Nonparametric Measurement of Long-Run Growth in Consumer Welfare

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  - ... settings in which demand composition varies with other (changing) observables?

Outline

Exact Measurement of Welfare Growth:

- Express welfare (real consumption) as expenditure under constant prices in base period

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Application to Measuring Average US Real Consumption Growth:

- Negative income elasticity of inflation in the US based on CEX/BLS data (new evidence prior to 2000)
- Sizable nonhom. correction over 1955-2019 (e.g., growth with 2019 base: 294% $\rightarrow$ 251%)

Prior Work

Nonhomotheticity Bias: highlights the importance of cov. bet. income elasticity & price change

Baqaee & Burstein (2021); Atkin, Faber, Fally, & Gonzelez-Navarro (2018); Fajgelbaum & Khandelwal (2016) & others

Inflation Inequality: documents cross-sectional variation bet. inflation and income

Argente & Lee (2021); Jaravel (2019); Kaplan & Schulhofer-Wohl (2017); Klick & Stockburger (2021); McGrahan & Paulson (2006); Hobijn & Lagakos (2005) & others

Classical Index Number Theory: approximating welfare change for flexible preferences

Diewert (1993); Pollak (1990); Diewert (1976); Samuelson & Swamy (1974) & many others

Nonparametric Welfare Measurement: general consumer heterogeneity (typically single good)

Hausman & Newey (2017, 2016); Blundell, Horowitz, & Parey (2017, 2012); Lewbell (2001) & others

Roadmap

Exact Measurement of Welfare Growth

Approximating Welfare Growth with Price Index Formulas

Empirics

Conclusion

Setting

Prices $\boldsymbol{p}_t$, expenditure y_t, and shares $\boldsymbol{s}_t$ rationalized by expenditure function $E\left(u; \boldsymbol{p}\right)$

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True cost-of-living index for real consumption c^b between periods t_0 and t (under base b):

$$\mathcal{P}^b_{t_0,t}\left(c^b\right) \equiv \frac{E(u; \boldsymbol{p}_t)}{E(u; \boldsymbol{p}_{t_0})} \qquad \text{such that } \; c^b = E(u; \boldsymbol{p}_b)$$

Nonhomotheticity Correction

Proposition: real consumption growth as correction to deflated nominal expenditure growth

$$\frac{d \ln c_t^b}{dt} = \frac{1}{1 + \Lambda_t^b \left(c_t^b\right)} \left(\frac{d \ln y_t}{dt} - \sum_i s_{it} \frac{d \ln p_{it}}{dt} \right)$$

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- Correction: elasticity of true index (from base b to t) w.r.t real consumption c^b

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Under nonhomothetic preferences (for $b < t$):

$\Lambda_t^b\left(c^b\right) < 0 \Rightarrow$ base-to-current cumulative inflation decreasing in real consumption

$\Rightarrow$ higher real consumption growth

Real Consumption Growth and the Choice of Base

Corollary: instantaneous real consumption growth for different base periods:

$$\frac{d \ln c_t^{b_2}}{d \ln c_t^{b_1}} = 1 + \frac{\partial \ln \mathcal{P}_{b_1,b_2}^{b_1}\left(c_t^{b_1}\right)}{\partial \ln c_t^{b_1}}$$

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- Intuition: food and mobile phones

Extension to Covariates | Illustrative Example

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Index Formulas

Observe sequences of prices, expenditures, & shares $(\boldsymbol{p}_t, y_t, \boldsymbol{s}_t)_{t=0}^{T}$ for $t \in \{0, 1, \cdots, T\}$

$$\max_{1 \leq i \leq I,\, 1 \leq t \leq T} |\Delta \ln p_{i,t}| \leq \Delta_p \qquad \max_{1 \leq t \leq T} |\Delta \ln y_t| \leq \Delta_y \qquad \Delta \equiv \Delta_p + \Delta_y < 1$$

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Price index formula: positive-valued function of $(\boldsymbol{p}_t, \boldsymbol{s}_t)$ and $(\boldsymbol{p}_{t+1}, \boldsymbol{s}_{t+1})$

$$\text{geometric}: \quad \ln \pi_{G,t} \equiv \sum_i s_{i,t} \Delta \ln p_{i,t} \qquad \text{Törnqvist}: \quad \ln \pi_{T,t} \equiv \sum_i \tfrac{1}{2}\left(s_{i,t} + s_{i,t+1}\right) \Delta \ln p_{i,t}$$

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Under homotheticity, index formulas approximate true index for any real consumption c

Diewert (1976)

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Key Assumption: data from collection households $n \in \{1, \cdots, N\}$ with identical preferences

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- Iterate the above steps for $t > b$, using $\ln \widehat{\mathcal{P}}_{b,t}(c) = \sum_{\tau=b}^{t-1} \ln \widehat{\mathcal{P}}_{\tau,\tau+1}(c)$

Full Iterative Algorithm

Accuracy of the Approximation

- Assumption: For all t, pdf of real consumption c bounded away from 0 over an interval $[\underline{c}, \overline{c}]$

Bounds on the Approximation Error of the First-Order Algorithm

If $\widetilde{E}^b(\cdot;\cdot)$ continuously differentiable of order $m \geq 5$, then for any b, as $N,\ K_N \to \infty$:

$$\Delta \ln c_t^n = \Delta \ln \widehat{c}_t^n \; + \; O\left(\Delta_p^2\right) \; + \; O_p\left(K_N^3\left(K_N^{1-m} + \sqrt{\frac{K_N}{N}} \cdot \Delta_p^4\right)\Delta\right)$$

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Sources of error:

1. Original Taylor-series approximation error $\underset{\Delta_p \to 0}{\longrightarrow} 0$
2. Approximation error $\mathcal{P}_{b,t}(\cdot)$ based on cross-sectional nonparametric estimation $\widehat{\mathcal{P}}_{b,t}(\cdot)$
   (i) Finite basis function error $\underset{K_N \to \infty}{\longrightarrow} 0$
   (ii) Finite sample error $\underset{K_N^7/N \to 0}{\longrightarrow} 0$

Illustrative Example

Extensions

Second-order Algorithm:

- Relies on the Törnqvist index instead of the geometric index
- Requires iterative procedure within each period
- Offers tighter error bounds

Error Bounds

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Algorithms for cases with covariates:

- First-order and Second-order algorithms for the cases involving (changing) consumer covariates
- Provide error bounds in each case

Roadmap

Empirics

Goals and Contributions:

- Evidence on the inequality inflation experienced across households (new evidence prior to 2000)
- Evaluate importance of nonhomotheticity correction in measuring real consumption growth in US
- Blueprint for including inflation inequality in Distributional National Accounts

  Piketty, Saez, & Zuncman (2018)

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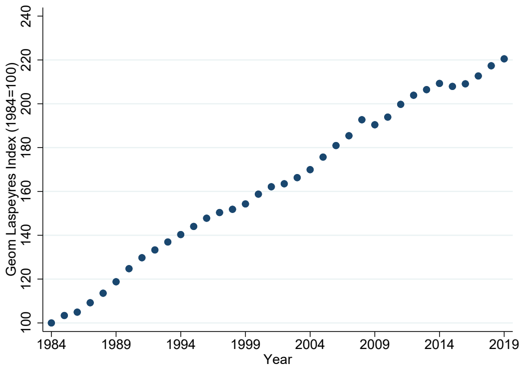
Linked Dataset:

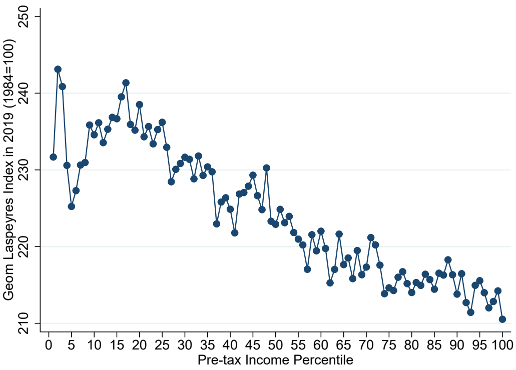
- Construct CEX-CPI crosswalk for $\approx$ 600 product categories (1984-2019)
- CEX household-level data aggregated by percentile of pre-tax income reweighted consistent with:
  - CEX official expenditure summary tables by income quintiles (1984-2019)
  - BEA data on aggregate consumption expenditure growth (1955-2019)

Inflation as Measured by Price Indices (1984-2019)

Chained Geometric w. Agg. Expenditure Shares



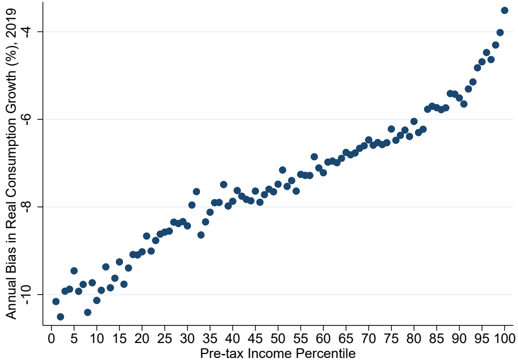
Chained Geometric by Income Group (Pecentile)



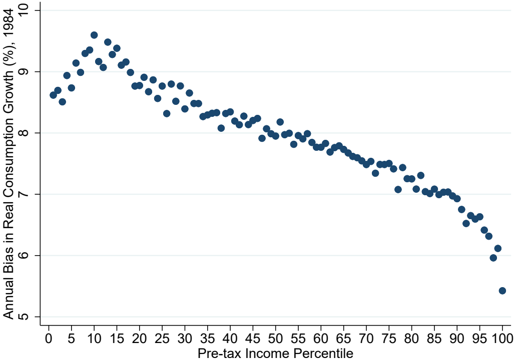
Pre vs. Post 1995

Annual Bias in Uncorrected Measures of Real Consumption Growth

Bias in 2019 (1984 as Base Prices)



Bias in 1984 (2019 as Base Prices)



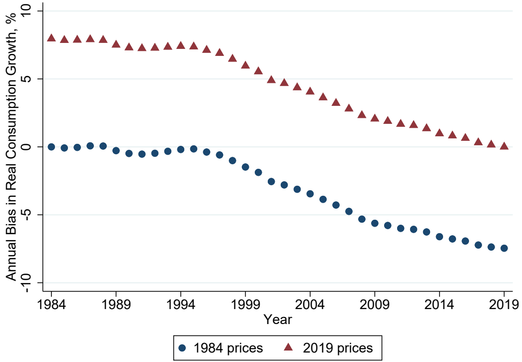
Annual bias in uncorrected measure of real consumption growth $g_t^n \equiv \Delta \ln y_t^n - \ln \pi_t^n$

$$g_t^n - \Delta \ln c_t^n \approx \left(\frac{\Lambda_{t+1}(c_t^n)}{1+\Lambda_{t+1}(c_t^n)} \right) \times g_t^n$$

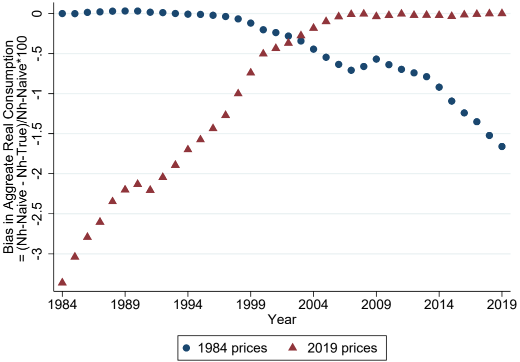
Further Details

Bias in the Aggreate

Annual Bias in Avg. Real Consumption Growth



Bias in the Level of Avg. Real Consumption



Long-Run Growth: Extending to 1955-2019

Annual CEX not available prior to 1984

Correction using 1984 shares in prior years:

- BLS inflation data (matched to CEX in 1984)
- BEA avg. nominal consumption expenditure
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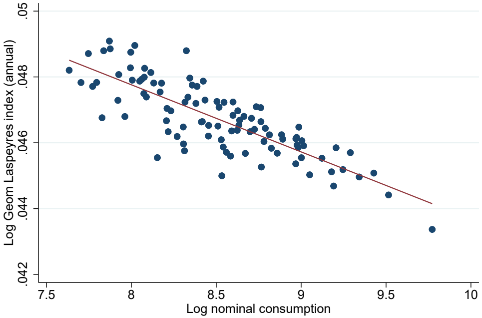
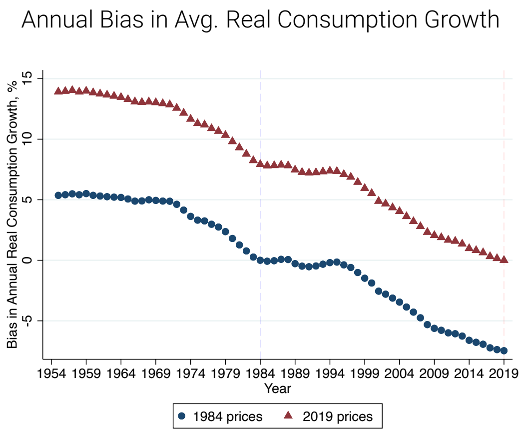
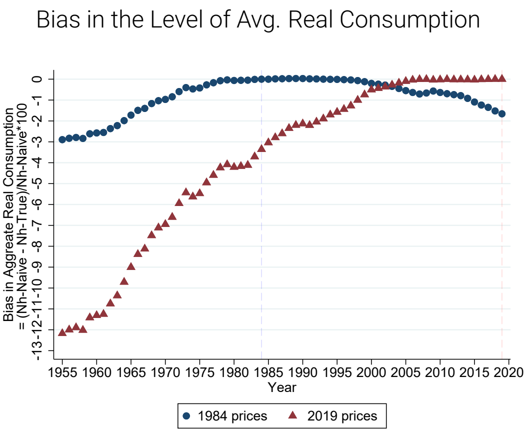
Inequality in Inflation Prior to 1984 (1955-1984)



The specification includes year fixed effects. The sample run from 1955 to 1983.
Each bin represents 1% of households.

