

# Predictable Forecast Errors in Full-Information Rational Expectations Models with Regime Shifts

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# Introduction

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- Many studies based on survey expectations of macro aggregates find that forecast errors are systematically predictable.
- Typically interpreted as evidence to reject the FIRE hypothesis
- Large literature introduces information frictions and/or departures from rational expectations to explain observed predictability of forecast errors.

# This paper

- We study forecast error in FIRE models with regime shifts
  - ▶ structural changes in the economic environment
  - ▶ changes in the stance of monetary or fiscal policy

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- We study forecast errors in FIRE models with regime shifts
- **Result 1:** Regime shifts imply ex-post predictable, regime-dependent forecast errors
  - ▶ Similar to “peso problem”, but analyzed in modern macroeconomic context
  - ▶ Implies **waves of over- and under-reaction** across rolling window samples, depending on sequence of ex-post realized regimes relative to agents’ ex-ante expectations
  - ▶ We confirm this implication with data from Survey of Professional Forecasters (SPF)

# This paper

- We study forecast errors in FIRE models with regime shifts
- **Result 1:** Regime shifts imply ex-post predictable, regime-dependent forecast errors
- **Result 2:** Regime-robust test of FIRE
  - ▶ Assume FIRE model with regime shifts as data-generating process = the null
  - ▶ Simulate distribution of predictability regression coefficients, incorporating uncertainty about sequence of realized regimes
  - ▶ Compare to predictability coefficients estimated in data = t-test of the null



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- **Result 1:** Regime shifts imply ex-post predictable, regime-dependent forecast errors
- **Result 2:** Regime-robust test of FIRE
- **Result 3:** Medium-scale DSGE-RS model implies that null of FIRE cannot be rejected
  - ▶ CEE / SW / JPT type model with monetary policy regime shifts as in Bianchi (2013)
  - ▶ Estimated on post-WW2 U.S. data

# This paper

- We study forecast errors in FIRE models with regime shifts
- **Result 1:** Regime shifts imply ex-post predictable, regime-dependent forecast errors
- **Result 2:** Regime-robust test of FIRE
- **Result 3:** Medium-scale DSGE-RS model implies that null of FIRE cannot be rejected
- **Lessons:**
  - ▶ Predictability of forecast errors is not sufficient to reject FIRE
  - ▶ Neither an endorsement of FIRE nor a rejection of alternative expectations theories
  - ▶ Different expectations theories should be evaluated as part of fully specified model that incorporates plausible regime shifts

# Roadmap

- **Reduced-form forecast error regressions**
- A simple univariate example
- Generalized framework
- Application with estimated medium-scale DSGE-RS model

## Forecast error regressions

- Survey-based forecasts of macro aggregates are often biased and forecast errors are autocorrelated (Mincer and Zarnowitz, 1969; Nordhaus, 1987; Croushore, 1998;...)
- Recent focus: forecast error regressions (Coibion and Gorodnichenko (2015); Kohlhas and Walther (2021))

$$y_{t+h} - F_t y_{t+h} = \alpha + \underset{<0}{\gamma} y_t + e_{t+h}$$

$$y_{t+h} - F_t y_{t+h} = \alpha + \underset{>0}{\delta} (F_t y_{t+h} - F_{t-1} y_{t+h}) + e_{t+h}$$

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- Used as motivation / benchmark for prominent theories of information frictions and/or departures from rational expectations.
  - ▶ Mankiw and Reis (2002); Bordalo et al. (2020), Angeletos et al. (2021); Kohlhas and Walther (2021)

## Forecast error regression estimates depend importantly on sample period

- Example: U.S. SPF data for inflation and output growth (Kohlhas and Walther, 2021)

