Misallocation under the Shadow of Death*

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November 27, 2022

Abstract

This study focuses on a slow exit process, known as a shadow of death, as a new factor of inefficient resource allocation in the macroeconomy. First, we develop an endogenous growth model that incorporates firms' R&D investment and distorted exit decisions. The model shows that the exit of firms in the market equilibrium is inefficiently slow from the social viewpoint even without distortions such as corporate subsidies, which further impede the exit process. Second, our empirical analysis using Japanese firm-level data confirms that exiting firms exhibit the shadow of death in a manner that is consistent with our model. Further, the degree of the shadow of death is related to our distortion measures such as corporate subsidies. Third, our simulation based on the calibrated model suggests that an increase in subsidies can help explain recent firm dynamics in Japan and worsen productivity growth and welfare, although the quantitative impacts of subsidies are limited.

Keywords: firm exit; shadow of death; reallocation; firm dynamics; economic growth

JEL classification: E22, L16, O31, O41

*This study was conducted as a part of the "Determinants of Firm Dynamics" project at the RIETI and under a joint research agreement between Hitotsubashi University and TSR Inc. Miyakawa, Oikawa, and Ueda are grateful for financial support from the JSPS Grant-in-Aid (21H00711, 21K01492, and 16KK0065, respectively). All the errors are our own.

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1 Introduction

Studies on misallocation argue that appropriate resource reallocation has a sizable impact on macroeconomic performance (Hopenhayn and Rogerson (1993); Lentz and Mortensen (2008); Restuccia and Rogerson (2008); Hsieh and Klenow (2009); Acemoglu et al. (2018); Edmond et al. (2018); Peters (2020); Miyakawa et al. (2022)). One of the main reallocation channels is through the exit of low-performing firms, which frees up employed resources for better-performing firms. However, a slow process of firm exit has been widely observed. One exemplifying observation is a "shadow of death": Exiting firms exhibit signs of exit—such as declines in productivity growth, sales, and profits—well before the actual exit (Griliches and Regev (1995); Olley and Pakes (1996); Golombek and Raknerud (2018)). Indeed, Japanese firm-level data show that the sales of exiting firms are significantly smaller than those of average firms and they decrease further toward the year of exit (Figure 1. Details are explained in Section 3.2).

A long shadow of death can have both static and dynamic effects. From a static perspective, it delays the reallocation of workers employed by firms that exit. From a dynamic perspective, it reduces the incentive for firms to increase productivity if they survive despite poor performance. This study aims to investigate whether and by how much aggregate productivity and welfare improve if firms destined to exit do indeed exit quickly from the market. To accomplish this, we perform three tasks. First, we develop a theoretical model that generates a shadow of death in equilibrium. We construct an R&D-driven endogenous growth model with heterogeneous firms, in which firms make R&D investment and exit decisions while the relative productivity of a non-R&D firm gradually declines over time. The process of loss of competitiveness from the time a firm stops R&D to the time it exits the market creates a shadow of death. The shadow of death becomes more gradual and longer when the industry-level R&D intensity is lower because the incentives for R&D effort decrease and exit is delayed. The model shows that shortening the shadow of death improves welfare, whereas the distortions that directly affect the exit decision, such as corporate subsidies, prolong the shadow of death.

Second, we document the facts pertaining to the shadow of death and examine the consistency of those facts with model implications by using firm-level data provided by one of the largest credit rating agencies (TSR Inc.) in Japan. We find evidence of the shadow of death and confirm that it has a significant relationship with the external environment faced by firms, such as corporate subsidies and the degree of development of the second-hand market. The relationship between R&D and sales indicates that the pace of decline in sales is magnified after the termination of R&D activities. This suggests that firms without R&D are left behind in the market, which is consistent with the model. Additionally, firms belonging to industries with higher distortions, which are exemplified by larger corporate subsidies or a less developed second-hand market for capital goods, exhibit smaller sales at the time of exit compared to that of non-exiting firms. This result implies that higher distortions prolong the length of the shadow of death. Furthermore, we find that industries with a higher level of the abovementioned distortions have larger sales when R&D stops vis-à-vis those of firms that continue R&D. This result again suggests that higher distortions prolong the length of the shadow of death.

Third, we implement simulations using a calibrated model based on Japanese data. The simulations allow us to quantitatively examine the macroeconomic impacts of distortions. Specifically, we simulate the effects of size-dependent subsidies and outside option values. The simulation results demonstrate that an increase in subsidies and/or a decrease in outside option values enable low productivity firms to survive longer; this decreases the entry/exit rate, increasing the length of the shadow of death and, in turn, decreasing welfare. It is important to note, however, that we also find that the effect of these distortions on real growth rate is not necessarily large. This suggests that the quantitative impacts of improved reallocation among firms in the left tail of the firm distribution are limited. Although such improvement in economic growth due to the reduction in distortion is still qualitatively meaningful, we should be cautious about the quantitative implication.

This study contributes to the literature on business dynamism. As summarized by Akcigit and Ates (2021), declining business dynamism—such as higher markups, lower entry and exit rates, and stagnant job creation—is observed in developed countries. Many of the theories that explain these phenomena refer to observed facts in the United States as the basis for their models. However, in the Japanese economy we examine in our empirical analysis, the market concentration rate has declined, rather than increased, along with declining business dynamism. It suggests that the U.S.-style explanation of declining business dynamism tied to the existence of GAFA and other giants that inhibit the innovations of other firms cannot be applied easily to

Japan. Furthermore, the entry/exit rate of the Japanese economy has been low by international standards, which has been attributed to the existence of zombie firms (Caballero et al. (2008)), increase in business closures due to the aging of corporate managers (Ito and Kato (2016); Tsuruta (2019); Xu (2019); Hong et al. (2020)), and shadow of death (Kiyota and Takizawa (2007) and Coad and Kato (2021)). These observations motivate us to analyze the left tail of firm-size distribution instead of the right tail, which is the focus of the giant firm story based on U.S. data. Our simulation results show that increased distortions not only prolong the shadow of death but also make the left tail thicker and decrease the market concentration rate.

Concerning a misallocation, this study focuses on a dynamic aspect rather than a static one. While Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) investigate whether resources are allocated to firms efficiently on the basis of their productivity, this type of static efficiency is satisfied in our model unless we introduce exit distortions exogenously. In our model, productivity is not given; R&D investment, entry, and exit are endogenous, as in Lentz and Mortensen (2008), Acemoglu et al. (2018), and Peters (2020). A dynamic misallocation occurs through inefficiencies with respect to R&D investment and entry. Since firms do not internalize the aggregate value of their innovation, firms that fall behind the frontier stop R&D investment too early. Moreover, an intertemporal knowledge spillover to entrants makes an entry in the market equilibrium too low. Furthermore, exit distortions, when given exogenously, discourage R&D investment further and make firms linger in the market too long, which lowers economic growth and worsens welfare.

There are many theoretical studies on endogenous exits, but our study differs in that it uses an R&D-driven endogenous growth model. While Hopenhayn (1992) and Luttmer (2007) consider exogenous productivity dynamics, our model incorporates the thresholds of R&D investment and exit, and productivity dynamics are endogenously determined depending on the size of firm sales. In Ericson and Pakes (1995), Olley and Pakes (1996), and Igami and Uetake (2020) not only exit but also investment is endogenous, and the effect of distortions is analyzed. Particularly, Ericson and Pakes (1995) show the existence of a coasting state in which there is neither investment nor exit, which overlaps with the theoretical properties derived in the present study. Our model differs from these in that it is a macroeconomic endogenous growth model of general equilibrium and focuses on the implications for the macroeconomy.

Although there are many empirical studies on the shadow of death, few studies relate it to firms' R&D investments and the external environment firms face, rooted in an explicit theoretical exposition. An exception is Blanchard et al. (2014), who demonstrate that sunk costs are a barrier to exit, consistent with our results. Rather than investigating firms that voluntarily exit, Yamakawa and Cardon (2017) study defaulting firms and find that investment of time and money prior to the point of distress delays the firm's exit. Rossi-Hansberg and Wright (2007) present contrasting evidence on the importance of the shadow of death; using U.S. data, they argue that it is observed only in very small and young establishments. However, in our sample of Japanese firms, we observe a shadow of death among firms who survive more than 10 years.

The rest of this paper is organized as follows. Section 2 introduces the model, Section 3 describes the empirical analysis, Section 4 simulates the model using a calibrated model based on Japanese data, and Section 5 concludes.

2 Model

To investigate the link from the length and intensity of the shadow of death to macroeconomic performance, we construct a model of endogenous growth with firm dynamics in which firm sales gradually decrease before exit.

2.1 Setup

2.1.1 Household

The representative household has the following preference

$$\int_0^\infty e^{-\rho t} \ln C_t \, dt, \tag{1}$$

where ρ represents the time preference and aggregate consumption at time t, C_t, is the composite of final goods $i \in [0, 1]$ such as

$$\ln C_t = \int_0^1 \ln Y_{it} di.$$

The expenditure in each industry is constant and normalized to one, that is, $P_{it}Y_{it} = 1$ for any i and t. Under the standard budget constraint, we have interest rate $r_t = \rho + g_t$ on a balanced growth path, where g_t is the aggregate consumption growth rate. We assume that each household supplies labor inelastically, and the total labor is L, which is constant over time.

2.1.2 Final Goods Firms

Final goods producers in industry i utilize an industry-specific set of intermediate goods. Denote $\mathcal{J}_{it}$ as the set of active firms that supply intermediate goods used in industry i at time t. Each intermediate good is monopolistically supplied by a single firm. Let n_{it} be the measure of $\mathcal{J}_{it}$. The production function of final goods i is given by

$$Y_{it} = n_{it}^{\varepsilon} \left[\int_{\mathcal{J}_{it}} x_{ijt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $\sigma > 1$ is the elasticity of substitution and $\varepsilon \in [-1/(\sigma-1), 0]$ determines the degree of love for variety, which has no impact on production at the lower bound of ε. Under perfect competition, a firm supplying final goods i maximizes its profit given by $P_{it}Y_{it} - \int_{\mathcal{J}_{it}} p_{ijt}x_{ijt}dj$.