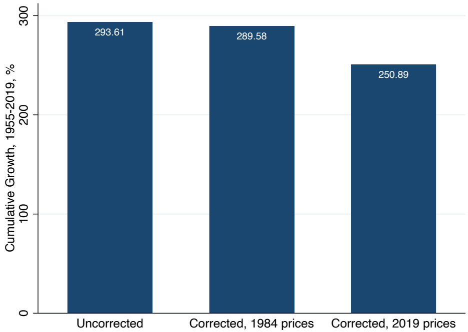
Long Run: Bias in Growth and Level of Real Consumption



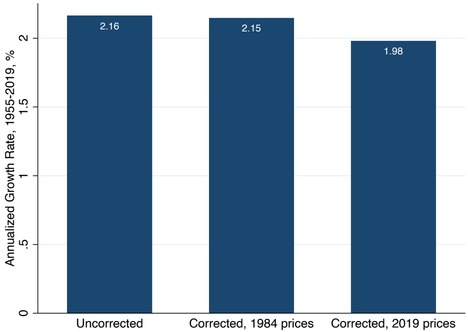


Long Run: Corrections to Cumulative Real Consumption Growth

Cumulative Growth (1955-2019)



Avg. Annualized Growth (1955-2019)



Robustness

Results robust to data construction and aggregation choices:

- Official CPI category-level expenditure weights used by BLS (19 categories, 1984-2019) Results
- Nielsen data (consumer packaged goods) (9131 products, 2004-2019)

Roadmap

Exact Measurement of Welfare Growth

Approximating Welfare Growth with Price Index Formulas

Empirics

Conclusion

Conclusion

Measuring real consumption growth:

- Provided theoretically-consistent correction for deflated nominal expenditure growth
- Characterized the precise dependence of real consumption growth on base year price

Contributions:

- Nonparametric approach to estimate income elasticity of inflation using cross-sectional data
- Analysis of the nonhomotheticity bias in measures of US power-war real consumption growth

Roadmap

Appendix

Extension to Covariates

Preferences depend on vector of covariates $\boldsymbol{x}$ with expenditure function $E\left(u; \boldsymbol{p}, \boldsymbol{x}\right)$

Extension to Covariates

Preferences depend on vector of covariates $\boldsymbol{x}$ with expenditure function $E(u; \boldsymbol{p}, \boldsymbol{x})$

Real Consumption: generalize to define $c_t^b \equiv E(u_t; \boldsymbol{p}_b, \boldsymbol{x}_t)$ and the growth follows:

$$\frac{d \ln c_t}{dt} = \frac{1}{1 + \Lambda_t(c_t; \boldsymbol{x}_t)} \left[\frac{d \ln y_t}{dt} - \sum_i s_{it} \frac{d \ln p_{it}}{dt} - \underbrace{\sum_k \Gamma_{kt}(c_t; \boldsymbol{x}_t) \frac{d \ln x_{kt}}{dt}}_{\text{Correction for covariate change}} \right]$$

- Nonhomotheticity correction and covariate-k correction:

$$\Lambda_t(c; \boldsymbol{x}) = \frac{\partial \ln \mathcal{P}_{b,t}(c; \boldsymbol{x})}{\partial \ln c}$$

$$\Gamma_{kt}(c; \boldsymbol{x}) = \frac{\partial \ln \mathcal{P}_{b,t}(c; \boldsymbol{x})}{\partial \ln x_k}$$

with generalized cost-of-living index defined as $\mathcal{P}_{t_0,t}(c; \boldsymbol{x}) \equiv \widetilde{E}(c; \boldsymbol{p}_t, \boldsymbol{x}) / \widetilde{E}(c; \boldsymbol{p}_{t_0}, \boldsymbol{x})$

back

Illustrative Example

Preferences: nhCES preferences calibrated with sectoral data

Comin, Lashkari, & Mestieri (2021)

$$E\left(u;\boldsymbol{p}_t\right) \equiv \left(\sum_{i\in\{a,m,s\}} \omega_i \left(u^{\varepsilon_i} p_{i,t}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \qquad (\sigma,\varepsilon_a,\varepsilon_m,\varepsilon_s) = (0.26, 0.2, 1, 1.65)$$

- Choose sectoral shifters ω_i to fit initial sectoral composition of US expenditure in 1953
- Compare against homothetic CES with $(\sigma,\varepsilon_a,\varepsilon_m,\varepsilon_s) = (0.26, 1, 1, 1)$

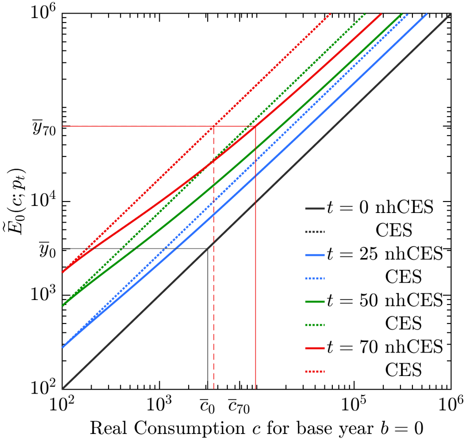
Inflation Patterns: consider positive and negative inflation/income-elasticity covariances

- Inflation in manufacturing = 3.2% (average US 1953-2019)
- Inflation in services/agriculture = $\pm 1\%$ relative to manufacturing

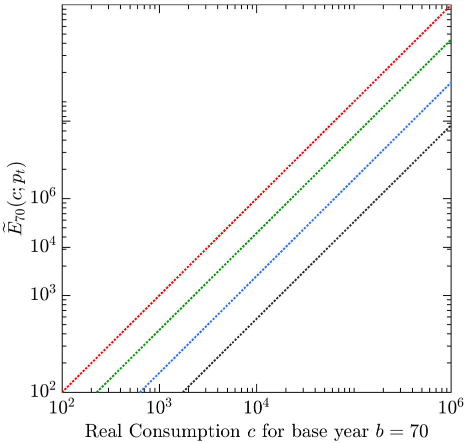
Income Heterogeneity: 1000 HHs with log-normal nom. expenditure (SE = 0.5) growing at 4.5%

The Evolution of the Expenditure Function

Initial Year as Base (Negative Covariance)



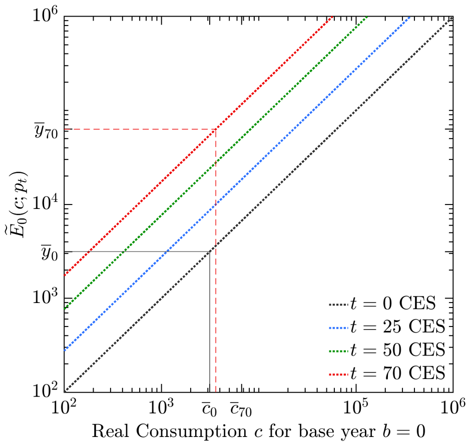
Last Year as Base (Negative Covariance)



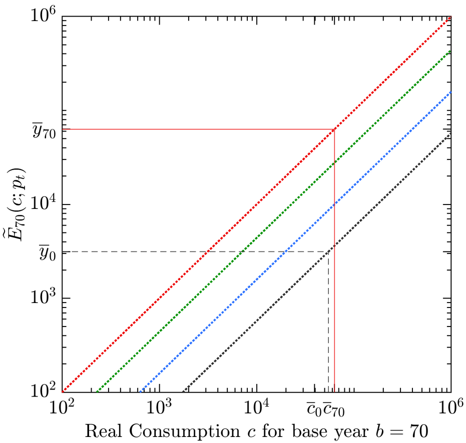
Positive Covariance

The Evolution of the Expenditure Function

Initial Year as Base (Negative Covariance)



Last Year as Base (Negative Covariance)