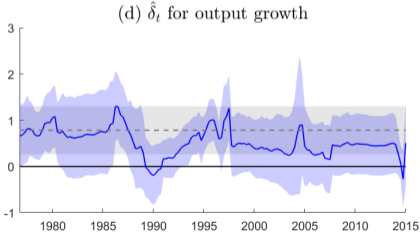
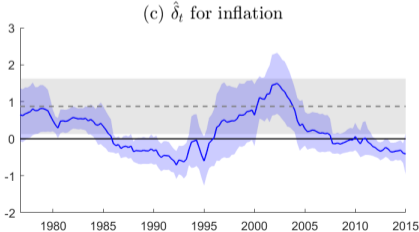
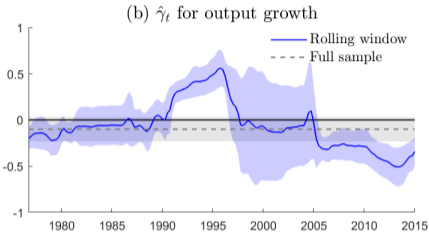
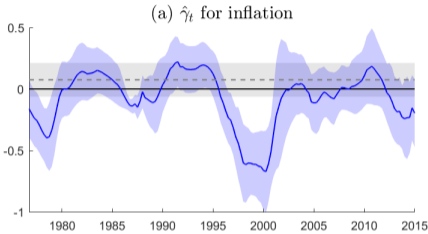
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	Inflation forecast error		Output growth forecast error	
	1970:4-2020:1	1982:3-2020:1	1970:4-2020:1	1982:3-2020:1
Current value ( $\gamma$ )	0.075 (0.070)	-0.131** (0.061)	-0.099 (0.067)	-0.075 (0.098)
Forecast revision ( $\delta$ )	0.876** (0.382)	0.104 (0.210)	0.784*** (0.264)	0.557* (0.310)

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# Waves of over- and under-reaction in 40 quarter rolling window samples



## Taking stock

- Forecast error predictability varies substantially and non-systematically across subsamples
- One interpretation: small sample uncertainty
- Alternative interpretation: regime shifts



# Roadmap

- Reduced-form forecast error regressions
- **A simple univariate example**
- Generalized framework
- Application with estimated medium-scale DSGE-RS model

## Univariate example without regime shifts

$$y_t = \beta \mathbb{E}_t y_{t+1} + \psi x_t \quad (1)$$

$$x_t = \phi x_{t-1} + \varepsilon_t \quad (2)$$

## Univariate example without regime shifts

$$y_t = \beta \mathbb{E}_t y_{t+1} + \psi x_t \quad (1)$$

$$x_t = \phi x_{t-1} + \varepsilon_t \quad (2)$$

- Full-information rational expectations solution

$$y_t = \frac{\psi}{1 - \beta\phi} x_t = a x_t$$

- $h$ -period ahead forecast error

$$y_{t+h} - \mathbb{E}_t y_{t+h} = a\phi^h x_t + a \sum_{\tau=1}^h \phi^{h-\tau} \varepsilon_{t+\tau} - a\phi^h x_t = a \sum_{\tau=1}^h \phi^{h-\tau} \varepsilon_{t+\tau}$$

⇒ Forecast errors are just accumulated i.i.d innovations = unpredictable

## Univariate example with Markov regime shifts

- Suppose  $\{\beta, \psi\}$  switch values across regimes  $s_t \in \{1, 2\}$  with transition matrix

$$P = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}$$

where  $p_{s_{t-1}s_t} = Pr(s_t | s_{t-1})$

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- Full-information rational expectations solution

$$y_t = a_{s_t} x_t$$

$$a_{s_t} = \begin{cases} a_1 & \text{if } s_t = 1 \\ a_2 & \text{if } s_t = 2 \end{cases}$$

with  $[a_1 \ a_2]' = (I_2 - \phi \beta P)^{-1} [\psi_1 \ \psi_2]'$

## Forecast errors

- $h$ -period ahead expectations

$$\mathbb{E}_t y_{t+h} = \phi^h (P_{s_t,1}^{(h)} a_1 x_t + P_{s_t,2}^{(h)} a_2 x_t)$$

