

Costs of Financing US Federal Debt: 1791-1933*

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Abstract

This paper uses a statistical model with drifting parameters to infer term structures of real and nominal yields on US federal bonds during the gold standard era from 1791-1933. Gold denominated yields trended downwards throughout the nineteenth century, falling below UK levels by the 1880s. Bonds near maturity carried a “liquidity premium” except during the height of the National Banking Era from 1880-1913. Long term price expectations were anchored until the late nineteenth century, even during the Civil War green-back devaluation. We note how rearrangements in monetary, financial, and fiscal institutions coincided with changes in US borrowing costs.

JEL CLASSIFICATION: E31, E43, G12, N21, N41

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1 Introduction

At pivotal moments in US history, leading political figures have proposed and sometimes implemented institutional changes designed to reduce the interest costs of servicing US federal debt. In his 1790 *Report on Public Credit*, Alexander Hamilton argued that the US government could reduce prevailing high *default-risk premium* on its debts by restructuring political institutions to sustain prospective net-of-interest surpluses. Treasury Secretary Salmon P. Chase and other proponents of the National Banking Acts of 1862-6 sought to enhance financial sector demand for US federal debt and thereby increase the *liquidity premium* the Treasury could earn on long maturity bonds. During the 1868 presidential campaign, President Andrew Johnson and many Democrats proposed to reduce debt servicing costs by redefining the unit of account from gold to inconvertible paper dollars (“greenbacks”). On the contrary, General Ulysses S. Grant asserted that not meeting gold obligations would actually raise prospective borrowing costs by increasing the *demonination risk premium*. Although these and other historical episodes have been much discussed in the literature, the lack of yield curve estimates for the pre-World War II period has prevented researchers from quantifying the impacts of such institutional and regulatory changes on US borrowing costs.

For a long but thin data set assembled by [Hall et al. \(2018\)](#), this paper uses a non-linear state space model with drifting parameters and stochastic volatility and a novel application of state-of-the-art sampling techniques to infer term structures of *yields* on US federal bonds from 1791 to 1933. At times, different types of currencies circulated, including gold coins, greenback paper dollars, and notes issued by state and federally chartered banks. To construct a consistent baseline estimate, we start by focusing on (nominal) yield curves for gold coin denominated US federal debt contracts because gold coins were in circulation through the entire period and dominant for most of it.¹ To approximate ex-ante *real* yield curves from 1791-2020, we combine our nominal yield curve estimates with estimates of inflation expectations from a related statistical model.

We infer a new collection of stylized facts about long term bond yields during the “gold standard era.” First, we find that long run inflation expectations were anchored around zero before the 1890s, so that gold denominated yields closely approximated real yields over this period. Second, yields trended downwards, with the 10-year gold yield dropping from around 8% in 1800 to around 2% in 1900. Third, until the 1880s US debt typically carried a risk premium relative to UK debt, the “safe-asset” of the era, but this reversed after 1905 when US yields became persistently lower than UK yields. Fourth, during nineteenth century wars, there were large spikes in real yields, but not during twentieth century wars. Together, these facts suggest that it wasn’t until the late nineteenth century that Hamilton’s hopes were fulfilled and the risk premium on the US debt ultimately evaporated. That transition coincided with a realignment of global finance that eventually led US government debt to replace UK debt as a global “safe-asset” during and after the years of the Bretton Woods arrangement.

We establish additional facts about the relative costs of short- and long-term funding during the “gold standard era.” We find a “short rate disconnect” for most of the nineteenth century, in the sense that government debts close to maturity traded at a 0.5 percentage point premium over short yields implied by prices of long maturity government securities. We interpret this as evidence of a significant liquidity premium on short debt. Yield curves typically sloped downward before the Civil War, with the 10-2 year spread averaging approximately -1 percentage points. The slopes of yield curves turned positive after the 1880s when the 10-2 year spread averaged approximately 1 percentage point. This pre-Civil War pattern contrasts with the twentieth century

¹George Washington and Alexander Hamilton introduced a gold standard in 1791 that was theoretically maintained until 1933, at which point Franklin Roosevelt accepted Irving Fisher’s advice to abandon the gold standard. [Edwards \(2018\)](#) describes how the US defaulted on its promises to pay gold dollars. [Rothbard \(2002\)](#) describes how Irving Fisher influenced President Roosevelt’s decision to leave the gold standard. The US also temporarily departed from the gold standard from 1862-1878 when gold and greenback dollars exchanged at a market-determined exchange rate.

when yield curves have been persistently upward sloping except for some relatively small inversions preceding recessions. We present evidence that these different patterns reflected the inflation process having changed from a near i.i.d. stochastic process before 1880 to become a more persistent, less volatile process with a higher long-run mean afterwards. This means that in the early years in our sample the major inflation risk was at long horizons, so that long-term bonds acted as a relatively better inflation hedge, while in later years short-term inflation risk became more important, making short-term bonds the better hedge.

Monetary and financial changes during and after the Civil War offer a case study about how institutional changes impacted government borrowing costs and highlight the benefit of estimating the full yield curve. We focus on several Civil War laws: the Legal Tender Act, signed by President Lincoln on February 25, 1862, the National Bank Act signed on February 25, 1863, and three subsequent National Bank Acts passed in 1864, 1865, and 1866. The Legal Tender Act authorized the government to issue legal tender, nonconvertible notes that soon came to be known as greenbacks.² The four National Bank Acts established a federally regulated banking system that issued bank notes backed by US federal debt. Both laws sought to expand the government’s debt capacity. During and after the Civil War, the federal government issued bonds denominated in both gold and greenbacks dollars, a situation that allows us to estimate investors’ expectations about the gold-greenback exchange rate and a greenback denominated yield curve. We infer a strong nominal anchor during the Civil War since investors seemed to have anticipated a fast return to greenback-gold parity despite a 60% depreciation in the greenback to gold exchange rate. An anticipated postwar appreciation of the greenback meant that greenback yields were persistently lower than the gold yields. We also find that the premium on short term debts disappeared on January 1, 1879 when the Treasury began pegging the greenback-gold exchange rate at 1. This suggests that Civil War monetary and banking reforms bolstered US federal debt capacity within 15 years of the end of the Civil War.

In parameterizing and estimating a stochastic process for yield curves on US federal bonds, we confront several challenges. The first is that nineteenth century macroeconomic data are unreliable. This prevents us from directly estimating a stochastic discount factor process that prices macroeconomic risks, especially at high frequency. For this reason, we follow a mongrel approach that specifies a general stochastic discount factor process with a law-of-one price restriction across maturities for each date, but does not explicitly impose the absence of arbitrage. This approach potentially nests a wide range of models, from affine asset pricing models to preferred habitat models, but using it means that we are restricted to estimating yield curves that bundle haircut risk and convenience premia into a “time-varying pricing kernel.”

A second challenge is that our data set is sparse in the cross-section dimension. We tackle this problem by adopting a time-varying version of a statistical model proposed by [Nelson and Siegel \(1987\)](#). Economists at policy institutions use a similar parameterisation, but in inferring a yield curve from observed prices and quantities they face a different challenge than we do. Because they have a superabundance of *cross-section* data on prices and quantities at each date, they solve an *overdetermined* inference problem. Our data are too sparse along the cross-section dimension to allow us to use even a *just-identified* version of the commonly used procedure. To confront this data deficiency, we enlist a prejudice or induction bias in the form of a parameterized statistical model of a panel having scattered missing observations. The data and statistical model tell us how much *smoothing* across time to do.

A third challenge is that nineteenth century US federal bonds often gave lenders and borrowers discretion over maturity dates, conversions, and other features. Our estimation assumes that agents priced bonds under perfect foresight about those discretionary contract features. To prevent such assumptions from influencing our inferences too much, we introduce bond-specific idiosyncratic pricing errors. This decreases the influence of

²[Lowenstein \(2022, ch. 5\)](#) describes political and financial forces that produced the Legal Tender Act.

peculiar bonds on our yield estimates while still informing us about situations when our assumptions prevent us from consistently pricing our *cross-section* of bonds using our pricing formulas.

Another challenge that we have overcome here is to infer posterior distributions associated with a complicated non-linear statistical model without relying on the particle filter or Gibbs sampling. We approximate posterior probabilities by deploying Hamiltonian Monte Carlo and No U-Turn sampling (HMC-NUTS).³ Our data set presents many technical difficulties—such as the changing number of observed assets, bonds that have payoff streams of varying lengths, periods without price observations, the relevant set of bond-specific pricing errors changing over time in complicated ways—that prevent us from applying the “standard” `Stan` toolkit and force us to code our log posterior functions from scratch. Our application of the `DynamicHMC.jl` package⁴ by Papp et al. (2021) can be used for other economic models with tractable likelihood functions that do not easily fit into the `Stan` framework.

Related Work In the spirit of Friedman and Schwartz (1963), we present a narrative history supported by data and statistics.⁵ There has been recent work compiling international historical interest rate series and examining long-term trends (e.g. Shiller (2015), Hamilton et al. (2016), Jordà et al. (2019), Schmelzing (2020), Officer and Williamson (2021), Chen et al. (2022)). An important data constraint is that these studies have limited coverage of US Federal yields. Instead, they use a commercial paper rate as a “short interest rate”⁶ and a “long market rate” from Homer and Sylla (2004) that combines yields-to-maturity on US Federal bonds pre-Civil War with yields-to-maturity on New England Municipal bonds and corporate bonds post-Civil War.⁷ By estimating the full yield curve on US Federal bonds, this paper opens up new and exciting questions about historical trends in government financing costs.

Technically, our work is related to Svensson (1995), Dahlquist and Svensson (1996), Cecchetti (1988), Annaert et al. (2013), Andreasen et al. (2019), Diebold and Li (2006) and Diebold et al. (2008) who, like Gürkaynak et al. (2007) and ourselves, implement versions of the parametric yield curve model of Nelson and Siegel (1987). Our non-linear state space model with drifting parameters and stochastic volatility builds on Cogley and Sargent (2005, 2015). Our analysis of events during the greenback period from 1862 to 1879 revisits issues presented in the landmark studies of Mitchell (1903, 1908) and Roll (1972). Computing posterior distributions implied by our data and our statistical model is a formidable task that we accomplish by using the HMC-NUTS algorithm of Hoffman and Gelman (2014) and Betancourt (2018). While this estimator has been used extensively in statistics, economic applications are scarce. Prominent exceptions are Bouscasse et al. (2021) who use it to study the evolution of productivity in England from 1250 to 1870 and Farkas and Tatár (2021) who estimate DSGE models with ill-behaved posterior densities.

³Hamiltonian Monte Carlo is named after mathematician and physicist William R. Hamilton, not US Secretary of the Treasury Alexander Hamilton.

⁴Written in `Julia` (Bezanson et al. (2017)), this package is a robust implementation of the HMC-NUTS sampler that mimics many aspects of `Stan`. An advantage of this package is that it allows the user to provide the Jacobian of the log-posterior manually and so one does not need to rely on automatic differentiation for a model with 7,500+ parameters.

⁵Our interpretations are shaped by a statistical model that we regard as an auxiliary model in the sense of Gallant and Tauchen (1996) in terms of how it would connect to an explicit structural model cast in terms of parameters that describe preferences, constraints, and information flows of purposeful agents inside the model.

⁶The Federal Reserve provides a commercial paper rate series from 1831-1930 and a call money rate series from 1857-1954.

⁷Homer and Sylla (2004) construct a time series of yields-to-maturity on 10-year federal bonds by taking the coupon rates on selected bonds that have approximately 10 years to maturity and trade close to par. However, most recent studies use a different, ‘composite series’ that combines the Homer and Sylla (2004) estimates for the period from 1798-1861 with the yield-to-maturity on a set of the New England municipal bonds for the period 1862-1899 and the yield-to-maturity on corporate bonds for the period 1900-1940. We provide evidence that this blended series underestimates costs of government financing during the Civil War and overestimates those costs afterwards because it calculates yields on high-grade state or corporate bonds rather than on US federal debt. See Siegel (1992), in particular Section 2.2.

Outline Section 2 describes data and provides historical context. Section 3 outlines a theory of zero-coupon bond yields, describes how we parameterise the gold dollar yield curve, and delineates an econometric strategy. Section 4 discusses some stylized facts about the “gold standard era” from 1791 to 1933. Section 5 argues that our parameterisation and identifying assumption are able to approximate observed bond prices well. Section 6 discusses statistical inferences about greenback dollar yield curves and gold-greenback price expectations during and after the Civil War. Section 7 is an epilogue that connects our results with recent data and discusses implications of studying long time series across different monetary and fiscal regimes.

2 Data Set and Historical Context

We have assembled prices, quantities, and descriptions of all securities issued by the US Treasury between 1776 and 1960.⁸ In Appendix A.1, we spotlight decisions about our data that we made to prepare for the statistical inferences presented in this paper. In this section, we provide historical context to help understand the data. We first discuss the characteristics of nineteenth century monetary policy, financial sector regulation, and treasury debt management. For reference, the major events are summarized in Table 2 in Appendix B. We then outline the challenges that these characteristics pose for yield curve estimation. These challenges shape specification and estimation strategies deployed in Section 3.

2.1 Monetary Policy and Financial Sector Regulation

1791-1862: Bimetallism, Banks of The US, and State Banks. Between April 1792 and February 1862, the US dollar was defined in terms of gold and silver (a “bimetallic” system).⁹ The federal government minted gold and silver coins but not paper notes. Instead, paper notes were created by the banking sector. Throughout the period, state legislatures chartered state banks, which could issue their own bank notes. Initially, the First (1791-1811) and Second (1816-1836) Banks of the United States operated at the national level. These banks were privately owned and undertook similar commercial operations to the state banks. However, they also had the special privileges of acting as the banker for the federal government (depositing tax revenue and making loans) and operating across state boundaries. Because tax revenues could be paid in state bank notes and were deposited in the First and Second Banks of the US, these banks effectively acted as a lender to the state banking system. This meant that the First and Second Bank of the US could influence state bank note and credit creation by setting the rate at which they redeemed their state bank notes into gold.¹⁰

The rechartering of the Second Bank of the US turned into a political struggle during the Presidency of Andrew Jackson (1829-1837). Andrew Jackson vetoed a bill to recharter the bank (1832), removed federal deposits from the bank (1833), and, ultimately, allowed the bank’s charter to expire (1836). In the subsequent decades (1837-1862), states expanded their banking sectors by allowing the automatic chartering of banks without requiring explicit approval from the state legislature. This period is often referred to as the “free banking era” and was perceived to be characterized by high bank risk taking and discounted state bank notes.

1862-1913: Greenbacks, Gold Standard, and the National Banking System. The outbreak of the Civil War in 1861 put significant strain on the monetary and financial systems, leading to major policy changes. In January 1862, state banks stopped honoring their legal obligation to convert their notes into specie (they “suspended”

⁸The data set (excluding any data taken from non-publicly accessible data sets) is available at the Github repository <https://github.com/jepayne/US-Federal-Debt-Public> and construction methods are explained in Hall et al. (2018).

⁹Prior to 1792, a dollar referred to a Spanish silver coin.

¹⁰See Hammond (1947) for a discussion of the operations of the First and Second Bank of the US.

convertibility). On February 25, 1862, Congress passed a Legal Tender Act that authorized the Treasury to issue 150 million dollars of a paper currency known as greenbacks that the government did not promise immediately to exchange for gold dollars. Subsequent acts authorized the Treasury to issue more notes, eventually totalling 450 million dollars. Investors could use greenbacks to purchase bonds from the federal government at their par values. Gold dollars continued to be used to settle international transactions and to pay US tariffs. From 1862 to December 31, 1878 paper notes (“greenbacks” or “lawful money”) traded at discounts relative to gold dollars (“gold” or “coin”). The greenback depreciated substantially during the Civil War and did not attain parity with gold until January 1, 1879, when the US Treasury started converting greenbacks into gold dollars one-for-one.

In addition, between 1863-6, Congress passed a collection of National Banking Acts, which established a system of nationally chartered banks and the Office of the Comptroller of the Currency. National banks faced restrictions on what loans they could make and were allowed to issue bank notes up to 90% of the minimum of par and market value of qualifying US federal bonds.¹¹ These national bank notes were intended to replace the state bank notes as a standardised currency that could be used across the country. In order to achieve this, Congress imposed a 10% annual tax on state bank notes, which was significantly greater than the 1% annual tax on national bank notes.

1913-1933: Establishment of Federal Reserve Bank. Bank runs and stock market crashes were a common feature of all different monetary and banking policy arrangements during the nineteenth century. There were country wide bank panics in 1819, 1827, 1857, 1873, 1893, and 1907 as well as many other local bank panics in New York and other financial hubs. In response, The Federal Reserve System was passed in 1913 to create a Federal Reserve Bank to act as a reserve money creator of last resort to prevent bank runs. Convertibility between gold and US notes at par prevailed through World War I and the 1929 stock market crash until 1933 when Franklin D. Roosevelt increased the paper price of gold and prohibited private US citizens from holding gold coins. For the purposes of this paper, we consider this the end of the gold standard in the US.

2.2 Nineteenth Century US Federal Bonds

Before World War I, the federal government issued bonds infrequently. New bond issues were often small. The US Congress, rather than the Treasury, designed each government security with the consequence that securities varied over time in terms of their coupon rates, denominations, lengths, units of account, tax exemptions, and call features. Before the 1920s, the federal government occasionally issued customized long term debt, mostly to finance wars, debt reschedulings, and specific infrastructure projects. As a result, between 1776 and World War I, the US Congress only authorized the Treasury to issue a total of approximately 200 distinct securities, with at most 8 distinct ones being authorized in any one year.

Between 1917 and 1939, Congress gradually delegated more and more decisions about designing US debt instruments to the Treasury and the Treasury gradually standardized security design. As a result, from 1920 to 1960 alone, the Treasury issued about 2500 securities with a wide range of maturities. Ultimately, this transformed the market for US Treasury securities into the world’s most liquid debt market with a collection of standardized securities at many maturities that allowed a large national debt to be issued and rolled over.

When gold and greenback dollars coexisted (1862-1878), different US Treasury bonds promised payments in different currencies. Some bonds promised all payments in gold (we refer to these as “gold dollar” bonds);

¹¹National banks could only operate one branch. They were restricted from making mortgages unless they were operating in rural areas, where they could make a limited range of loans collateralized by agricultural land. Initially, qualifying bonds were United States government registered bonds bearing interest in coupons of 5% or more. In addition to needing to hold federal bonds, the National Banks also needed a minimum quantity of equity capital that depended on the population in the town where they operated.

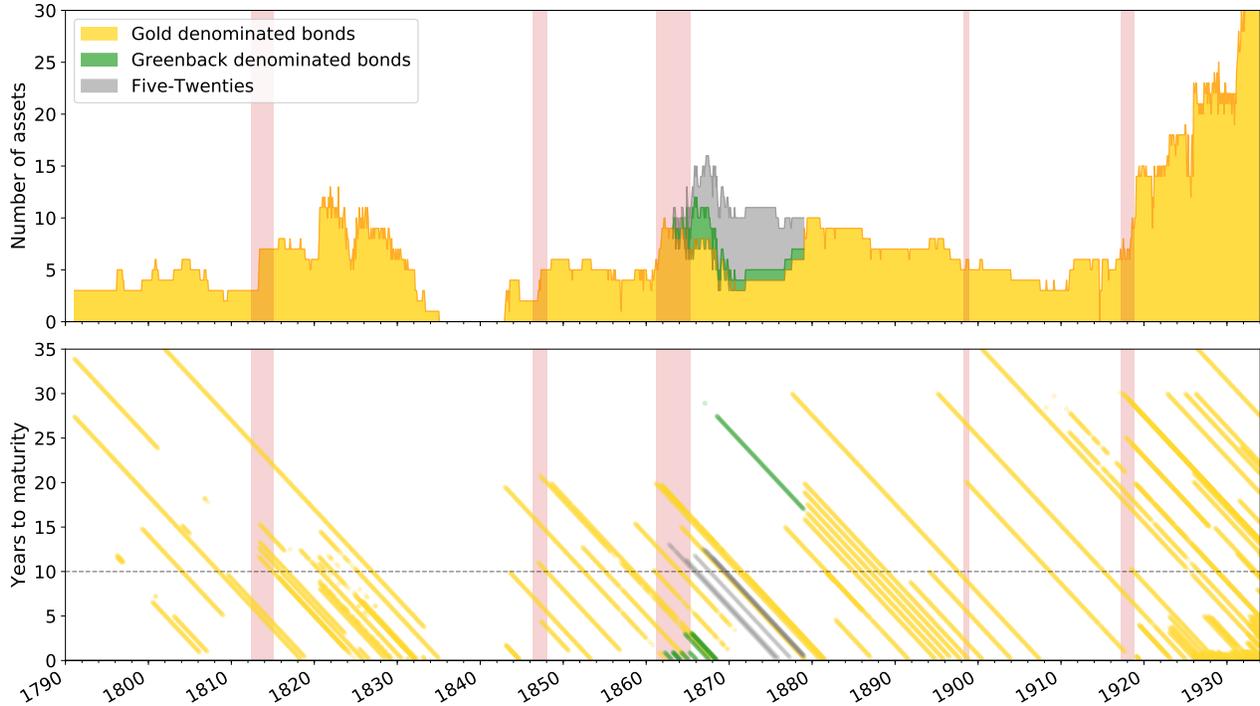


Figure 1: The top panel depicts the number of securities with observed prices each month. The bottom panel depicts maturities (in years) of observed securities. Darker lines indicate overlapping securities. Red bars correspond to wars.

other bonds promised all payments in greenbacks (we refer to these as “greenback dollar” bonds); and yet other bonds offered coupons in gold but left ambiguous whether the principal would be paid in gold or greenbacks (we refer to these as “ambiguously” denominated bonds). While bonds denominated in different currencies present an opportunity because they allow us to estimate both gold and greenback dollar nominal yield curves, the difficulty is that we observe only 9 greenback dollar bonds and only 6 ambiguously denominated bonds. Consequently, we will focus on the gold dollar bonds to obtain our baseline yield curve estimate. When we turn to estimating the greenback dollar yield curve in Section 6, we will build on this baseline gold dollar yield curve.

2.3 Inference Challenges

Skilled researchers have estimated yield curves on US federal debt in the post-WW2 period, by which time federal debts had become standardised, and government bonds for sale had become plentiful. We estimate yield curves starting in 1791 and so have to confront estimation challenges posed by peculiar structures of US federal bond markets before 1920. This requires us to address the following issues.

- (1) *How should we handle periods that provide sparse or inaccurate macroeconomic data?*

In principle, we could attempt to use historical macroeconomic data to estimate a model of the stochastic discount factor that prices macroeconomic risks. However, we are skeptical about the quality of nineteenth century macroeconomic data, especially at high frequency. For this reason, we estimate a flexible model that sidesteps directly specifying a stochastic discount factor process. As we outline in Section 3, this approach still allows us to infer US federal costs of finance via debt issues without recovering a stochastic discount factor. We leave for future work an attempt to use noisy historical macroeconomic data to infer

a stochastic discount factor process directly.

(2) *How should we handle periods with sparse bond data?*

The federal government issued few securities during the nineteenth century so we have few price observations. We see this in Figure 1, which shows monthly time series for the number of securities with observed prices and times to maturity (in years) of all outstanding bonds. The gold color scheme represents gold denominated bonds. The green color scheme represents greenback denominated bonds. The gray color scheme represents the five-twenties, which had an ambiguous denomination. There were often fewer than 5 price observations in a given period, often only for bonds with long maturities. We have no prices in the late 1830s because no federal securities were outstanding then. This means that while we have “big data,” our unbalanced sample prevents us from applying commonly used techniques from the yield curve estimation literature. Instead, we must posit a statistical model that lets us learn about yields at all dates simultaneously by pooling information across time periods. We do this by first imposing a time-varying Nelson and Siegel (1987) style parameterisation of the gold dollar yield curve in Section 3.1.2 and then “smoothing” parameter updates in Section 3.3.1.

(3) *How should we handle peculiar bonds?*

Throughout our sample, many US Treasury securities had custom features such as indefinite maturities associated with call or conversion options. In principle, we could attempt to construct custom pricing formulas for each bond using the universe of possible derivatives. Instead, we start by arbitrarily “converting” idiosyncratic contract features into those of more standard bonds. We do this by *ex post* imputing perfect foresight about call dates and other discretionary components of the contracts. For bonds that specified a window during which the government could call the bond or bond holders could redeem the bond, we impose that agents priced the bonds knowing their actual future redemption dates.¹² For bonds that could be converted into other bonds, we assume that agents priced these bonds knowing the cash flows of associated new bonds after conversions. We inspect and refine these assumptions by studying bond-specific pricing errors introduced in Section 3.3.2.

(4) *How should we handle haircut risk and convenience yields?*

Today, US federal debt is often assumed to carry no haircut risk. That assumption is implausible for much of the nineteenth century. After Alexander Hamilton’s 1790 re-financing had concluded, the US federal government did not again impose haircuts on federal creditors, but it faced several crises during the nineteenth century (e.g., the War of 1812 and the Civil War) that investors interpreted as threatening to leave the US federal government unable or unwilling to service its debts completely. State governments also recurrently defaulted during the nineteenth century, and the Confederate States of America defaulted. We address this difficulty by packaging haircut risk into the pricing kernel by imposing that haircut risk is common across all federal government bonds and ultimately by estimating prices of risky government promises. We tell how we do this in Section 3.1.1.

In addition, the limited money supply and incompleteness of nineteenth century asset markets suggests that there were also convenience yields on some government liabilities, particularly short-term bonds that

¹²More specifically, we impose perfect foresight in the following way. For callable bonds, we set the maturity date to the date at which the government called the bond. For redeemable long term bonds (greater than 3 years maturity) and/or bonds that pay regular coupons, we set the maturity date to the last date at which any of the bonds could be redeemed. For redeemable short term bonds (less than 3 years maturity) that pay coupons at maturity, we set the maturity date to the last date at which bonds were issued plus the duration of the bond and match total coupon payments with bond duration. For example, for a 1-year bond, this means we impose that only one year’s coupons are paid at redemption, regardless of the date at which the bond was redeemed.

could be used for payments. We address this by dropping bonds with maturities less than one year and then inspecting whether the pricing kernel that fits long maturity bonds can also successfully price the short end of the yield curve. We discuss this in Section 4.2

3 Statistical Models

In this section, we outline our strategy for estimating the gold denominated yield curve. We start by describing our parameterisation of the stochastic process of the gold dollar yield curve. A key feature of our approach is to impose a tight parametric structure *across maturities*, while letting the yield curve parameters vary *across time* in a flexible way but nonetheless allows to pool information across time periods. This gives rise to a non-linear state space model described in Section 3.3. Finally, we conduct “laboratory experiments” to illustrate what is required from our sample for our statistical model to overcome data scarcity.