Positive Covariance

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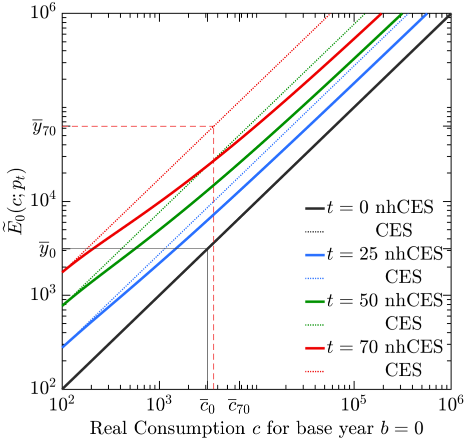
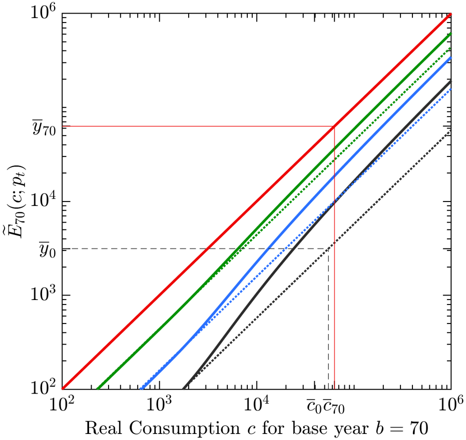


Last Year as Base (Negative Covariance)



Positive Covariance

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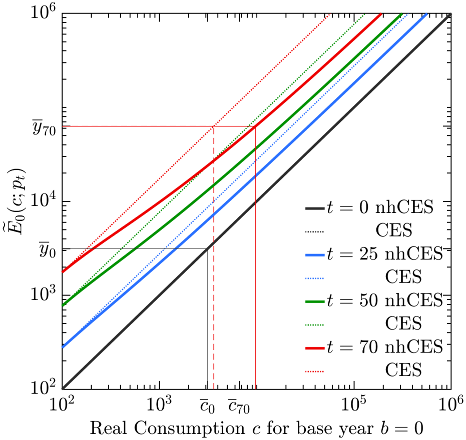

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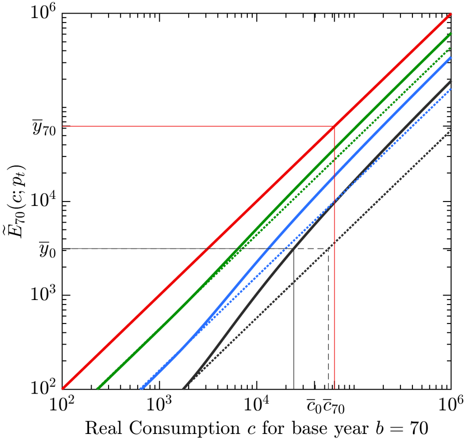

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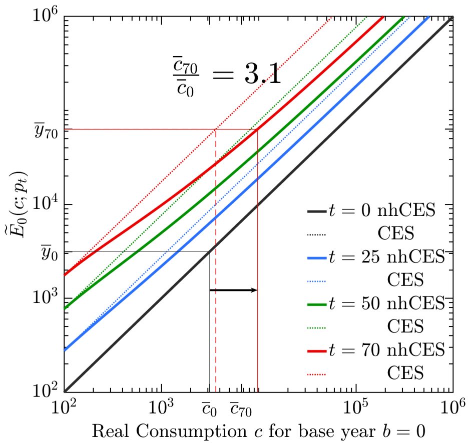

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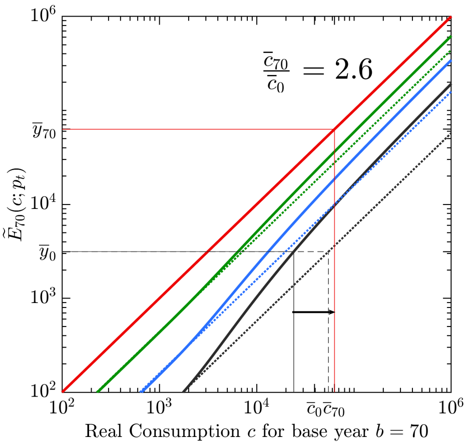
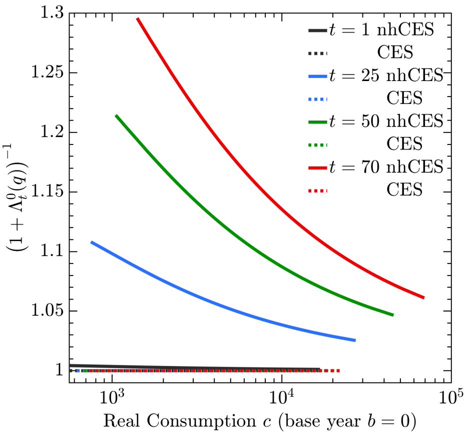


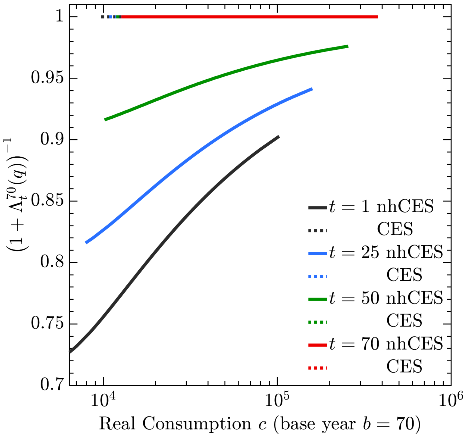
Positive Covariance

Nonhomotheticity Correction

Initial Year as Base (Negative Covariance)



Last Year as Base (Negative Covariance)



Positive Covariance Back

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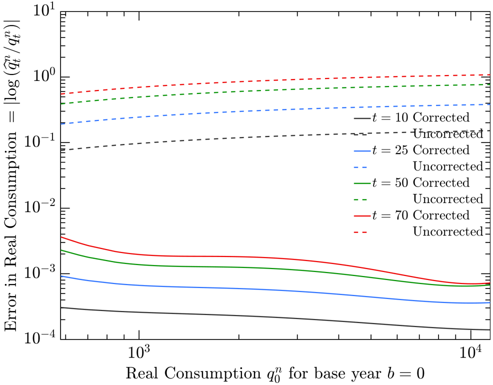
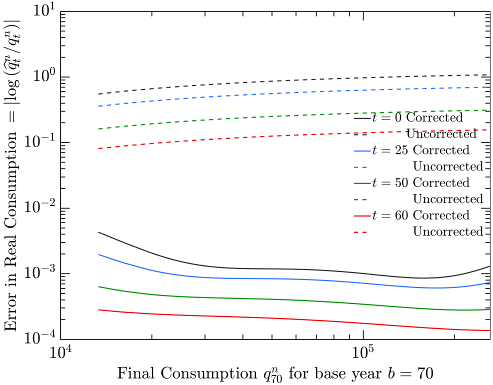
Illustrative Example: Approximation Errors

Initial Year as Base (Negative Covariance)



Last Year as Base (Negative Covariance)

