Scaling Auctions as Insurance: A Case Study in Infrastructure Procurement

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Motivation: American Infrastructure Investment

- Major responsibility of the public sector
- Approximately 2.4% of GDP annually
  - $416 billion total (in 2014)
  - $165 billion on highways/bridges
- Major area of Public-Private Risk-Sharing
  - Construction typically contracted to private firms
  - Small firms are often used (and sometimes advantaged)
  - Increasing interest in long-term private management of public works

Uncertainty and Risk Aversion

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  - Construction firms are often "mom and pop"

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- Small firms
  - Construction firms are often "mom and pop"
  - Public procurement contracts are large relative to earnings
- Construction projects (and especially repairs) are uncertain
  - Additional damage/requirements are often discovered once work has started
  - Changing conditions generate shocks to costs, work load, etc.

Uncertainty and Risk Aversion

Auction design with risk averse agents is different

- Revenue equivalence does not hold

$\Rightarrow$ First price auctions dominate second price

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  - Pre-committing to paying for adjustments
  - Setting caps on the amount of risk allowed in a bid

MassDOT Scaling Auctions

- Auctions to procure construction + maintenance for Massachusetts bridges
  - \$100 million annual budget
  - \$3.7 billion backlog
- Scaling auctions: used extensively in infrastructure procurement

  $\Rightarrow$ "Bid Express" software used by 39 different state DOTs

Scaling Auctions

- Gov't elicits unit bids for every "item" involved in a project
- Winner is evaluated on the sum of unit bids x DOT quantity estimates
- Winner is paid based on quantities actually used

Uncertainty and Risk Sharing

- Two sources of strategic uncertainty:
  1. Chance of winning the auction
  2. Chance of bidding "incorrectly" [ex-post]

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- Two sources of strategic uncertainty:
  1. Chance of winning the auction
  2. **Chance of bidding "incorrectly" [ex-post]**

  $\Rightarrow$ Scaling auctions partially "insure" bidders against #2
    - DOT underestimates are covered (in principle)
    - Bidders can optimize "portfolio" of risks

Presentation Outline

- Argue that risk aversion is important for
  1. (Simple) Empirical Predictions
  2. Identification/Estimation of the Bidder's Problem
  3. Market Design
- Present a model for equilibrium bidding under uncertainty + risk aversion
- Show evidence + estimates from MassDOT bridge auctions
- Discuss policy recommendations from simulated counterfactuals

A Portfolio Model of Scaling Auction Bidding

- A project consists of:
  - *Ex-Ante* (Estimated) Quantities: $q_1^e, \ldots, q_T^e$
  - *Ex-Post* (Actual) Quantities: $q_1^a, \ldots, q_T^a$
  - Market-Rate Unit Costs: $c_1, \ldots, c_T$
  - Features (project manager ID, project type, etc.): X

A Portfolio Model of Scaling Auction Bidding

- A project consists of:
  - DOT Quantity Estimates: $q_1^e, \ldots, q_T^e$
  - Actual Quantities: $q_1^a, \ldots, q_T^a$
  - Market-Rate Unit Costs: $c_1, \ldots, c_T$
  - Features (project manager ID, project type, etc.): X

- Bidders:
  - (Independent Private) Types
  - Identical beliefs about ex-post quantities (no winner's curse)
  - Cannot influence ex-post quantities (no moral hazard)

A Portfolio Model of Bidding: 1D Types & CARA Risk Aversion

- Risk Aversion:
  - Bidders are risk averse, w/ private CARA utility:

$$u_i(\pi) = 1 - exp(-\gamma_i \pi)$$

- Efficiency Types:
  - Bidders have private "efficiency" cost types:

$$c_{i,t} = \alpha_i \cdot c_t \text{ for every } t$$

- Information Structure:
  - Bidders get a public noisy signal of the ex-post quantity of each item:

$$q_t^b = q_t^a + \epsilon_t \text{ where } \epsilon_t \sim \mathcal{N}(0, \sigma_t^2)$$

Bidder Profits

Each bidder i maximizes her expected utility subject to risk aversion:

$$\mathbb{E}[u(\pi(\mathrm{b}^i, \alpha^i))] = \underbrace{\left(1 - \mathbb{E}_{q^a}\left[\exp\left(-\gamma_i \sum_{t=1}^{T} q_t^a \cdot (b_t^i - \alpha^i c_t)\right)\right]\right)}_{\text{Expected Utility Upon Winning}} \times \underbrace{\mathrm{Prob}\left\{s^i < s^j \text{ for all } j \neq i\right\}}_{\text{Probability of Winning}}$$

where $s^i = \sum_{t=1}^{T} b^i q_t^e$ is the score implied by b^i.