3.1 Gold Dollar Yield Curves

At a given date, a term structure of interest rates is a list of yields on zero-coupon bonds of maturities $j = 1, 2, \dots, J$. Because the US government did not issue zero-coupon securities of all maturities, we must approximate zero-coupon yield curves on US Treasury debt indirectly from observed prices of a limited set of federal government securities having differing coupons, par values, and maturity dates. As discussed in Section 2, inference is especially challenging before World War I because the Treasury issued bonds infrequently; bonds contained default risk; and bonds differed in currency denomination, their tax exemptions, their investor redemption options, and government call options. To make progress, we impose a collection of assumptions that allow us to parameterise stochastic processes for yield curves on US federal bonds. In Section 3.1.1, we discuss the theoretical restrictions that we impose on the pricing kernel. In Section 3.1.2, we outline our parameterisation of the yield curve. In Section 3.1.3, we bring all of our assumptions together to construct pricing equations for the yield curve.

3.1.1 Identifying Assumption

Consider a setting with a composite consumption good, gold dollars, and a collection of zero-coupon bonds that promise payouts in gold dollars. We use gold dollars as numeraire. Suppose that a particular j -maturity zero-coupon bond, indexed by i , promises to pay $\bar{m}_{t+j}^{(i)}$ units of gold at time $t + j$. Let $p_t^{(i,j)}$ denote the price of such a bond in units of gold dollars. We invoke one assumption throughout.

Assumption 1. For each $t \geq 0$, there exists a time-varying process ψ that acts as a pricing kernel for all government bonds in the sense that¹³:

$$p_t^{(i,j)} = \mathbb{E}_t \left[\left(\frac{\psi_{t+j}}{\psi_t} \right) \bar{m}_{t+j}^{(i)} \right].$$

We do not impose a particular pricing kernel structure because we have insufficient data to identify an appropriate structure for the US during the nineteenth century. Instead, we specify a pricing kernel that is sufficiently flexible to incorporate environments in which there is stochastic discount factor (SDF) that is consistent with the parametric specification of the yield curve we introduce in the following section.¹⁴

¹³Technically, we have a process at each time, $\psi^{(t)}$. However, with some abuse of notation, we drop the explicit superscript on the pricing kernel to simplify the expressions.

¹⁴We specify a separate kernel process at each t rather than a single kernel to ensure that our asset pricing formulas are compatible

Assumption 1 expresses our key identifying restriction: within each time period, there is no cross-sectional variation in how government promises of bond repayment are priced. Although it is not required for our estimation approach, it will be helpful to keep in mind the following example model where the SDF and default risk are modeled explicitly. Suppose that a real SDF process, S , exists and that the i -th j -maturity zero-coupon government bond pays a stochastic fraction, $\xi_{t+j}^{(i)}$, of its *promised* payment, and provides convenience benefit, $\eta_{t+j}^{(i)}$, proportional to the size of bond flows. In addition, let $e_t^{(g)}$ denote the quantity of consumption goods that can be exchanged for one gold dollar at time t (i.e., the consumption goods price of gold dollars). In this case, the general pricing formula becomes:

$$p_t^{(i,j)} = \mathbb{E}_t \left[\left(\frac{S_{t+j}}{S_t} \right) \left(\frac{e_{t+j}^{(g)}}{e_t^{(g)}} \right) \xi_{t+j}^{(i)} \eta_{t+j}^{(i)} \bar{m}_{t+j}^{(i)} \right]$$

with pricing kernel $\psi_{t+j}^{(i)}/\psi_t^{(i)} = (S_{t+j}/S_t) \left(e_{t+j}^{(g)}/e_t^{(g)} \right) \xi_{t+j}^{(i)} \eta_{t+j}^{(i)}$. So, in this specification, our pricing kernel packages together inflation risk, haircut risk, and convenience benefits. Assumption 1 then asserts that all government bonds have the same haircut risk and convenience benefits. That is, for all i , t , and j , $\xi_{t+j}^{(i)} = \xi_{t+j}$ and $\eta_{t+j}^{(i)} = \eta_{t+j}$. Observe that we allow haircut risk and convenience benefits to vary with the maturity, j , just not by individual bond. We will return to versions of this specific kernel structure in later sections of the paper when we discuss haircut risk, convenience yields, and currency exchange risk in more detail. Our approach lets us impose a minimal structure sufficient to answer the questions that interest us.

Aside: Did Congress issue revenue bonds? Before the Second Liberty Bond Act of 1917, Congress sequentially directed the Treasury to sell particular securities, one issue at a time, and restricted spending the proceeds on specific purposes (e.g., wars, refinancing of outstanding debt, infrastructure). However, Congress has always allowed the Treasury to use general revenue to make coupon and principal payments rather than dedicate specific revenue sources (e.g., tolls or land sales). In the language of municipal finance, Treasury bonds were and continue to be general obligation bonds, not revenue bonds. Consequently, had the Treasury explicitly defaulted on its debt, it is not clear that some securities would have been senior to others. For this reason, we start with Assumption 1 and then inspect pricing errors for violations.

3.1.2 Parameterisation of Gold Dollar Yield Curves

Influenced by data limitations, we impose additional assumptions to facilitate approximating yield curves. For convenience, we define the gold price of a government promise to one gold dollar at time $t + j$ (packaged with any haircut risk and convenience benefit specific to government bonds) as:

$$q_t^{(j)} := \mathbb{E}_t \left[\frac{\psi_{t+j}}{\psi_t} \right]$$

with the version of the [Nelson and Siegel \(1987\)](#) parameterisation that we adopt in Section 3.1.2. Several authors have studied how to reconcile time-varying versions of the [Nelson and Siegel \(1987\)](#) parametric yield curve specification with an arbitrage free asset pricing theory and a single SDF process that would be associated with it. Our model is closest to [Diebold et al. \(2005\)](#), which imposes an autoregressive structure on the NS factors. [Krippner \(2015\)](#) shows that this specification is an “eigenvalue approximation” to an arbitrage free affine asset pricing model, but [Björk and Christensen \(1999\)](#) and [Filipović \(1999\)](#) argue that this approximation cannot be formalized. To deal with this issue, we assume a sequence of stochastic discount factor processes. An alternative approach would be to use an augmented yield curve specification proposed by [Christensen et al. \(2011\)](#). We do not choose this alternative approach here because we want to stay close to the [Diebold et al. \(2005\)](#) specification that is more commonly used in applied work.

and transform this bond price into a zero-coupon yield of maturity j using the formula:

$$y_t^{(j)} := -\frac{\log q_t^{(j)}}{j}.$$

We then define a gold dollar yield curve as an array $\mathbf{y}_t := \{y_t^{(j)}\}_{j=0}^\infty$ and parameterise this object directly. Because we have a long time series but few cross-sectional observations, in Assumption 2, we impose a parameterisation that is parsimonious in the term dimension but rich in the time dimension.

Assumption 2. The j -th component $y_t^{(j)}$ of a gold dollar yield curve takes the form

$$y_t^{(j)} = \beta_{0,t} \left[\left(\frac{1 - \exp(-j\tau)}{j\tau} \right) (j\tau - 2) \right] + (\beta_{1,t} + \beta_{2,t}) \left[\frac{1 - \exp(-j\tau)}{j\tau} \right] - \beta_{2,t} \exp(-j\tau) \quad (3.1)$$

with parameters $\beta_t := [\beta_{0,t}, \beta_{1,t}, \beta_{2,t}]'$ and τ .

This specification is in the spirit of [Nelson and Siegel \(1987\)](#) and has a number of desirable features.¹⁵ First, it is flexible enough to generate “typical yield curve shapes” (e.g., monotonic, humped, and S-shaped curves). Second, it ensures that yields converge as maturity goes to $+\infty$, with $\beta_{0,t}$ parameterizing the asymptote. Third, distinct parameters shape different parts of the yield curve: $\beta_{1,t}$ at the short end of the yield curve, $(\beta_{2,t}, \tau)$ for medium-term yields. Fourth, it is compatible with estimates of recent yield curves.¹⁶

Parameter transformation: The efficiency of the Hamiltonian Monte Carlo algorithm that we will use to sample from our model’s posterior distribution is known to be sensitive to parameterisation: the vector field that guides the sampler toward the “typical region” of the parameter space is *not* invariant to re-parameterisation. Experimenting with alternative parameterisations led us strongly to favor one that uses logarithmic transformations of the β_t vector. However, simply using $\log \beta_{i,t}$ would implicitly force $\beta_{i,t}$ to be strictly positive. To avoid this, we instead define the parameter vector

$$\lambda_t := \left[\log(\beta_{0,t} - \underline{\beta}), \log(\beta_{1,t} - \underline{\beta}), \log(\beta_{2,t} - \underline{\beta}) \right]',$$

where we set the lower-bound value, $\underline{\beta}$, sufficiently low so that the sampled $\{\beta_{i,t}\}$ paths do not get close to their respective boundaries.¹⁷

3.1.3 Pricing Formulas

Suppose that at time t we observe prices on an integer number M_t of *coupon-bearing* government bonds. A given bond, i , promises a sequence of gold dollar coupon and principal payments $\{\bar{m}_{t+j}^{(i)}\}_{j=1}^\infty$. We allow $\bar{m}_{t+j}^{(i)}$ to be zero. Most bonds have finite maturities so we let $J_t^{(i)}$ denote the remaining number of periods with non-zero payments.¹⁸ Let $p_t^{(i)}$ denote the price of a coupon-bearing gold dollar bond in terms of gold.

¹⁵Strictly speaking, this is a different parameterisation than the one originally advocated by [Nelson and Siegel \(1987\)](#), who used $y_t^{(j)} = b_0 + (b_1 + b_2) \frac{1 - \exp(-\tau j)}{\tau j} - b_2 \exp(-\tau j)$, but, of course, the Nelson-Siegel parameterisation matches ours with $\beta_0 = b_0$, $\beta_1 = b_0 + b_1$, and $\beta_2 = b_0 + b_2$. We prefer our parameterisation because the sampler that we use to estimate our model appears to handle it more easily than the one by [Nelson and Siegel \(1987\)](#).

¹⁶For example, [Gürkaynak et al. \(2007\)](#) use this form for the period 1961-1980. After 1980, they use an extension proposed by [Svensson \(1994\)](#) to allow for a second hump in the yield curve regarded as a “convexity effect”. Recent work by [Liu and Wu \(2021\)](#) suggests that estimating yield curves using a non-parametric approach captures more local variation. We are unable to use a similar approach due to data limitations. However, in section 5 and appendix G, we show that our pricing errors are consistently small along bond, time, and maturity dimensions.

¹⁷In practice we set $\underline{\beta}$ arbitrarily and check (*ex post*) whether the posterior sample contains any element that is in the neighborhood of $\underline{\beta}$. If we find such an element, we redo the estimation with a lower value. For the results of this paper we use $\underline{\beta} = -100$.

¹⁸In case of perpetual *consols*, $J_t^{(i)} = \infty$.

Assumption 3. The law of one price holds for government bonds.

To account for differences in maturities and coupons, we view each coupon-bearing bond i as a basket of zero-coupon securities and use the standard pricing formula:

$$p_t^{(i)} = \sum_{j=1}^{\infty} q_t^{(j)} \overline{m}_{t+j}^{(i)}. \quad (3.2)$$

To prepare the way for expressing asset prices as inner products of price and quantity vectors, let:

- $\mathbf{q}_t := \{q_t^{(j)}\}_{j=1}^{\infty}$ denote a sequence of gold dollar zero-coupon bond prices
- $\overline{\mathbf{m}}_t^{(i)} := \{\overline{m}_{t+j}^{(i)}\}_{j=1}^{\infty}$ denote a sequence of promised *coupon and principal* payments in gold dollars and it is understood that $\overline{m}_{t+j}^{(i)} = 0$ for $j > J_t^{(i)}$.

Observe that Assumption 2 parameterises \mathbf{q}_t , so that we can write it as $\mathbf{q}(\lambda_t, \tau)$. This allows us to represent bond prices as

$$p_t^{(i)} = \left\langle \mathbf{q}(\lambda_t, \tau), \overline{\mathbf{m}}_t^{(i)} \right\rangle \quad (3.3)$$

where $\langle \cdot \rangle$ denotes an inner product (on the space of real sequences). Section 3.3 will elaborate on how we use information about $p_t^{(i)}$ and $\overline{m}_t^{(i)}$, together with equations (3.1) and (3.3), to infer parameters $\{\lambda_t\}$ and τ , that pin down our gold dollar yield curve.

3.2 Real Yield Curves

So far, we have focused on a “nominal” pricing kernel and a yield curve denominated in gold coins. We can also define a “real”, i.e., gold inflation adjusted, pricing kernel, $\hat{\psi}$, price, $\hat{q}^{(j)}$, and yield, $r_t^{(j)}$, by:

$$\psi_t := e_t^{(g)} \hat{\psi}_t \quad \hat{q}^{(j)} := \mathbb{E}_t \left[\frac{\hat{\psi}_{t+j}}{\hat{\psi}_t} \right] \quad r_t^{(j)} := - \frac{\log \hat{q}_t^{(j)}}{j}$$

Ultimately, we are interested in estimating real yields. Constructing an *ex ante* real yield curve series requires estimating inflation expectations at various horizons. To this end, we define gold inflation between period t and $t + j$ as $\pi_t^{(j)} := \log \left(e_t^{(g)} / e_{t+j}^{(g)} \right)$ and use the following “risk-neutral” approximation of the *ex-ante* real yield:

$$r_t^{(j)} := y_t^{(j)} + \frac{1}{j} \log \mathbb{E}_t \left[\exp \left(- \pi_t^{(j)} \right) \right] \quad (3.4)$$

Researchers have developed sophisticated techniques for estimating inflation expectations (the second term in expression 3.4) incorporating macroeconomic data and theory. In principle, we could implement similar tools for the post WW1 period. However, the lack of reliable macroeconomic data limits our ability to extend these techniques back into the nineteenth century. Instead, we follow our “macro-theory-lite” approach and use a flexible statistical model of inflation with random walk stochastic volatility and drifting mean and persistence to obtain inflation expectations that are consistently estimated throughout our entire sample. Appendix D describes details of our procedure—a step separate from our strategy for estimating the gold yield curve that we turn to now.

3.3 Estimation Strategy

Using our pricing formulas from Section 3.1.3 we build a statistical model for which a likelihood function can be derived. To this end, we introduce two types of Gaussian shocks: (1) shocks that move parameter vector λ_t over time, and (2) pricing errors for each bond. These building blocks give rise to a richly parameterised—yet tractable—non-linear state space model. To estimate its (more than 7,500) parameters, we apply Bayesian Markov Chain Monte Carlo (MCMC) methods. We specify weakly informative prior distributions for the model’s hyper-parameters (see Appendix E) with the specific purpose of *regularizing* our estimator and facilitating smooth operation of the sampling algorithm. We do not aim to choose these priors to summarize our subjective beliefs faithfully, rather we view them as tools that help our statistical model make reliable inference about the objects we care about.

3.3.1 Pooling Across Time

Assumption 2 explicitly makes the yield curve parameters λ_t time-dependent, which raises the question: how do the different components relate to each other over time? A widely used yield curve estimate—available for the period after 1960—by [Gürkaynak et al. \(2007\)](#) assumes no intertemporal dependence among the four parameters in (3.1): they estimate a different set of β s and τ for each t using only bond prices available at date t . Therefore, nothing learned about the yield curve at any one date contributes to their estimates for other dates. Unlike the post-WW2 period, prior to the First World War, price data are sparse and coverage varies across time. Consequently, we need to pool information over time to estimate a time series of yield curve parameters. To this end, we introduce a multilevel (a.k.a. an hierarchical) statistical model.

At any given point in time, the shape of the gold dollar yield curve is summarized by four parameters: τ which we assume to be time-invariant and the three parameters in λ_t whose time variation we model with a flexible stochastic process specified in Assumption 4 below.

Assumption 4. Parameter vector λ_t follows:

$$\lambda_{t+1} = \bar{\lambda}_t + \varrho(\lambda_t - \bar{\lambda}_t) + \Sigma_t^{\frac{1}{2}} \varepsilon_{\lambda,t+1}, \quad \varepsilon_{\lambda,t+1} \sim \mathcal{N}(\mathbf{0}, \mathbb{I}_3), \quad \forall t \geq 0,$$

where ϱ is a 3×3 matrix, Σ_t is a covariance matrix decomposed as¹⁹

$$\Sigma_t = \Xi_t \Omega \Xi_t, \tag{3.5}$$

where Ω is the time-invariant correlation matrix and Ξ_t is a diagonal matrix containing the marginal standard deviations $\sigma_t := [\sigma_{1,t}, \sigma_{2,t}, \sigma_{3,t}]'$ that follow:

$$\log \sigma_{t+1} = \log \sigma_t + \Xi_\sigma \varepsilon_{\sigma,t+1}, \quad \varepsilon_{\sigma,t+1} \sim \mathcal{N}(\mathbf{0}, \mathbb{I}_3), \quad \forall t \geq 0,$$

where Ξ_σ is a positive definite diagonal matrix. Elements of the vector $\bar{\lambda}_t$ follow independent random walks:

$$\bar{\lambda}_{t+1} = \begin{cases} \bar{\lambda}_t + \bar{\Xi} \varepsilon_{\bar{\lambda},t+1} & \text{if } t = k\Delta \text{ for } k \in \mathbb{N} \\ \bar{\lambda}_t & \text{otherwise} \end{cases}, \quad \varepsilon_{\bar{\lambda},t+1} \sim \mathcal{N}(\mathbf{0}, \mathbb{I}_3), \quad \forall t \geq 0,$$

where $\bar{\Xi}$ is a positive definite diagonal matrix and $\Delta \geq 1$ is the frequency at which $\bar{\lambda}_t$ updates.

Four characteristics of this model deserve further discussion:

¹⁹This decomposition implies $\sigma_i := \Xi_{i,i} = \sqrt{\Sigma_{i,i}}$ and $\Omega_{i,j} = \frac{\Sigma_{i,j}}{\sigma_i \sigma_j}$ for $i, j \in \{1, 2, 3\}$. See [Barnard et al. \(2000\)](#).

- (i) Parameter matrix Σ_t governs how evidence about a yield curve at one date affects inferences about yield curves at other dates. The closer are two dates to each other, the more correlated are the associated yield curves, with Σ_t capturing what “close” means.²⁰ The limit $\Sigma \rightarrow 0$ corresponds to *complete pooling*: here the yield curve is assumed to be fixed over time so that each observation has an equal influence as all other dates. Contrary situations in which $\Sigma \rightarrow \infty$ call for *no pooling*: there is no relationship between adjacent parameter estimates, we use only period t information to estimate period t yield curve parameters as in [Gürkaynak et al. \(2007\)](#). By inferring Σ from the data, we learn how much pooling across time we should do to improve estimates in light of intertemporal imbalances in data availability. In this context, “stochastic volatility” means that the amount of pooling can be time varying throughout the sample.
- (ii) We allow shocks to different components of λ_t to be correlated. This enables us to infer relatively precise estimates of the short end of the yield curve throughout the whole sample period. Assuming that different parts of the yield curve follow correlated but time-invariant dynamics allows us to transmit what we learn about co-movements between short- and long-term yields from years when many maturities are outstanding (as in the second half of the 19th century) to years when data about short-term yields are scarce (as in the early 20th century).
- (iii) The yield curve parameter processes are decomposed into permanent and temporary components: the vector $\bar{\lambda}_t$ denotes the slow moving “long-run mean” to which λ_t reverts and the matrix ϱ governs the rate at which this mean reversion occurs. We refer to $\bar{\lambda}_t$ as the “low-frequency” component and $\lambda_t - \bar{\lambda}_t$ as the “temporary” component of λ_t . We impose this structure to allow for potential mean reversion in the yield curve without imposing a common mean across the entire period from 1791-1933.
- (iv) The long-run mean $\bar{\lambda}_t$ follows a random walk with updates at frequency Δ . As $\Delta \rightarrow \infty$, the frequency of parameter updates goes to zero, providing a state-space model with time-invariant long-run mean $\bar{\lambda}$. Setting $\Delta > 1$ to low values is a compromise between identifying the long-run mean with high accuracy and letting it move over time. Effectively, we divide the period of interest into subperiods of equal length Δ and assume complete pooling within subperiods and partial pooling across subperiods.

3.3.2 Bond-specific Pricing Errors

In addition to the obvious difficulties arising from transcribing and time aggregating price quotations from newspapers, we have other reasons to believe that, in certain periods, particular bonds violate the collection of assumptions that we imposed in Section 2.3: perfect foresight about discretionary contract components and common haircut risk and convenience yields across all government bonds. We address this issue by introducing *bond specific* pricing errors, modelled as random variables with Gaussian distribution (in Assumption 5) and by using our statistical model to estimate error distribution parameters jointly with yield curve parameters.

Assumption 5. Each bond has a pricing error with the following stochastic properties: errors on bond i are independent from errors on other bonds, and the distribution of errors on bond i is a time-invariant Gaussian distribution with mean 0 and standard deviation $\sigma_m^{(i)}$.

Introducing these errors enables our statistical model to decrease the influence of peculiar bonds on our yield estimates while still informing us about situations when our collective assumptions prevent us from consistently

²⁰One might be inclined to call this procedure “stochastic smoothing” because consecutive λ_t vectors are linked by a sequence of random variables $\{\varepsilon_{\lambda,t}\}$. Alternatively, one could define a deterministic smoothing function that specifies the sequence $\{\lambda_t\}$ in terms of parameters λ_0 and Σ , mimicking frequently used averaging techniques like a simple moving-average. Modeling the sequence $\{\lambda_t\}$ as a stochastic process allows our algorithm to choose from a much richer set of smoothing functions.

pricing the *cross-section* of bonds using our pricing formulas. We view the imposition of a non-bond specific pricing kernel in Assumption 1 as our key identification assumption. If Assumption 1 were violated and some bonds had idiosyncratic haircut risk $\xi^{(i)}$ or convenience yield $\eta^{(i)}$ processes, then we would expect to see large estimates of $\sigma_m^{(i)}$ for some of the observable bonds. We use this as a diagnostic tool to inspect whether we should change the way we treat the cash flows from particular bonds, exclude particular bonds from the estimation of certain yield curves, or divide bonds into new subgroups according to their common characteristics and re-estimate a yield curve for each group separately.²¹ If yield curve estimates differ across the subgroups, then we can interpret the difference as the yield premium arising from the specific characteristic.

3.3.3 A Nonlinear State Space Model of Bond Prices

Using these building blocks, we can write our nonlinear state space model as:

$$\begin{aligned}
\tilde{p}_t^{(i)} &= \langle \mathbf{q}(\lambda_t, \tau), \bar{\mathbf{m}}_t^{(i)} \rangle + \sigma_m^{(i)} \varepsilon_t^{(i)} && \text{gold bonds} \\
\lambda_{t+1} &= \bar{\lambda}_t + \varrho(\lambda_t - \bar{\lambda}_t) + \Sigma_t^{\frac{1}{2}} \varepsilon_{\lambda, t+1} && \text{yield curve parameters} \\
\log \sigma_{t+1} &= \log \sigma_t + \Xi_\sigma \varepsilon_{\sigma, t+1} && \text{stochastic volatility} \\
\bar{\lambda}_{t+1} &= \begin{cases} \bar{\lambda}_t + \bar{\Xi} \varepsilon_{\bar{\lambda}, t+1}, & \text{if } t = k\Delta \text{ for } k \in \mathbb{N} \\ \bar{\lambda}_t & \text{otherwise} \end{cases} && \text{long-run mean} \\
\text{with } \varepsilon_t^{(i)} &\sim \mathcal{N}(0, 1) \quad \forall i, \quad \varepsilon_{\lambda, t} \sim \mathcal{N}(\mathbf{0}, \mathbb{I}_3) \quad \varepsilon_{\bar{\lambda}, t} \sim \mathcal{N}(\mathbf{0}, \mathbb{I}_3) \quad \varepsilon_{\sigma, t} \sim \mathcal{N}(\mathbf{0}, \mathbb{I}_3), \forall t \geq 1
\end{aligned}$$

where $\tilde{p}_t^{(i)}$ denotes the *observed* period- t price of bond i in terms of gold. The posterior distribution of this model is obtained by adding up the Gaussian log-likelihoods associated with the independent shocks and combine them with priors described in Appendix E.

3.3.4 Methodological Contribution

Alternative to Particle Filtering: Estimating the model in Section 3.3.3 involves a complicated filtering problem due to the non-linear nature of bond prices and the existence of stochastic volatility. A standard approach to such non-linear filtering problems is to use some version of particle filtering. However, thanks to the length and other complexities of our data set, the well-known drawbacks of particle filters, such as sample degeneracy and impoverishment, become particularly acute in our case. We deploy an alternative strategy and approach the problem as a high-dimensional statistical model by “treating latent variables as parameters.”²² From this viewpoint, the model has more than 7,500 parameters. To cope with such a high-dimensional parameter space, we use Hamiltonian Monte Carlo with a “No-U-Turn Sampler” of Hoffman and Gelman (2014), along with subsequent developments described in Betancourt (2018). The basic idea of the method is to use slope information about the log-likelihood to devise an efficient Markov Chain Monte Carlo sampler. This method can attain a nearly i.i.d. sample from the posterior by proposing moves to distant points in the parameter space through (an approximately) energy conserving simulated Hamiltonian dynamic.

Bond-specific pricing errors for classification: In theory, idiosyncratic bond characteristics, such as denomination, flexible maturities, and conversion options, would require custom pricing formulas for each bond. In practice, such a procedure is impractical while it is a priori unclear that all features are equally important for bond pricing. To decide which bond characteristics warrant special treatments, we devise a “cost-benefit

²¹This error inspection process spiritually joins Hansen and Jagannathan (1997).

²²We use quotation marks because in the Bayesian paradigm there is no clear distinction between latent variables and parameters.

analysis” tool in the form of bond-specific pricing errors: starting with presuming that all bonds can be priced with a common time-varying pricing kernel, we look for patterns in the estimated pricing errors, the idea being that misjudgments in our bond classification would show up as large, cluster-specific relative pricing errors. This approach helped us identify two types of bonds that require special treatment: (i) greenback-denominated bonds, and (ii) bonds that are close to maturity. As for the greenback bonds, we devise custom pricing formulas detailed in Subsection 6.1.²³ As for bonds with short maturity, we suspect the large relative pricing errors are due to a liquidity premium that emerges from the relative ease in which such bonds could be used for transactions. We deal with this misclassification by dropping prices of bonds that are less than one year to maturity from the sample that we use to estimate our yield curve. In Subsection 4.2 we use the residual pricing errors on these bonds as a proxy for the liquidity premium on money-like federal liabilities.