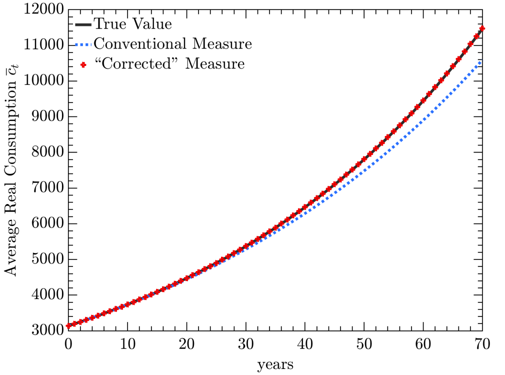


Positive Covariance

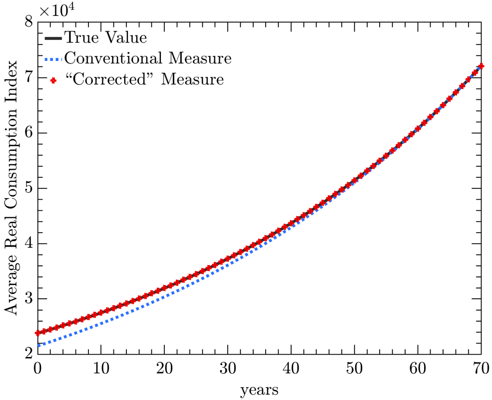
2nd Order Algorithm

Illustrative Example: Average Real Consumption

Initial Year as Base (Negative Covariance)

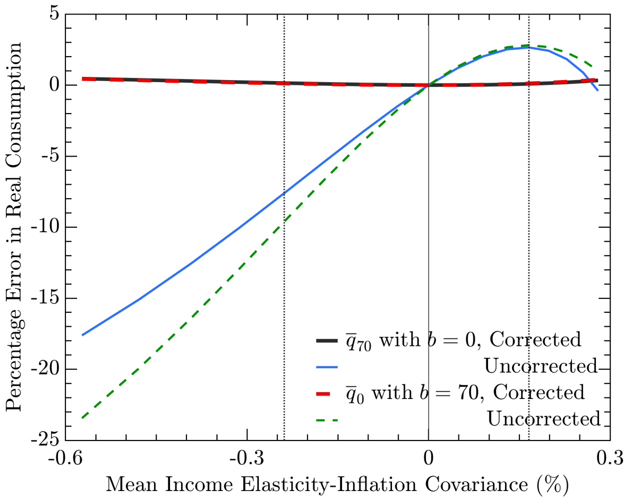


Last Year as Base (Negative Covariance)



Positive Covariance

Illustrative Example: Error vs. Covariance



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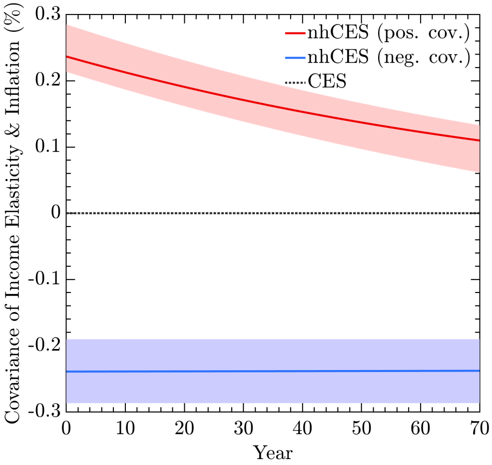
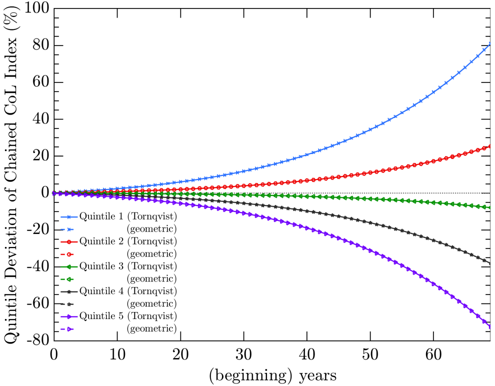
Patterns of Inflation

Evolution of the Covariance



Chained Cumulative Price Indices by Quintile



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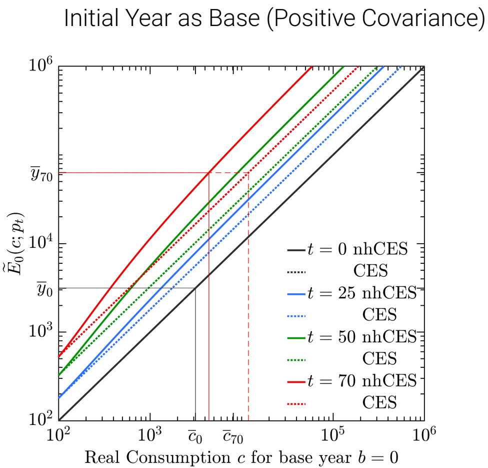
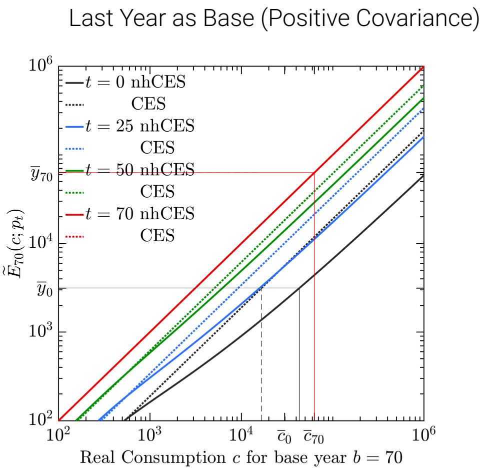
The Evolution of the Expenditure Function





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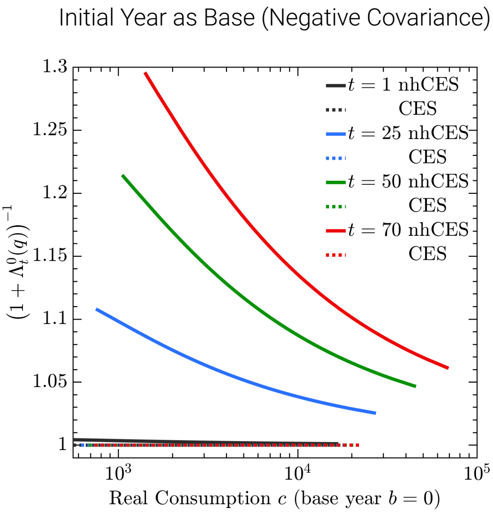
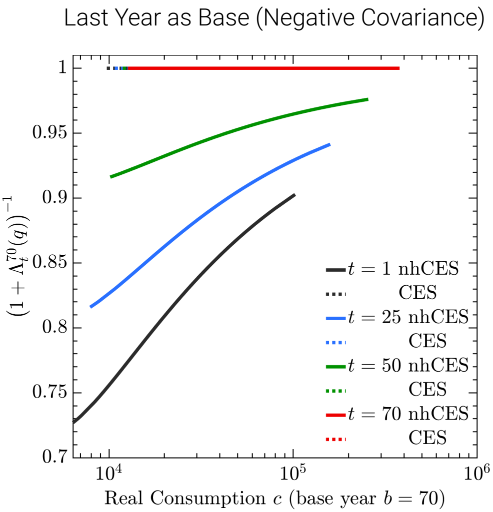
Nonhomotheticity Correction





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First-order Algorithm

Series of log-power functions $\left(f_k\left(x\right) \equiv \left(\ln x\right)^k\right)_{k=0}^{K_N}$ for data from N consumers:

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   (i) Next-period $\widehat{\mathcal{P}}_{b,t+1}(;\cdot)$: run OLS of $\left\{\ln \pi_{G,t}^n\right\}_n$ on $\{(f_k(\widehat{c}_t^n))_k\}_n \Rightarrow$ coefficients $(\widehat{\alpha}_{k,t})_{k=0}^{K_N}$

$$\ln \widehat{\mathcal{P}}_{b,t+1}(c) \equiv \ln \widehat{\mathcal{P}}_{b,t}(c) + \sum_{k=0}^{K_N} \widehat{\alpha}_{k,t} f_k(c)$$

