Scaling Auctions as Insurance: A Case Study in Infrastructure Procurement

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Motivation: American Infrastructure Investment

- Major responsibility of the public sector

- Approximately 2.4% of GDP annually
  - $416 billion total (in 2014)
  - $165 billion on highways/bridges

- Major area of Public-Private Risk-Sharing
  - Construction typically contracted to private firms
  - Small firms are often used (and sometimes advantaged)
  - Increasing interest in long-term private management of public works
Uncertainty and Risk Aversion

- Small firms
  - Construction firms are often “mom and pop”
Uncertainty and Risk Aversion

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Uncertainty and Risk Aversion

- Small firms
  - Construction firms are often “mom and pop”
  - Public procurement contracts are large relative to earnings
- Construction projects (and especially repairs) are uncertain
  - Additional damage/requirements are often discovered once work has started
  - Changing conditions generate shocks to costs, work load, etc.
Uncertainty and Risk Aversion

Auction design with risk averse agents is different

- Revenue equivalence does not hold

⇒ First price auctions dominate second price
Uncertainty and Risk Aversion

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⇒ Auctioneer can gain by paying to reduce risk:
  - Investing in reducing uncertainty
  - Pre-committing to paying for adjustments
  - Setting caps on the amount of risk allowed in a bid
MassDOT Scaling Auctions

- Auctions to procure construction + maintenance for Massachusetts bridges
  - $100 million annual budget
  - $3.7 billion backlog

- Scaling auctions: used extensively in infrastructure procurement
  => "Bid Express" software used by 39 different state DOTs
Scaling Auctions

- Gov’t elicits unit bids for every “item” involved in a project
- Winner is evaluated on the sum of unit bids x DOT quantity estimates
- Winner is paid based on quantities actually used
Uncertainty and Risk Sharing

Two sources of strategic uncertainty:

1. Chance of winning the auction
2. Chance of bidding "incorrectly" [ex-post]
Uncertainty and Risk Sharing

- Two sources of strategic uncertainty:
  1. Chance of winning the auction
  2. **Chance of bidding “incorrectly”** [ex-post]

  ⇒ Scaling auctions partially “insure” bidders against #2

  ▶ DOT underestimates are covered (in principle)

  ▶ Bidders can optimize “portfolio” of risks
Presentation Outline

- Argue that risk aversion is important for
  1. (Simple) Empirical Predictions
  2. Identification/Estimation of the Bidder’s Problem
  3. Market Design

- Present a model for equilibrium bidding under uncertainty + risk aversion

- Show evidence + estimates from MassDOT bridge auctions

- Discuss policy recommendations from simulated counterfactuals
A Portfolio Model of Scaling Auction Bidding

A project consists of:

- Ex-Ante (Estimated) Quantities: $q_e^1, \ldots, q_e^T$
- Ex-Post (Actual) Quantities: $q_a^1, \ldots, q_a^T$
- Market-Rate Unit Costs: $c_1, \ldots, c_T$
- Features (project manager ID, project type, etc.): $X$
A Portfolio Model of Scaling Auction Bidding

A project consists of:

- DOT Quantity Estimates: $q^e_1, \ldots, q^e_T$
- Actual Quantities: $q^a_1, \ldots, q^a_T$
- Market-Rate Unit Costs: $c_1, \ldots, c_T$
- Features (project manager ID, project type, etc.): $X$

Bidders:

- (Independent Private) Types
- Identical beliefs about ex-post quantities (no winner’s curse)
- Cannot influence ex-post quantities (no moral hazard)
A Portfolio Model of Bidding: 1D Types & CARA Risk Aversion

- **Risk Aversion:**
  - Bidders are risk averse, w/ private CARA utility:

\[
u_i(\pi) = 1 - \exp(-\gamma_i \pi)\]

- **Efficiency Types:**
  - Bidders have private “efficiency” cost types:

\[c_{i,t} = \alpha_i \cdot c_t \text{ for every } t\]

- **Information Structure:**
  - Bidders get a public noisy signal of the ex-post quantity of each item:

\[q^b_t = q^a_t + \epsilon_t \text{ where } \epsilon_t \sim \mathcal{N}(0, \sigma^2_t)\]
Each bidder $i$ maximizes her expected utility subject to risk aversion:

$$
\mathbb{E}[u(\pi(b^i, \alpha^i))] = 
\left(1 - \mathbb{E}_{q^a}\left[\exp\left(-\gamma_i \sum_{t=1}^{T} q^a_t \cdot (b^i_t - \alpha^i c_t)\right)\right]\right) \times
\text{Prob}\{s^i < s^j \text{ for all } j \neq i\}
$$

where $s^i = \sum_{t=1}^{T} b^i q^e_t$ is the score implied by $b^i$. 