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- $h$ -period ahead forecast error

$$y_{t+h} - \mathbb{E}_t y_{t+h} = \underbrace{\frac{(-1)^{s_{t+h}-1} (a_1 - a_2) (1 - P_{s_t, s_{t+h}}^{(h)}) \phi^h}{a_{s_t}}}_{\gamma_{s_t, s_{t+h}}^{(h)} \neq 0} y_t + \underbrace{a_{s_{t+h}} \sum_{\tau=1}^h \phi^{h-\tau} \varepsilon_{t+\tau}}_{\xi_{t+h}}$$

$\Rightarrow$  assume  $a_1 > a_2$  w.l.o.g.:  $\gamma_{s_t,1}^{(h)} > 0$  and  $\gamma_{s_t,2}^{(h)} < 0$  for  $s_t \in \{1, 2\}$

## Implications for reduced-form regressions

- Consider sample  $\{y_t\}_{t=1}^{T+1}$  conditional on regime sequence  $\{s_t\}_{t=1}^{T+1}$  and estimate

$$y_{t+1} - F_t y_{t+1} = \gamma y_t + e_{t+1}$$



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- Expected regression coefficient implied by univariate model (absent small sample bias)

$$\mathbb{E} \left[ \gamma \mid \{s_t\}_{t=1}^{T+1} \right] = \frac{\mathbb{E} \left[ (y_{t+1} - \mathbb{E}_t y_{t+1}) y_t \mid \{s_t\}_{t=1}^{T+1} \right]}{\mathbb{E} \left[ y_t^2 \mid \{s_t\}_{t=1}^T \right]}$$

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where conditional sample frequencies  $f_{ij} = \frac{\sum_{t=1}^T \mathbf{1}(s_t=i, s_{t+1}=i)}{\sum_{t=1}^T \mathbf{1}(s_{t+1}=i)} \gtrless p_{ij}$

## Implications for reduced-form regressions

- ① For finite sequence  $\{s_t\}_{t=1}^{T+1}$  characterized by  $f_{11}$  and  $f_{22}$  such that

$$g(f_{11}, f_{22}) > 0 \Leftrightarrow \mathbb{E} \left[ \gamma \mid \{s_t\}_{t=1}^{T+1} \right] > 0 \Rightarrow \text{agents look like they under-react to } y_t$$

$$g(f_{11}, f_{22}) < 0 \Leftrightarrow \mathbb{E} \left[ \gamma \mid \{s_t\}_{t=1}^{T+1} \right] < 0 \Rightarrow \text{agents look like they over-react to } y_t$$

Illustration

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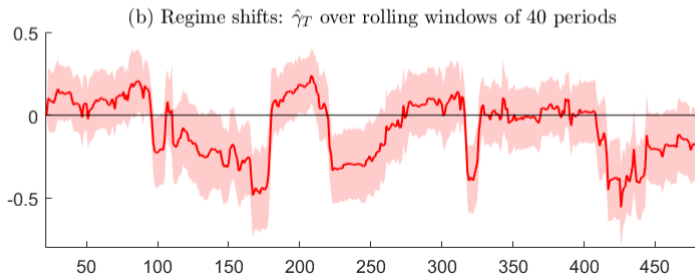
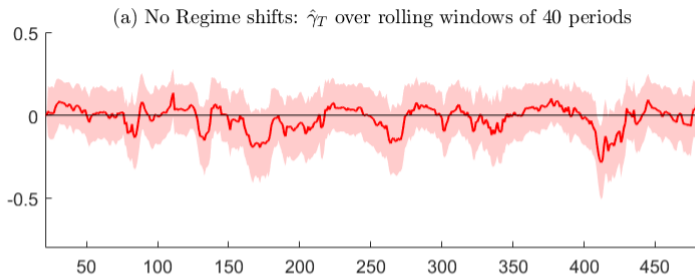
- 2 For  $T \rightarrow \infty$  and  $f_{ij} \rightarrow p_{ij}$  for any  $i \in \{1, 2\}$

$$g(f_{11}, f_{22}) \rightarrow 0 \Leftrightarrow \mathbb{E} \left[ \gamma \mid \{s_t\}_{t=1}^{T+1} \right] \rightarrow \mathbb{E}[\gamma] = 0$$

