supply

Parametrizing Pooling Technology

Total trip time:

$$T = \underbrace{w(l)}_{\text{Waiting time}} + \underbrace{0.5 \cdot \tau}_{\text{Batch Length}} + \underbrace{T^0}_{\text{Direct time}} + \underbrace{P(q)}_{\text{Match prob.}} \cdot \underbrace{d(q)}_{\text{Expected detour}},$$

where l is the number of idle drivers, $w(l)$ and $d(q)$ are decreasing, $P(q)$ is increasing.

Suppose that density of **trips with detour distance** x is proportional to

$$\underbrace{\alpha}_{\text{Efficiency}} \cdot \underbrace{q}_{\text{Requests}} \cdot \underbrace{\tau}_{\text{Batch length}} \cdot \frac{1}{\eta} \cdot x^{\underbrace{\eta-1}_{\text{Skewness}}},$$

then $x \sim$ **Weibull** (also shortest path btn. vertices on random graph; Bauckhage et al., 2013).

- Only match if detour $< \bar{x}$.
- $P(q)$ is the CDF of the shortest detour $1 - \exp(-\alpha \cdot q \cdot \tau \cdot \bar{x}^\eta)$.
- Expected detour $d(\bar{x}, \alpha, q, \tau)$: mean of detour, truncated at \bar{x}

Supply Utilization

How much **driver time** does a trip take on average?

$$T^B = \underbrace{(1 - F(\bar{x}; \alpha, q, \tau))}_{\text{Prob. of no match}} \cdot \underbrace{T^0}_{\text{Busy time if unmatched}} + \underbrace{F(\bar{x}; \alpha, q, \tau)}_{\text{Match prob.}} \cdot \underbrace{\frac{1}{2} \cdot (T^0 + d(\bar{x}, \alpha, q, \tau))}_{\text{Busy time if matched}}$$

The number of idle drivers is:

$$I = L - q \cdot T^B.$$

Waiting time (distance to the closest driver):

- Avg. waiting time is $w(I) = \frac{\rho}{\sqrt{I}}$ (Arnott 1996).

Additional **assumptions**:

- Optimal match radius depends on density: $\bar{x}(q) = \delta \cdot q^{-\beta}$
- Batch length τ fixed at one minute.
- Set ρ so that avg. ride-hailing time is 4 min

