Real Exchange Rate Dynamics Beyond Business Cycles*

Dan Cao[†]   **Martin D.D. Evans**   **Wenlan Luo**[‡]

Georgetown University   Georgetown University   Tsinghua University

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Abstract

We examine the determinants of medium-term movements in real exchange rates. Using US-UK data over the past 200 years, we find that the real exchange rate co-moves with GDP and the co-movements are significantly affected by banking crises. This relationship can be rationalized by an extension of the IRBC model with persistent productivity shocks and incomplete markets. Using a new global solution method, we demonstrate that the transmission of productivity shocks depends critically on the proximity of a national economy to its international borrowing limit. The mechanism differs from the Harrod-Balassa-Samuelson effect which does not generate these state dependent responses.

Introduction

Research on the behavior of real exchange rates spans a broad frequency spectrum. At one end, macroeconomic models emphasize how changing interest rates (driven by monetary policy and other variables) induce monthly and quarterly variations in real exchange rates via their impact on nominal exchange rates in the presence of price-stickiness (see, e.g., Engel et al., 2008).[1] At the other end of the spectrum, following Harrod (1933), Balassa (1964) and Samuelson (1964) (HBS), a large empirical literature examines how very low frequency or secular movements in real exchange rates are linked to sectoral shifts in productivity and other factors. This paper focuses on the behavior of real exchange rates in the less studied center of the frequency spectrum; that is the middle ground between business-cycles and the very long run. In particular, we examine how medium-term co-movements in real exchange rates and GDP vary with international financial conditions.

We begin by documenting the empirical importance of medium-term co-movements in the real exchange rate and GDP (per capita) using more than 200 years of US and UK data. We show that shocks driving the cyclical components of GDP have persistent exchange-rate effects that last for approximately ten years, and that these shocks account for a significant fraction of the real US-UK depreciation rate over horizons ranging from five to fifteen years. Our empirical estimates also show that shocks producing an increase in US (UK) GDP *generally* induce an initial real appreciation (depreciation) of the US dollar which is amplified during the next few years. We then empirically examine how medium-term co-movements in real exchange rates and GDP vary with international financial conditions. Using a chronology from Reinhart and Rogoff (2008), we show that annual co-movements in GDP and the real exchange rate depend on whether either the US or UK was

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[1]More recent work such as Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021) highlights the importance of financial intermediaries and financial frictions in the short-run variations of nominal exchange rates.

in a banking crisis. In particular, we find that the response of the real exchange rate to shocks that affect cyclical GDP differs between non-crisis and crisis years. In the absence of a crisis, a shock that increases the cyclical component of US (UK) GDP produces an initial real appreciation (depreciation) of the US dollar, which is amplified over the next one to two years, before slowly disappearing over the next decade. The effects of shocks in crisis years are quite different. The initial exchange-rate effects of the shock are reversed (i.e., depreciations replace appreciations, and vise versa), there is no amplification, and the effects die more quickly.

Next, we compare these empirical findings with the theoretical predictions of International Real Business Cycle (IRBC) models (Backus et al., 1992). In particular, we examine whether the medium-term co-movements in GDP and the real exchange rate are consistent with the international transmission and propagation of productivity shocks under different international financial conditions. Our theoretical model is based on the standard IRBC models: There are two countries, which we call the US and UK, each populated by a large number of households. Each country hosts a representative firm that produces a tradable good using domestic labor and capital. The firm hires labor and undertakes investment to maximize its total value to its shareholders. Households consume a basket of US and UK tradable goods, provide labor to domestic firms, and hold a portfolio of financial assets. However, we depart from the standard IRBC models in two important dimensions. First, the model is in annual frequencies and productivity in each country follows co-integrated unit root processes. These processes generate the persistent responses of output and real exchange rates to the productivity shocks, as seen in our empirical analyses. Second, we relax the complete markets/complete risk-sharing assumption made in Backus et al. (1992) by limiting international financial asset trades. Households can trade equity issued by domestic firms and internationally-traded bonds, but they cannot trade equity issued by foreign firms. We solve the

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model globally using a novel solution method and the global solution allows us to examine how the responses of real exchange rate to productivity shocks vary with prevailing financial conditions, i.e., the proximity of a national economy to its international borrowing limit.

Our analysis reveals the following characterization of the transmission mechanism. When international risk-sharing is incomplete, a positive productivity shock to US firms will generally raise the wealth of US households (who hold the firms' equity) more than UK households. This, in turn, increases the demand for the baskets of US and UK traded goods by all households. On the production side, the productivity shock raises the marginal product of US capital and thereby creates an incentive for US firms to undertake more physical investment. So the transmission mechanism must reconcile higher investment by US firms with greater consumption by US and UK households. This reconciliation takes place via movements in the relative prices of US and UK goods faced by US and UK households. In particular, the relative price of US goods rises for UK households so that they substitute UK for US goods in their consumption baskets. This reduces world demand for US goods, thereby facilitating US investment, and increases world demand for the UK good. Consequently, US net exports fall. The change in relative prices also reduces the marginal revenue product of UK capital, so the increase in demand for UK goods is accommodated at the existing level of production by a fall in UK investment. Thus, taken together, the productivity shock produces an immediate change in relative prices, which is reflected in a real appreciation of the US dollar, that facilitates a rise in US investment and a fall in UK investment accompanied by a deterioration in the US net foreign asset position. Relative prices also adjust through time as the effects of the productivity shock diminish. These adjustments facilitate the return of the US net foreign asset position and the US and UK capital stocks to the levels consistent with the long-run levels of productivity.

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The operation of the transmission mechanism relies on several structural features: the persistence of productivity shocks, the elasticity of substitution between domestic and foreign goods in households' consumption baskets, and the impediments to risk-sharing. It also only operates when US households' existing asset holdings enable them to freely borrow internationally. When households are close to or at their international borrowing limit, US positive productivity shocks induce a real depreciation of the US dollar rather than an appreciation. As US households cannot borrow to finance more investment and consumption, the increase in demand for US goods is outweighed by the rise in the supply of US goods, which depresses the relative price of US goods inducing the real depreciation.

This analysis of the transmission mechanism squares with our empirical findings. In the years without crises, the co-movements in GDP and real exchange rates are consistent with the IRBC transmission of productivity shocks away from the international borrowing limit. In years with either a US or UK banking crisis, which hinders international borrowing and lending, the co-movements in GDP and the real exchange rate are consistent with the operation of the IRBC transmission mechanism near the borrowing constraint.

This paper contributes to very large literatures that study the behavior of real exchange rates and international macroeconomics using IRBC models.[2] We therefore focus on papers from these literatures that are most directly relate to our study. Our empirical analysis of the US-UK real exchange rate builds on Lothian and Taylor (2008) insofar as we use US and UK per capita GDP to identify the long-run real exchange rate.[3] Consistent with Ricci et al. (2013), our empirical model takes the form of an error-correction model, but we also allow for state-dependency to capture the

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[2]For a survey of the real exchange-rate literature see Froot and Rogoff (1995). Important early contributions in international macroeconomics using IRBC models include; Backus et al. (1992), Baxter and Crucini (1995) and Kehoe and Perri (2002).

[3]Other contributions to the literature on the long-term determinants of real exchange rates include De Gregorio et al. (1994), Froot and Rogoff (1995), Goldfajn and Valdes (1999), Chen and Rogoff (2003), Cashin et al. (2004), and Galstyan and Lane (2009), among others.

effects of banking crises. Our theoretical analysis is most closely related to work on the exchange-rate implications of IRBC models. Backus and Smith (1993) and Kollmann (1995) pointed out that real exchange rates should be perfectly correlated with relative cross-country consumption under complete risk-sharing, but the empirical correlation is negative or close to zero for many country pairs. This finding is commonly referred to as the Backus-Smith puzzle. Subsequent research has explored how incomplete risk-sharing affects the correlation (see, e.g., Corsetti et al. (2008), Kollmann (2012), Benigno and Thoenissen (2008), and Bodenstein (2008)).[4]

Within this literature, our analyses are most closely related to the analyses in Corsetti et al. (2008) with important distinctions. On the empirical front, we use longer time series which allow us to focus on the medium-run frequencies while Corsetti et al. focus on the business cycle frequencies. We also document the importance of financial conditions in real exchange rate dynamics. On the theoretical front, Corsetti et al. show that, under incomplete markets, a positive (highly persistent) US productivity shock can generate an improvement of the US terms-of-trade and real exchange rate appreciation in different versions of the IRBC models when the elasticity of substitution between traded goods is high or low enough (solving simultaneously the Backus-Smith puzzle). While the authors only emphasize the role of the wealth effect on consumption in their paper, we find that investment, and international borrowing, plays an important role in the dynamics of terms-of-trade and real exchange rate. In particular, an economy with fixed capital with exactly the same productivity processes and market incompleteness would experience worsen terms-of-trade and real exchange rate depreciation. More importantly, as in the empirical analyses, we focus on the medium run effects of productivity shocks on real exchange rate by altering the standard IRBC model with non-stationary productivity processes. Our global solution method allows us to solve the model

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[4]Recent research on the Backus-Smith puzzle that departs from the canonical IRBC setup, but maintaining the complete markets assumption, includes Colacito and Croce (2013) and Kollmann (2016) which use recursive utilities instead separable utilities.

accurately and it to uncover the state-dependent responses of real exchange rate to productivity shocks, as in our empirical analyses. This state-dependency is not present in all IRBC models linking productivity shocks and real exchange rates. For example, in an extension of our model with traded and non-traded goods, the HBS effect implies a persistent appreciation of the real exchange rate when a positive productivity shock hits a country's traded good sector (due to the changes in the relative prices between traded and non-traded goods). However, this effect does not vary with the country's distance to its international borrowing limit.

Our focus on the medium-run is also shared by Rabanal and Rubio-Ramírez (2015), who note that most of the variation in real exchange rates occurs at lower than business-cycle frequencies. Finally, our analysis extends existing research on solving international macro models with incomplete risk-sharing and portfolio choice. Existing methods developed by Devereux and Sutherland (2010), Tille and van Wincoop (2010) and Evans and Hnatkovska (2012) approximate the equilibrium around the steady state,[5] whereas we adapt and extend the global solution method in Cao (2018) to accommodate endogenous capital accumulation, imperfect substitution between traded and non-traded goods, non-stationary productivity process, and portfolio choices, which require up to six continuous state variables. The use of a global solution method allows us to examine how the exchange-rate implications of productivity shocks critically depend on the economy's proximity to the borrowing constraint.

The paper is structured as follows. We begin, in Section 1, with our analysis of the joint GDP and real exchange rate dynamics in US-UK data. Section 2 presents our benchmark model and the transmission mechanism. Section 3 examines how different features of IRBC model contribute to

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[5]These solution methods have been used to study external adjustment (see, e.g., Tille and van Wincoop, 2010 and Evans, 2014) and origins of home bias in international portfolio holdings (see, e.g., Coeurdacier and Gourinchas, 2008, Hnatkovska, 2009, Engel and Matsumoto, 2009, Coeurdacier et al., 2010, Devereux and Sutherland, 2010, and Coeurdacier and Rey, 2012).

the transmission mechanism. Section 4 concludes.

1   Empirical Analysis

We focus on the medium-term behavior of real exchange rates; that is in the middle ground between business-cycles and the very long run. For this purpose, our analysis uses more than 200 years of annual data (1802 - 2016) on exchange rates, prices and GDP from the US and UK. These data extend the time series of exchange rates and prices constructed by Lothian and Taylor (1996). The data for per capita GDP is obtained from The Maddison Project (2018). In the first part of this section, we present a time-series model that characterizes the medium-term co-movements in the data. We then examine how these co-movements depend on financial conditions.

For the purpose of this study, we define the real exchange rate as the relative price of the basket of goods consumed in the UK in terms of the price of a basket of goods consumed in the US. Mathematically, the real exchange rate in period t is calculated as

$$\mathcal{E}_t = \frac{S_t \hat{P}_t}{P_t}, \tag{1}$$

where P_t and $\hat{P}_t$ are US and UK period-t price indices and S_t denotes the period-t nominal exchange rate measured as the price of a UK pound in US dollars. (Hereafter we use hats to denote UK variables.) According to this definition, a depreciation (appreciation) in the real value of the US dollar corresponds to a rise (fall) in $\mathcal{E}_t$.

The first step in constructing our empirical model accounts for the fact that both the US and UK economies have undergone secular changes during the past 200 years which have produced long-run exchange-rate effects. Consistent with the empirical literature, these long-run effects appear

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to be well-represented by cointegration between the log real exchange rate $\varepsilon_t = \log \mathcal{E}_t$ and log per capita real GDP in the US and UK, $y_t = \log Y_t$ and $\hat{y}_t = \log \hat{Y}_t$. In particular, we estimate the cointegrating relation to be

$$\varepsilon_t = \underset{(0.367)}{0.624} + \underset{(0.118)}{0.558}\hat{y}_t^{Trend} - \underset{(0.079)}{0.439}y_t^{Trend}, \tag{2}$$

where y_t^{Trend} and $\hat{y}_t^{Trend}$ are the trends in y_t and $\hat{y}_t$ computed from the HP filter. Equation (2) shows the cointegrating coefficient estimates and their standard errors computed by DOLS following Stock and Watson (1993).[6] Estimating the cointegration regression between the log real exchange rate and the logs of per capital GDP rather than the HP trends produces very similar results. We prefer to use the HP trends because the long-run level for ε_t implied by the cointegration estimates has less year-by-year variation, which seems more economically plausible. Figure 1 plots the log real exchange rate ε_t and these long-run estimates ε_t^{LR} over our sample period. We are agnostic about the underlying economic reason for cointegration in the data. It could represent the long-term effects of sectoral shifts in productivity, changes in the terms of trade, government consumption, trade policy or a structural shift. Our focus is instead on the medium-term variations in ε_t that are not included in ε_t^{LR}.[7]

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[6]Statistical tests reveal that we cannot reject null of a unit root in ε_t , y_t^{Trend} and $\hat{y}_t^{Trend}$ individually, and we can reject the null of no cointegration between the three variables. In particular, the ADF tests for the null hypothesis that ε_t contains a unit root produce p-values of 0.167 and 0.451, with and without a constant, respectively, while the KPSS test for the null hypothesis that ε_t is stationary produces a p-value of less than 0.01. In the case of y_t^{Trend} and $\hat{y}_t^{Trend}$, the ADT tests produce p-values of over 0.99, and the KPSS tests produce p-values of less than 0.01. The ADF test for the null hypothesis of no cointegration produces a p-value of less than 0.01. We obtain very similar results using per capita GDP, y_t and $\hat{y}_t$, rather than the HP trends. We do not impose the restriction that the coefficients on y_t^{Trend} and $\hat{y}_t^{Trend}$ have equal and opposite signs because a test for the null of this restriction produces a p-value of less than 0.01. This apparent lack of symmetry in the estimated cointegrating vector is not surprising in view of the different evolution of the UK and US economies over the past 200 years (such as different ratios of traded to non-traded content in the two nations' baskets and different CPI compositions).

[7]Much of the research on the long-term determinants of real exchange rates originates with the Harrod-Balassa-Samuelson (HBS) model (Harrod, 1933 Balassa, 1964 and Samuelson, 1964). For example, Lothian and Taylor (2008) use the HBS model as the rationale for defining the long-run US-UK rate in terms of the difference between US and UK log per capita GDP. Other contributions by De Gregorio et al. (1994), Chen and Rogoff (2003), and Cashin et al. (2004), among others, focus on the terms of trade. The role of government consumption is considered in Froot and Rogoff (1995) and Galstyan and Lane (2009); while Goldfajn and Valdes (1999) study the effects of trade policy.

Figure 1: US/UK Real Exchange Rate
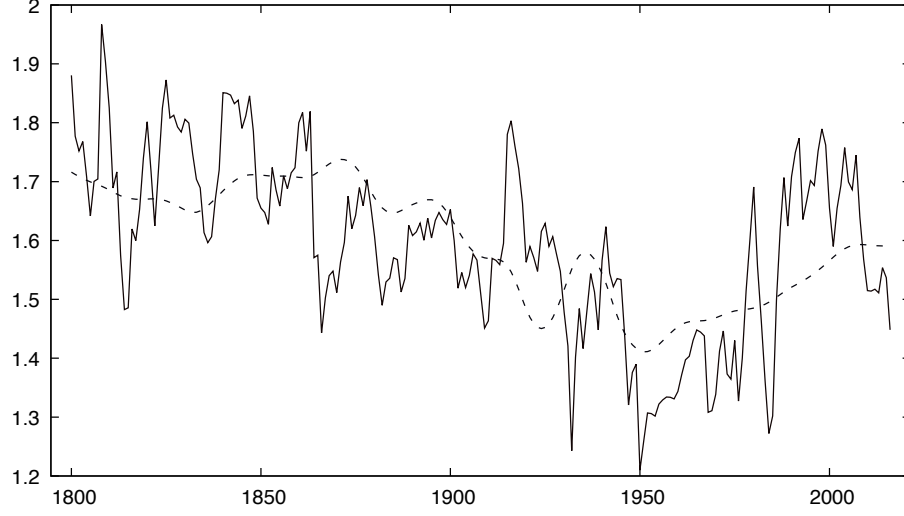
Notes: Log Real Exchange Rate ε_t (solid) and log long-run level ε_t^{LR} implied by cointegration (dashed): annual data 1880-2016.

Our empirical model comprises three equations: one for the annual real depreciation rate $\Delta\varepsilon_t = \varepsilon_t - \varepsilon_{t-1}$, and one each for the cyclical component of US and UK per capita GDP computed from the HP filter; denoted by y_t^{Cycle} and $\hat{y}_t^{Cycle}$, respectively

$$\Delta\varepsilon_t = \begin{bmatrix} \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} y_t^{Cycle} \\ \hat{y}_t^{Cycle} \end{bmatrix} + \gamma gap_{t-1} + e_t, \tag{3}$$

$$\begin{bmatrix} y_t^{Cycle} \\ \hat{y}_t^{Cycle} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} y_{t-1}^{Cycle} \\ \hat{y}_{t-1}^{Cycle} \end{bmatrix} + \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} \Delta\varepsilon_{t-1} + \begin{bmatrix} v_t \\ \hat{v}_t \end{bmatrix}, \tag{4}$$

where $gap_t = \varepsilon_t - 0.624 - 0.558\hat{y}_t^{Trend} + 0.439 y_t^{Trend}$ is the error-correction term from the cointegrat-

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Canzoneri et al. (1999) examine the effects of relative labor productivities and Lane and Milesi-Ferretti (2004) and Gourinchas and Rey (2007) examine the effects of net foreign asset positions. Ricci et al. (2013) examine all these factors in a panel cointegration model covering 24 years and 48 countries.