Computational issues: While `Stan` might seem an obvious choice for the task at hand—it is a well-developed software that efficiently implements the HMC-NUTS sampler—non-trivial features of our data set make it inconvenient for our purposes. Some of the main technical difficulties we face are: (1) the number of observed assets changes over time, (2) each bond has a payoff stream of varying length, (3) periods without price observations, (4) the set of bond-specific pricing errors that are relevant at a given period t changes over time in a complicated fashion, etc. To tackle these difficulties, we code the log posterior function of our model from scratch and feed it into the `DynamicHMC.jl` package by [Papp et al. \(2021\)](#) which is a robust implementation of the HMC-NUTS sampler mimicking many aspects of `Stan`. An important advantage of this package is that it allows the user to provide the Jacobian of the log-posterior manually. Not having to rely on automatic differentiation for a model with 7,500+ parameters cuts running time by several orders of magnitude. In most cases, we use the recommended (default) tuning parameters for the NUTS algorithm.

3.4 Laboratory Experiment

Our parameterisation can capture a wide range of yield curve shapes. However, as was shown in Figure 1, we want to infer yield curve parameters from relatively few price observations, with most observed prices being for long term bonds. How can we recover short yields? To show how pooling information over time can help with this matter, we conduct a “laboratory experiment”: taking a particular yield curve process (in line with our state space model in Subsection 3.3.3) as given, we use it to price four bonds with known characteristics (maturity, coupons, pricing error), then perform our econometric procedure, and compare our posterior yield estimates to the true values that generated our artificial data. We investigate two scenarios:

Case 1: long term bonds with maturity dates that are distributed relatively evenly over the sample period

Case 2: there is an extended period without bonds that mature in less than 10 years

We create bonds that are “representative” of our sample in the sense that they are long term. Here information about short yields must be recovered from prices of bonds that were originally long term but are now approaching maturity.

The rows of Figure 2 depict the outcomes of the two scenarios. The red lines are the true 1-year (middle column) and 10-year yields (right column) that were used to generate prices of the four bonds, the characteristics of which are depicted in the left column. The blue lines depict the posterior median and the shaded blue area depicts the 90% interquantile range of the posterior distribution. Even though we have few price observations

²³Similarly, we found evidence that special treatment is needed for the 5-20s whose principal denomination was ambiguous during and after the Civil War. We drop these bonds from our sample and leave the construction of 5-20s-specific pricing formulas for future research.

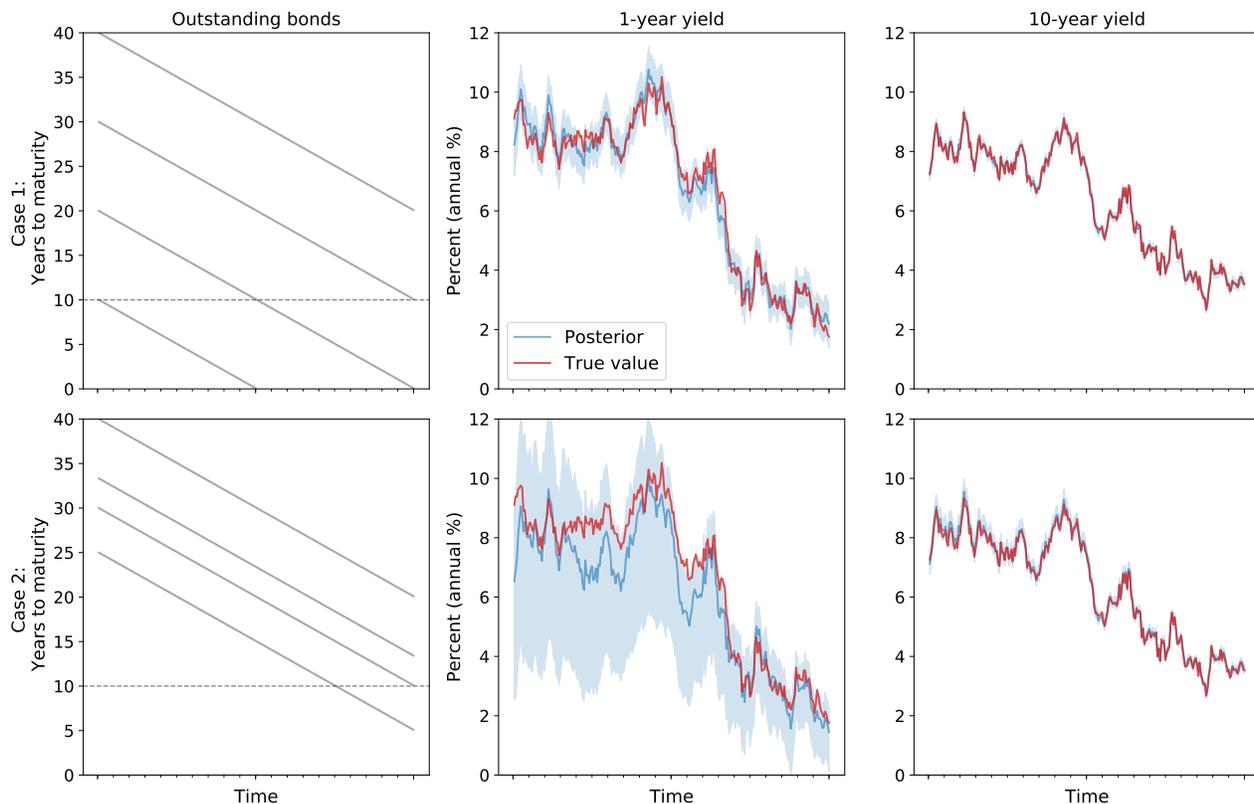


Figure 2: Comparison of Posteriors to True Values.

Artificial samples with 4 bonds ($T = 20$ year). **Case 1: [top row]** (i) 6 % (semi-annual), 10 year maturity, $\sigma_m^{(i)} = 3$; (ii) 3 % (semi-annual), 20 year maturity, $\sigma_m^{(i)} = 2$; (iii) 5 % (semi-annual), 30 year maturity, $\sigma_m^{(i)} = 1$; (iv) 2 % (semi-annual), 40 year maturity, $\sigma_m^{(i)} = 4$. **Case 2: [bottom row]** (i) 6 % (semi-annual), 25 year maturity, $\sigma_m^{(i)} = 3$; (ii) 3 % (semi-annual), 33 year maturity, $\sigma_m^{(i)} = 2$; (iii) 5 % (semi-annual), 30 year maturity, $\sigma_m^{(i)} = 1$; (iv) 2 % (semi-annual), 40 year maturity, $\sigma_m^{(i)} = 4$.

for bonds with short maturity, the algorithm nevertheless does a good job of recovering the true 1-year yield under the first scenario (Case 1). That is, at least when the common pricing kernel assumption is a good description of the data, observing a few long term bonds can be sufficient to recover the short end of the yield curve as long as the maturity dates of the observed bonds are distributed relatively uniformly over time. This is what our model’s ability to pool information buys us.

To illustrate this point, Case 2 represents a scenario when all four bonds mature beyond 20 years and shorter term securities are not issued in the meantime,²⁴ so our model has little chance to utilize information about short yields. The result is depicted in the bottom row of Figure 2. As can be seen, the algorithm can still recover the true 10-year yield (it can observe bonds close to 10-years in the second half of the sample) but it has much more trouble trying to recover the 1-year yield. The posterior 90% interquartile range is large, and the posterior median departs significantly from the true value for many periods. This illustrates that the structure of our Nelson-Siegel parameterisation does not automatically generate tight posteriors. We do need some observations of prices for short maturity bonds to recover the yield curve.

²⁴This scenario describes the last decade of the eighteenth century well, during which we observe only the three “Hamilton bonds.”

4 Stylized Facts about the Gold Standard Era: 1791-1933

In this section, we use our estimate of the gold denominated US federal yield curve to establish a collection of stylized facts: (i) US debt traded at a premium to UK debt until the late nineteenth century, (ii) a “short rate disconnect” existed until the late nineteenth century, in the sense that debts close to maturity traded at a premium, (iii) long term inflation expectations were close to zero until the 1880s so that long term gold denominated yields can be interpreted as real yields, and (iv) the slope of the yield curve was typically negative before the Civil War and positive afterwards. We discuss how these facts reflect changes in the US federal government’s reputation, monetary policy, and financial regulation.

4.1 Long term yields fell to UK rates and were high in wars

Figure 3 depicts selected long term yield estimates. The solid black and grey lines depict the median of our posterior estimate for the 10-year, gold denominated, zero coupon yield. We use black for periods with price observations for bonds with maturity within 1 to 10 years so the estimate can be considered an interpolation. Otherwise, we use grey. We show the 25 year gold dollar zero-coupon yield for dates before 1800 because we only have price observations for very long term bonds during that period. The grey bands around the posterior median depict the 90% interquartile range.

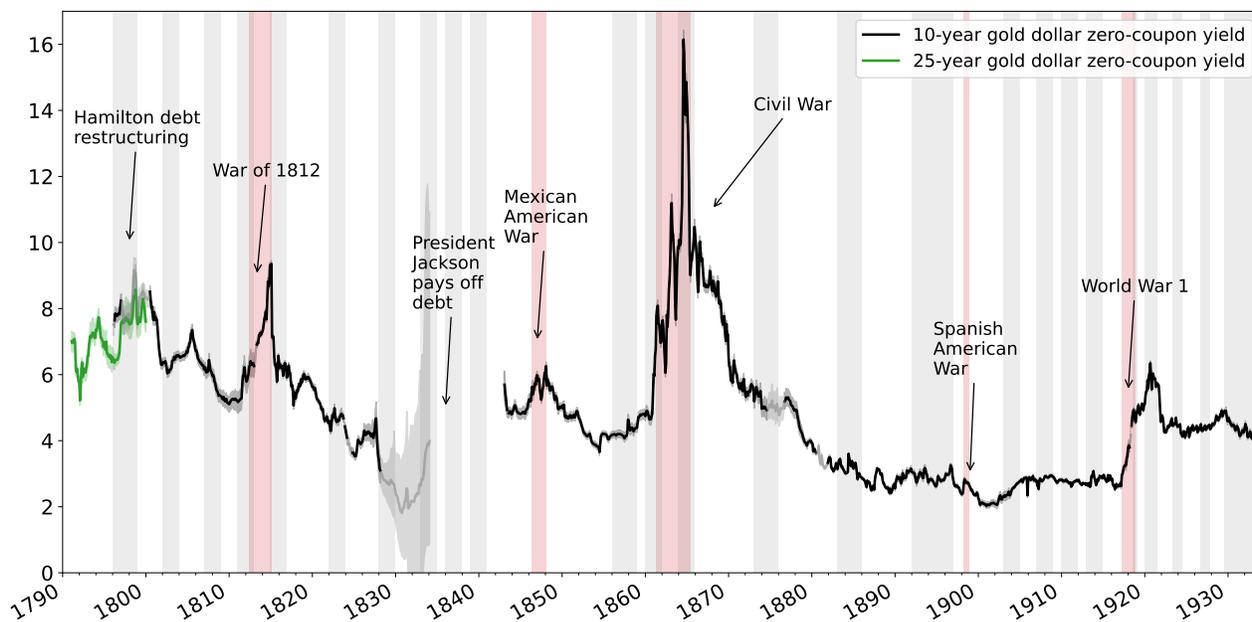


Figure 3: Long-Term Yield Estimates.

The solid black line depicts the mean of our posterior estimate for the 10-year, gold denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquartile range. The light gray intervals depict recessions as dated by [Davis \(2006\)](#) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

Long yields trended downward throughout the nineteenth century interrupted by sharp (but temporary) increases during times of war and financial turmoil. Our approximating yields are quite volatile during the 1790s when secondary markets in Treasury securities were thin. Yields fell steadily from January 1791 to March 1792 when a financial panic caused sharp drops in bond prices and corresponding increases in yields. Long-term

yields remained high for the remainder of the decade and spiked at 9% in August 1798, one month after the Congress authorized a 15-year loan paying an 8% coupon to cover increased military spending at the outbreak of the Quasi-War with France. Yields trended downward thereafter, and by 1803 the US government was able to issue at par a \$11.25 million 15-year loan with a 6% percent coupon to finance the Louisiana Purchase. The sharpest increases come during the War of 1812 and the Civil War. During the War of 1812, the 10-year zero-coupon yield spiked to over 9%. A big source of funds for this war was the Treasury's issuing of five long-term loans with face values totaling \$66 million. Resistance to the war mainly from Federalists in the Northeast and a failure to replace lost customs revenue with internal taxes forced the Treasury to sell these bonds at deep discounts. Bayley (1882) reports that two of these loans were sold at 12% discounts, and a third was sold at a 20% discount. Those officially-stated discounts understate the true discounts, since for payment the Treasury accepted at face value bank notes whose market values had sunk substantially below par. The Treasury again had trouble selling new bonds at par during the Civil War, leading to much higher yields. Our ten-year gold yield estimate reaches a peak of 16% near the end of the Civil War.

One explanation for the sustained decrease in long yields is a fall in the risk premium on US federal debt securities. After the American War for Independence, the Continental Congress owed approximately \$52 million in foreign loans to France, Spain, and Holland, loan office and debt certificates to the American public, and unpaid interest. The Congress confronted substantially higher long term yields than the UK even though the UK then had a high debt-to-GDP ratio. This situation spawned a lively debate in the US about whether and how to service wartime debts. Treasury Secretary Alexander Hamilton argued in his *1790 Report on Public Credit*

For when the credit of a country is in any degree questionable, it never fails to give an extravagant premium, in one shape or another, upon all the loans it has occasion to make. Nor does the evil end here; the same disadvantage must be sustained upon whatever is to be bought on terms of future payment.

Ultimately, Hamilton and others persuaded Congress to repay the foreign debt at face value and issue new bonds to refinance the domestic certificates and interest in arrears. Hamilton claimed that by following through on this policy the US could eventually acquire a reputation for servicing its debts that would reduce US interest rates to the lower levels than paid by the UK government.

We use Figure 4 to quantify whether and when Hamilton's hopes were fulfilled. The figure compares yields-to-maturity on gold denominated UK consols to yields-to-maturity on *hypothetical* gold denominated US consols that promise the same coupon flows as the UK consols.²⁵ We plot a yield-to-maturity on gold denominated UK consols because almost all UK government bonds were consols, so that is the only UK yield that can be reliably estimated. First, notice that the hypothetical US consol yield exhibits a downward trend, falling from close to 8% at the beginning of the nineteenth century to around 2% at the end of the century. Second, notice that the US yield was typically higher than the UK yield until the 1880s with a temporary convergence during the 1820s. Third, notice that the US yield was persistently lower than the UK yield after 1900. This suggests that the combination of the federal government's having serviced War of Independence IOUs, admittedly with substantial haircuts to domestic creditors, and having completely retired all debt by the mid 1830s, along with activities of the First and Second Bank of the United States made significant progress toward realizing Hamilton's hopes. However, the reemergence of the spread between US and UK debt during the period from 1840-1880 suggests that it wasn't until the introduction of the National Banking Act and the reestablishment of gold-greenback

²⁵The UK consol yield is the series "Spliced consol yield 1753-2015, corrected for Goschen's conversion issues" from Thomas and Dimsdale (2017). The hypothetical, gold denominated US consols promise the same annuity coupon payments as those used in the UK consol yield series

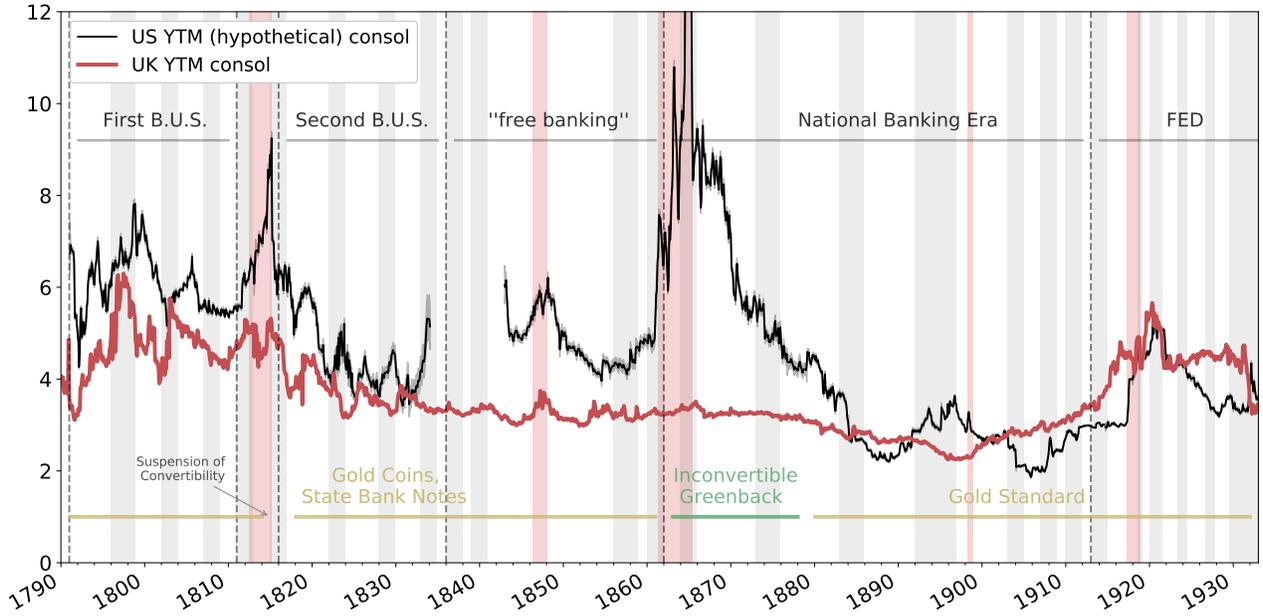


Figure 4: US and UK Long Term Yields.

The solid black line depicts the mean of our posterior estimate for the yield-to-maturity on hypothetical gold denominated US consols that promise the same coupon flows as the UK consols. The grey bands around the posterior mean depict the 90% interquantile range. The solid green line depicts the mean of our posterior estimate for the 10-year, gold denominated, zero coupon yield. The pale green bands around the posterior mean depict the 90% interquantile range. The green line depicts the UK long-term yield (implied by the 3% consol price) from [Thomas and Dimsdale \(2017\)](#). The light gray intervals depict recessions as dated by [Davis \(2006\)](#) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

parity that Hamilton’s vision was fully realized.

The difference between the yield-to-maturity on the UK consol and the hypothetical US consols most likely reflects different haircut risks. UK debt was considered a “safe-asset” during the nineteenth century, whereas many military and political incidents probably induced investors to regard nineteenth century US debt to be risky. In Appendix C.1, we state conditions under which one can interpret the difference between US and UK consol yields as reflecting the risk premium on US federal debt. Under this interpretation, Figure 4 suggests that US federal debt traded with a risk premium until the late nineteenth century when it became an alternative “safe-asset”. Evaporation of those risk-premia signals a realignment of global finance that ultimately led US government debt to replace UK debt as a global “safe-asset” during and after the years of the Bretton Woods arrangement.²⁶

We can better understand the trend decline in the yields by studying the “low-frequency” component of the yield curve. We do this in Figure 28 in Appendix I, which depicts both our posterior estimate for the 10-year yield and long run mean yield. We interpret the long run mean as reflecting the impact of long term structural changes in the economy and interpret the difference between the yield and its long run mean as reflecting the impact of temporary events such as wars and economic crises. As can be seen, the low frequency component of the yield curve declines steadily until the 1880s when it stabilizes around 2%. This suggests a possible structural break during the National Bank Act. We explore this possibility further in the subsequent sections.

²⁶[Chen et al. \(2022\)](#) describe this realignment in detail focusing on its fiscal implications for both the UK and US.

4.2 Premium on short term bonds (“short-rate disconnect”)

Analysts have argued that the modern US federal debt yield curve exhibits a “short-term disconnect,” in the sense that short term bonds are over-priced relative to a hypothetical price implied by pricing kernels that successfully price bonds at longer maturities.²⁷ In Figure 5, we use our statistical model to examine whether a similar short-rate disconnect existed during the nineteenth century. The pale blue dots depict the difference between model-implied and observed yield-to-maturities for bonds with *less than one year* to maturity. Because our model was estimated using bonds with maturity greater than 1 year, these dots represent an “out-of-sample” fit at the short end of the yield curve. The solid blue line depicts the 15-year centered moving average of these blue dots. The orange solid line depicts the 15-year centered moving average of the difference between model-implied and observed yield-to-maturities for bonds with *more than one year* to maturity. Evidently, pricing errors average out for bonds with long maturities but are systematically positive for extended periods for bonds close to maturity.²⁸ In particular, until the 1880s, bonds close to maturity traded with a premium in a range of 0.5 to 1.0 percentage points. The premium effectively disappeared from the 1880s until the First World War before reappearing in the 1920s. We interpret this as strong evidence that there has been a short rate disconnect through most US history, with a period towards the end of the nineteenth century when the short rate disconnect disappeared.

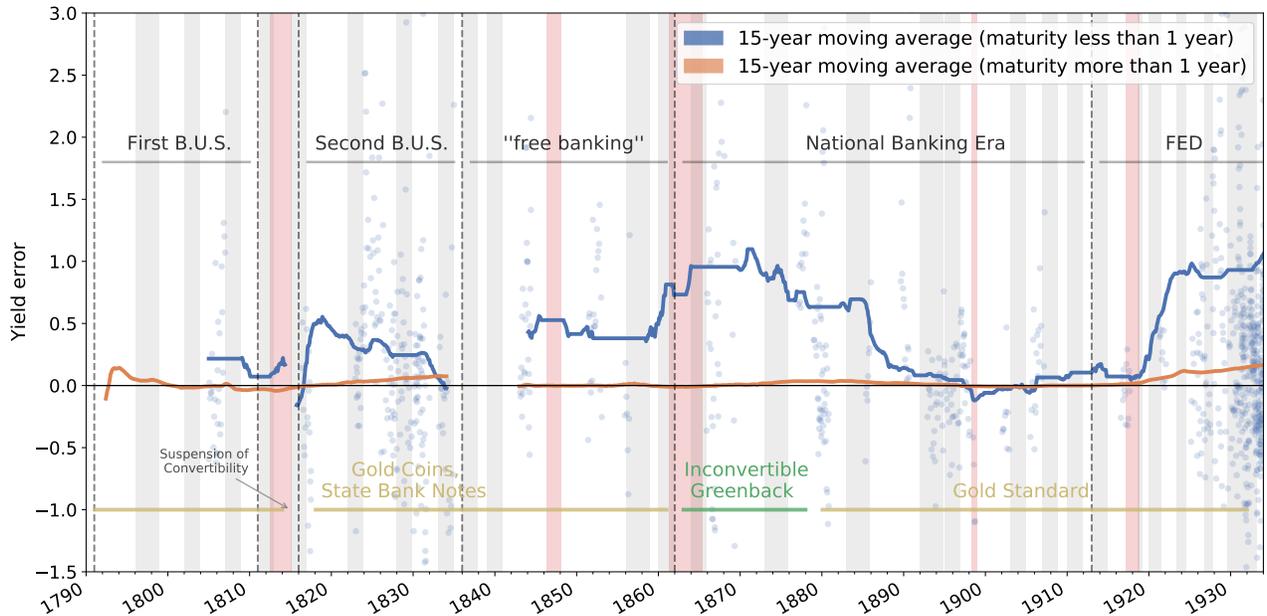


Figure 5: Short Rate Disconnect

Pale blue dots depict the difference between model-implied and observed yield-to-maturities for bonds with *less than one year* to maturity. The solid blue line depicts the 15-year centered moving average of these dots excluding yield errors with magnitude greater than 4 (to handle potential outliers from data issues). The orange solid line depicts the 15-year centered moving average of the difference between model-implied and observed yield-to-maturities for bonds with *more than one year* to maturity. The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

²⁷For instance, see Duffee (1996), Greenwood et al. (2018), and Lenel et al. (2019).

²⁸In appendix I, we present and discuss evidence that the pricing errors also average out within a finer set of maturity bins. We take this as evidence that the short rate disconnect is not driven by the inability of the Nelson and Siegel (1987) parameterization to fit the maturities that we include in our estimation.

The evolution of the short rate disconnect probably indicates how successfully different money and banking eras were able to create a stable currency throughout the nineteenth century. Before the Civil War, the money supply was limited to gold dollars and state bank notes, the latter typically trading at volatile discounts that reflected inconsistent and opaque backing. This is consistent with the persistent premium we observe in the first half of the nineteenth century. In particular, the short rate disconnect is large early in the tenure of the Second Bank of the US, which coincides with the Second Bank of the US's attempt to restrict bank money creation and credit.