**Expected Utility Upon Winning**

**Probability of Winning**
Bidder Profits

Each bidder $i$ maximizes her expected utility subject to risk aversion:

$$E[u(\pi(b^i, \alpha^i))] =$$

$$\left(1 - E_{\epsilon} \left[ \exp \left( -\gamma_i \sum_{t=1}^{T} (q^b_t - \epsilon_t) \cdot (b^i_t - \alpha^i c_t) \right) \right] \right) \times$$

Expected Utility Upon Winning

$$\text{Prob}\left\{ s^i < s^j \text{ for all } j \neq i \right\}$$

Probability of Winning

where $s^i = \sum_{t=1}^{T} b^i q^e_t$ is the score implied by $b^i$. 
Equilibrium Bidding

1. Each $\alpha^i$ chooses the optimal score $s(\alpha^i)$ s.t.:
\[
\frac{\partial \mathbb{E}[u(\pi(\tilde{s}, \alpha^i))]}{\partial s} \bigg|_{\tilde{s} = s(\alpha^i)} = 0
\]

2. For each $(\alpha^i, s)$, $b^i$ maximizes the certainty equivalent of profits:
\[
\max_{b^i} \left[ \sum_{t=1}^{T} q_t^b (b^i_t - \alpha^i c_t) \right. \\
\left. - \frac{\gamma^i \sigma_t^2}{2} (b^i_t - \alpha^i c_t)^2 \right]
\]

s.t. $\sum_{t=1}^{T} b^i_t q_t^e = s$
Model Predictions: Equilibrium Bids

1. Each $\alpha^i$ chooses the optimal score $s(\alpha^i)$ s.t.:

$$\frac{\partial \mathbb{E}[u(\pi(\tilde{s}, \alpha^i))]}{\partial s} \bigg|_{\tilde{s} = s(\alpha^i)} = 0$$

2. For each $(\alpha^i, s^i)$, $b^i$ maximizes the certainty equivalent of profits:

$$b^*_{i,t}(s^i) = \alpha^i c_t + \frac{q_b^t}{\gamma_i \sigma^2_t} + \frac{q^e_t}{\sigma^2_t} \left( s^i - \sum_{p=1}^{T} \left[ \alpha^i c_p q^e_p + \frac{q_b^p q^e_p}{\gamma_i \sigma^2_p} \right] \right)$$

[Detailed Assumptions]
Model Predictions

- Items w/ higher expected quantities (overruns) \( \left( \frac{\mathbb{E}[q^2_t]}{q^2_t} \right) \Rightarrow \) higher markups

- Items with higher variance \( (\sigma^2_t) \Rightarrow \) lower (absolute) markups

- Generally, bid skewing is a function of:
  - Levels of risk aversion \( (\gamma) \)
  - Level of risk \( (\mathbb{E}[q^2], \sigma^2) \)
  - Level of competition

Example
Data and Reduced Form
MassDOT Data

- Years: 1998-2016
- Type: Highway and Bridge, Construction and Maintenance
- Number of Projects: 440 (bridge only)
- Winning bids, losing bids, and DOT cost estimates
- Types of material, DOT quantity estimates, and amount of each material actually used
- Other information about project managers, general project location, dates of work, etc.
## Summary Statistics

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<thead>
<tr>
<th>Statistic</th>
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<th>Pctl(25)</th>
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</tr>
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Prediction 1: Items that overrun in quantity more are overbid more:

\[ \uparrow \frac{q_t^a - q_t^e}{q_t^e} \implies \uparrow \frac{b_t - c_t}{c_t} \]
Can Massdot Bidders Predict Over-Runs?