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ing regression. Equation (3) relates the current real depreciation rate to the cyclical components of GDP in the US and UK and the lagged error-correction term. Equation (4) characterizes the joint dynamics of the cyclical components of GDP in terms of past cyclical GDP and the real depreciation rate. These equations allow us to empirically characterize the joint dynamics of the real exchange rate and GDP in a parsimonious manner. Estimating alternative specifications that include, for example, lagged depreciation rates in (3) and error-correction terms in (4), produce statistically insignificant coefficients on the additional variables.

Panel I of Table 1 reports the OLS estimates of (3) and (4). Both regression equations appear well-specified; the majority of the coefficient estimates are highly statistically significant and the is no evidence of residual correlation. Column (i) reports the estimates of the depreciation rate equation in (3). The estimates imply that shocks that raise cyclical GDP in the US tend to also produce a real appreciation of the US dollar, whereas shocks that increase cyclical GDP in the UK produce a real depreciation. Columns (ii) and (iii) report the estimates of the GDP equations (4). Here we see that there is significant cross-country dependency via lagged GDP in the case of the US equation and via the lagged real depreciation rate in the UK equation.

To provide more perspective on the implications of the model estimates, Figure 2 plots the dynamic response of the log real exchange rate to a shock that induces a one percent increase in either y_t^{Cycle} or $\hat{y}_t^{Cycle}$, together with 90 percent confidence bands.[8] The plots show that positive shocks to the cyclical components of US and UK GDP have opposite initial effects on the real exchange rate; producing a real appreciation (depreciation) when the shock affects US (UK) GDP.

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[8]The goal here is simply to characterize the joint dynamics of the $\Delta\varepsilon_t$, y_t^{Cycle} and $\hat{y}_t^{Cycle}$ rather than to identify the effects of particular economic shocks. Mathematically, (3) and (4) can be rewritten as a Vector Error Correction Model (VECM) for $\Delta\varepsilon_t$, y_t^{Cycle} and $\hat{y}_t^{Cycle}$ with zero restrictions on several of the coefficients. The plots in Figure 2 correspond to the impulse responses functions (IRFs) from the constrained VECM to positive v_t and $\hat{v}_t$ shocks. We have also computed (IRFs) that account for the estimated correlation between v_t and $\hat{v}_t$ shocks, using a standard Cholesky identification scheme with different orderings. The IRFs from these identification schemes are very similar to those plotted in Figure 2. The confidence bands are calculated by sampling the estimated coefficients from their joint asymptotic distribution and generating IRFs using the same procedure.

Table 1: Model Estimates

I: Estimates Equation	$\Delta\varepsilon_t$	y_t^{Cycle} (US)	$\hat{y}_t^{Cycle}$ (UK)
	(i)	(ii)	(iii)
Regressors			
y^{Cycle} (US)	-0.262*	0.496***	-0.003
	(0.147)	(0.082)	(0.052)
$\hat{y}^{Cycle}$ (UK)	0.526***	0.278***	0.536***
	(0.159)	(0.099)	(0.095)
gap	-0.175***		
	(0.034)		
$\Delta\varepsilon$		0.043	0.063***
		(0.033)	(0.024)
SEE	0.064	0.033	0.025
R^2	0.137	0.388	0.324

II: Variance Contribution	Horizon (years)				
	1.	5.	10.	15.	20.
GDP Contribution	0.062	0.167	0.217	0.192	0.144
(standard error)	(0.019)	(0.024)	(0.030)	(0.036)	(0.025)
Trend GDP Contribution	0.011	0.036	0.082	0.099	0.077
(standard error)	(0.007)	(0.012)	(0.019)	(0.027)	(0.023)

Notes: Panel I reports estimates of (3) and (4). Estimates are computed by OLS from annual data 1802-2016 (215 observations), and robust standard errors are shown in parenthesis below the coefficient estimates. Statistical significance at the 10, 5 and 1% levels is indicated by *, ** and ***, respectively. Panel II reports the estimated contribution of US and UK GDP (trend and cyclical components) to the variance of the real depreciation rate over horizons from one to 20 years, and the variance contributions of the trend components in US and UK GDP alone. See footnote 9 for details.

These effects are amplified in the next one to two years as they produce feedback effects on cyclical GDP in both countries, before slowly disappearing over the next decade.

The persistence of the dynamic responses in Figure 2 is also reflected in the variance decompositions for the real depreciation rate reported in Panel II of Table 1. The upper row of statistics shows the contribution of cyclical and trend GDP to the variance of the real depreciation rate over

Figure 2: Real Exchange Rate Response
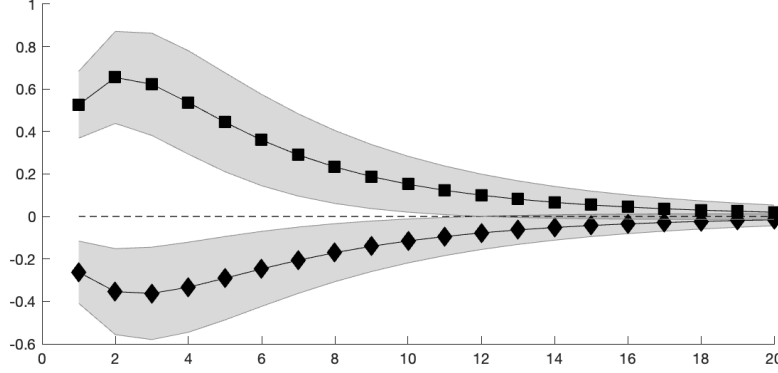
Notes: The figure plots the dynamic response of the log real exchange rate to a shock that induces a one percent increase in either y_t^{Cycle} (US cyclical GDP: diamonds) or $\hat{y}_t^{Cycle}$ (UK cyclical GDP: squares). The shaded areas show 90 per cent confidence bands computed by Monte Carlo simulation.

horizons ranging from one to twenty years.[9] These statistics show that GDP contributes most to the variations in the real exchange rate at around the ten-year horizon (the exact maximum contribution is estimated to be 22 percent at 11 years). The cyclical components of GDP account for most of these contributions. When we recalculate the variance contributions without the cyclical components (i.e., imposing the restriction that $\beta_1 = \beta_2 = 0$), the variance contributions of GDP fall by at least 50 percent, as is shown by the lower row of statistics.[10]

To this point, our empirical analysis reveals that variations in cyclical GDP become more eco-

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[9]To compute these variance contributions, we use the estimates of (3) and (4) to decompose the time series for ε_t into three components: one driven by the v_t and $\hat{v}_t$ shocks to y_t^{Cycle} and $\hat{y}_t^{Cycle}$, denoted by ε_t^{Cycle}; one driven by the changes in y_t^{Trend} and $\hat{y}_t^{Trend}$, denoted by ε_t^{Trend}; and one driven by the e_t shocks, denoted by ε_t^e. By construction, $\varepsilon_t = \varepsilon_t^{Cycle} + \varepsilon_t^{Trend} + \varepsilon_t^e$, so the variance of the k-year depreciation rate can be written as $Var(\Delta^k \varepsilon_t) = Cov(\Delta^k \varepsilon_t, \Delta^k \varepsilon_t^{Cycle}) + Cov(\Delta^k \varepsilon_t, \Delta^k \varepsilon_t^{Trend}) + Cov(\Delta^k \varepsilon_t, \Delta^k \varepsilon_t^e)$, where Δ^k denotes the k'th difference operator. The variance contribution of the cyclical and trend GDP components is $\left(Cov(\Delta^k \varepsilon_t, \Delta^k \varepsilon_t^{Cycle}) + Cov(\Delta^k \varepsilon_t, \Delta^k \varepsilon_t^{Trend}) \right) / Var(\Delta^k \varepsilon_t)$, which we compute as the slope coefficient from a regression of $\Delta^k \varepsilon_t^{Cycle} + \Delta^k \varepsilon_t^{Trend}$ on $\Delta^k \varepsilon_t$. The variance contribution of the trend GDP components is computed analogously from the regression of $\Delta^k \varepsilon_t^{Trend}$ on $\Delta^k \varepsilon_t$. Standard errors for the variance contributions are computed from the regressions with the Newey-West estimator which allows for heteroskedasticity and overlapping data.

[10]Our results are consistent with the finding in Rabanal and Rubio-Ramírez (2015) that most of the variation in real exchange rates occurs at lower than business-cycle frequencies. While they used spectral methods to decomposed the variance of the real exchange rates by frequency, we focus on the contribution of shocks that also drive cyclical GDP.

nomically important as proximate drivers of the real depreciation rate beyond business-cycle frequencies. Shocks driving the cyclical components of the GDP appear to have a sizable and persistent impact on the real US-UK exchange rate and account for more than ten percent of the variations in the real depreciation rate over horizons ranging from five to fifteen years. Of course, these results characterize the co-movements of GDP and the real exchange rate over more than 200 years, during which time there has been substantial evolution in the international economic and financial systems. So we next investigate whether these co-movements have varied with changes in financial conditions, exchange-rate regimes, and other economic conditions.

Our first investigation is motivated by the idea that financial conditions affect how economic shocks are transmitted within countries and internationally. We pursue this in the next section by showing how the transmission and propagation of productivity shocks in a variety of IRBC models critically depend on the proximity of either country to its international borrowing limit. In anticipation of this analysis, we investigate here whether the co-movements of GDP and the real exchange rate vary in years where international borrowing and lending were impaired by banking crises. For this purpose, we construct a dummy variable s_t that equals one in years when there is either a US or UK banking crisis, based on the chronology in Reinhart and Rogoff (2008, Table A.3). We then re-estimate equations (3) and (4) allowing for the coefficients to vary with s_t. In the case of the cyclical GDP equations (4), there was no statistically significant evidence of state-dependency in any of the coefficients. However, in the case of the real depreciation rate equation (3), there are statistically significant differences in all the coefficients between crisis and non-crisis years.

Panel I of Table 2 reports the estimates of the depreciation equation (3) allowing for the effects of crises. The coefficient estimates in the left-hand column, computed in years without a crisis,

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Table 2: State-Dependent Model Estimates

I: Estimates		Real Depreciation Rate Equation: $\Delta\varepsilon$	
		No Crisis	Difference in Crisis
Regressors			
y^{Cycle} (US)		-0.294*	0.806**
		(0.155)	(0.386)
$\hat{y}^{Cycle}$ (UK)		0.609***	-1.702***
		(0.163)	(0.589)
gap		-0.148***	-0.313***
		(0.034)	(0.097)
SEE	0.063		
R^2	0.183		

II: Variance Contribution		Horizon (years)				
		1.	5.	10.	15.	20.
GDP Contribution		0.068	0.185	0.234	0.201	0.149
(standard error)		(0.021)	(0.026)	(0.032)	(0.038)	(0.026)
Trend GDP Contribution		0.009	0.032	0.075	0.091	0.072
(standard error)		(0.006)	(0.012)	(0.018)	(0.027)	(0.023)

Notes: Panel I reports estimates of the real depreciation rate equation

$$\Delta\varepsilon_t = \left(\begin{bmatrix} \beta_1 & \beta_2 \end{bmatrix} + \begin{bmatrix} \beta_1^{\mathrm{diff}} & \beta_2^{\mathrm{diff}} \end{bmatrix} s_t\right) \begin{bmatrix} y_t^{Cycle} \\ \hat{y}_t^{Cycle} \end{bmatrix} + \left(\gamma + \gamma^{\mathrm{diff}} s_t\right) gap_{t-1} + e_t,$$

where $\beta_1^{\mathrm{diff}}, \beta_2^{\mathrm{diff}}$ and γ^{diff} identify the difference in the corresponding coefficient between crisis and non-crisis years. The left-hand column reports OLS estimates of β_1, β_2 and γ, and the right-hand column reports estimates of $\beta_1^{\mathrm{diff}}, \beta_2^{\mathrm{diff}}$ and γ^{diff}. Robust standard errors are shown in parenthesis. Estimates are computed from annual data 1802-2016 (215 observations). Statistical significance at the 10, 5 and 1% levels is indicated by *, ** and ***, respectively. Panel II reports the estimated contribution of US and UK GDP (trend and cyclical components) to the variance of the real depreciation rate over horizons from one to 20 years, and the variance contributions of the trend components in US and UK GDP alone. See footnote 9 for details.

are similar to those in Table 1. In particular, these estimates imply that in the absence of a banking crisis, shocks that raise US GDP tend to also produce a real appreciation of the US dollar, whereas shocks that increase UK GDP produce a real depreciation. The right-hand column reports the estimated difference between the coefficients in crisis and non-crisis years. As the table shows, all of these differences are statistically significant at conventional levels. The immediate

exchange-rate effects of a shock in a crisis are determined by the sum of the coefficients in each row. These estimated coefficients imply that shocks producing an increase in USD GDP induce a real depreciation of the dollar, and shocks increasing UK GDP are accompanied by a real appreciation.

The estimates in Table 2 also have implications for the persistence of co-movements in cyclical GDP and the real exchange rate because the coefficients on the error-correction term differ significantly between crisis and non-crisis years. To appreciate these effects, Figure 3 plots the dynamic response of the log real exchange rate to shocks that increase cyclical US and UK GDP in crisis and non-crisis years.[11] The responses in non-crisis years are similar to those in Figure 2. The initial impact of a shock is amplified for a couple of years, and then dissipates slowly. The response patterns are quite different in crisis years. Not only are the initial exchange-rate effects of the shock reversed (i.e., depreciations replace appreciations, and vise versa), but there is no amplification and the effects die more quickly.

Crises do not appear to materially affect how the cyclical and trend components of GDP contributed to the variance of the depreciation rates across different horizons. Panel II of Table 2 reports the variance contributions of GDP to depreciation rates allowing for the effects of crises. These estimated contributions are similar to those in Table 1. Even though the exchange-rate implications of shocks differ between crisis and non-crisis years, crises occur relatively infrequently (crisis years cover only seven percent of the sample period), so the variance decompositions largely reflect the co-movements in GDP and depreciation rates in non-crisis years.

Given the long span of our data, there are other potential sources of state-dependency in the co-movements of GDP and the real exchange rate. In particular, it is possible that the co-movements

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[11]We estimate that the probability of remaining in a crisis is 0.69 and out of a crisis is 0.98. Figure 3 plots the dynamic response to the shock in a crisis and non-crisis year incorporating these transition probabilities into the responses for the years that follow. Thus we show responses conditioned on the *initial* crisis/no-crisis state, not those conditioned on remaining in a crisis/non-crisis state.

Figure 3: Conditional Real Exchange Rate Responses
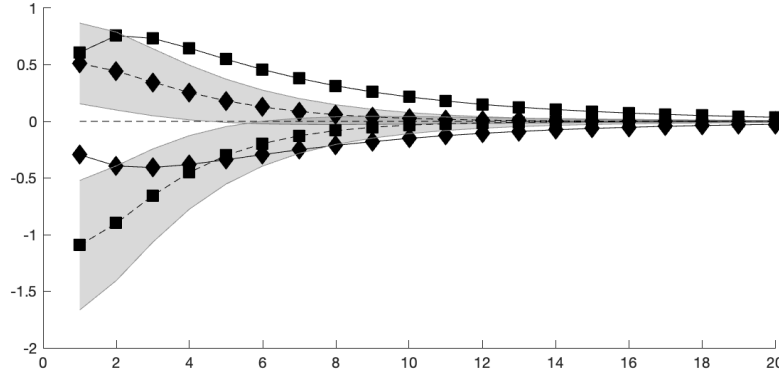
Notes: The figure plots the dynamic response of the log real exchange rate to a shock that induces a one percent increase in either y_t^{Cycle} (US cyclical GDP: diamonds) or $\hat{y}_t^{Cycle}$ (UK cyclical GDP: squares). Solid plots for responses in no-crisis years, dashed plots for crisis years. The Shaded areas show 90 percent confidence bands around the crisis-year responses computed by Monte Carlo simulation.

varied according to whether the US and UK were on or off the Gold Standard, or changed after the collapse of the Bretton Woods System. In Appendix A we investigate these possibilities by testing for state-dependency in the estimates of (3) and (4) associated with these events (using analogous methods to the one described above). We found no statistically significant evidence in either case. It does not appear that our banking crises results are proxying for these changes in the international financial system.

In summary, the estimates in Table 1 and IRFs in Figure 2 show that the co-movements in the US-UK real exchange rate and US and UK GDP over the past two centuries have differed according to whether one or both countries were experiencing a banking crisis. We interpret these findings as prima facie evidence that financial conditions affect how economic shocks are transmitted within countries and internationally. In the next section, we present a theoretical framework that helps explain the empirical findings.

2 The Transmission Mechanism

We now examine whether the transmission and propagation of productivity shocks in a variety of IRBC models would produce co-movements in real exchange rates and GDP consistent with the US-UK data. Our analysis is centered on a benchmark model based on the the IRBC model in Backus et al. (1992) with two symmetric countries and two traded goods. Each country is populated by a large number of identical utility-maximizing households and a large number of identical firms subject to recurrent aggregate productivity shocks.

We depart from Backus et al.'s model in two important dimensions. First, the model is in annual frequencies and productivity in each country follows co-integrated unit root processes. These processes help us generate the persistent responses of output and real exchange rates to the productivity shocks, as seen in our empirical analyses. Second, we relax the complete markets and complete risk-sharing assumption made in Backus et al. (1992) by limiting international financial asset trades. Households can trade equity issued by domestic firms and internationally-traded bonds, but they cannot trade equity issued by foreign firms.

2.1 Benchmark Model

The benchmark model features two countries with each populated by representative households and firms. We introduce their decision problems before proceeding with the equilibrium concept, solution, and calibration.

Households   We refer to the two countries in our models as the US and UK. The US is populated by a continuum of identical households (mass 1) with preferences defined over a consumption basket of traded goods. In particular, the expected utility of a representative US household in period t is

given by

$$\mathbb{U}_t = \mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta^i \frac{1}{1-\gamma} C_{t+i}^{1-\gamma} \right],$$

where $\gamma > 0$ is the coefficient of relative risk-aversion and $1 > \beta > 0$ is the subjective discount factor. $\mathbb{E}_t$ denotes expectations conditioned on period t information. C_t is the consumption index defined over two consumption goods; a US-produced good, C_t^{US}, and a UK-produced good, C_t^{UK}. We assume that the index takes the CES form so $C_t = \mathcal{C}(C_t^{\mathrm{US}}, C_t^{\mathrm{UK}})$ where $\mathcal{C}$ is defined as

$$\mathcal{C}(a,b) = \left(\alpha^{\frac{1}{\theta}} a^{\frac{\theta-1}{\theta}} + (1-\alpha)^{\frac{1}{\theta}} b^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \tag{5}$$

with elasticity parameter θ, and share parameter α for the US good. We also assume consumption home-bias: $\alpha > 1/2$. P_t is the associated consumption price index for US households in US dollars

$$P_t = \left(\alpha (P_t^{\mathrm{US}})^{1-\theta} + (1-\alpha) (P_t^{\mathrm{UK}})^{1-\theta} \right)^{\frac{1}{1-\theta}}, \tag{6}$$

where P_t^{US} and P_t^{UK} are the dollar prices at which the US households purchase US and UK goods respectively. US households are also endowed with one unit of labor and derive nominal wage income of W_t from working for US firms.