The first-order condition with respect to x_{ijt} leads to the following demand function for intermediate goods j in industry i:

$$x_{ijt} = n_{it}^{\varepsilon(\sigma-1)} P_{it}^{\sigma} Y_{it} p_{ijt}^{-\sigma}. \tag{3}$$

2.1.3 Intermediate Goods Firms

Each intermediate goods firm produces differentiated goods with a linear production function in labor as $x_{ijt} = z_{ijt}\ell_{ijt}$ and a fixed cost of κ_o in labor units, where z_{ijt} represents the idiosyncratic productivity of intermediate goods firm j in industry i, and ℓ_{ijt} is its employment. The firm maximizes profits $\pi_{ijt} = (p_{ijt} - w_t/z_{ijt})x_{ijt} - \kappa_o w_t$, subject to demand function (3), where κ_o is a parameter that represents fixed costs for production per unit of time in times of labor and w_t is the wage; this yields the monopoly price $p_{ijt} = \frac{\sigma}{\sigma-1}\frac{w_t}{z_{ijt}}$. By substituting the optimal x_{ijt} into equation (2), we obtain the industry-level price of final goods i as follows:

$$P_{it} = \frac{\sigma}{\sigma-1}\frac{w_t}{n_{it}^{\varepsilon} Z_{it}}, \tag{4}$$

where we define the industry-level productivity as

$$Z_{it} \equiv \left[\int_{\mathcal{J}_{it}} z_{ijt}^{\sigma-1} dj \right]^{\frac{1}{\sigma-1}} . \tag{5}$$

From $P_{it}Y_{it} = 1$, the employment and profit for intermediate goods firm j in industry i are expressed as

$$\ell_{ijt} = \frac{\sigma-1}{\sigma} \frac{s_{ijt}}{w_t}, \tag{6}$$

$$\pi_{ijt} = \frac{s_{ijt}}{\sigma} - \kappa_o w_t, \tag{7}$$

where we define the relative productivity of firm j as

$$s_{ijt} \equiv \left(z_{ijt}/Z_{it} \right)^{\sigma-1} , \tag{8}$$

which satisfies $\int_{\mathcal{J}_{it}} s_{ijt} dj = 1$. Note that s_{ijt} is equal to the sales of intermediate goods firm j in industry i, that is, $s_{ijt} = p_{ijt}x_{ijt}$. As firms' decisions depend not on z_{ijt} but on s_{ijt}, we focus on the dynamics of relative productivity, s_{ijt}, below.

2.1.4 R&D by Incumbents

Each intermediate goods firm can improve its own productivity, z_{ijt}, by R&D investment, which entails the fixed costs of κ_r per unit of time in terms of labor. Success in R&D leads to productivity improvement by the rate of $\gamma > 0$ with probability λdt such that

$$z_{ijt+dt} = \begin{cases} (1+\gamma)\, z_{ijt} & \text{w.p. } \lambda dt, \\ z_{ijt} & \text{w.p. } 1 - \lambda dt. \end{cases} \tag{9}$$

Thus, the expected growth rate of z_{ijt} equals $\lambda\gamma$ if R&D investment is carried out, and z_{ijt} remains unchanged without R&D. By contrast, the relative productivity, s_{ijt}, is always changing over time. Since $z_{ijt}^{\sigma-1}$ increases by the rate of $\gamma_\sigma \equiv (1+\gamma)^{\sigma-1} - 1$ when succeeding in R&D, the expected growth rate of $z_{ijt}^{\sigma-1}$ is $\lambda\gamma_\sigma$. Let θ_{it} be the growth rate of $Z_{it}^{\sigma-1}$; then we obtain the expected growth rate of relative productivity as

$$\mathbb{E}_t \frac{\dot{s}_{ijt}}{s_{ijt}} = \begin{cases} \lambda\gamma_\sigma - \theta_{it} & \text{if } \chi_{ijt} = 1, \\ -\theta_{it} & \text{if } \chi_{ijt} = 0, \end{cases} \tag{10}$$

where χ_{ijt} is the indicator that takes the value of 1 when firm j in industry i invests in R&D at time t. The relative productivity of a firm gradually declines according to the aggregate productivity growth with occasional increases with the individual R&D if the firm invests. Otherwise, the relative productivity monotonically decreases over time.[1]

Now, we define the Hamilton–Jacobi–Bellman (HJB) equation for the value of intermediate goods firm, $v(s_{ijt,}\theta_{it,}w_t)$. Given the law of motion for relative productivity (10), we have

$$\begin{aligned} r_t v(s_{ijt}, \theta_{it}, w_t) = \max \Big\{ 0, \ & \frac{s_{ijt}}{\sigma} - \kappa_o w_t \\ & + \max_{\chi \in \{0,1\}} \mathbb{E}_t \left\{ v_s(s_{ijt}, \theta_{it}, w_t) \dot{s}_{ijt}|_{\chi=0}, \ -\kappa_r w_t + v_s(s_{ijt}, \theta_{it}, w_t) \dot{s}_{ijt}|_{\chi=1} \right\} \\ & + v_\theta(s_{ijt}, \theta_{it}, w_t) \dot{\theta}_{it} + v_w(s_{ijt}, \theta_{it}, w_t) \dot{w}_t \Big\} . \end{aligned} \tag{11}$$

A firm exits the market when the firm value reaches the lower bound of zero. The exit condition is modified in the next section to incorporate exit distortions.

We consider the R&D decision for a given θ_{it} that appears in the second line of equation (11). Intuitively, R&D investment in the current instant increases the expected profit in the next instant by $s(\lambda\gamma_\sigma - \theta_{it})dt/\sigma$, while the profit decreases by $s\theta_{it}dt/\sigma$ without R&D. The expected return from R&D increases in s, and its cost is independent of s. Hence, a firm with a higher s has a greater incentive to make R&D investment, yielding a unique threshold $\hat{s}_{it}$ such that a firm invests in R&D if and only if its s is greater than $\hat{s}_{it}$. The next proposition presents the best cutoff strategy for R&D investment, taking θ_{it} as given.

Proposition 1. *Given $\theta_{it} \geq 0$, there exists a unique threshold $\hat{s}_{it} > 0$ above which a firm invests in R&D. $\hat{s}_{it}$ satisfies*

$$v_s\left(\hat{s}_{it}, \theta_{it}, w_t\right) \hat{s}_{it} = \frac{\kappa_r w_t}{\lambda\gamma_\sigma}. \tag{12}$$

[1] We refer to $\theta \equiv (\dot{Z^{\sigma-1}})/Z^{\sigma-1}$ as the aggregate growth rate although the true aggregate productivity growth is $\dot{Z}/Z = \theta/(\sigma-1)$.

All proofs are in Online Appendix A. Given R&D threshold $\hat{s}$, Figure 2 describes a pattern of firm dynamics. When relative productivity s is higher than $\hat{s}$, a firm makes R&D investment and succeeds in raising s with a probability. If the firm fails, s decreases because the aggregate productivity, Z_{it}, increases over time. When s is lower than $\hat{s}$, a firm stops R&D investment, and s decreases monotonically until the firm exits the market when s reaches $\bar{s}$, which is the exit threshold derived in the next subsection. We interpret the declining phase of relative productivity when approaching $\bar{s}$ as the shadow of death.

2.1.5 Firm Exit and Entry

Exit Threshold. Since a non-R&D firm's relative productivity, s_{ijt}, has a negative trend, the non-R&D firm is destined to exit the market at some point. Let $\bar{s}_{it}$ be the threshold for exit. Note that $\bar{s}_{it} < \hat{s}_{it}$ because $v_s(\bar{s}_{it}, \theta_{it}, w_t) = 0$, which is derived from the smooth-pasting condition, implying no R&D incentive at $\bar{s}_{it}$ in equation (11). Then, we have

$$0 = \frac{\bar{s}_{it}}{\sigma} - \kappa_o w_t + v_n(\bar{s}_{it}, \theta_{it}, w_t)\dot{\theta}_{it} + v_w(\bar{s}_{it}, \theta_{it}, w_t)\dot{w}_t, \tag{13}$$

which implicitly determines $\bar{s}_{it}$.

The exit rate in industry i in period t, say δ_{it}, is

$$\delta_{it} = \theta_{it}\bar{s}_{it}f_{it}(\bar{s}_{it}), \tag{14}$$

where f_{it} be the density function, with the associated distribution, F_{it}.

Entrants' R&D. Potential entrants enter an industry with a new variety of intermediate goods by paying a fixed cost of κ_e in labor units. We assume that they draw relative productivity s from a continuous distribution $F_e(s)$ with the associated density of $f_e(s)$. We assume that an entrant with $s < \bar{s}_{it}$ immediately exits. Then, the free entry condition implies

$$\int_{\bar{s}_{it}}^{\infty} v(s, \theta_{it}, w_t) dF_e = \kappa_e w_t. \tag{15}$$

Let $\mu_{it} n_{it}$ be the measure of firms that invests in R&D to come up with a new variety. Then actual entry rate is $\mu_{it}\left[1 - F_e\left(\bar{s}_{it}\right)\right]$. The change in the measure of intermediate goods firms in industry i is given by

$$\dot{n}_{it} = \left(\mu_{it}\left[1 - F_e\left(\bar{s}_{it}\right)\right] - \delta_{it}\right) n_{it}. \tag{16}$$

2.1.6 Aggregate Productivity Growth

The aggregate productivity grows through incumbents' R&D and entry/exit. Entry and exit contribute to aggregate productivity growth by replacing the bottom firms with entrants whose average productivity must be higher than exiting firms. The aggregate productivity growth, θ_{it}, is determined by

$$\theta_{it} = n_{it}\left[\lambda\gamma_\sigma \int_{\hat{s}_{it}}^{\infty} s dF_{it} + \mu_{it}\int_{\bar{s}_{it}}^{\infty} s dF_e - \delta_{it}\bar{s}_{it}\right], \tag{17}$$

where the first term in the parenthesis represents the contribution from incumbents' R&D and the remaining terms represent the contribution through entry/exit.

2.1.7 Labor Market

Demand for labor consists of four terms: variable demand for intermediate goods production, fixed demand for intermediate goods production, fixed demand for R&D, and fixed demand for entry. The labor market is cleared according to

$$\begin{aligned} L &= \int_0^1 \int_{\mathcal{J}_{it}} \ell_{ijt} dj di + \kappa_o \int_0^1 n_{it} di + \kappa_r \int_0^1 n_{it}\left(1 - F_{it}\left(\hat{s}_{it}\right)\right) di + \kappa_e \int_0^1 \mu_{it} n_{it} di \\ &= \frac{\sigma - 1}{\sigma w_t} + \int_0^1 n_{it}\left[\kappa_o + \kappa_r\left(1 - F_{it}\left(\hat{s}_{it}\right)\right) + \kappa_e \mu_{it}\right] di, \end{aligned} \tag{18}$$

where we use $\int_{\mathcal{J}_{it}} s_{ijt} dj = n_{it}\int_{\bar{s}_{it}}^{\infty} s dF_{it} = 1$.