Winning Bidders Over-Bid on Items that Over-Run

\[
\text{beta} = 0.0886, \quad \text{s.e.} = 0.0104
\]
Prediction 2: Items that are more uncertain have lower markups
Absolute Markups Decrease with Item Variance

- More Evidence
- Quantity Overrun Histogram
How Material is the Risk in Our Setting?
(A Structural Estimation)
Structural Model of Bidding (Overview)

- Model of optimal bidding:
  - Bidders observe a noisy signal of each item’s quantity
  - Bidders differ by a private cost-multiplier $\alpha_{i,n}$
  - Bidders are risk averse w/ a private CARA coeff $\gamma_{i,n}$

- Estimate parameters:
  - Statistical model for item quantity signals
  - Economic model of optimal bidding for
    - (a) Bidders’ Cost Types
    - (b) Bidders’ CARA Coefficients
Uncertainty and Risk Aversion

Auction design with risk averse agents is different

- Revenue equivalence does not hold

⇒ First price auctions dominate second price

⇒ Auctioneer can gain by paying to reduce risk:
  - Pre-committing to paying for adjustments ⇒ Lump sum would cost 86.8% more on median
  - Investing in reducing uncertainty
  - Setting caps on the amount of risk allowed in a bid
Uncertainty and Risk Aversion

Auction design with risk averse agents is different

⇒ Auctioneer can gain by paying to reduce risk:
  ▶ Pre-committing to paying for adjustments ⇒ Lump sum would cost 86.8% more on median
  ▶ **Investing in reducing uncertainty** ⇒ Best Case Scenario: save ≈ 12.6% on median
  ▶ Setting caps on the amount of risk allowed in a bid
Uncertainty and Risk Aversion

Auction design with risk averse agents is different

⇒ Auctioneer can gain by paying to reduce risk:
  ▶ Pre-committing to paying for adjustments ⇒ Lump sum would cost 86.8% more on median
  ▶ Investing in reducing uncertainty ⇒ Best Case Scenario: save ≈ 12.6% on median
  ▶ Setting caps on the amount of risk allowed in a bid ⇒ A minimum bid of 25% the market rate saves ≈ 3.2%, reducing the uncertainty premium by a quarter.
Thank You!

Thank You!
Certainty Equivalents Balance Linear Profits Against Risk Variance

\[ a \quad b_{\text{concrete}} = 12 \text{ and } b_{\text{cone}} = 19 \]

\[
\begin{align*}
12 \times (0) + 16 \times (1) & - \frac{0.05 \times 2}{2} \times (0)^2 - \frac{0.05 \times 1}{2} \times (1)^2 = 15.98 \\
\text{Expection of Profits} & \quad \text{Variance of Profits}
\end{align*}
\]

\[ \text{or} \]

\[ b \quad b_{\text{concrete}} = 40 \text{ and } b_{\text{cone}} = 5 \]

\[
\begin{align*}
12 \times (28) + 16 \times (-13) & - \frac{0.05 \times 2}{2} \times (28)^2 - \frac{0.05 \times 1}{2} \times (-13)^2 = 84.58 \\
\text{Expection of Profits} & \quad \text{Variance of Profits}
\end{align*}
\]

\[ \text{Math for Score } = 1000 \]
Distribution of Quantity Over/Under-Runs

Back to Summary Stats
Distribution of Quantity Over/Under-Runs
The Top Two Bidders Bid Similarly on Average
Top Two Bids are Especially Close on Items that Don’t Go Unused
The Proportion of Revenue from each Item Decreases with its Variance
Bid Revenue Proportion Definition

%Δ Proportion Revenue from $t = \frac{\sum_p b_t q^a_p - \sum_p c_t q^e_p}{\sum_p c_t q^e_p} \times 100$
Ex-Post Overruns are Lower for Higher Value Projects
Estimating A Model of Quantity Uncertainty

For each item $t$ in auction $n$:

- Predict best-fit of ex-post quantity given:
  - DOT estimate $q_{t,n}^e$
  - Item-Auction features $X_{t,n}$

- Estimate using Hamiltonian Monte Carlo

- Output:
  - Predicted quantity: $\hat{q}_{t,n}^b$
  - Residual variance: $\hat{\sigma}_{t,n}^2$
Estimation: Quantity Signal Model

\[ q_{t,n}^a = \beta_0 q_{t,n}^e + \beta \vec{q} X_{t,n} + \eta_{t,n} \]

where

\[ \eta_{t,n} \sim \mathcal{N}(0, \hat{\sigma}^2_{t,n}) \]

and

\[ \hat{\sigma}_{t,n} = \exp(\beta_{0,\sigma} q_{t,n}^e + \vec{\beta}_{\sigma} X_{t,n}) \].
Quantity Signal Model Prediction Fit

