Why aging induces deflation and secular stagnation.[*][†]

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Abstract

We show that an aging population distribution produces persistent downward pressure on the inflation rate, real and nominal interest rates, and output in a quantitative overlapping generations model calibrated to Japanese data. Aging creates persistent upward pressure on aggregate demand for physical assets and nominal government bonds and this latter effect puts downward pressure on the inflatino rate. The reaction of monetary policy to deflationary pressure influences the size and timing of inflation and real economic activity via an asset substitution or Tobin effect transmission channel.

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1 Introduction

Many high income economies have experienced secular declines in real interest rates, the inflation rate and per–capita output growth in the years leading up to the outset of the pandemic in 2020. While the specific start date of these declines varies across countries these empirical regularities which we will collectively refer to as secular stagnation are a global phenomenon that have been occurring in advanced economies in North America, Europe and Asia. What are the forces that are driving secular stagnation? Will they continue to exert downward pressure on interest rates, the inflation rate and output-growth in future years?

This paper investigates the quantitative significance of a demographic transition to an older population–age distribution in accounting for these macroeconomic outcomes with data from Japan. Japan is in the midst of a particularly large demographic transition. Japan experienced a large increase in fertility rates in the period following World War II and these cohorts are now transitioning into retirement. Japan has also experienced large declines in the total fertility rate which fell from 2.14 in 1970 to 1.26 in 2015. Finally, Japan has also experienced a modern health miracle. Life expectancies have increased from 60 years for Japanese born in 1950 to 84 years for individuals born in 2014. Japan's population is already falling and using data from the Japanese government we project that the level of the Japanese 21+ population will decline from 102 million in the year 2015 to 80 million in 2055 and that the 70+ share of the population will increase from 23 percent to 38 percent[1]. Japan is not unique. Lower fertility rates, aging of the babyboomers and longer life expectancies are a common phenomenon in many high income countries as documented in e.g. Yoon et al. (2014).

Our results suggest that aging is a quantitatively important source of secular stagnation and that it will continue to put downward pressure on interest rates, the inflation rate and per capita output in future years. Using the quantitative OLG model of Braun and Ikeda (2021), we find that Japan's demographic transition to an older age distribution puts steady and quantitatively significant downward pressure on the inflation rate, interest rates and output growth in the first 25 years of a transition that starts in the year 2014.

Our perspective that demographic change is inducing strong downward pressure on interest rates, inflation and output is accepted in some policy making circles (see e.g. Nishimura (2011) or Shirakawa (2021)), but is controversial among academic economists.

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[1]see Table 1

For instance, Mian et al. (2021), provide empirical evidence that higher inequality is the main source of the secular decline in the U.S. natural interest rate and argue that aging of the babyboomers is of secondary importance in accounting for the secular decline in the U.S. natural interest rate.

Another reason why the aging hypothesis for secular stagnation is controversial is because it is at odds with standard theory about the relationship between population growth rates, real interest rates and inflation. According to the modified golden rule a slower population growth rate is associated with a lower real interest rate on capital. Then a no arbitrage argument implies that a lower population growth rate should also be associated with a higher inflation rate to equate the real return on nominal government liabilities with the real return on capital.

Bullard et al. (2012) offer a political economy explanation for how aging could induce a *decline* in the inflation rate. They observe that older households have no labor income and consume instead out of their savings. Older households prefer a lower inflation rate because it increases the real return on their holdings of nominally denominated assets. If the fraction of the old increases and one assumes that their political influence also increases, then policy makers may take actions to push the price level down. This hypothesis explains why aging might produce deflationary pressure, but it also implies that periods of deflation should also be periods with high real interest rates. This implication of their hypothesis is at odds with the fact that both the inflation rate and real interest rates have been falling at the same time in Japan and other high income countries.

A final puzzling aspect of secular stagnation is that monetary and fiscal authorities have taken actions to counteract these macroeconomic trends. In Japan, the policy nominal interest rate (uncollateralized overnight call rate) was reduced from 6% to nearly zero between 1990 and 1999 and a range of other quantitative easing measures have been taken since then. Fiscal stimulus has also been significant. Japan's government net debt–output ratio has increased by more than a factor of 15 since the year 1990 and will likely grow in future years.[2]

The starting point for understanding why aging induces secular stagnation is to understand how aging influences the aggregate demand for assets. Our model features a liquidity premium. Households hold portfolios of liquid and illiquid assets and a household's demand

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[2]Our measure of net government debt is gross general government debt securities plus net loan liabilities of central and local governments minus general debt held by the central and local governments, the Fiscal Loan Fund, the Bank of of Japan and net assets of social security trust funds.

for each type of asset varies with its age. Young households borrow liquid assets and use them to purchase illiquid assets like homes and durable goods. Older households, in contrast hold positive amounts of both assets because mortality risk increases with age and pension benefits don't fully replace their previous labor earnings while retired. We fit the age-profile of household asset allocations in the model to Japanese data. Then we show that aging produces persistent increases in aggregate asset demand (in partial equilibrium) for both assets, but that the increase in the demand for liquid assets is particularly large.

Next we allow prices to adjust to clear markets and government policy to react to the demographic shock. Nominal government debt constitutes a large share aggregate liquid asset supply and it follows that if per capita nominal assets are held fixed, aging puts downward pressure on the price level. Aging also reduces the size of the working-aged population and this induces capital deepening which in conjunction with higher aggregate demand pushes the real interest rate on illiquid assets down. However, the size and the timing of these responses as well as the output response depends on how government policies respond to aging.

Both monetary and fiscal policy influence prices and real economic activity in our model. We posit a nominal interest rate role for the monetary authority and the deflationary pressure induced by aging results in a lower policy nominal interest rate. A lower nominal interest rate transmits to the real sector in two ways. First, we model New Keynesian (NK) nominal price rigidities. This channel turns out to be largely irrelevant for our results. Second, our model features an asset substitution transmission channel of monetary policy as in Tobin (1969) and more recently Hu et al. (2019). This transmission channel is important for our results. In our OLG framework reducing the nominal interest rate influences the composition of household asset demand. A lower nominal interest rate reduces the real return on government debt and other liquid securities and this induces households to tilt their asset allocations towards illiquid assets. This policy reaction influences the size and persistence of the response of the inflation rate, real interest rates and aggregate investment, but also induces negative wealth effects on middle aged and older households who respond by reducing their consumption.

Changes in the supply of nominally denominated government debt also influences prices and real economic activity. In our model households require a higher real return on government debt to induce them to hold more of it and the real interest rate for liquid securities increases when government debt is increased. Households save more but also substitute their savings away from physical assets and private investment is crowded out. Still, the

increase in household asset demand induced by aging is so large that the model continues to produce secular stagnation during the first 25 years of the transition even when we consider large increases in the supply of government debt.

Goodhart and Pradhan (2017) argue that aging will raise the equilibrium real interest rate and increase the inflation rate in future years and Juselius and Takáts (2018) provide empirical evidence that the combination of a lower share of younger population cohorts and a higher share of older cohorts will create inflationary pressure in future years. Our model is a good laboratory for understanding the quantitative significance of their results.

Household demand in our model increases in the first stage of the transition but at horizons of about 50 years, aggregate asset demand for government debt starts to decline. Thus, the second stage of the transition is characterized by rising real interest rates and a rising inflation rate. In fact, both variables eventually overshoot their terminal steady-state values. The intuition for these properties of our model is related to Sargent and Wallace (1981) who analyze how open market operations alter the time-profile of asset demand. It turns out that the increase in the inflation rate in the later stage of our transition gradual and the peak inflation rate is much smaller than its trough. The later stages of our transition are unpleasant for another reason. A higher inflation rate is not sufficient to balance the government budget constraint and government transfers also fall.

The remainder of our paper proceeds as follows. In Section 2 we use a simple OLG model to illustrate the workings of the asset substitution channel and show how the re-actions of fiscal and monetary policy to a demographic shock influences prices and real economic activity. Then Section 3 describes the quantitative model and Section 4 provides an overview of the model calibration. Our main results are reported in Section 5. Section 6 discusses some robustness checks and Section 7 contains our concluding remarks.

2   2-period OLG Model

Three of the main economic mechanisms underlying our results can be illustrated using a 2-period flexible price OLG model. First, the price level is determinant in our model and it is influenced by both monetary and fiscal policy. The mechanism that determines the price level is different from the fiscal theory of the price level (FTPL) and as we explain below it is more difficult to account for our secular stagnation observations under the FTPL. Second, monetary policy affects real economic activity in our flexible price model due to an asset substitution or Tobin effect. Third, an increase in the nominal stock of

government debt is non-neutral when monetary policy follows a Taylor rule. Our method for determining the price-level is based on Sargent and Wallace (1981), Hagedorn (2018) and Hu et al. (2019). In fact, the 2-period model analyzed here is essentially the model of Hu et al. (2019). We extend their model by assuming that monetary policy is endogenous and allowing population and the stock of nominal debt to vary over time.

2.1 The model

Environment Time is discrete and continues forever: $t = 0, 1, ..$ There is a single good that can be consumed or used to produce new capital. The economy is populated by households who are either young or old. In period t, N_t of young households are born. In the next period, the young generation becomes old and exits the economy in the end of the period. Hence, in period t, there are N_t young households and N_{t-1} old households. The growth rate of the population of young households or "fertility rate" is $n_t = N_t/N_{t-1}$. In the initial period $t = 0$, there are old households with population N_{-1} and each old household is endowed with capital a_{-1} and nominal government bonds $d_{-1}^n \equiv P_{-1}d_{-1}$, where P_{-1} is the price level and d_{-1} is government bonds in real units. In aggregate, the initial capital stock is $K_0 = a_{-1}N_{-1}$ and the initial nominal government debt is $D_{-1}^n = d_{-1}^n N_{-1}$.

Households Households born in period t supply one unit of labor inelastically and earn real wage w_t. They have two ways to save: they can invest in capital which fully depreciates after being used in production in period $t+1$ or they can acquire nominal government bonds. Only old households consume. This assumption isolates the asset substitution channel of monetary policy because asset demand doesn't depend on the interest rate and allows us to show in a transparent way how monetary and fiscal policy influence the price level. The problem of a household born in period t is to maximize consumption c_{t+1} subject to

$$a_t + d_t = w_t \tag{1}$$

$$c_{t+1} = R_{t+1}^k a_t + R_t \frac{P_t}{P_{t+1}} d_t + \xi_{t+1} \tag{2}$$

When young, the household earns w_t and allocates its earnings to capital a_t and government bonds d_t. When old, the household receives the return on capital, $R_{t+1}^k a_t$, the return on government bonds, $R_t(P_t/P_{t+1})d_t$, and a lump-sum transfer ξ_{t+1}, where R_t denotes the

nominal interest rate.[3] An interior solution for a_t and d_t satisfies the arbitrage condition between investing in capital and government bonds:

$$R_{t+1}^k = R_t \frac{P_t}{P_{t+1}} \tag{3}$$

Firms Perfectly competitive firms produce the single good according to $Y_t = K_t^\alpha N_t^{1-\alpha}$. Capital depreciates fully after production, and the returns on capital and wages are given, respectively, by

$$R_t^k = \alpha K_t^{\alpha-1} N_t^{1-\alpha} = \alpha k_t^{\alpha-1} \tag{4}$$

$$w_t = (1-\alpha) K_t^\alpha N_t^{-\alpha} = (1-\alpha) k_t^\alpha \tag{5}$$

where $k_t \equiv K_t/N_t$ is capital per young household.[4] Substituting equation (4) into equation (3) yields the Fisher equation:

$$\alpha k_{t+1}^{\alpha-1} = R_t \frac{P_t}{P_{t+1}}. \tag{6}$$

Government The government consists of a central bank and a fiscal authority. The central bank sets the nominal interest rate R_t on government debt and the fiscal authority issues one-period nominal debt D_t^n and makes transfers ξ_t to the old.[5] In period t, the government faces the following budget constraint

$$\xi_t = \frac{d_t^n n_t}{P_t} - R_{t-1} \frac{d_{t-1}^n}{P_t} \tag{7}$$

expressed in per capita terms where $d_t^n = D_t^n/N_t$.