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Equilibrium Bidding

1. Each α^i chooses the optimal score $s(\alpha^i)$ s.t.:

$$\frac{\partial \mathbb{E}[u(\pi(\tilde{s}, \alpha^i))]}{\partial s}\Big|_{\tilde{s}=s(\alpha^i)} = 0$$

2. For each (α^i, s), b^i maximizes the **certainty equivalent** of profits:

$$\max_{\mathrm{b}^i} \left[\sum_{t=1}^{T} \underbrace{q_t^b(b_t^i - \alpha^i c_t)}_{\text{Expectation of Profits}} - \underbrace{\frac{\gamma_i \sigma_t^2}{2}(b_t^i - \alpha^i c_t)^2}_{\text{Variance of Profits}} \right]$$

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Model Predictions: Equilibrium Bids

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$$b^*_{i,t}(s^i) = \alpha^i c_t + \frac{q^b_t}{\gamma_i \sigma^2_t} + \frac{q^e_t}{\sigma^2_t \sum_{p=1}^{T} \left[\frac{(q^e_p)^2}{\sigma^2_p}\right]} \left(s^i - \sum_{p=1}^{T} \left[\alpha^i c_p q^e_p + \frac{q^b_p q^e_p}{\gamma_i \sigma^2_p} \right] \right)$$

Detailed Assumptions

Model Predictions

- Items w/ higher expected quantities (overruns) $\left(\frac{\mathbb{E}[q_t^a]}{q_t^e}\right) \Rightarrow$ higher markups
- Items with higher variance $(\sigma_t^2) \Rightarrow$ lower (absolute) markups
- Generally, bid skewing is a function of:
  - Levels of risk aversion (γ)
  - Level of risk $(\mathbb{E}[q^a], \sigma^2)$
  - Level of competition

Example

Data and Reduced Form

MassDOT Data

- Years: 1998-2016
- Type: Highway and Bridge, Construction and Maintenance
- Number of Projects: 440 (bridge only)
- Winning bids, losing bids, and DOT cost estimates
- Types of material, DOT quantity estimates, and amount of each material actually used
- Other information about project managers, general project location, dates of work, etc.

Summary Statistics

Statistic	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Project Length (Estimated)	1.53 years	0.89 years	0.88 years	1.48 years	2.01 years
Project Value (DOT Estimate)	$2.72 million	$3.89 million	$981,281	$1.79 million	$3.3 million
# Bidders	6.55	3.04	4	6	9
# Types of Items	67.80	36.64	37	67	92
Ex-Post Overruns	−$26,990	$1.36 million	−$208,554	$15,653	$275,219
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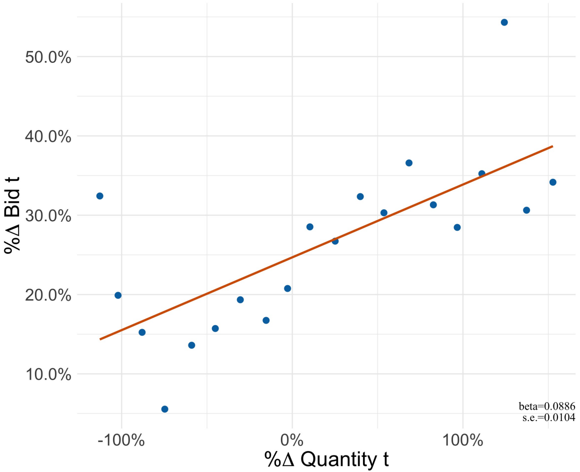
Prediction 1: Items that overrun in quantity more are overbid more:

$$\uparrow \frac{q_t^a - q_t^e}{q_t^e} \Rightarrow \ \uparrow \frac{b_t - c_t}{c_t}$$