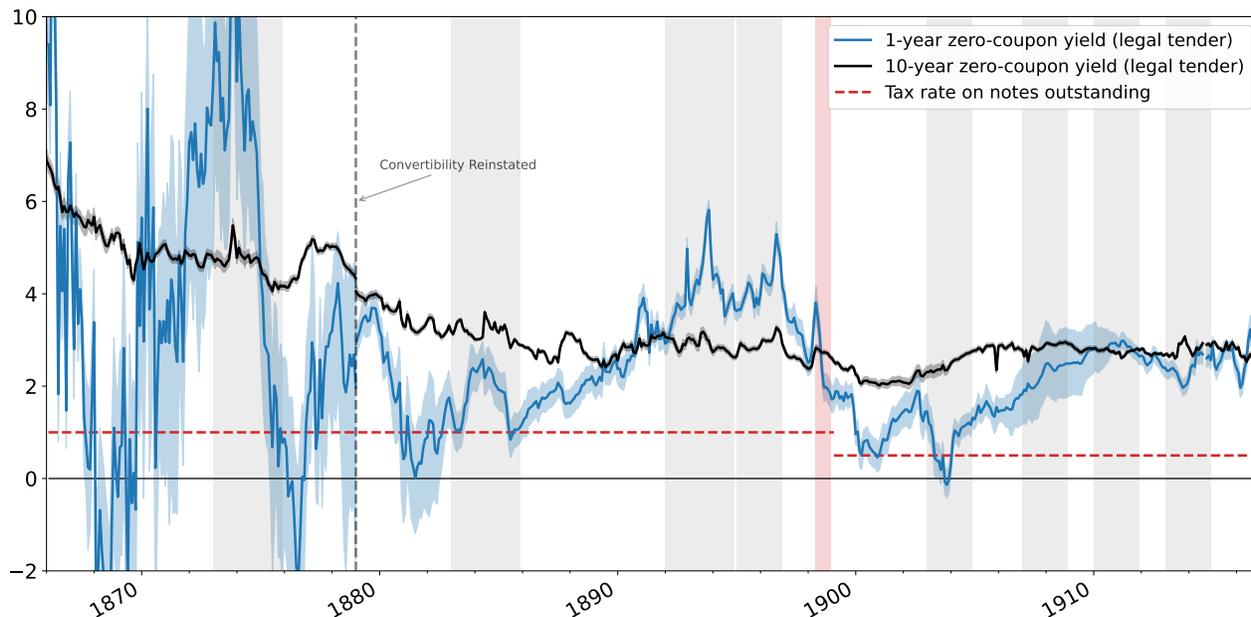


Figure 6: Comparison to Tax Rates on Outstanding Bank Notes

The red dashed line depicts the tax rate on note issuance. The solid black line depicts the mean of our posterior estimate for the 10-year, legal tender, zero coupon yield. The grey bands around the posterior mean depict the 95% interquartile range. The solid blue line depicts the mean of our posterior estimate for the 1-year, legal tender, zero coupon yield. The light blue bands around the posterior mean depict the 95% interquartile range. The black dashed line depicts the tax rate on note issuance. The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

During the Civil War, the government started issuing paper greenbacks (in 1861) and passed three National Banking Acts (in 1862, 1865, and 1866) that established a system of federally regulated national banks. National banks could issue standardized bank notes backed by their holdings of US federal bonds.²⁹ From 1862-1878, greenback dollars traded at a significant discount to gold dollars. On January 1, 1879, the government began converting greenbacks into gold dollars at par. Key goals of the National Banking Acts were to increase supplies of liquid assets and to increase financial sector demand for long term US federal debt so that the government could borrow at a lower cost. Our estimates enable a novel evaluation of how successfully those Acts achieved their goals. Translated into our calculations, the National Banking Acts sought to eliminate the short rate

²⁹Technically, national banks could issue bank notes for circulation according to the following rules. Banks had to deposit certain classes of US Treasury bonds as collateral for note issuance. Permissible bonds were US federal registered bonds bearing coupons of 5% or more. Deposited bonds had to be at least one-third of the bank's capital (not less than \$30,000). Banks could issue bank notes up to an amount of 90% of the maximum of the market value of the bonds and the par value of the bonds. The 90% value was changed to 100% in 1900. Before 1900, the banks had to pay 1.0% tax on the notes they had issued. After 1900, they had to pay a 0.5% tax.

disconnect and decrease long term yields. Figures 4 and 28 indicate that neither goal was immediately achieved in the 1860s but that both goals were largely achieved by the 1880s. This timing pattern offers suggestive evidence that parity between greenbacks and gold helped get the National Banking Act to work as desired, which in turn suggests that bank note issuance had initially been restrained by risks associated with currency devaluation. This allows us to shed light on a long standing puzzle about the issuance of bank notes. Researchers have argued that there was persistent under-issuance of national bank notes during the National Banking Era because the yield on eligible treasuries did not consistently fall to the tax rate on notes outstanding (see [Champ et al. \(1994\)](#), [Champ and Wallace \(2003\)](#), [Champ \(2007\)](#)). We show in Figure 6 that our estimates confirm this observation; short term and long term yields were typically above the tax rates. However, as acknowledged by many researchers, comparing yields to the tax rate on note issuance is a misleading way to test the National Banking Act because many other forces could have shifted levels of yield curves. Our analysis shows that, if we focus on the short rate disconnect rather than a the spread relative to the tax rate, then it appears that the National Bank Act achieved considerable success once parity between greenbacks and gold was restored.

After the introduction of the Federal Reserve System (FED) and World War I, the short-rate disconnect reemerge. This likely reflects the Treasury’s decision to introduce Certificates of Indebtedness—interest-bearing securities with less than 1 year maturity—to smooth the mismatch between quarterly tax receipts and the Liberty Bonds’ irregular coupon and principal payments. Most blue dots that we see in Figure 5 during the FED era correspond to these certificates. [Garbade \(2012\)](#) describes how three innovations by the New York FED in 1920 increased the liquidity of these certificates relative to other government bonds: (i) they started to raise discount rates that forced the Treasury to let certificates be traded below par, (ii) they extended credit to all banks and dealers via repurchase agreements (“repos”) secured by certificates, and (iii) they offered a service for “wire transfer” of certificates.³⁰ This increase in the relative liquidity of the Certificates of Indebtedness is consistent with a short rate disconnect reappearing. The large variation in the short rate disconnect across the different regulation eras suggests that it can be thought of as reflecting a government design choice.

4.3 Inflation anchor until 1880s

The previous subsections discussed nominal gold denominated yield curves. To understand the real cost of financing, we need to estimate inflation expectations and a real yield curve. As mentioned in Subsection 3.2 and explained in detail in Appendix D, we estimate a flexible statistical model of inflation with stochastic volatility, drifting mean and persistence to obtain inflation expectations.³¹ The posterior distributions of conditional moments implied by this model are depicted in Figure 7. The top panel shows conditional inflation expectations: color grey refers to long term expectations (permanent component of inflation), color blue represents inflation expectations one year ahead. The grey line in the bottom plot depicts the posterior median estimate for the model implied 5 year ahead conditional inflation volatility.³²

We infer slow moving changes within the gold standard era. Throughout most of the nineteenth century, gold inflation expectations were anchored around zero or negative (especially between 1810-1850). However, this did not mean *stable* inflation: wars, recessions, and panics were associated with sharp increases in inflation volatility. The story starts to change in the 1880s when long-run gold inflation expectations started to become positive and inflation volatility dropped. In this sense, we transition from a period with large but temporary

³⁰The certificates were the precursors of the *zero-coupon* Treasury Bills that the Treasury started to issue in 1929.

³¹Our model is similar in spirit to the one used by [Cogley and Sargent \(2015\)](#), except that the mean-reverting part of inflation is more flexibly parameterized. In addition, we use an inflation series that combines wholesale prices with CPI whereas [Cogley and Sargent \(2015\)](#) use wholesale price inflation from 1850 to 2012. [Cogley and Sargent \(2015\)](#) also include measurement errors.

³²We plot the *annualized* conditional volatility defined as $\sigma_{\pi,t}^{(j)} := \sqrt{\frac{1}{j} \left(\mathbb{E}_t \left[\exp \left(2\pi_t^{(j)} \right) \right] - \mathbb{E}_t \left[\exp \left(\pi_t^{(j)} \right) \right]^2 \right)}$.

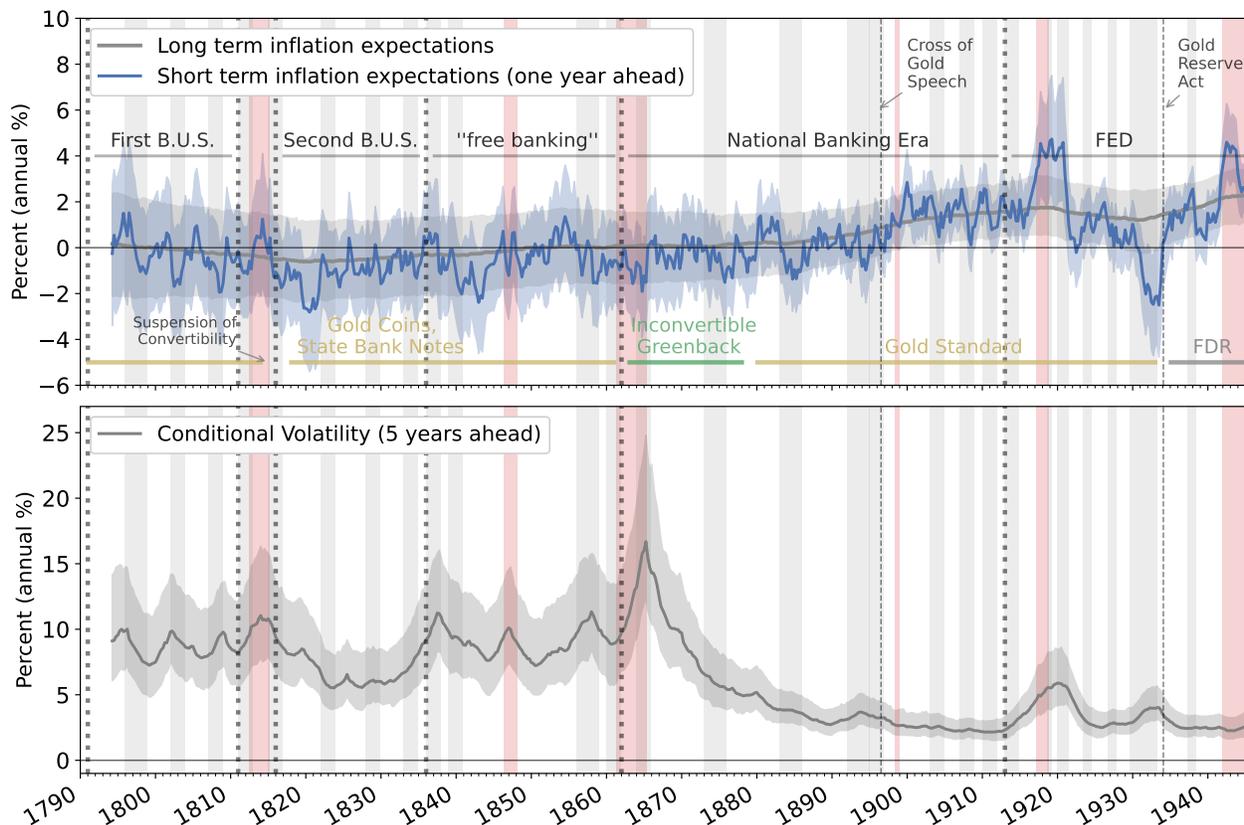


Figure 7: Smoothed Conditional Moments of Inflation

Top plot: The solid grey line depicts the posterior median estimate for the permanent component of inflation. The solid blue line depicts the posterior median estimate for one year ahead inflation expectations implied by our statistical model. Bands around the posterior medians depict 90% interquantile ranges. *Bottom plot:* The solid grey line depicts the posterior median estimate for the smoothed, annualized conditional inflation volatility 5 years ahead. The light bands around the posterior medians depict 90% interquantile ranges. The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars. Conditional moments are “smoothed” in the sense that they are derived from a posterior for parameters in period t that conditions on the full sample (not on the sample up to period t).

inflation shocks to a period where shocks primarily hit the permanent component, implying an increase in inflation persistence.³³

A possible source of the elimination of deflation and the decrease in inflation volatility is the introduction of the National Banking Act in 1863-5, which created a supply of national bank notes, backed by federal government debt, that could act as a stable currency. This is in sharp contrast to the pre-Civil War period, which was characterized by restrictions on the quantity and high volatility in the quality of bank money supply. In addition, there was a large gold rush in California from 1848-1855 that significantly increased the supply of gold in the US. A possible source of the increase in long run inflation expectations in the late nineteenth century was the strong support from elements of both major political parties for returning to a bimetallic gold-silver standard at a mint price ratio of 16-1 when when the market price ratio had become much higher. Prospects of a return to bimetalism at an exchange rate that overvalued silver naturally made investors fear inflation (See Friedman (1990a), Friedman (1990b), Velde and Weber (2000), Weiss (2020), and Fulford and Schwartzman

³³These findings are broadly consistent with Benati (2008) who shows that whenever a monetary regime has a clearly defined nominal anchor inflation is only weakly persistent.

(2020)).³⁴

Short-run inflation expectations spiked to over 4% per annum during World War I but stabilized at around 1% per annum soon afterwards. At the same time, long term inflation expectations moved little and stayed around 1-1.5%. That pattern may reflect that the US was one of the few Western countries to not formally abandon the gold standard during the war. The next major change came in 1933 when President Roosevelt signed the Gold Reserve Act that, at least for US citizens, effectively took the US off the gold standard. Short term inflation expectations immediately increased by approximately 3 percentage points and remained positive throughout the rest of the 20th century.

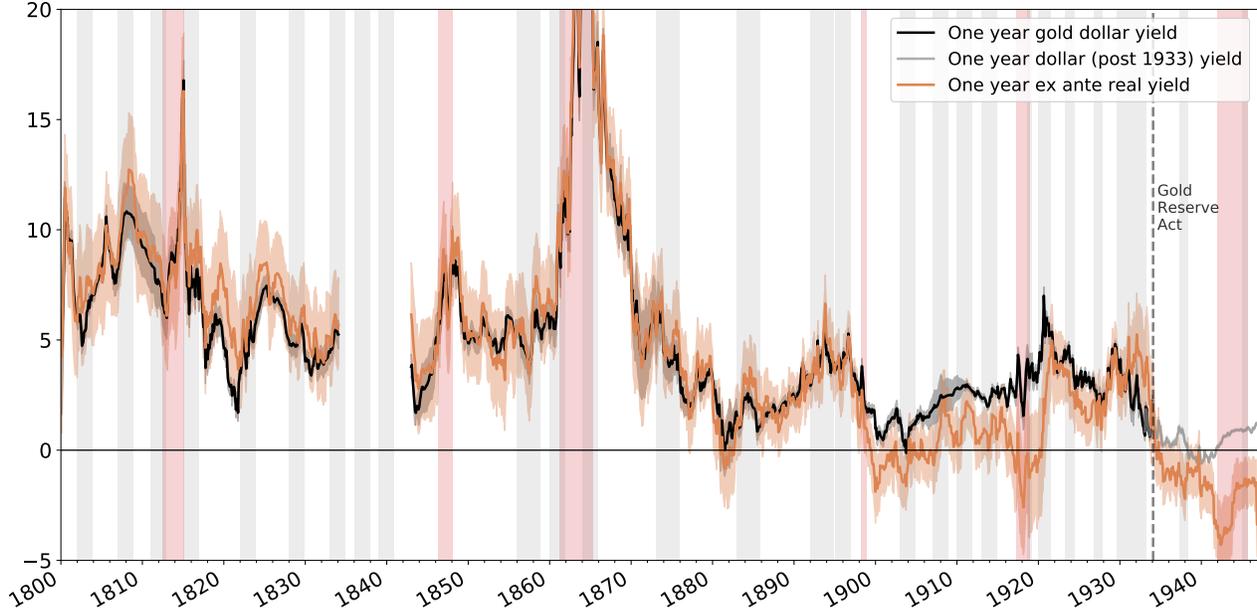


Figure 8: Short-Term Nominal and *ex ante* Real Yield Estimates.

The black solid line depicts the posterior median estimate for the one year, gold denominated, zero-coupon yield. The gray solid line depicts the one year dollar zero-coupon yield after convertibility has been suspended by the Gold Reserve Act. The orange solid line depicts the posterior median estimate for the one year *ex ante* real zero-coupon yield. The light bands around the posterior medians show 90% interquartile ranges. The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars.

We use our estimates of conditional inflation expectations to calculate the *ex ante* real yield curve defined in Subsection 3.2. Figure 8 depicts our results for the 1-year yield. The black and gray lines depict nominal yields (for gold before the Gold Reserve Act (1933) and for dollars afterwards). The orange line depicts the 1-year ex-ante real zero-coupon yield. As can be seen, even at the one year horizon, the gold yield is typically a good proxy for the real yield during the nineteenth century, except during the deflation in the 1820s. This changes in 1896, the year of the “Cross of Gold” speech, when elevated inflation expectations push the ex-ante real yield below the nominal yield. In fact, real yields go negative during the early twentieth century, World War I, Roosevelt’s New Deal, and World War II. The negative real yields during crises in the twentieth century stand in striking contrast to the nineteenth century when the federal government faced very high real yields during the War of 1812 and the Civil War.

³⁴The “free silver” movement advocated for the introduction of silver dollars. The 1896 Democratic presidential nominee William Jennings Bryan made the case for silver coinage in his *Cross of Gold speech*. The Democratic Party made free silver central to its 1896 presidential campaign.

4.4 Slope of yield curve switched signs

While the previous sections studied the level of the yield curve, in this section, we turn to the slope. The top plot in Figure 9 depicts the 10-year gold dollar yield minus the 2-year yield, which we refer to as the term spread. A positive term spread indicates an upward sloping yield curve (i.e., longer maturity bonds have higher rates), while a negative term spread indicates an inverted yield curve (i.e., shorter maturity bonds have higher rates). As can be seen, the term spread was typically negative before the Civil War and positive afterwards, with major decreases during the War of 1812, the Mexican-American War, and the Civil War.

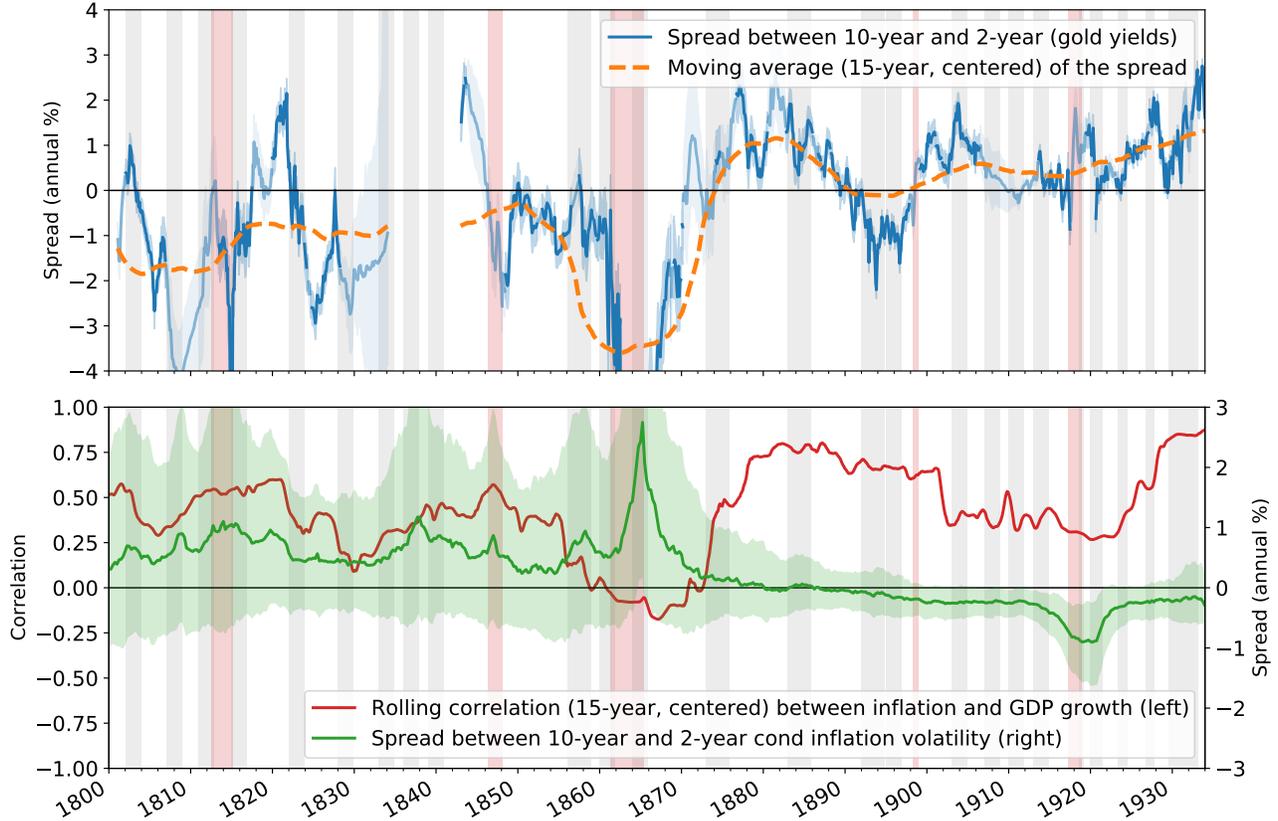


Figure 9: Term Spread and Inflation Risk

Top plot: The solid blue line depicts the posterior median for the yield on 10-year, gold denominated, zero coupon US government bonds minus the yield on 2-year, gold denominated, zero coupon US government bonds. The pale blue bands around the posterior median depict the 90% interquartile range. The orange dashed line depicts the 15-year centered moving average of the term spread (posterior median). *Bottom plot:* The green solid line (right axis) depicts the posterior median for the difference between the 10-year ahead (smoothed) annualized conditional inflation volatility and the 2-year ahead (smoothed) annualized conditional inflation volatility. The pale green bands around the posterior median depict the 90% interquartile range. The red solid line (left axis) depicts 15-year centered rolling correlation between inflation and GDP per capita. The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars.

A possible explanation for why the slope of the yield curve switched signs is related to a striking change in the inflation dynamics. The green solid line in the bottom plot in Figure 9 shows the change in the relative forecastability of inflation at long and short horizons, as measured by the difference between the 10- and 2-year ahead conditional inflation volatility.³⁵ Positive values indicate that inflation was harder to predict at the 10-

³⁵More precisely, we plot the posterior distribution of the statistic $\sigma_{\pi,t}^{(10)} - \sigma_{\pi,t}^{(2)}$. Apart from the difficulty of predicting inflation, the wide posterior band reflects the significant parameter uncertainty underlying our estimates.

year horizon than at the 2-year horizon. Negative values indicate the opposite. We see that long term inflation became relatively easier to predict following the Civil War³⁶ and that this change coincides with the sign switch of the slope of the yield curve from negative to positive. This suggests that the term spread becoming positive might be connected to the decrease in the long run “inflation risk.” This relationship would be consistent with asset pricing theory if the inflation risk premium was negative. That is, if federal gold bonds provided a good hedge against inflation. The red line in the bottom plot, depicting the rolling correlation between inflation and real GDP growth per capita, provides suggestive evidence that this was indeed the case. The correlation between GDP growth and inflation appears to be positive in the gold era which is consistent with inflation risk premium being negative.

An extensive literature has studied the slope of the yield curve in the modern period and argued that a negative slope is a strong predictor of recessions. In Appendix H, we connect our estimate of the historical nominal yield curve with existing estimates for the post-WW2 period and plot the slope of yield curve from 1800 to 2020. We find that the persistence in the slope of the yield curve is substantially lower following the end of the Bretton-Woods system. In Appendix H, we replicate studies of the forecasting power of the slope of the yield curve for the nineteenth century period and find some evidence that changes in the slope have some predictive power, especially after the Civil War.

5 Statistical Fits

In this section, we argue that the common pricing kernel assumption—accompanied with our flexible parametric statistical model—provides a reasonable summary of the available bond price data. Three observations justify this claim: (1) mean pricing errors are generally small for all bonds that we include for the estimation of gold dollar yield curves; (2) our long term yield estimates line up with other respectable series during “non-emergency” periods; and (3) yield-to-maturities of observed bonds concentrate around our estimated par yield curves.

5.1 Small pricing errors

An important aspect of our approach is the assumption of bond-specific pricing errors. This allows the algorithm to decide if certain bonds are likely to violate our common pricing kernel assumption. The black crosses in Figure 10 depict *mean absolute pricing errors* for each bond included in the analysis. It is computed as the time average of the absolute difference between observed prices and posterior median price forecasts. We see that our gold dollar yield curve estimates the prices of the included bonds fairly well, which is a sign of a good in-sample fit and the fact that imposing a common pricing kernel provides a reasonably good description of the gold dollar bonds with maturities larger than 1 year. Further inspection of the errors reveals that accurately pricing the cross-section of bonds simultaneously is most difficult in wartimes. Figure 24 in Appendix I depicts the time series of mean absolute pricing error, i.e., the *cross-sectional* average of the absolute difference. The largest errors are associated with the War of 1812, the Civil War, and the First World War.

Similarly, the estimated standard deviations of bond-specific pricing errors, $\sigma_m^{(i)}$, are also small. The boxplots in Figure 10 depict summary statistics of the corresponding posterior distributions. The relative magnitude of these estimates is indicative of how influential certain bonds are on the estimated yield curve. Our algorithm assigns relatively less “weight” to bonds with large estimated $\sigma_m^{(i)}$ values. Figure 10 shows that the set of bonds

³⁶Mechanically, this comes from the stunning fact that the persistence and long run mean of inflation increased while inflation volatility fell *simultaneously* after Resumption. Because gross inflation $\exp(\pi_t^j)$ is modeled as a *log-normal* random variable, finding the exact source of the change is difficult, but inflation becoming more persistent is certainly a key factor.

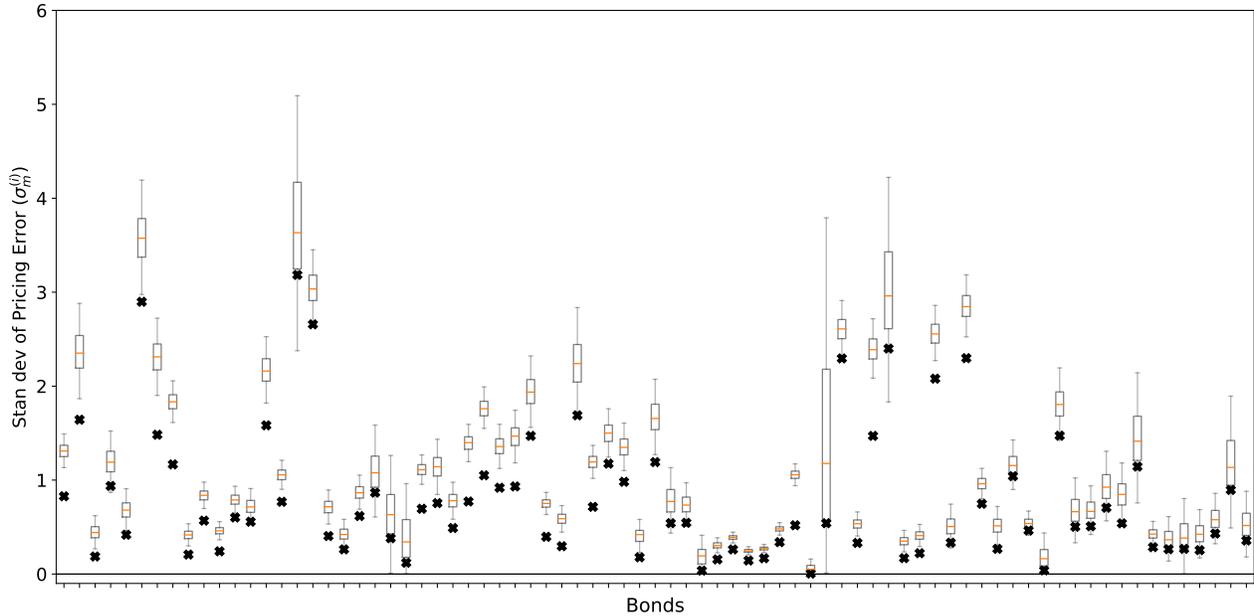


Figure 10: Posterior distributions of bond-specific pricing error variances

Each boxplot represents the interquartile range (IQR) and median (orange line) of the posterior distribution of $\sigma_m^{(i)}$ —that is the standard deviation of the bond-specific pricing error—for each bond used in the estimation. Black crosses represent *mean absolute price errors* computed from the difference between observed and model-implied prices for each bond.

with relatively little influence more or less coincides with the bonds with the highest mean absolute pricing error.