Law of motion for capital The aggregate capital stock evolves according to $K_{t+1} = a_t N_t$. Using the budget constraint (1), the law of motion for capital can be expressed in per capita terms as

$$n_{t+1} k_{t+1} = w_t - d_t. \tag{8}$$

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[3]The initial old households consume what they earn as $c_0 = R_0^k a_{-1} + R_{-1} d_{-1}^n/P_0 + \xi_0$.

[4]In the ensuing analysis it is convenient to express period t per capita variables in terms of the population of young households in period t.

[5]Transfers are negative here because they finance interest payments on government debt. But, we use this notation to be consistent with the notation in the quantitative model.

Market clearing   The market clearing condition for period t government debt is

$$\frac{d_t^n}{P_t} = d_t \qquad (9)$$

Clearing in the goods market implies $c_t N_{t-1} + K_{t+1} = Y_t$ or

$$\frac{c_t}{n_t} + n_{t+1} k_{t+1} = k_t^\alpha. \qquad (10)$$

Combining equations (5), (8), and (9) yields

$$\frac{d_t^n}{P_t} + n_{t+1} k_{t+1} = (1 - \alpha) k_t^\alpha \qquad (11)$$

which is the aggregate asset market clearing condition. The right hand side is asset demand, which by design depends on wages, but *not* on the real interest rate between period t and $t + 1$. The left hand side of the expression is asset supply.

Equilibrium   Given $\{R_t, d_t^n\}$, observe that the equilibrium sequence $\{k_t, P_t\}$ is determined by the Fisher equation, (6) and the asset market clearing condition, (11). This strategy for deriving the equilibrium price level is similar to Sargent and Wallace (1981) and Hagedorn (2018) and identical to Hu et al. (2019). We refer to this scheme as the asset-market theory of the price level (ATPL).

Definition 1 (ATPL equilibrium) *Given the initial capital k_0, the initial nominal obligation $R_{-1} D_{-1}^n$, a sequence of fertility rates $\{n_t\}$, and a sequence of policy variables $\{d_t^n, R_t\}$, a competitive equilibrium for this economy consists of a sequence of prices $\{P_t, R_t^k, w_t\}$, a set of allocations $\{c_t, k_t, d_t\}$ and a sequence of lump-sum transfers $\{\xi_t\}$ that satisfy the firms' optimality conditions (4) and (5), the Fisher equation (6), the government budget constraint (7), and the market clearing conditions (9)-(11).*

2.2   Analytical results

We now examine the effects of monetary policy, fiscal policy and demographic changes on the real economy.

Proposition 1 (Non-neutrality of money) *In an ATPL equilibrium, monetary policy is not neutral.*

Proof. See Appendix A.1. ∎

It is easiest to illustrate the nature of the non-neutralities induced by a change in the nominal interest rate as well as the effects of a lower fertility rate and higher stock of government debt by considering a steady state ATPL equilibrium. Suppose (without loss of generality) that the per capita stock of nominal debt is constant. Then equation (11) implies that the price level is also constant or the gross inflation rate is unity, $\pi = 1$. Since the central bank sets the nominal interest rate, the Fischer equation, (6) implies that the capital stock is given as $k = (\alpha/R)^{1/(1-\alpha)}$. Now consider an increase in R. The previous expression implies that the capital stock falls to equate the real returns on capital and government debt. Then equation (11) implies that the price level falls to clear the asset market. Observe next that changes in the stock of nominal debt and the fertility rate have no real effects in a ATPL steady state equilibrium: $\partial k/\partial d^n = 0$ and $\partial k/\partial n = 0$. To ascertain how the price level responds, note that aggregate demand for real government debt, $d = (1-\alpha)k^\alpha - nk$, is independent of the stock of nominal debt. It then follows from equation (11) that an increase in the steady state stock of nominal debt increases the price level: $\partial P/\partial d^n > 0$. Finally, consider how the price level responds to a change in the steady state fertility rate. Aggregate demand for real government debt is decreasing in the fertility rate. Then, using equation (11) we see that the price level is increasing in the fertility rate: $\partial P/\partial n > 0$. In other words, a drop in the fertility rate decreases the price level. These final two results are summarized in the following proposition.

Proposition 2 (Demographics and fiscal policy) *In the ATPL steady state, a lower fertility rate decreases the price level: $\partial P/\partial n < 0$; a higher issuance of the per capita nominal government debt increases the price level: $\partial P/\partial d^n > 0$.*

Proof. See Appendix A.2. ∎

The results reported in Propositions 1 and 2 are premised on the assumption of an ATPL equilibrium and the properties of our model are quite different in a FTPL equilibrium. In Appendix A.3, we analyze an FTPL equilibrium and show that changes in the nominal interest rate are neutral and a lower fertility rate increases the inflation rate in that equilibrium. The main reason for this distinction is that government transfers are held fixed in the FTPL equilibrium and monetary policy doesn't induce redistribution across generations. In the ATPL, in contrast, government transfers are endogenous and change when the central bank alters the nominal interest rate. We will see that the ability of our

quantitative model to account for the secular stagnation observations relies heavily on the asset substitution channel of monetary policy. In other words, an important maintained hypothesis in the analysis that follows is that monetary policy induces redistribution.

2.3 Impulse response analysis

Hu et al. (2019) conduct a dynamic theoretical and numerical analysis of shocks to monetary policy and find that an increase in the (exogenous) nominal interest rate crowds out private capital formation and puts downward pressure on prices. Their result suggests that the policy rule pursued by the central bank during a demographic transition will influence the trajectory of the inflation rate and real allocations. We now document that this is the case by computing impulse response functions (IRFs) to a lower fertility shock under two alternative assumptions about the central bank's interest rate targeting rule.

We consider two different monetary policy rules: a fixed interest rate rule and an inflation targeting rule. Specifically the central bank sets the policy interest rate according to

$$R_t = R + \phi_\pi \left(\frac{P_t}{P_{t-1}} - 1 \right). \tag{12}$$

The fixed interest rate rule corresponds to $\phi_\pi = 0$ and the inflation targeting rule corresponds to $\phi_\pi > 1$. Under this policy rule, equation (6) becomes

$$P_{t+1} = \left[\frac{R + \phi_\pi \left(\frac{P_t}{P_{t-1}} - 1 \right)}{\alpha k_{t+1}^{\alpha-1}} \right] P_t. \tag{13}$$

It will be helpful to refer to his equilibrium condition in the discussion that follows.

2.3.1 Reaction of monetary policy to a lower fertility rate

Assume that the economy is in steady state in $t = 0$ and consider a situation in which the fertility rate suddenly decreases to $n < n_0$ in the beginning of period $t = 1$ and stays at the new lower level for all $t = 1, 2...$ For simplicity, assume $\pi = 1$ in the initial steady state and consider a baseline fiscal policy such that $d_t^n = d^n$. The amount of the nominal bonds is normalized to unity: $d^n = 1$. We assume that the nominal interest rate is the same between the initial and final steady states: $R_0 = R$.

Figure 1 plots the initial steady-state $t = 0$ and then impulse responses when the economy is hit by a sudden decrease in the fertility rate from n_0 to n in the beginning of

Figure 1: Responses to a decline in the fertility rate: alternative monetary policy rules.
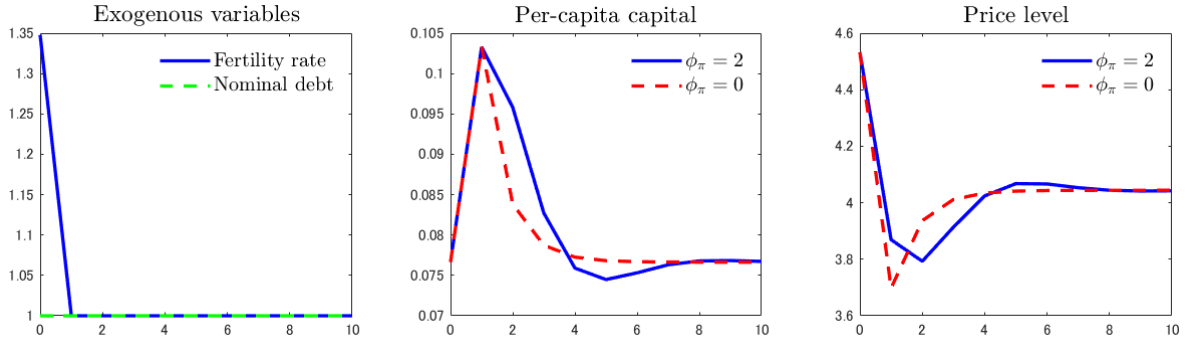
period $t = 1$ under the two monetary policy rules: $\phi_\pi = 0$ and $\phi_\pi = 2$.[6]

As shown in Figure 1, the (exogenous) fertility rate drops in $t = 1$. In the case of $\phi_\pi = 0$ (red dashed lines), the price level falls sharply in $t = 0$ and then converges to the terminal steady state from below. The capital stock, in contrast, increases on impact and transitions to the new steady state from above. Computing the transition requires numerical methods (we use a shooting algorithm). However, considerable intuition for the results can be gathered by inspecting the two equilibrium conditions that determine the capital stock and the price level. Consider first the asset market clearing condition, (11) in period 1. The wage rate and nominal debt are predetermined in period 1, but the fertility rate is now lower. It follows that the real supply of assets has to increase. This adjustment occurs in two ways. Households leave the period with more capital, so that k_2 increases. However, capital and government debt are perfect substitutes and have the same real return moving forward. In particular, equation (13) with $\phi_\pi = 0$ is

$$\alpha k_{t+1}^{\alpha-1} = \frac{R}{\pi_{t+1}} \tag{14}$$

and it follows that the price level falls on impact in period 1 as shown in Figure 1 to compensate households for the lower future real returns on government debt.

Allowing monetary policy to respond to changes in the inflation rate ($\phi_\pi = 2$) affects the asset supply responses in period 1 and asset demand (wages) from period 2 on. The price level now declines less in period 1, but more in periods 2 and 3 compared to the $\phi_\pi = 0$ scenario and the capital stock is now higher in periods 2 and 3.

These two examples illustrate that how monetary policy responds to a lower fertility rate

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[6]The rest of the parameterization in this computational example is: $\alpha = 0.3$, $n_0 = (1.01)^{30}$, $n = 1$ and $R = (1.02)^{30}$.

Figure 2: Responses to a permanent decline in the fertility rate and a permanent increase in the stock of nominal government debt: alternative monetary policy rules.
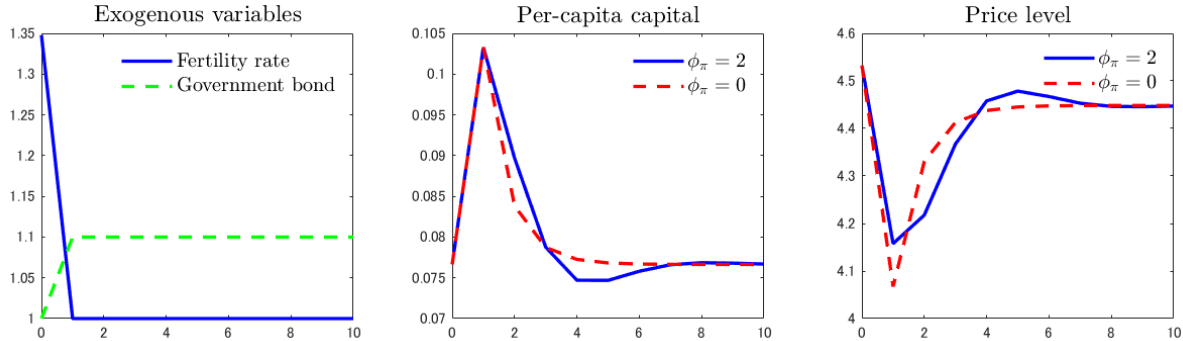
influences both the size of the price response in the impact period and also the dynamics of the inflation rate and capital stock in subsequent periods.