2.2 Stationary Equilibrium

We assume that the industries are symmetric and focus on the stationary equilibrium of this economy. Below, we exclude industry and time subscripts. Let $F(s)$ be the stationary distribution of relative productivity s with an associated density of $f(s)$.

Since $\hat{s}$ and $\bar{s}$ are constant over time and $\dot{\theta} = \dot{w} = 0$ in a stationary state, the

HJB equation (11) becomes

$$rv(s,\theta,w) = \max\left\{0,\ \frac{s}{\sigma} - \kappa_o w + \max\left\{0,\ \lambda\gamma_\sigma s v_s(s,\theta,w) - \kappa_r w\right\} - \theta s v_s(s,\theta,w)\right\}. \tag{19}$$

Then, we obtain the exit and R&D thresholds as in the next proposition.

Proposition 2. *In a stationary state with a given $\theta > 0$, the thresholds for exit and R&D are uniquely determined and satisfy*

$$\bar{s} = \sigma\kappa_o w = \frac{(\sigma-1)\kappa_o}{L_X}, \tag{20}$$

$$\frac{1}{r+\theta}\left(\frac{\hat{s}}{\bar{s}} - \left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}}\right) = \frac{\kappa_r/\kappa_o}{\lambda\gamma_\sigma}. \tag{21}$$

Moreover, $\hat{s}$ increases in θ, ceteris paribus.

Equation (21) is derived from the smooth-pasting condition at the R&D threshold. Since a firm commits to no R&D below the threshold, the firm value for $s \leq \hat{s}$ can be easily calculated. Moreover, because R&D is based on free decision making, firm value is smoothly connected to that in the region $s > \hat{s}$. In other words, the marginal firm value is continuous at $\hat{s}$. This property provides the condition (21). Note that it pins down the ratio, $\hat{s}/\bar{s}$, which determines the length of the shadow of death.[2]

In the stationary state, the measure of firms, n, is constant so that, from equations (14) and (16),

$$\mu = \frac{\theta\bar{s}f(\bar{s})}{1-F_e(\bar{s})}.$$

Then, from equation (17), the aggregate productivity growth in the stationary state, θ, is expressed as

$$\theta = \left[1 - n\bar{s}f(\bar{s})\left\{\frac{\int_{\bar{s}}^{\infty} s dF_e}{1-F_e(\bar{s})} - \bar{s}\right\}\right]^{-1} n\lambda\gamma_\sigma \int_{\hat{s}}^{\infty} s dF. \tag{22}$$

The essential source of the aggregate productivity growth, or the slope of the shadow of death, is the incumbents' R&D intensity that appears in the last term of

[2]Although it is not an important difference, strictly speaking, $\hat{s}/\bar{s}$ represents the "minimum" length of the shadow of death, while an "observed" shadow of death includes the periods in which a firm invests but fails in R&D, as depicted in Figure 2. Ericson and Pakes (1995) refer to $s \in [\bar{s}, \hat{s})$ as the coasting state.

equation (22), $n\lambda\gamma_\sigma \int_{\hat{s}}^{\infty} sdF$. The first bracket indicates that the effect of the R&D intensity is amplified by the replacement of exiting firms by entrants.

Other variables in the stationary equilibrium are characterized as follows. Real output and consumption satisfy

$$C_t = Y_t = n^{\varepsilon} L_X Z_t,$$

which grows at the rate of $g = \theta/(\sigma - 1)$. The real interest rate is determined by $r = \rho + g$. The labor market is cleared according to

$$L = L_X + n\left[\kappa_o + \kappa_r\left(1 - F\left(\hat{s}\right)\right) + \kappa_e \mu\right], \tag{23}$$

where

$$n = \left[\int_{\bar{s}}^{\infty} sdF\right]^{-1}.$$

Finally, assuming that the economy is in the stationary equilibrium at the initial state, we can calculate welfare as

$$U = \frac{\ln Y_0}{\rho} + \frac{g}{\rho^2}. \tag{24}$$

2.3 Equilibrium Shadow of Death is Inefficiently Long

There are two kinds of inefficiencies in this model. One is in R&D decision making. This contrasts with Hopenhayn (1992), where productivity growth is determined by exogenous stochastic processes and, thus, the equilibrium is efficient.

The social planner maximizes the utility of the representative household, equation (1), subject to production function (2), productivity growth (9), resource constraint (18), and the dynamics of n by choosing the exit threshold $\bar{s}$, R&D threshold $\hat{s}$, production workers L_X, and entry μ.

To compare the shadow of death in the market equilibrium and the socially optimal allocation, we focus on deriving the optimal conditions for $\bar{s}$ and $\hat{s}$, leaving the full derivation of the socially optimal solution to Appendix A.3. Let F^* be the stationary distribution of s in the socially optimal solution. Further, let L_X^*, n^*, and Y_t^* be the socially optimal production workers, a measure of firms, and aggregate output, respectively, in the stationary state.

First, the exit threshold, $\bar{s}$, is analogous to that in the market equilibrium because the social planner stops operating the firm if its contribution to the aggregate productivity is not commensurate with its fixed operational cost. From equation (2) with the efficient labor allocation, $\ell_j = s_j L_X$, for a given L_X, the loss of output by stopping one unit of firms at $\bar{s}$ is

$$-\frac{1}{n^* f^*(\bar{s})} \left.\frac{\partial Y_t}{\partial \bar{s}}\right|_{Y_t = Y_t^*} = \frac{\sigma \bar{s} Y_t^*}{\sigma - 1}.$$

The gain from stopping such a firm is

$$\frac{Y_t^*}{L_X^*} \left(\bar{s} L_X^* + \kappa_o\right),$$

where Y_t^*/L_X is the value of labor in terms of output. Thus, at the optimal choice of exit threshold, $\bar{s}^*$, we have

$$\bar{s}^* = \frac{(\sigma - 1)\kappa_o}{L_X^*}, \tag{25}$$

which indicates the same relation of $\bar{s}$ to L_X as in the market equilibrium. In this sense, there is no inefficiency in exit decisions and any gap between $\bar{s}^*$ and $\bar{s}$ in equilibrium is attributed to a difference between L_X and L_X^*.

Second, the optimal R&D threshold, $\hat{s}^*$, should equate the expected marginal reward for R&D investment and its cost. Because $1/L_X$ is the utility value of one unit of worker, the cost of R&D in the socially optimal solution is κ_r/L_X^*, on one hand. On the other hand, the expected value from R&D by a firm at the border is

$$\frac{1}{\rho Y_t^*} \times \lambda \gamma \hat{z} \left.\frac{\partial Y_t}{\partial z_j}\right|_{z_j = \hat{z}, Y_t = Y_t^*} = \frac{\lambda \gamma \hat{s}}{\rho},$$

where the first term in the above equation stands for the marginal value of output and the second term is the expected output growth from R&D by a firm at the border, $\hat{z} \equiv (\hat{s})^{\frac{1}{\sigma-1}} Z$. By equating the benefit and loss from R&D, the optimal R&D threshold satisfies

$$\hat{s}^* = \frac{\rho(\sigma - 1)\kappa_r}{\lambda \gamma_\sigma L_X^*}. \tag{26}$$

Because both thresholds linearly depend on the value of labor (or $1/L_X$), we can easily compare the lengths of shadows of death in the market equilibrium and the socially optimal allocation. The following proposition shows that the shadow of death is inefficiently long in the market equilibrium.

Proposition 3. *The market equilibrium has a wider range of firms that are not engaged in R&D, that is,*

$$\frac{\hat{s}}{\bar{s}} > \frac{\hat{s}^*}{\bar{s}^*}.$$

Comparing the R&D thresholds in the market equilibrium and the socially optimal allocation, equations (12) and (26), respectively, the source of inefficiency is the gap between $\sigma v_s(\hat{s}, \theta, w)$ and $1/\rho$, where the former is the marginal private value from R&D and the latter is the marginal social value of R&D. The social planner evaluates R&D returns in terms of absolute productivity, z, because it enhances output and welfare. However, the target for a private firm is relative productivity, s, because it determines profits. Even if a firm succeeds in R&D, the relative advantage will quickly diminish in an environment where many rival firms continue to invest in R&D. In other words, the aggregate productivity growth, θ, draws down the reward for R&D from the private viewpoint. Hence, R&D in the market equilibrium is lower relative to the optimal allocation such that the shadow of death lengthens in the market equilibrium.

This effect is highlighted when we consider the case in which there is almost no aggregate R&D activity, namely θ is very small. From the firm value in the R&D-inactive region and the smooth-pasting at $\hat{s}$, we have

$$\sigma v_s(\hat{s}, \theta, w) = \frac{1}{r+\theta}\left(1 - \left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}-1}\right) \to \frac{1}{r} = \frac{1}{\rho} \qquad \text{as } \theta \to 0.$$

Therefore, the private choice of R&D becomes consistent with the socially optimal allocation, namely $\hat{s}/\bar{s} = \hat{s}^*/\bar{s}^*$, when the aggregate R&D is sufficiently small such that the difference between relative and absolute productivity is negligible.

The second source of inefficiency in this model is due to intertemporal knowledge spillover. New entrants draw relative productivity s, not absolute productivity z, so the entrants benefit from aggregate productivity growth. Hence, entry in the market equilibrium is too low relative to the socially optimal level. This inefficiency does not affect the length of shadow of death, $\hat{s}/\bar{s}$, while it affects the speed of sales declines within the shadow of death period (i.e., the slope of the shadow of death in Figure 2), θ, implying that exiting firms exit more quickly in the socially optimal stationary state than in the market equilibrium.

The two types of inefficiencies can be alleviated by R&D and entry subsidies, which decline the fixed costs, κ_r and κ_e, to make the private values closer to the social values (See Appendix A.3.3). While each of these is an interesting policy issue, we focus on the impact of distortionary firm-exits in the following analysis to investigate misallocation caused by firm exit behaviors. The main takeaway from the analysis here is that, if an exit distortion lengthens the shadow of death, it never improves welfare because a shadow of death is too long in the market equilibrium.