First-order Algorithm

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$$\ln \widehat{\mathcal{P}}_{b,t+1}(c) \equiv \ln \widehat{\mathcal{P}}_{b,t}(c) + \sum_{k=0}^{K_N} \widehat{\alpha}_{k,t} f_k(c)$$

   (ii) Next-period $\widehat{c}_{t+1}^n$ using $\widehat{\Lambda}_{t+1}(c) \equiv \frac{\partial \widehat{\mathcal{P}}_{b,t+1}(c)}{\partial \ln c}$

$$\ln \widehat{c}_{t+1}^n = \ln \widehat{c}_t^n + \frac{1}{1 + \widehat{\Lambda}_{t+1}(\widehat{c}_t^n)} \left(\Delta \ln y_t^n - \ln \pi_{G,t}^n\right)$$

Back

Second-order Algorithm Accuracy

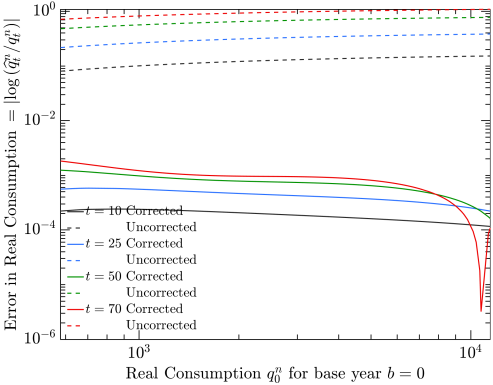
Approximation of the Second-Order Algorithm

$$\ln \mathcal{Q}_{RC}^{n}\left(t, t+1; \boldsymbol{p}_b\right) = \ln\left(\frac{\widehat{q}_{t+1}^{n}}{\widehat{q}_{t}^{n}}\right)$$
$$+ O\left(\Delta^{3} + \epsilon\right) + O_p\left(K_N^3\left(\sqrt{\frac{K_N}{N}}\left(\Delta^3 + K_N^{4-m}\right)^2 + K_N^{1-m}\right)\Delta\right)$$

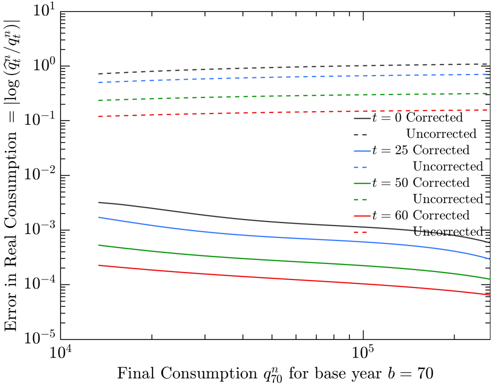
Back

Illustrative Example: Approximation Errors

Initial Year as Base (Positive Covariance)



Last Year as Base (Positive Covariance)



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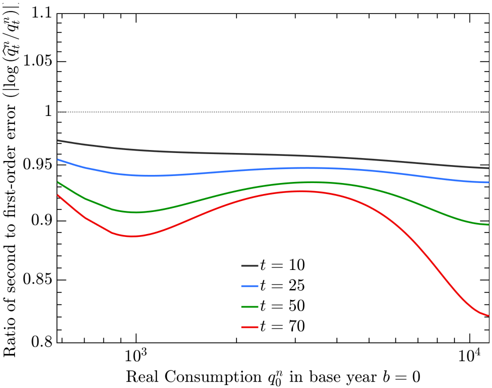
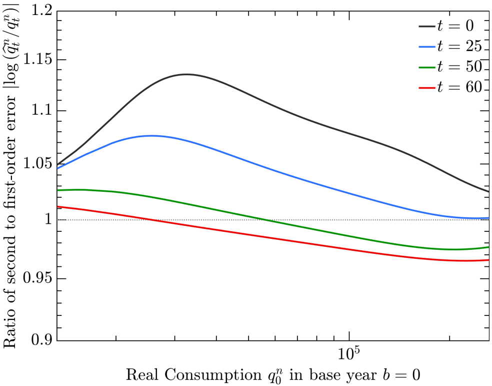
Illustrative Example: 1st vs. 2nd Order Approximation Errors

Initial Year as Base (Negative Covariance)



Last Year as Base (Negative Covariance)



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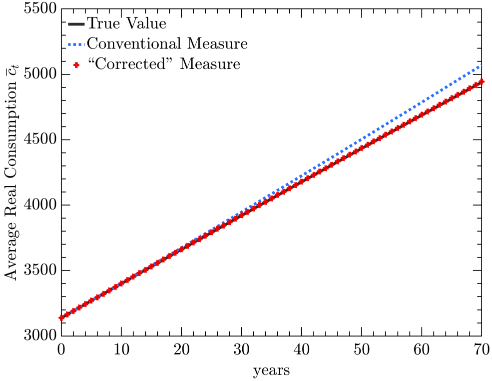
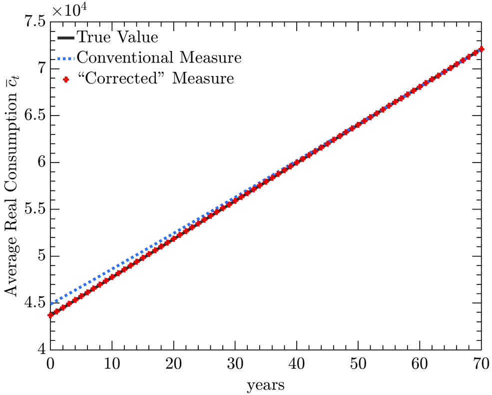
Illustrative Example: Average Real Consumption

Initial Year as Base (Positive Covariance)



Last Year as Base (Positive Covariance)



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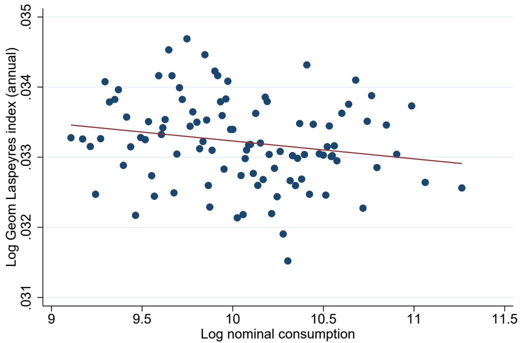
Inequality in Inflation Measured by Price Indices

Weaker Negative Income-Inflation Relation

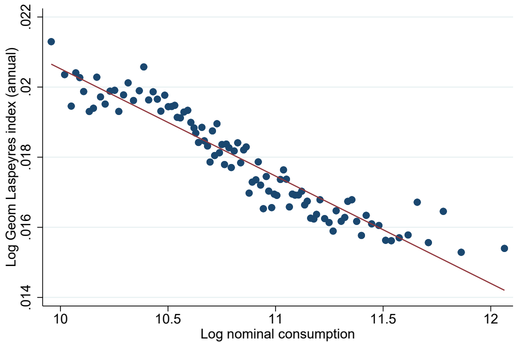
1984-1995



The specification includes year fixed effects. The sample run from 1984 to 1995.
Each bin represents 1% of households.