In our benchmark model, US households can trade equity shares of the representative US firm and a bond. The bond payoff is denominated in the US price index so it is risk-free from the perspective of US households. The budget constraint of US households is therefore given by

$$P_t C_t + Q_t \chi_t + Q_t^b b_t \leq (D_t + Q_t) \chi_{t-1} + P_t b_{t-1} + W_t, \tag{7a}$$

where: χ_t is the number of shares of equity issued by US firms held by the US household; Q_t is

the nominal price of equity issued by US firms; D_t is the nominal dividend paid by US firms; b_t is the number of bonds held by the US household; Q_t^b is nominal price of a bond (in US dollars). Dividing both sides of the budget constraint (7a) by P_t, we obtain the constraint in real terms

$$C_t + q_t \chi_t + q_t^b b_t \leq (d_t + q_t)\chi_{t-1} + b_{t-1} + w_t, \tag{7b}$$

where q_t, q_t^b, and w_t are the corresponding real values of Q_t, Q_t^b, and W_t. We also impose exogenous constraints on real bond holdings

$$b_t \geq \underline{b}\xi_t, \tag{8}$$

where ξ_t depends on the levels of productivity in both countries to ensure the (relative) stationarity of the model. We define ξ_t in (13) below.

Similarly, the UK is populated by a continuum of households (mass 1). The preferences and optimization problem of the UK households are identical to those of US households with the $\hat{}$ notation for UK variables. For example, the UK consumption index is denoted by $\hat{C}_t = \mathcal{C}(\hat{C}_t^{\mathrm{UK}}, \hat{C}_t^{\mathrm{US}})$ where $\mathcal{C}$ is given in (5) and $\hat{C}_t^{\mathrm{UK}}$ and $\hat{C}_t^{\mathrm{US}}$ denote the consumption of UK and US-produced goods by UK households. The price index for UK households in pounds is given by

$$\hat{P}_t = \left(\hat{\alpha}(\hat{P}_t^{\mathrm{US}})^{1-\theta} + (1-\hat{\alpha})\,(\hat{P}_t^{\mathrm{UK}})^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{9}$$

where $1 - \hat{\alpha} = \alpha > 1/2$ is the UK home-bias parameter.

We assume that the law of one price holds for both traded goods, so $P_t^{\mathrm{UK}} = S_t \hat{P}_t^{\mathrm{UK}}$ and $P_t^{\mathrm{US}} = S_t \hat{P}_t^{\mathrm{US}}$.[12] Let $p_t^{\mathrm{US}} = P_t^{\mathrm{US}}/P_t$ and $p_t^{\mathrm{UK}} = P_t^{\mathrm{UK}}/P_t$ denote the relative prices of US and UK goods

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[12]To simplify the presentation of our mechanism, we abstract away from other real wedges such as distribution costs and trade costs. Crucini and Yilmazkuday (2014) document the important of these costs in the cross-section and might serve as a good starting point if we extend our model to include these wedges.

faced by US households, and $\hat{p}_t^{\mathrm{US}} = \hat{P}_t^{\mathrm{US}}/\hat{P}_t$ and $\hat{p}_t^{\mathrm{UK}} = \hat{P}_t^{\mathrm{UK}}/\hat{P}_t$ denote the relative prices faced by

UK households. Then, by the definition of real exchange rate in (1), $p_t^{\mathrm{US}} = \mathcal{E}_t \hat{p}_t^{\mathrm{US}}$ and $p_t^{\mathrm{UK}} = \mathcal{E}_t \hat{p}_t^{\mathrm{UK}}$.

From the definitions of P_t and $\hat{P}_t$ in (6) and (9), we obtain $1 = \alpha(p_t^{\mathrm{US}})^{1-\theta} + (1-\alpha)(p_t^{\mathrm{UK}})^{1-\theta}$ and

$1 = \hat{\alpha}(\hat{p}_t^{\mathrm{US}})^{1-\theta} + (1-\hat{\alpha})(\hat{p}_t^{\mathrm{UK}})^{1-\theta}$. Combining these equations with (1) produces the following

expression for the real exchange rate

$$\mathcal{E}_t = \left(\frac{1-\alpha}{1-\hat{\alpha}} + \frac{(\alpha - \hat{\alpha})(\hat{p}_t^{\mathrm{US}})^{1-\theta}}{1-\hat{\alpha}} \right)^{\frac{1}{\theta-1}}. \tag{10}$$

Notice that this expression for $\mathcal{E}_t$ is decreasing in $\hat{p}_t^{\mathrm{US}}$ when there is consumption home bias ($\alpha > \hat{\alpha}$).

In other words, a depreciation of the real exchange rate lowers the relative price of US goods facing

UK households when there is consumption home bias.

Firms There is a single industry in each country, and each industry is populated by a continuum of

identical firms distributed on the interval [0,1]. A representative US firm owns all of its capital stock,

K_t, and hires labor L_t to produce output of US goods, Y_t, according to $Y_t = F(A_t, K_t, L_t)$, where

F is a constant-returns-to-scale production function, and A_t denotes the state of productivity. The

output of UK goods by a representative UK firm, $\hat{Y}_t$, is given by an identical production function

using its own capital, $\hat{K}_t$, and hiring labor $\hat{L}_t$, with productivity $\hat{A}_t$.

Firms in each country choose production and investment plans to maximize their total value to

shareholders. In particular, a representative US firm solves

$$\max_{\{D_{t+i}, I_{t+i}, K_{t+i+1}\}} \mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} \frac{D_{t+i}}{P_{t+i}} \right]$$

s.t.

$$D_t = P_t^{\text{US}} F(A_t, K_t, L_t) - W_t L_t - P_t^{\text{US}} I_t \quad \text{and} \quad K_{t+1} = (1 - \delta)K_t + I_t,$$

where: I_t is investment, $1 > \delta > 0$ is the depreciation rate, and λ_t is the stochastic discount factor of the firm's shareholders. Since equity can only be traded in domestic asset markets, the firm's shareholders are US households, so $\lambda_t = C_t^{-\gamma}$.

After substituting out investment, the problem of a representative US firm can be rewritten in real terms as

$$\max_{\{K_{t+i+1}, L_{t+i}\}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} \left\{ p_t F(A_{t+i}, K_{t+i}, L_{t+i}) - w_{t+i} L_{t+i} - p_t [K_{t+i+1} - (1 - \delta)K_{t+i}] \right\}.$$

The representative UK firm solves a similar problem with $\hat{\ }$ variables.

As in the IRBC literature starting with Backus et al. (1992), we assume that production functions are Cobb-Douglas:

$$F(A, K, L) \equiv A K^{\eta} L^{1-\eta}, \tag{11}$$

with $0 \leq \eta < 1$. However, unlike Backus et al. (1992), to capture the low frequency dynamics of productivity, we use the unit-root specification from Rabanal et al. (2011), instead of Backus et al.'s stationary specification:

$$\log A_t = \log A_{t-1} + \varrho \log \left(\frac{\hat{A}_{t-1}}{A_{t-1}} \right) + \epsilon_t, \tag{12a}$$

and

$$\log \hat{A}_t = \log \hat{A}_{t-1} + \varrho \log \left(\frac{A_{t-1}}{\hat{A}_{t-1}} \right) + \hat{\epsilon}_t, \tag{12b}$$

with $\varrho > 0$, where ϵ_t and $\hat{\epsilon}_t$ are mean-zero, I.I.D. shocks with standard deviations $std(\epsilon_t) = std(\hat{\epsilon}_t) = \sigma_\epsilon$.

It is important that the borrowing limit remains economically relevant even though productivity in each country follows a non-stationary process. To this end, we assume that the process for ξ_t used to determine borrowing limit in (8) follows

$$\log \xi_t = \rho_\xi \log(\xi_{t-1}) + (1 - \rho_\xi)\frac{\log(A_t) + \log(\hat{A}_t)}{2}, \tag{13}$$

where $1 > \rho_\xi > 0$. This specification implies that ξ_t is co-integrated with $\log(A_t)$ and $\log(\hat{A}_t)$ ensuring that the borrowing limit remains relevant as productivity grows or shrinks over time. We also choose a large value for ρ_ξ so that short-run fluctuations in productivity have very small effects on ξ_t.

Equilibrium We study a standard sequential competitive equilibrium: in equilibrium, prices and allocations are such that allocations solve the households' and firms' optimization problems and all markets clear in each history of shocks. Numerically, we solve for sequential competitive equilibrium with a particular recursive structure: a recursive equilibrium is a sequential competitive equilibrium in which prices and allocations are (single-valued) functions of two exogenous state variables; the levels of US and UK productivity $(A_t, \hat{A}_t)$, and three endogenous state variables; the US and UK capital stocks, and US bond holdings $(K_t, \hat{K}_t, b_{t-1})$. In our benchmark model, the only internationally traded asset is the US bond. This bond is in zero net supply, so $0 = b_t + \hat{b}_t$ by market clearing. Thus b_{t-1} identifies the US net foreign asset position at the start of period t.

To calculate the moments from the solution of the model, we use the concept of a stationary recursive equilibrium in Duffie et al. (1994) and Cao (2020). A stationary recursive equilibrium is a recursive equilibrium with a stationary ergodic distribution resulting from the transition functions for the normalized state variables, $\left(\frac{\hat{A}_t}{A_t}, \frac{\xi_t}{A_t}, \frac{K_t}{A_t}, \frac{\hat{K}_t}{A_t}, \frac{b_{t-1}}{A_t}\right)$, in our benchmark model. We study the

equilibrium dynamics of variables with impulse response functions (IRFs) that are calculated by simulating draws from the ergodic distribution. Precise definitions for these equilibrium concepts and a description of the global solution method are given in Appendix B.

Calibration, Moments, and Distributions   Since we focus on the equilibrium dynamics beyond business cycles, we use annual frequencies for our model (instead of the quarterly frequencies used in the IRBC literature starting with Backus et al., 1992).[13] We use the preference and technology parameters from Corsetti et al. (2008) who provide annual estimates of these parameters. The parameters are given in Table 3. We use the elasticity of substitution $\theta = 5$ in the benchmark calibration. This elasticity is in the mid-range of the estimates found in the trade literature as reported in Broda and Weinstein (1996) and more recently Imbs and Mejean (2015), which contains a comprehensive survey of the literature on trade elasticity estimations. For the borrowing limit, we use $\underline{b} = -0.5$. The calibration of this lower bound implies that households can borrow up to approximately 20% of their income (including capital and labor income).

Table 3: **Benchmark Calibration**

Parameters	Descriptions	Values
β	discount factor	0.98
γ	risk aversion	2
$\alpha = 1 - \hat{\alpha}$	home consumption share	0.9
θ	consumption elasticity	5
δ	depreciation rate	0.06
η	capital share	0.36
$\underline{b}$	borrowing limit	-0.5
ϱ	co-integrated factor	0.007
σ_ϵ	std of productivity innovation	0.01
ρ_ξ	persistence of latent process	0.95

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[13]In Appendix F, we use the quarterly productivity processes estimated in Heathcote and Perri (2002) and Rabanal et al. (2011).

It is useful to briefly describe some of the characteristics of the equilibrium in our benchmark model. To this end, Table 4 compares the variability of consumption, investment and productivity in post-war US data (available from the St Louis Fed's database, FRED), and the equilibrium of the benchmark model. We follow the standard practice (Backus et al., 1992) of comparing the standard deviation of each variable relative to the standard deviation of output. The benchmark model's statistics are calculated using long time series (20,000 years) simulated from the ergodic distribution of the recursive equilibrium. For comparison purposes, we also report statistics from the equilibrium of an IRBC model with complete risk-sharing that has all the core features of our benchmark except the restrictions on asset trade.

Table 4: **Business Cycle Moments**

	Relative Variability: $Std(X)/Sdt(Y)$		
$X =$	Consumption	Investment	Productivity
Data	0.87	3.86	0.46
Benchmark	0.82	2.24	1.57
Complete Markets	0.60	2.39	1.57

As the table shows, the benchmark model comes close to replicating the relative variability of consumption in the data, but somewhat understates (overstates) the relative variability of investment (productivity). These discrepancies between the data and model are most likely due to the simplicity of our specification with respect to consumption and production.[14] They do not appear to be primarily attributable to the restrictions on asset trade because the relative variability of

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[14]Recall that, to highlight the transmission mechanism in our model, we choose to omit many ingredients that would bring the business cycles moments closer to those in the data, such as non-traded goods or distribution services. Corsetti et al. (2008) show that these ingredients are important quantitatively. For example, we use the standard deviation for productivity shocks from the traded good sectors in Corsetti et al. (2008), which implies larger output volatility than in the data. In the data, output includes both traded and non-traded goods.

investment and productivity under complete risk-sharing are almost identical.[15] However, as we would expect, the variability of consumption under complete risk-sharing is considerably lower than in our benchmark model.

Figure 4: Histograms of Normalized State Variables
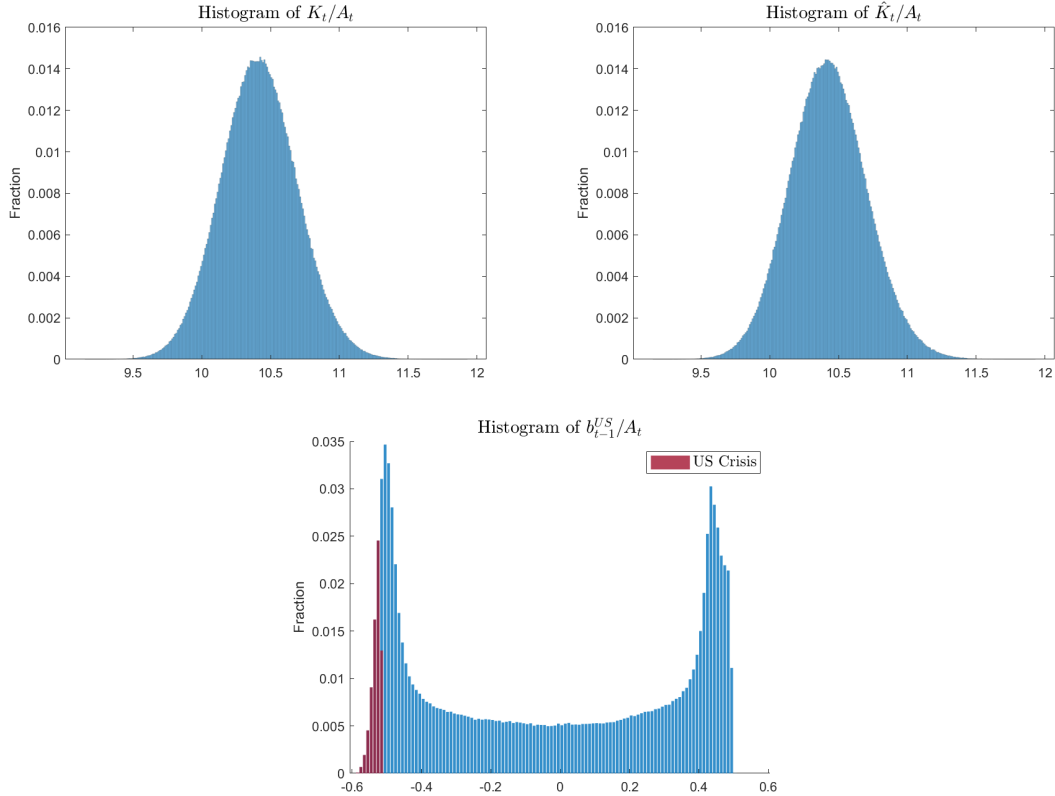
Note: This figure is generated by the solution of the baseline model with parameters given in Table 3.

We identify the recursive equilibrium in the benchmark model with three endogenous state variables: the US and UK capital stocks, and US bond holdings; $K_t, \hat{K}_t$ and b_{t-1}. Figure 4 shows the marginal distributions of these endogenous state variables computed by simulation from the

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[15]The relative volatility of productivity is really measuring the volatility of equilibrium output because productivity is exogenous.

stationary recursive equilibrium. The distributions for capital are unimodal and symmetric. In contrast, the distribution for b_{t-1}, which identifies the US net foreign asset (NFA) position, is bimodal with peaks at the US and UK borrowing limits. This feature of the equilibrium reflects the fact that shocks have persistent effects on the international distribution of bond holding away from the borrowing limits: it appears in all the IRBC models we study with incomplete risk-sharing.[16] As we shall see, the proximity of an economy to its international borrowing limit has important implications for the transmission and propagation of productivity shocks.

2.2 The Transmission Mechanism in the Benchmark Model

In this section, we examine the transmission and propagation of productivity shocks in the benchmark model. We first show that *under most circumstances* the transmission mechanism produces initial co-movements in the real exchange rate and output that are consistent with the empirical evidence from the US-UK data, presented in Section 1. Next, we demonstrate that a very different transmission mechanism operates in the model when either the US or UK is close to or at the international borrowing constraint. This mechanism is consistent with the empirical evidence in Section 1 when either country is in a banking crisis.

General Transmission According to our empirical model, shocks raising the cyclical component of US GDP produced a real appreciation in the US dollar, whereas shocks increasing the cyclical component of UK GDP produced a real depreciation (see Figure 2). We now compare these implications with the dynamic responses of output and the real exchange rate to productivity shocks in the benchmark model.

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[16]The existing literature removes the feature by altering the model with stationary cardinal utility (Corsetti et al., 2008) or with quadratic bond-holding cost (Heathcote and Perri, 2002). These modifications make it possible to solve the model using local, linearization methods. But they also remove the state-dependent equilibrium properties, which we focus on in the present paper.