5.2 Long yields line up with available series

The 10 year yield is the part of the yield curve that has attracted the most attention from historians so there are some previous estimates that can be used for comparison. Most papers in the literature use the “Federal Government Bonds: Selected Market Yields” series of [Homer and Sylla \(2004\)](#), computed as the coupon rate on US federal bonds that have approximately 10 years to maturity and were trading close to par. Figure 11 depicts our estimates of ten-year gold dollar zero-coupon yields along with the US long term yield series of [Homer and Sylla \(2004\)](#).

Evidently, our estimates typically follow the [Homer and Sylla \(2004\)](#) series, except that we estimate substantially higher yields during the War of 1812 and the Civil War. In particular, our ten-year gold yield estimate reaches a peak of 16% near the end of the Civil War, which is substantially higher than the [Homer and Sylla \(2004\)](#) series peak of 6% at the start of the war. The following observations suggest that our estimate of yields during this period is more plausible than those of [Homer and Sylla \(2004\)](#).³⁷ Starting in 1862, all US Treasury bonds could be purchased with greenback dollars, including bonds with coupons and principal payments denominated in diverse units of account, some in greenbacks, others in gold dollars. The value of the greenback fluctuated with battlefield and political news, and all Treasury bond prices deviated substantially from par.

³⁷[Homer and Sylla \(2004\)](#) themselves caution against using their estimates for the Civil War period stating on page 303, “. . . the tables of bond yields for the years 1863 to 1870 do not provide a reliable picture of long-term interest rates.” This is because there were no federal bonds trading with a gold price of par and so they are forced to estimate the yield as the gold coupon rate for bonds trading with a greenback price of par. We can capture greater variation in the yield curve because we use the universe of US Treasury bonds at monthly frequency whereas [Homer and Sylla \(2004\)](#) use the subset of these bonds that are trading at par.

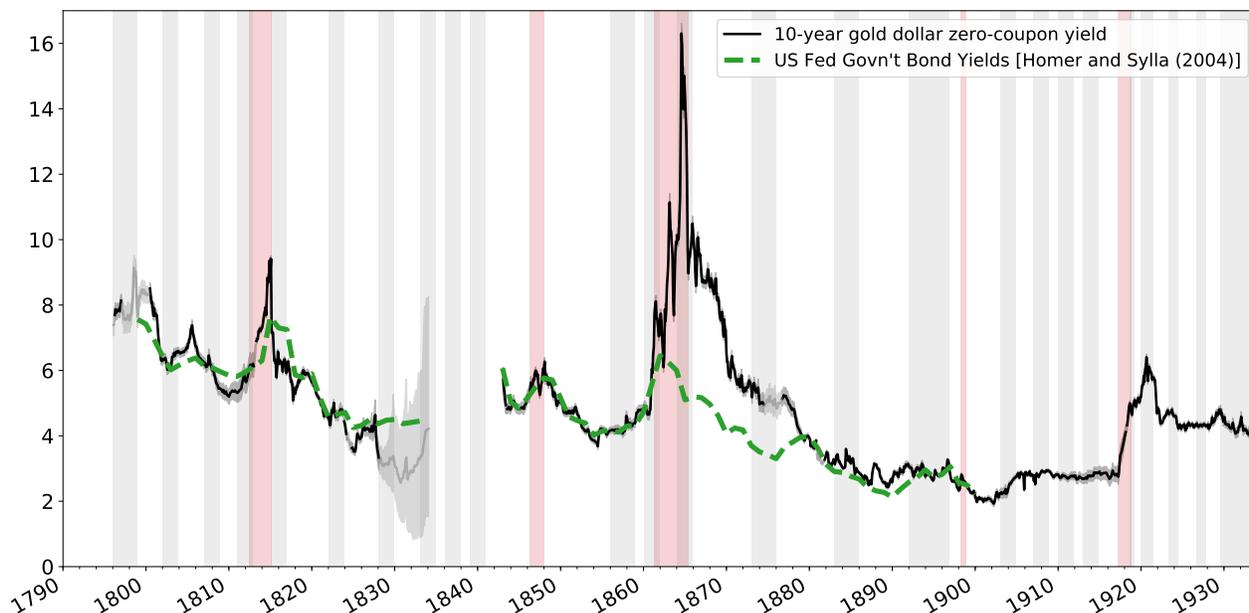


Figure 11: Comparison to the long-run yield of [Homer and Sylla \(2004\)](#)

The solid black line depicts the mean of our posterior estimate for the 10-year, gold denominated, zero coupon yield. The dashed grey line depicts the mean of our posterior estimate for the 10-year, dollar denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquartile range. The dashed green line depicts the ‘Federal Government Bonds: Selected Market Yields’ series from Table 38 of [Homer and Sylla \(2004\)](#). The light gray intervals depict recessions as dated by [Davis \(2006\)](#) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

For example, during the summer of 1864, when re-election of President Abraham Lincoln was in doubt, 100 greenback dollars could be purchased for as few as 40 gold dollars. Consequently, during that time Treasury bonds that promised to pay 6 percent coupons in gold dollars could be purchased for 40 percent of par, implying long-term yields in excess of 15 percent.³⁸

We find it reassuring that our estimate aligns with [Homer and Sylla \(2004\)](#) during “non-emergency” periods because there are good reasons to think that their estimates should be a good approximation to the 10 year yield. Their approach calculates an average yield to maturity for 10 year bonds, which should be similar to the 10 year zero-coupon yield when the yield curve is relatively flat.³⁹ Except during and after the Civil War, the average duration of outstanding bonds was close to 10 years and the average market trading price is close to par and [Homer and Sylla \(2004\)](#) have a large data set.⁴⁰ For these reasons, we consider the general congruence between our estimated 10-year yields and “long-term federal government bond yields” in [Homer and Sylla \(2004\)](#) as a reassuring check on the plausibility of our findings. In Appendix F.1, we report comparisons to other historical estimates and discuss why they might differ from our gold denominated zero-coupon yields.

³⁸In his State of the Union Address on December 9, 1868, President Andrew Johnson said: "It can not be denied that we are paying an extravagant percentage for the use of the money borrowed, which was paper currency, greatly depreciated below the value of coin. This fact is made apparent when we consider that bondholders receive from the Treasury upon each dollar they own in Government securities 6 per cent in gold, which is nearly or quite equal to 9 per cent in currency; that the bonds are then converted into capital for the national banks, upon which those institutions issue their circulation, bearing 6 per cent interest; and that they are exempt from taxation by the Government and the States, and thereby enhanced 2 per cent in the hands of the holders. We thus have an aggregate of 17 per cent which may be received upon each dollar by the owners of Government securities." Our estimate of the ten-year gold dollar zero-coupon yield in December 1868 is 8 percent; our calculations do not include interest earned by national banks and don't account for the tax exemption.

³⁹We discuss the relationship between the zero-coupon yield curve and the yield to maturity in Appendix C.2.

⁴⁰Bonds typically traded close to par because the government set coupon rates to ensure an issue price of par.

Aside: Comparing our yield estimates to that of [Homer and Sylla \(2004\)](#) sheds new light on an economic history literature that, starting with [Evans \(1985, 1987\)](#), has concluded that during the nineteenth century there was no strong association between interest costs and deficits. To conclude that, previous papers used the composite series in Figures 22 and 23 in Appendix F.1. Our analysis indicates that those series substantially underestimates increases in yields on US federal debt during episodes of large nineteenth century government deficits. One way to reconcile our analysis with this literature would be to argue that yields on US municipal and corporate bonds were not highly correlated with surpluses even though yields on US federal bonds were. We leave a detailed analysis of nineteenth century municipal and corporate yields for future work.

5.3 Observed yields-to-maturities are close to estimated par yield curves

Our third argument as to why our estimates are plausible is based on the fact that the Congress and the Treasury often aimed to set coupon rates on new bonds so that initially they would sell at par. That outcome would make their yields-to-maturities equal their coupon rates. This practice implies that we should expect observed yield-to-maturities to be close to the so called *par yield curve*; a curve that shows the required coupon rate for any bond with maturity j to sell at par. This object is a non-linear, one-to-one function of the zero-coupon yield curve, therefore, we can use our estimated model to see how well observed yield-to-maturities line up with the estimated par yield curves at least in “non-emergency” periods when issuing new bonds at par was feasible.

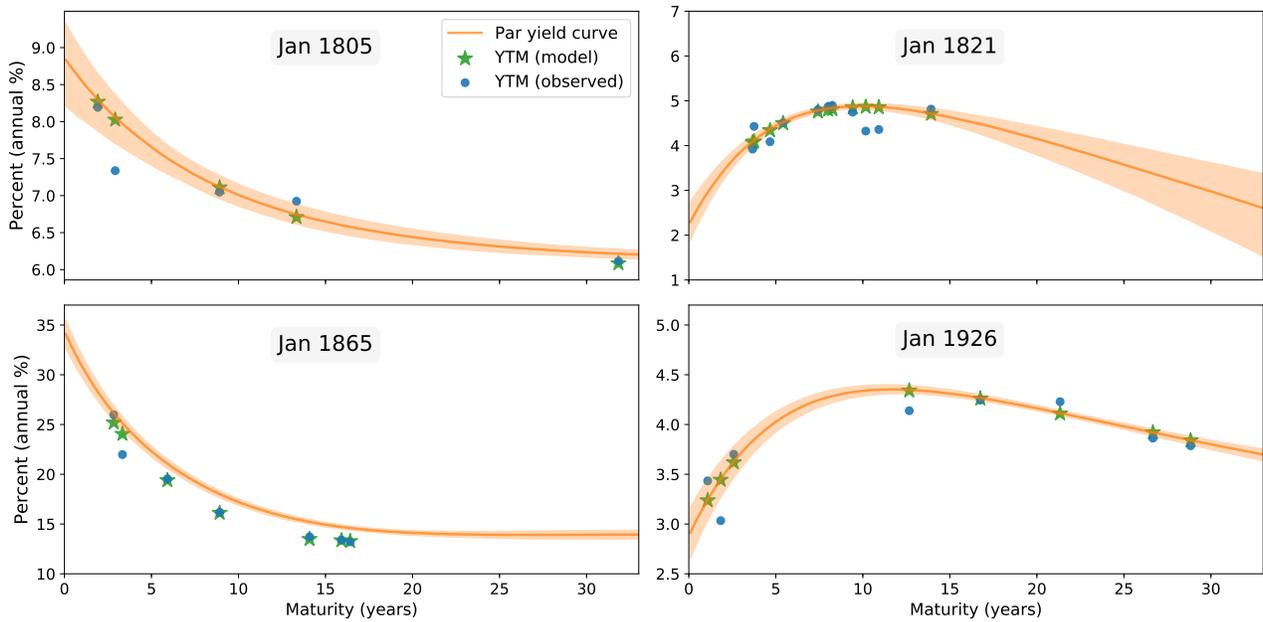


Figure 12: Par Yield Curve Estimates vs. Yield-to-Maturities.

The solid orange lines depict the median of our posterior for the gold dollar par yield yield curve at four specific dates (in gray boxes). The light orange bands around the posterior median depict the 95% interquartile ranges. Blue dots represent observed yield-to-maturities for bonds that are outstanding at the given period. Green stars depict model implied yield-to-maturities for the same bonds—computed from the posterior median price forecasts.

The subplots of Figure 12 depict estimated par yield curves (orange lines) at dates that are more or less representative of certain sub-periods of our sample. Observed and model implied yield-to-maturities for the outstanding bonds are represented by blue dots and green stars, respectively. The close proximity of the dots and stars is indicative that the fit of our model is quite good across the whole maturity spectrum: our model is

able to replicate a wide variety of yield curve shapes and succeeds in capturing the fact that yields at the long end of the maturity spectrum is often lower than yields at medium horizons irrespective of how short-term yields behave.⁴¹ Moreover, comparing the blue dots to the estimated par yield curves illustrate that the Congress' objective to sell bonds at par was often achieved (see the subplots for 1805, 1821 or 1926).

Changes in market conditions, however, frustrated this objective during important episodes in US history. Thus, at times of financial distress during the War of 1812 and the Civil War, Treasury debt sold at deep discounts; and during disagreements between the President and the Congress, like those in the 1890s, the Treasury issued bonds with coupon rates exceeding current yields, so that bonds sold at a premium.⁴² Indeed, the bottom left subplot in Figure 12 shows that in the last months of the Civil War, the par yield curve systematically deviated from the blue dots even though the model implied yield-to-maturities (stars) closely approximates the observed yield-to-maturities (dots).

In remarks at a 2010 Minneapolis Fed conference, Professor V.V. Chari offered an “accounting tail wags the dog” explanation of why Congresses often wanted only to market new bonds that would sell “at par”.⁴³ Chari’s explanation was that Congresses viewed themselves as stuck with Alexander Hamilton’s peculiar accounting rules that told them to measure total government debt by simply adding up *undiscounted* par values of all outstanding debts, ignoring coupon values. That accounting system could provide good approximations to the value of debt only if bonds traded at or near par values.

6 The Greenback Era: 1862-1878

The period from 1862-1878, during which greenback dollars circulated at a volatile discount to gold, offers a particularly interesting case study on how government credibility can survive major monetary policy changes and impact government borrowing costs. Figure 13 shows the greenback to gold exchange rate as well as the price of bonds that promised payment in gold. As can be seen, bond prices only start to track the greenback to gold exchange rate close to maturity. This suggests that investors did not expect a persistent deviation from greenback to gold parity, despite the dramatic devaluation of the greenbacks during the Civil War. In this sense, we find there was a strong nominal anchor. We formalize this intuition in this section by using price data for greenback paying bonds and data on the gold to greenback exchange to estimate expectations about future exchange rates and a greenback denominated yield curve.

6.1 Greenback Dollar Yield Curves

While bonds denominated in different currencies present an opportunity because they allow us to estimate both gold and greenback denominated nominal yield curves, the fact that we observe only 9 greenback dollar bonds means that our sparse cross-section problem is worse for estimating the greenback yield curve. While in principle we could pose a parameterised greenback dollar yield curve similar to that captured by Assumptions 1-5, data limitation prevents us from doing so. We instead proceed by positing additional, admittedly strong assumptions that let us use information about *both* gold and greenback dollar bonds to estimate greenback yield curves.

⁴¹In other words, allowing for a “hump” in the yield curve is often necessary.

⁴²In 1895, after a run drained 40% of the Treasury’s Gold Reserve Fund, President Grover Cleveland sought to issue debt to purchase the gold needed to replenish these reserves. But proponents of bimetallism in Congress blocked new borrowing. Accepting advice from J.P. Morgan’s lawyers, the Cleveland Administration bypassed Congress and used some Civil War-era legislation to issue 30-year bonds bearing 4 percent coupons, at a time when the 10-year zero-coupon yield was below 3 percent. The controversy surrounding the issuance of these bonds helped inspire William Jennings Bryan’s “Cross of Gold” Speech at the 1896 Democratic Convention. See [Chernow \(2001, ch5\)](#) for details.

⁴³Chari was responding to the content of a draft version of [Hall and Sargent \(2011\)](#), which documented differences between the US government accounting method and an alternative mark-to-market method.

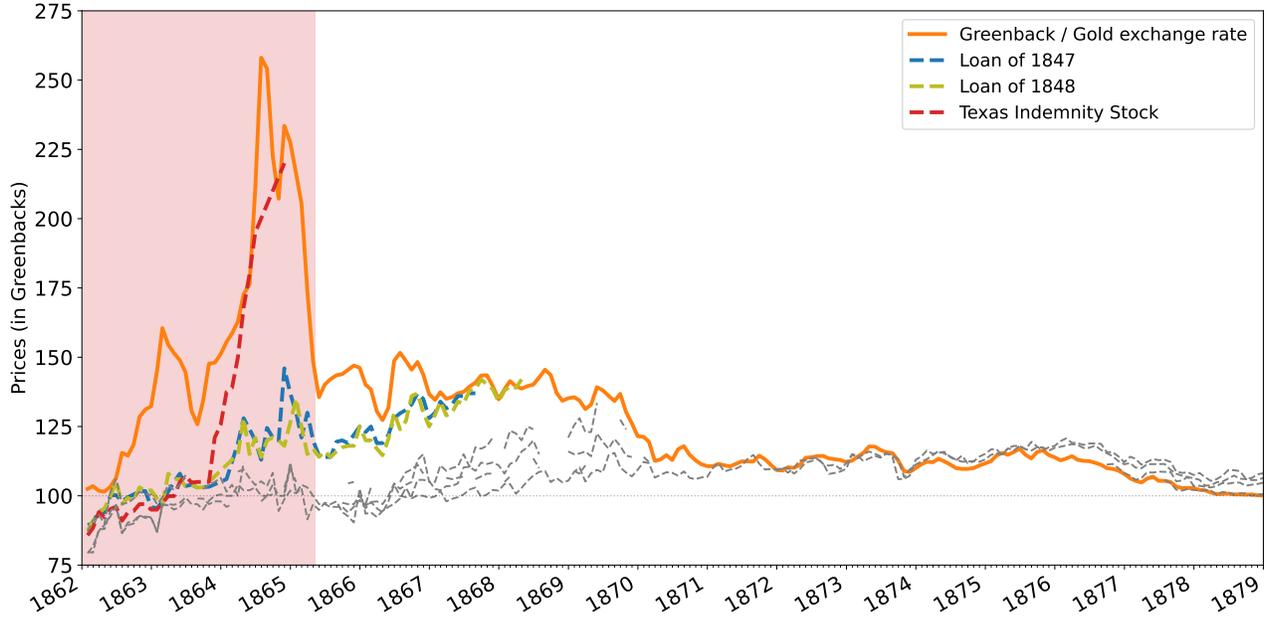


Figure 13: Prices of Gold and Bonds: 1860-1880.

The solid orange line depicts the greenback to gold exchange rate (expressed as the number of greenback dollars required to purchase 100 gold dollars). The dashed lines depict observed prices (denominated in greenbacks) for the outstanding bonds. The grey lines depict prices of bonds that matured after 1868. The light red interval depicts the Civil War.

Identifying assumption: Our common pricing kernel process introduced in Assumption 1 is denominated in gold dollars. In order to be able to price government promises to greenback dollars, we change the denomination of ψ . Let $e_t^{(d)}$ denote the quantity of consumption goods that can be exchanged for one greenback dollar at time t (i.e., the consumption goods price of greenback dollars) and let $P_t := e_t^{(d)}/e_t^{(g)}$ denote the quantity of gold dollars that can be exchanged for one greenback dollar at time t (i.e., the gold-greenback exchange rate at time t). In general, the joint dynamics between the pricing kernel and the two exchange rate processes determine how greenback dollar yields are related to gold dollar yields. The following strong assumption simplifies this relationship significantly and makes our problem tractable.

Assumption 6. Let $v_t = \{P_t, e_t^{(g)}\}$ denote observable macroeconomic price variables at time t . Then, the gold price of a government promise to one *greenback dollar* at time $t + j$ is:

$$q_t^{(j,d)} = \mathbb{E}_t \left[\left(\frac{\psi_{t+j}}{\psi_t} \right) \left(\frac{P_{t+j}}{P_t} \right) \right] = q_t^{(j)} \frac{\mathbb{E}[P_{t+j}|v_t]}{P_t}$$

where, as before, $q_t^{(j)}$ is in units of t -period gold dollar per time $(t + j)$ -period *gold* dollar.

If we impose the specification of the pricing kernel $\psi_{t+j}/\psi_t = (S_{t+j}/S_t) \left(e_{t+j}^{(g)}/e_t^{(g)} \right) \xi_{t+j}\eta_{t+j}$, then we can interpret this assumption in terms of relationships among supposed components of ψ . Literally, assumption 6 says that, during the Civil War, all the relevant information for forecasting P_{t+j} is contained in observable prices. Since the major driver of the greenback-to-gold exchange rate was most likely “default” due to wartime losses, this assumption can be interpreted as saying that all information relevant for forecasting financial effects of the war is contained in the greenback-to-gold exchange rate and the price level. Although this is a strong assumption, greenback money issuance during the Civil War was closely tied to Union fortunes on the battlefield,

particularly over a medium term horizon. We also find it encouraging that the greenback-to-goods exchange rate tracks the gold yield curve during big Civil War changes.

Parameterisation of Greenback Dollar Yield Curves: Expressing this relationship in terms of zero-coupon yields of maturity j leads to an “interest rate parity” formula:

$$y_t^{(j,d)} - y_t^{(j)} = -\frac{1}{j} \log \left(\frac{\mathbb{E}_t[P_{t+j}]}{P_t} \right)$$

That is, conditional on having a proxy for the gold dollar zero-coupon yield curve, we can make inferences about the greenback dollar yield curve by simply estimating the expected future movements in the gold-greenback exchange rate. To estimate this conditional moment, in the next subsection, we introduce a simple yet flexible statistical model parameterized by a vector θ_t which is allowed to vary over time. Using this parameterisation, we define the “conversion multiplier” $z^{(j)}(\theta_t) := \mathbb{E}_t[P_{t+j}]$ that converts $(t+j)$ -period greenback dollars to $(t+j)$ -period gold dollars.

Pricing formulas: Suppose that a given bond i promises a sequence of greenback dollar coupon and principal payments $\{\bar{m}_{t+j}^{(i,d)}\}_{j=1}^{\infty}$, where we allow $\bar{m}_{t+j}^{(i,d)}$ to be zero. As before, introduce extra notation for the sequences:

- $\mathbf{z}_t := \{z_t^{(j)}\}_{j=1}^{\infty}$ denote a sequence of greenback dollar “conversion multipliers”
- $\bar{\mathbf{m}}_t^{(i,d)} := \{\bar{m}_{t+j}^{(i,d)}\}_{j=1}^{\infty}$ denote a sequence of promised *coupon and principal* payments in greenback dollar and it is understood that $\bar{m}_{t+j}^{(i,n)} = 0$ for $j > J_t^{(i)}$.

This allows us to write the gold dollar price of any greenback dollar bond as the inner product:

$$p_t^{(i)} = \left\langle \mathbf{q}(\lambda_t, \tau) \odot \mathbf{z}(\theta_t), \bar{\mathbf{m}}_t^{(i,d)} \right\rangle$$

where \odot denotes a Hadamard (element-wise) product. The next subsection will elaborate on how we use prices of greenback bonds along with exchange rates P_t and $e_t^{(g)}$ to infer parameters $\{\theta_t\}$, that pin down our greenback dollar yield curves *conditional on* the gold dollar yield curves, parameterized by $\{\lambda_t\}$ and τ .

6.2 Inference Strategy

Estimation of greenback dollar yield curves for the period between 1862 and 1879 requires us to infer a new object from data: the sequence of conversion multipliers \mathbf{z}_t defined as the conditional expectation of gold-greenback exchange rates. We model this conditional expectation by a bivariate state-space model with time varying long-run mean and persistence parameters specified in Assumption 7.

Assumption 7. Joint dynamics of exchange rates $v_t := [P_t, e_t^{(g)}]'$ obey a state-space model:

$$\begin{aligned} v_{t+1} &= \mu_t + x_t + F\varepsilon_{v,t+1} \\ x_{t+1} &= A_t x_t + K\varepsilon_{v,t+1} \end{aligned} \quad \varepsilon_{v,t+1} \sim \mathcal{N}(\mathbf{0}, \mathbb{I}_2), \quad \forall t \geq 0$$

where x_t is a 2-vector hidden state with a given initial x_0 , F and K are 2×2 matrices with F being lower

triangular. Parameters μ_t and A_t follow drift-less random walks with shocks that arrive every Δ months:

$$\begin{aligned} \mu_{t+1} &= \begin{cases} \mu_t + \Xi_\mu \varepsilon_{\mu,t+1} & \text{if } t = k\Delta \text{ for } k \in \mathbb{N} \\ \mu_t & \text{otherwise} \end{cases}, & \varepsilon_{\mu,t+1} &\sim \mathcal{N}(\mathbf{0}, \mathbb{I}_2) \\ \text{vec}(A_{t+1}) &= \begin{cases} \text{vec}(A_t) + \Xi_A \varepsilon_{A,t+1} & \text{if } t = k\Delta \text{ for } k \in \mathbb{N} \\ \text{vec}(A_t) & \text{otherwise} \end{cases}, & \varepsilon_{A,t+1} &\sim \mathcal{N}(\mathbf{0}, \mathbb{I}_4), \end{aligned}$$

where Ξ_μ and Ξ_A are positive definite diagonal matrices.

As before, we introduce Δ to economize on the number of model parameters and to avoid over-fitting the few greenback denominated bonds we observe. As $\Delta \rightarrow \infty$, the frequency of parameter updates goes to zero, providing a linear state-space model with time-invariant long-run mean μ and persistence A .⁴⁴ An advantage of this formulation is that while it nests a first-order vector autoregressive process (VAR), it is more flexible due to the presence of the latent variables x_t that allow for first-order moving-average dynamics. By setting $\Delta \geq 1$ to low values, we let the long-run mean and persistence of v_t move over time. In terms of the notation introduced before, we can collect parameters of the exchange rate model into the vector:

$$\theta_t := [\mu_t', \text{vec}(A_t)', \text{vec}(F)', \text{vec}(K)']'.$$

We write our complete model of bond prices in the following compact form:

$$\begin{aligned} \hat{p}_t^{(i)} &= \langle \mathbf{q}(\lambda_t, \tau), \bar{\mathbf{m}}_t^{(i)} \rangle + \sigma_m^{(i)} \varepsilon_t^{(i)} && \text{gold bonds} \\ \hat{p}_t^{(i)} &= \langle \mathbf{q}(\lambda_t, \tau) \odot \mathbf{z}(\theta_t), \bar{\mathbf{m}}_t^{(i,d)} \rangle + \sigma_m^{(i)} \varepsilon_t^{(i)} && \text{greenback bonds} \\ \lambda_t & \text{ from Assumption 4} && \text{yield curve parameters} \\ \theta_t & \text{ from Assumption 7} && \text{expectation parameters} \\ \text{with } \varepsilon_t^{(i)} & \sim \mathcal{N}(0, 1) \quad \forall i, \forall t \geq 1 \end{aligned}$$

where $\hat{p}_t^{(i)}$ denotes the *observed* period- t price of bond i in terms of gold. We believe that Assumptions 6-7 impose much more stringent restrictions on the data than our assumptions supporting the estimation of the gold dollar yield curve. To defend our baseline gold dollar yield estimates against the influence of these less trusted assumptions, we choose not to estimate the above model in one step. Instead, we proceed in two steps:

1. *Gold yield curve:* Using prices on gold dollar bonds and priors described in Appendix E, draw a random sample from the posterior distribution of the gold yield curve model in Subsection 3.3.3. Approximate the joint posterior distribution of $\{\lambda_t\}$ and τ with a (correlated) Gaussian distribution.
2. *Greenback yield curve:* Treat the joint posterior distribution of $\{\lambda_t\}$ and τ as a “second-stage” prior—along with priors for θ_0 , Ξ_A and Ξ_μ described in Appendix E—and combine it with prices on greenback dollar bonds and the observed series of exchange rates $\{v_t\}$ to characterize the “second-stage” posterior of $\{\theta_t\}$, $\{x_t\}$, Ξ_A , Ξ_μ , and $\{\sigma_m^{(i)}\}$.