2.3.2 Response of government borrowing to lower fertility

In our quantitative model we will consider how changes in the supply of nominal government debt during a demographic transition influence the inflation rate and real economic activity. We saw in the steady state analysis that money matters, but that nominal government debt is neutral. It follows that changes in the size of nominal government debt only have real effects in this economy if they trigger a response by the monetary authority. Figure 2 reports a scenario in which the fertility rate falls and nominal government debt issue is permanently increased by 10 percent in period 1. Consider the $\phi_\pi = 0$ scenario first. The negative response of the price level in period 1 is much smaller now as compared to Figure 1 and the terminal steady-state has a higher price level. However, the trajectory of the capital stock is identical to Figure 1 because prices are flexible. However, when the monetary policy interest rate rule is endogenous higher government debt crowds out private capital formation. Comparing Figures 1 and 2, we see that the scenario with high government debt has a lower capital stock in periods 2-5 when $\phi_\pi = 2$.

In our quantitative model we introduce costly adjustments of illiquid assets and prices and these frictions influence the timing and size of the responses shown here. Still, monetary policy, fiscal policy and real economic activity already have rich interactions in this simple flexible price model in the ATPL equilibrium.

3 Quantitative Model

We conduct our quantitative analysis using the model of Braun and Ikeda (2021) and readers are referred to that paper for more details on the motivation and discussion of our modeling assumptions. That paper also contains a detailed analysis of the short-run properties of the model. Here we highlight the main features of the model and focus our discussion on how household saving decisions change during a demographic transition and how the model accounts for the empirical observations that motivate our analysis.

We consider an OLG economy with representative cohorts. Households can save and/or borrow two assets that differ in terms of their liquidity services. Illiquid assets offer a higher return but are costly to acquire and sell. Liquid assets offer a lower return but are costless to adjust. Depending on where households are in their lifecycle, they choose to borrow liquid assets to purchase illiquid assets or hold positive amounts of both assets.

3.1 Demographic structure

The economy has an OLG structure that evolves in discrete time with a period length of one year. Let j denote the age of the individual as $j = 1, ..., J$. We start keeping track of individuals at age 21 and individuals survive until at most age 120. Thus, model age of $j = 1$ corresponds to age 21, model age $J = 100$ corresponds to age 120 and $J = 100$ cohorts are active in a given year. Let $N_{j,t}$ be the number of individuals of age j in period t, then the population age distribution in period t is given by the $J \times 1$ vector $\mathbf{N}_t \equiv [N_{1,t}, ..., N_{J,t}]'$.

The dynamics of population are governed by

$$\mathbf{N}_{t+1} = \begin{bmatrix} n_{1,t} & 0 & 0 & \dots & 0 & 0 \\ \psi_{1,t} & 0 & 0 & \dots & 0 & 0 \\ 0 & \psi_{2,t} & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & 0 & \dots & \psi_{J-1,t} & 0 \end{bmatrix} \mathbf{N}_t \equiv \mathbf{\Gamma}_t \mathbf{N}_t,$$

where $n_{1,t}$ is the gross population growth rate of age-1 households between periods t and $t+1$, which we will henceforth refer to as the net fertility rate[7], and $\psi_{j,t}$ is the conditional

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probability that a household of age j in period t survives to period $t+1$. It follows that the aggregate population in period t is given by $N_t = \sum_{j=1}^{J} N_{j,t}$ and that the population growth rate is given by $n_t = N_{t+1}/N_t - 1$. Finally, the unconditional probability of surviving from birth in period $t - j + 1$ to age $j = 2, ..., J$ in period t is:

$$\Psi_{j,t} = \psi_{j-1,t-1}\Psi_{j-1,t-1}$$

where $\Psi_{1,t} = 1$ for all t.

3.2 Households

Individuals are organized into households. Each household consists of one individual (adult) and children. The number of children varies with the age of the adult and the age of the household is indexed by the age of the adult. Adults face mortality risk and have no bequest motives. At the beginning of each period the adult learns whether she will die at the end of the current period and this rules out accidental bequests. Let $z_{j,t}^i \in \{0,1\}$ index the survival state for the ith household where a value of zero denotes the death state. Death is the only source of idiosyncratic risk faced by households and there are only two types of households in any cohort: surviving households ($z_{j,t}^i = 1$) and non-surviving households ($z_{j,t}^i = 0$).

Households work, consume, and save for retirement. A household of age j in period t earns an after-tax wage rate of $(1 - \tau^w)w_t\epsilon_j$, where τ^w denotes a labor-income tax rate and ϵ_j is the efficiency of labor of an age-j household.[8] All cohorts face the same age-efficiency profile and the efficiency index ϵ_j is assumed to drop to zero for all $j \geq J_r$, where J_r is the mandatory retirement age.

Households provision for retirement by acquiring liquid and illiquid assets. They may save and/or borrow using either asset and the liquid asset is nominally denominated because monetary policy directly controls the nominal interest rate on liquid assets in this economy. The liquid asset earns the nominal interest rate R_{t-1} between period $t-1$ to t and its after-taxed real return is given by $\tilde{R}_{t-1}/\pi_t$, where $\tilde{R}_{t-1} = 1 + (1 - \tau^a)(R_{t-1} - 1)$. The real return on illiquid assets in period t is R_t^a and its after-taxed return is $\tilde{R}_t^a = 1 + (1 - \tau^a)(R_t^a - 1)$.

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one period to the next.

[8]Given that there is only one type of heterogeneity in a cohort, to conserve on notation we do not explicitly index the identity of each household of age j in period t in the ensuing discussion unless it is required to avoid confusion.

From the perspective of the household the only distinction between liquid and illiquid assets is that households face costs of adjusting their holdings of illiquid assets as in Aiyagari and Gertler (1991) and Kaplan and Violante (2014). When we parameterize our model, we follow Kaplan et al. (2018) and include physical assets such as homes and durable goods and illiquid financial assets such as equities in our measure of illiquid assets. So the adjustment costs can be interpreted as representing service flows to the financial service sector when, for instance, a household purchases or sells a home. Following Kaplan et al. (2018), we also abstract from the service flow of utility services provided by physical assets. Thus, the benefit from holding illiquid assets is entirely pecuniary in our model.

Adjustment costs on holdings illiquid assets are given by

$$\chi(a_{j,t}, a_{j-1,t-1}, z) = \begin{cases} \frac{\gamma_a(z)}{2}(a_{j,t} - a_{j-1,t-1})^2, & a_{j-1,t-1} > 0 \\ \frac{\gamma_a(z)}{2}a_{j,t}^2, & a_{j-1,t-1} = 0 \end{cases} \tag{15}$$

where $a_{j,t}$ denotes the holdings of illiquid assets in the end of period t and $\gamma_a(z) \geq 0$ is a parameter that governs the size of the adjustment costs for $z = z_{j,t}^i \in \{0,1\}$. These costs have two main features. First, they vary with the level of the change in assets. Second, they depend on whether the household experiences the death event in the current period. In Braun and Ikeda (2021) we show that this two parameter model of financial frictions allows us to match the main features of the age profiles of liquid asset holdings and illiquid asset holdings in Japanese data.

Given these definitions, the decisions of a surviving household of age-j in period t (i.e., a household with $z_{j,t}^i = 1$) are constrained by:

$$(1 + \tau^c)c_{j,t} + a_{j,t} + \chi(a_{j,t}, a_{j-1,t-1}, 1) + d_{j,t}$$
$$\leq \tilde{R}_t^a a_{j-1,t-1} + \frac{\tilde{R}_{t-1}}{\pi_t}d_{j-1,t-1} + (1 - \tau^w)w_t \epsilon_j h_{j,t} + b_{j,t} + \xi_t, \tag{16}$$

where $c_{j,t}$ is total household consumption for a household of age j in period t, τ^c is a consumption tax rate, $d_{j,t}$ denotes holdings of the liquid asset, expressed in terms of the final good, at the end of period t, $h_{j,t}$ denotes hours worked, $b_{j,t}$ denotes public pension (social security) benefits, ξ_t is a lump–sum government transfer, and $\chi(\cdot)$ is the transaction cost of adjusting individual holdings of the illiquid asset.[9] We wish to emphasize that there

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[9]We are omitting here the dependence of individual choices on the survival event to save on notation. Formally, we have for $z_{j,t}^i \in \{0,1\}$: $c_{j,t}(z_{j,t}^i)$, $a_{j,t}(z_{j,t}^i)$, $d_{j,t}(z_{j,t}^i)$, and $h_{j,t}(z_{j,t}^i)$. In what follows this

are no ad hoc restrictions on borrowing of surviving households. They are free to borrow against their future earnings and they are also free to take leveraged long positions on illiquid assets, which have a higher return in equilibrium. The only constraint on borrowing of surviving households is the natural borrowing constraint.

If instead the household is in its final period of life ($z_{j,t}^i = 0$), the event is publicly observed by lenders and borrowing is not possible. Thus, the optimal strategy for the household is to consume all of its income and wealth during the current period

$$(1+\tau^c)c_{j,t} = \tilde{R}_t^a a_{j-1,t-1} + \frac{\tilde{R}_{t-1}}{\pi_t}d_{j-1,t-1} + (1-\tau^w)w_t\epsilon_j h_{j,t} + b_{j,t} + \xi_t - \chi(0, a_{j-1,t-1}, 0). \quad (17)$$

The period utility function for a household of age j in period t is given by

$$u(c_{j,t}, h_{j,t}; \eta_j) = \frac{\eta_j \left(c_{j,t}/\eta_j\right)^{1-\sigma}}{1-\sigma} - \frac{\upsilon}{1+1/\nu}h_{j,t}^{1+1/\nu}, \quad (18)$$

where $\sigma > 0$ is the inverse of the elasticity of inter-temporal substitution, $\nu > 0$ governs the Frisch elasticity of labor supply, $\upsilon > 0$ is a labor dis-utility parameter, and η_j is a family scale, which we assume is time-invariant. In the model, children are essentially age-specific deterministic demand shocks to household consumption.

We assume that working-age households belong to a labor union. The union respects their marginal utilities and wages are flexible. We analyze the symmetric equilibrium. Thus, hours worked are identical for all workers in period t, $h_{j,t} = \bar{h}_t$ for all $j < J_r$ with $\bar{h}_t$ given by

$$(1-\tau^w)\bar{\epsilon}_t w_t = \upsilon\bar{\lambda}_t^{-1}\bar{h}_t^{\frac{1}{\nu}} \quad (19)$$

where $\bar{\lambda}_t$ is the weighted average of the marginal utilities of working households and $\bar{\epsilon}$ is the weighted average of the efficiency of labor. This specification implies that workers who experience an aggregate or idiosyncratic shock are unable to self-insure by adjusting their hours worked differently from the average worker. Household earnings vary by age because the efficiency of a worker's labor depends on the worker's age. The interested reader is referred to Braun and Ikeda (2021) for more details.

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dependence is only made explicit when required.

The household's optimal choices are given by the solution to

$$U_j(a_{j-1,t-1},d_{j-1,t-1}, z_{j,t}) = \max_{\{c_{j,t},a_{j,t},d_{j,t}\}} \left\{ u\left(c_{j,t},\bar{h}_t;\eta_j\right) \right.$$
$$\left. +\beta z_{j,t}\left[(1-\psi_{j+1})U_{j+1}(a_{j,t}, d_{j,t}, 0) + \psi_{j+1}U_{j+1}(a_{j,t}, d_{j,t}, 1)\right] \right\}, \tag{20}$$

subject to equations (16) and (17) for $z_{j,t} \in \{0,1\}$ and for $j = 1,..J-1$, and $z_{J,t} = 0$, where $\beta > 0$ is the preference discount factor and ψ_{j+1} is the conditional probability that a household of age $j+1$ survives to the next period.[10] Note that we have imposed no restrictions on the sign or magnitude of asset holdings beyond the natural borrowing constraint. It is thus conceivable, for instance, that households might want to borrow both types of assets. However, in equilibrium, the return on illiquid assets exceeds the return on liquid assets and all private borrowing will be in the form of liquid assets.