Our global method produces a set of non-linear equations that characterize the equilibrium dynamics in the model, so the dynamic effects of productivity shocks depend on the prevailing values of the endogenous state variables. To account for this form of state-dependency, we simulate the equilibrium effects of the productivity shocks over 50 periods starting from 5000 values for the endogenous state variables that are randomly chosen from their joint ergodic distribution. We then compute the average dynamic effect of the productivity shock for each of the 50 periods across the simulations. We refer to these averages as the Unconditional Impulse Response Functions (UIRFs) because the dynamic effects of the productivity shocks are not conditioned on particular values of the prevailing endogenous state variables.
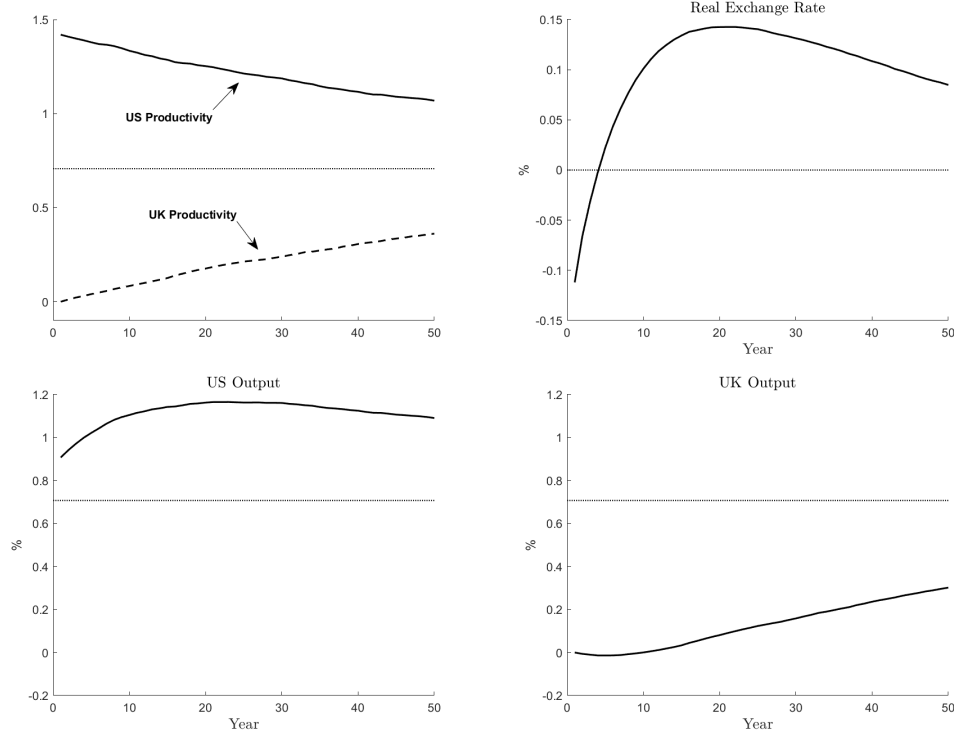
Figure 5 plots the UIRFs for log US productivity, the log real exchange rate, and the logs of US and UK output, induced by a positive US productivity shock.[17] The plots show that while the productivity shock produces an immediate and persistent rise in US output, it has minimal effects on UK output (there is a small fall and then rise in the years following the shock). The shock also produces an immediate and persistent real appreciation of the US dollar, which lasts for approximately 5 years. The medium run effects of the productivity shock on US output and the real exchange rate are consistent with the empirical responses in Figure 2.

At first glance, the immediate exchange-rate effects of productivity shocks shown in Figure 5 may seem counterintuitive. Since positive US productivity shocks increase the production of US goods, it seems that the relative price of these goods should fall, which would be reflected in a depreciation of the real exchange rate (see equation (10)). To uncover the flaw in this intuition, we need to understand how productivity shocks affect the demand for US (and UK) goods. Recall that households are prohibited from holding foreign equity, which inhibits risk-sharing. As a consequence, productivity shocks to US firms have larger wealth effects on US households than UK households

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[17]The UIFR for productivity is a standard IRF because the process in (12) is linear.

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Figure 5: UIRFs for US productivity, real exchange rate, and US, UK output

(even though all households choose their portfolios optimally). This means that a positive US productivity shock increases the world consumption demand for US goods relative to UK goods *in the absence of any change in relative prices* (because household preferences exhibit home bias in consumption). Productivity shocks also affect the investment demand for goods by firms through their impact on the marginal product of capital. In particular, because a positive shock produces a persistent increase in US productivity (see Figure 2), it also creates a strong incentive for US firms to increase investment when the shock occurs. In sum, therefore, the initial effect of a positive US productivity shock is to increase the demand for US goods for both consumption and investment

Figure 6: IRFs for US NFA and US, UK consumption and investment
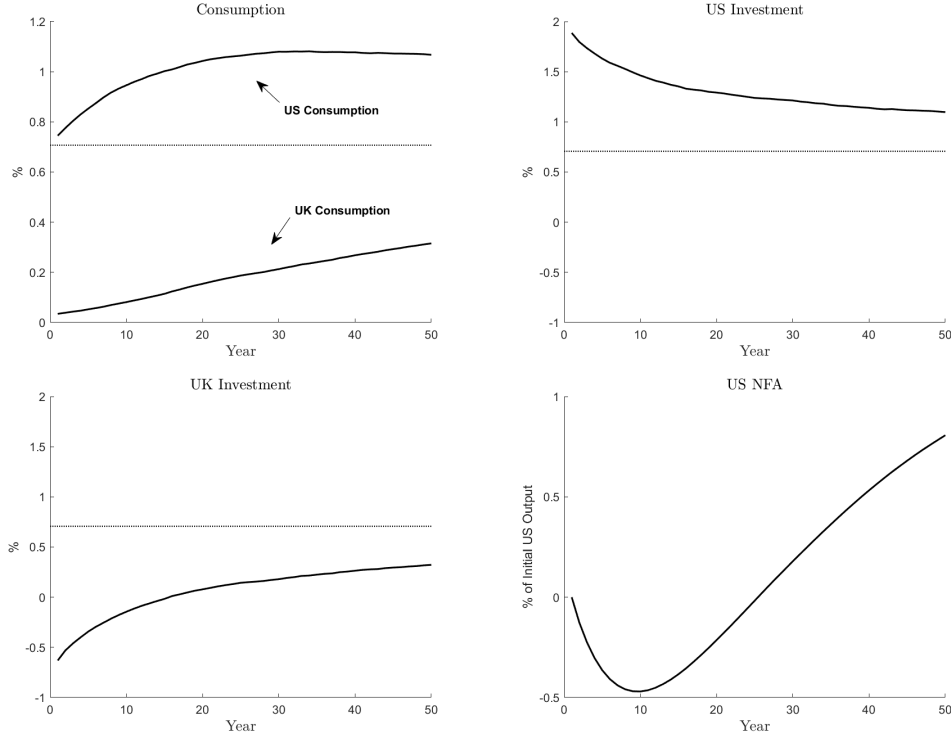
Note: This figure is generated by the solution of the baseline model with parameters given in Table 3. The horizontal dotted line plots the long-run trend.

purposes. Of course, the productivity shock also makes existing capital (and labor) more productive, which increases the supply of US goods. While it is *possible* that the higher supply of US goods matches the increase in demand, so markets will clear without any change in relative prices, this is not the case in our calibration of the benchmark model. Instead, the relative price of US goods rises to reduce the consumption demand for US goods so that total demand matches the production capacity of US firms while they undertake more investment. The real appreciation of the dollar induced by the positive US productivity shock reflects this change in relative prices.

Figure 6 provides more information on the transmission of productivity shocks. The upper panel

plots the UIRFs for the US and UK consumption indices and US real investment. These plots show that the real appreciation of the dollar reconciles the differing wealth effects of the productivity shock on US and UK households produced by incomplete risk sharing with the desire of US firms to increase real investment. The increase in the relative price of US goods induces both US and UK households to substitute UK goods for US goods in their consumption baskets, which facilitates market clearing in the US goods market, but it also increases the world consumption demand for UK goods. Since there is no immediate change in the productive capacity of UK firms, this increase in demand must be accommodated by a fall in UK investment, as is shown by the UIRF in the lower left-hand panel of Figure 6. This takes place because the expected increase in UK consumption following the shock (shown in the upper left-hand plot) raises the required real return on UK capital. Thus, in effect, the appreciation of the real exchange rate facilitates a shift in the pattern of international real investment towards US production and away from UK production.[18]

The US productivity shock also has implications for trade flows and the US NFA position (b_{t-1} in the US budget constraint (7b)). In particular, the initial higher consumption of UK goods by US households produces a US trade deficit and a deterioration in the US NFA position. This is shown by the UIRF for the US NFA position plotted in the lower panel of Figure 6. Then, as US investment falls relative to output, the real exchange rate depreciates and consumption shifts back towards US goods, producing a US trade surplus. As the plot shows, the shock has persistent effects on US NFA position with a deterioration that lasts for close to ten years.

The response of the US NFA position in Figure 6 also makes clear that in order to garner the full benefit of a US productivity shock, US households must be able to borrow internationally,

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[18]It is clear from the discussion that if US firms can substitute UK goods for US goods when undertaking investment in their capital stock, as assumed in Backus et al. (1993) and the subsequent literature, then our transmission mechanism is weakened. Backus et al. (1993) assume that investment goods have exactly the same home-bias and substitution patterns as consumption goods. However, Burstein et al. (2004) find that investment goods have very different domestic-imported contents from consumption goods. Our model features this dissimilarity between investment and consumption.

or equivalently run a trade deficit. Because the UIRFs in Figures 5 and 6 are average responses computed from the entire ergodic distribution of the endogenous state variables, they do not reflect the transmission of productivity shocks when the US NFA position is close to the lower bound of the ergodic distribution shown in Figure 4. We consider this scenario next.

Transmission Near the Borrowing Constraint The previous analyses highlight the importance of the households' ability to borrow to take advantage of positive productivity shocks in real exchange rate dynamics. The global solution allows us to study what happen in regions of the state space where the borrowing ability is hindered, i.e., where households are close to their international borrowing limit. An empirical proxy for these regions is the banking crises documented in Section 1. These are times where the primary debt markets are impaired. Financial institutions in the U.S. or in the U.K. are significantly constrained in issuing or underwriting new debts to households. Hence, households act as if they are being close to the borrowing limit in our model and are constrained in their ability to borrow more.

Following the international literature on sudden stops such as Mendoza (2010) and Bianchi (2011), we classify banking crisis in the model as states in which US NFA position lies below the 7% percentile of its marginal distribution. The threshold is chosen so that the frequency, and hence the implied persistence of the crisis states, are similar to those of banking crises in Reinhart and Rogoff (2008)'s US and UK samples. In order to study the effects of US productivity shocks in these states, we compute a second set of impulse response functions by simulations. We simulate the equilibrium effects of the productivity shocks over 50 periods starting from 5000 values for the endogenous state variables but now they are randomly chosen from a portion of their joint ergodic distribution where the US NFA position lies below the crisis threshold. We then compute the average dynamic effect of the productivity shock for each of the 50 periods across the simulations.

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We refer to these averages as the Conditional Impulse Response Functions (CIRFs) because the dynamic effects of the productivity shocks are conditioned on US households being in the proximity of their international borrowing constraint.

Figure 7 plots the CIRFs and the UIRFs for log output, the log real exchange rate, log consumption, US and UK investment, and the US NFA position induced by a positive US productivity shock. The plots in the upper left-hand panel show that proximity to the borrowing constraint does not significantly affect how US productivity shocks affect either US or UK output. In particular, there are no visible differences between the CIRF and UIRF for US output up to five years following the shock. In contrast, the upper right-hand panel shows that proximity to the borrowing limit changes the impact of the shock on the real exchange rate; producing an initial real depreciation of the US dollar rather than an appreciation. The plot also shows that proximity to the borrowing constraint dampens the real exchange-rate response over the next ten years.

To understand why the exchange rate effects of the productivity shock are so different in the proximity of borrowing constraint, recall that at prevailing relative prices, the initial effect of the shock is to increases consumption and investment demand for US goods beyond the productivity capacity of US firms. When US households are able to borrow internationally, markets clear because the relative price of US goods rises so that US households substitute UK for US goods in their consumption basket. Since this adjustment mechanism produces a US trade deficit, it is inoperable when US households are at the borrowing constraint. Instead, US households are restricted from consuming more UK goods that cannot be purchased from the proceeds of higher US exports to the UK. The relative price of UK goods must rise to reconcile this balanced trade restriction with the wealth effect of the productivity shock on US households. The middle panel of Figure 7 shows that the shock produces an increase in US households' composite consumption via the wealth effect, so US

Figure 7: IRFs for US real exchange rate, NFA, and US, UK output, consumption, investment
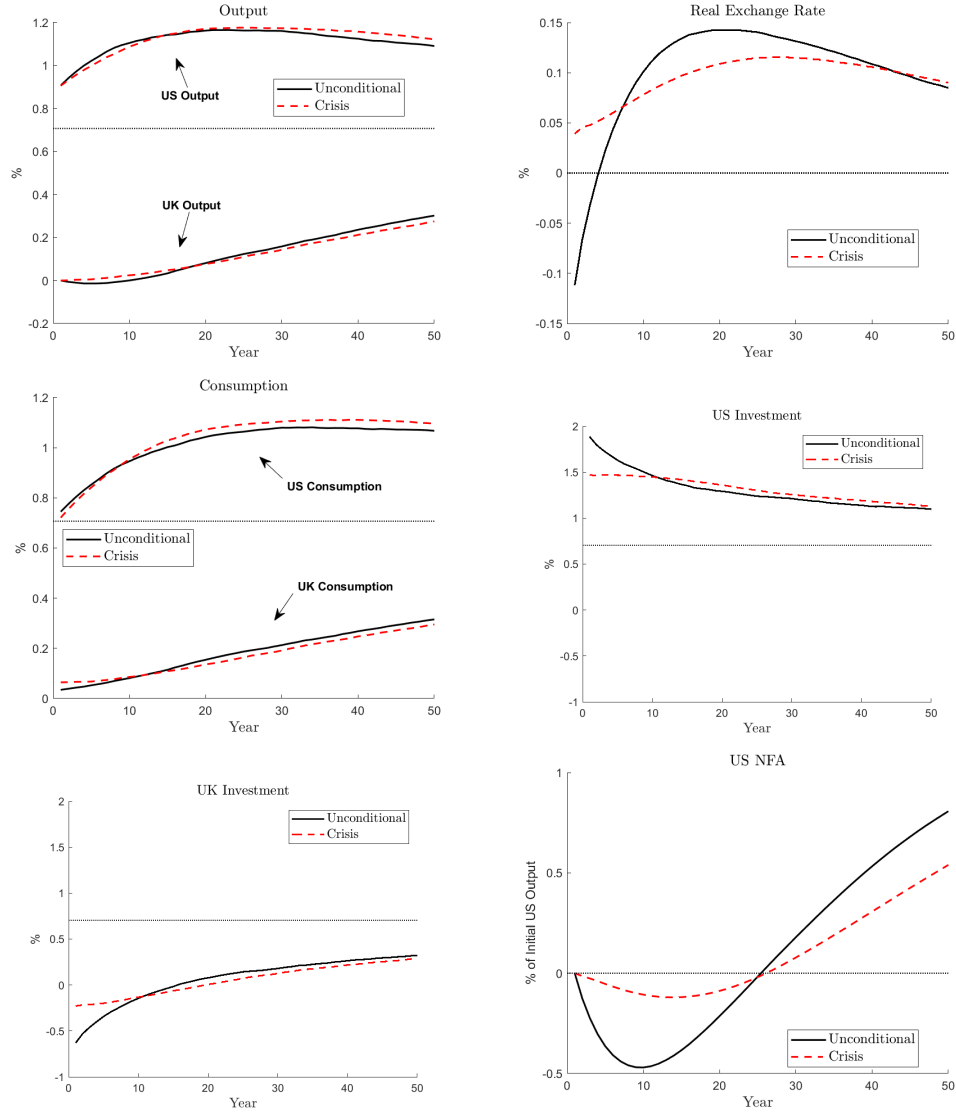
Note: This figure is generated by the solution of the baseline model with parameters given in Table 3. The solid lines are UIRFs and the dashed lines are CIRFs, conditional on US NFA normalized by US productivity being in the fifth percentile of its marginal distribution. The horizontal dotted line plots the long-run trend.

households need to shift the composition of their consumption basket from UK to US goods to avoid

running a trade deficit. Of course, this shift means that less of the prevailing productive capacity of

US firms is available for real investment, so the higher demand for US investment produced by the productivity shock must be tempered by an increase in the required return on capital. These effects can be seen by comparing the CIFR and UIFR for US investment and consumption in the middle panel of Figure 7. The productivity shock induces a smaller increase in US investment and higher *growth* in US consumption. The lower panel of Figure 7 shows two further implications. First, the initial fall in UK investment is reduced because the borrowing constraint limits export demand.[19] Second, the international borrowing constraint prevents the US NFA position from falling in the immediate aftermath of the shock and dampens its rise in subsequent years. Indeed, a comparison of the CIRFs and UIRFs for US consumption and the NFA position makes clear that proximity to the international borrowing constraint limits the ability of US households to inter-temporally smooth consumption.

In summary, our analysis shows that the transmission mechanism for productivity shocks depends critically on the ability of a country benefiting from the shock to borrow internationally. When there are no restrictions on further borrowing, the exchange rate moves to facilitate the international reallocation of investment to its most productive use, but when further borrowing is restricted, the real exchange rate moves to mitigate the effects of the restriction on welfare.

3   Additional Inspections of the Mechanism

In this section, we examine the role of different modeling ingredients in the transmission mechanism described in the last section. We first discuss the role of trade elasticities in the next subsection. Then, we examine the role of financial market structure and other ingredients such as the produc-

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[19]The appreciation of the dollar also lowers the value of UK bond holdings, so that the wealth effect of the productivity shock on UK households is smaller; see the CIRF and UIRF for UK composite consumption in the middle panel of Figure 7.

tivity process, capital accumulation, and solution methods. Due to space constraints, we relegate many of the details in these analyses to the appendix.

3.1 The Role of Trade Elasticities

The transmission mechanism in the benchmark model relies on the high elasticity θ between traded goods (i.e., the ability of both US and UK households to substitute US goods for UK goods in their consumption baskets). To better understand the role of this elasticity, we solve and simulate the models for different values of θ. Figure 8 plots the unconditional and conditional responses of the log real exchange rate when a positive productivity shock hits the US. The figure shows that θ needs to be larger than 2 in order for the exchange rate to appreciate on average. In addition, θ needs to be larger than 4 for the appreciation to last more than 4 years, consistent with the medium run variation documented in the empirical section.

