Optimal Urban Transportation Policy: Evidence from Chicago

Milena Almagro, Juan Camilo Castillo, Nathaniel Hickok, Felipe Kup, and Tobias Salz
Motivation
Some Facts about Public Transit in the US

The current state of public transit:

- Customers only pay about 24% of the trip cost directly through fares.
  Source: newgeography.com
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→ Is current urban transit running **efficiently**?
Challenges in urban transportation policy

For a city government that operates and regulates urban transportation

People travel using different transportation modes

− Heterogeneous: car, bus, subway, taxi, ride-hailing

Challenges:

− Externalities: congestion, environmental, network
− Limited budget
− Distributive role of transportation

Policy levers:

− Prices and capacities of public transit
− Congestion pricing for private transit
Urban Transportation

Main focus of this project

City planner’s problem:

- Maximize:
  
  Consumer Surplus – Costs – Externalities

- Setting prices and capacities (frequencies) across modes → decentralized equilibrium

- Subject to budget constraint:
  
  Revenue – Costs ≤ City Budget

Accounting for:

- Heterogeneity across consumers and locations

- Response of private providers: taxi/ride-hailing drivers, (later: optimal response of ride-hailing platforms)

Scope: conditional on current infrastructure (short-run!)
This Project

Research Questions:

1. What are the optimal prices and capacities/frequencies of an urban transit system?
2. What are the welfare gains and distributional effects relative to the current state?
This Project

Research Questions:

1. What are the optimal *prices* and *capacities/frequencies* of an urban transit system?
2. What are the welfare gains and distributional effects relative to the current state?

Strategy:

- Model of a city planner
  - Discrete choice *mode demand* (McFadden 1974, Berry et al. 1995)
  - Transportation *technology*: cost, trip times, and congestion/network externalities
- Estimation: construct novel dataset for all relevant modes for *Chicago*
- Counterfactuals to determine optimal policy and measure welfare effects
Preview of Results

Preliminary!

The city should:

1. Lower price of public transit by $\sim 90\%$, decrease capacity by 20%-30%
2. Charge congestion/carbon tax on private cars $\sim 0.25$/km

Tax achieves a larger welfare gain ($\sim 6$/M/week vs $\sim 2$/M/week)
   - Leads more people to switch
   - But with huge, regressive decrease in CS ($\sim 35$/M/week)

Ride hailing prices only slightly higher than socially optimal
   - Market power markup plays the role of a Pigouvian tax, no need for additional tax
Literature Review


2. **Spatial Equilibrium Models**:
   - **Empirics**: Frechète et al. (2019), Buchholz (2021), Brancaccio et al. (2020), Arora et al. (2020), Castillo (2020), Buchholz et al. (2020), Cairncross et al. (2021), Rosaia (2021), Leccese (2021), Brancaccio et al. (2021), Kreindler (2022), Durrmeyer and Martínez (2022)

3. **Transportation in the long run**: Tsivanidis (2018), Allen and Arkolakis (2020), Barwick et al. (2021)

4. **Geo-location Data and Mobility**: Miyahuchi et al. (2020), Glaeser et al. (2020), Couture et al. (2021)

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⇒ Budget constrained city with heterogeneous agents, optimal prices and capacities
Roadmap

1. Data

2. Model and Estimation
   2.1 City Planer
   2.2 Demand
   2.3 Transportation Technology

3. Counterfactuals
Roadmap

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Data Sources


Raw data sets:

1. Individual cell phone location records: 40% of all devices.
2. Universe of public transit trips through MIT-CTA partnership.
3. Universe of taxi and ride hailing (pooled + single rides) trips from the city of Chicago.
4. Block level census data.
5. 2019 Chicago transit survey for validation and calibration.

Goal: Hourly flows, prices, and travel times by mode across community areas + micro moments.

Car trips are identified as:

\[
\text{Car Trips} = \text{Cell Phone Trips} - \text{Public Transit Trips} - \text{Ride-hailing Trips} - \text{Taxi Trips}
\]

Cell phone data

CTA data
Combined vs. Survey Data: Flows Across Community Areas

Heatmap, survey data

Heatmap, combined data
Mode Market Shares

Market share by mode (survey)

- Car: 0.7
- Bus: 0.1
- Ride hail: 0.05
- Taxi: 0.01
- Train: 0.1
Heterogeneity across locations

Mode choice by income
Heterogeneity across locations

Mode choice correlates with accessibility
Utilization of Buses during Weekdays

The city runs 129 bus routes with 1,864 buses.
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Model

Outline

There are 3 parts:

1. Commuters make mode choices: \((\text{prices and times}) \mapsto (\# \text{ of people choosing each mode})\)

2. Transportation technology: \((\# \text{ of people choosing each mode}) \mapsto (\text{times and social cost})\)

3. City government: chooses prices and capacity of each mode, trades off welfare and budget (Ramsey)

Next few slides: model with one O-D pair, one time period, government-run modes

- Main empirical model: CA by CA by hour of the week, privately owned modes
Model

Outline

Demand for mode $j$: $D_j(p, t)$

- $p_j$: price
- $t_j$: Total time (wait + travel time)
Model

Outline

Demand for mode \( j \): \( D_j(p, t) \)
- \( p_j \): price
- \( t_j \): Total time (wait + travel time)

Transportation technology \( j \) is described by three functions of quantity and capacity (\( q, \kappa \)):

1. \( \tau_j(q_j, \kappa_j) \): Wait + travel times (congestion, bus/train routing, taxi/ride-hail matching)
2. \( C(q, \kappa) \): Cost (fuel, depreciation, labor)
3. \( E(q, \kappa) \): Environmental externalities
Model

Outline

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- $t_j$: Total time (wait + travel time)

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3. $E(q, \kappa)$: Environmental externalities

$\kappa_j$: Capacity of mode $j$, determines waiting times

- Taxis, ride-hailing, shared: Number of drivers working. More idle drivers $\rightarrow$ lower times.
- Buses, trains: Route frequency. More buses $\rightarrow$ lower times.
Model
City Government and Equilibrium

City government chooses prices and capacities \((p, \kappa)\). Equilibrium \((q, t)\) such that:

\[
q = D(t, p) \quad \text{and} \quad t = \tau(q, \kappa)
\]

- For any \((q, \kappa)\), there is a unique \(p(q, \kappa)\) that satisfies the equilibrium conditions.

Government’s problem (Ramsey):

\[
\max_{(q, \kappa)} \begin{cases} \text{Consumer Surplus} & \frac{U(t, p)}{} - p(q, \kappa) \cdot q + p(q, \kappa) \cdot q - C(q, \kappa) - E(q, \kappa) \\ \text{Gov. revenue \ Profits} & \text{Welfare} \end{cases} \quad \text{s.t.} \quad p(q, \kappa) \cdot q - C(q, \kappa) \geq B
\]
Optimal Pricing

Lagrangian for government’s problem:

\[
\max_{q,\kappa} U(q, \kappa) - C(q, \kappa) - E(q, \kappa) + \lambda \cdot \left\{ \sum_j p_j(q, \kappa) \cdot q_j - C(q, \kappa) - B \right\}
\]

First-order conditions

1. Quantity:

\[
p_j = \frac{\partial C}{\partial q_j} - \frac{1}{1 + \lambda} \cdot \frac{\partial U}{\partial t_j} \cdot \frac{\partial t_j}{\partial q_j} + \frac{1}{1 + \lambda} \cdot \frac{\partial E}{\partial q_j} - \frac{\lambda}{1 + \lambda} \cdot \sum_k q_k \cdot \frac{\partial p_k}{\partial q_j}
\]

2. Capacity:

\[
\frac{\partial C}{\partial \kappa_j} = \frac{1}{1 + \lambda} \cdot \frac{\partial U}{\partial t_j} \cdot \frac{\partial t_j}{\partial \kappa_j} - \frac{1}{1 + \lambda} \cdot \frac{\partial E}{\partial \kappa_j} + \frac{\lambda}{1 + \lambda} \cdot \sum_k q_k \cdot \frac{\partial p_k}{\partial \kappa_j}
\]
Optimal Pricing

\[ p_j - \tilde{C}_j = \tilde{E}_j - \frac{1}{\lambda} \left( \tilde{E}_j - \sum_{k} q_k \cdot \Omega_{kj} - \sum_{k} (\tilde{u}_k^T - \bar{u}_k^T) \cdot \tilde{T}_{kj} \right) \]

- \( p_j \) is the price
- \( \tilde{C}_j \) is the marginal cost
- \( \tilde{E}_j \) is the marginal externality
- \( \sum_{k} \tilde{u}_k^T \cdot \tilde{T}_{kj} \) represents network effects
- \( \frac{1}{\lambda} \left( \tilde{E}_j - \sum_{k} q_k \cdot \Omega_{kj} - \sum_{k} (\tilde{u}_k^T - \bar{u}_k^T) \cdot \tilde{T}_{kj} \right) \) comprises market power distortion and Spence distortion.