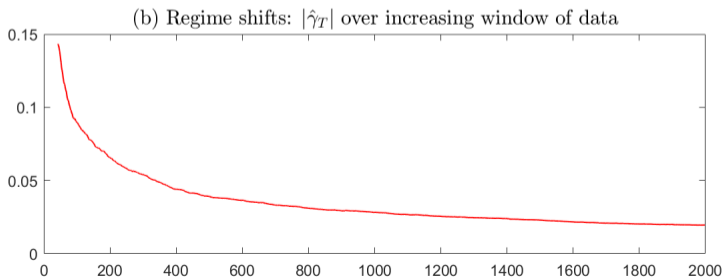
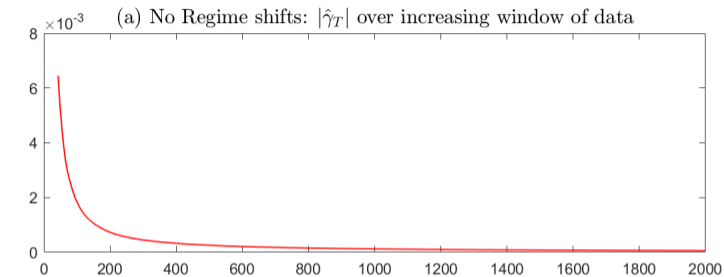
## Waves of over- and under-reaction across rolling window samples

- 1 Set  $a_1 = 2$ ,  $a_2 = 0.5$  and  $p_{11} = p_{22} = 0.7$
- 2 Simulate univariate model for  $T = 500$  periods and compute implied 1-period ahead forecast
- 3 Estimate  $y_{t+1} - F_t y_{t+1} = \alpha + \gamma y_t + e_{t+1}$  for rolling windows of 40 periods
- 4 Bias-correct OLS estimate  $\hat{\gamma}_T$  for each rolling window

# Waves of over- and under-reaction across rolling window samples



# Average predictability of ex-post forecast errors by sample size



## Regime-robust test of FIRE

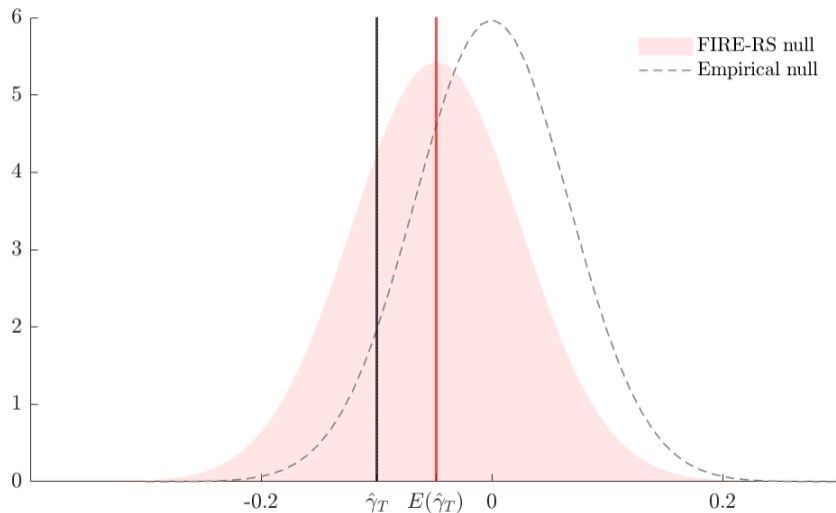
- Assume FIRE model with regime shifts as data-generating process = the null
- Estimate data-generating process: parameters and smoothed regime probability evolution
- Simulate  $i = 1, \dots, N$  samples of size  $T$  and estimate  $\hat{\gamma}_T^i$ 
  - ▶  $T$  - sample size of SPF data being tested for FIRE
- Compute probability that  $\hat{\gamma}_T^i$  exceeds  $\hat{\gamma}_T = t$ -test of the null