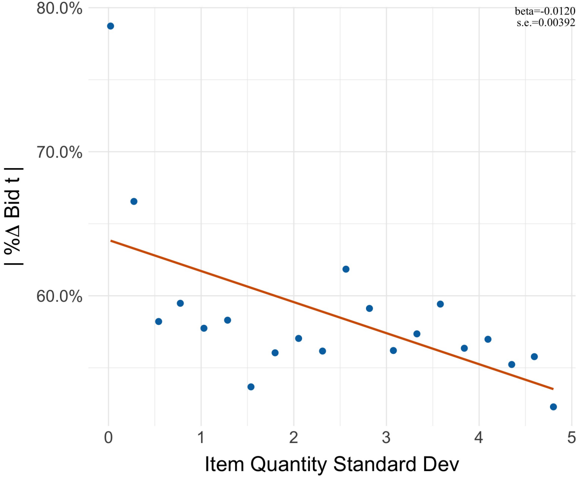
Can Massdot Bidders Predict Over-Runs?

Winning Bidders Over-Bid on Items that Over-Run



Prediction 2: Items that are more uncertain have lower markups

Absolute Markups Decrease with Item Variance



More Evidence

Quantity Overrun Histogram

How Material is the Risk in Our Setting?
(A Structural Estimation)

Structural Model of Bidding (Overview)

- Model of optimal bidding:
  - Bidders observe a noisy signal of each item's quantity
  - Bidders differ by a **private cost-multiplier** $\alpha_{i,n}$
  - Bidders are risk averse w/ a **private CARA coeff** $\gamma_{i,n}$
- Estimate parameters:
  - Statistical model for item quantity signals
  - Economic model of optimal bidding for
    (a) Bidders' Cost Types
    (b) Bidders' CARA Coefficients

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$\Rightarrow$ **Auctioneer can gain by paying to reduce risk:**

  - **Pre-committing to paying for adjustments** $\Rightarrow$ Lump sum would cost 86.8% more on median
  - Investing in reducing uncertainty
  - Setting caps on the amount of risk allowed in a bid

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- Pre-committing to paying for adjustments $\Rightarrow$ Lump sum would cost 86.8% more on median
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- Setting caps on the amount of risk allowed in a bid

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- Pre-committing to paying for adjustments $\Rightarrow$ Lump sum would cost 86.8% more on median
- Investing in reducing uncertainty $\Rightarrow$ Best Case Scenario: save $\approx$ 12.6% on median
- **Setting caps on the amount of risk allowed in a bid** $\Rightarrow$ A minimum bid of 25% the market rate saves $\approx$ 3.2%, reducing the uncertainty premium by a quarter.

Thank You

Thank You!

Certainty Equivalents Balance Linear Profits Against Risk Variance

a $b_{\text{concrete}} = \$12$ and $b_{\text{cone}} = \$19$

$$\underbrace{12 \times (\$0) + 16 \times (\$1)}_{\text{Expection of Profits}} - \underbrace{\frac{0.05 \times 2}{2} \times (\$0)^2 - \frac{0.05 \times 1}{2} \times (\$1)^2}_{\text{Variance of Profits}} = \$15.98$$

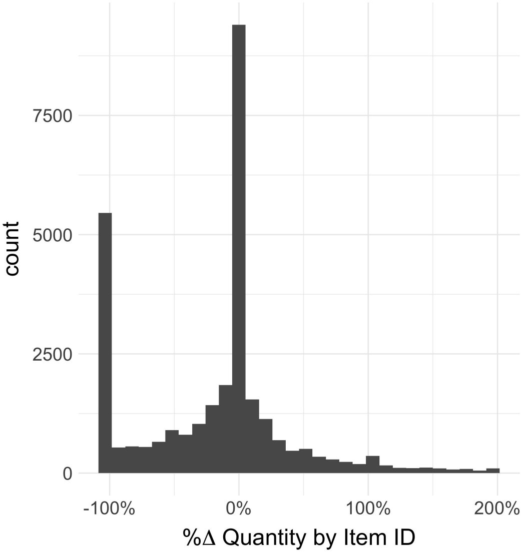
or

b $b_{\text{concrete}} = \$40$ and $b_{\text{cone}} = \$5$

$$\underbrace{12 \times (\$28) + 16 \times (-\$13)}_{\text{Expection of Profits}} - \underbrace{\frac{0.05 \times 2}{2} \times (\$28)^2 - \frac{0.05 \times 1}{2} \times (-\$13)^2}_{\text{Variance of Profits}} = \$84.58$$