2.4 Exit Distortions

The previous subsection shows that the equilibrium is inefficient even without exit distortions. This inefficiency would increase further when distortions exist in terms of firm exit decisions. Here, we consider such distortions in a model-oriented manner. When we assume that the elasticity of substitution is common across industries, the equilibrium exit threshold, $\bar{s}_i = \sigma\kappa_{o,i}w$, is common across firms and industries after controlling fixed costs, $\kappa_{o,i}w$. As depicted in Figure 3, using the estimation result in the subsequent section, we observe a dispersion of exit thresholds, suggesting that firms face some extent of idiosyncratic exit distortion. Hence, we introduce exit distortions as a wedge of exit decisions of firms such that

$$\bar{s}_{ij} = \tau_{ij}\sigma\kappa_{o,i}w, \tag{27}$$

where $\tau_{ij} \geq 0$ represents the degree of exit distortion and $\tau_{ij} = 1$ indicates no distortion. The next proposition summarizes an individual firm's response to such a distortion.

Proposition 4. *Suppose that the economy is at a stationary state, and an individual firm receives constant K per unit of time in addition to the flow profit. Then, this firm chooses exit and R&D thresholds, $\bar{s}_\tau$ and $\hat{s}_\tau$, respectively, such that*

$$\bar{s}_\tau = \tau\sigma\kappa_o w,$$
$$\frac{1}{r+\theta}\left(\frac{\hat{s}_\tau}{\bar{s}_\tau} - \left(\frac{\hat{s}_\tau}{\bar{s}_\tau}\right)^{-\frac{r}{\theta}}\right) = \frac{1}{\tau}\frac{\kappa_r/\kappa_o}{\lambda\gamma_\sigma},$$

where $\tau = 1 - \frac{K}{\kappa_o w}$. Both $\bar{s}_\tau$ and $\hat{s}_\tau$ monotonically increase in τ. Moreover, $\hat{s}/\bar{s}$ decreases in τ.

This proposition covers a variety of different types of distortions. First, a subsidy to firms implies that $K > 0$ and $\tau < 1$, and τ decreases as K increases. A subsidized firm chooses $\bar{s}_\tau < \bar{s}$ and $\hat{s}_\tau < \hat{s}$, while $\hat{s}_\tau/\bar{s}_\tau > \hat{s}/\bar{s}$. In other words, the firm stays in the market for a longer period under a subsidy policy even when it loses competitiveness. Note that $K < 0$ implies a tax, and the situation is symmetric.

Second, Proposition 4 also holds when firms have a non-zero outside option value. Intuitively, an increase in outside option is parallel to a reduction in the subsidy. Suppose that a firm obtains a value of ξ/r immediately after exit. By defining $\tau = 1+\xi/(\kappa_o w)$, Proposition 4 is applicable. A greater outside option value leads to earlier exit and a shorter shadow of death. A relevant case of outside options is the resale value of a firm's equipment. If the secondary market for equipment is well-developed, exiting firms may be able to obtain large sums by selling their equipment. This leads to $\xi > 0$ (and $\tau > 1$). It should be noted that outside option values not only affect the exit decision but also change the R&D decision because a firm has less incentive to escape the shadow of death when facing an increase in its value (see Online Appendix A.1 for more details). We can also consider negative outside option values ($\tau < 1$); in which case, firms stay for a longer period even with negative profit flows. One example of negative outside option is direct or indirect exit costs. Barriers to re-starting a business or re-entering the workforce at another firm is another example. Particularly, aging of managers decreases outside option values because the aged find it hard to be hired in a new job.

One example of a distortion that deviates from the setting in Proposition 4 is a size-dependent subsidy policy. Suppose that a firm can obtain a flow subsidy of K if its sales volume is below $\tilde{s}$, an exogenous threshold set by the government. We assume $\tilde{s} \in [\bar{s}, \hat{s})$ in equilibrium. In this case, the R&D threshold is determined by the smooth-pasting condition of the firm value as follows:

$$v(s) = \int_0^{\frac{1}{\theta}\log\frac{s}{\bar{s}}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} - \tau\kappa_o w\right)dt - \int_0^{\frac{1}{\theta}\log\frac{s}{\tilde{s}}} e^{-rt}K dt \qquad \text{for } s \in [\tilde{s}, \hat{s}].$$

Then, at $\hat{s}$, we have

$$\frac{\tau}{r+\theta}\left[\frac{\hat{s}}{\bar{s}} - \left\{1 + \frac{r+\theta}{\theta}\frac{1-\tau}{\tau}\left(\frac{\tilde{s}}{\bar{s}}\right)^{\frac{r}{\theta}}\right\}\left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}}\right] = \frac{\kappa_r/\kappa_o}{\lambda\gamma_\sigma}. \tag{28}$$

Unlike in the case of a uniform subsidy, greater subsidy K increases, rather than decreases, $\hat{s}_\tau$ under the size-dependent subsidy; this is because a decline in sales volume results in support from the government, leading to lower incentive to invest in R&D. See Online Appendix A.2 for more details.

It should be noted that the analysis in this subsection is based on the decisions of individual firms for a given aggregate state such as firm distribution, industry-level productivity growth, and so on. Numerical analysis is needed when many firms are affected by distortions, which could be idiosyncratic or aggregate, because the resultant behaviors of $\bar{s}$ and $\hat{s}$ also depend on changes in the stationary state.

3 Empirical Evidence

In this section, we provide empirical facts on the shadow of death using firm-level data for Japan; through this, we aim to check whether our model is consistent with the data.

3.1 Data

We use firm-level data provided by TSR, which is one of the largest credit rating companies in Japan and a counterpart to the Dun & Bradstreet in the United States. The data contain information on firm sales from 2001 to 2019 and on exits from 2008 to 2019. The number of firm observations is around 0.8 to 0.9 million per year. According to the Economic Census of 2016, the total number of firms in Japan is 3.9 million; thus, the TSR data cover more than 20% of all firms in Japan.[3] Regarding firm exit, 10,000 firms out of the 0.8 to 0.9 million firms exit from the market per year according to our dataset. Such exits amount to an annual exit rate of 1.1% to 1.3%. The reasons for firm exit are classified into closure, dissolution, bankruptcy (default), merger, or others by TSR.[4] Among those exit reasons, we focus on closure and dissolution, which we term "voluntary closure." This is because exit through bankruptcy and merger is associated with different mechanisms from those described

[3]Hong et al. (2020) show that the TSR data resemble the Census data in terms of geographic coverage and firm size. See Miyakawa et al. (2021) who use the same TSR data to study the effects of the COVID-19 pandemic on firm exit.

[4]According to TSR, closure is defined as the stopping of business without officially declaring dissolution when a firm is solvent (assets exceed debts), and dissolution is defined as a procedure of ending a corporate entity by declaration at a legal bureau.

in our theoretical model, and the records of voluntary closure account for around 90% of the total exit records in our dataset.

3.2 Pre-Exit Firm Dynamics

Our model shows that if firms do not make R&D investment, their sales continue to decrease until they exit from the market. We investigate whether such a pattern is observed in the data.

The baseline specification we use for the empirical analysis on pre-exit firm dynamics is as follows:

$$\log\left(\text{sales}_{j,t}\right) = \alpha + \sum_{h=0}^{H} \beta_h \mathbb{1}\left(\text{exit}_{j,t+h}\right) + \eta_{I_j,t} + \varepsilon_{j,t}, \tag{29}$$

for firm j in industry I_j, to which firm j belongs, and year t. Sales are in a nominal term. The explanatory variable $\mathbb{1}\left(\text{exit}_{j,t+h}\right)$ takes the value of 1 if firm j exits in year $t+h$ and zero otherwise, and $\eta_{I_j,t}$ accounts for the industry-year-specific fixed effects.

Coefficient β_h captures the relative sales of an exiting firm (i.e., how much larger are the sales of an exiting firm compared to the average sales of non-exiting firms) as of h years prior to its exit. As $\eta_{I,t}$ controls for industry-level sales in each year, we can interpret β_h as the relative sales share of an exiting firms, which corresponds to the sales share s_{ijt} of intermediate goods firm j in industry i in year t introduced in our theoretical model. We expect β_h to be negative because exiting firms tend to be small, as predicted by the model. Specifically, we have the exact relationship between β_1 and exit threshold $\bar{s}$, that is, $\beta_1 = \log(\bar{s}/\text{average sales})$. Further, a change in β_h (e.g., $\beta_1 - \beta_h$) indicates the speed of decline in sales for exiting firms, which is expected to be negative and corresponds to minus one times the industry productivity growth, $-\theta$, based on the model.

The following should be noted. We include $\eta_{I,t}$ to control for the effect of business cycles. Given that firms are dropped from our observations once they exit the market, the data are essentially unbalanced. Firm entry after 2001 (the beginning year of our observation period) also makes our data unbalanced. While we mainly use unbalanced data for our empirical analysis, as a robustness check, we also present the results based on relatively more balanced data that comprise firms that have survived for at least 10 years. The industry classification is based on TSR's original industry code, which categorizes each firm into around 100 industries.

First, Table 1 summarizes the estimation results of β_h for the unbalanced and balanced panel data, and Figure 1 shows the point estimates of β_h with 95% confidence intervals. As immediately observed, the sales of exiting firms are significantly smaller than those of average firms and they decrease toward the year of exit; this is consistent with the story of the shadow of death. The results are robust to the choice of unbalanced or balanced panel data.

Second, Figure 3 depicts the distribution of the relative sales volume of exiting firms in the year prior to the exit (i.e., β_1) estimated for each industry. Specifically, we estimate equation (29) for each industry with time fixed effects η_t rather than $\eta_{I,t}$, and the estimated β_1 and α are transformed to the ratio of $\bar{s}$ to fixed costs, where $\bar{s}$ is calculated as $\exp(\beta_1 + \alpha)$ and fixed costs are the sum of selling, general, and administrative expenses (see Online Appendix C.4 for details). The vertical axis is the number of industries exhibiting the ratio of $\bar{s}$ to fixed costs corresponding to each bin. Heterogeneity is observed in the relative size of exiting firms across industries.

One concern associated with the aforementioned results is that such a sales pattern is mechanically driven by aging owners, which is considered one of the most important issues for small and medium enterprises (SMEs) in Japan (Ito and Kato (2016); Tsuruta (2019); Xu (2019); Hong et al. (2020)). It is likely that owners reduce the size of the enterprises toward their retirement simply because of aging, a mechanism that differs from our model, which supposes that the fundamental factor behind R&D and exit is firm productivity. Given this concern, in Figure 4, we estimate coefficient β_h for the firms that have survived for at least 10 years and with the age of the owner ranging from 15 to 65 years. The figure confirms that the results presented above are robust even when we exclude aged firm owners. Furthermore, we have confirmed that the estimation results are robust to the inclusion of firm-level fixed-effects, the quantile regression, and the use of labor productivity instead of sales.[5]

3.3 R&D Investment and Firm Dynamics

Thus far, we have shown that firm sales dynamics toward the exit threshold are consistent with the data. In the present subsection, we further investigate whether the theoretical threshold associated with the termination of R&D is empirically sup-

[5]We do not present these results due to space constrains. The results are provided upon request.

ported. To this end, we measure the termination of R&D activities and document the dynamics before and after this termination. According to the model, we expect that, toward the point of R&D termination, the relative sales size of firms that eventually terminate their R&D activities decreases over time compared to those of firms that continue R&D. Moreover, the relative sales size of these non-R&D firms is expected to decrease further after R&D termination. In this subsection, we aim to confirm these empirical patterns by using Japanese firm-level panel data.