\( \lambda \) is a parameter that balances the importance of market power and Spence distortions.
Optimal Pricing

\[
p_j - \tilde{C}_j = \tilde{E}_j - \sum_k \tilde{u}_k^T \cdot \tilde{T}_{kj} + \frac{\lambda}{1 + \lambda} \left\{ - \tilde{E}_j - \sum_{k \in J} q_k \cdot \Omega_{kj} - \sum_k (\tilde{u}_k^T - \bar{u}_k^T) \cdot \tilde{T}_{kj} \right\}
\]

Notation:

- \( \tilde{C}_j = \frac{\partial C_i}{\partial q_j} + \frac{\partial C}{\partial k_j} \frac{k_j}{q_j} \): Mg. cost of a trip, keeping capacity per trip \( \frac{k_j}{q_j} \) constant

- \( \tilde{E}_j = \frac{\partial E_i}{\partial q_j} + \frac{\partial E}{\partial k_j} \frac{k_j}{q_j} \): Mg. externality of a trip, keeping capacity per trip \( \frac{k_j}{q_j} \) constant

\[\Rightarrow \quad \text{“Standard” Pigouvian tax/subsidy}\]
Optimal Pricing

\[ p_j - \tilde{C}_j = \tilde{E}_j - \sum_k \tilde{u}_k^T \cdot \tilde{T}_{kj} + \frac{\lambda}{1 + \lambda} \left\{ -\tilde{E}_j - \sum_{k \in J} q_k \cdot \Omega_{kj} - \sum_k (\tilde{u}_k^T - \tilde{u}_k^T) \cdot \tilde{T}_{kj} \right\} \]

**Notation:**
- \( \tilde{u}_j^T = \frac{\partial CS}{\partial t_j} \): How does CS change if the time of mode \( j \) increases?
- \( \tilde{T}_{kj} = \frac{\partial T_j}{\partial q_k} + \frac{\partial T_j}{\partial \kappa_k} q_k \): How does mode-\( j \) time change if mode-\( j \) trips and capacity increase? Two effects:
  1. Congestion: negative externality
  2. Returns to scale: positive externality (Arnott, 1996)

\[ \Rightarrow \text{ Tax modes that cause congestion, subsidize modes with increasing returns to scale} \]
Optimal Pricing

\[ p_j - \tilde{C}_j = \tilde{E}_j - \sum_k \tilde{u}_k^T \cdot \tilde{T}_{kj} + \frac{\lambda}{1 + \lambda} \left\{ -\tilde{E}_j - \sum_{k \in J} q_k \cdot \Omega_{kj} - \sum_k (\tilde{u}_k^T - \bar{u}_k^T) \cdot \tilde{T}_{kj} \right\} \]

Notation:

- \( \Omega \): Inverse Jacobian of demand (~ inverse price elasticities)
- \( \tilde{u}_j^T \): If \( t_j \) increases by 1%, how much does utility of marginal commuters change?
Optimal Pricing

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Notation:
- \( \Omega \): Inverse Jacobian of demand (~ inverse price elasticities)
- \( \tilde{u}_j^T \): If \( t_j \) increases by 1%, how much does utility of marginal commuters change on avg.?

Government cares about budget, so it behaves like a monopolist to some extent:
- Underweights externalities
- Market power markup: higher price for less elastic modes
- Caters to marginal rather than average consumer
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Demand

Market Definition:

– $m =$ community area $a$ to $a'$ during hour $h$
– Exogenous arrival rate $\lambda_m$ of travelers

Choices:

$$j^* = \arg\max_{j \in \mathcal{J}_i \cup \{0\}} u^i_{mj}$$

Utility of agent $i$:

$$u^i_{mj} = \xi_{mj} - \alpha^i_T \cdot T_{ij} - \alpha^i_p \cdot p_{ij} + \epsilon^i_j,$$

Estimation: Instrument prices with inflow of trips

1. Value of time: $\sim$ $75/\text{h}$ on avg, higher for high income
2. Taxi/ride hailing elasticities $\sim 2$
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Transportation technology

Three parts:

1. Congestion
2. Public transit (bus/subway)
3. Private transit (taxis/ride hailing)
Congestion

Given vector $q$ of all trips, what is the travel time from $a$ to $a'$ using mode $j$?