In the benchmark model, we choose θ equals to 5. While this is within the range of estimates in the literature, one might argue that it is relatively large. However, the benchmark model features only traded goods, while the empirical literature considers a wide range of traded goods with different elasticities of substitution. To reflect that kind of heterogeneity, we consider an extension of the benchmark model with both traded and non-traded goods with much lower elasticity between these goods. This structure effectively lowers the average elasticity between US and UK goods. In particular, we assume that US households consume a basket of US non-traded goods and US and UK traded goods:

$$C_t = \tilde{\mathcal{C}} \left(C_t^{\mathrm{NT}}, \mathcal{C} \left(C_t^{\mathrm{US}}, C_t^{\mathrm{UK}} \right) \right). \tag{14}$$

The aggregator between US and UK traded goods is given by $\mathcal{C}(.)$ described in the benchmark

Figure 8: Comparative Statics in Trade Elasticity
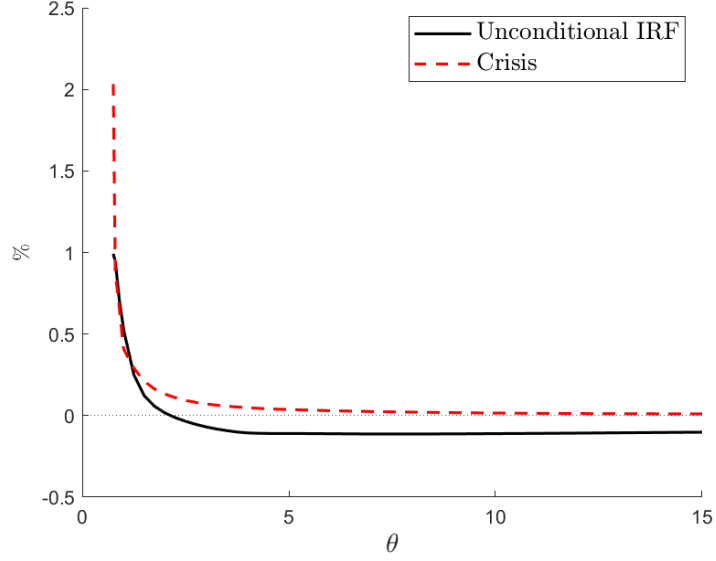
Notes: The figure plots the impact responses of real exchange rate as functions of the trade elasticity, θ.

model, (5) and the aggregator $\tilde{\mathcal{C}}(.)$ between non-trade and traded components is given by

$$\tilde{\mathcal{C}}(a,b) = \left(\lambda^{\frac{1}{\kappa}} a^{\frac{\kappa-1}{\kappa}} + (1-\lambda)^{\frac{1}{\kappa}} b^{\frac{\kappa-1}{\kappa}}\right)^{\frac{\kappa}{\kappa-1}}. \tag{15}$$

Given these aggregators, the consumption price index P_t for US households in US dollars becomes

$$P_t = \left(\lambda(P_t^{\mathrm{T}})^{1-\kappa} + (1-\lambda)(P_t^{\mathrm{NT}})^{1-\kappa}\right)^{\frac{1}{1-\kappa}}, \tag{16}$$

where P_t^{NT} is the dollar price of non-traded goods and P_t^{T} is the price index for traded goods

$$P_t^{\mathrm{T}} = \left(\alpha(P_t^{\mathrm{US}})^{1-\theta} + (1-\alpha)(P_t^{\mathrm{UK}})^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{17}$$

36

where P_t^{US} and P_t^{UK} are the dollar prices at which the US households purchase US and UK traded-goods respectively.

The addition of non-traded goods opens the door to another channel through which productivity shocks affect the real exchange rate. In particular, the real exchange rate now is determined by

$$\mathcal{E}_t = \frac{S_t \hat{P}_t}{P_t} = \underbrace{\left[\frac{\left((1-\hat{\alpha}) + \hat{\alpha}\mathcal{T}_t^{1-\theta} \right)^{\frac{1}{1-\theta}}}{\left(\alpha + (1-\alpha)\mathcal{T}_t^{1-\theta} \right)^{\frac{1}{1-\theta}}} \right]}_{\text{ToT}} \underbrace{\left[\frac{\left(\hat{\lambda} + \left(1-\hat{\lambda}\right)(\hat{P}_t^{\mathrm{NT}}/\hat{P}_t^{\mathrm{T}})^{1-\kappa} \right)^{\frac{1}{1-\kappa}}}{\left(\lambda + (1-\lambda)(P_t^{\mathrm{NT}}/P_t^{\mathrm{T}})^{1-\kappa} \right)^{\frac{1}{1-\kappa}}} \right]}_{\text{HBS}}, \qquad (18)$$

where $\mathcal{T}_t = \frac{P_t^{\mathrm{UK}}}{S_t \hat{P}^{\mathrm{US}}} = \frac{P_t^{\mathrm{UK}}}{P_t^{\mathrm{US}}}$ are the terms of trade (ToT). The first term on the right-hand-side (which is equal to $S_t \hat{P}_t^{\mathrm{T}}/P_t^{\mathrm{T}}$) is solely a function of $\mathcal{T}_t$, so we call this the ToT component:[20] The second term on the right-hand-side depends on the relative prices of traded and non-traded goods in both countries. HBS emphasize the importance of this term in determining real exchange rate in the long run. Therefore, we call this the HBS component. We will use this decomposition for $\mathcal{E}_t$ to analyze the factors that affect the dynamics of real exchange rate in our model.

In this model, we assume that the firms in the non-traded sectors have the same Cobb-Douglas production function as the ones in the traded sectors: $F^{\mathrm{NT}}(K,L) = A^{\mathrm{NT}} K^{\eta} L^{1-\eta}$, with exactly the same productivity:$A_t^{\mathrm{NT}} = A_t$. The other details of the model are given in Appendix G. We keep θ at 5 but pick a much lower value for the elasticity between traded and non-traded goods, $\kappa = 0.75$, in line with the estimates in the literature.

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[20]To derive this expression, we use the fact that

$$\frac{S_t \hat{P}_t^{\mathrm{T}}}{P_t^{\mathrm{T}}} = \frac{S_t \hat{P}_t^{\mathrm{US}} \left(\hat{P}_t^{\mathrm{T}}/\hat{P}_t^{\mathrm{US}} \right)}{P_t^{\mathrm{US}} \left(P_t^{\mathrm{T}}/P_t^{\mathrm{US}} \right)} = \frac{\left(\hat{P}_t^{\mathrm{T}}/\hat{P}_t^{\mathrm{US}} \right)}{\left(P_t^{\mathrm{T}}/P_t^{\mathrm{US}} \right)}.$$

Combing this expression with the definitions for P_t^{T} , $\hat{P}_t^{\mathrm{T}}$, and $\mathcal{T}_t$ produces the component shown in (18).
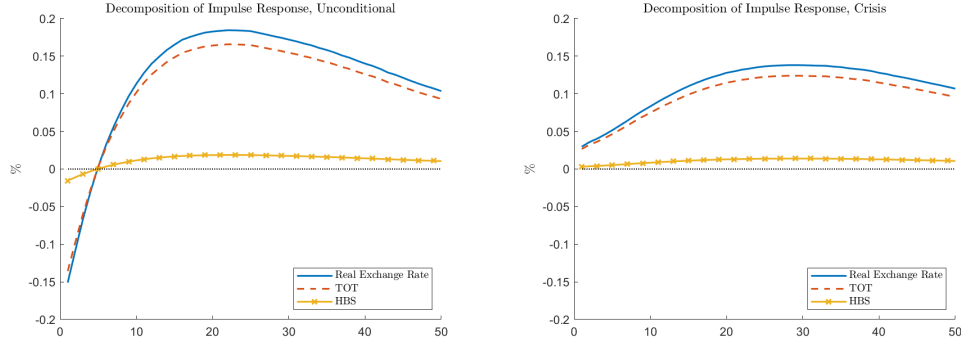
Figure 9: IRFs for US real exchange rate, NFA, and US, UK output, consumption, investment
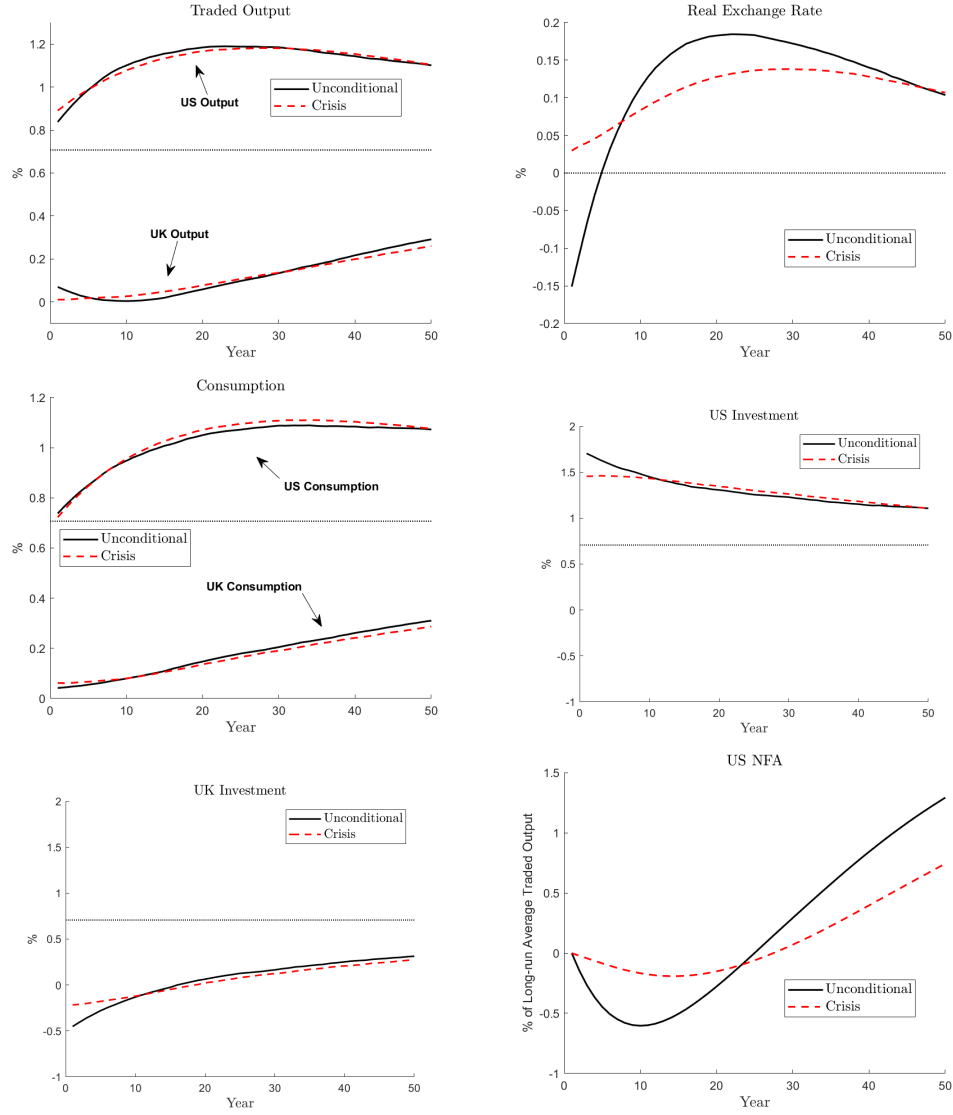
Note: This figure is generated by the solution of the baseline model with parameters given in Table 3. The solid lines are UIRFs and the dashed lines are CIRFs, conditional on US NFA normalized by US productivity being in the fifth percentile of its marginal distribution. The horizontal dotted line plots the long-run trend.

Figure 9 shows that the unconditional and conditional IRFs are similar to the ones in Figure 5 and Figure 7 for the benchmark model. In particular, real exchange rate appreciates unconditionally

Figure 10: IRFs for real exchange rate and its decomposition

Note: This figure is generated by the solution of the baseline model with parameters given in Table 3. The solid lines are UIRFs and the dashed lines are CIRFs, conditional on US NFA normalized by US productivity being in the fifth percentile of its marginal distribution. The horizontal dotted line plots the long-run trend.

but depreciates near the borrowing limit. The magnitude of real exchange rate responses are larger in this model than in the benchmark model (the average depreciation at impact is -0.16% in this model compared to -0.12% in the benchmark model). Figure 10 shows that most of the response of real exchange rate is driven by the term of trade effect. The HBS effect is small because the productivity is the same between the traded and non-traded sectors.

In Appendix G, we consider an alternative specification in which productivity shocks directly hit the traded-good sector and only affect productivity in the non-traded good sector with a significant lag. In this specification, the variation in real exchange is driven mostly by the HBS effect and the magnitude of the variation is larger. However, the real exchange rate response does not depend significantly on whether the countries are close to their borrowing limits. Even near the borrowing limit, a positive productivity shocks in the US leads a real appreciation of the US dollar.

3.2 The Role of Financial Market Structure and Other Ingredients

Financial markets play a crucial role in the transmission mechanism described in the benchmark model. Indeed, previous results show that the transmission mechanism changes as countries get closer to their borrowing limits. Now we further examine the role of the financial market structure by considering three variants on our benchmark specification: one with complete markets, one with financial autarky, and one where households can trade two bonds internationally. In the two bond model, households face a more substantive portfolio choice problem of allocating their wealth between domestic equity, US bonds and UK bonds. A complete description of these models can be found in Appendix D.

Figure 11 plots the UIRFs of endogenous variables including output and the real exchange rate from a positive US productivity shock for the benchmark model and the three variants. These plots show that the degree of risk-sharing has small effects on the behavior of output. The UIRFs are essentially indistinguishable between the models with international trade in one or two bonds, complete markets, or financial autarky. The UIRFs also show that the impact of the shock on the real exchange rate is similar in the models with one or two bonds; there is an initial real appreciation of the dollar that lasted for 4 years followed by depreciation. In contrast, there is an initial real depreciation of the dollar in the model with complete financial markets and financial autarky, which persists for a very long time. The response of the exchange rate under financial autarky is similar to the response we examined in the benchmark model when US households were close to the international borrowing limit. Complete markets allow for much more risk-sharing between the two countries as reflected in the comparable rise in UK consumption relative to US consumption and the accommodating depreciation of the real exchange rate.

There are significant differences between the UIRFs for US and UK consumption across the

Figure 11: UIRFs under Different Market Structures
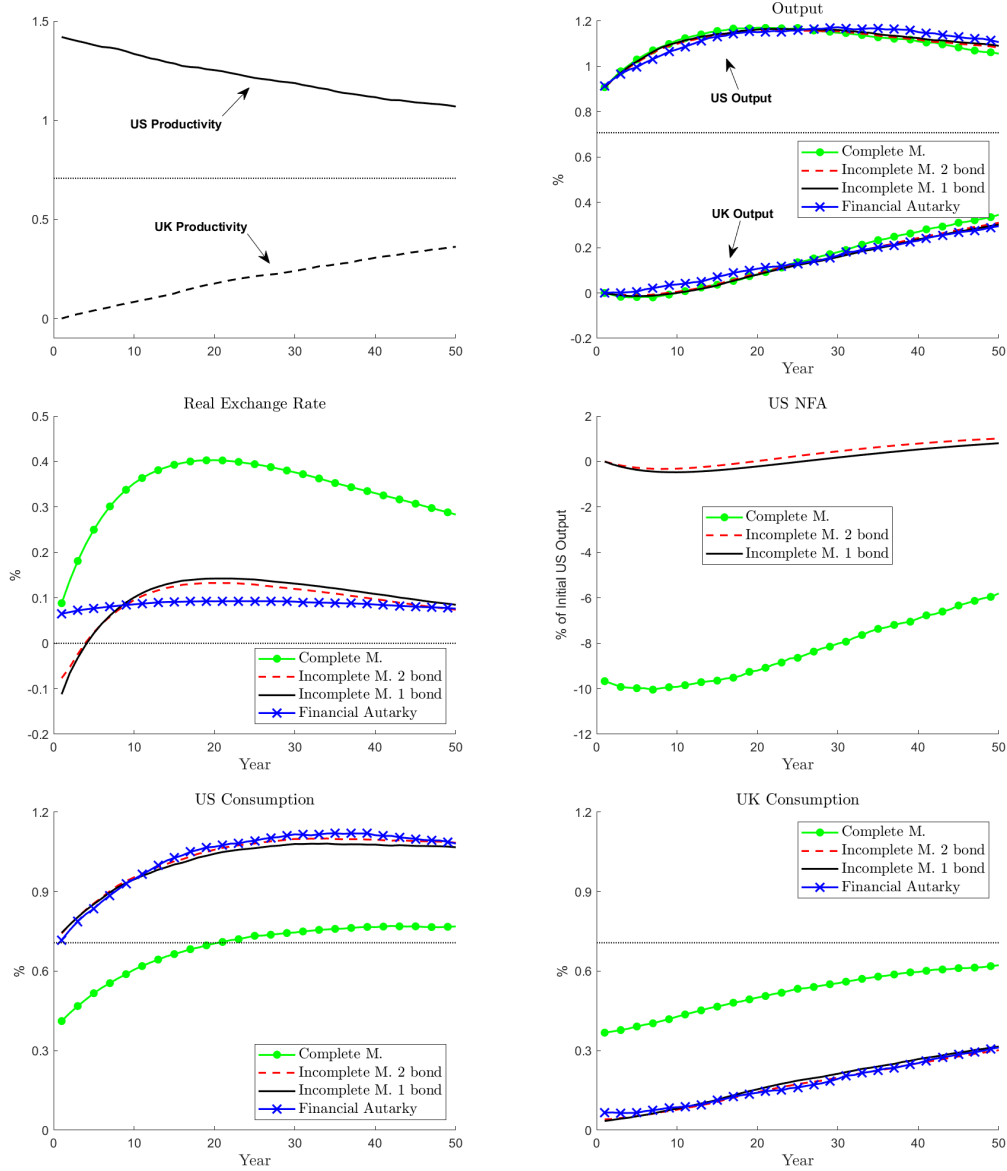
Note: This figure is generated by the solution of the baseline model with parameters given in Table 3 and under different financial market structures. The horizontal dotted line plots the long-run trend.

models with financial autarky and incomplete markets (with one or two bonds) versus complete markets. Figure 11 shows that under financial autarky the effects of the US productivity shock are almost entirely confined to US consumption because the real depreciation of the dollar keeps the US demand for UK goods essentially unchanged. Productivity shocks have similar effects on US and UK consumption in the models with international asset trade in one or two bonds. In both models, the initial effect of the shock is to increase US consumption more than UK consumption because the wealth effects are concentrated in US equity. In the complete markets model, the wealth effects of the productivity shock are more evenly distributed between US and UK households, so the shock has a smaller (larger) initial impact on US (UK) consumption than when markets are incomplete. In this case, the UIRFs for US and UK consumption appear very similar. The analysis also reveals that co-movements in the real exchange rate and consumption depend on the degree of risk-sharing. When risk-sharing is impaired (i.e., in either the one or two bond models), positive US productivity shocks induce a real appreciation and a rise in US consumption relative to UK consumption. More generally, in our benchmark specification productivity shocks induce a negative correlation between the real depreciation rate $\Delta \varepsilon_{t+1}$ and relative consumption growth $\Delta \ln C_{t+1} - \Delta \ln \hat{C}_{t+1}$. Table 5 shows that the correlation between relative consumption growth and the real depreciation rate is significantly negative under incomplete markets. The correlation is similar to the estimated correlation of -0.39 documented in Corsetti et al. (2008) using recent US and UK annual data. Of course, as Backus and Smith (1993) originally stressed, the correlation is one under complete markets. It is also close to one under financial autarky.