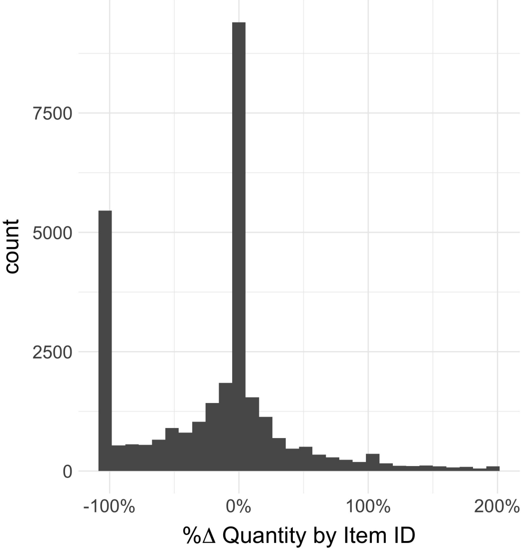
▸ Math for Score = \$1000

Distribution of Quantity Over/Under-Runs



Back to Summary Stats

Distribution of Quantity Over/Under-Runs



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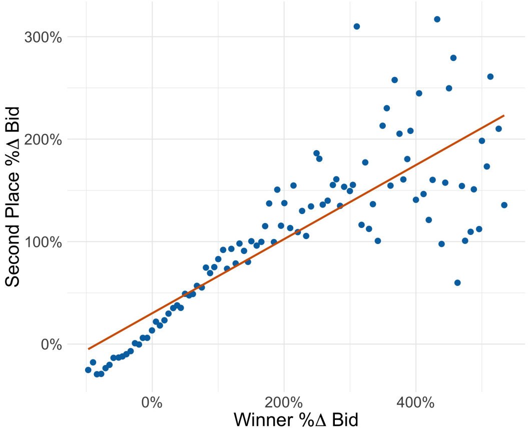
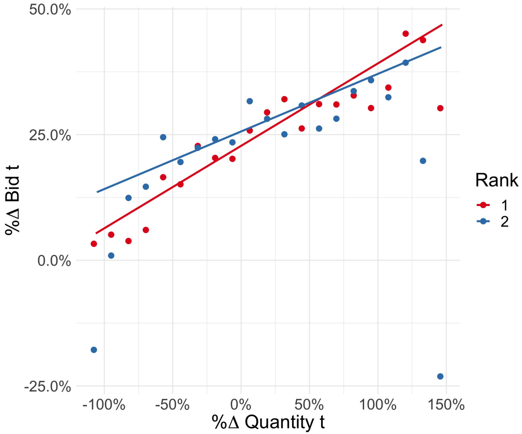
The Top Two Bidders Bid Similarly on Average