Supply

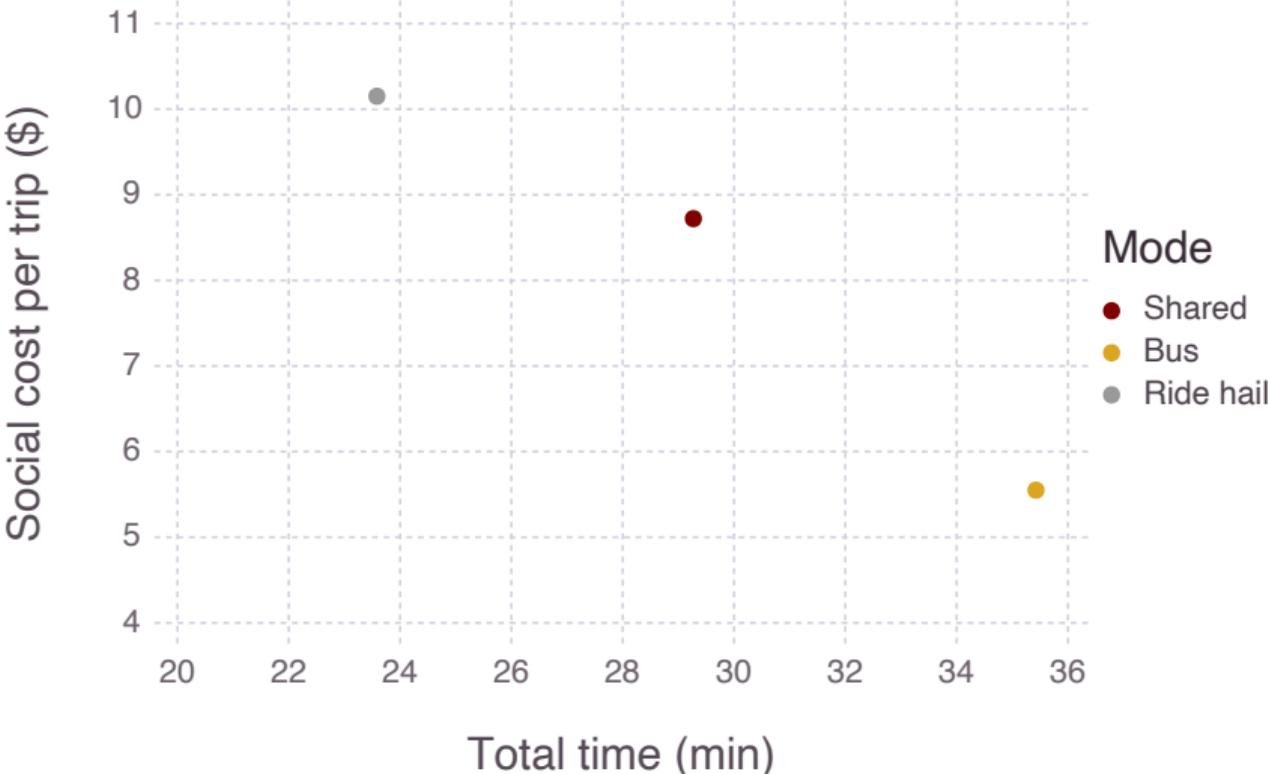
From model to the data

Given parameters $\alpha, \eta, \beta, \delta$, we can construct

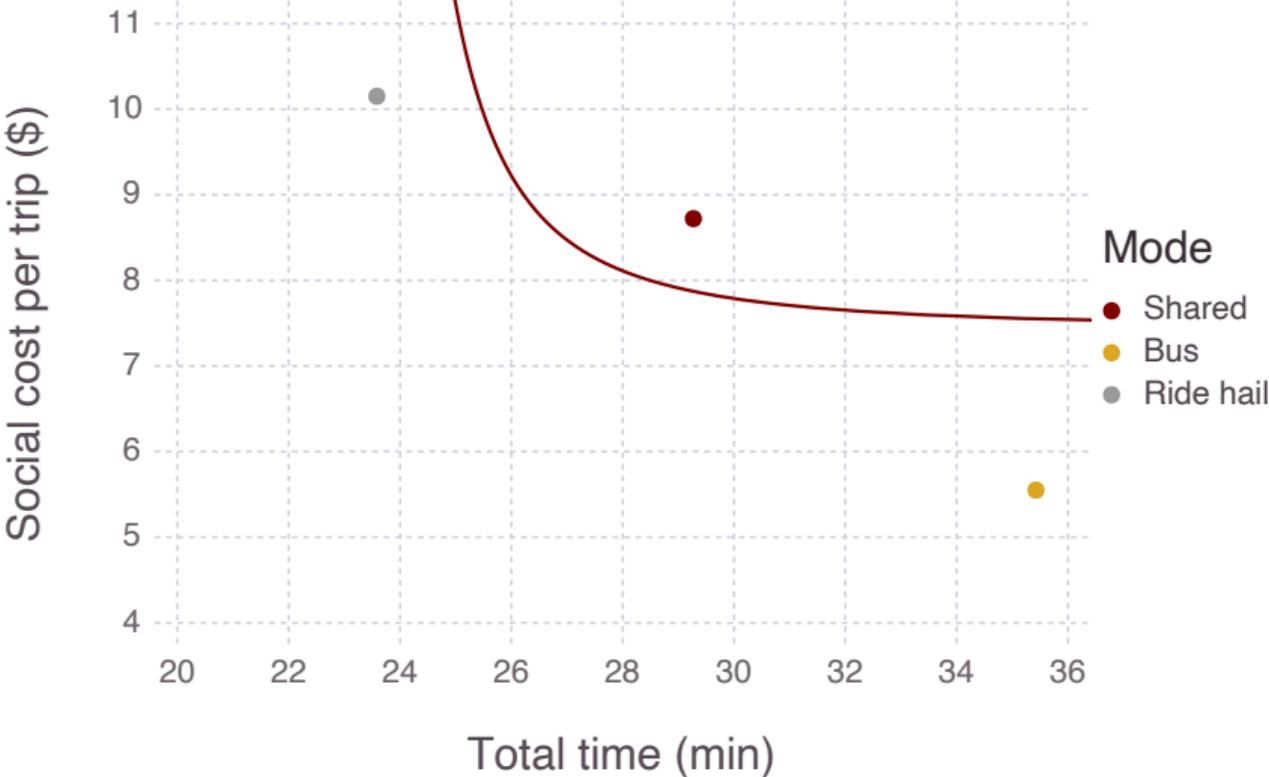
- $P(q)$ of being pooled $1 - \exp(-\alpha \cdot q \cdot \tau \cdot \bar{x}^\eta)$.
- $E(q)$ Expected detour $d(\bar{x}, \alpha, q, \tau)$