3.3 Production of goods and services

The production of goods and services is organized into four sectors.

Final good sector. Firms in this sector are perfectly competitive and combine a continuum of intermediate goods, $\{Y_t(i)\}_{i\in(0,1)}$, to produce a homogeneous final good Y_t, using the production technology: $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{\theta}}di\right]^\theta$ with $\theta > 1$. Let $P_t(i)$ denote the price of intermediate good i, and P_t denote the price of the final good. Final good firms are price takers in input markets and it follows that demand for intermediate good i is:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta}{\theta-1}} Y_t. \tag{21}$$

The final good is either consumed by households or used as an input in the capital good sector.

Intermediate goods sector. Firms in this sector are monopolistically competitive and each firm produces a unique good indexed by $i \in (0,1)$. Intermediate goods firm i produces $Y_t(i)$ by combining capital $K_t(i)$ and effective labor $H_t(i)$ with a Cobb-Douglas production

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[10]There is a theoretical possibility that adjustment costs on illiquid assets could exceed the size of beginning of period illiquid assets. Our strategy for parameterizing the adjustment costs on illiquid assets rules this possibility out.

function:

$$Y_t(i) = K_t(i)^\alpha H_t(i)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{22}$$

Intermediate goods firm i faces demand curve (21), and sets its price $P_t(i)$ to maximize profits subject to a quadratic price adjustment cost function. In a symmetric equilibrium, the condition can be expressed as

$$(\pi_t - 1)\pi_t = \frac{1}{\gamma}\frac{\theta}{\theta-1}(mc_t - 1) + \Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}(\pi_{t+1} - 1)\pi_{t+1}, \tag{23}$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate. Equation (23) is the New Keynesian Phillips curve that relates the current inflation rate π_t to the real marginal cost mc_t and the future inflation rate π_{t+1}. In a symmetric equilibrium the aggregate output is

$$Y_t = K_t^\alpha H_t^{1-\alpha}, \tag{24}$$

where K_t denotes the aggregate capital and H_t denotes the aggregate effective labor. The aggregate total profits of intermediate goods firms in period t, $\Omega_t \equiv \int_{i\in(0,1)} \Omega_t(i)di$, are given by

$$\Omega_t = \left[\theta - mc_t - \frac{\gamma}{2}(\pi_t - 1)^2\right]Y_t. \tag{25}$$

Capital good sector. Capital good firms are perfectly competitive and use a linearly homogeneous production technology to produce capital. The representative firm purchases $(1-\delta)K_t$ units of old (depreciated) capital from the mutual fund and I_t units of the final good from the final good firms, and uses the two inputs to produce K_{t+1} units of new capital that is sold back to the mutual fund. Then, the conventional investment identity obtains:

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{26}$$

Mutual fund sector. Our economy has two types of illiquid assets – capital and shares in intermediate goods firms – and there is no aggregate uncertainty in the model after time-zero. Thus, a no arbitrage argument implies that the return on the two illiquid assets is the same in all periods except possibly time-zero when their returns will differ if an aggregate time-zero shock occurs. We allocate ownership and the potential time-zero capital gains and losses among households by assuming that households invest in a mutual fund produced by perfectly competitive financial service firms. Each firm holds the market portfolio of the

two illiquid assets and pays households the market return on illiquid assets.

To derive the market return on illiquid assets note that the return on capital in period t is given by

$$R_t^k = r_t^k + 1 - \delta. \tag{27}$$

The one period return from investing one unit of the period $t - 1$ final good into shares is

$$R_t^v = \frac{\Omega_t + V_t}{V_{t-1}}, \tag{28}$$

where V_t is the share price. We assume that the return on capital and equity is subject to a corporate tax as well as an asset income tax paid by households. Liquid assets, in contrast, will consist primarily of government debt in equilibrium and are taxed once at the household level. To reduce the notational burden, we assume that corporate taxes are paid by the mutual fund. Let τ^k denote the corporate tax rate. Then, perfect competition leads to the arbitrage conditions:

$$R_t^a - 1 = (1 - \tau^k)(R_t^k - 1) = (1 - \tau^k)(R_t^v - 1). \tag{29}$$

for all $t > 0$. From this no-arbitrage restriction the share price is given by

$$V_t = \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{R_{t+j}^k} \right) \Omega_{t+i}. \tag{30}$$

Hence, the discount factor $\Lambda_{t,t+1}$ in equation (23) is given by $\Lambda_{t,t+1} = 1/R_{t+1}^k$.

We analyze the evolution of our economy during a demographic transition by solving a two point boundary problem. The terminal condition is determined by a steady-state and the initial condition is an initial age-distribution and an initial age-asset distribution.

Equation (29) does not obtain in period $t = 0$ because the response of the price-system generally induces distinct capital gains and losses on shares in intermediate goods firms and capital.

3.4   Government

The government consists of a central bank and a fiscal authority.

Central bank. The central bank sets the nominal interest rate R_t following a simple rule

that depends on the current inflation rate and the past nominal interest rate:

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + (1 - \rho_r)\phi_\pi \log(\pi_t), \tag{31}$$

where R is a constant. The parameter ρ_r governs the inertia of the nominal interest rate, and the parameter $\phi_\pi > 1$ captures the central bank's stance on inflation. A high ϕ_π implies a strong anti-inflation stance and vice versa.

Fiscal authority. The fiscal authority raises revenue by taxing consumption, labor income, capital income, and mutual funds. Total tax revenue is

$$T_t = \sum_{j=1}^{J} \left[\tau^c \bar{c}_{j,t} + \tau^{ka}(R_t^k - 1)a_{j-1,t-1} + \tau^a \frac{(R_{t-1} - 1)}{\pi_t} d_{j-1,t-1} + \tau^w w_t \epsilon_j \bar{h}_t \right] N_{j,t}, \tag{32}$$

where $\bar{c}_{j,t} = \psi_{j,t} c_{j,t}(1) + (1 - \psi_{j,t}) c_{j,t}(0)$ is the average consumption by surviving and non-surviving households and $\tau^{ka} = \tau^a + \tau^k - \tau^a \tau^k$ is the total tax rate on illiquid assets.

Let D_t^n denote the face value of nominal government debt issued in period t. Then aggregate government expenditures consist of government purchases G_t, nominal interest payments on its debt, net of new issuance, $(R_{t-1}D_{t-1}^n - D_t^n)/P_t$, subsidies to intermediate goods firms, $\tau^f Y_t = (\theta - 1)Y_t$, public pension benefits $B_t \equiv \sum_{j=J_r}^{J} b_{j,t} N_{j,t}$, and lump–sum transfers to households, $\Xi_t \equiv \sum_{j=1}^{J} \xi_t N_{j,t}$. It follows that the government flow budget constraint is given by

$$G_t + \frac{R_{t-1}D_{t-1}^n - D_t^n}{P_t} + \tau^f Y_t + B_t + \Xi_t = T_t \tag{33}$$

and the government bond market clearing condition is given by

$$\frac{D_t^n}{P_t} = D_t \equiv \sum_{j=1}^{J} \bar{d}_{j,t} N_{j,t}, \tag{34}$$

where $\bar{d}_{j,t} = \psi_j d_{j,t}(1) + (1 - \psi_j) d_{j,t}(0)$ is the average government bond holdings by surviving and non-surviving households.[11]

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[11]Because $d_{j,t}(0) = 0$, the aggregate bond can be arranged as

$$D_t \equiv \sum_{j=1}^{J} \left[\psi_j d_{j,t}(1) + (1 - \psi_j) d_{j,t}(0) \right] N_{j,t} = \sum_{j=1}^{J} \psi_j d_{j,t}(1) N_{j,t} = \sum_{j=1}^{J} d_{j,t}(1) N_{j+1,t+1}.$$

20

We will consider two different scenarios for the time path of government debt. It proves easier to isolate the impact that household asset demand has on real interest rates, the nominal interest rate and the inflation rate if we assume that the supply of per capita nominal government debt is constant. However, we also consider scenarios where the supply of nominal government debt varies over the transition.

For a given time path of nominal government debt, we close the government budget constraint by varying the size of the lump–sum transfer, ξ_t. Benefits from the pubic pension program are modeled in the same way as Braun et al. (2009). A household starts receiving a public pension benefit at the mandatory retirement age of J_r. The real size of the benefit during the household's retirement is constant at a level that is proportional to its average real wage income before retirement:

$$
b_{j,s+j-1} = \begin{cases} 0 & \text{for } j = 1, ..., J_r - 1 \\ \lambda \left(\frac{1}{J_r-1} \sum_{j=1}^{J_r-1} w_{s+j-1} \epsilon_j \bar{h}_{s+t-j} \right) & \text{for } j = J_r, ..., J, \end{cases} \tag{35}
$$

where λ is the replacement ratio of the pension benefit and s is the household's birth year. Thus, the public pension system implicitly assumes perfect inflation indexation of pension benefits.

3.5   Competitive equilibrium

In the analysis that follows, we assume that at the beginning of time zero households observe the future evolution of the demographic distribution and have perfect foresight about the subsequent evolution of prices and government policy reactions.[12] Consequently, our definition of equilibrium is a perfect foresight competitive equilibrium. More details on the defintion of equilibrium can be found in Braun and Ikeda (2021).

4   Model parameterization

4.1   Demographic transition

We assume that new households are formed at age 21 and the size of the household is parameterized in the same way as Braun et al. (2009). In the model individuals face mandatory retirement at age 68 ($J_r = 48$). This is two years older than the age where one

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[12]Boppart et al. (2017) provide a justification for using this approach in heterogeneous agent economies.

can qualify for full public pension benefits in Japan and is chosen to be consistent with the effective labor-market exit age in 2014 for Japan estimated by the OECD.[13] Finally, the maximum lifespan is set to 120 years ($J = 100$).

Table 1 reports summary statistics for Japan's age distribution and population at 10 year intervals. We limit attention to the 21+ population to make the data consistent with the workings of our model. The combined impact of aging of the babyboomers, lower fertility rates and longer life expectancies are already putting downward pressure on the age 21+ population and this pressure will increase in future years. During this transition the percentage share of the 70+ population in the total 21+ population will increase from 23% to 38% according to our estimates which are based on data from the National Institute of Population and Social Security Research (IPSS).

Table 1: Age distribution and 21+ population by decade

| | year | | | | |
Age/pop 21+	2015	2025	2035	2045	2055
Under 30	0.11	0.11	0.10	0.09	0.09
30–39	0.15	0.12	0.12	0.12	0.11
40–49	0.18	0.16	0.13	0.13	0.13
50–59	0.15	0.17	0.17	0.14	0.14
60–69	0.18	0.14	0.18	0.17	0.15
70+	0.23	0.29	0.31	0.35	0.38
Pop 21+ (millions)	102	100	95	88	80

*Data source: Our estimates using data from the National Institute of Population and Social Security Research (IPSS).

4.2 Other model parameters

Table 2 reports the entire parameterization of the model. Complete details on our calibration strategy can be found in Braun and Ikeda (2021).

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[13]See Pensions at a Glance OECD, 2015.