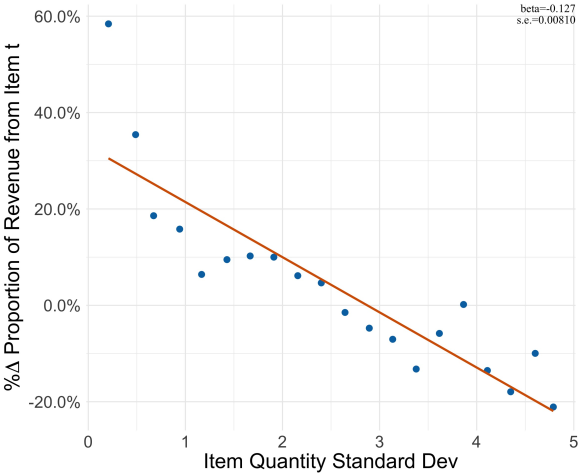
Back to Bidder Comparison

Top Two Bids are Especially Close on Items that Don't Go Unused



▸ Back to Bidder Comparison

The Proportion of Revenue from each Item Decreases with its Variance



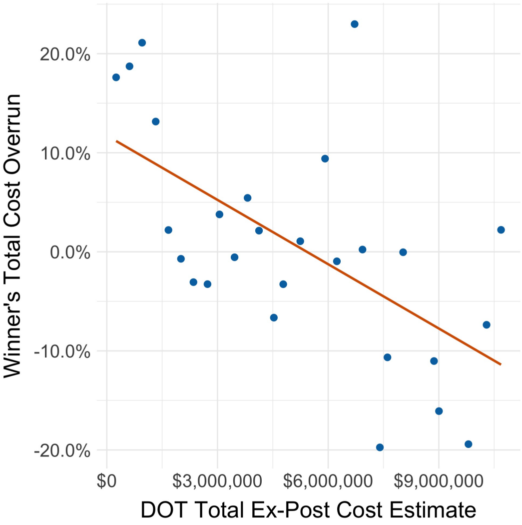
▸ Definition ▸ More Evidence ◂ Back

Bid Revenue Proportion Definition

$$\%\Delta \text{ Proportion Revenue from } t = \frac{\frac{b_t q_t^a}{\sum_p b_p q_p^a} - \frac{c_t q_t^e}{\sum_p c_t q_p^e}}{\frac{c_t q_t^e}{\sum_p c_t q_p^e}} \times 100$$

▸ Back to Bin Scatter

Ex-Post Overruns are Lower for Higher Value Projects



Back to Rev Proportion Scatter

Estimating A Model of Quantity Uncertainty

For each item t in auction n:

- Predict best-fit of ex-post quantity given:
  - DOT estimate $q^e_{t,n}$
  - Item-Auction features $X_{t,n}$
- Estimate using Hamiltonian Monte Carlo
- Output:
  - Predicted quantity: $\widehat{q^b_{t,n}}$
  - Residual variance: $\hat{\sigma}^2_{t,n}$

Back Details Model Fit

Estimation: Quantity Signal Model

$$q^a_{t,n} = \beta_{0,q} q^e_{t,n} + \vec{\beta}_q X_{t,n} + \eta_{t,n}$$

where

$$\eta_{t,n} \sim \mathcal{N}(0, \hat{\sigma}^2_{t,n})$$

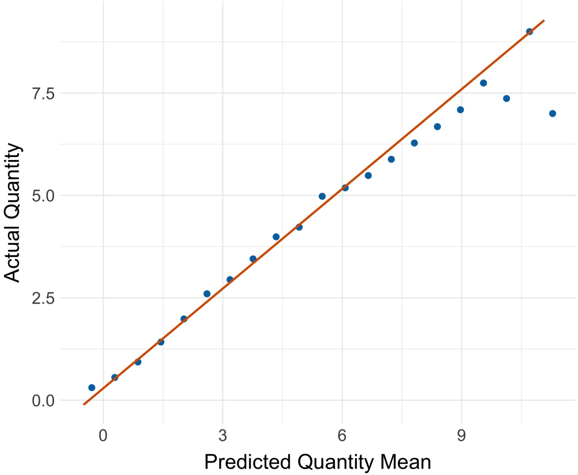
and

$$\hat{\sigma}_{t,n} = \exp(\beta_{0,\sigma} q^e_{t,n} + \vec{\beta}_\sigma X_{t,n}).$$

Back to Quantity Model Overview

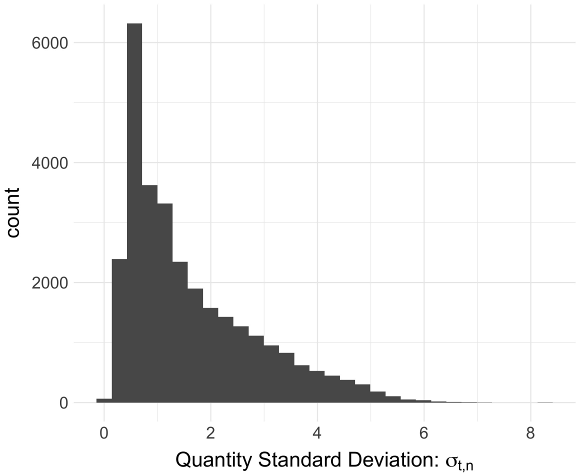
Quantity Signal Model Prediction Fit

Predicted Item Quantities Against Realized Quantities (Bin Scatter)