Predicted Item Quantities Against Realized Quantities (Bin Scatter)
Quantity Signal Model Residual Standard Deviations

![Graph showing the distribution of quantity standard deviations. The x-axis represents the quantity standard deviation, and the y-axis represents the count. The distribution is skewed right with a peak near 0.](image)

[Back to Quantity Model Overview]
Second Stage Model Fit
Predicted Bids Against Actual Bids
Equilibrium Model Fit
Predicted Winners’ Scores Against Actual Scores
Assumptions

- Bidder $i$’s costs are fully characterized by a 1-D type $\alpha^i$ s.t.
  \[ c^i_t = \alpha^i c_t^o \text{ for all } t. \]
- All bidders have \textit{the same} coefficient of absolute risk aversion $\gamma$.
- All bidders observe \textit{the same} vector of quantity signals $\{q^b_t\}_{t=1,...,T}$.
- Bidders have common, rational expectations over the distributions of quantity signals + scores.
- The number of bidders is commonly known prior to bidding.
Equilibrium Bidding

1. Each $\alpha^i$ chooses the optimal score $s(\alpha^i)$ s.t.:

$$\frac{\partial E[u(\pi(\tilde{s}, \alpha^i))]}{\partial s}\bigg|_{\tilde{s}=s(\alpha^i)} = 0$$

2. For each $(\alpha^i, s)$, $b^i$ maximizes the certainty equivalent of profits:

$$\max_{b^i} \left[ \sum_{t=1}^{T} q_t^b (b^i_t - \alpha^i c_t) \right. - \frac{\gamma \sigma^2_t}{2} \left( b^i_t - \alpha^i c_t \right)^2$$

Expectation of Profits

$$\left. - \frac{\gamma \sigma^2_t}{2} \left( b^i_t - \alpha^i c_t \right)^2 \right]$$

Variance of Profits

s.t. \[ \sum_{t=1}^{T} b^i_t q_t^e = s \]
<table>
<thead>
<tr>
<th>Prize</th>
<th>Prize for 50-50 to Equal 50%</th>
<th>Certainty Equivalent for 50-50 Bet to Win/Lose Prize Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>10.001</td>
<td>-0.002</td>
</tr>
<tr>
<td>100</td>
<td>100.115</td>
<td>-0.23</td>
</tr>
<tr>
<td>1,000</td>
<td>1,011.771</td>
<td>-22.992</td>
</tr>
<tr>
<td>10,000</td>
<td>11,504.674</td>
<td>-2,223.188</td>
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Counterfactual: What if We Eliminate Risk?

DOT Savings in Dollars

Histogram in Percent
Removing Bidder Mis-Estimation in the Baseline

DOT Savings from Eliminating Risk

Histogram in Percent
## Baseline

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<td>Net DOT Savings</td>
<td>$2,145.37</td>
<td>$24,704.09</td>
<td>$-9,354.61</td>
<td>$2,203.49</td>
<td>$13,987.89</td>
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<tr>
<td>% DOT Savings</td>
<td>0.70%</td>
<td>4.25%</td>
<td>$-1.02%</td>
<td>0.23%</td>
<td>1.60%</td>
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<td>Bidder Gains</td>
<td>$6.64</td>
<td>$145.87</td>
<td>$3.76</td>
<td>$17.61</td>
<td>$43.35</td>
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$q^b_t = q^a_t$

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<td>Net DOT Savings</td>
<td>$172,513.80</td>
<td>$165,129.50</td>
<td>$61,569.34</td>
<td>$125,187.10</td>
<td>$226,318.90</td>
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<td>% DOT Savings</td>
<td>13.74%</td>
<td>9.05%</td>
<td>7.18%</td>
<td>11.98%</td>
<td>18.25%</td>
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<td>Bidder Gains</td>
<td>$19.16</td>
<td>$124.55</td>
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Ex-Post Overruns (Data)
### Counterfactual: What if an Additional Bidder Enters?