Table 2: Parameterization of the model

Parameter	Description	Value
Demographics		
J_r	Retirement age	48 (Age 68)
J	Maximum lifespan	100 (Age 120)
$\{\psi_j\}_{j=1}^{J}$	Survival probabilities	Braun and Joines (2015)
Technology		
θ	Gross markup	1.05
γ	Price adjustment cost	41.2
α	Capital share parameter	0.30
δ	Depreciation rate	0.102
$\gamma_a(0)$	Cost of adjusting illiquid assets in death year	0.0723
$\gamma_a(1)$	Cost of adjusting illiquid assets in non-death year	0.203
Preferences		
σ	Inverse elasticity of intertemporal substitution	3
ν	Frisch labor supply elasticity	2
υ	Preference weight on leisure	6.9
β	Preference discount factor	0.996
Monetary Policy		
ρ_r	Interest rule persistence	0.35
ϕ_π	Interest rule inflation elasticity	2
Fiscal Policy		
τ^c	Consumption tax rate	0.05
τ^k	Corporate tax rate	0.35
τ^a	Tax rate on asset income	0.2
τ^w	Tax rate on labor income	0.232
τ^f	Subsidy to intermediate goods firms	$\theta - 1$
G/Y	Government share of output	0.16
λ	Public pension replacement ratio	0.094
D/Y	Net government debt output ratio	1.23

5   Results

5.1   Household asset demand during the demographic transition: partial equilibrium and general equilibrium

Our first step in analyzing the macroeconomic effects of aging is to show that asset demand initially increases during a demographic transition and that demand for liquid assets increases by more than demand for illiquid asset during this period.

One reason this is the case is that the age-groups who have the highest asset holdings and also the highest liquid asset holdings initially go up.

Table 3: Net worth, liquid and illiquid asset holdings by age relative to income of households aged 50–59.

Age	Net worth	Liquid assets	Illiquid assets
Under 30	0.01	-0.63	0.64
30–39	0.88	-0.85	1.73
40–49	2.85	0.19	2.65
50–59	5.54	2.23	3.31
60–69	7.27	3.63	3.64
70+	4.16	0.94	3.22

Notes: Our estimates using data from the NSIFE and FSIE from 2014. See Braun and Ikeda (2021) for more details.

To measure the strength of this mechanism consider Table 3 which reports net worth and the steady-state age-asset profile of households in our model. This profile is a steady-state age profile and was calibrated to Japanese survey data from 2014. Liquid asset holdings are negative for younger age groups because these age groups are borrowing liquid assets on net. Net worth increases with age up until retirement which occurs at age 68 in the model and then declines thereafter. The variation in holdings of liquid assets over the lifecycle is particularly large. Younger households borrow liquid assets to purchase illiquid physical and financial assets. Households close to age 68, in contrast, hold large amounts of both liquid and illiquid assets. Then during retirement households draw down both assets.

The second fact can be seen in Table 1. The fraction of households with high asset holdings and high relative holdings of liquid assets increases persistently. During the initial stages of the transition to an older age distribution, the fraction of households with high asset holdings and high liquid asset holdings increases while the fraction of households with small assets and negative holdings of liquid assets declines. The fraction of households aged 50–70 increases from 0.33 in the year 2014 to 0.35 in 2044 while the fraction of households under age 39 declines from 0.26 in the year 2014 to 0.22 in the year 2044. Table 4 shows how these changes in the population distribution affect aggregate demand for liquid and illiquid assets. The changes in aggregate asset demand reported in the table are partial equilibrium in the sense that they only recognize changes in population-age distribution and use the steady-state age-profile of savings plans.

The dynamics of the population distribution in our model are pinned down by an initial age-distribution, fertility rates in each year, and life-expectancies for each birth cohort. Table 4 helps to understand the contribution of these factors. The "Aging of

Table 4: Partial equilibrium aggregate demand for liquid and illiquid assets.

Demographic Scenario	Liquid assets	Year	Illiquid assets	Year
Aging of Babyboomers	19.83	2038	2.32	2029
Longer life expectancy	0.63	2045	0.07	2044
Lower fertility rates	24.12	2065	6.18	2067
Baseline	27.1	2043	5.24	2053

Notes: The table reports the maximum increase in each type of asset expressed as a percentage of initial assets and the year where the maximum increase occurs. "Aging of Babyboomers" reflects changes in the population distribution due to aging of the babyboomer cohorts only. "Longer life expectancy" reflects changes in the age distribution do to higher survival rates only. "Lower fertility rates" reflects changes in the birth rate of 21 year olds only and "Baseline" incorporates all three channels.

babyboomers" scenario, holds fertility rates and survival probabilities fixed in all years at their terminal values and uses the initial 2014 age distribution as the initial condition. In the "Longer life expectancy" scenario survival probabilities vary by birth cohort and gradually increase over time, but the age distribution and fertility rates are fixed in all periods. The "Lower fertility rate" scenario considers the case where fertility rates gradually decline and the initial age distribution and survival probability age-profiles are set to their terminal values. Finally, the "baseline" demographic scenario incorporates all three factors. Each demographic factor in isolation and collectively induce persistent increases in liquid and illiquid asset holdings with the peak increase occurring at least 29 years in the future. The strongest factor is the aging of the babyboomers but lower fertility rates is also large although the peak deviation of this second factor occurs much later. The second important feature of the results is that aging has a much larger impact on the aggregate demand for liquid assets as compared to the aggregate demand for illiquid assets. As households enter retirement they prefer to hold a larger share of their portfolios in liquid assets because, the replacement rate provided by pension income is less than one. Moreover, they face an elevated mortality risk and can avoid some of the costs of liquidating their holdings of illiquid assets in their death year if they tilt their portfolio towards liquid assets.

5.2   General equilibrium results

The partial equilibrium analysis shows that aging puts steady and persistent upward pressure on asset demand for about 30 years when using the terminal steady–state price system. However, it is not clear whether this result survives when prices adjust in each period to

clear markets. In addition, price adjustments induce reactions in monetary policy under our assumption that the central bank follows an interest rate targeting rule and price changes affect government revenues, so fiscal policy also adjusts. We now turn to consider simulating general equilibrium scenarios. We assume perfect foresight in all periods except time zero. In 2014 households in the model discover that the population distribution is going to evolve over time and they adjust their consumption, savings and labor supply plans accordingly. When making these adjustments they fully anticipate the future evolution of prices. These assumptions allow us to use global sequence methods to compute an equilibrium.[14,][15]

Before discussing the results we wish to emphasize that our objective is to understand whether demographic forces are large enough, in isolation, to induce secular stagnation which we define as steady and persistent declines in interest rates, the inflation rate and output. We are not attempting to make forecasts about the future course of the Japanese economy. Instead our results are best understood as impulse responses to a particular shock and we are interested in the shape of the IRFs during the first stage of the transition. Demographics evolve gradually over time and are very persistent and we will see that they induce gradual changes in macroeconomic activity, but the cumulative effects of aging can be large.

When solving for the general equilibrium version of the model we specify an exogenous sequence of nominal government debt. The baseline specification assumes that the stock of per capita nominal debt is held fixed in each period and that lump-sum taxes adjust to satisfy the government budget constraint in each period. This assumption allows us to isolate the important role that the monetary policy response plays in accounting for the secular stagnation observations that motivate this paper. In section 5.2.2 we consider scenarios where the stock of nominal government debt increases during the demographic transition. The monetary authority is assumed to follow a nominal interest rate targeting rule with a serial correlation coefficient ρ_r set to 0.351 and an inflation elasticity, $\phi_\pi = 2$.

Figure 3 reports results for the baseline scenario. These results indicate that Japan's demographic transition induces steady downward pressure on the real interest rate, the nominal interest rate and inflation rate and per capital output. The real interest rate declines from 3.4 percent in 2015 to 1.2 percent in 2040 and the inflation rate falls from

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[14]Chen et al. (2006) find that their results are robust to the model of expectations formation for a demographic shock in a representative agent model.

[15]Computing an equilibrium for a given parameterization of the model can take several days on a 2020 vintage Mac Pro with 16 cores using Matlab's parallel toolbox.

−0.32 in 2015 to a low of −1.41 percent in 2035. It is also interesting to see that the model produces a large increase in the government debt-output ratio. It rises from 1.16 in 2015 to 1.84 in 2040. Finally, the model also produces a gradual but steady decline in per capita output at the average rate of −0.23 percent per year between 2015 and 2040.[16]

We now turn to inspect the mechanisms that are inducing these responses. The declines in the real interest rate on illiquid assets and the inflation rate suggest that aging continues to induce persistent increases in asset demand when we assume that markets clear. In the baseline GE simulation, private demand for assets peaks later and at a higher level. The peak increase in private demand for liquid assets is 68.9% in the GE simulation as compared to 27.1% in the PE scenario and the peak occurs in 2077 in the GE scenario as compared to 2043 in the PE scenario. Demand for illiquid assets peak earlier (2030) in the GE simulation as compared to 2053 in the PE simulation. The peak increase is also a bit higher at 6.1% in the GE scenario as compared to 5.2% for the PE scenario.

The reason why liquid asset demand increases and the inflation rate falls is particularly easy to understand under our assumption that per capita government nominal debt is constant. The liquid asset market clearing condition (34) in our model can be written as:

$$\frac{d_t^n}{P_t} = \frac{\sum_{j=1}^J \bar{d}_{j,t} N_{j,t}}{N_t}.$$

(36)

where d_t^n is per capita nominal government debt which is fixed in this simulation, P_t is the price level and the right hand side is real household demand for liquid assets. Our analysis of the 2–period OLG model suggests that the reason why the price level is falling is because private net demand for liquid assets is increasing.

Higher demand for liquid assets can explain why the inflation rate is falling but, it is still not clear why the model produces concurrent declines in the real interest rate and the inflation rate. In fact, the Fisher equation predicts a negative relationship between the inflation rate and the marginal product of capital.

To get a better idea of why both the real interest rate and the inflation rate are moving down together it is helpful to refer to the Fisher relationship in our quantitative model

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[16]Our model parameterization assumes that the state of technology is constant over time, so this decline is (approximately) the deviation in output from trend.

which is approximately given by[17]

$$\frac{1 + R_t}{1 + \pi_{t+1}} = \frac{\alpha k_t^{\alpha-1} + 1 - \delta + \gamma_a \Delta a_{j+1,t+1}}{1 + \gamma_a \Delta a_{j,t}}. \tag{37}$$

for a household of age j in period t.

This version of the Fischer equation has a wedge between the marginal product of capital and the real interest rate on liquid assets. The size of it depends on the size of the liquidity premium which is governed by the parameter γ_a and also the household's age specific survival probability. Note also that the aggregate Fisher relationship depends on the population-distribution. Finally, as we illustrated in Section 2 the reaction of monetary policy to deflation also influences real returns on liquid and illiquid assets. We now turn to report counterfactuals that are designed to provide more information about how the model accounts for the secular stagnation facts.

### 5.2.1 The reaction of monetary policy to aging

In our quantitative model an aging population distribution induces increases in the demand for liquid assets and this puts downward pressure on the inflation rate. Figure 3 shows that monetary policy reacts to this deflationary pressure by lowering the nominal interest rate. How does this policy response influence macroeconomic outcomes? To answer this question we consider a scenario where the central bank doesn't react to the deflationary pressure by setting ϕ_π to zero. Figure 4 reports the results. Monetary policy has a profound effect on macroeconomic outcomes. Under a fixed nominal interest rate targeting rule, the inflation rate plummets on impact by nearly 10 percent and then recovers in subsequent periods. Aging continues to push the real interest rate down between the years 2019 and 2040. However, output increases gradually up until the year 2028. The debt–output ratio increases much more rapidly, peaking at 1.6 in 2028 and then gradually declines thereafter.[18] We illustrated in Section 2 that the specification of the central bank's policy rule affected the trajectories of the price level and capital accumulation. The results from the quantitative model show that the nominal interest rate targeting rule plays a central role the model's ability to account for the persistent concurrent declines in the real interest rate and the inflation rate and also for below trend growth in per capita output.

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[17]The following expression is approximate because it assumes that households don't observe their death event at the beginning of their final period of life.