One issue regarding this empirical examination is how to measure the termination of R&D activities. In the following analysis, we define a dummy variable assigned to firm j and time t, that is, $\mathbb{1}\left(R\&D_{j,t,t+h'} = 0\right)$, which takes the value of 1 if firm j makes no R&D investment from year t to $t + h'$ ($h' \geq 0$) and zero otherwise. Then, when $\mathbb{1}\left(R\&D_{j,t,t+h'} = 0\right)$ takes the value of 1, we consider that firm j terminates its R&D activity at time t. In this manner, we identify the timing of firms' R&D termination in a retrospective way. This reflects our presumption that R&D investment could be lumpy (Whited (2006)). Given investment lumpiness, it is sensible to consider that a firm stops R&D investment only when it does not make R&D investment for a certain duration ($h' + 1$ years).

While employing long h' seems to be a better approach, there is a tradeoff from setting a greater value of h'. Namely, as we use a longer h', firms in the datasets are biased to larger firms that report records for several consecutive years. We rely on the data to set an appropriate h'. From our dataset, we can measure the probability of observing a positive R&D after consecutive $h' + 1$ years of non-R&D activities. If this probability is high for a certain h', we need to set a reasonably longer length than h' because observing $h' + 1$ years of non-R&D activities might not indicate the "true" termination of R&D activities. In our dataset, the probability of observing a positive R&D after observing one year of non-R&D activities (i.e., $h' = 0$) is merely 0.33% and that after two years of non-R&D activities (i.e., $h' = 1$) is merely 0.30%. Although the probability becomes slightly lower as h' becomes longer, it is a minute change. This evidence indicates that for identifying zero-R&D, it is sufficient to use $h' + 1 = 1$ or at most $h' + 1 = 2$.

Using $h' + 1 = 1$ or $h' + 1 = 2$, we estimate the following equation for $h = -5, -4, \cdots, 4, 5$:

$$\log\left(\text{sales}_{j,t}\right) = \gamma + \delta_h \mathbb{1}\left(R\&D_{j,t-h,t-h+h'} = 0\right) + \eta_{I_j,t} + \varepsilon_{j,t}. \tag{30}$$

We are interested in coefficient δ_h, which captures the relative sales of non-R&D firms compared to (i.e., how much larger are the sales of a non-R&D firm) the average sales of R&D firms as of $|h|$ years before/after R&D stoppage. Specifically, when h is negative (positive), δ_h captures the relative sales as of $|h|$ years before (after) the termination of R&D investment. As $\eta_{I,t}$ controls for industry-level sales in each year, we can interpret δ_h as the relative sales share of non-R&D firms, which corresponds to the sales share s_{ijt} of intermediate goods firm j in industry i in year t introduced in our theoretical model. In the regression, the firms that made no R&D investment are excluded, while the firms that stopped and restarted R&D investment are included.

According to the model, δ_h for $h \simeq 0$ has an exact relationship with R&D threshold $\hat{s}$, that is, $\delta_0 = \log(\hat{s}/\text{average sales for R\&D firms})$, and δ_h should be negative. Further, a change in δ_h for a positive h (e.g., $\delta_h - \delta_0$) indicates the speed of change in sales for non-R&D firms, which is expected to be negative and corresponds to minus one times the industry productivity growth, $-\theta$, based on the model. We should note that a change in δ_h for a negative h (e.g., $\delta_0 - \delta_h$) indicates the speed of change in sales for R&D firms terminating their investment in the near future, which is again expected to be negative but the size of the change in sales is smaller than that of the change in δ_h for a positive h. This is because R&D firms have a chance to increase their sales by improving their productivity.

Figure 5 presents the dynamics of sales before and after the termination of R&D investment. The left (right) panel corresponds to the case of $h' = 1$ ($h' = 2$). Both panels are consistent with the model: The estimated δ_h is significantly negative and decreases after R&D termination for a positive h. When h is negative, there is no clear decrease in δ_h; this is not necessarily inconsistent with the model because the sample before R&D termination is a mixture of successes and failures in R&D.

We have confirmed that the results reported above are robust to the use of other measures of intangible investment as R&D. We have also implemented the Probit estimation for "exit" and "zero R&D" to investigate s and $\hat{s}$ from a different perspective. We use the following specification:

$$\mathbb{1}\left(\text{Event}_{j,t}\right) = \Phi\left(\text{sales, sales growth, profit/sales,industry FE, year FE}\right) + \varepsilon_{j,t}.$$

The estimation result shows that a decrease in sales increases the probabilities of both exiting the market and terminating R&D. See Appendix B.2 for details. It also shows that the probability of terminating R&D is much higher than that of exiting for a given level of sales. This suggests that as sales decrease, firms are likely to terminate R&D and then eventually exit the market. Finally, we observe a dispersion in the data-based $\hat{s}$, similar to that in $\bar{s}$ in Figure 3, by regressing equation (30) and plotting estimated δ_h for each industry.[6]

3.4 Distortions and Firm Dynamics

In the previous two subsections, we have indicated a heterogeneity in terms of the estimated β_h and δ_h across industries. In this subsection, we empirically examine whether such heterogeneity in our estimates is statistically associated with distortions specifically measured for each industry and year. To this end, we estimate equations (31) and (32) by using industry-specific distortion measures, that is, $\text{distortion}_{i,t}$, which is explained later. We include $\eta_{I_j,t}$, accounting for the industry-year-specific fixed effects corresponding to the industry to which firm j belongs:

$$\log\left(\text{sales}_{j,t}\right) = \alpha + \beta_h \mathbb{1}\left(\text{exit}_{j,t+h}\right) + \beta_h^D \mathbb{1}\left(\text{exit}_{j,t+h}\right) \times \text{distortion}_{I_j,t} + \eta_{I_j,t} + \varepsilon_{j,t}, \quad (31)$$

$$\begin{aligned} \log\left(\text{sales}_{j,t}\right) = {} & \gamma + \delta_h \mathbb{1}\left(R\&D_{j,t-h,t-h+h'} = 0\right) \\ & + \delta_h^D \mathbb{1}\left(R\&D_{j,t-h,t-h+h'} = 0\right) \times \text{distortion}_{I_j,t} + \eta_{I_j,t} + \varepsilon_{j,t}. \end{aligned} \quad (32)$$

For simplicity, we use a single lag structure indicated by a certain h instead of including multiple $h's$ for $\text{exit}_{j,t+h}$ or $\text{R\&D}_{j,t-h,t-h+h'}$.

We are mainly interested in whether β_h^D and δ_h^D are significantly different from zero, and if so, their signs are consistent with the following predictions of our model. First, a distortion should decrease $\bar{s}$; thus, it is expected to lower $\beta_h + \beta_h^D$. In other words, β_h^D is expected to be negative. Second, a distortion should increase the gap of $\hat{s}/\bar{s}$. In other words, $\beta_h^D - \delta_h^D$ is expected to be negative.

To measure distortions, we construct the following two variables. The first variable is a distortion associated with the subsidy. We employ industry-level time-variant information for the subsidy and indirect tax obtained from the input-output table published by the Japanese Ministry of Internal Affairs and Communications. Following Beason and Weinstein (1996), we compute the rate of net subsidies (subsidies less

[6] In Online Appendix B, we provide descriptive statistics to show relationships between R&D intensity, exit probability, and sale growth, which confirms the robustness of our empirical analysis and the consistency with our model.

indirect taxes) as a percentage of value-added for each sector. Since those data are recorded only for 1995, 2000, 2005, 2011, and 2015, we map them to the years used in our dataset in the following manner. The subsidy measured in 1995 is for 2000, 2000 for 2001 to 2005, 2005 for 2006 to 2011, 2011 for 2012 to 2015, and 2015 for 2016 and onward in our dataset.[7]

The second variable is capital resalability, which is associated with post-exit outside option values and therefore with the inverse of a distortion. We employ the industry×year-level information on capital investment, which is collected by the Japanese Cabinet Office as a part of the System of National Accounts. As the data are divided into capital investment on new and used assets, we can compute the ratio of the latter to the sum of capital investment on new and used assets for industry×year. If this ratio is high, it means that the resalability of capital assets in a specific industry and year is high. Given that these data are only available from 2006, we map the years of resalability to the years used in our dataset in the following manner. The resalability measured in 2006 is for 2000 to 2007, and that measured in year t ($t \geq 2007$) is for $t + 1$ in our dataset.

Table 2 summarizes the estimation results. The top table represents the case in which a distortion is measured by the subsidy. As before, β_h and δ_h are negative. More importantly, as the model predicts, the coefficient associated with the interaction term of $\text{exit}_{j,t+h} \times \text{distortion}_{i,t}$, β_h^D, is negative. This suggests that the relative sales of exiting firms prior to the exit become smaller as the distortion associated with the subsidy increases. Moreover, the coefficient associated with the interaction term of $(R\&D_{j,t-h,t-h+h'} = 0) \times \text{distortion}_{i,t}$, δ_h^D, is positive. This suggests that the relative sales of firms that stop R&D become larger as the distortion associated with the subsidy increases. The fact that $\beta_h^D - \delta_h^D$ is negative suggests that as the distortion associated with the subsidy increases, the length of the shadow of death becomes longer. This is consistent with the model prediction.