Approach: Model city as directed graph of community areas
- Edges $e$ are adjacent community areas
- Agents in market $m$ take a route $r_m$, a sequence of edges

During hour $h$, the congestion on edge $e$ is

$$Q_{eh} = \sum_j \sum_{r \text{ s.t. } e \in r} b_j q_{rhj}$$

Travel times:

By edge: $\tau_{ehj} = A_{ej} Q_{ehj}^{\alpha_j}$

By market: $T_{mj}^{\text{travel}} = \sum_{e \in r_{mj}} \tau_{ehj}$
Congestion estimation

Data: travel times and vehicle flows between adjacent CAs at the hourly level

Binscatters with o-d pair fixed effects:

We calibrate congestion elasticities based on this data

- Later: capture nonlinearity with more flexible functional form
Public transit (bus/subway)

Agents in market $m = (a, a', h)$ take a bus/train route $R_m$

Total time for agent $i$ in market $m = (a, a', h)$, who takes route $R$:

$$T_i = \underbrace{T_{i\text{\_walk,1}}}_{\text{See below}} + \underbrace{T_{R\text{\_wait}}}_{\text{Congestion}} + \underbrace{T_{R\text{\_travel}}}_{\text{Congestion}} + \underbrace{T_{i\text{\_walk,2}}}_{\text{Congestion}}$$

Let $\kappa_{R_h}$ be the frequency of route $R$. Then the waiting time is

$$T_{R\text{\_wait}} = \frac{1}{\kappa_{R_h}}$$

Cost and externality:

$$C_{R_h} = c_R \kappa_{R_h} \quad E_{R_h} = e_R \kappa_{R_h}$$
Private transit (taxis and ride hailing)

Total time in private mode $j$ in market $m = (a, a', h)$ is $T_{mj}^{\text{wait}} + T_{mj}^{\text{travel}}$. See below for waiting time. Congestion in market $m = (a, a', h)$ is $T_{mj}^{\text{wait}} / \text{dcurly.alt3} + T_{mj}^{\text{travel}} / \text{dcurly.alt4}$. Congestion is due to high demand at the origin $a$.

Waiting time depends on $L_{ahj}$, the number of available drivers at the origin $a$:

$$T_{mj}^{\text{wait}} = A_{aj} L_{ahj}^{-\phi_j}$$

The distribution of available drivers arises from a model of driver movements (Buchholz, 2021; Rosaia, 2020):

- There is a total number of drivers $\kappa_{hj}$
- Travelers who chose mode $j$ matched to drivers, who later become available at drop off
- Available drivers relocate $\rightarrow$ tendency to move towards high earnings neighborhoods

23/30
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Counterfactuals

**Benchmark:** Status quo

**Counterfactuals:** City government maximizes welfare by adjusting:

1. *Transit:* Prices and frequencies of public transit
2. *Tax:* Congestion tax
3. *Transit + tax:* Prices and frequencies of public transit + congestion tax
4. *Social planner:* Prices and frequencies of public transit + congestion charge + price of Uber and taxis

We consider alternatives with and without budget constraint
## Counterfactual Results

<table>
<thead>
<tr>
<th>Status quo</th>
<th>Bus + subway w/ budget</th>
<th>subway prices, congestion</th>
<th>subway prices, congestion</th>
<th>Only congested</th>
<th>Social planner variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>-5.2% uber_price_change</td>
</tr>
<tr>
<td>0.0%</td>
<td>-104.5%</td>
<td>-88.4%</td>
<td>-93.4%</td>
<td>-83.1%</td>
<td>-93.4% bus_price_change</td>
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<tr>
<td>0.0%</td>
<td>-107.6%</td>
<td>-91.0%</td>
<td>-96.4%</td>
<td>-86.6%</td>
<td>-96.4% train_price_change</td>
</tr>
<tr>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>26.0%</td>
<td>26.1%</td>
<td>26.4% car_surcharge</td>
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<td>-23.3%</td>
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<td>-23.3% bus_capacity_change</td>
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<td>-29.1%</td>
<td>-27.8% train_capacity_change</td>
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<tr>
<td>0.000</td>
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<td>7.960</td>
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<td>0.000</td>
<td>0.000</td>
<td>37.361</td>
<td>37.421</td>
<td>37.842</td>
</tr>
</tbody>
</table>
Substitution patterns

Change in the number of trips

Mode

Bus Train Taxi Uber Car Stay

Label

- Bus + subway prices
- Bus + subway prices w/ budget constraint
- Bus + subway prices, congestion tax
- Only congestion tax
- Social planner

26/30
Distributional effects

Changes in consumer surplus relative to status quo
Distributional effects
Changes in consumer surplus relative to status quo

Congestion price
Δ car trips
Δ stay

Public transit prices + capacity
Δ car trips
Δ stay
Future Steps
Much Work Ahead!