▸ Back to Quantity Model Overview

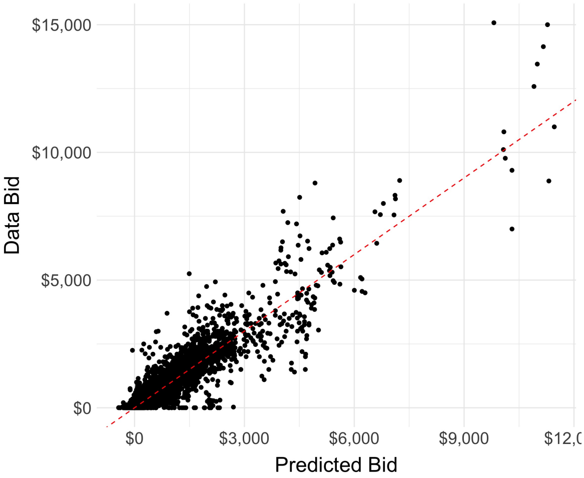
Quantity Signal Model Residual Standard Deviations



Back to Quantity Model Overview

Second Stage Model Fit

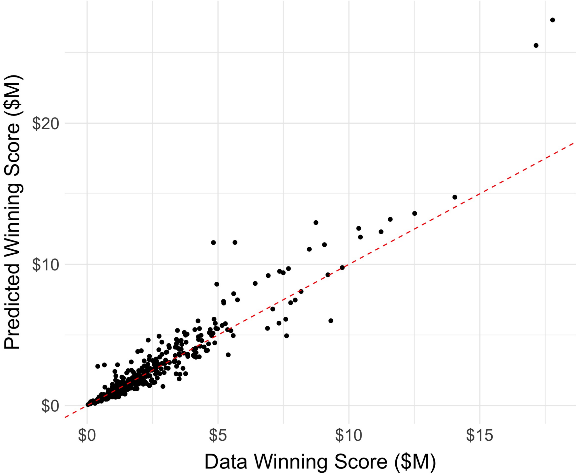
Predicted Bids Against Actual Bids



Back to Results

Equilibrium Model Fit

Predicted Winners' Scores Against Actual Scores



Back to Results

Assumptions

- Bidder i's costs are fully characterized by a 1-D type α^i s.t.

$$c_t^i = \alpha^i c_t^o \text{ for all } t.$$

- All bidders have *the same* coefficient of absolute risk aversion γ
- All bidders observe *the same* vector of quantity signals $\{q_t^b\}_{t=1,\ldots,T}$
- Bidders have common, rational expectations over the distributions of quantity signals + scores
- The number of bidders is commonly known prior to bidding

▸ Optimal Bid Program ◂ Auction Characterization

Equilibrium Bidding

1. Each α^i chooses the optimal score $s(\alpha^i)$ s.t.:

$$\frac{\partial \mathbb{E}[u(\pi(\tilde{s}, \alpha^i))]}{\partial s}\Big|_{\tilde{s}=s(\alpha^i)} = 0$$

2. For each (α^i, s), b^i maximizes the **certainty equivalent** of profits:

$$\max_{\mathrm{b}^i} \left[\sum_{t=1}^{T} \underbrace{q_t^b(b_t^i - \alpha^i c_t)}_{\text{Expectation of Profits}} - \underbrace{\frac{\gamma \sigma_t^2}{2}(b_t^i - \alpha^i c_t)^2}_{\text{Variance of Profits}} \right]$$

$$\text{s.t.} \sum_{t=1}^{T} b_t^i q_t^e = s$$

▸ Auction Characterization

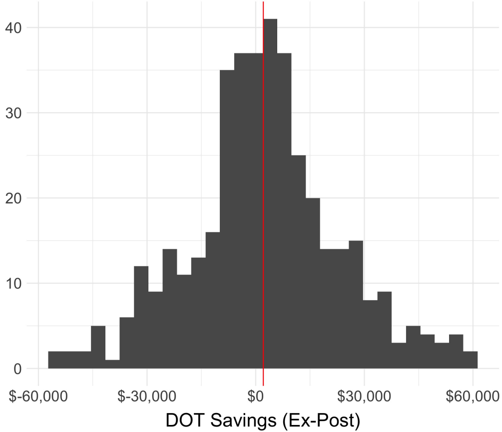
Certainty Equivalents by Scale for $\widehat{\gamma} = 0.046$

Prize	Prize for 50-50 to Equal 50%	Certainty Equivalent for 50-50 Bet to Win/Lose Prize Value
1	1	0
10	10.001	-0.002
100	100.115	-0.23
1,000	1,011.771	-22.992
10,000	11,504.674	-2,223.188

Estimation Results

Counterfactual: What if We Eliminate Risk?