The bottom table represents the case in which a distortion is measured by capital

[7]While the data obtained from the input-output table do not have any information regarding the contents of the subsidy, we can roughly see the breakdown of the budgets associated with each ministry. For example, in the case of Ministry of Economy, Trade and Industry, out of its total annual budget (fiscal year 2021), 920 billion JPY, more than half, is used for energy policy. The rest of the budget is allocated to mostly the salary of the public offers and subsidy. Most of the items are recorded as the subsidy in their budgets target SMEs, which is consistent with our empirical results. Beason and Weinstein (1996) argue that the public role in R&D support is limited for Japan, although R&D subsidies influence R&D and exit decisions differently from size-dependent subsidies.

resalability. As before, β_h and δ_h are negative. Here, if an improvement in capital resalability corresponds to an increase in an outside option value, capital resalability should influence exit and R&D in the opposite direction to the subsidy. The estimation result indicates that β_h^D is indeed positive. This means that firms tend to exit earlier (later) as the resalability becomes higher (lower), which is consistent with our theoretical prediction. However, unlike our prediction, δ_h^D is in fact positive. This suggests that higher resalability, which we interpret as smaller distortion, induces firms to stop their R&D relatively earlier. Furthermore, the negative number for $\beta_h^D - \delta_h^D$ means that when facing higher (lower) resalability, the shadow of death is longer (shorter). One possible explanation for this inconsistency between the model prediction and the empirical findings regarding the response of $\hat{s}$ to this type of distortion is as follows: Capital resalability makes investment in tangible assets, which can be sold in the secondary market, more profitable than investment in R&D, which is problematic to sell in the secondary market. If this is the case, higher resalability would decrease the incentive for R&D investment.

These empirical results show that there exists heterogeneity in terms of exit and R&D decisions across industries, part of which can be explained by distortions.

4 Quantitative Investigations

An immediate question that follows from the preceding discussions is how large the inefficiency associated with the shadow of death is. In Japan, we observe a very low rate of firm entry and exit. Does this inactive entry/exit suggest that the channel we consider in this study is unimportant or that some kinds of distortions are creating a long and inefficient shadow of death and depressing the entry and exit rate? To answer these questions, in this section, we simulate the effects of distortions on the economy. We use the model introduced in Section 2 that is calibrated to the Japanese economy based on the TSR data we used in Section 3. For data fitting, we add the firm exit rate due to exogenous shocks, $\bar{\delta}$. This modification does not change the essence of the model. See Online Appendices C.1 and C.2 for details.

4.1 Calibration

The unit of time is year. We set $\rho = 0.05$,$\sigma = 4.3$, $L = 1$, and $\kappa_e = 9$. The value of σ is chosen so that the markup ratio is 1.3, which is consistent with Hall (2018)'s estimate. The value of κ_e is fixed because it does not appear to influence targeted variables independently of other parameter values such as κ_o and κ_r. We assume a normal distribution for $\log(s)$ of potential entrants, where the mean is normalized to zero and the standard deviation is 1.402. The latter value is estimated by using the distribution of the log sales of young firms that are less than three years old based on their establishment date in the TSR data. We further assume that size-dependent subsidy τ takes five values, rather than one, so that firm distribution becomes smooth. Specifically, τ is assumed to follow a normal distribution. We set the value of $\tilde{s}$ so that the size-dependent subsidy is distributed to firms with sales $s < \tilde{s} < \hat{s}$. While the size of the standard deviation for τ, which is set at 0.1, has little impact on our results, the size of the mean matters, as we present simulation results below by changing the mean value. We calibrate the mean of τ to 0. The other five calibrated parameters are $\lambda = 0.037$, $\bar{\delta} = 0.0028$, $\gamma = 0.11$, $\kappa_o = 0.055$, and $\kappa_r = 0.035$. To calibrate these six parameters, we target the following six variables: the probability of positive sales growth for R&D firms relative to non-R&D firms (equivalent to λ), the exit rate of R&D firms (equivalent to $\bar{\delta}$), the entry rate (equivalent to δ), the share of fixed costs in sales (related to κ_o), the share of R&D costs in sales for R&D firms (related to κ_r), and the ratio of the R&D threshold to the exit threshold (equivalent to $\hat{s}/\bar{s}$). Table 3 shows the calibration results and suggests the goodness of fit.

In the table, we also present three untargeted variables, namely, the ratio of the mean of sales for all firms to that for entrants, the ratio of the standard deviation of sales for all firms to that for entrants, and the speed of change in sales for non-R&D firms (equivalent to $-\theta$). The simulation yields a slightly lesser speed of change in sales for non-R&D firms, $\theta = 0.033$, where the data suggest $\theta = 0.040$. Note that the length of time in the shadow of death is given by $\log(\hat{s}/\bar{s})/\theta$. Thus, the length of time in the shadow of death in the simulation is slightly longer than that the data suggest. Further, the simulation yields smaller and larger values for the ratios of the mean and standard deviation, respectively, of sales for all firms to that for entrants. See Online Appendices C.3 and C.4 for details on the variables based on both the data and model.

4.2 Simulation Results

We simulate the effects of distortions on the economy by changing one of two distortion measures, τ, that is, the size-dependent subsidy or outside option values. Figure 6 shows the simulation results when we change the degree of size-dependent subsidy $1-\tau$ from -0.2 to 0.2. An increase along the horizontal axis corresponds to an increase in the subsidy, which enables low productivity firms to survive longer, in turn yielding a downward slope of exit threshold $\bar{s}$. The decrease in the exit rate is accompanied by a decrease in the entry rate. The slope of R&D threshold $\hat{s}$ is positive, and thus, the subsidy increases the gap $\hat{s}/\bar{s}$, which decreases real growth rate g. Consequently, welfare, measured in units of consumption, decreases. Figure 7 depicts a change in firm-size distribution for various τ's, where the line width becomes thinner as the subsidy increases. The figure shows that an increased subsidy increases the proportion of low productivity (sales) firms, which contributes to a decrease in wage (w), a decrease in profits, and a decrease in market concentration (Herfindahl-Hirschman Index, HHI).

Next, Figure 8 presents the simulation results when we change outside option value $1-\tau$ from -0.2 to 0.2. An increase along the horizontal axis corresponds to an decrease in outside option value. The simulation results are similar to those for the size-dependent subsidy. One notable difference from the case of the size-dependent subsidy is that the slope of $\hat{s}$ is negative, and the increase in the gap $\hat{s}/\bar{s}$ is smaller.

It should be emphasized that the effect of distortions on the economy is not large. The figure shows that the the entry rate and real growth rate change only by the order of 10^{-3} and 10^{-4}, respectively. Thus, the elimination of distortions should not be suggested for promoting R&D investment and raising the real growth rate, at least in terms of firm entry/exit.[8]

We also calculate the socially optimal state that was discussed in Section 2.3, where the details are relegated to Appendix A.3. We confirm that R&D threshold $\hat{s}$ decreases considerably from the market equilibrium, while exit threshold $\bar{s}$ does not change much. The gap $\hat{s}/\bar{s}$ decreases, shortening the length of the shadow of death. Further, entry rate μ increases. Consequently, real growth and welfare increase. The numerical simulation suggests that real growth rate g increases by around one percentage point. This magnitude can be interpreted as sizable; however, it also

[8]Edmond et al. (2018) argue that subsidizing entry is not an effective tool and that size-dependent policies aimed at reducing concentration and markups need to be viewed with caution.

implies that the promotion of R&D investment and entry can contribute to real growth by, at most, this magnitude.

In summary, the results imply that an increase in the subsidy and/or a decrease in outside option value reduces welfare through lower entry and a prolonged shadow of death. Further, they can help explain firm dynamics in Japan to some extent, manifesting as a decrease in market concentration and entry.

5 Concluding Remarks

In this study, we focused on the slow process of firm exit as the basis of inefficient resource allocation and declining business dynamism in Japan. We conducted an empirical analysis using Japanese firm data and constructed a model that incorporates R&D and exit—including the shadow of death. Through this, we showed that various support measures mainly for SMEs could prolong the life span of firms that should be eliminated from the market, distorting the effective allocation of resources and worsening welfare.

The most important point to be considered in future analysis is the transition process. In the model, the analysis was limited to the steady state. It did not consider the short-term transition process of the economy when the external environment changes, and it did not assume any frictions in the movement of workers between firms. In reality, however, the movement of workers from firms that are exiting or are on the verge of exiting to firms with higher productivity is not smooth, and frictional unemployment is likely to occur. It will be necessary to develop a more sophisticated model that incorporates realistic transition processes, and simultaneously, it would be appropriate to discount the implications for social welfare of the subsidy reductions simulated in this study.

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Table 1: Pre-exit Firm Dynamics

Pre-exit dynamics	Unbalanced			Firms surviving for at least 10 years		
	Coef.	s.e.		Coef.	s.e.	
β_1	-1.526	0.005	***	-1.773	0.012	***
β_2	-1.378	0.005	***	-1.656	0.012	***
β_3	-1.290	0.005	***	-1.583	0.012	***
β_4	-1.218	0.005	***	-1.533	0.013	***
β_5	-1.148	0.005	***	-1.486	0.015	***
β_6	-1.086	0.005	***	-1.440	0.017	***
β_7	-1.034	0.005	***	-1.395	0.020	***
β_8	-0.981	0.005	***	-1.360	0.025	***
β_9	-0.941	0.005	***	-1.393	0.041	***
β_{10}	-0.913	0.005	***			
β_{11}	-0.885	0.006	***			
β_{12}	-0.863	0.006	***			
β_{13}	-0.834	0.007	***			
β_{14}	-0.809	0.008	***			
β_{15}	-0.780	0.008	***			
β_{16}	-0.750	0.010	***			
Fixed-effect						
Year×Industry		yes			yes	
Number of observations	16,491,824			2,620,854		
Adj R-squared	0.1893			0.1733		

Notes: Coefficient β_h captures the relative sales of an exiting firm as of h years prior to its exit.

Table 2: Distortions and Firm Dynamics

(i) Distortion: Net subsidy/Value-added

	Pre-exit dynamics						Pre/post-R&D termination dynamics					
	$h=1$			$h=3$			$h=-1$, $h'=1$			$h=1$, $h'=1$		
	Coef.	s.e.		Coef.	s.e.		Coef.	s.e.		Coef.	s.e.	
β_h	-1.393	0.011	***	-1.278	0.012	***						
β_h^D	-0.401	0.134	***	-0.492	0.148	***						
δ_h							-0.889	0.021	***	-0.934	0.023	***
δ_h^D							0.416	0.204	**	0.544	0.219	**
Fixed-effect												
Year×Industry	yes			yes			yes			yes		
Number of observations	9,064,930			6,983,006			80,344			70,021		
Prob>F	0.0000			0.0000			0.0000			0.0000		
Adj R-squared	0.1585			0.1373			0.3810			0.3844		

(ii) Distortion: Capital investment on used assets / Total capital investment

	Pre-exit dynamics						Pre/post-R&D termination dynamics					
	$h=1$			$h=3$			$h=-1$, $h'=1$			$h=1$, $h'=1$		
	Coef.	s.e.		Coef.	s.e.		Coef.	s.e.		Coef.	s.e.	
β_h	-1.493	0.018	***	-1.421	0.019	***						
β_h^D	0.259	0.067	***	0.494	0.073	***						
δ_h							-1.305	0.036	***	-1.332	0.039	***
δ_h^D							1.269	0.154	***	1.181	0.166	***
Fixed-effect												
Year×Industry	yes			yes			yes			yes		
Number of observations	4,756,232			3,577,931			49,401			43,321		
Prob>F	0.0000			0.0000			0.0000			0.0000		
Adj R-squared	0.1393			0.1420			0.3614			0.3633		

Notes: Coefficient β_h captures the relative sales of an exiting firm as of h years prior to its exit. Coefficient δ_h captures the relative sales of a firm as of $|h|$ years before/after R&D stoppage. Coefficients β_h^D and δ_h^D represent those on the interaction terms of the exit and R&D stoppage dummies, respectively, × distortions.