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<tbody>
<tr>
<td>Net DOT Savings</td>
<td>$82,583.25</td>
<td>$87,568.51</td>
<td>$22,296.89</td>
<td>$49,335.35</td>
<td>$103,379.50</td>
</tr>
<tr>
<td>% DOT Savings</td>
<td>8.90%</td>
<td>8.45%</td>
<td>2.06%</td>
<td>5.65%</td>
<td>13.47%</td>
</tr>
<tr>
<td>Bidder Certainty Equivalent</td>
<td>$2,315.80</td>
<td>$1,524.88</td>
<td>$1,264.95</td>
<td>$1,959.42</td>
<td>$3,135.44</td>
</tr>
</tbody>
</table>

[Back to Histogram]
A Simple Example

Suppose a project requires only two inputs: concrete and traffic cones

<table>
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<tr>
<th></th>
<th>DOT Estimates $q^e$</th>
<th>Bidders Expect $q^b$</th>
<th>Noise Var $\sigma^2$</th>
<th>Bidder Cost $\alpha \times c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>10</td>
<td>12</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>Traffic Cones</td>
<td>20</td>
<td>16</td>
<td>1</td>
<td>18</td>
</tr>
</tbody>
</table>
Only the Total Score Matters for Winning

- The winning contractor has the lowest total bid

- Contractor’s probability of winning is the same if she bids:
  a. $12 \times 10$ tons + $19 \times 20$ cones = $500$
  or
  b. $40 \times 10$ tons + $5 \times 20$ cones = $500$
Unit Bids (at a Score) Determine Profits

- The winning contractor has the lowest total bid

- Contractor’s probability of winning is the same if she bids:
  a. $12\times10$ tons + $19\times20$ cones = $500
     or
  b. $40\times10$ tons + $5\times20$ cones = $500

- Contractor’s expected utility upon winning is different:
  a. $\text{CE}(12, 19) = 15.98$
     or
  b. $\text{CE}(40, 5) = 84.58$
The Utility-Maximizing Bid Spread Depends on the Score

Score = $500

Score = $1000
The Extent of Total Skewing Depends on the Competitive Score

Score = $1000

Certainty Equivalent

Unit Bid on Traffic Cones
Equilibrium Bidding

1. Each $\alpha^i$ chooses the optimal score $s(\alpha^i)$ s.t.:

$$\frac{\partial \mathbb{E}[u(\pi(\tilde{s}, \alpha^i))]}{\partial s}|_{\tilde{s}=s(\alpha^i)} = 0$$

2. For each $(\alpha^i, s)$, $b^i$ maximizes the certainty equivalent of profits:

$$\max_{b^i} \left[ \sum_{t=1}^{T} q^b_t (b^i_t - \alpha^i c_t) - \frac{\gamma\sigma^2_t}{2} (b^i_t - \alpha^i c_t)^2 \right]$$

s.t. $\sum_{t=1}^{T} b^i_t q^e_t = s$
Certainty Equivalents Balance Linear Profits Against Risk Variance

\[\begin{align*}
\text{a} \quad b_{\text{concrete}} &= $98 \text{ and } b_{\text{cone}} = $1 \\

12 \times ($86) + 16 \times (-$17) &- 0.05 \times 2 \times ($86)^2 - 0.05 \times 1 \times (-$17)^2 = $383 \\
\text{Expection of Profits} &= $760 \\
\text{Variance of Profits} &= -$377
\end{align*}\]
Certainty Equivalents Balance Linear Profits Against Risk Variance

Score = $1000

a  \( b_{\text{concrete}} = $98 \) and \( b_{\text{cone}} = $1 \)

\[
\begin{align*}
12 \times ($86) + 16 \times (-$17) & - \frac{0.05 \times 2}{2} \times ($86)^2 - \frac{0.05 \times 1}{2} \times (-$17)^2 = $383 \\
\text{Expection of Profits} &= $760 \\
\text{Variance of Profits} &= -$377
\end{align*}
\]

or

b  \( b_{\text{concrete}} = $50 \) and \( b_{\text{cone}} = $25 \)

\[
\begin{align*}
12 \times ($38) + 16 \times ($7) & - \frac{0.05 \times 2}{2} \times ($38)^2 - \frac{0.05 \times 1}{2} \times ($7)^2 = $495 \\
\text{Expection of Profits} &= $658 \\
\text{Variance of Profits} &= -$73
\end{align*}
\]