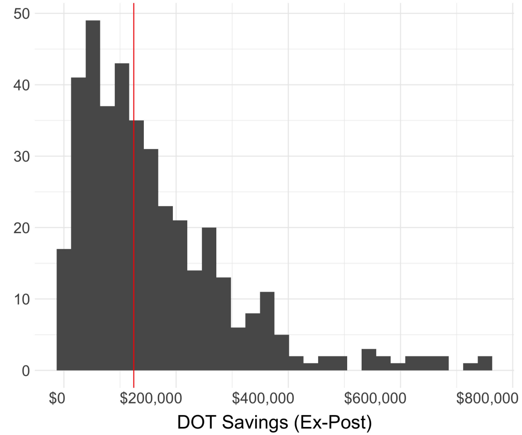
DOT Savings in Dollars



▸ Histogram in Percent

Removing Bidder Mis-Estimation in the Baseline

DOT Savings from Eliminating Risk



Histogram in Percent

Counterfactual: What if We Eliminate Risk?

Baseline

Statistic	Mean	St. Dev.	25%	Median	75%
Net DOT Savings	$2,145.37	$24,704.09	− $9,354.61	$2,203.49	$13,987.89
% DOT Savings	0.70%	4.25%	−1.02%	0.23%	1.60%
Bidder Gains	$6.64	$145.87	$3.76	$17.61	$43.35

$$q_t^b = q_t^a$$

Statistic	Mean	St. Dev.	25%	Median	75%
Net DOT Savings	$172,513.80	$165,129.50	$61,569.34	$125,187.10	$226,318.90
% DOT Savings	13.74%	9.05%	7.18%	11.98%	18.25%
Bidder Gains	$19.16	$124.55	−$8.48	$4.81	$37.64

Back

Summary Statistics

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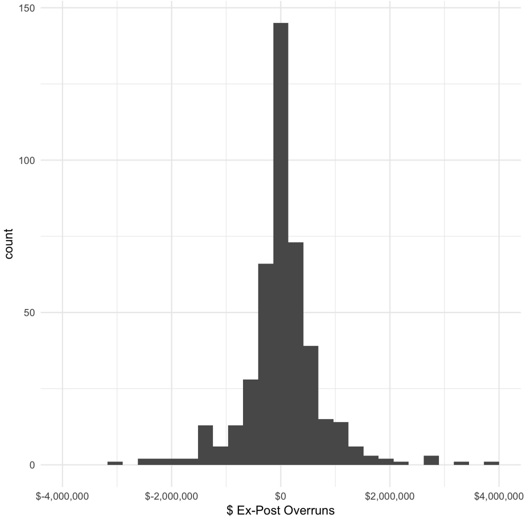
Ex-Post Overruns (Data)



Summary Statistics

Counterfactual: What if an Additional Bidder Enters?

Statistic	Mean	St. Dev.	25%	Median	75%
Net DOT Savings	$82,583.25	$87,568.51	$22,296.89	$49,335.35	$103,379.50
% DOT Savings	8.90%	8.45%	2.06%	5.65%	13.47%
Bidder Certainty Equivalent	$2,315.80	$1,524.88	$1,264.95	$1,959.42	$3,135.44

Back to Histogram

A Simple Example

- Suppose a project requires only two inputs: concrete and traffic cones

	DOT Estimates q^e	Bidders Expect q^b	Noise Var σ^2	Bidder Cost $\alpha \times c$
Concrete	10	12	2	12
Traffic Cones	20	16	1	18