Table 3: Calibration

		Data	Simulation
Targeted moments			
	Prob. of sales share increase for R&D firms	0.037	0.037
	Prob of exit for R&D firms	0.0028	0.0028
	Entry rate	0.006 (0.051)	0.016
	Share of fixed costs in sales	0.050	0.047
	Share of R&D costs in sales for R&D firms	0.028	0.030
	Ratio of R&D threshold to exit threshold	4.080	4.091
Untargeted moments			
	Ratio of the mean of sales of all firms to entrants	0.971	0.667
	Ratio of the SD of sales of all firms to entrants	0.534	0.691
	Speed of sales change for non R&D firms	-0.040	-0.033

Notes: The entry rate in parentheses is derived from the Annual Report on Employment Insurance by the Ministry of Health, Labour and Welfare.

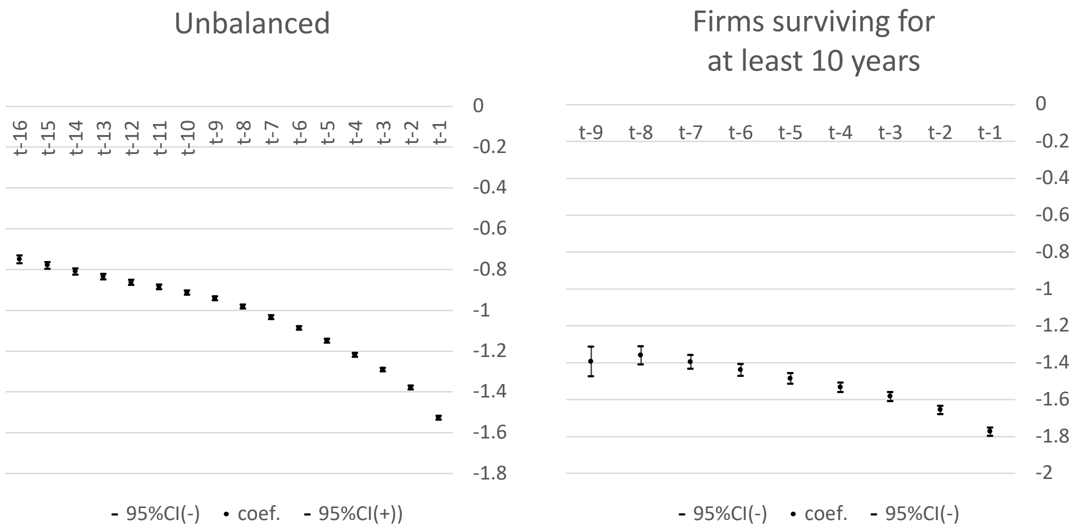

Figure 1: Pre-exit Firm Dynamics: Estimate β_{t-h}

Note: The figure shows coefficient β_{t-h} for each h, which captures the relative sales of an exiting firm as of h years prior to its exit.

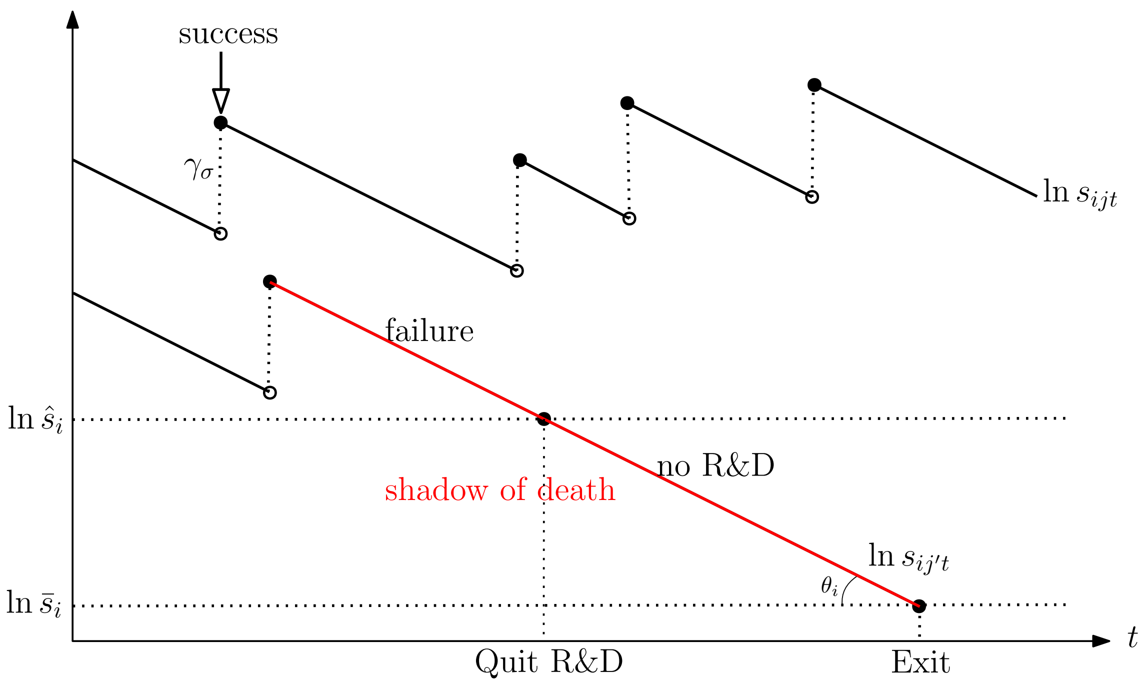

Figure 2: Dynamics of Relative Productivity (Sales)

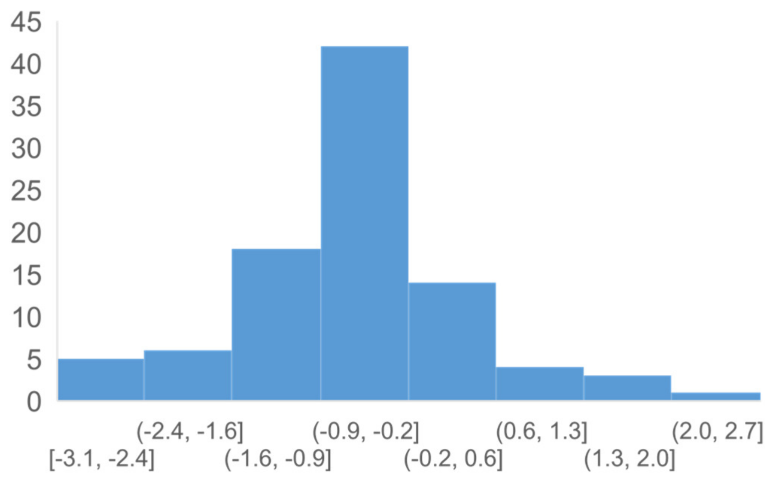

Figure 3: Dispersion of Exit Thresholds

Note: The horizontal axis indicates $\bar{s}$ over fixed costs, where $\bar{s}$ is calculated as $\exp(\beta_1 + \alpha)$ for the regression of equation (29). The vertical axis is the number of industries.

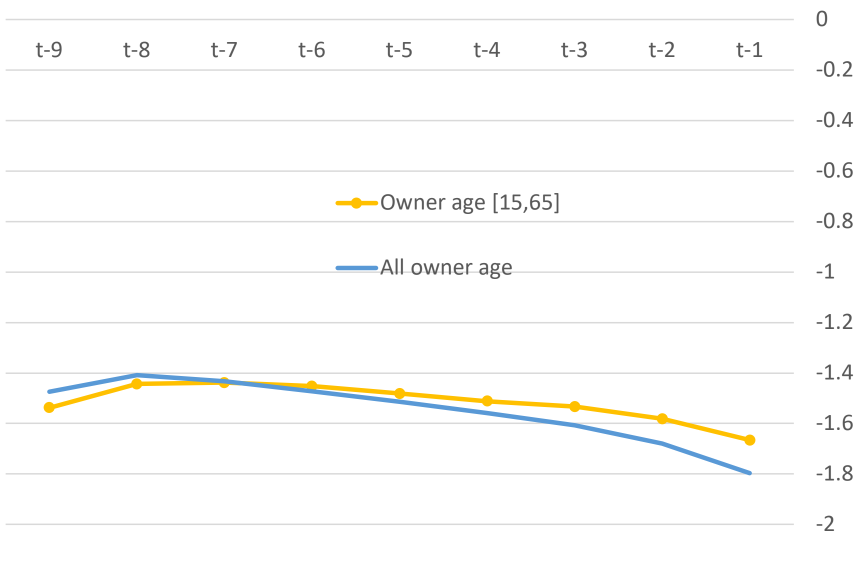

Figure 4: Pre-exit Firm Dynamics: Dependence on the Ages of Owners

Note: The figure shows coefficient β_{t-h} for each h, which captures the relative sales of an exiting firm as of h years prior to its exit.