Stronger Negative Income-Inflation Relation

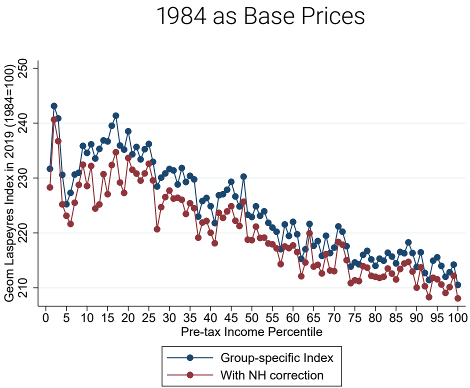
Post-1995

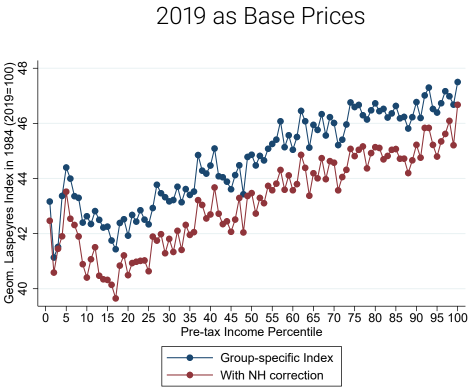


The specification includes year fixed effects. The sample run from 1995 to 2019.
Each bin represents 1% of households.

Back

Nonhomotheticty Correction and the Consumption Deflator



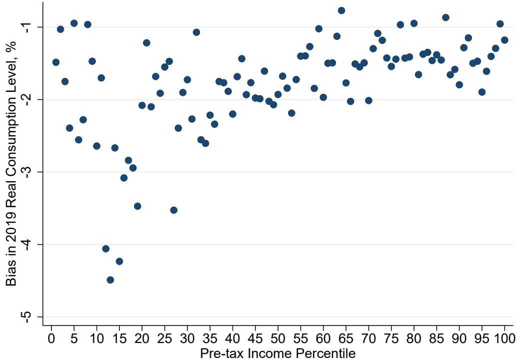


Chained index formula ($\prod_\tau \pi^n_\tau$) vs. the corrected deflator ($\frac{y^n_t}{c^n_t}$)

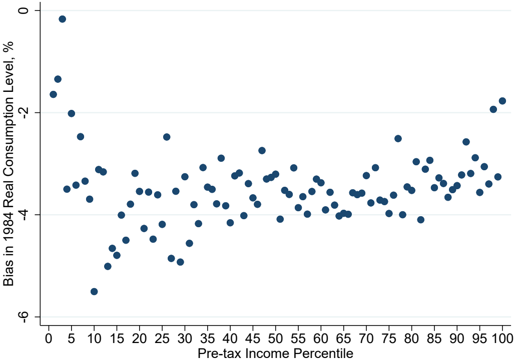
Back | Level of Real Consumption | Cumulative Growth | Comparison w. Group-Specific Indices

Bias in the Level of Real Consumption

Bias in 2019 (1984 as Base Prices)



Bias in 1984 (2019 as Base Prices)



Bias in uncorrected measure of real consumption $y_t^n / \prod_{\tau=0}^{t} \pi_\tau^n$ relative to c_t^n

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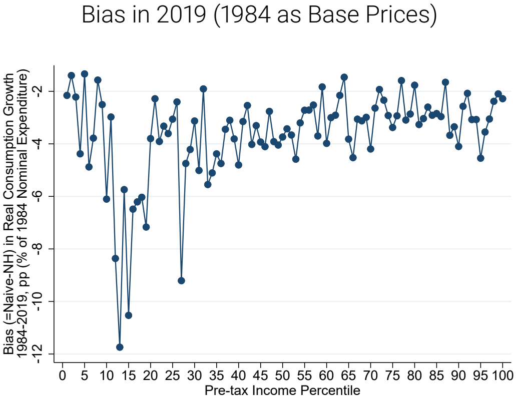
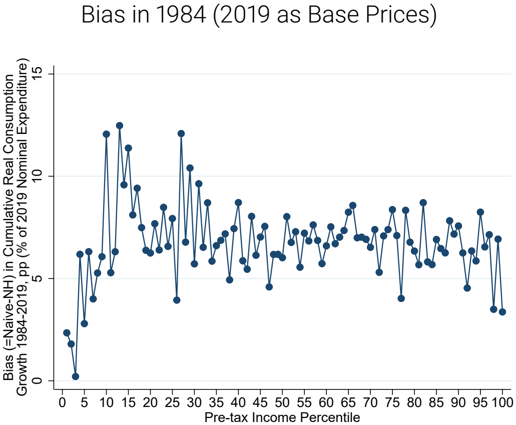
Bias in Cumulative Real Consumption Growth



This graph shows that economic growth, expressed in 1984 prices,
is underestimated over time absent the NH correction.



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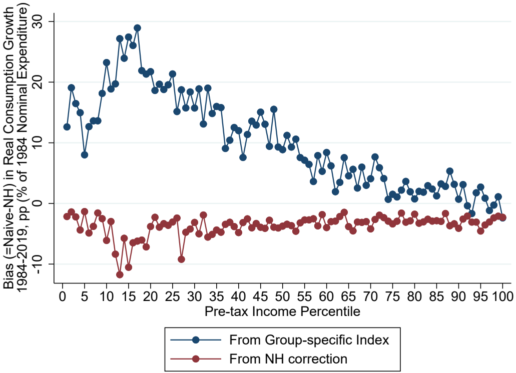
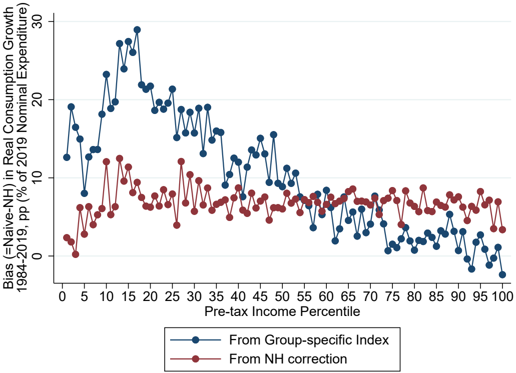
Nonhomotheticity Bias vs. Group-Specific Index Bias

Bias in 2019 (1984 as Base Prices)



Bias in 1984 (2019 as Base Prices)



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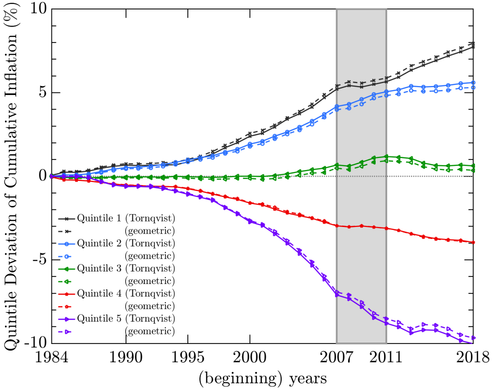
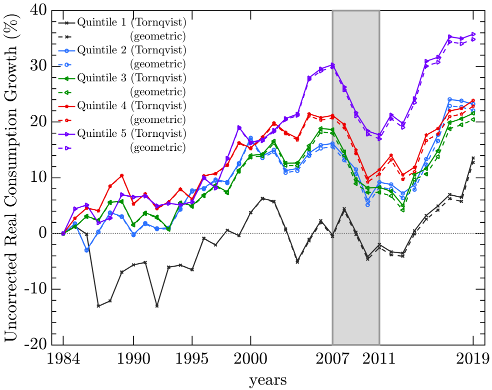
Official BLS Data: Uncorrected Price Indices