\implies Both functions $P(q)$ and $E(q)$ can be directly constructed from the data!

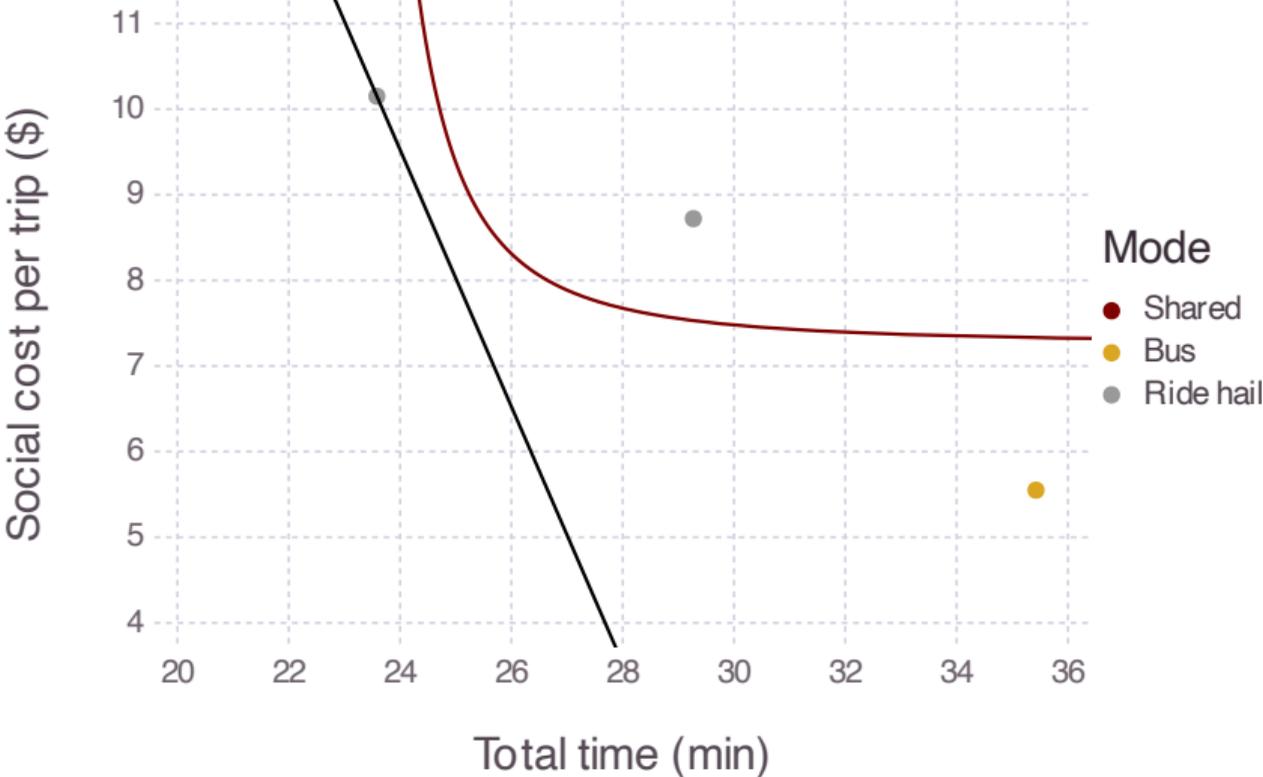
Improving current technologies



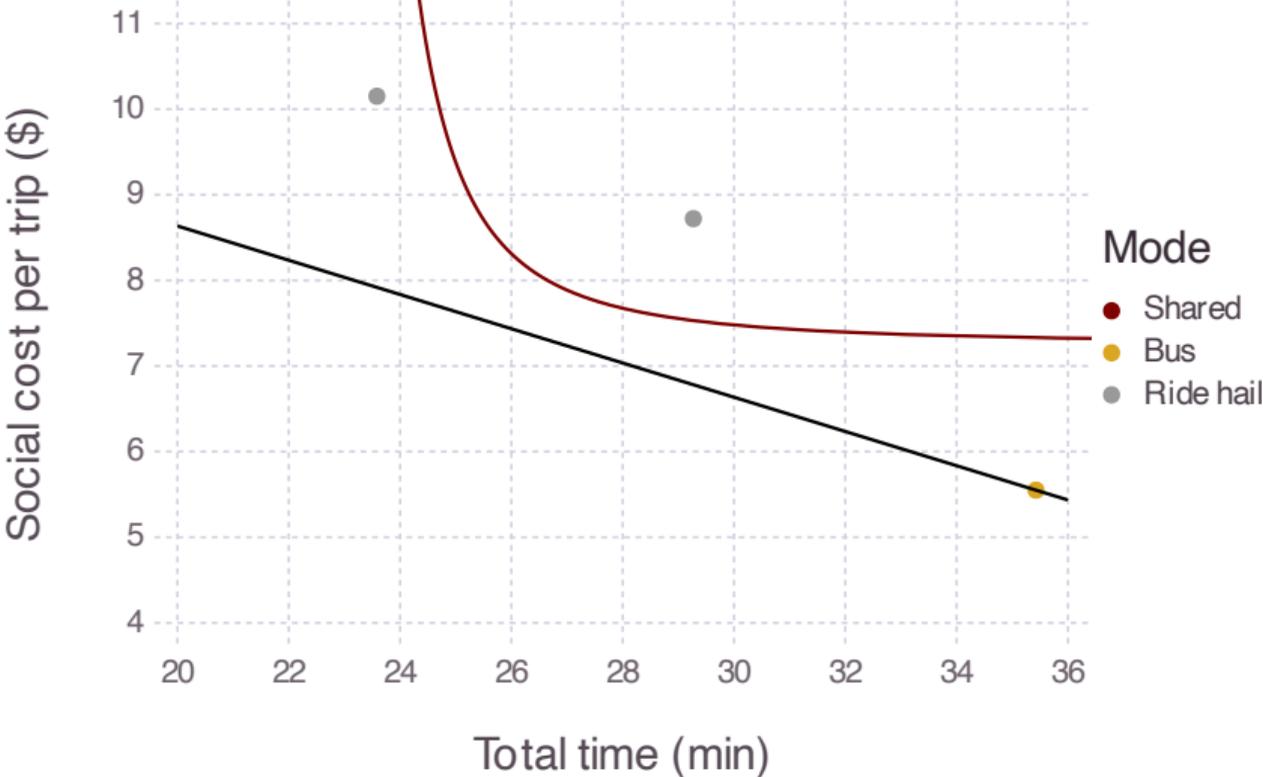
Improving current technologies



Improving current technologies



Improving current technologies



Improving current technologies