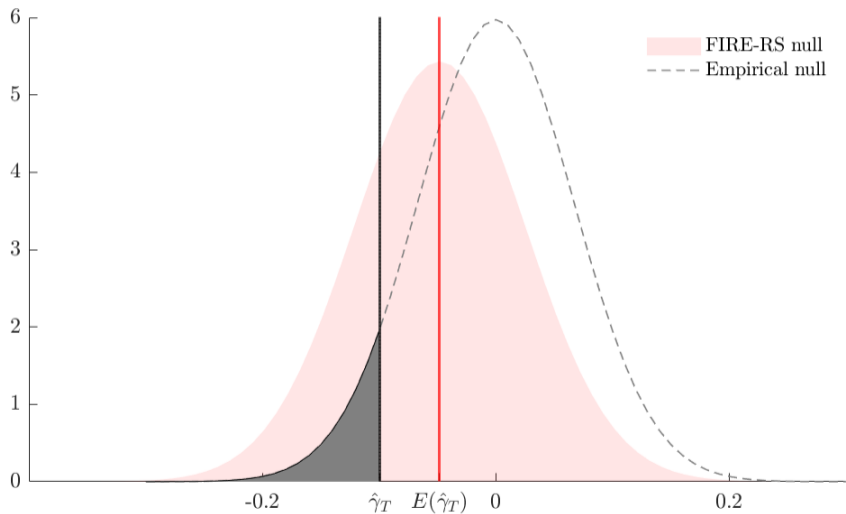
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- Two important features:
  - 1 Incorporate uncertainty about regime realizations by drawing regimes  $\{s_t^i\}_{t=1}^{T+h}$  from  $\hat{P}_r(s_t | \mathcal{Y}_T)$  implied by model and observed data  $\mathcal{Y}_T$  Output growth volatility regimes
  - 2 Naturally delivers robust standard error of finite sample distribution of  $\hat{\gamma}_T$

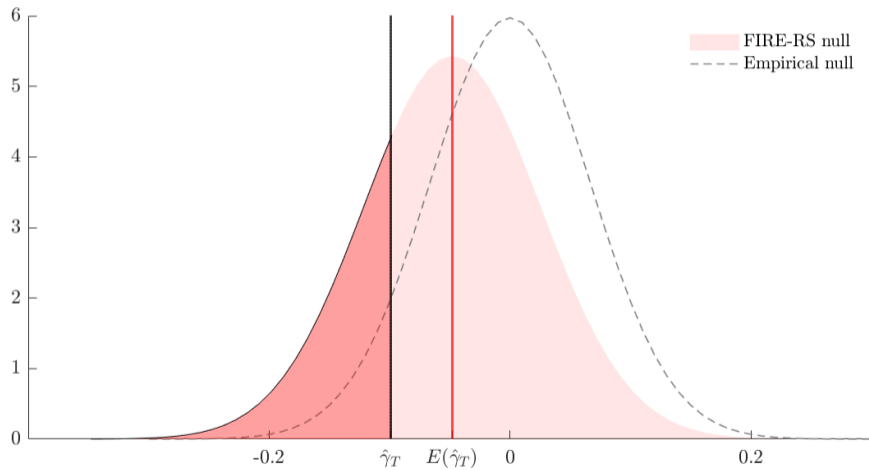
# Test for $\gamma$ with univariate model estimated on output growth data