Figure 11 also provides a perspective on why the consumption responses to productive shocks differ in the models with complete and incomplete markets. The figure plots the UIRFs for the US NFA position in the model with complete markets and in the models with one and two inter-

Table 5: International Co-movements

Economy	RER-Output Corr $corr\left(\Delta \log Y_{t+1}, \Delta \log \mathcal{E}_{t+1}\right)$	Backus-Smith Corr $corr\left(\Delta \log C_{t+1} - \Delta \log \hat{C}_{t+1}, \Delta \log \mathcal{E}_{t+1}\right)$
Data		
US vs. UK		Negative
Specifications		
Complete Markets	0.519	1.000
Incomplete Markets (2 bonds)	-0.478	-0.707
Incomplete Markets (1 bond)	-0.444	-0.632
Financial Autarky	0.705	1.000

nationally traded bonds. Here we see that a positive US productivity shock has little impact on the US NFA position in the models with incomplete markets.[21] So the wealth effects of the shock originate from the capital gains of households' domestic equity holdings. In the complete market model, the productivity shock lowers the US NFA position, which dampens the wealth effect on US households, and amplifies the effect on UK households.[22] Intuitively, households hold a portfolio of state-contingent securities that provides a hedge against adverse domestic productivity shocks (because there is home bias in consumption), so US households enjoy a smaller capital gain in response to positive US productivity shocks than UK households. As a consequence, the shock produces an increase in UK consumption almost as large as US consumption.

We conclude by briefly noting the results of some additional robustness exercises. First, to understand the importance of endogenous investment and capital accumulation in the transmission mechanism, we study an exchange economy with exactly the same productivity processes and financial market structures in Appendix E. In this exchange economy, the real exchange immediately depreciates in response to positive productivity shocks regardless of financial market structures.

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[21]In the benchmark model, NFA positions are defined in terms of US bond holdings, so an appreciation of the real exchange rate implies a capital gain on UK NFA, but has no effect on US NFA. In the two bond model, an appreciation of the real exchange rate represents a capital loss on US holdings of UK bonds, and a capital gain on UK holdings of US bonds. The UIRF for US NFA shows that these valuation effects are typically very small.

[22]Under complete markets, households trade state-contingent Arrow securities which pay off depending on the realization of productivity shocks. The US NFA position is defined as the ex-post payoff of US households' Arrow security portfolio. The exact notation and definition are given in Appendix D.

Second, we study a quarterly version of the model with stationary productivity processes (as in the standard RBC literature) in Appendix F. Under incomplete markets, this version of our model delivers a real exchange rate appreciation at the impact of a productivity shock near the borrowing limit but the appreciation lasts for only one or two quarters.

4   Conclusions

This paper has examined how medium-term movements in real exchange rates and GDP vary with international financial conditions. Empirically, we have shown that shocks producing cyclical variations in GDP account for a significant fraction of the real US-UK depreciation rate over horizons ranging from five to fifteen years. We then showed that productivity shocks can induce co-movements in the real exchange rates, GDP and consumption that are consistent with the data in a variety of IRBC models. One important finding to emerge from this analysis is that the co-movements depend on prevailing financial conditions; more specifically the proximity of a country to its international borrowing constraint. We find evidence consistent with this form of state-dependency in the US-UK data.

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APPENDIX

A   Additional Empirical Analyses

The empirical analyses in Section 1 takes very long time series for the US and the UK. There are major historical developments over the time period that might potentially relevant for international finance and trade, and hence the medium run dynamics of real exchange rate. These includes changes in exchange rate regimes (the sterling as the dominant currency, the gold standard, and the Bretton Woods system) and the world wars. In this appendix, we examine whether our state-dependence results depend on these different regimes. To do so, we re-estimate equations (3) allowing for the coefficients to vary with the regimes. That is, the equation is modified to

$$\Delta\varepsilon_t = \left(\begin{bmatrix} \beta_1 & \beta_2 \end{bmatrix} + \sum_{\text{regime}\in\{1,2,3,4\}} \begin{bmatrix} \beta_{1,\text{regime}}^{\text{diff}} & \beta_{2,\text{regime}}^{\text{diff}} \end{bmatrix} s_t \right) \begin{bmatrix} y_t^{\text{Cycle}} \\ \hat{y}_t^{\text{Cycle}} \end{bmatrix} + \begin{bmatrix} \gamma + \gamma^{\text{diff}} s_t \end{bmatrix} gap_{t-1} + e_t,$$

where the four regimes are, respectively: (1) 1802-1913, the sterling as the dominant currency; (2) 1914-1948, the war times; (3) 1949-1971, the Bretton Woods system; (4) 1972-2016, the post Bretton Woods system. Column (1) of Table 6 replicates the estimates that are reported in Table 2 of the paper. Column (2) reports the estimates for the modified equation. The regime-dependent effects are captured by the coefficients of the GDP-crisis-regime triple interaction terms.[23] As shown in the table, none of the estimated coefficients of the triple interaction terms are statistically significant, and the F-statistics for the joint test that all triple interaction coefficients are zero is 0.51 (p-value=0.67). This result suggests that the null hypothesis that there are no structural breaks is not rejected.

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[23]Not all coefficients are estimated since in some regimes there is not a single crisis.

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Table 6: Testing for structural breaks

	Real Depreciation Rate Equation: $\Delta\varepsilon$	
	(1) Baseline	(2) Testing breaks
Regressors		
y^{Cycle}(US)	-0.294*	-0.296*
	(0.155)	(0.158)
$\times crisis$	0.806**	1.481
	(0.386)	(0.935)
$\times crisis \times 1(1914-1948)$		-
$\times crisis \times 1(1949-1971)$		-
$\times crisis \times 1(1972-2016)$		1.601
		(2.350)
y^{Cycle}(UK)	0.609***	0.609***
	(0.163)	(0.167)
$\times crisis$	-1.702***	-2.147***
	(0.589)	(0.625)
$\times crisis \times 1(1914-1948)$		-0.705
		(0.737)
$\times crisis \times 1(1949-1971)$		-
$\times crisis \times 1(1972-2016)$		-1.111
		(3.233)
gap	-0.148***	-0.144***
	(0.034)	(0.0355)
$\times crisis$	-0.313***	-0.316***
	(0.097)	(0.0995)
R-squared	0.183	0.188
F-statistics (all triple interaction coefs = 0)	-	0.51

B Equilibrium Definitions and Solution Method

In this appendix, we provide a more detailed description of the equilibrium and the global solution method.

B.1 Equilibrium

We use a standard definition of sequential competitive equilibrium: Given initial state variables $K_0, b_{-1}, \hat{K}_0, \hat{b}_{-1} = -\hat{b}_1$, a sequential competitive equilibrium (SCE) consists of *stochastic* sequences of allocations

$$\left\{ C_t^{\text{US}}, C_t^{\text{UK}}, \chi_t, b_t, K_{t+1}, L_t, \hat{C}_t^{\text{US}}, \hat{C}_t^{\text{UK}}, \hat{\chi}_t, \hat{b}_t, \hat{K}_{t+1}, \hat{L}_t \right\}, $$

and prices

$$\left\{ p_t^{\text{US}}, p_t^{\text{UK}}, q_t, q_t^b, \hat{p}_t^{\text{US}}, \hat{p}_t^{\text{UK}}, \hat{q}_t, \mathcal{E}_t \right\},$$

such that the allocations solve households' and firms' optimization problems and markets clear.

Market clearing in the two goods markets requires that

$$Y_t = A_t K_t^{\eta} L_t^{1-\eta} = C_t^{\text{US}} + \hat{C}_t^{\text{US}} + K_{t+1} - (1-\delta)K_t, \quad \text{and}$$

$$\hat{Y}_t = \hat{A}_t \hat{K}_t^{\eta} \hat{L}_t^{1-\eta} = C_t^{\text{UK}} + \hat{C}_t^{\text{UK}} + \hat{K}_{t+1} - (1-\delta)\hat{K}_t.$$

Labor is supplied inelastically by domestic households. We normalize the labor supply to unity in both counties so labor market clearing requires,

$$1 = L_t \qquad \text{and} \qquad 1 = \hat{L}_t.$$

Finally, the total equity issued by US and UK firms is normalized to unity, and bonds are in zero net supply, so the market clearing conditions in the four financial markets are

$$1 = \chi_t, \qquad 1 = \hat{\chi}_t, \qquad \text{and} \qquad 0 = b_t + \hat{b}_t.$$

In order to solve for SCEs numerically, we look for recursive equilibria with a natural state space that includes the following variables:

$$x_t = \left(\frac{\hat{A}_t}{A_t}, \frac{\xi_t}{A_t}, \frac{K_t}{A_t}, \frac{\hat{K}_t}{A_t}, \frac{b_{t-1}}{A_t} \right).$$

We normalize the state variables by A_t to keep the state variables stationary. Notice that from the financial market clearing conditions, equity holdings are always equal to 1 and the bond holdings of UK households are implied by the bond holdings of US households. In a recursive equilibrium, the allocations and prices are functions of the state variables.

Definition 1. A recursive equilibrium (RE) is a SCE in which the allocations and prices, appropriately normalized, are functions of the state variable x_t:

$$z_t = \mathcal{Z}(x_t)$$

for each equilibrium variable z

$$z = \left(\frac{C^{\text{US}}}{A}, \frac{C^{\text{UK}}}{A}, \frac{b'}{A}, \frac{K'}{A}, \frac{\hat{C}^{\text{US}}}{A}, \frac{\hat{C}^{\text{UK}}}{A}, \frac{\hat{b}'}{A}, \frac{\hat{K}'}{A}, p^{\text{US}}, p^{\text{UK}}, \frac{q}{A}, q^b, \hat{p}^{\text{US}}, \hat{p}^{\text{UK}}, \hat{q}, \mathcal{E} \right), \tag{19}$$

where b', K' and $\hat{b}', \hat{K}'$ stand for next period bond holdings and capital stocks.

For later reference, we use the notation $\mathcal{K}(x), \hat{\mathcal{K}}(x), \mathcal{B}(x)$ for the $K', \hat{K}', b'$ components of $\mathcal{Z}(x)$. In Subsection B.2, we present the solution method used to compute these REs. A related equilibrium concept is the stationary recursive equilibrium which corresponds to a recursive equilibrium with a stationary distribution over the state variables (see Duffie et al., 1994 and Cao, 2020).

Definition 2. An stationary recursive equilibrium is a RE with a stationary distribution Δ over the state variable $x \in \mathbb{R}^5$ such that Δ is a fixed point of the transition function $\mathcal{F}$ implied by the RE policy functions:

$$\left(\mathcal{K}(x), \hat{\mathcal{K}}(x), \mathcal{B}(x) \right),$$

and exogenous stochastic law of motion for A and $\hat{A}$.

In an stationary recursive equilibrium, by the Ergodic Theorem, the long-run moments implied by the model can be computed as deterministic functionals of the policy functions $\mathcal{Z}$'s, transition function, $\mathcal{F}$, and stationary distribution Δ.

B.2 Global Solution Method

We solve for REs using policy-function iteration, a global method developed in Cao (2018) and Cao and Nie (2017). The method is well-suited for dynamic stochastic general equilibrium (DSGE) models with portfolio choices.[24] We extend the method to allow for endogenous capital accumulation, imperfect substitutions between traded goods, and up to five continuous state variables.[25] In the recent international finance literature, Rabitsch et al. (2015) and Stepanchuk and Tsyrennikov

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[24]Earlier work using policy-function iterations for DSGE economies includes Coleman (1990) and Judd et al. (2000).

[25]The incomplete markets two-bond model with co-integrated random walk productivity processes in Appendix D features five continuous state variables: relative productivity, two capital stocks, and two bond positions. Because of the presence of the two bonds, US and UK households need to solve portfolio choice problems. The bonds' returns are also close to being collinear because the variation of the real exchange rate is small when the elasticity of substitution, θ, is high.

(2015) also solve a two-country model with portfolio choice using policy-function iteration, but they only consider endowment economies, which allow them to describe the solution with a single continuous state variable (the wealth share). Coeurdacier et al. (2019) allow for up to three continuous state variables (two capital stocks and a bond position) but their model does not feature portfolio choice or imperfect substitution between traded goods. We describe in detail our global solution method below.

The optimization problems facing households and firms are concave maximization problems (concave objectives and convex constraints). Therefore, to guarantee that the allocations solve the agents' optimization problems, it is necessary and sufficient that the allocations satisfy the first-order conditions (F.O.C.s) at equilibrium prices. The Lagrange multiplier λ_t on the US households' budget constraint, (7b), is shown below to be equal to their marginal utility, and let μ_t denote the multiplier on the US households' borrowing constraint, (8). For US households, the F.O.C.s are:

$$\{C_t\} : C_t^{-\gamma} = \lambda_t,$$

$$\{b_t\} : -\lambda_t q_t^b + \beta \mathbb{E}_t[\lambda_{t+1}] + \mu_t = 0,$$

$$\{b_t, \mu_t\} : \mu_t(b_t - \underline{b}\xi_t) = 0, \mu_t \geq 0 \qquad \text{and}$$

$$\{\chi_t\} : -q_t\lambda_t + \beta \mathbb{E}_t[\lambda_{t+1}(q_{t+1} + d_{t+1})] = 0.$$

Similarly, we obtain the F.O.C.s for the UK households using the hat variables.

For US firms, the F.O.C in K_{t+1} (I_{t+1}) is:

$$\mathbb{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{p_{t+1}^{\text{US}}}{p_t^{\text{US}}} \left(\eta A_{t+1} K_{t+1}^{\eta-1} + 1 - \delta \right) - 1 \right] = 0,$$

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and for UK firms, the F.O.C. in $\hat{K}_{t+1}$ $(\hat{I}_{t+1})$ is:

$$\mathbb{E}_t \left[\beta \frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t} \frac{\hat{p}_{t+1}^{\text{UK}}}{\hat{p}_t^{\text{UK}}} \left(\eta \hat{A}_{t+1} \hat{K}_{t+1}^{\eta-1} + 1 - \delta \right) - 1 \right] = 0.$$

To solve for the the recursive equilibrium, we look for policy functions, $\{\mathcal{P}^{(n)}\}_{n=1}^N$ that map from state variables

$$x = \left(\frac{\hat{A}}{A}, \frac{\xi}{A}, \frac{K}{A}, \frac{\hat{K}}{A}, \frac{b}{A} \right),$$

to allocations and prices z given in (19), as well as the Lagrangian multipliers $\lambda, \mu, \hat{\lambda}, \hat{\mu}$. The Lagrange multipliers appear in the households' F.O.C.s to ensure that the allocations are optimal solutions to the households' consumption-saving/investment problems.

We initialize the policy function sequence with $\mathcal{P}^{(1)}$ which corresponds to the equilibrium in the 1-period version of the model. Assuming that we have solved for $\mathcal{P}^{(n)}$, we look for $\mathcal{P}^{(n+1)}$ by solving the system of equilibrium equations at each collocation point of the exogenous and endogenous state variables. The system of equations involves the 12 unknowns listed above and 12 equations (F.O.C.s, market-clearing conditions, relative prices and real exchange-rate restrictions). $\mathcal{P}^{(n)}$ corresponds to the equilibrium in the n-period version of the model and the limit of $\{\mathcal{P}^{(n)}\}$ corresponds to the equilibrium of the infinite-horizon model.[26]

To accommodate the high-dimension of the problem, we resort to the adaptive sparse grid method developed in Ma and Zabaras (2009) and recently applied in economic applications by Brumm and Scheidegger (2017). A unique aspect of the problem is that we need to solve for equilibria with multiple assets and portfolio choice in an environment with incomplete markets where there is trade in both US and UK denominated bonds (Appendix D). This problem is challenging

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[26]See Duffie et al. (1994) and Cao (2018, 2020) for examples of existence proofs using the limit of equilibria in finite-horizon economies.

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since returns to assets in certain regions of the state space are close to collinear, requiring the Jacobian matrix of the equilibrium system to be evaluated very accurately. To tackle this problem, we adopt an automatic differentiation method (Baydin et al., 2017) which allows us to evaluate the Jacobian matrix up to machine precision with speed comparable to analytical gradients.[27] Nevertheless, to solve the models accurately for many different calibrations requires intensive computation, which we undertake with numerical implementations in C++ and parallel programing on a 48 CPU core machine.

Ergodic Distribution and Impulse Response Functions   In order to compute the ergodic distribution over the (exogenous and endogenous) state variables

$$x = \left(\frac{\hat{A}}{A}, \frac{\xi}{A}, \frac{K}{A}, \frac{\hat{K}}{A}, \frac{b}{A} \right),$$

we simulate 100 samples for $21,000$ periods using the nonlinear policy functions

$$\left(\mathcal{K}(x), \hat{\mathcal{K}}(x), \mathcal{B}(x) \right)$$

from the RE solution.[28] We drop the first $1,000$ burn-in periods from each sample. Therefore, in total we have 2 million observations for the state variables which we use to construct the histogram of the ergodic distribution.

We use impulse response functions to understand the transmission mechanism in our model. To compute the unconditional impulse response functions (UIRFs), we pick $N = 500,000$ draws from the ergodic distribution (the last $5,000$ period draws from each of the 100 samples) and produce

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[27]We use an efficient (Jacobian) gradient-based equation solver that respects boxed bounds with an interior-point method, following Powell (1970), Coleman and Li (1996), and Bellavia et al. (2012).

[28]By the Ergodic Theorem, one long sample is enough to approximate the ergodic distribution but using 100 samples allows us to parallelize the simulations.

two replica sets of the draws. Starting from each draw

$$\left(x^{(n)}\right)_{n=1}^{N}$$

in the first set, we simulate forward 50 periods (years, or quarters depending on the model). We discretize the innovations of the productivity processes, ϵ_t and $\hat{\epsilon}_t$, using three-point distributions. From each draw in the second set, we set ϵ_t in the initial period to the highest level, and simulate forward 50 periods. The impulse response of a variable of interest, among the policy variables z, is the difference of the averages of the variable values from the two sets.

To compute the conditional impulse response functions (CIRFs), we pick a subset of N draws that satisfies the conditionality (e.g., b lies in the bottom 7% of its marginal ergodic distribution). Then we simulate forward the two sets of draws and calculate the difference of the averages of the variable of interest as we do for the UIRFs.