Only the Total Score Matters for Winning

- The winning contractor has the lowest total bid
- Contractor's probability of winning is the same if she bids:

  a $12*10 tons + $19*20 cones = $500

  or

  b $40*10 tons + $5*20 cones = $500

Unit Bids (at a Score) Determine Profits

- The winning contractor has the lowest total bid
- Contractor's probability of winning is the same if she bids:
  a \$12*10 tons + \$19*20 cones = \$500

  or

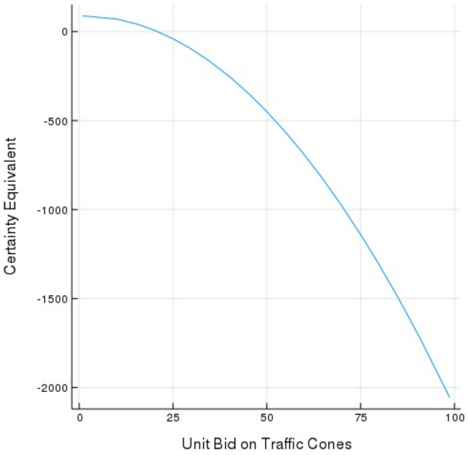
  b \$40*10 tons + \$5*20 cones = \$500
- Contractor's expected utility upon winning is different:
  a CE(\$12, \$19) = \$15.98

  or

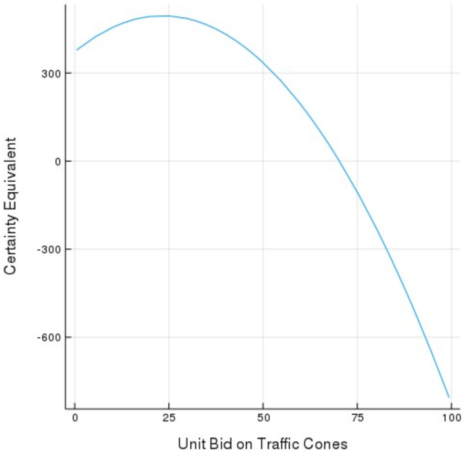
  b CE(\$40, \$5) = \$84.58

The Utility-Maximizing Bid Spread Depends on the Score

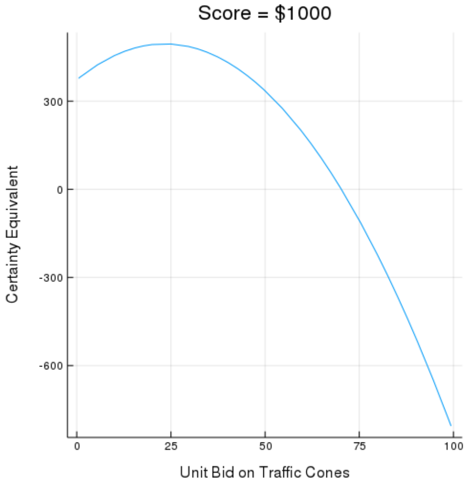
Score = \$500



Score = \$1000



The Extent of Total Skewing Depends on the Competitive Score



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Certainty Equivalents Balance Linear Profits Against Risk Variance

a $b_{\text{concrete}} = \$98$ and $b_{\text{cone}} = \$1$

$$\underbrace{12 \times (\$86) + 16 \times (-\$17)}_{\text{Expection of Profits} = \$760} - \underbrace{\frac{0.05 \times 2}{2} \times (\$86)^2 - \frac{0.05 \times 1}{2} \times (-\$17)^2}_{\text{Variance of Profits} = \text{-}\$377} = \$383$$

Certainty Equivalents Balance Linear Profits Against Risk Variance

Score = \$1000

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$$\underbrace{12 \times (\$86) + 16 \times (-\$17)}_{\text{Expection of Profits} = \$760} - \underbrace{\frac{0.05 \times 2}{2} \times (\$86)^2 - \frac{0.05 \times 1}{2} \times (-\$17)^2}_{\text{Variance of Profits} = \text{-}\$377} = \$383$$

or

b $b_{\text{concrete}} = \$50$ and $b_{\text{cone}} = \$25$

$$\underbrace{12 \times (\$38) + 16 \times (\$7)}_{\text{Expection of Profits} = \$658} - \underbrace{\frac{0.05 \times 2}{2} \times (\$38)^2 - \frac{0.05 \times 1}{2} \times (\$7)^2}_{\text{Variance of Profits} = \text{-}\$73} = \$495$$

Math for Score = \$500 | Back