[18]The maximum debt–output ratio in the baseline is 2.2 and occurs in the year 2074

Figure 3: Baseline macroeconomic responses to aging in Japan
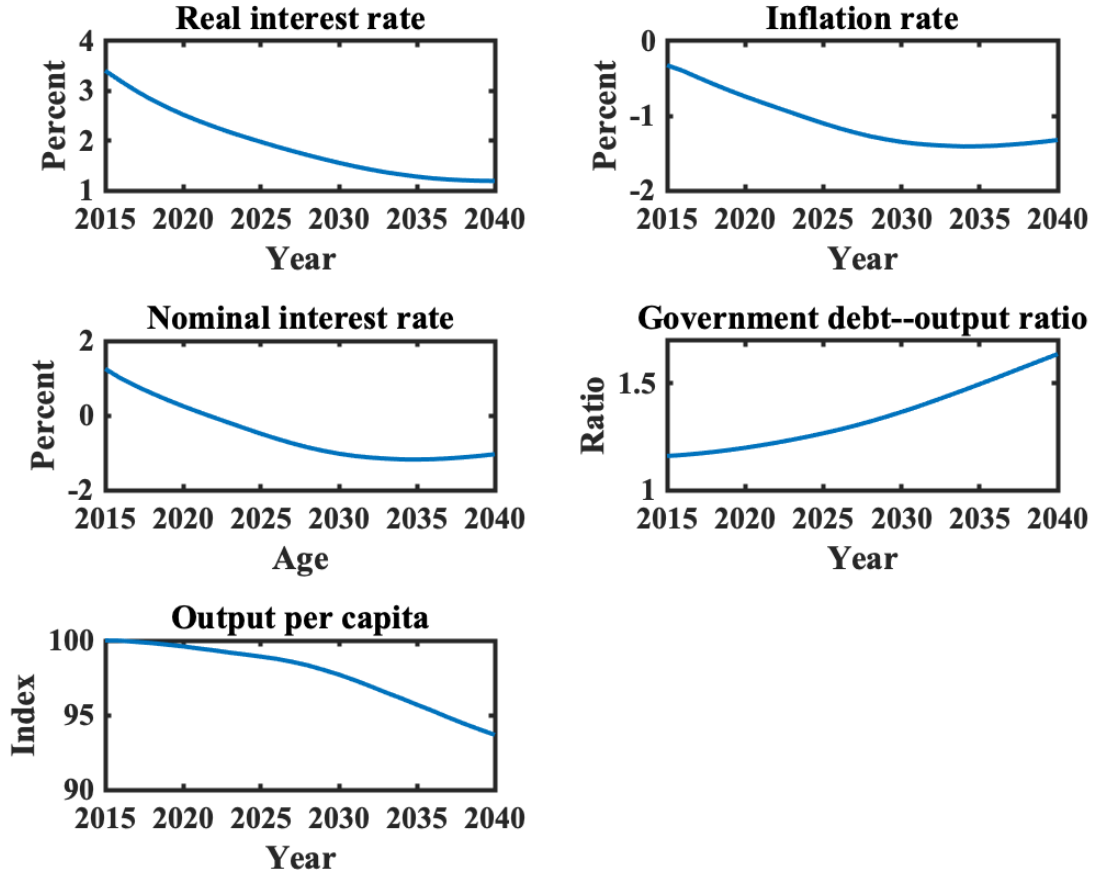
Lowering the nominal interest rate attenuates and propagates the downward pressure on the price level induced by aging and creates asset substitution effects that put downward pressure on real interest rates and make it more difficult for households to provision for their retirement.

To get a better idea about the differences between the two equilibria consider Table 5 which reports some of the main macroeconomic variables for the year 2040 expressed in terms of deviations from the terminal steady–state. Observe that the real return on both illiquid and liquid assets is higher in this year in the constant R equilibrium. The capital stock and investment are lower in this equilibrium, but this economy has more wealth. Aggregate illiquid assets and aggregate real holdings of liquid assets are both higher in the constant R equilibrium (the price level is lower). The sign of household consumption varies by age, but aggregate consumption is higher relative to the baseline and aggregate

Table 5: Prices and aggregate allocations in the year 2040 under alternative monetary policy rule.

Scenario	C	K	H	$\bar{h}$	Y	w	r^a	r^d
Baseline	-0.4	13.9	-5.9	3.1	-0.4	5.78	-1.65	-1.37
Constant R	1.6	10.4	-2.1	7.2	1.5	3.64	-1.05	-0.59

Notes: H is aggregate labor input in efficiency units and $\bar{h}$ is hours per worker. All variables are percentage deviations from steady-state except the two interest rate variables which are differences percent from steady-state.

investment is lower. The first effect is larger and this is why output is higher in 2040 in the constant R equilibrium.[19] Another way to see why output is higher in the constant R equilibrium is to consider the two inputs of production. The capital stock is smaller in this equilibrium, but labor input in efficiency units, H, is much less depressed relative to the baseline. To understand why labor input is less depressed observe that aggregate labor supply, $\bar{h}$, is higher in the constant R equilibrium. Aggregate labor supply in our model is a marginal-utility based weighted average of labor supply by age (see equation 19). A relatively large share of older workers have been negatively impacted by the history of prices they have experienced up to this date. They have low consumption in 2040 and prefer to work harder and their preferences are receiving more weight in the labor supply aggregator.

This is a good point to compare our results with previous results in Bullard et al. (2012) and Katagiri et al. (2020). Even though both papers use OLG frameworks to analyze the effect of demographics on the inflation rate, monetary policy in the settings they consider doesn't affect the inflation rate or real allocations. More importantly, both frameworks have the property that the inflation rate is negatively correlated with the real interest rate. In their models money is a perfect substitute for another interest bearing asset and households only hold both assets if money provides the same real return as the interest bearing asset. Thus, if the real return on the other asset increases, the price level has to fall because money is not an interest bearing asset.

In our model, in contrast, monetary policy transmits to the real economy in two ways. First, our model has nominal price rigidities that are the hallmark of New Keynesian economics. In particular, intermediate goods firms face quadratic price adjustment costs.[20]

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[19]Note that in the figures output is reported as an index relative to its 2015 level whereas in Table 5 it is expressed as a percentage deviation from its terminal steady-state value.

[20]As Rupert and Šustek (2019) document it doesn't automatically follow that an easier monetary policy

Figure 4: Macroeconomic responses to aging in Japan $\phi_\pi = 0$
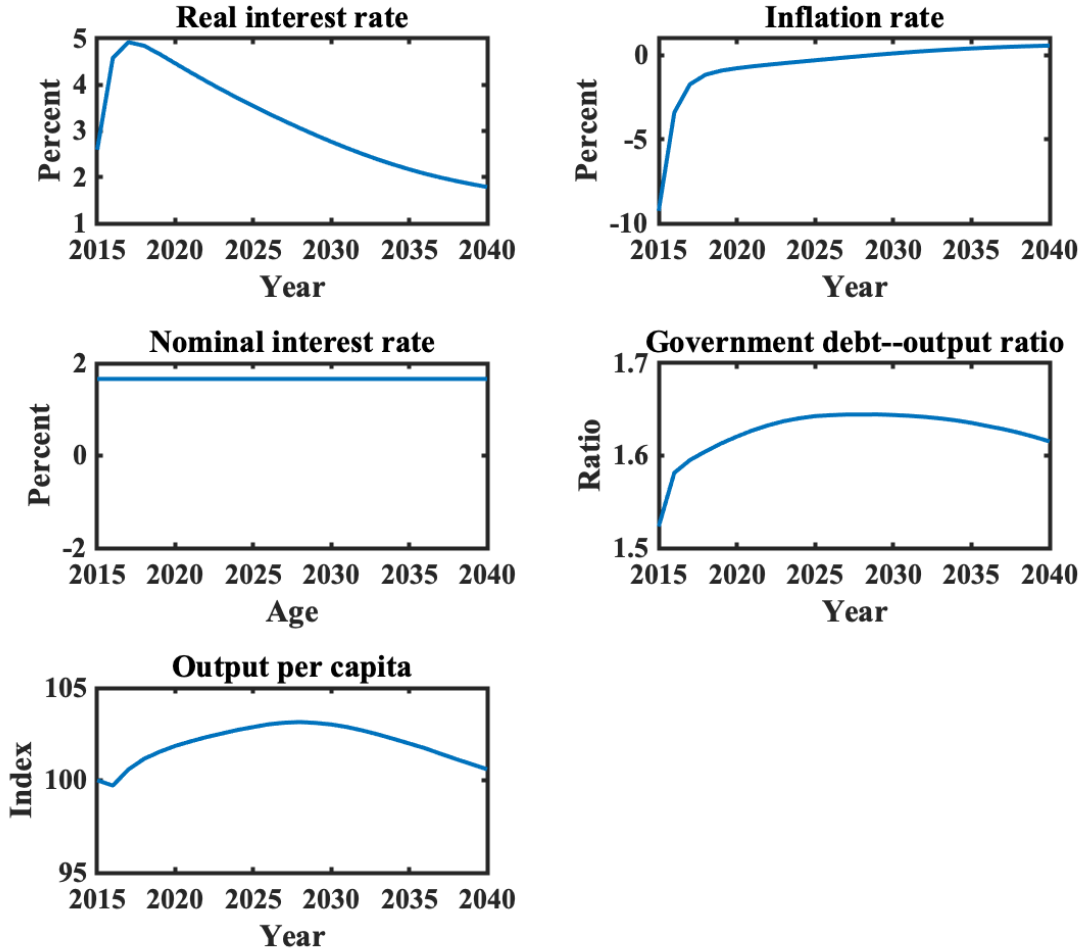
Second, in our OLG framework monetary policy affects the real interest rate on liquid and illiquid assets via the asset substitution channel as we explained in Section 2 using a 2–period OLG model. Braun and Ikeda (2021) find that nominal price rigidities help the model to reproduce empirical SVAR evidence on the signs and magnitudes of a variety of aggregate variables to monetary policy shocks. However, here we are considering a shock to the age distribution and for this shock the asset substitution channel is more important and nominal rigidities are largely irrelevant. This result can be ascertained by comparing the results in Figure 3 with the results in Figure 5 which reports impulse responses to Japan's aging shock under the assumption that prices are flexible. A comparison of the

lowers the real interest rate.

Figure 5: Macroeconomic responses to aging in Japan with flexible prices
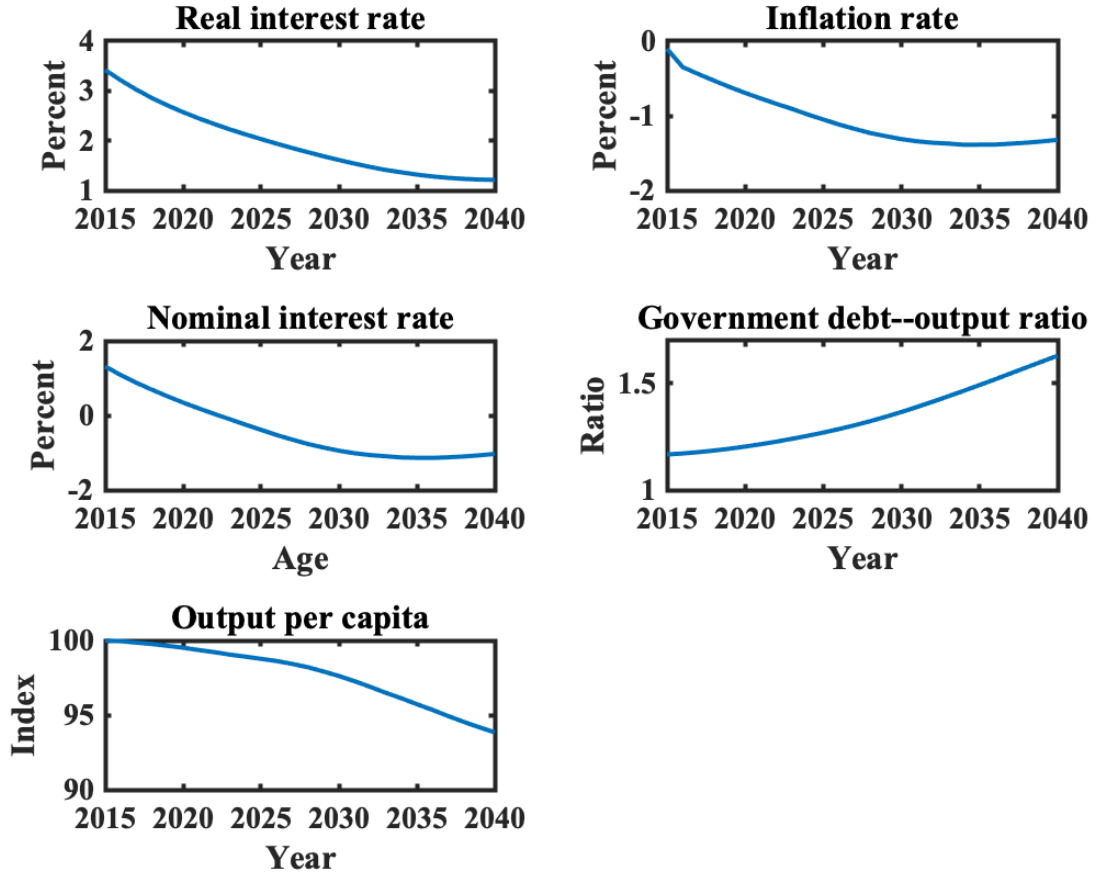
two figures indicates that the responses are virtual indiscernible.[21] Thus, the reason why monetary policy is having such a profound influence on the macroeconomic outcomes is because of the large Tobin effect. In our model the actions taken by the central bank to offset deflationary pressure is pushing the real interest rate on liquid assets down and via the asset substitution effect this is also depressing the real return on illiquid assets.