C Trade Elasticities

Figure 8 shows that θ affects the response of real exchange rate to productivity shocks. In particular, the real exchange rate appreciates upon a positive productivity shock only if θ is sufficiently high. Figure 12 provides additional insights by displaying the IRFs (solid lines) for the benchmark value of θ, $\theta = 5$, against the IRFs (dashed lines) for a lower value of θ, $\theta = 0.75$. While the dynamics of outputs are similar in the two cases, the dynamics of real exchange rate differ significantly.

Similar to Figure 8, we plot the average correlation between output growth rate, $corr\left(\Delta Y_{t+1}, \Delta \log \mathcal{E}_{t+1}\right)$,

Figure 12: Unconditional IRFs for models with different trade elasticity
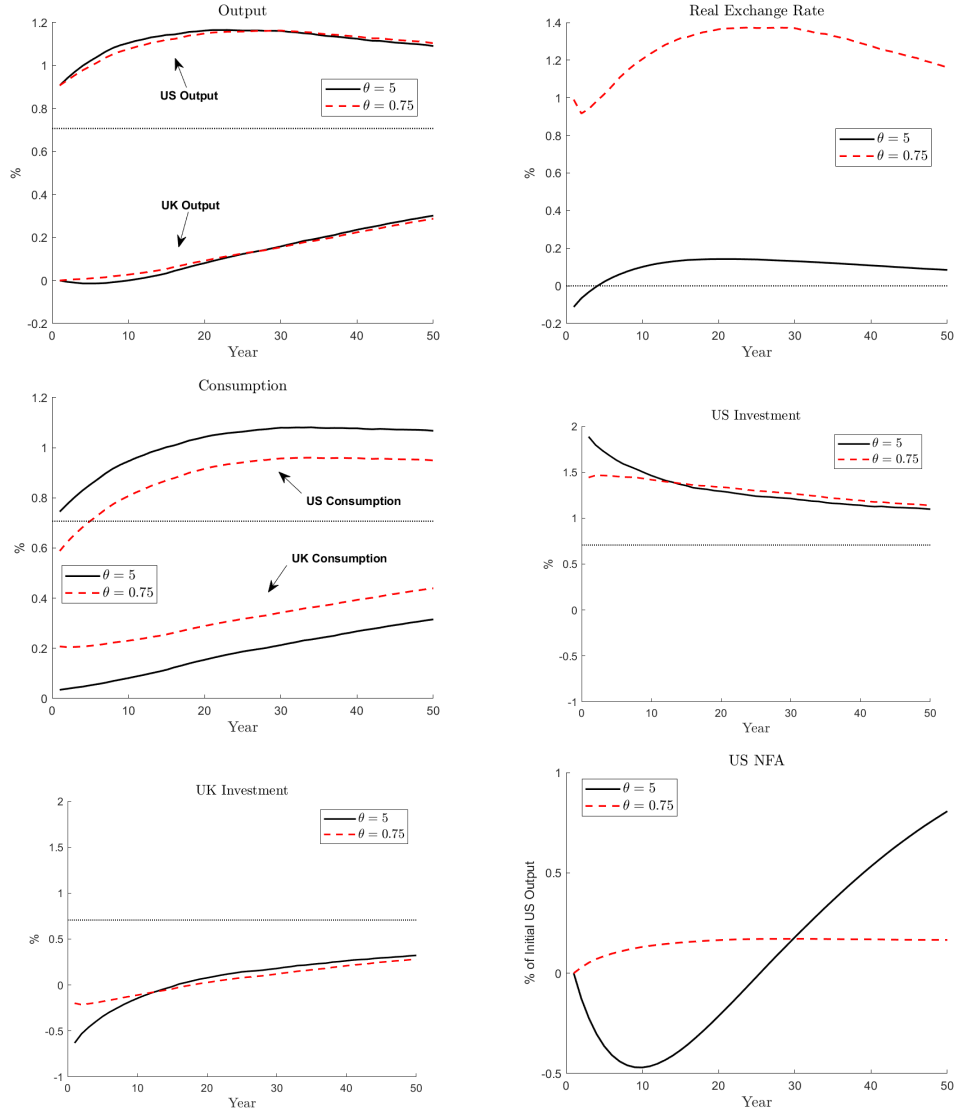
Note: This figure compares the unconditional IRFs for the benchmark model ($\theta = 5$) and the model with low trade elasticity ($\theta = 0.75$). The horizontal dotted line plots the long-run trend.

and the depreciation rate in Figure 13 and the Backus-Smith correlation,

$$corr\left(\Delta \log C_{t+1} - \Delta \log \hat{C}_{t+1}, \Delta \log \mathcal{E}_{t+1}\right),$$

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Figure 13: Comparative Statics in Trade Elasticity
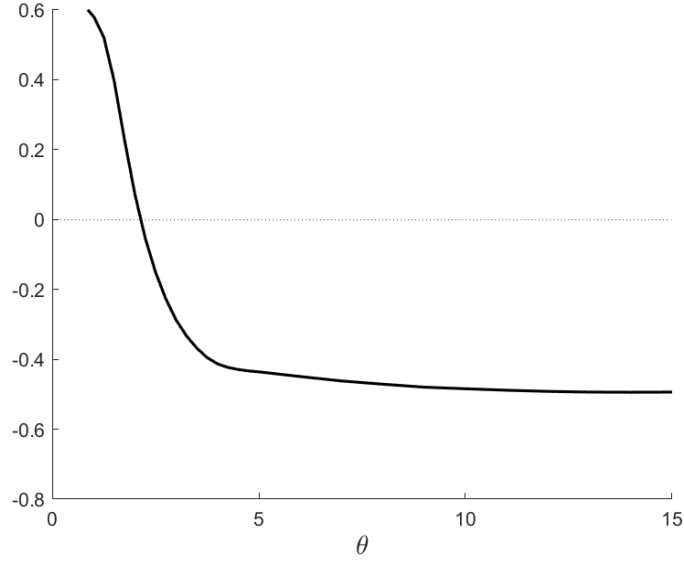
Notes: The figure plots the correlation between relative changes in output and depreciation rate as a function
of the trade elasticity, θ.

in Figure 14. These correlations turn negative when θ exceeds 2.5.

D Alternative Market Structures

In this appendix, we examine the equilibrium dynamics under different financial market struc-
tures: complete markets, incomplete markets with one or two bonds, and financial autarky. Under
financial autarky, US and UK households can trade in the good markets but not in financial markets.
Therefore, the current account must be zero at all times.

Figure 14: Comparative Statics in Trade Elasticity
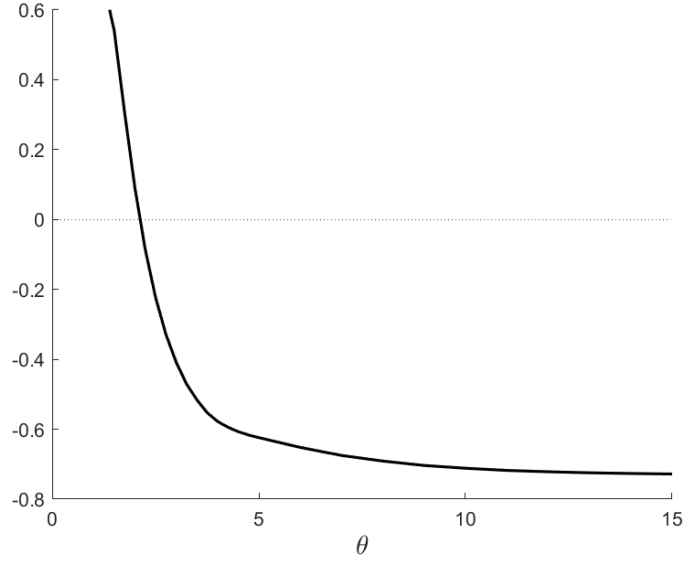
Notes: The figure plots the Backus-Smith correlation as a function of the trade elasticity, θ.

D.1 Complete Markets

Under complete markets, households have access to the complete set of state-contingent Arrow securities, besides domestic equity. At time t and history z^t, let $\mathcal{Q}_t^A(z_{t+1}|z^t)$ denote the price of the Arrow security that pays off one US dollar at $t+1$ if and only if state z_{t+1} is realized. Similarly let $\hat{\mathcal{Q}}_t^A(z_{t+1}|z^t)$ denote the price of the Arrow security that pays off one UK pound at $t+1$ if and only if state z_{t+1} is realized. To simplify notation, we omit the history dependence. It follows from no-arbitrage that

$$\hat{\mathcal{Q}}_t^A(z_{t+1}) = \mathcal{Q}_t^A(z_{t+1})S_{t+1}(z_{t+1})/S_t.$$

Given the Arrow securities and prices, the US households' budget constraint, (7a), becomes

$$P_t C_t + Q_t \chi_t + \sum_{z_{t+1}} \mathcal{Q}_t^A(z_{t+1}) \mathcal{A}_t(z_{t+1}) \leq (D_t + Q_t)\chi_{t-1} + \mathcal{A}_{t-1}(z_t) + W_t L_t,$$

where $\mathcal{A}_t(z_{t+1})$'s are the holdings of state-contingent Arrow securities which payoff in US dollar. Let $a_{t-1} = \frac{\mathcal{A}_{t-1}(z_t)}{P_t}$ denote the real holding and $q_{t+1}^A(z_{t+1}) = \mathcal{Q}_t^A(z_{t+1})\frac{P_{t+1}(z_{t+1})}{P_t}$ denote the real price of Arrow securities. The US households' budget constraint can now be written in real terms as

$$C_t + q_t \chi_t + \sum_{z_{t+1}} q_{t+1}^A(z_{t+1}) a_t(z_{t+1}) \leq (d_t + q_t)\chi_{t-1} + a_{t-1}(z_t) + w_t L_t.$$

The US NFA at time t is defined as $a_{t-1}(z_t)$. The UK households' nominal and real budget constraints under complete markets are defined similarly using $\hat{\ }$ variables.

D.2 Incomplete Markets with Two Bonds

In the benchmark model, the two countries are not symmetric because we assume that the international bond is denominated in a particular currency (USD). In this subsection, we consider a completely symmetric model in which there are two bonds: a US bond and a UK bond. The US bond payoff is denominated in the US price index so that it is risk-free from the perspectives of US households. The UK bond payoffs is denominated in the UK price index $\hat{P}_t$, defined similarly to P_t. So the budget constraint of the US households, (7a), is modified to:

$$P_t C_t + Q_t \chi_t + Q_{\text{US},t}^b b_t^{\text{US}} + S_t \hat{Q}_{\text{UK},t}^b b_t^{\text{UK}} \leq (D_t + Q_t)\chi_{t-1} + P_t b_{t-1}^{\text{US}} + S_t \hat{P}_t b_{t-1}^{\text{UK}} + W_t,$$

where b_t^{US} is the number of US bonds held by the US household; $Q_{\mathrm{US},t}^b$ is nominal price of a US bond (in US dollars); b_t^{UK} is the number of UK bonds held by the US household; $\hat{Q}_{\mathrm{UK},t}^b$ is the nominal price of an UK bond (in pounds). Dividing both sides of the nominal budget constraint by P_t, we obtain the constraint in real terms

$$C_t + q_t \chi_t + q_{\mathrm{US},t}^b b_t^{\mathrm{US}} + \mathcal{E}_t \hat{q}_{\mathrm{UK},t}^b b_t^{\mathrm{UK}} \leq (d_t + q_t)\chi_{t-1} + b_{t-1}^{\mathrm{US}} + \mathcal{E}_t b_{t-1}^{\mathrm{UK}} + w_t,$$

where $q_t, q_{\mathrm{US},t}^b$, and w_t are the corresponding real values of $Q_t, Q_{\mathrm{US},t}^b$, and W_t; $\hat{q}_{\mathrm{UK},t}^b = \hat{Q}_{\mathrm{UK},t}^b / \hat{P}_t$ is the real price of UK bonds in the UK. We also impose exogenous constraints on real bond holdings

$$b_t^{\mathrm{US}}, b_t^{\mathrm{UK}} \geq \underline{b}\xi_t.$$

The US NFA at time t is defined as $b_{t-1}^{\mathrm{US}} + \mathcal{E}_t b_{t-1}^{\mathrm{UK}}$.

The budget constraints of the UK households in nominal and real terms are defined similarly using $\hat{}$ variables. Notice that we do not assume that there are transaction costs associated with holding foreign bonds (see, e.g., Ghironi et al., 2009), or similarly that the domestic return on foreign bonds exogenously depends on the level of international debt.

E   Exchange Economy

To understand the importance of endogenous investment and capital accumulation in our transmission mechanism, we consider a variant of our baseline model in which capital stocks are fixed at their steady-state values. This is effectively an endowment economy. Figure 15 displays the IRFs for the main equilibrium variables under different market structures. Despite the high elasticity of

substitution and persistent endowment shocks, the real exchange rate depreciates upon an increase in endowment in the US. A greater supply of US traded goods depresses its price relative to UK traded goods. The results are robust across all financial market structures.

Figure 15: UIRFs in an Exchange Economy and under Different Market Structures
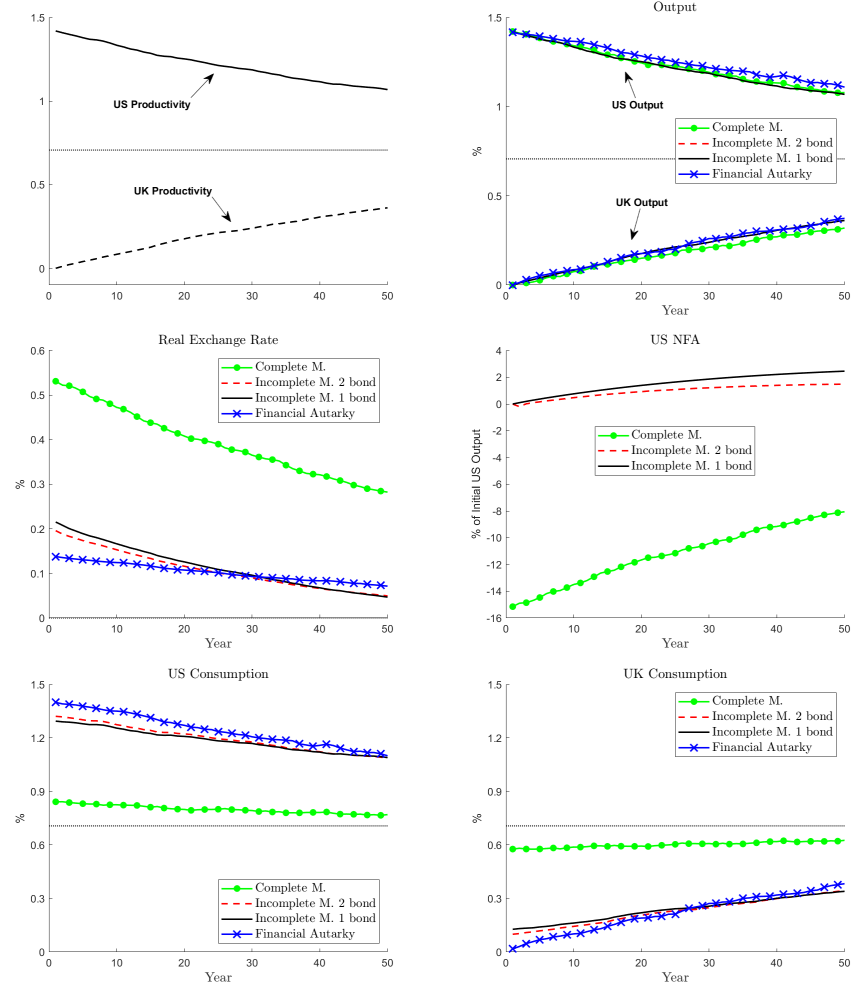
Note: This figure is generated by the solution of the exchange economy under different financial market structures with parameters given in Table 3 and capital stock fixed at the steady-state value of the baseline model.

F   Alternative Productivity Processes

The annual, non-stationary productivity process (12) plays an important role in the persistent responses of real exchange rate in the benchmark model. In the previous version of our paper, we studied a quarterly version of our model with the standard productivity process used in the IRBC literature. In particular, following Backus et al. (1992), we assumed that log productivity in each country follows stationary AR(1) processes:

$$\log A_t = \rho \log A_{t-1} + \epsilon_t$$

$$\log \hat{A}_t = \rho \log \hat{A}_{t-1} + \hat{\epsilon}_t,$$

with $0 < \rho < 1$, where ϵ_t and $\hat{\epsilon}_t$ are I.I.D productivity shocks. We also use constant borrowing limits, $b_t, \hat{b}_t \geq \underline{b}$. This specification also requires a higher elasticity of substitution $\theta = 10$. Other parameters of the model are given in Table 7.

Table 7: **Benchmark Calibration**

Parameters	Descriptions	Values
β	discount factor	0.96
γ	risk aversion	2
$\alpha = 1 - \hat{\alpha}$	home consumption share	0.72
θ	consumption elasticity	10
δ	depreciation rate	0.1
η	capital share	0.36
$\underline{b}$	borrowing limit	-1
$corr(A_t, \hat{A}_t)$	productivity correlation	0
ρ	productivity persistence	0.82
σ	productivity shock std.	0.022

Figure 16 shows that on average real exchange rate appreciates when a positive productivity

shocks hits the US. In addition, when the US is near its borrowing limit (we define crisis states using the 7% threshold as Section 2), real exchange rates depreciates. These dynamics are also consistent with the empirical results in Section 1. However, the (unconditional) appreciation is rather short-lived. As shown the upper right panel, it lasted for only one quarter. This is inconsistent with the persistent appreciation estimated in Section 1.

Another important difference between the model with stationary and non-stationary processes is that under complete markets, the stationary model produces a negative correlation between relative changes in output and real depreciation rate, as shown in Table 8, while the correlation is positive in the benchmark model (Table 5). Non-stationary productivity processes accentuates the importances of financial market structures.