Aside: Did bondholders’ beliefs change? Following Cogley and Sargent (2005) and Cogley (2005), we interpret time-variation in θ_t as bondholders’ “changing beliefs” induced by shifts in fiscal-monetary policy

⁴⁴This fixed parameter model has been used by Piazzesi and Schneider (2007) and Szöke (2021) (among others) to estimate conditional moments of inflation and consumption growth in consumption-based asset pricing models.

rules. During and after the Civil War, the direction of US monetary-fiscal policies recurrently either shifted markedly or seemed to be on the verge of swerving onto another course. We cope with this situation by positing a shifting law of motion for the relative value of greenback dollars. We assume that financial market participants understood that policies were drifting and sought to adapt their beliefs accordingly. The vector θ_t represents their period- t beliefs about the currency price processes. We assume that the pricing formulas hold on a date-by-date basis, i.e., although agents keep updating their beliefs, they treat the updated θ_t as if it would remain constant forever. [Kreps \(1998\)](#) incorporates such behavior in his ‘anticipated utility’ model.

6.3 The Nominal Anchor

Our approach allows us to infer how investors’ expectations about the greenback-dollar exchange rate evolved during and after the Civil War. Figure 14 shows our estimate for the expected Gold/Greenback exchange rate 10 years into the future at each date. As can be seen, 10 year exchange rate expectations moved very little during the Civil War. In this sense, there was a very strong “nominal anchor” throughout the Civil War.

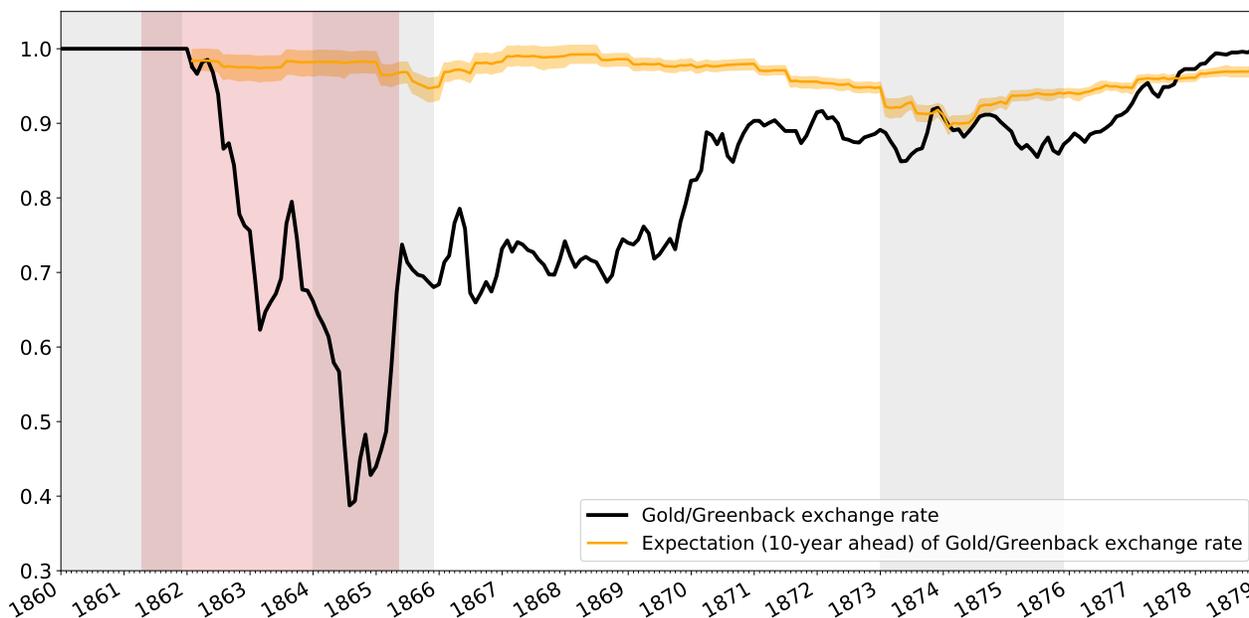


Figure 14: Nominal Anchor

The black line shows the path of the gold/greenback exchange rate, P_t . The orange line shows the median of our posterior estimate for the expected Gold/Greenback exchange rate 10 years into the future at each date. The orange shaded area is the 90% interquantile range for our estimate.

We elaborate on this point in figure 15, which shows expected gold/green back exchange rate paths at different dates during the Civil War. On each plot, a black line shows the path of the gold/greenback exchange rate, P_t , up until a particular date, the gray line shows the continuation of the realized gold/greenback exchange rate after that date, and the orange line shows our estimates of investors’ expectations about paths of the gold/greenback exchange. Evidently, throughout the War (1861-65), investors expected a rapid return to the gold standard in the post war period. This was true even during the large drops in the value of the greenback that occurred in 1863 and 1864 in response to bad news from the war front. Thus, even in the face of very high greenback inflation during the War, expectations of a rapid resumption of greenback convertibility at par seemed to prevail. However, after the War, bond holders became less optimistic about a rapid return to gold.

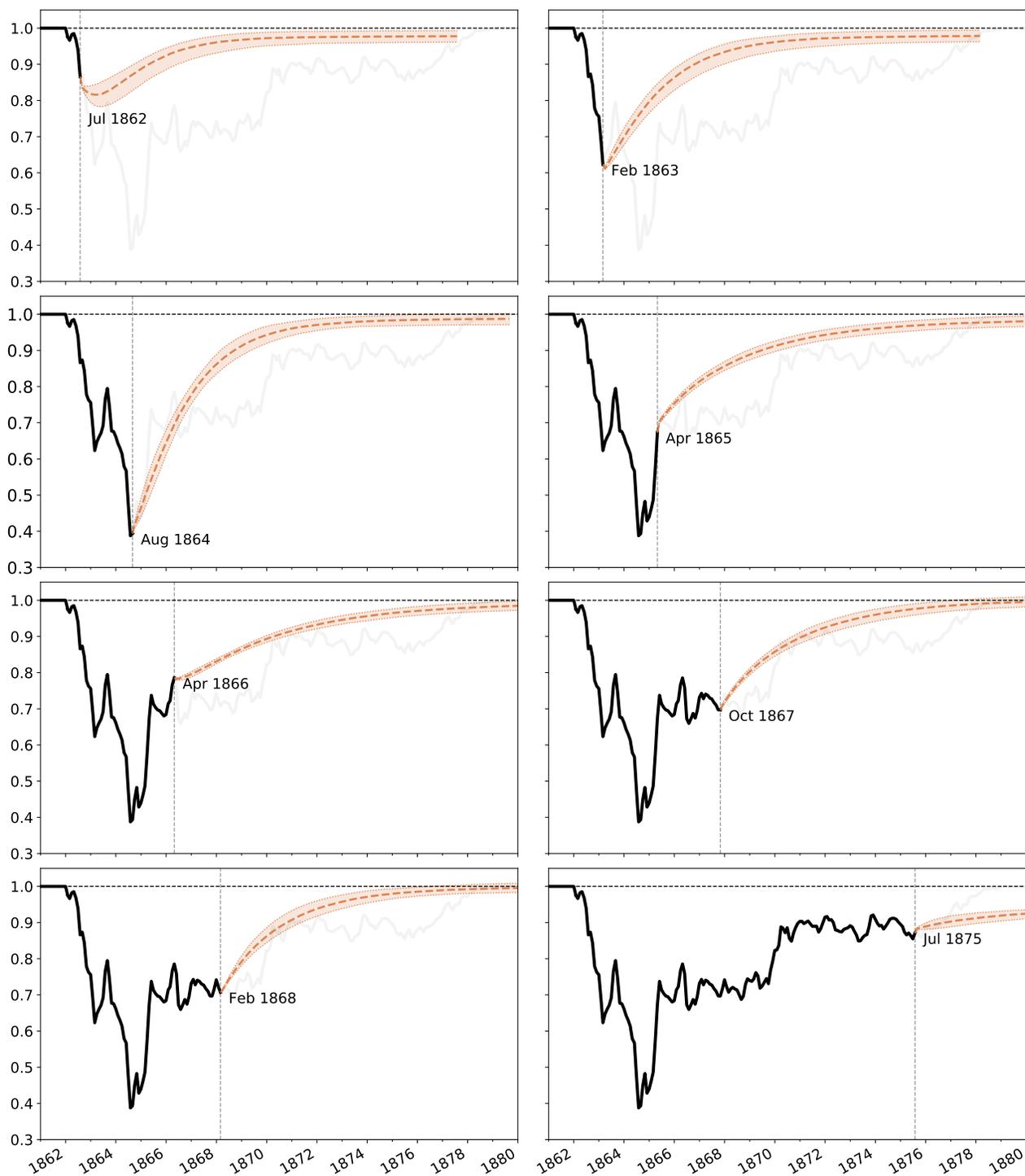


Figure 15: Evolution of Gold/Greenback Exchange Rate Expectations

On each plot, the black line shows the path of the gold/greenback exchange rate, P_t , up until a particular date. The gray line shows the continuation of the realized gold/greenback exchange rate after highlighted date. The dashed orange line shows our model's estimate of investors' expectations about the path of the gold/greenback exchange. The orange shaded area is the 90% interquartile range.

It is enlightening to stare at the post-war panels with a copy of Dewey (1922, pp. 340-345) in hand and to seek explanations for this pattern there in terms of fiscal-monetary decisions made by the Congress and Treasury. Dewey (1922, pp. 340-352) described unfoldings of political struggles about how and whether to service or to tax bond holders or outright to default on US bonds. After describing tentative steps initially taken in early 1866 to retire greenbacks, Dewey tells how Congress postponed measures designed to return to the gold standard. On page 340 he writes "... a great opportunity was lost, for public sentiment in the winter of 1866 would have sustained a more rapid contraction; the country at large was expecting it, and the deed might have been accomplished if Congress had had enough courage." Our estimates indicate that by the mid-1870s investors thought that discrepancies between gold and greenback prices would persist almost indefinitely.

6.4 Greenback dollar yields were lower than gold dollar yields

The estimated greenback and gold 10-year yields and slopes are shown together in Figure 16. The greenback denominated yield is systematically below the gold denominated yield. The results from Section 6.3 suggest that this is because investors expected a return to the gold standard post Civil War and so expected greenbacks to appreciate in value. This meant that they were willing to accept a low greenback yield.⁴⁵ Unlike for the gold yield curve, the slope of the greenback yield curve became positive during the Civil War monetary expansion and negative shortly before the recession of 1873. Interestingly, this behaviour is consistent with the behavior of the post World War II nominal yield curves.

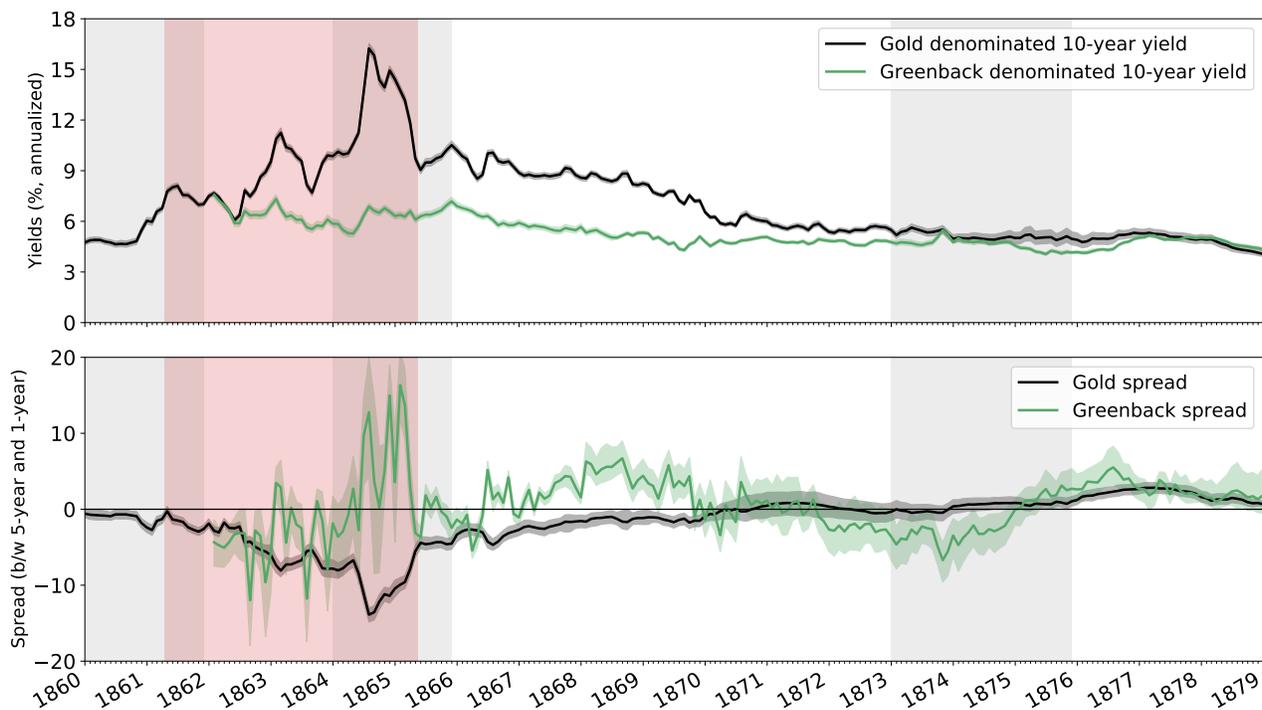


Figure 16: Greenback Yield Curves.

The black line is the ten-year gold denominated zero-coupon yield curve. The green line is the 10-year greenback denominated zero-coupon yield curve. The light gray intervals depict recessions as dated by Davis (2006).

The low greenback yields during the Civil War indicate the powerful influence of beliefs about government

⁴⁵Roll (1972) makes a similar point when he discusses the greenback yield through this period.

commitment on asset pricing. After the Civil War, President Andrew Johnson and much of the Democratic party proposed to reduce debt servicing costs by redefining the unit of account from gold to greenbacks that were then trading at a substantial discount relative to gold. That units-of-account sleight of hand was contested in the 1868 election. During the 1868 Presidential election campaign, the Republican party and its candidate General Ulysses S. Grant promised to sustain the practice of servicing federal debts in gold dollars that Alexander Hamilton had proposed in 1790. Grant won. President Grant said this at his inauguration:

A great debt has been contracted in securing to us and our posterity the Union. The payment of this, principal and interest, as well as the return to a specie basis as soon as it can be accomplished without material detriment to the debtor class or to the country at large, must be provided for. To protect the national honor, every dollar of Government indebtedness should be paid in gold, unless otherwise expressly stipulated in the contract. Let it be understood that no repudiator of one farthing of our public debt will be trusted in public place, and it will go far toward strengthening a credit which ought to be the best in the world, and will ultimately enable us to replace the debt with bonds bearing less interest than we now pay. U. S. Grant, March 1869

7 Concluding Remarks and Epilogue (1933-2020)

We have established stylized facts about US federal debt financing costs from 1791 to 1933. These facts shed light on a long process of adopting and adapting a “Hamiltonian program” for organizing monetary, financial, and fiscal institutions. We conclude by offering a narrative assessment of outcomes. We focused on the years before 1933 because we believe that FDR’s Gold Reserve Act embraced an alternative vision for organizing monetary, financial, and fiscal institutions. Nonetheless, we believe that studying our time series statistics dating to the founding of the US offers lessons about challenges that confront contemporary macroeconomists and policy makers. Taking such a long-term perspective and using a flexible auxiliary statistical model as our guide allows us to study which “stylized facts” reflect more enduring economic forces and which reflect peculiar outcomes from today’s monetary, financial, and fiscal policy eras. That provides us with a framework for evaluating macroeconomic theories that aspire to represent consequences of forces that are invariant to changes in institutional arrangements. We present tentative thoughts about some things that have endured and others that have not between 1791 and 2020.

7.1 The Hamiltonian Program

In three reports, 34 year old Alexander Hamilton advocated a project to improve the fiscal capacity of the federal government. His project sought to smooth medium frequency government expenditure surges by sustaining a reputation for timely debt service (*1790 Report on the Public Credit*); to foster a bimetallic stable national currency (*1791 Report on the Establishment on a Mint*); and to charter a monopoly federal bank that would help finance high-frequency government expenditure fluctuations (*1790 Report on the National Bank*).

From 1790-1829 a sequence of actions unfolded that were designed to implement Hamilton’s vision. The federal government restructured Revolutionary War IOUs, established a gold dollar, restricted states’ ability to issue paper currency, and introduced the First and Second Banks of the United States to serve as fiscal agent of the federal government and to regulate state banks’ creation of money and credit. We have described evidence that these reforms reduced the spread between US and UK yields by the 1820s, suggesting that federal policy makers did foster an improved reputation for servicing federal debts. The reforms also had at least some success in currency management. We find little persistence in inflation during the early nineteenth century, and

inflation volatility declined significantly during the tenure of the Second Bank of US (1816-36). However, we also see persistent deflation and high liquidity premia on short-duration federal bonds, suggesting that growth rates of stocks of currency fell short of growth rates of real GDP. That may have been caused by low rates of growth of state bank notes that were restrained by state-bank-note-buying programs of the Second Bank of the US, and that neither the First nor the Second Bank of the US issued enough of a national currency.

But these accomplishments did not endure. Following political struggles about the role of the Second Bank of the US and Andrew Jackson's veto of the bank's charter, during the 1840s we watch spreads between US and UK yields widen back to 1790s levels and eventually well above 1790s levels during the Civil War. Ultimately, difficulties of financing the Civil War persuaded the Union to restart a Hamiltonian program by establishing the National Banking System (1862-5) and sustaining gold-greenback parity after January 1, 1879. Our estimates of a "short rate disconnect" offer novel indirect evidence about how those policies affected the money supply. We find that the money-like premium on short term government debt declined significantly in the 1880s and stayed relatively low until after World War I. During the 1880s, the US yield again converged to the UK yield. Although US yields again temporarily rose above UK yields in the 1890s, by the turn of the twentieth century, US yields were well below UK yields, portending the emergence of US debt as a global safe asset in the twentieth century. We interpret this package of evidence as indicating that by the late nineteenth century significant progress had been made towards implementing Hamilton's vision.

7.2 Epilogue: 1933-2020

Cost of Financing Wars: An outcome of nineteenth century reforms was that, by the early twentieth century, the US federal government could finance large deficits at low or negative real yields. See Figure 17, which plots our estimates of 5-year *ex ante* real yields on US Treasuries, our estimates of 5-year nominal zero-coupon yields on US Treasuries, and US surpluses as percentages of GDP.⁴⁶ Evidently, large deficits during the War of 1812 and the Civil War coincided with high real yields. That pattern stands in stark contrast to the US experience during the twentieth century when it financed large deficits during WW1, WW2, the Depression, and the Great Recession at low real yields.

This figure sheds light on a historical contest between two founding fathers: Alexander Hamilton and Thomas Jefferson. During the late 18th and early 19th centuries, the UK serviced high debt-GDP ratios at low interest rates. US statesmen disagreed about whether the US could and should foster a similar outcome. One of Alexander Hamilton's motivations for his reform "program" was to ensure the US could on occasions run large deficits to finance wars and build infrastructures. By contrast, Thomas Jefferson advocated low federal taxes and spending and a limited federal borrowing capacity, partly to prevent the US from supporting a standing army and becoming entangled in foreign adventures. Figure 17 assesses the success of both Hamilton and Jefferson as advocates and prophets. Hamilton's hopes of low interest rate deficit financing were eventually realized in the early twentieth century. However, as Jefferson feared, the achievement of a low financing cost regime coincided with the nation's introducing a big standing army and more frequently waging foreign wars.

Interest Rates For Policy and Macroeconomic Modeling: Recently, there has been a lively discussion about a "trend decline" in "real rates" over the past 40 years. Figure 18 puts recent declines in historical context. The blue line corresponds to the 1-year real yield, a key variable in contemporary macroeconomic models, the orange line corresponds to the 10-year real yield, and the black line corresponds to the combination of our 1-year nominal yield estimates with recent zero-coupon yield curve estimates. After the slow decline in

⁴⁶We combine our nominal yield curve estimates for 1790-1947 with the zero-coupon yield estimates of McCulloch and Kwon (1993) covering the period 1947 - 1961 and the estimates of Gürkaynak et al. (2007), which is available since 1961.

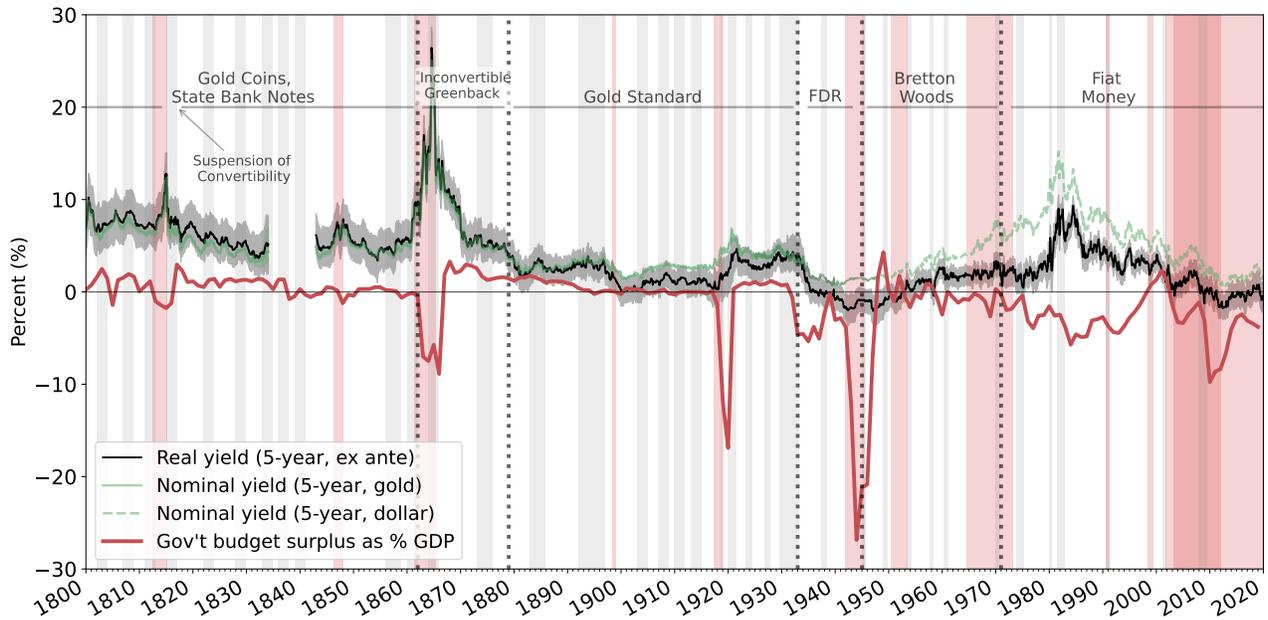


Figure 17: US Budget Surpluses and *ex ante* Real Bond Yields

The solid black line depicts the posterior median estimate for the 5-year, gold denominated, zero coupon yield. The grey bands around the posterior median depict the 90% interquartile range. The solid green line depicts the posterior median for the 5-year gold denominated yield. The dashed green line depicts the combination of our posterior median estimate for the 5-year dollar (post 1933) yield with the zero-coupon yield estimates of [McCulloch and Kwon \(1993\)](#) and [Gürkaynak et al. \(2007\)](#). The solid red line shows US surplus as a percentage of GDP. The light gray intervals depict recessions as dated by [Davis \(2006\)](#) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars.

the nineteenth century, short term real yields on government debt were typically close to zero throughout the twentieth century and frequently negative.⁴⁷ The 1980-1990 period that witnessed prolonged high inflation and high *ex ante* real rates stands out as an exception. Evidently, during the 1970s inflation started to exhibit random walk-like behavior,⁴⁸ indicating that an “inflation anchor” started to drift. By not recognizing drifting long run inflation expectations, the FED’s putative “tightening” during the early 1970s resulted in flat or slightly decreasing *ex-ante* real yields. That made it harder to bring down inflation expectations in the 1980s and 1990s during Volcker’s and Greenspan’s tenures as FED Chairmen. Nevertheless, a lesson from this episode is that, in principle, a *well-managed* fiat regime can re-stabilize long run inflation expectations without necessarily promising some form of gold backing.

Our long time series is consistent with a view in the asset pricing literature that yields on government debt are strongly influenced by inflation risk premia (e.g. [Piazzesi and Schneider \(2007\)](#), [Rudebusch and Swanson \(2012\)](#), [Campbell et al. \(2020\)](#)). We conjecture that if we were to estimate a statistical model with sufficient structure to allow us to construct an inflation risk-premium-free measure of a real rate on government debt, we would infer that it was close to zero (and often negative) throughout the twentieth century. This would suggest that a sizable portion of the recent “trend decline” in real rates was attributable to the Fed’s effort to re-stabilize long run inflation expectations during the 1980s and 1990s. This poses pressing questions as to what is a “correct” proxy for a “shadow rate” for macroeconomic modeling.

⁴⁷Recent papers, such as ([Schmelzing, 2020](#)), have documented a long term average decline in interest rates around the world. Our estimates suggests that the US contributed to this trend decline during the nineteenth century. However, during the twentieth century, yields on US debt followed a different trend, possibly because of the privileged role that US debt has come to play.

⁴⁸See e.g. [Cogley and Sargent \(2005\)](#), [Stock and Watson \(2007\)](#), and [Benati \(2008\)](#).