5.2.2 Higher government debt in an aging society

Up to this point we have held the supply of per capita nominal government debt fixed. This was a deliberate choice because it made it easier to inspect the mechanisms that allow our model to account for the secular stagnation facts. In addition, the baseline specification

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[21]The impact responses are different in the two specifications. But, we have omitted the impact responses to emphasize the secular properties of the two specifications.

also reproduces the fact that aging in Japan has been associated with a large increase in the debt–output ratio. Still, the increase in the debt–output ratio in the baseline specification is due entirely to changes in the real value of nominal debt and part of the increase in Japan's debt–output ratio is due to larger issue of nominal debt. In our model a higher supply of government debt is inflationary. This can be seen by inspection of equation 36 and the results from the 2–period model. Thus, it is a quantitative question whether the deflationary sources induced by aging are large enough to overwhelm the inflationary sources induced by higher debt issue.

Figure 6 considers a scenario where the time-profile of government issue is assumed to gradually increase in tandem with the increase in aggregate private demand for liquid-assets. The debt–output ratio in 2015 is 1.13, which is close to its value in the baseline scenario (1.15). However, the debt–output ratio rises more rapidly over time in the high debt scenario and is 31 percent higher (2.1) than its baseline value of 1.6 in the year 2040.

Higher issuance of government debt attenuates the downward pressure on the price level induced by higher private demand for liquid assets, but the model still predicts that aging is deflationary. The lowest value of the inflation rate in the baseline scenario was -1.4 percent. In this scenario its minimum value is -1 percent and it hits this floor in the year 2032. An inflation rate of -1 percent is not exceptionally low. For instance, the Japanese core CPI (CPI less food and energy) experienced year over year declines in excess of 1 percent in 2001, 2009 and 2021. Thus, our main result that aging induces deflationary pressure also obtains even if we posit large increases in the stock of government debt during the transition.

In the baseline scenario 100 percent of the increase in the real stock of government debt between the years 2015 and 2040 was due to revaluation effects induced by deflation. Here the majority of the increase in the real stock of government over this same time period is due to higher nominal debt issuance and only 41 percent of the increase in is due to revaluation effects.

Accommodating households' increased demand for liquid securities, higher government debt issuance attenuates deflationary pressure and this in turn attenuates the decline in the nominal interest rate engineered by the central bank. The nominal interest rate still falls over time but its minimum value is now only -0.36 percent per annum as compared to -1.2 percent in the baseline scenario. This in turn weakens the asset substitution effect. The minimum value of the real interest rate in liquid assets is 0.64 percent in the year of 2034 in the high debt scenario. In the baseline, in contrast, the real return on liquid assets

Figure 6: Macroeconomic responses to aging in Japan with higher government debt
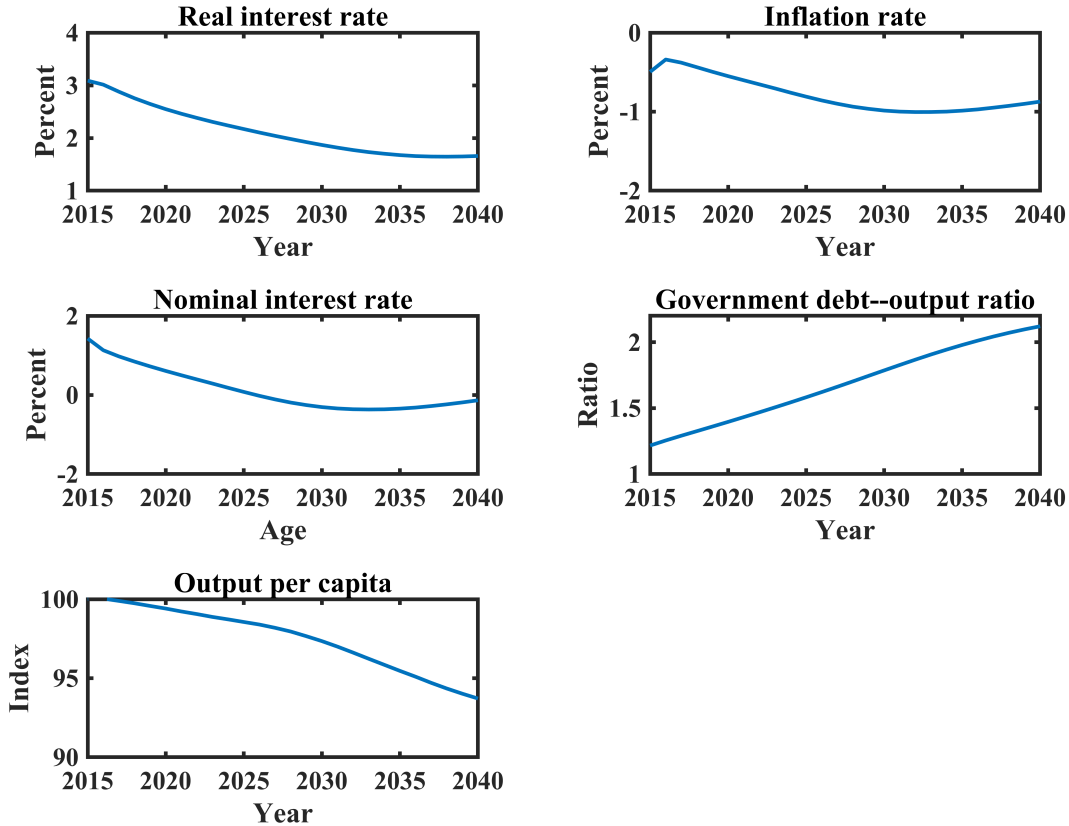
falls to a minimum of 0.23 percent in 2035.

With less downward pressure on the real interest rate on liquid assets, the interest rate on illiquid assets also adjusts to induce households to continue to hold both securities. The real interest rate still declines in this scenario but the magnitude of the peak decline is smaller. It falls to a low 1.65 percent in the year 2039 here as compared to a low of 1.2 percent in the year 2035 in the baseline scenario.

Finally, observe that the output declines are of about the same magnitude in the two scenarios. On the one hand, households have higher returns on their savings in the scenario with higher government debt and aggregate consumption falls less in response to the demographic shock as compared to the baseline. On the other hand, higher government debt issuance crowds out private investment and this offsets most of the consumption gains.

5.2.3 Unpleasant monetary arithmetic

Up to this point the focus of our analysis has been on documenting of the macroeconomy during the first 25 years of the transition to an older age distribution. We have seen that aging induces downward pressure on the inflation rate at this horizon. However, in our economy household's demand for liquid assets is downward sloping and they only hold more real government debt if they are compensated for it with a higher real interest rate. This imposes constraints on both the monetary and fiscal authorities because their joint reactions to changes in private asset demand determine the price level in our economy. These properties of our model are similar in spirit to arguments first made in Sargent and Wallace (1981) and indeed as we show next, our economy has the property that the initial period of deflation that we have documented is followed by a subsequent episode of inflation. Figure 7 displays the entire transition. Perhaps the most noteworthy feature of it is that the inflation rate experiences overshooting. It rises above its steady-state level and then returns to the steady–state from above. The reason for this result is that private asset demand peaks during the transition and after this peak it declines. Table 4 shows that the date of the peak for the partial equilibrium baseline scenario is 2043. The period of peak asset demand for the baseline general equilibrium specification occurs in the year 2074 which is the same year that the inflation rate changes sign in Figure 7.

Goodhart and Pradhan (2017) have argued that aging will produce inflation in future years and Juselius and Takáts (2018) provide empirical evidence that the combination of a lower share of younger population cohorts and a higher share of older cohorts will create inflationary pressure in future years. Our model produces a period of overshooting of the inflation rate during the transition. But, the onset of inflation is very gradual and the peak is not particularly large in the baseline specification. The inflation rate doesn't peak until 2100 and the peak inflation rate is only 0.79 percent. The baseline specification holds government debt fixed. But, the conclusion is the same if we consider the higher public debt scenario instead. In this simulation the peak inflation rate is even smaller, 0.53 %, and the peak occurs in 2097. The reason why overshooting is smaller in the high government debt simulation is because this scenario has less deflation in the first 25 years of the transition.

Overshooting in the inflation rate in Figure 7 is accompanied by overshooting in the nominal interest rate and the real interest rate. As household demand for assets falls, both the price level and the real interest rate have to increase to induce households to hold the aggregate stocks of government debt and illiquid assets. The response of the monetary

Figure 7: Baseline macroeconomic responses to aging in Japan at longer horizons
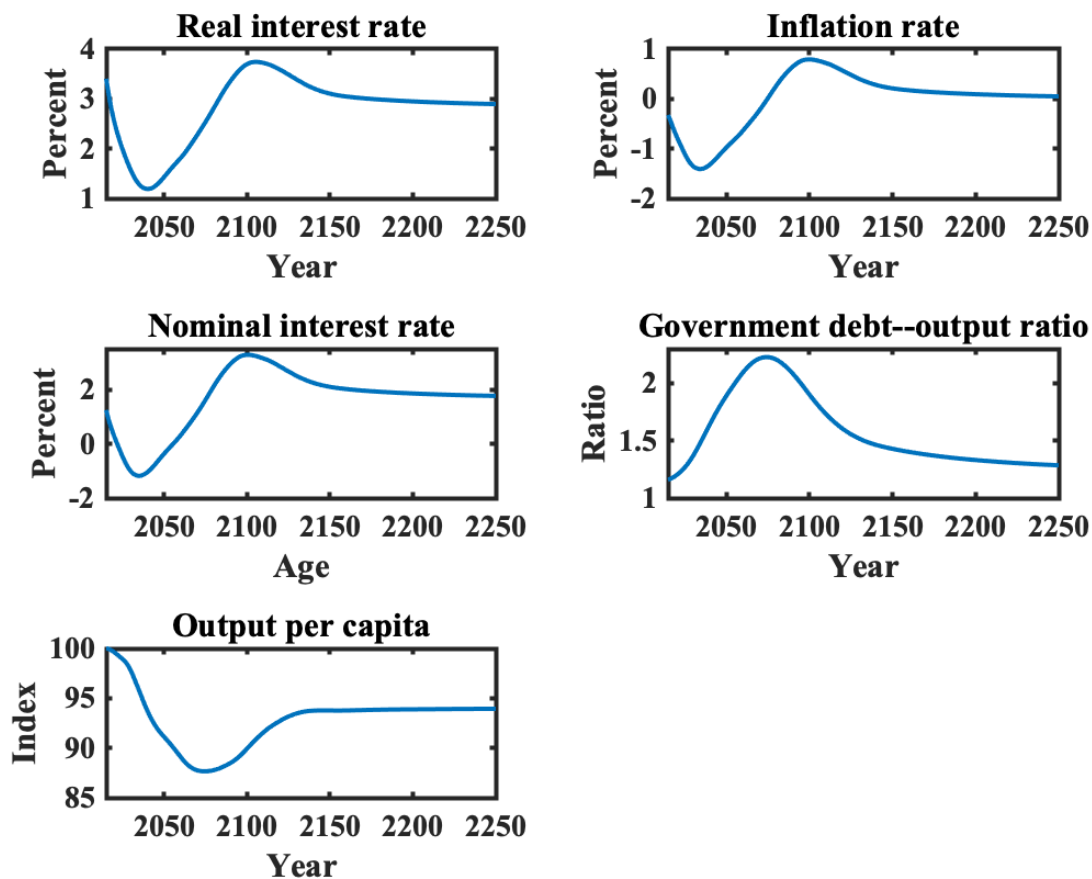
authority magnifies these real interest responses for the reasons we have explained above.