Table 8: International Co-movements

Economy	RER-Output Corr $corr\left(\Delta \log Y_{t+1}, \Delta \log \mathcal{E}_{t+1}\right)$	Backus-Smith Corr $corr\left(\Delta \log C_{t+1} - \Delta \log \hat{C}_{t+1}, \Delta \log \mathcal{E}_{t+1}\right)$
Data		
US vs. UK		Negative
Specifications		
Complete Markets	-0.133	1.000
Incomplete Markets (2 bonds)	-0.223	-0.234
Incomplete Markets (1 bond)	-0.208	-0.211
Financial Autarky	0.681	1.000

To shed more light on these results, Figure 17 shows the IRFs for the main equilibrium variables under different market structures in this specification. The equilibrium dynamics are very similar under incomplete markets with one bond or two bonds. Under complete markets, the real exchange rate appreciates when productivity increases in the US, as in the incomplete markets specification. However, US consumption increases by less than UK consumption, leading to a positive correlation between the relative consumption growth and the real depreciation rate (Table 8). Under financial autarky, the real exchange rate depreciates, similar to the responses under incomplete markets with

Figure 16: IRFs for the model with stationary productivity processes
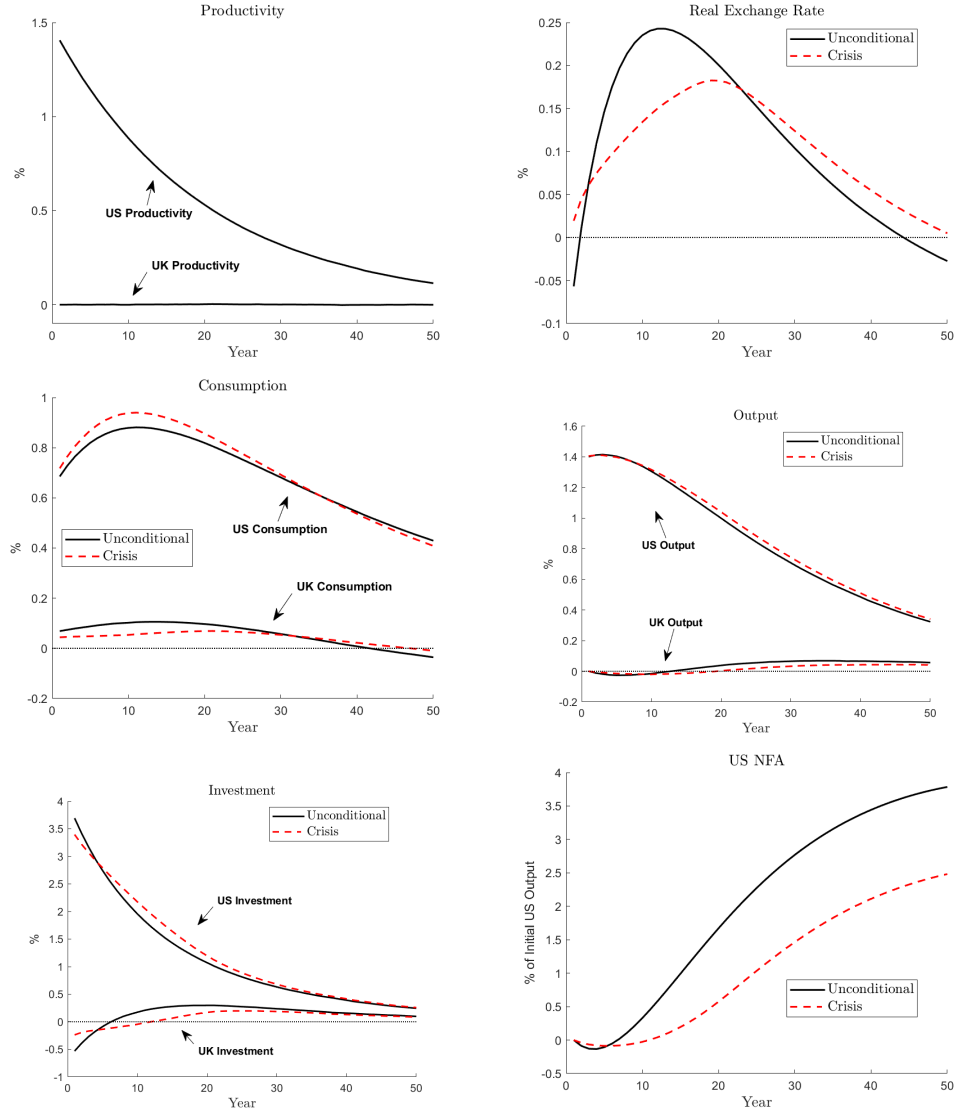
Note: This figure plots the IRFs of the model with stationary productivity processes that are symmetric and independent across countries, described by $\log(A_t) = \rho_A \log(A_{t-1}) + \epsilon_t$, with $\rho_A = 0.82$, and $std(A_t) = 0.022$. The IRFs are the responses to the US productivity shock described by the top left panel.

one bond and when the US households are close to their borrowing limit.

Figure 17: UIRFs under Different Market Structures
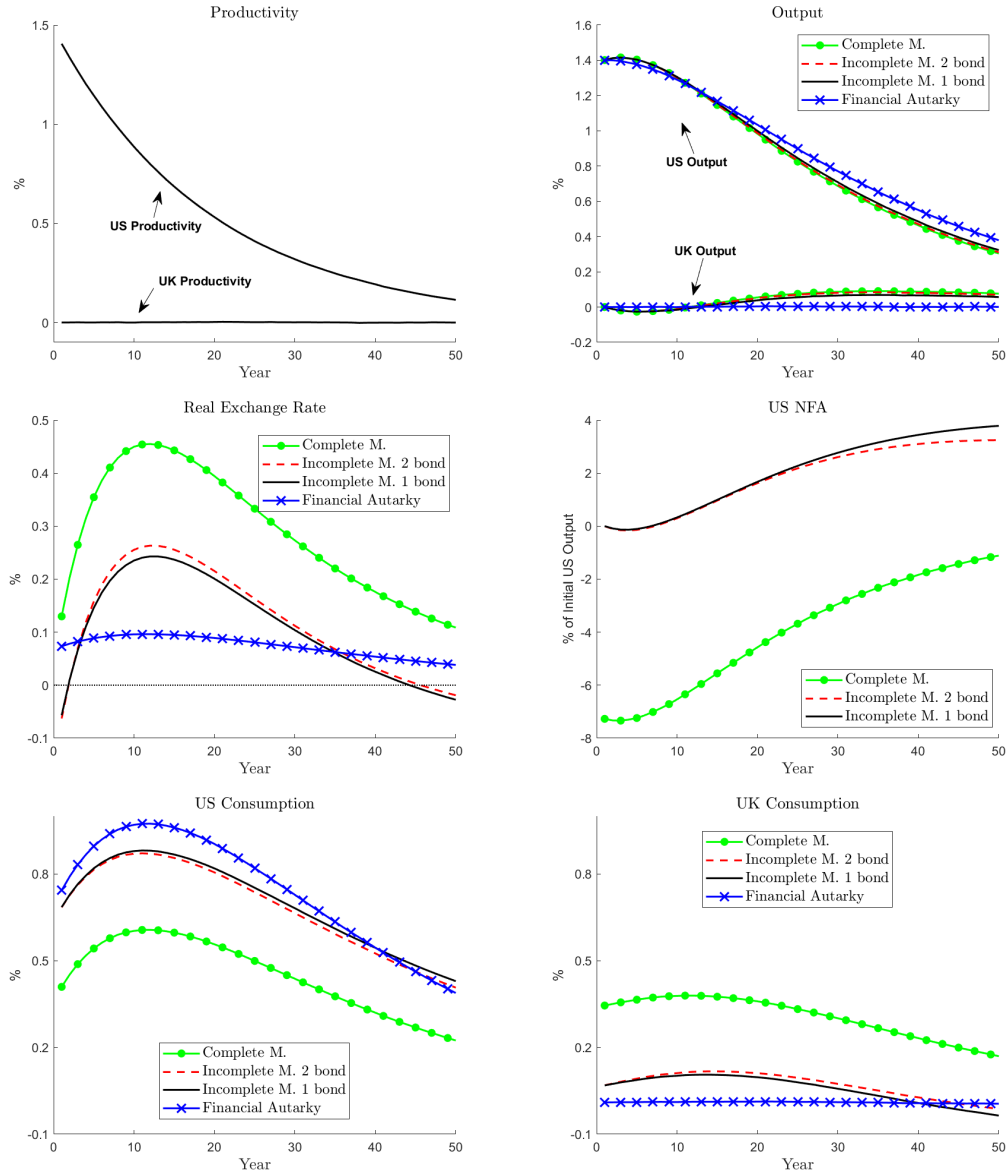
Note: This figure is generated by the solution of the baseline model with parameters given in Table 3 and under different financial market structures.

G Traded and Non-Traded Goods

In this appendix, we present the details from the extension with both traded and non-traded goods described in Subsection 3.1. We also examine the HBS effects when productivity shocks affects the traded goods sectors more than the non-traded goods sectors.

G.1 Model Ingredients

US households consume a basket of US non-traded goods, C_t^{NT}, US traded goods, and UK traded goods. The aggregators are given in (14) and (15) which gives rise to price indices (16) and (17). To simplify the exposition, we separate production firms from investment firms. US traded goods firms use capital and labor in production:

$$Y_t^{\text{US}} = A_t \left(K_t^{\text{US}} \right)^{\eta} \left(L_t^{\text{US}} \right)^{1-\eta}.$$

Capital and labour are hired in competitive factor markets, capital with a nominal rental rate of R_t, and labor with a nominal wage rate of W_t. Firms maximize profits

$$\Pi_t^{\text{US}} = P_t^{\text{US}} Y_t^{\text{US}} - W_t L_t^{\text{US}} - R_t K_t^{\text{US}},$$

which implies that

$$R_t = (1-\eta) P_t^{\text{US}} A_t \left(K_t^{\text{US}} \right)^{\eta-1} \left(L_t^{\text{US}} \right)^{1-\eta},$$

and

$$W_t = \eta P_t^{\text{US}} A_t \left(K_t^{\text{US}} \right)^{\eta} \left(L_t^{\text{US}} \right)^{-\eta}.$$

We also assume that there is free entry into the traded goods sector which drives profits to zero in equilibrium.

Non-traded goods firms now use capital and labor in production:

$$Y_t^{\text{NT}} = A_t^{\text{NT}} \left(K_t^{\text{NT}}\right)^{\eta} \left(L_t^{\text{NT}}\right)^{1-\eta}.$$

Again capital and labour are hired in competitive factor markets and firms maximize profits

$$\Pi_t^{\text{NT}} = P_t^{\text{NT}} Y_t^{\text{NT}} - W_t L_t^{US} - R_t K_t^{US},$$

which implies that

$$R_t = \eta P_t^{\text{NT}} A_t^{\text{NT}} \left(K_t^{\text{NT}}\right)^{\eta-1} \left(L_t^{\text{NT}}\right)^{1-\eta},$$

$$W_t = (1-\eta) A_t^{\text{NT}} \left(K_t^{\text{NT}}\right)^{\eta} \left(L_t^{\text{NT}}\right)^{-\eta}.$$

We also assume that there is free entry into the non-traded goods sector which drives profits to zero in equilibrium.

There is a separate representative firm in each country that produces capital to be used by domestic traded and non-traded firms. The capital producing firm combines domestic traded and non-traded goods to produce capital using the accumulation technology

$$K_{t+1} = K_t(1 - \delta) + I_t,$$

where investment uses both traded and non-traded goods

$$I_t = \mathcal{I}(I_t^{\text{US}}, I_t^{\text{NT}}),$$

with

$$\mathcal{I}(a, b) \equiv \left(\varphi a^{\frac{\xi-1}{\xi}} + (1 - \varphi)^{\frac{1}{\xi}} b^{\frac{\xi-1}{\xi}} \right)^{\frac{\xi}{\xi-1}}.$$

The price index for aggregate investment is

$$P_t^{\text{I}} = \left(\varphi (P_t^{\text{US}})^{1-\xi} + (1 - \varphi) (P_t^{\text{NT}})^{1-\xi} \right)^{\frac{1}{1-\xi}}.$$

The capital-producing firm maximizes dividends using the domestic households' stochastic discount factor $\lambda_t = C_t^{-\sigma}$:

$$\max_{\{D_{t+i}, I_{t+i}, K_{t+i+1}\}} \mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} \frac{D_{t+i}}{P_{t+i}} \right],$$

subject to

$$D_t = R_t \cdot K_t - P_t^{\text{I}} I_t.$$

The demand for each type of investment good is

$$I_t^{\text{US}} = \varphi \left(\frac{P_t^{\text{US}}}{P_t^I} \right)^{-\xi} I_t \qquad \text{and} \qquad I_t^{\text{NT}} = (1 - \varphi) \left(\frac{P_t^{\text{NT}}}{P_t^I} \right)^{-\xi} I_t.$$

Competitive equilibrium requires that labor and capital markets clear

$$K_t^{\text{US}} + K_t^{\text{NT}} = K_t,$$

$$L_t^{\text{US}} + L_t^{\text{NT}} = 1.$$

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The goods market clearing conditions now includes investment demand

$$Y_t = C_t^{\text{US}} + \hat{C}_t^{\text{US}} + I_t^{\text{US}},$$

$$Y_t^{\text{NT}} = C_t^{\text{NT}} + I_t^{\text{NT}}.$$

G.2 Harrod-Balassa-Samuelson Effect

In Subsection 3.1, we consider the case in which the productivity processes in traded and non-traded good sectors are perfectly correlated, $A_t^{\text{NT}} = A_t$ and $\hat{A}_t^{\text{NT}} = \hat{A}_t$. Now we consider an alternative specification in which these processes are relatively uncorrelated:

$$A_t^{\text{NT}} = \hat{A}_t^{\text{NT}} = \xi_t.$$

This specification is similar to the scenario studied by HBS in which productivity shocks only hits traded good sectors. Figure 18 shows that, in this specification, on average, the real exchange rate also appreciates when a positive productivity shock hits the US and the appreciation is persistent. This unconditional result is similar to the result in Subsection 3.1 (Figure 9). However, when the US is near its borrowing limit, real exchange rates still appreciates, while it depreciates in Subsection 3.1 and in our empirical analysis. To better understand the results, we use the decomposition (18).
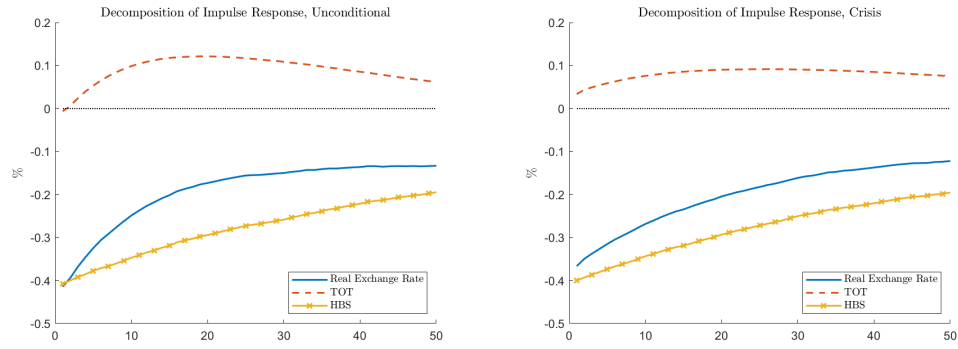
Figure 19 displays the decomposition. Real exchange rate appreciation is driven by the HBS effect and the HBS effect dominates the ToT effect in this specification. However, near the borrowing limit, the HBS effect does not reverse its sign and hence, we still see that the real exchange rate appreciates.

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Figure 18: IRFs for US real exchange rate, NFA, and US, UK output, consumption, investment
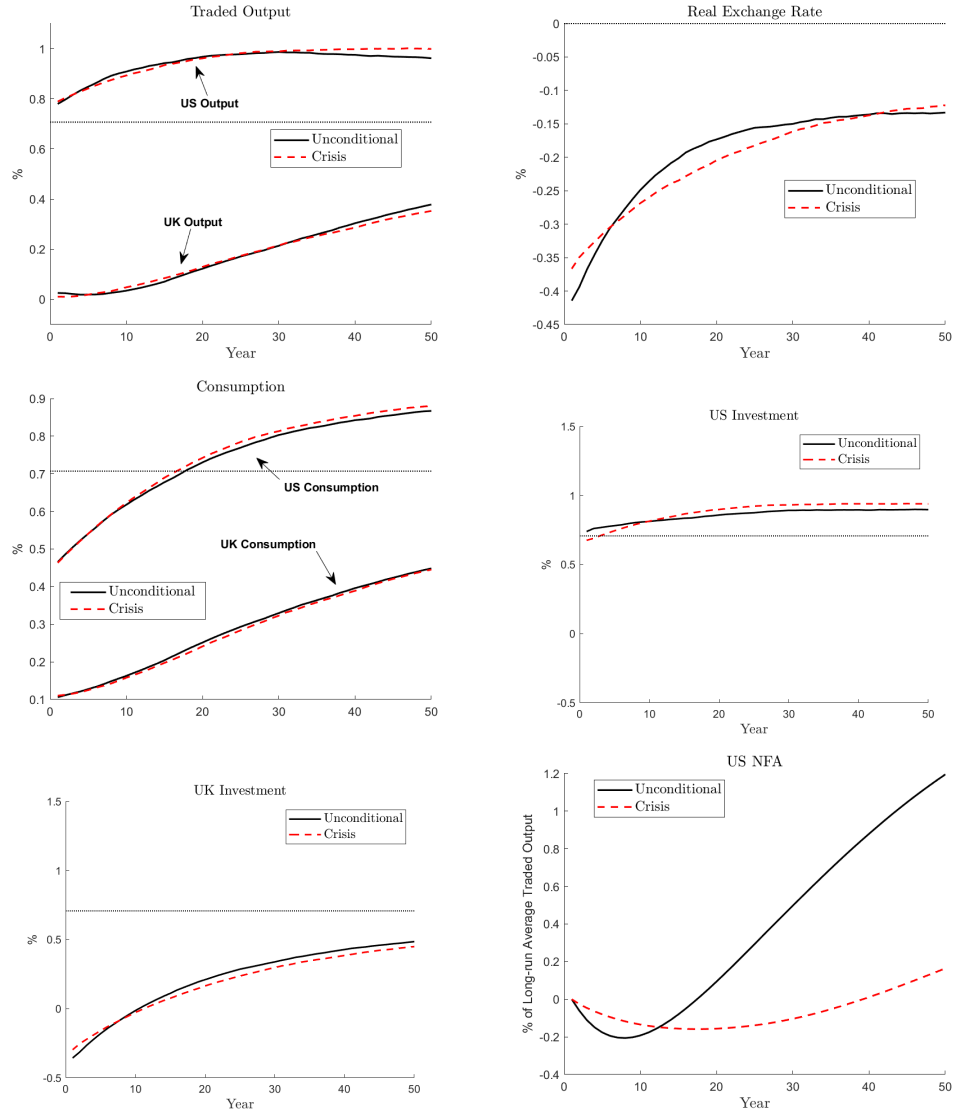
Note: This figure is generated by the solution of the baseline model with parameters given in Table 3. The solid lines are UIRFs and the dashed lines are CIRFs, conditional on US NFA normalized by US productivity being in the seventh percentile of its marginal distribution. The horizontal dotted line plots the long-run trend.

Figure 19: IRFs for real exchange rate and its decomposition

Note: This figure is generated by the solution of the baseline model with parameters given in Table 3. The solid lines are UIRFs and the dashed lines are CIRFs, conditional on US NFA normalized by US productivity being in the seventh percentile of its marginal distribution. The horizontal dotted line plots the long-run trend.