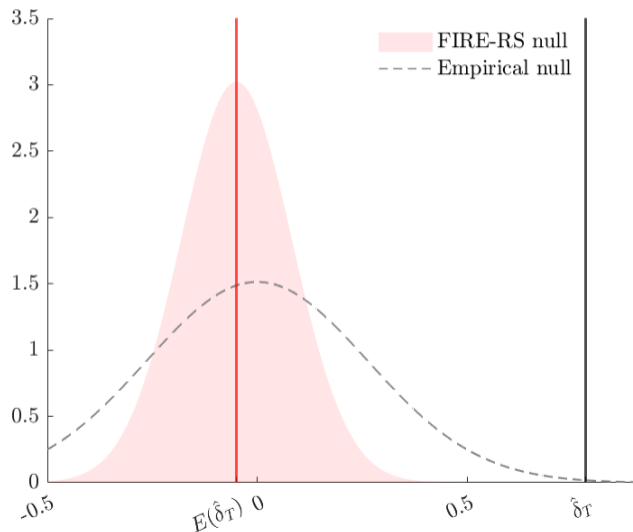
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# Test for $\gamma$ with univariate model estimated on output growth data



# Test for $\delta$ with with univariate model estimated on output growth data



# Roadmap

- Reduced-form forecast error regressions
- A simple univariate example
- **Generalized framework**
- Application with estimated medium-scale DSGE-RS model

## Generalized framework

- Consider multivariate regime-shift FIRE model with minimum state variable solution

$$\underbrace{X_t}_{n_x \times 1} = A_{s_t} X_{t-1} + B_{s_t} \underbrace{\epsilon_t}_{n_\epsilon \times 1}$$

$$\underbrace{Y_t}_{n_y \times 1} = \Psi_0 + \Psi_1 \underbrace{X_t}_{n_x \times 1}$$

- As in univariate case, suppose that  $A_{s_t}$ ,  $B_{s_t}$  switch values across regimes  $s_t \in \{1, 2\}$
- Important:  $A_{s_t}$  is generally not a diagonal matrix

## Relation of ex-post forecast errors with current information

- As in univariate example,  $h$ -period ahead forecast errors are predictable

$$Y_{t+h} - \mathbb{E}_t Y_{t+h} = \underbrace{\Gamma_{t,t+h}}_{\neq 0} X_t + \xi_{t+h}$$



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- Two key differences to univariate example**

- $\Gamma_{t,t+h}$  is complicated function of entire sequence of regime realizations  $\{s_\tau\}_{\tau=t}^{t+h}$
- $\Gamma_{t,t+h}$  is not diagonal  $\Leftrightarrow$  elements of  $Y_{t+h} - \mathbb{E}_t Y_{t+h}$  are related to entire vector  $X_t$

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$\Rightarrow y_{t+h} - F_t y_{t+h} = \alpha + \gamma y_t + e_{t+h}$  does not have structural counterpart  $\Rightarrow$  reduced-form regression is subject to omitted variable bias

## Relation of ex-post forecast errors with ex-ante forecast revisions

- Recall that

$$y_{t+h} - F_t y_{t+h} = \alpha + \underset{<0}{\gamma} y_t + e_{t+h}$$

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- General regime-shift FIRE model implies

$$Y_{t+h} - \mathbb{E}_t Y_{t+h} = \underbrace{\Delta_{t,t+h}}_{\neq 0} (\mathbb{E}_t X_{t+h} - \mathbb{E}_{t-1} X_{t+h}) + \underbrace{\Lambda_{t,t+h}}_{\neq 0} X_{t-1} + \xi_{t+h}$$

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$\Rightarrow y_{t+h} - F_t y_{t+h} = \alpha + \delta (F_t Y_{t+h} - F_{t-1} Y_{t+h}) + e_{t+h}$  is s.t omitted variable bias

$\Rightarrow$  No guarantee that  $sign(\mathbb{E} [\hat{\gamma}_T | \{s_t\}_{t=1}^{T+1}]) = sign(\mathbb{E} [\hat{\delta}_T | \{s_t\}_{t=1}^{T+1}])$

## Taking stock

- FIRE models with regime shifts generate predictable, regime-dependent forecast errors
- In generalized linear framework, predictability of  $y_{t+h} - F_t y_{t+h}$  depends on (i) entire sequence of regime realizations  $\{s_\tau\}_{\tau=t}^{t+h}$ ; and (ii) entire state vector  $X_t$
- In finite samples, regime shifts imply waves of (what appears like) over- and under-reaction to current information ( $y_t$ ) and forecast revisions ( $F_t Y_{t+h} - F_{t-1} Y_{t+h}$ )
- Can realistic FIRE model with regime shifts generate forecast errors predictability consistent with reduced-form evidence?