The decline in asset demand has another impact on the fiscal situation of the government. We assume that lump–sum transfers are adjusted each period to clear the government's budget constraint and public transfers decline as private asset demand falls in later stages of the transition and approach their terminal steady-state from below.

6   Robustness

We have also performed simulations to investigate the robustness of our main result that the transition to an aging population distribution produces a period of secular stagnation. For instance, we have considered scenarios where we impose a zero-lower bound on the nominal interest rate. Once this lower bound is hit, the dynamics of the economy resemble those

of the ($\phi_\pi = 0$) scenario reported in Figure 4. In particular, the co-movements between the inflation rate and the real interest rate turn negative. We interpret this result to mean that unconventional monetary policy such as quantitative easing (QE) has been successful in steering the effective nominal interest rate into negative territory.

We have also explored how the starting date of the demographic transition impacts our conclusions by considering a scenario where the demographic transition starts earlier. If we assume that the news about Japan's demographic transition to an older population distribution arrives in 1983 instead (see Figure 8. This scenario abstracts any other shocks. But, we find it interesting that the model initially produces a boom period with rising real interest rates, high inflation and above trend output growth. However, the model subsequently produces a protracted episode of falling real interest rates, deflation and declining output.

7 Conclusion

In this paper we have shown that a demographic transition to an older age distribution along the lines that Japan is facing now induces strong and persistent downward pressure on real interest rates, the inflation rate and output. Both the response of monetary policy and the transmission channel of monetary policy are important for our results. Our results suggest that Tobin effects are more important than nominal rigidities for understanding the transmission channel of monetary policy for this type of shock and that how monetary policy responds to it matters for aggregate economic activity. In our future work we plan to investigate how welfare of different birth cohorts is impacted by an aging population and consider the properties of welfare enhancing monetary and fiscal policies.

Figure 8: Macroeconomic responses if demographic shock arrives in 1983.
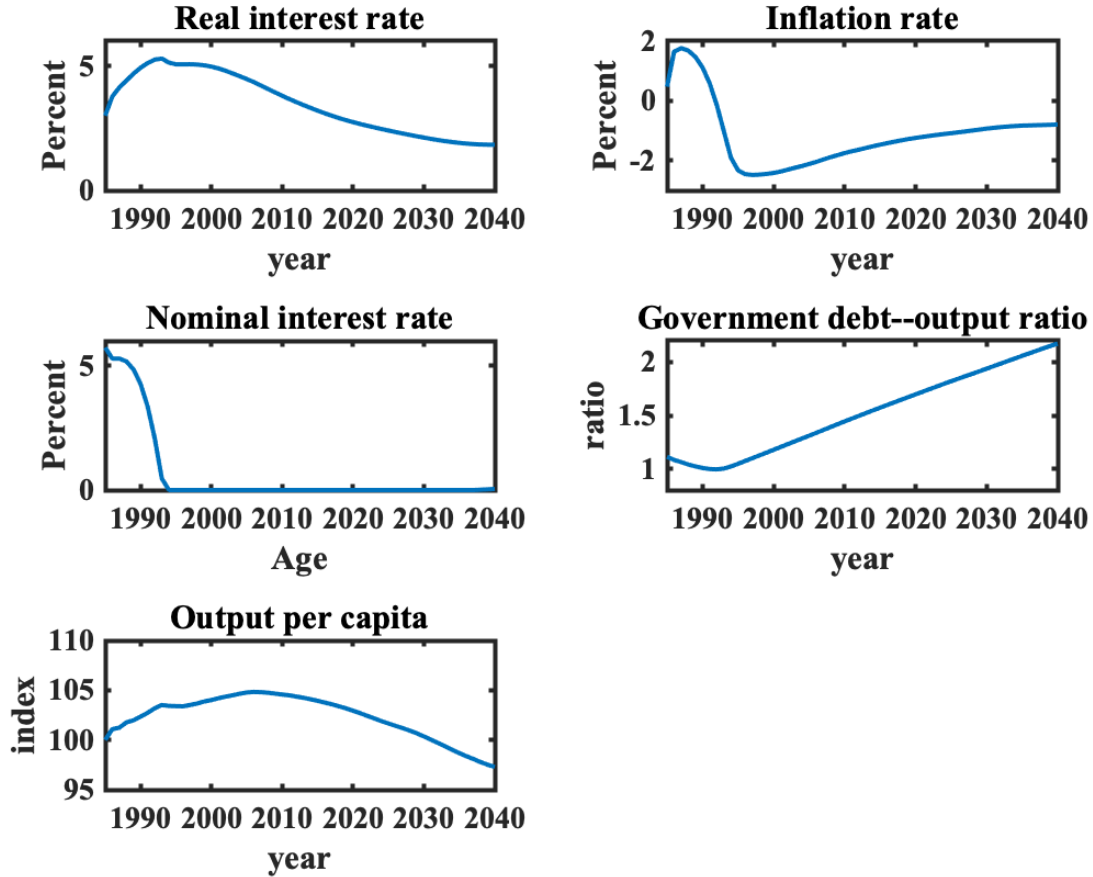
Notes: This simulation imposes the zero-lower bound on the policy nominal interest rate.

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Appendix (For online publication)

A Analytical Model

A.1 Proof of Proposition 1

In the ATPL equilibrium, the set of two endogenous variables $\{k_{t+1}, P_t\}$ is governed by two equations (6) and (11), which are reproduced here for convenience.

$$\alpha k_{t+1}^{\alpha-1} = R_t \frac{P_t}{P_{t+1}} \tag{A.1}$$

$$\frac{d_t^n}{P_t} + n_{t+1} k_{t+1} = (1-\alpha) k_t^{\alpha} \tag{A.2}$$

Once the equilibrium P_0 is determined, equation (A.2) determines k_{t+1} and equation (A.1) determines P_{t+1} for $t = 0, 1, ...,$. In equation (A.1), the nominal interest rate R_t affects the price level P_{t+1}, which in turn affects k_{t+2} as can be seen from equation (A.2). Hence, the setting of the nominal interest rate can affect the real economy. With $\{k_{t+1}, P_t\}$ on hand, ξ_t is given by equation (7), d_t is given by equation (9), and c_t is given by equation (10).

A.2 Proof of Proposition 2

In the ATPL steady state, the gross inflation rate is unity, $\pi = 1$, since the issuance of the per capita nominal government bonds is assumed to be constant overt time. From equation (6), the capital stock is given by $k = (\alpha\pi/R)^{1/(1-\alpha)}$. The price level in steady state is determined by equation (11) as

$$\frac{d^n}{P} = d = (1-\alpha) k^{\alpha} - nk$$

Since k is independent of n, it follows that $\partial P/\partial n > 0$. It is also obvious that $\partial P/\partial d^n > 0$.

A.3 The FTPL version of the model

We start by defining the FTPL equilibrium of our model.

Definition 2 (FTPL equilibrium) *Given the initial capital k_0, the initial nominal obligation $R_{-1} D_{-1}^n$, a sequence of fertility rates $\{n_t\}$, and a sequence of policy variables $\{\xi_t, R_t\}$, a competitive equilibrium for this economy consists of a sequence of prices $\{P_t, R_t^k, w_t\}$, a set of allocations $\{c_t, k_t, d_t\}$ and a sequence of nominal government bonds $\{d_t^n\}$ that satisfy the firms' optimality conditions (4) and (5), the Fisher equation (6), the government budget constraint (7), and the market clearing conditions (9)-(11).*

Solving the government budget constraint (7) forward while using equation (4) yields the standard equation for the FTPL as

$$\frac{R_{-1}d^n_{-1}}{P_0} = (-\xi_0) + \sum_{t=1}^{\infty} \left(\prod_{\ell=1}^{t} \frac{n_{\ell-1}}{R_\ell^k} \right) (-\xi_t)$$

with the transversality condition $\lim_{t\to\infty} \left[\prod_{\ell=1}^{t}(n_\ell/R_\ell^k) \right] d_t = 0$ imposed. This equation holds in both the ATPL and the FTPL, but a difference is that in the FTPL $\{\xi_t\}$ is set exogenously and $\{d^n_t\}$ is determined endogenously to satisfy the government budget constraint, while in the ATPL $\{d^n_t\}$ is set exogenously and $\{\xi_t\}$ is determined endogenously.

First, we show that monetary policy has no real effect in the FTPL equilibrium. Combining equations (7) and (11) in the initial period yields

$$\frac{R_{-1}d^n_{-1}}{P_0} = (-\xi_0) + n_0 \left[(1-\alpha)k_0^\alpha - n_1 k_1 \right] \tag{A.3}$$

From period $t = 1$ onward, this equation can be written by using equations (6) and (11) as

$$\alpha k_t^{\alpha-1} \left[(1-\alpha)k_{t-1}^\alpha - n_t k_t \right] = (-\xi_t) + n_t \left[(1-\alpha)k_t^\alpha - n_{t+1}k_{t+1} \right] \tag{A.4}$$

Once the equilibrium P_0 is determined, k_1 is given by equation (A.3) and k_{t+1} is given by equation (A.4) for $t = 2, 3, ...$ Since the equilibrium P_0 is determined in such a way that k_{t+1} converges to its steady state value, the real economy is independent of the nominal interest rate. With $\{k_{t+1}, P_t\}$ on hand, d^n_t is given by equation (7), d_t is given by equation (9), and c_t is given by equation (10).

Next, we show that a lower fertility rate increases inflation in the FTPL steady state. In the FTPL steady state, combining equations (6), (7) and (6) yields

$$\left(\alpha k^{\alpha-1} - n \right) \left[(1-\alpha)k^\alpha - nk \right] = (-\xi) \tag{A.5}$$

Assume ξ is set such that $k^* < k < k_{\text{gold}}$, where $k^* \equiv [(1-\alpha)\alpha/n]^{1/(1-\alpha)}$ is the level of capital that maximizes the demand for government bonds, and $k_{\text{gold}} \equiv (\alpha/n)^{1/(1-\alpha)}$ is the golden-rule level of capital. Totally differentiating equation (A.5) with respect to k and n yields

$$\frac{\partial k}{\partial n} = \frac{(1-\alpha)k^\alpha - nk + (\alpha k^{\alpha-1} - n)k}{\alpha(\alpha-1)k^{\alpha-2}((1-\alpha)k^\alpha - nk) + (\alpha k^{\alpha-1} - n)((1-\alpha)\alpha k^{\alpha-1} - n)}$$

$$= -\frac{d + (R^k - n)k}{\alpha(1-\alpha)k^{\alpha-2}d + (R^k - n)(n - (1-\alpha)\alpha k^{\alpha-1})}$$

where $d = (1-\alpha)k^{\alpha} - nk$. The numerator is positive since we consider the dynamic efficient economy: $R^k - n > 0$. The denominator is also positive since we limit our attention to $k > k^*$. Since $\partial k / \partial n < 0$ and the marginal return of capital is decreasing in k, it follows that $\partial R^k / \partial n > 0$. Since $R^k = R^n / \pi$ and R^n is constant, set by the central bank, the inflation rate is decreasing in n: $\partial \pi / \partial n < 0$.