Refine estimation:
- Add demand heterogeneity
  - Car ownership as choice set variation
  - Include micro variation in access
- Flexible congestion and driver supply models

Exploit fine spatial resolution
- More sophisticated policy levers
- More results on heterogeneity

Additional counterfactuals
- Endogenous response by ride hailing platforms
Conclusions

- Construct new dataset of all trips across all modes for a major urban area.
- Compute optimal transit **prices**, **capacities**, and **subsidies** with government budget constraints.
- Preliminary results:
  - Public transit should be further subsidized and its frequency should be reduced
  - Congestion/carbon taxes → large welfare gains, large CS decrease
  - Markup of ride hailing platforms serves as a Pigouvian tax
Thank you!
Supply
Parametrizing Pooling Technology

Total trip time:

\[ T = w(I) + 0.5 \cdot \tau + T^0 + P(q) \cdot d(q), \]

where \( I \) is the number of idle drivers, \( w(I) \) and \( d(q) \) are decreasing, \( P(q) \) is increasing.
Supply
Parametrizing Pooling Technology

Total trip time:

\[ T = \underbrace{w(I)}_{\text{Waiting time}} + \underbrace{0.5 \cdot \tau}_{\text{Batch Length}} + \underbrace{T^0}_{\text{Direct time}} + \underbrace{P(q) \cdot d(q)}_{\text{Match prob.} \cdot \text{Expected detour}} , \]

where \( I \) is the number of idle drivers, \( w(I) \) and \( d(q) \) are decreasing, \( P(q) \) is increasing.

Suppose that density of trips with detour distance \( x \) is proportional to

\[ \frac{\alpha}{\eta} \cdot \frac{q}{\tau} \cdot \frac{1}{\eta} \cdot x^{\eta-1} , \]

then \( x \sim \text{Weibull} \) (also shortest path btn. vertices on random graph; Bauckhage et al., 2013).
Supply
Parametrizing Pooling Technology

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Suppose that density of trips with detour distance \( x \) is proportional to

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then \( x \sim \text{Weibull} \) (also shortest path btn. vertices on random graph; Bauckhage et al., 2013).

- Only match if detour < \( \bar{x} \).
- \( P(q) \) is the CDF of the shortest detour \( 1 - \exp(-\alpha \cdot q \cdot \tau \cdot \bar{x}^{\eta}) \).
- Expected detour \( d(\bar{x}, \alpha, q, \tau) \): mean of detour, truncated at \( \bar{x} \)
Supply

Utilization

How much **driver time** does a trip take on average?

\[
T^B = (1 - F(\bar{x}; \alpha, q, \tau)) \cdot \frac{T^0}{\text{Busy time if unmatched}} + F(\bar{x}; \alpha, q, \tau) \cdot \frac{1}{2} \cdot (T^0 + d(\bar{x}, \alpha, q, \tau))
\]

**The number of idle drivers is:**

\[
I = L - q \cdot T^B.
\]

Waiting time (distance to the closest driver):

- Avg. waiting time is \( w(I) = \frac{\rho}{\sqrt{I}} \) (Arnott 1996).

**Additional assumptions:**

- Optimal match radius depends on density: \( \bar{x}(q) = \delta \cdot q^{-\beta} \)
- Batch length \( \tau \) fixed at one minute.
- Set \( \rho \) so that avg. ride-hailing time is 4 min
Supply

*From model to the data*

Given parameters $\alpha$, $\eta$, $\beta$, $\delta$, we can construct

- $P(q)$ of being pooled $1 - \exp(-\alpha \cdot q \cdot \tau \cdot \bar{x}^\eta)$.
- $E(q)$ Expected detour $d(\bar{x}, \alpha, q, \tau)$

$\implies$ Both functions $P(q)$ and $E(q)$ can be directly constructed from the data!
Improving current technologies

![Graph showing the relationship between total time (in minutes) and social cost per trip ($). The graph includes data points for Shared, Bus, and Ride hail modes.](image-url)
Improving current technologies

![Graph showing the relationship between total time and social cost per trip across different modes: Shared, Bus, and Ride hail.](image-url)
Improving current technologies

![Graph showing total time vs. social cost per trip (in $) for different modes: Shared, Bus, and Ride hail. The graph plots total time in minutes on the x-axis and social cost per trip in dollars on the y-axis. The graph includes data points and lines for each mode, illustrating the relationship between time and cost.]
Improving current technologies

![Graph showing the relationship between total time (min) and social cost per trip ($) for different modes: Shared, Bus, and Ride hail.

- **Shared**
- **Bus**
- **Ride hail**

The graph indicates that as total time increases, the social cost per trip decreases for all modes.
Improving current technologies

![Graph showing the relationship between total time and social cost per trip for different modes of transportation. The graph includes lines for Shared, Bus, and Ride hail modes.]