# Roadmap

- Reduced-form forecast error regressions
- A simple univariate example
- Generalized framework
- **Application with estimated medium-scale DSGE-RS model**

## Medium-scale DSGE-RS model

- Model a la Christiano et al. (2005), Smets and Wouters (2007), Justiniano et al. (2011)
  - ▶ Habit persistence in consumption
  - ▶ Investment adjustment cost and variable capital utilization
  - ▶ Staggered nominal price and wage contracts
  - ▶ Monetary policy rule
  - ▶ Several real and nominal shocks



## Regime-robust test of FIRE with medium-scale DSGE-RS model

- Model a la Christiano et al. (2005), Smets and Wouters (2007), Justiniano et al. (2011)
- Regime shifts in monetary policy rule as in Bianchi (2013)

$$R_t = \phi_{s_t}^R R_{t-1} + (1 - \phi_{s_t}^R) \left( \phi_{s_t}^{\Delta y} \Delta y_t + \phi_{s_t}^{\pi} \pi_t \right) + v_t$$

- Estimate model on post-war US macro data Policy rule parameters estimates
- Simulate  $N$  data samples of size  $T$ , taking into account uncertainty about sequence of regime realizations Evolution of monetary regimes
- Estimate distribution of forecast error regression estimates from simulated data
- Compute t-test of null that observed estimates were generated by DSGE-RS model

# Null of DSGE-RS model cannot be rejected at high significance levels

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*Panel A:*  $y_{t+h} - F_t y_{t+h} = \alpha + \gamma y_t + e_{t+h}$

	$\hat{\gamma}_T$	$\mathbb{E}[\hat{\gamma}_T]$	$\sigma_{\hat{\gamma}_T}$	$p$ -value $H_0 : \hat{\gamma}_T = \mathbb{E}[\hat{\gamma}_T]$
Inflation	0.086	-0.031	0.086	0.179
Output growth	-0.086	-0.005	0.058	0.166

*Panel B:*  $y_{t+h} - F_t y_{t+h} = \alpha + \delta(F_t y_{t+h} - F_{t-1} y_{t+h}) + e_{t+h}$

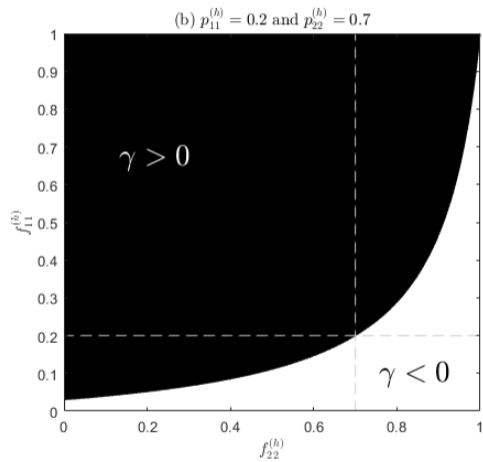
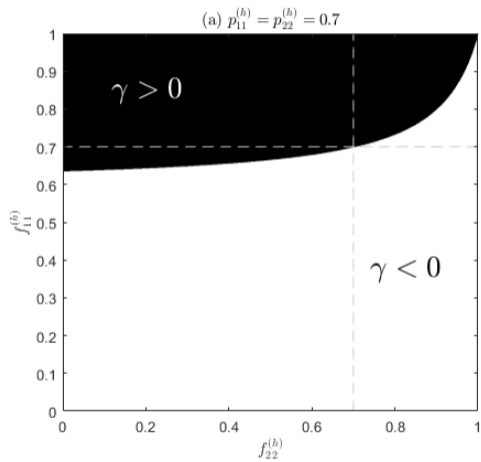
	$\hat{\delta}_T$	$\mathbb{E}[\hat{\delta}_T]$	$\sigma_{\hat{\delta}_T}$	$p$ -value $H_0 : \hat{\delta}_T = \mathbb{E}[\hat{\delta}_T]$
Inflation	0.925	0.754	0.088	0.052
Output growth	0.837	0.689	0.168	0.370