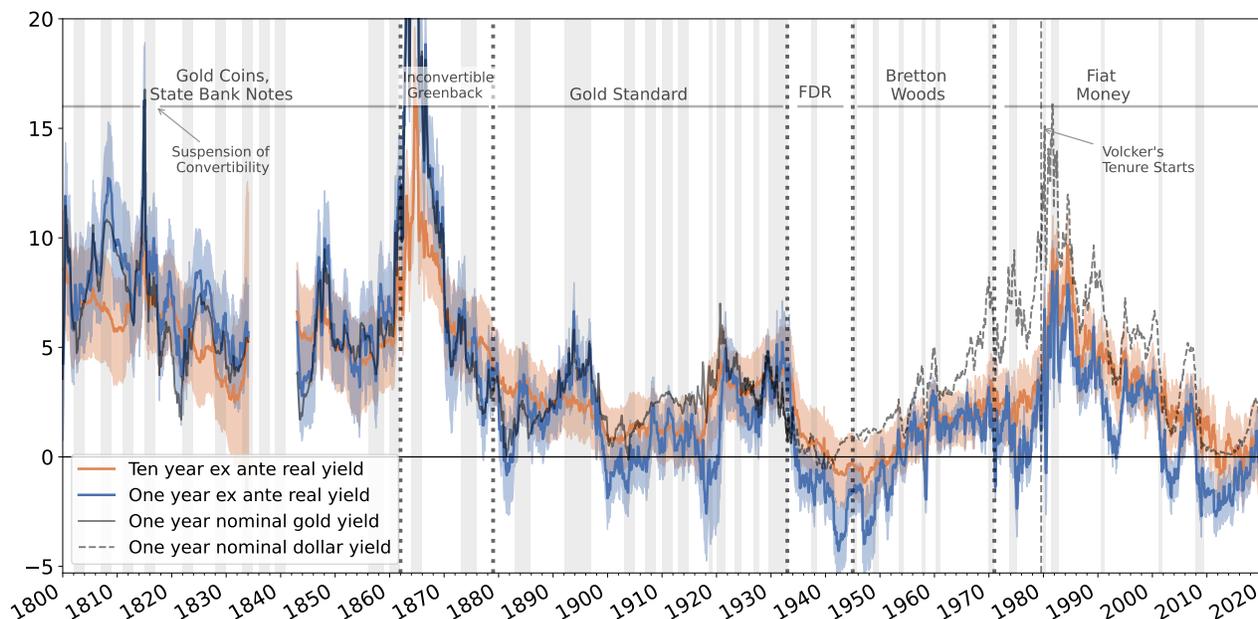


Figure 18: *Ex ante* Real Yields and “the” Nominal Interest Rate

The solid orange line depicts the posterior median estimate for the 10-year *ex ante* real zero coupon yield. The solid blue line depicts the posterior median estimate for the 1-year *ex ante* real zero coupon yield. The bands around the posterior medians depict 90% interquartile ranges. The solid black line depicts the posterior median for the 1-year gold denominated yield. The black dashed line depicts the combination of our posterior median estimate for the 1-year dollar (post 1933) yield with the zero-coupon yield estimates of McCulloch and Kwon (1993) and Gürkaynak et al. (2007). The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter.

Evolving Purposes of Monetary Policy Makers: A much discussed feature of the 2007-9 financial crisis was the combination of output decline and deflationary pressures. To many contemporary researchers, the positive correlation between output growth and inflation seemed to be a historical anomaly. Figure 19 provides a long term perspective that challenges this view by showing that a positive correlation was actually the historical norm until World War II. The top plot of figure 19 shows that the rolling correlation between per capita output growth and inflation was positive from 1790 to 1933, except for the Civil War period when the two series became uncorrelated.⁴⁹ This relationship changes dramatically following World War II, when the correlation becomes significantly negative due to a series of low inflation booms and the “stagflation” of the 1970s and 1980s.⁵⁰

We suggest that these changes reflect how different administrations have balanced trade-offs between lowering federal borrowing costs, price stability, and financial stability. Before the Civil War, the government prioritized decreasing the cost of government financing and keeping trend inflation low. It implemented this by adhering to the gold standard and, part of the time, via monopoly powers given to the First and Second Banks of the US. The middle and bottom plots of Figure 19 show that these policies came at the costs of volatile inflation, long run deflation, and relatively frequent financial crises. This suggests an economy characterized by downturns with bank crises, in which households demand more gold by seeking to convert state bank notes into gold, which in turn forces state banks to demand more gold. As a result, we see gold appreciation (deflation) in the

⁴⁹Figure 19 plots the 15 year rolling correlation, but our finding is robust to other horizons. In fact, one can easily spot the changing co-movement between inflation and output growth per capita by inspecting the raw series, which also appear in figure 19.

⁵⁰The figure shows the irony that a “Phillips curve” prevailed for approximately 150 years but then abruptly broke down just when economists discovered it in the late 1950s.

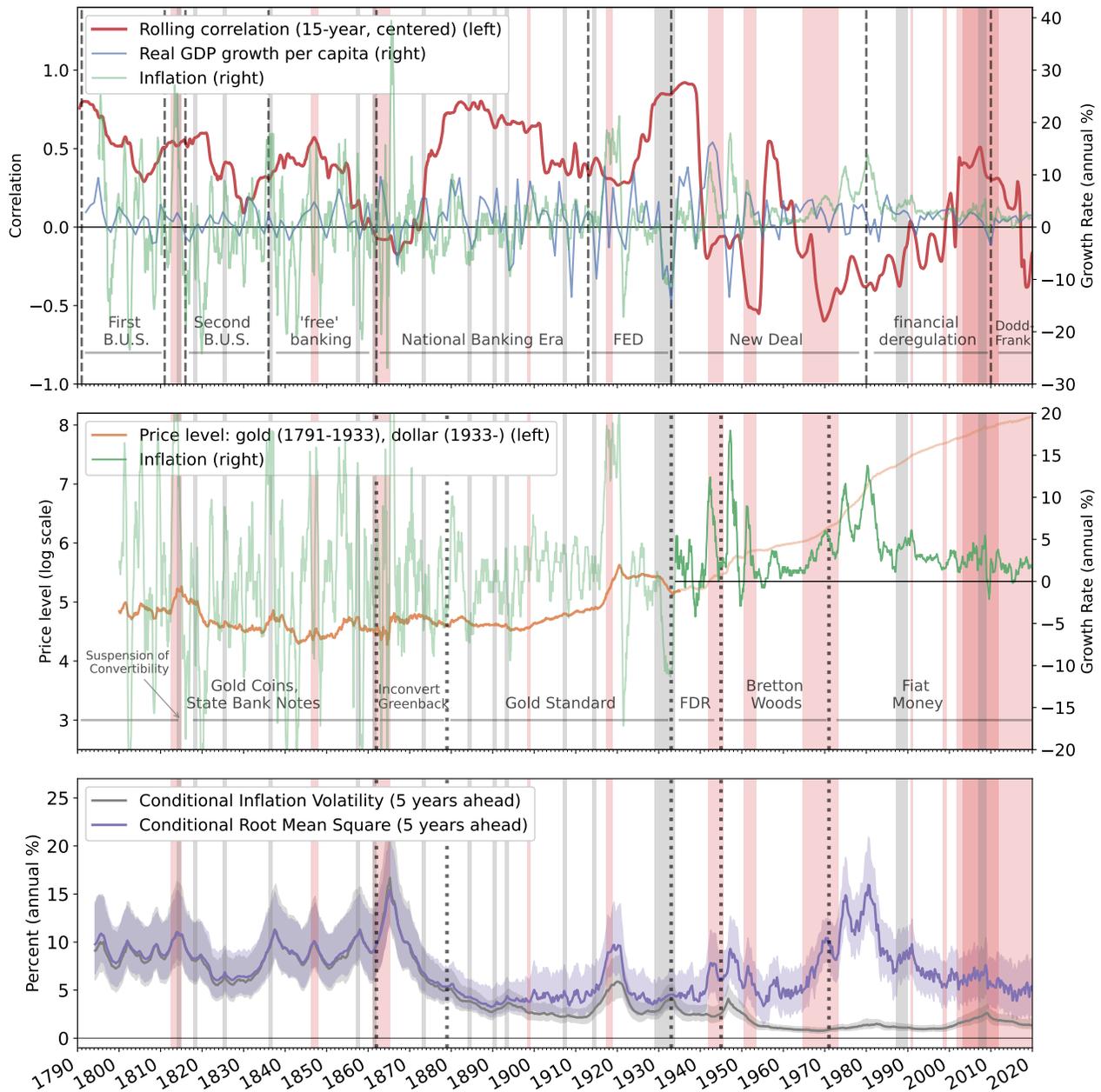


Figure 19: Output and Inflation.

Top plot: The pale blue line depicts annual real GDP growth per capita, the pale green line depicts our annual inflation series, both measured on the right axis. The red thick line shows the 15-year (centered) rolling correlation between the blue and green series, measured on the left axis. *Middle plot:* The orange line depicts our combined log price index (left axis). It measures the gold price of goods before 1933 and the price of goods in dollar after 1933. The green line shows the annual growth rates (inflation) of the orange line (right axis). We use different level of transparency to highlight which series is more informative in which subperiod. *Bottom plot:* The solid grey line depicts the posterior median estimate for the 5-year-ahead smoothed, annualized conditional inflation volatility. The solid purple line depicts the posterior median estimate for the 5-year-ahead smoothed conditional root mean square statistic—a measure of conditional second moment of inflation—used by Cogley and Sargent (2015) to quantify ‘price instability’ (as opposed to unpredictability). The light bands around the posterior medians depict 90% interquartile ranges. The light gray intervals depict banking crises from Reinhart and Rogoff (2009). The light red intervals depict wars.

midst of recessions. In this sense, under the gold standard, frequent financial crises generated strong positive co-movement between output growth and inflation.

After the Civil War, the government had similar priorities but, now armed with more powerful tools under its new institutions, focused more on lowering inflation volatility. The key new institution was the National Banking System, which allowed the government to earn a higher convenience yield on long term government debt and also to stabilize the market value of broad money. That the government accomplished those purposes is indicated by the evaporation of the spread between US and UK debt yields, the elimination of the “short rate disconnect,” the substantial decrease in inflation volatility, and the stabilization of the trend price level. Although the National Banking Acts did restrict bank lending, they did not create a government-run lender-of-last-resort backstop for the financial system. The system still experienced large financial crisis shocks and the positive relationship between output growth and inflation continued.

During the first half of the twentieth century the government’s priorities changed. Concerns about ensuring financial and business cycle stability increased while concerns about ensuring price stability decreased as the government used inflation taxes to lower its borrowing costs especially during wars. Institutional changes accompanied these shifting priorities. Created by the Federal Reserve Act of 1913 and beginning to operate in late 1914, the Federal Reserve Bank was empowered to act as a lender-of-last-resort to member banks. During the 1930s, Franklin D. Roosevelt’s New Deal devalued the dollar relative to gold, introduced national deposit insurance, passed the Glass-Steagall Banking Act of 1933, and established the Federal National Mortgage Association (“Fannie Mae”) to insure a large fraction of bank issued mortgage loans. After World War II, the Fed focused more and more on taming business cycles. We see these changes reflected in the increase in long run inflation expectations and the relative stability of the financial sector from 1933 through to 2007.⁵¹ A plausible explanation for the changed correlation between output growth and inflation during 1940-2000 was the government’s decision to refocus from price stability to financial stability during the middle of the twentieth century. The government’s priorities change again towards the end of the century when it embarks on a program of financial deregulation. During this period the correlation between inflation and output increased and eventually became positive again in the early decades of the twenty-first century.

Our data and auxiliary statistical model have helped us detect how coincident arrangements for regulating financial institutions and administering monetary and fiscal policies have impinged on costs of government finance. To understand more about connections between arrangements and outcomes, we plan to construct structural macroeconomic models that make contact with statistics from our auxiliary model.⁵²

⁵¹It is enlightening to compare frequencies of gray bands in Figure 19 before and after the New Deal.

⁵²As in footnote 5, the distinction between auxiliary and structural comes from [Gallant and Tauchen \(1996\)](#).

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A Data Appendix (For Online Publication)

Here we provide details about how the data sets were constructed. We first outline which bonds are included and excluded in the different estimation exercises. We then describe the assumptions made in constructing of the cash flow series. Finally, we discuss the recession bands that we use. Some of these points have already been referenced in the main text but we collect the assumptions here for completeness.

A.1 Summary of Data Sources

We combined existing historical databases with transcription from the digital archives of newspapers and government reports. Table 1 summarizes the different data sources that we have used throughout the paper. The data set for bond prices and quantities is available at the Github repository <https://github.com/jepayne/US-Federal-Debt-Public> and construction methods are explained in [Hall et al. \(2018\)](#).⁵³ In this subsection of the appendix, we spotlight decisions about our data that we made to prepare for the statistical inferences presented in this paper.

Our bond price data are monthly. When available, we use the closing price at the end of each month. However, if a closing price is not available, then we use an average of high and low prices or an average of bid and ask prices. The sources for the price data from 1776 to 1839 are *Global Financial Data*, [Razaghian \(2002\)](#), and [Sylla et al. \(2006\)](#). Prices from 1840 to 1859 are from [Razaghian \(2002\)](#), *The New York Times*, and *Global Financial Data*. Prices from 1860 to 1925 are from the *Commercial and Financial Chronicle*, *Global Financial Data*, [Martin \(1886\)](#), *Merchants' Magazine and Commercial Review*, the *New York Times*, and *US Treasury Circulars*. When overlaps occurred, data were taken from the US Treasury Circulars. Prices from 1919 to 1925 are from "United States Govt. Bonds" tables in the *New York Times*. Prices after 1925 are taken from the *CRSP US Treasury Database*.⁵⁴ Data on contractually promised dollar payments come from [Bayley \(1882\)](#) for the period from 1790-1871 and from [U.S. Department of the Treasury \(2015\) Monthly Statements of the Public Debt](#) for the period from 1872-1960.

The quantity data are quarterly from 1776 to 1871 and monthly thereafter. All quantity entries record the quantity outstanding on the last business day of the period. The quantities outstanding from 1790 to 1871 are imputed from the issue and redemption series reported by [Bayley \(1882\)](#). We cross-checked these quantities against quantity outstanding series reported in [Register's Office \(1886\)](#). After 1871 our source for quantity outstanding series is the [U.S. Department of the Treasury \(2015\) Monthly Statements of the Public Debt](#). The call data are from *Annual Reports of the Secretary of Treasury* for various years. Data on Treasury securities held in government accounts are from *Banking and Monetary Statistics 1914-1941* prior to 1941 and from *Treasury Bulletin* thereafter.⁵⁵

We require data on greenback-gold dollar exchange rates to estimate the greenback and real yield curves. For the gold-to- greenback exchange rate, we use Greenback price data from [Mitchell \(1908\)](#)⁵⁶ for the period from 1862-1878 during which greenbacks and gold dollars both circulated. For the gold-to-goods exchange rate, we combine several series. For the period from 1800-1860, we use the wholesale price index from [Warren et al. \(1932\)](#). For the period from 1860-1913, we use the General Price Level Index from the *NBER Macroeconomics Database*⁵⁷. For the period from 1913-2020, we use the Consumer Price Index from the U.S. Bureau of Labor Statistics. Finally, we use the GDP series from [Officer and Williamson \(2021\)](#).

⁵³Only data from publicly available data sets are posted on the GitHub page.

⁵⁴See <http://www.crsp.com/products/research-products/crsp-us-treasury-database>.

⁵⁵See [Board of Governors of the Federal Reserve System \(1943\)](#) and [Register's Office \(1886\)](#).

⁵⁶See Table 2

⁵⁷See <https://www.nber.org/research/data/nber-macrohistory-iv-prices>

Table 1: Summary of Data Sources

Series	Period	Frequency	Source
Bond prices	1776-1839	M	Razaghian (2002) , Sylla et al. (2006) and <i>Global Financial Data</i> .
	1840-1859	M	Razaghian (2002) , <i>The New York Times</i> , and <i>Global Financial Data</i>
	1860-1925	M	<i>Commercial & Financial Chronicle</i> , <i>Global Financial Data</i> , <i>Merchant’s Magazine</i> , <i>The New York Times</i> , <i>US Treasury Circulars</i> , and Martin (1886) .
	1925-1960	M	<i>CRSP US Treasury Database</i> .
Quantities	1790-1871	Q	Bayley (1882) .
	1872-1960	M	U.S. Department of the Treasury (2015) .
Contract Info.	1790-1960		1790-1871 from Bayley (1882) .
			1872-1960 from U.S. Department of the Treasury (2015) .
Gold/Goods	1800-1860	M	Wholesale Price Index (Warren/Pearson)
Exchange Rate	1860-1913	M	U.S. Index of the General Price Level (NBER Macrohistory: Series NBER 04051)
	1913-2020	M	CPI (BLS)
GDP	1790-2020	A	Officer and Williamson (2021)
Gold/Greenbacks	1862-1878	M	Yale SOM ICF dataset
Exchange Rate			

¹ Repository for bond time series: <https://github.com/jepayne/US-Federal-Debt-Public>

A.2 Exclusion of Bonds

The estimation of the non-linear state space model of gold bond prices defined in section 3.3.3 excludes all bonds that paid coupons and/or principals in any denomination other than gold. It also excludes short term Treasury notes that the US issued during the War of 1812. [Bayley \(1882\)](#) lists these notes as the Treasury Notes of 1812, Treasury Notes of 1813, Treasury Notes of March 1814, Treasury Notes of December 1814, and the Small Treasury Notes of 1815. These notes were used for payments well after their earliest redemption date and so probably earned a convenience yield. It also excludes the Panama Canal bonds, which we were not able to price consistently with the rest of the bonds, suggesting that they they have a different pricing kernel.

For the estimation of the non-linear state space model of greenback bonds defined in section 6.1 we exclude the following bonds, which [Bayley \(1882\)](#) documents had ambiguous denominations for the repayment of the principal: the “Five-Twenties of March 1864”, the “Five-Twenties of June 1864”, the “Five-Twenties of 1865”, the “Consols of 1865”, the “Consols of 1867”, and the “Consols of 1868”.

A.3 Construction of cash-flows

In order to estimate the yield curve, we need to construct the currency flows promised by each bond. For many of the early bonds in the sample, both the coupon dates and the maturity date have ambiguity because the bond information is imprecise and because it unclear whether newspaper prices are ex or cum dividend. For the coupon dates, we used the following rule. If [Bayley \(1882\)](#) lists exact coupon dates, then we use those dates. Otherwise, we identify the coupon dates from cyclical decreases in the price series at the frequency of coupon payment. We interpret these decreases as the price impact of the coupon payment.

For the maturity dates, we used the following rules. For bonds with an explicit maturity date, we set the maturity to that date. For the three Hamilton bonds (which Bayley (1882) lists as *Six Per Cent Stock of 1791*, *Deferred Six Per Cent Stock of 1791*, and the *Three Per Cent Stock of 1791*), which were issued as annuities but ultimately redeemed early, we impose that investors had perfect foresight about the early redemption and set the maturity date to be date at which greater than 90% of the outstanding bonds had been redeemed. For bonds with a redemption window, we calculate the minimum of the date at which 90% of the outstanding bonds had been redeemed and date at which the bonds started to trade at par value. We then set the maturity date to be closest coupon payment date to that minimum calculation. For bonds that converted into different bonds, we set the maturity date to be maturity of the bond into which it is converted.

A.4 Construction of Recession bands

For the 1796-1914 period we use recession dates from Davis (2006). These are derived solely from the Davis (2004) annual industrial production index. The Davis index incorporates 43 annual series in the manufacturing and mining industries in a manner similar to the Federal Reserve Board’s present-day industrial production index. For this reason, we regard it as an improvement over earlier more qualitative approaches of dating pre-World War I business cycles. Since the data used to date peaks and troughs is annual, the methodology is quite simple: A year immediately preceding an absolute decline in the aggregate level of Davis’s industrial production index defines a peak, and the last consecutive decline following a peak defines a trough (Davis, 2006). For the 1915-present period we use recession dates from the NBER.

B Historical Time Line (For Online Publication)

The text references many changes to monetary and financial regulation. In this section, we collect those events into a historical timeline, which is shown in table 2. The time line is broken up into a collection a collection of banking “eras”. The first era is from 1791-1836, during which the First and Second Banks of the US operated alongside state banks. The second era is from 1837-1962, during which state banks could automatically gain bank charters without a congressional review process, often referred to as the “free banking” era. The third era is from 1863-1913, during which the federal government chartered national banks that issued bank notes backed by US federal government debt. The fourth era is from 1913-1933, during which the Federal Reserve Bank was introduced to act as lender-of-last resort to the banking sector. The fifth era is from 1934-1980, during which the New Deal financial regulations were in place. The sixth era is from 1980s-2009, during which the New Deal financial regulations were gradually unwound. Finally, there is the era from 2010 to the present day, during which the Dodd-Frank Act another financial crisis legislation are in place.

Table 2 Time Line of Monetary and Financial Events

1791	• Congress charters the First Bank of the US. The bank is privately owned. It operates as a commercial bank but also has the special privileges of acting as banker for the federal government (storing tax revenue and making loans) and being able to operate across states. It shares responsibility with state banks for bank note issuance. It influences state bank money and credit issuance by setting the rate at which it redeems state notes collected as tax revenue into gold.
1792	• Coinage Act of 1792. Authorizes the US to issue a new currency, the US gold dollar.

1811	Charter of the First Bank of the US expires and is not renewed.
1812-5	War of 1812. Convertibility to bank notes to gold is suspended. Government issues Treasury Notes to finance the war.
1816	Congress charters the Second Bank of the U.S.
1819	Panic of 1819. Cotton prices fall, farms go bankrupt, and banks fail.
1832	Jackson vetoes bill to recharter Second Bank.
1833	Jackson removes federal deposits from Second Bank of the US
1834	Coinage Act of 1834. Changes the ratio of silver to gold from 15:1 to 16:1.
1836	Charter of the Second Bank of the US expires and is not renewed. The Second Bank becomes a private corporation.
<hr/>	
1837	“Free Banking” Era begins. Michigan Act allows the automatic chartering of banks (without requiring explicit approval from state legislature) that issue bank notes backed by specie (gold and silver coins). Over the next few years, other states pass similar laws.
1837	Panic of 1837. Sharp decrease in real estate prices leads to large bank losses. In New York, every bank suspends payment in gold and silver coinage. Many banks fail.
1857	Coinage Act of 1857. Foreign coins can longer be legal tender.
1857	Panic of 1857. Railroad company stocks drop sharply. Ohio Life Insurance and Trust company fails, which prompts a collapse in stock prices and widespread failures across mercantile firms.
<hr/>	
1861-5	Civil War.
1862	Legal Tender Act. Authorizes the federal government to use nonconvertible greenback paper dollars to pay its bills.
1863-4	The National Bank Acts. The National Currency Act (1863) and The National Bank Act (1864) establish a system of nationally chartered banks and the Office of the Comptroller of the Currency. National banks can issue national bank notes up to 90% of the minimum of par and market value of qualifying US federal bonds. Limit on aggregate national bank note issuance is \$300 million. Banks must pay a 1% annual tax per on outstanding national bank notes backed by US federal bonds. State banks must start paying a 2% annual tax on state bank notes.
1865-6	Additional National Bank Acts. State banks must start paying a 10% annual tax on state bank notes.
1870	Limit on aggregate national bank note issuance increases to \$354 million.
1873	Bank panic of 1873. Widespread failure of railroad firms leads to stock market crash and bank failures. Jay Cooke and Company goes bankrupt.
1875	Congress repeals limit on aggregate national bank note issuance.
1879	US Treasury starts to promise to convert greenbacks to dollars one-for-one.

1893	Bank panic. A combination of falling commodity prices, oversupply of silver, and a fall in US Treasury gold reserves prompted a run on bank deposits.
1896	Cross of Gold Speech. Democratic presidential candidate William Jennings Bryan gives a speech in favor of allowing unlimited coinage of silver into money demand (“free silver”).
1900	Tax on national bank notes backed by US federal bonds paying coupons less than or equal to 2% is reduced to 0.5% per annum.
1900	Gold Standard Act. The gold dollar becomes the standard unit of account (further restricting the possibility of “free silver”).
1907	Panic of 1907. The Knickerbocker Trust Company collapses prompting a bank run. J.P. Morgan organizes New York bankers to provide liquidity to shore up the banking system.
<hr/>	
1913	Federal Reserve Act. Establishment of the Federal Reserve Bank to act as a reserve money creator of last resort during financial panics.
1914-8	World War I.
1917	2nd Liberty Loan Act establishes a \$15 billion aggregate limit on the amount of government bonds issued.
1929	Stock market crash and start of the Great Depression.
1929	US issues first Treasury Bill.
<hr/>	
1933	Banking Act (“Glass-Steagall Act”). Establishes the Federal Deposit Insurance Corporation (FDIC). Separates commercial and investment banking. Introduces cap on deposit interest rate (“Regulation Q”).
1933	President Roosevelt issues an Executive Order requiring people and businesses to sell their gold to the government at \$20.67 per ounce.
1934	Gold Reserve Act.
1934	National Housing Act. Establishes the Federal Savings and Loan Insurance Corporation (FSLIC).
1935	The last national bank notes are replaced by Federal Reserve notes.
1938	Amendment to the National Housing Act established the Federal National Mortgage Association (FNMA), commonly known as Fannie Mae.
1939-45	World War II.
1942	The Treasury and Federal Reserve agree to fix the yield curve on Treasury securities.
1944	Bretton Woods Agreement.
1951	Treasury-Fed Accord ends the fixed yield curve on Treasury securities and establishes the Fed’s policy independence from fiscal concerns.
1968	Housing and Urban Development Act of 1968. Creates the Government National Mortgage Association (GNMA), commonly known as Ginnie Mae.

1966	Fed applies Regulation Q to impose deposit rate ceiling for the first time.
1971	US effectively terminates the Bretton Woods system by ending the convertibility of the US dollar to gold.
1977	Congress issues the Fed with the dual mandate to “promote effectively the goals of maximum employment, stable prices, and moderate long term interest rates”.
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1980	Depository Institutions Deregulation and Monetary Control Act of 1980 starts to phase out Regulation Q.
1986-1989	Savings and loan crisis.
1994	Riegle-Neal Interstate Banking and Branching Efficiency Act. Allows banks to operate across states.
1999	Gramm–Leach–Bliley Act. Repeals provisions of the Glass-Steagall Act that prohibited a bank holding company from owning other financial companies.
2007-9	Great Financial Crisis.
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2010	Dodd-Frank Wall Street Reform and Consumer Protection Act.