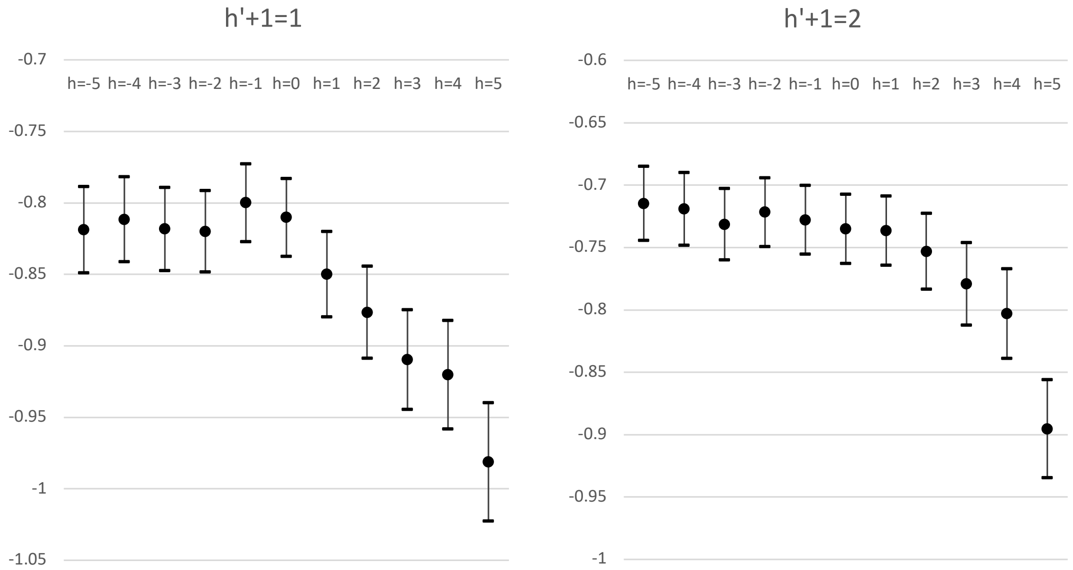

Figure 5: Firm Dynamics Before/After R&D Stoppage

Note: The figure shows coefficient δ_h for each h, which captures the relative sales of a firm as of $|h|$ years before/after R&D stoppage. In the left and right panels, we consider that a firm stops R&D investment when it makes no R&D investment for $h' + 1 = 1$ and 2 years, respectively.

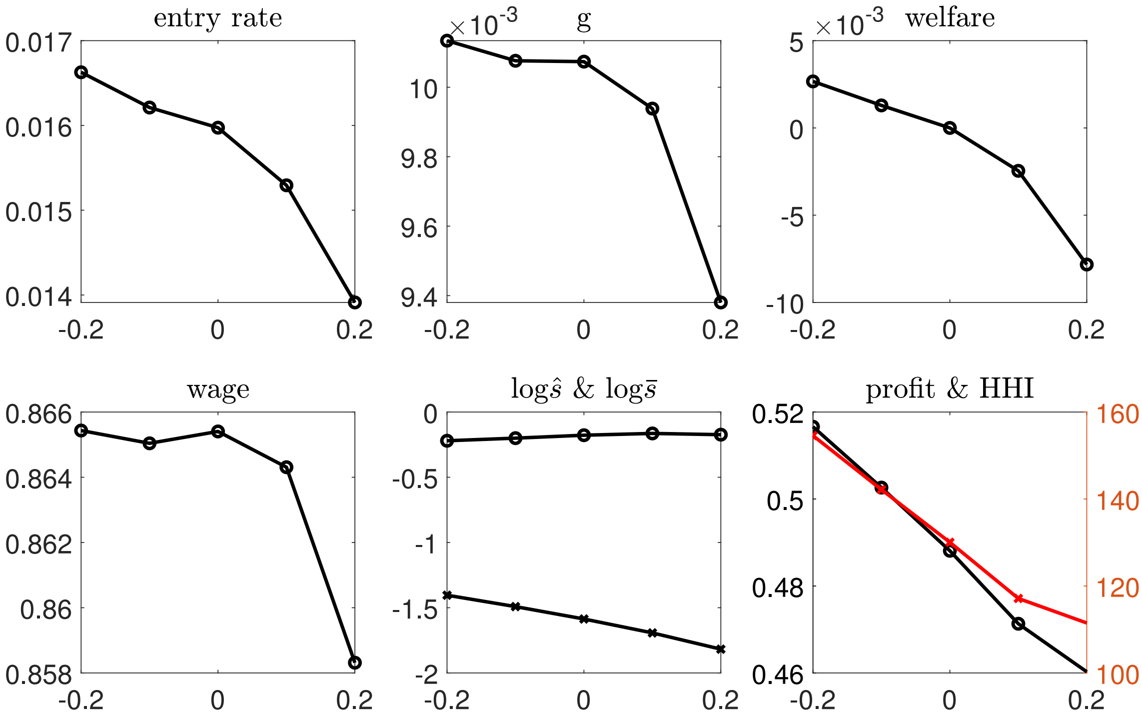

Figure 6: Effects of a Size-Dependent Subsidy on the Macroeconomy

Note: The horizontal axis represents subsidy $1-\tau$; $\bar{s}$ and $\hat{s}$ are expressed in logarithm as the line with crosses and the line with circles, respectively; and the HHI is indicated as the red line with crosses on the right axis.

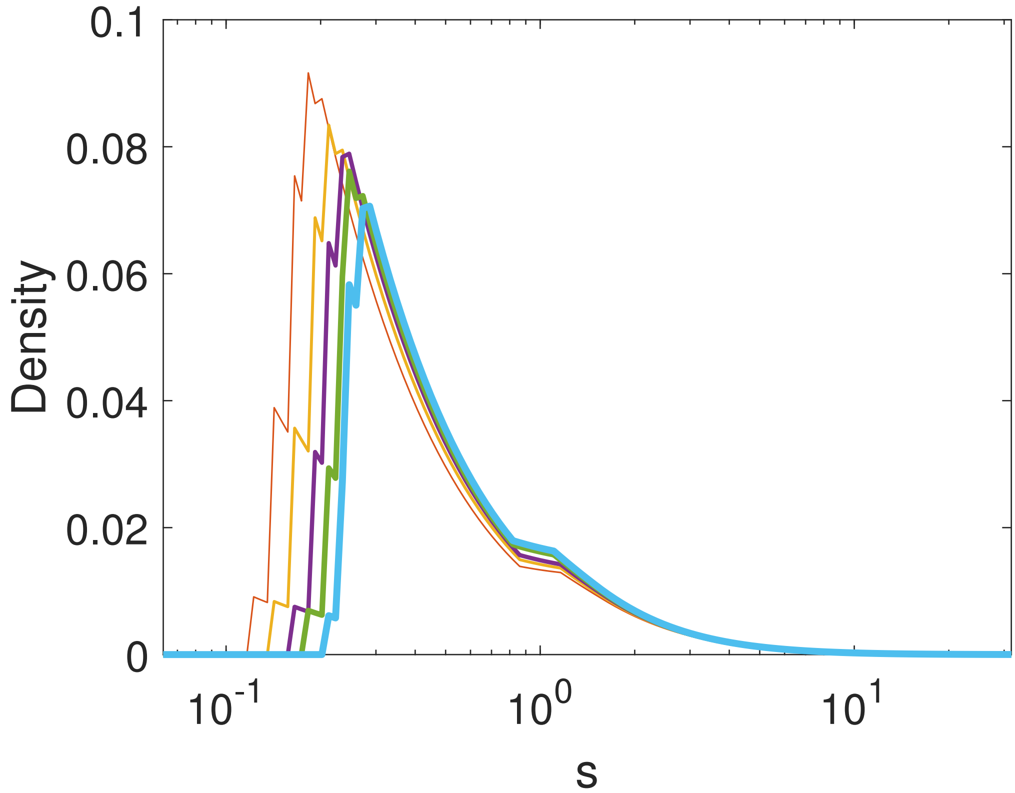

Figure 7: Effects of a Size-Dependent Subsidy on Firm-Size Distribution

Note: Firm distribution is drawn for various values of subsidy $(1-\tau)$, where the horizontal axis represents sales s. The line width becomes thinner as the subsidy increases.

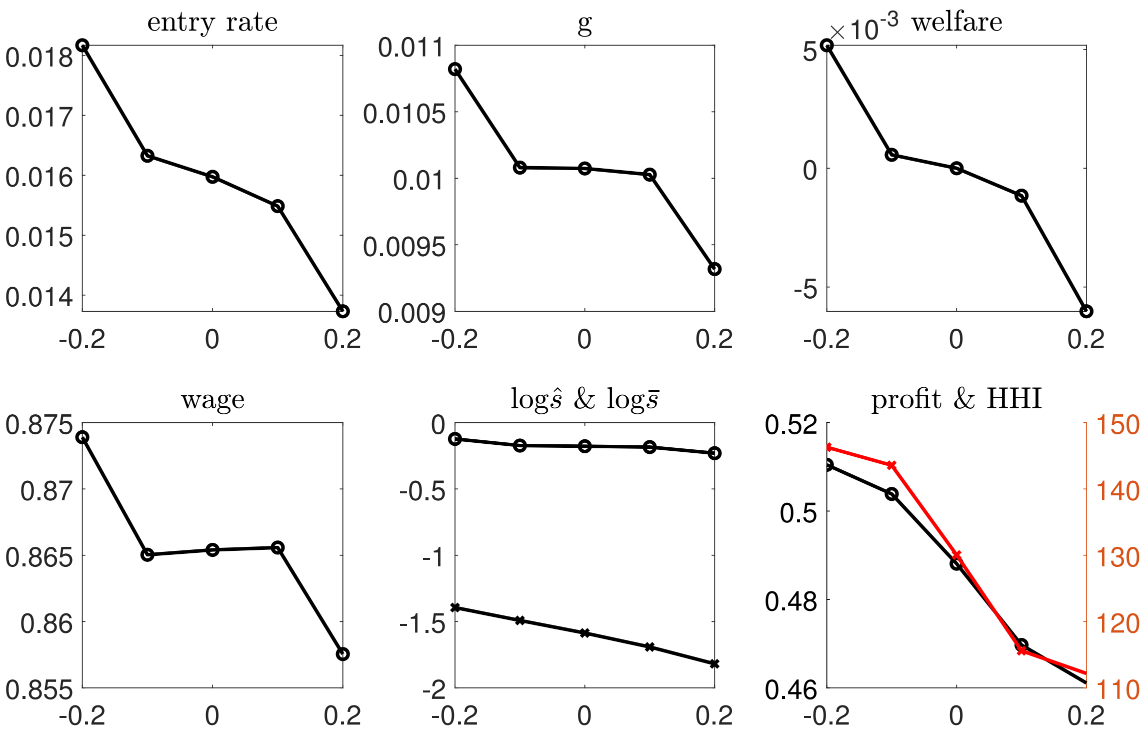

Figure 8: Effects of Outside Options on the Macroeconomy

Note: The horizontal axis represents outside option value $1 - \tau$; $\bar{s}$ and $\hat{s}$ are expressed in logarithm as the line with crosses and the line with circles, respectively; and the HHI is indicated as the red line with crosses on the right axis.