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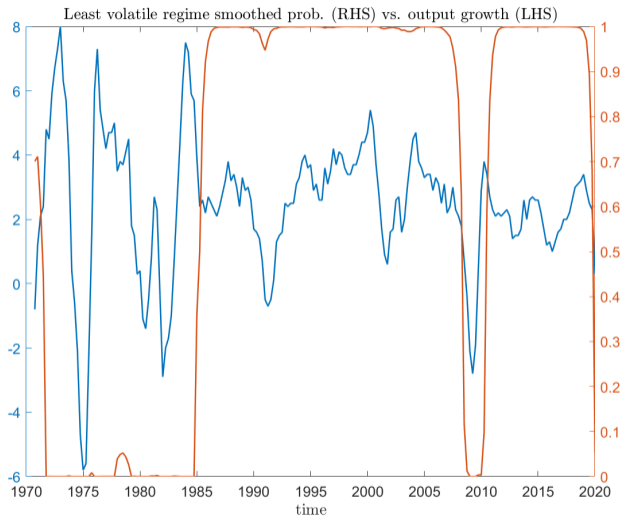
## Concluding Remarks

- FIRE models with regime shifts imply predictable, regime-dependent forecast errors.
- We propose a regime-robust test and fail to reject FIRE at high significance levels.
- Neither an endorsement of FIRE nor a rejection of alternative expectations theories.
  - ▶ Forecast error predictability regressions provide valuable moments
  - ▶ Apply regime-robust test for whatever is your favorite model
  - ▶ FIRE or non-FIRE; different assumptions about regime shifts and learning;...

# Illustration



# Estimated output growth volatility regimes



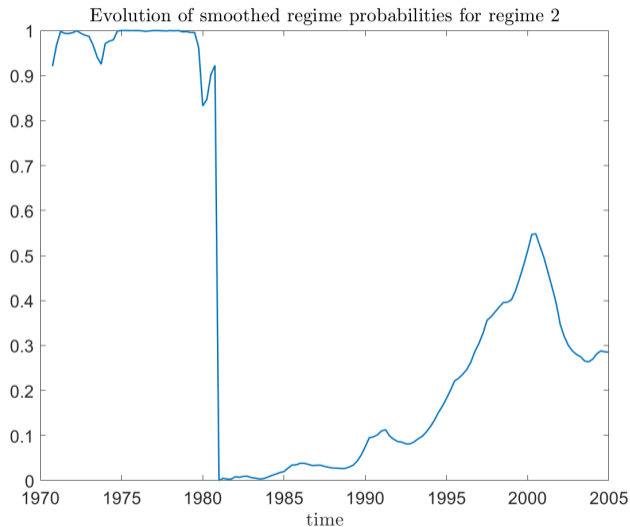
## Estimation and implied monetary policy regime

- Compute minimum state solution under FIRE
- Bayesian estimation on post-war U.S. data for (output growth, consumption growth, investment growth, wage growth, labor hours, inflation, Fed funds rate)
- Posterior mode estimates of monetary policy rule and transition matrix

	$\rho_{s_t}^R$	$\phi_{s_t}^{\Delta y}$	$\phi_{s_t}^{\pi}$	$P(1, s_t)$	$P(2, s_t)$
$s_t = 1$	0.68	0.29	2.07	0.979	0.034
$s_t = 2$	0.77	0.33	1.10	0.021	0.966

- Compute smoothed regime probabilities (Kim and Nelson, 1999) and infer implied regime  $s_t = i$  if  $Pr(s_t = i) > 0.5$

# Estimated monetary policy regimes (regime 2 = least aggressive regime)



# Waves in simulated vs SPF forecast error regression coefficients