Quantile-Level Data: lower inflation for higher income quantiles

Quantile Deviation of Price Indices



Quantile Uncorrected Real Consumption



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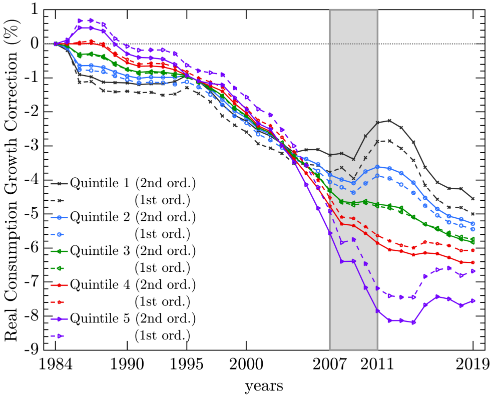
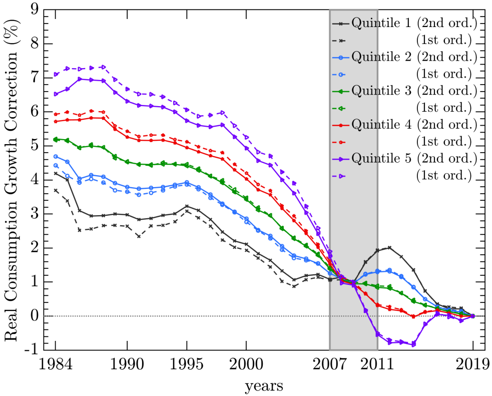
Official BLS Data: Bias without Correction

Quantile-Level ($K = 2$): annual λ_t^n error as share of common measure of growth

Initial Period as Base ($b = 1984$)



Final Period as Base ($b = 2019$)



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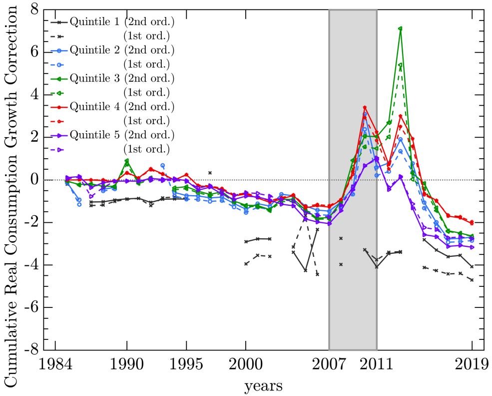
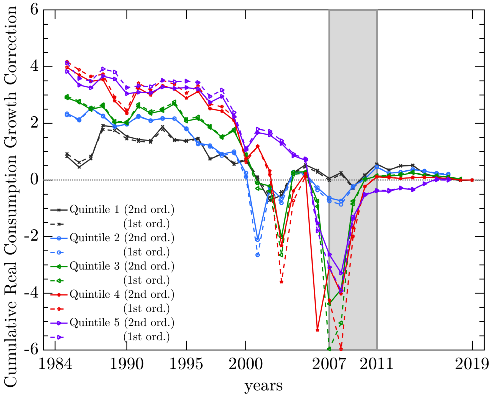
Official BLS Data: Bias without Correction

Quantile-Level ($K = 2$): cumulative $\lambda^n_{C,t}$ error as share of common measure of growth

Initial Period as Base ($b = 1984$)



Final Period as Base ($b = 2019$)



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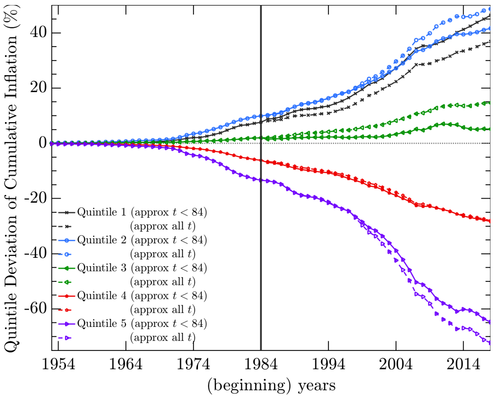
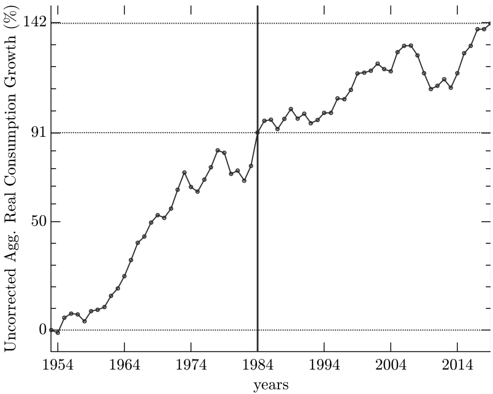
Official BLS Data: Long-Run Growth without Correction

Quintile Deviation of Price Indices



Avg. Uncorr. Real Consumption Growth



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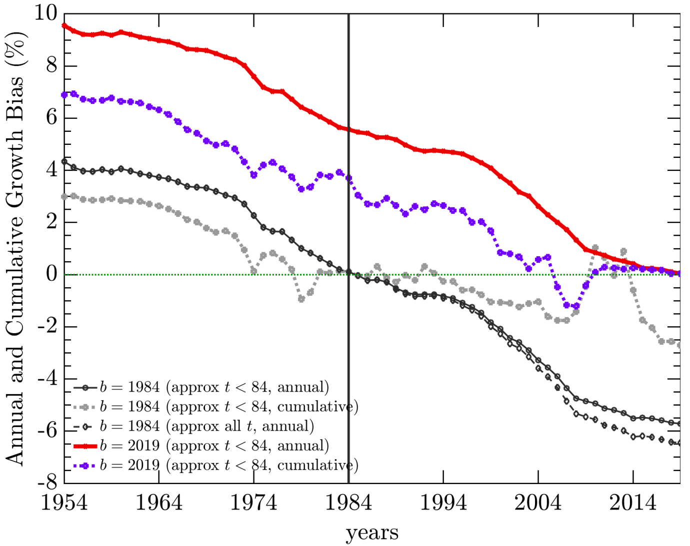
Official BLS Data: Bias in Long-Run Growth



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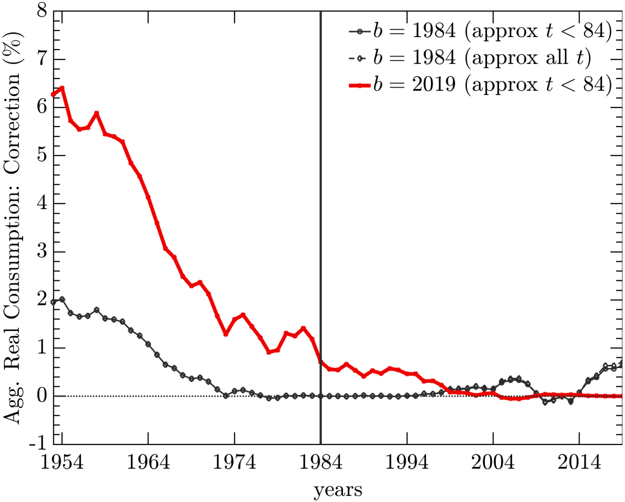
Long-Run Growth: Bias in Levels in the Long Run



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