C Additional Theory (For Online Publication)

C.1 Inferring risk premia from the US-UK yield spread

The yield-to-maturity on an annuity with gold coupon payments \bar{m} and price p_t is the rate \bar{y}_t that solves:

$$p_t = \sum_{j=1}^{\infty} \exp(-\bar{y}_t)^j \bar{m} \quad (\text{C.1})$$

Let $\bar{q}_t := \exp(-\bar{y}_t)$. In lemma 1 in Appendix C.2, we show that combining equation (C.1) with equation (3.2) gives the following expression for the yield-to-maturity:

$$\bar{q}_t = 1 - \frac{1}{\sum_{j=0}^{\infty} q_t^{(j,g)}}$$

Let lowercase letters represent US prices and yields and let capital letters represent UK prices and yields. Then from corollary 1 in Appendix C.2, we have that the difference between the US and UK consol yields is:

$$\bar{y}_t - \bar{Y}_t \approx \frac{\sum_{j=0}^{\infty} (q_t^{(j,g)} - Q_t^{(j,g)})}{\left(\sum_{j=0}^{\infty} q_t^{(j,g)}\right) \left(\sum_{j=0}^{\infty} Q_t^{(j,g)}\right)} \quad (\text{C.2})$$

where \bar{y}_t and $q_t^{(j,g)}$ are yields-to-maturity and zero-coupon prices in the US and \bar{Y}_t and $Q_t^{(j,g)}$ are yield-to-maturity and zero-coupon prices in the UK. If, for simplicity, we impose the following structure on the pricing

kernel:

$$p_t^{(i,j)} = \mathbb{E}_t \left[\left(\frac{S_{t+j}}{S_t} \right) \left(\frac{e_{t+j}^{(g)}}{e_t^{(g)}} \right) \xi_{t+j}^{(i)} \bar{m}_{t+j}^{(i)} \right]$$

where we have implicitly assumed that there is no convenience yield on UK or US debt. If, in addition, we impose that haircut risk is zero in the UK implies, then the spread between US and UK zero-coupon bond prices is given by:

$$q_t^{(j,g)} - Q_t^{(j,g)} = \mathbb{E}_t \left[\frac{S_{t+j}}{S_t} \right] \left(\underbrace{\mathbb{E}_t \left[\frac{e_{t+j}^{(n)}}{e_t^{(n)}} \right]}_{\text{Expected gold inflation in US}} \underbrace{\mathbb{E}_t [\xi_{t+j}]}_{\text{US haircut probability}} \underbrace{\left(1 + \text{Cov}_t \left(\frac{S_{t+j}}{S_t}, \frac{\xi_{t+j}}{\mathbb{E}_t \left[\frac{S_{t+j}}{S_t} \right]} \right)}_{\text{Risk premium on US haircut risk}} \right)}_{\text{Expected gold inflation in UK}} \right)$$

If gold inflation expectations were similar in the US and UK during the gold standard⁵⁸, then we can interpret the difference between the US and UK consol yields in figure 4 as reflecting the risk premium on US federal debt.

Estimating haircut risk: In principle, we could attempt to use UK yields to estimate haircut risk. However, we face the major challenge of only observing the prices of UK consols. This means that, to make progress, we would need to impose a one-dimensional functional parameterisation of $\mathbb{E}_t[\xi_{t+j}]$. Here is one way to do this. Suppose that government haircuts are governed by a two-state Markov Chain with default as an absorbing state. Let \mathbf{p}_t be bondholders' perceived probability of default in period t and assume that they use the two-state Markov Chain to forecast future cash-flows. For simplicity, suppose that upon default, government bonds pay 0. These assumptions imply $\mathbb{E}_t[\xi_{t+j}] = (1 - \mathbf{p}_t)^j$. In addition, suppose that bondholders' are risk-neutral in the sense that $\text{cov}_t \left(\frac{S_{t+j}}{S_t}, \xi_{t+j}^{(i)} \right) = 0$. In this special case, we have:

$$q_t^{(j,n)} - Q_t^{(j,g)} = \underbrace{\mathbb{E}_t \left[\frac{e_{t+j}^{(n)}}{e_t^{(n)}} \right]}_{\text{Expected gold inflation in US}} \underbrace{(1 - \mathbf{p}_t)^j}_{\text{US haircut probability}} - \underbrace{\mathbb{E}_t \left[\frac{E_{t+j}^{(n)}}{E_t^{(n)}} \right]}_{\text{Expected gold inflation in UK}}$$

which we could combine with equation (C.2) to estimate a haircut probability \mathbf{p}_t . Of course, this particular example imposes strong assumptions and ignores the possibility of varying convenience yields on US and UK federal debt. We leave the complicated task of resolving the estimation of haircut risk to future work.

C.2 Connection Between Yields on Finite-Horizon Zero-Coupon Bonds and Yield-To-Maturity

Some analysts have expressed historical long-term interest rates as yields-to-maturity rather than zero-coupon yields. In this appendix, we discuss the connection between the different types of yields. A yield-to-maturity (a.k.a. an internal rate of return) is defined as a fixed discount rate, $\bar{y}^{(i,n)}$, that equates the currency n bond price to the present discounted value of its promised currency n payments. Thus, the dollar n yield-to-maturity

⁵⁸We have not estimated inflation expectations during the nineteenth century in the UK, but this seems like a reasonable prior given that both countries were on the gold standard.

on bond i with payments in currency n and maturity $J^{(i)}$ is the rate $\bar{y}_t^{(i,n)}$ that solves:

$$p_t^{(i,n)} = \sum_{j=1}^{J^{(i)}} \exp\left(-\bar{y}_t^{(i,n)}\right)^j \bar{m}_{t+j}^{(i,n)}$$

To compare to the zero-coupon prices, let $\bar{q}_t^{(i,n)} := \exp\left(-\bar{y}_t^{(i,n)}\right)$. The bond price can be expressed in terms of $\bar{q}_t^{(i,n)}$ as:

$$p_t^{(i,n)} = \sum_{j=1}^{J^{(i)}} \left(\bar{q}_t^{(i,n)}\right)^j \bar{m}_{t+j}^{(i,n)}. \quad (\text{C.3})$$

Lemma 1. Consider a bond with $J^i = \infty$ and $\bar{m}_{t+j}^{(i,n)} = \bar{m}^{(n)}$ (i.e. a fixed coupon annuity in currency n). Denote the yield-to-maturity on such a bond by $\bar{y}_t^{(n)}$ and the associated price by $\bar{q}_t^{(n)} := \exp(-\bar{y}_t^{(n)})$. Then $\bar{q}_t^{(n)}$ can be expressed in terms of zero-coupon yields as:

$$\bar{q}_t^{(n)} = 1 - \frac{1}{\sum_{j=0}^{\infty} q_t^{(j,n)}} \quad (\text{C.4})$$

Proof. From equation (C.3), we have that the price of the fixed coupon annuity is:

$$\bar{p}_t^{(n)} = \sum_{j=1}^{\infty} \left(\bar{q}_t^{(n)}\right)^j \bar{m}^{(n)} = \bar{m}^{(n)} \sum_{j=1}^{\infty} \left(\bar{q}_t^{(n)}\right)^j = \bar{m}^{(n)} \left(\frac{1}{1 - \bar{q}_t^{(n)}} - 1\right)$$

From equation (3.2) we also have the expression:

$$\bar{p}_t^{(n)} = \sum_{j=1}^{\infty} \bar{q}_t^{(j,n)} \bar{m}^{(n)} = \bar{m}^{(n)} \left(\sum_{j=0}^{\infty} \bar{q}_t^{(j,n)} - 1\right)$$

where $\bar{q}_t^{(0,n)} = 1$. Equating the expressions gives that:

$$\frac{1}{1 - \bar{q}_t^{(n)}} = \sum_{j=0}^{\infty} \bar{q}_t^{(j,n)}$$

and rearranging gives the desired result. \square

Corollary 1. Let lowercase letters represent US prices and yields and let capital letters represent UK prices and yields. Then the difference between the US and UK consol yields is

$$\bar{y}_t - \bar{Y}_t \approx \frac{\sum_{j=0}^{\infty} \left(q_t^{(j,g)} - Q_t^{(j,g)}\right)}{\left(\sum_{j=0}^{\infty} q_t^{(j,g)}\right) \left(\sum_{j=0}^{\infty} Q_t^{(j,g)}\right)}$$

Proof. Using equation (C.4), we have that:

$$\bar{y}_t = \log(\bar{q}_t^{(n)}) = \log\left(1 - \frac{1}{\sum_{j=0}^{\infty} q_t^{(j,n)}}\right) \approx -\frac{1}{\sum_{j=0}^{\infty} q_t^{(j,n)}}$$

and so:

$$\begin{aligned}\bar{y}_t - \bar{Y}_t &\approx -\frac{1}{\sum_{j=0}^{\infty} q_t^{(j,n)}} + \frac{1}{\sum_{j=0}^{\infty} Q_t^{(j,n)}} \\ &= \frac{\sum_{j=0}^{\infty} (q_t^{(j,g)} - Q_t^{(j,g)})}{\left(\sum_{j=0}^{\infty} q_t^{(j,g)}\right) \left(\sum_{j=0}^{\infty} Q_t^{(j,g)}\right)}\end{aligned}$$

□

Equation (C.3) indicates that the yield-to-maturity on a coupon-bearing bond is some kind of *weighted average* of zero-coupon yields, with cash-flow payments serving as weights. For the case of an annuity, the average is unweighted and reduces to equation (C.4). Because a principal payment is typically substantially larger than the coupon payments, the maturity-related zero-coupon yield gets the largest weight in the average. As a result, a yield-to-maturity on a J -maturity bond can approximate a J -period zero-coupon yield, although the quality of approximation depends on details of a bond's promised payment stream. The only exact equality is that a yield-to-maturity on a j -period zero-coupon bond coincides with the j -period zero-coupon yield, $y_t^{(j,n)}$.

D State-Space Model of Inflation Expectations (For Online Publication)

We estimate inflation expectations between 1794-2020 by applying a univariate state-space model with drifting coefficients and stochastic volatility. The underlying data are our combined inflation series described in Appendix A.1. During the temporary suspension of gold convertibility (1862-1879), the General Price Level Index expresses greenback inflation, so we convert it into gold inflation by using the gold/greenback exchange rate P_t . The estimates in the paper are based on quarterly inflation, however, our key findings are robust to estimating the model using monthly or annual inflation.

Let π_{t+1} denote the logarithm of quarterly price change between period t and $t+1$. We model this variable with the following state-space model with stochastic volatility, changing long-run mean and (infrequently) changing persistence parameter:

Assumption 8. Quarterly inflation π_t obeys a state-space model:

$$\begin{aligned}\pi_{t+1} &= \alpha_t + x_t^\pi + \sigma_{\pi,t} \varepsilon_{\pi,t+1} & \varepsilon_{\pi,t+1} &\sim \mathcal{N}(\mathbf{0}, 1), \quad \forall t \geq 0 \\ x_{t+1}^\pi &= \rho_t x_t^\pi + \sigma_x \varepsilon_{\pi,t+1}\end{aligned}\tag{D.1}$$

where x_t^π is a hidden state with a given initial x_0 . Parameters α_t and $\sigma_{\pi,t}$ follow random walks:

$$\begin{aligned}\alpha_{t+1} &= \alpha_t + \sigma_\alpha \varepsilon_{\alpha,t+1} & \varepsilon_{\alpha,t+1} &\sim \mathcal{N}(\mathbf{0}, 1) \\ \log \sigma_{\pi,t+1} &= \log \sigma_{\pi,t} + \sigma_{\sigma_\pi} \varepsilon_{\sigma_\pi,t+1} & \varepsilon_{\sigma_\pi,t+1} &\sim \mathcal{N}(\mathbf{0}, 1)\end{aligned}$$

while the persistence parameter ρ_t follows a random walk with infrequent shocks:

$$\rho_{t+1} = \begin{cases} \rho_t + \sigma_\rho \varepsilon_{\rho,t+1} & \varepsilon_{\rho,t+1} \sim \mathcal{N}(\mathbf{0}, 1) & \text{if } t = k\Delta \text{ for } k \in \mathbb{N} \\ \rho_t & & \text{otherwise} \end{cases}.$$

Our baseline estimates set $\Delta = 4$, i.e., the persistence of quarterly inflation can change once every year. Model (D.1) posits that j -period ahead logged inflation, $\sum_{i=1}^j \pi_{t+i}$, is a normal random variable, implying that j -period ahead gross inflation, $\Pi_t^{(j,n)}$, is log-normal. Using the model-implied conditional mean and variance of $\sum_{i=1}^j \pi_{t+i}$, one can derive an estimate for $\mathbb{E}_t \left[\exp \left(-\pi_t^{(j,n)} \right) \right]$ that goes into formula (3.4). We estimate this model using the same HMC-NUTS sampler that we use for our yield curve model.

Priors: We use independent Gaussian priors for σ_x and the initial parameters α_0 and ρ_0 :

$$\sigma_x \sim \mathcal{N}(0, 0.5), \quad \alpha_0 \sim \mathcal{N}(0, 1), \quad \rho_0 \sim \mathcal{N}(0, 0.5)$$

For the initial standard deviation $\sigma_{\pi,0}$, we use a log-normal prior $\sigma_{\pi,0} \sim \log \mathcal{N}(0.015, 0.01)$. For the standard deviations σ_α , σ_{σ_π} , and σ_ρ , we use a *common* exponential prior with the rate parameter tuned so that *a priori* the probability that $\sigma_i > 0.3$ is lower than 5%. The corresponding prior mean is 0.1.

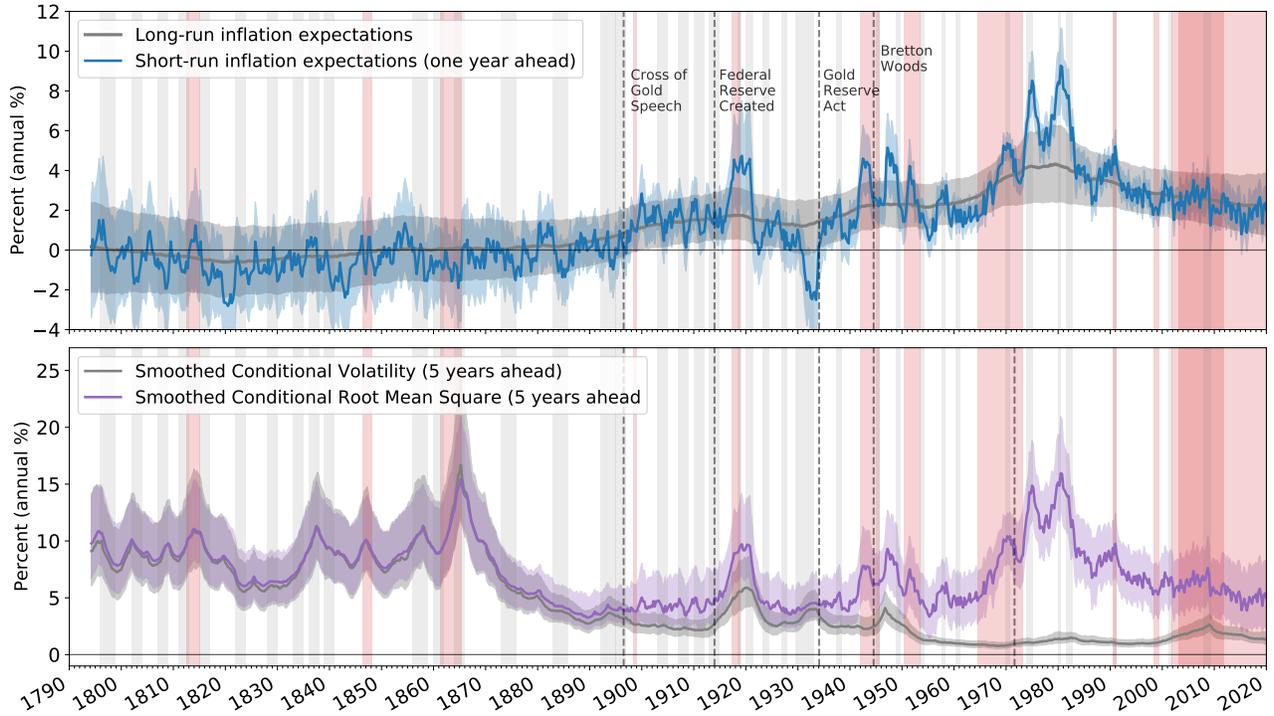


Figure 20 Smoothed Conditional Moments of Inflation

Top plot: The solid grey line depicts the posterior median estimate for the permanent component of inflation. The solid blue line depicts the posterior median estimate for period t one year ahead inflation expectations implied by our statistical model. Bands around the posterior medians depict 90% interquartile ranges. *Bottom plot:* The solid grey line depicts the posterior median estimate for the 5-year-ahead smoothed, annualized conditional inflation volatility. The solid purple line depicts the posterior median estimate for the 5-year-ahead smoothed conditional root mean square—a measure of conditional second moment of inflation—used by Cogley and Sargent (2015) to quantify ‘price instability’ (as opposed to unpredictability). The light bands around the posterior medians depict 90% interquartile ranges. The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars.

Results: The posterior distributions of conditional moments implied by this model are depicted in Figure 20. The top panel shows conditional inflation expectations: color grey refers to long term expectations (permanent component of inflation), color blue represents inflation expectations one year ahead. The grey line in the bottom

plot depicts the posterior median estimate for the model implied 5 year ahead conditional inflation volatility. We plot the *annualized* conditional volatility defined as

$$\sigma_{\pi,t}^{(j)} := \sqrt{\frac{1}{j} \left(\mathbb{E}_t \left[\exp \left(2\pi_t^{(j)} \right) \right] - \mathbb{E}_t \left[\exp \left(\pi_t^{(j)} \right) \right]^2 \right)}.$$

The purple line in the bottom plot depicts the posterior median estimate for the 5-year-ahead smoothed conditional root mean square statistic—a measure of conditional second moment of inflation—used by [Cogley and Sargent \(2015\)](#) to quantify ‘price instability’ (as opposed to unpredictability). In this case, the conditional root mean square statistic can be written as

$$crms_{\pi,t}^{(j)} := \sqrt{\frac{1}{j} \mathbb{E}_t \left[\exp \left(2\pi_t^{(j)} \right) \right]}.$$

It is worthwhile comparing our estimates here with what [Goodfriend and King \(2005\)](#) describe as Paul Volcker’s incredible disinflation.

E Priors (For Online Publication)

Gold Dollar Yield Curve: Assumptions 2 and 4 give rise to a flexible model of the gold denominated yield curve process that is pinned down by a small set of hyper-parameters. We specify a prior on τ and the initial (time 0) λ vector that effectively determines an “average yield curve” for the whole sample period. We use log-normal prior for τ and independent log-normal priors for the three entries of the initial λ vector that implies the prior distribution for the initial yield curve shown in the left panel of Figure 21. Our prior imposes a flat “average yield curve,” i.e., for all maturities the prior mean is 10% with standard deviation of around 5%. More precisely, the underlying priors are:

$$\lambda_{0,0} \sim \log \mathcal{N}(10 - \underline{\beta}, 6), \quad \lambda_{1,0} \sim \log \mathcal{N}(10 - \underline{\beta}, 6), \quad \lambda_{2,0} \sim \log \mathcal{N}(10 - \underline{\beta}, 15), \quad \tau \sim \log \mathcal{N}(60, 60).$$

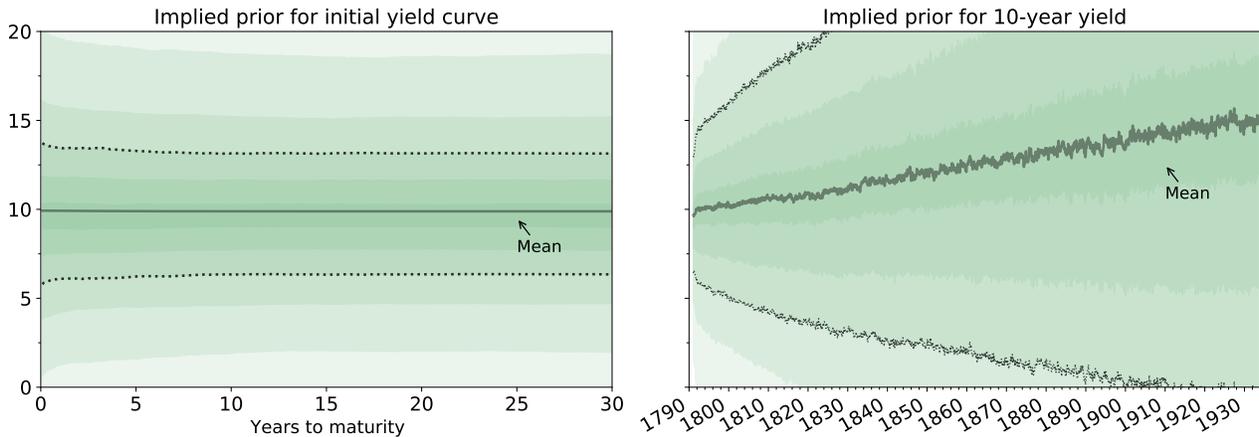


Figure 21 Implied prior distribution of the initial yield curve and the 10-year zero-coupon yield.

The solid grey lines depict the mean, dotted lines depict the 25% and 75% percentiles of the prior distribution. Shaded areas represent interquantile ranges so that dark areas are indicative of concentrated prior probability.

While the “average yield curve” influences our posterior distribution in the early part of the sample, it is much less influential later due to the random walk component in λ_t . The right panel of Figure 21 illustrates how the prior mean and “prior coverage bands” for the 10-year yield grow over time. How much our prior for λ_0 affects the posterior distribution for later periods depends mainly on our priors on $\{\bar{\lambda}_t\}$, ϱ , and $\{\Sigma_t\}$ that we specify as follows:

- For the correlation matrix Ω we use the LKJ prior with a concentration parameter $\eta = 5$, which is a unimodal but fairly vague distribution over the space of correlation matrices. For η values larger than 1, the LKJ density increasingly concentrates mass around the unit matrix, i.e., favoring less correlation.⁵⁹
- For the initial standard deviations σ_0 we use independent log-normal priors: $\sigma_{i,0} \sim \log \mathcal{N}(0.05, 0.1)$.
- We use *common* exponential priors on the standard deviation in the diagonal of Ξ_σ , with the rate parameter tuned so that *a priori* the probability that $\sigma_\sigma^{(i)} > 0.15$ is lower than 5%. The prior mean is 0.05.
- We use independent normal priors on the entries of ϱ . The prior mean is chosen as a diagonal matrix with diagonal entries $[0.8, 0.8, 0.8]$ while we set standard deviation of 0.3 for all 9 entries of ϱ .
- We use independent log-normal priors for the three entries of the initial $\bar{\lambda}_0$ (permanent component of λ):

$$\bar{\lambda}_{0,0} \sim \log \mathcal{N}(10 - \underline{\beta}, 6), \quad \bar{\lambda}_{1,0} \sim \log \mathcal{N}(10 - \underline{\beta}, 6), \quad \bar{\lambda}_{2,0} \sim \log \mathcal{N}(10 - \underline{\beta}, 15).$$

- We use *common* exponential priors on the standard deviation in the diagonal of $\bar{\Xi}$, with the rate parameter tuned so that *a priori* the probability that $\bar{\sigma}^{(i)} > 0.15$ is lower than 5%. The prior mean is 0.05.

Pricing Errors: We use *common* exponential priors on the standard deviation of pricing errors, $\sigma_m^{(i)}$, with the rate parameter tuned so that *a priori* the probability that $\sigma_m^{(i)} > 30$ is lower than 5%. Prior mean is 10.

Model of Exchange Rates: We use independent Gaussian priors for all components of θ_0 except for F .

- For entries of the initial long-run mean vector μ_0 and matrix K , we set the mean of the Gaussian prior to the point estimates coming from estimating a time-invariant version of the model in Assumption 7 using data for 1862-1863. We set the standard deviations so that the prior allows for reasonably large deviations from these point estimates.⁶⁰ This procedure guarantees that the prior distribution concentrates on sensible parameter values, but because the estimation is based on a short stretch of data, the location of the parameters is only weakly restricted.
- For entries of the initial persistence matrix A_0 we set a prior that assumes mildly positive auto-correlations for both entries of x_t while being agnostic about the cross-terms.⁶¹ Observe that we do not explicitly restrict A_t to be a stable matrix, but use a prior that pushes the initial A_0 matrix in the direction of the “stable region.”
- Parameter matrix F is lower-triangular that we parameterize as follows. First, similar to (3.5), we decompose the covariance matrix FF' into correlation coefficients and marginal variances $FF' = \Xi_F \Omega_F \Xi_F$, where Ξ_F is a diagonal matrix containing the marginal standard deviations and Ω_F is the corresponding

⁵⁹See Lewandowski et al. (2009). The LKJ distribution is defined by $p(\Omega|\eta) \propto \det(\Omega)^{\eta-1}$. For $\eta = 1$, this is a uniform distribution.

⁶⁰In particular, we set $\mu_0[1] \sim \mathcal{N}(1, 1)$, $\mu_0[2] \sim \mathcal{N}(1.23, 1)$, and $K[1, 1] \sim \mathcal{N}(0.03, 0.05)$, $K[2, 1] \sim \mathcal{N}(-0.04, 0.05)$, $K[1, 2] \sim \mathcal{N}(0.0, 0.05)$, $K[2, 2] \sim \mathcal{N}(0.03, 0.05)$.

⁶¹In particular, we set $A_0[1, 1] \sim \mathcal{N}(0.9, 0.1)$, $A_0[2, 1] \sim \mathcal{N}(0, 1)$, $A_0[1, 2] \sim \mathcal{N}(0, 1)$, $A_0[2, 2] \sim \mathcal{N}(0.9, 0.1)$.

correlation matrix. Matrix F can be written as $F = \Xi_F L\Omega_F$, where $L\Omega_F$ is the lower-triangular Cholesky factor of Ω_F such that $(L\Omega_F)(L\Omega_F)' = \Omega_F$. For the standard deviations in the diagonal of Ξ_F we use log-normal priors (independent across components): $\sigma_F^{(1)} \sim \log \mathcal{N}(0.02, 0.01)$ and $\sigma_F^{(2)} \sim \log \mathcal{N}(0.04, 0.01)$. For the Cholesky factor $L\Omega_F$ we use the LKJ prior with concentration parameter $\eta_F = 2$.

- We assume that Ξ_μ and Ξ_A are diagonal matrices, i.e., shocks to the components of μ_t and A_t are independent. For their standard deviations we use a *common* exponential prior (independent across components) with the rate parameter tuned so that *a priori* the probability that $\sigma_i > 0.06$ is lower than 5%. The prior mean is 0.02.

F Additional Estimates For Gold Yield Curve (For Online Publication)

F.1 Comparison to Other Historical Estimates

The [Homer and Sylla \(2004\)](#) series depicted in figure 11 is not the long-term US bond series that is commonly used in the economic history literature. Instead, researchers⁶² have typically used a ‘composite series’ that combines the [Homer and Sylla \(2004\)](#) estimates for the period from 1798-1861 with the yield-to-maturity on the New England Municipal bond for the period 1862-1899 and the yield-to-maturity on corporate bonds for the period 1900-1940.⁶³ Figure 22 plots this composite series alongside our 10-year yields. Our estimates diverge post 1861 when the composite series stops using US federal debt prices. We estimate a much higher long-term yield during the war and a lower long-term yield in the late nineteenth century. Possible sources for these discrepancies are that federal debt carried a greater default risk during the Civil War and that, after the war, National Banking Era protocols stimulated demands for federal bonds as reserves against National Bank Notes.

⁶²For example, [Officer and Williamson \(2021\)](#), [Shiller \(2015\)](#), [Jordà et al. \(2019\)](#), and [Hamilton et al. \(2016\)](#).

⁶³It is not obvious that during the 19th century municipal debt was a safer investment than federal debt. Until the 1934 Gold Reserve Act, the federal government had never defaulted. In contrast, eight states and one territory defaulted in 1830s and 1840s and ten states defaulted in 1870s and 1880s. These state defaults are discussed in [McGrane \(1935\)](#) and [English \(1996\)](#).

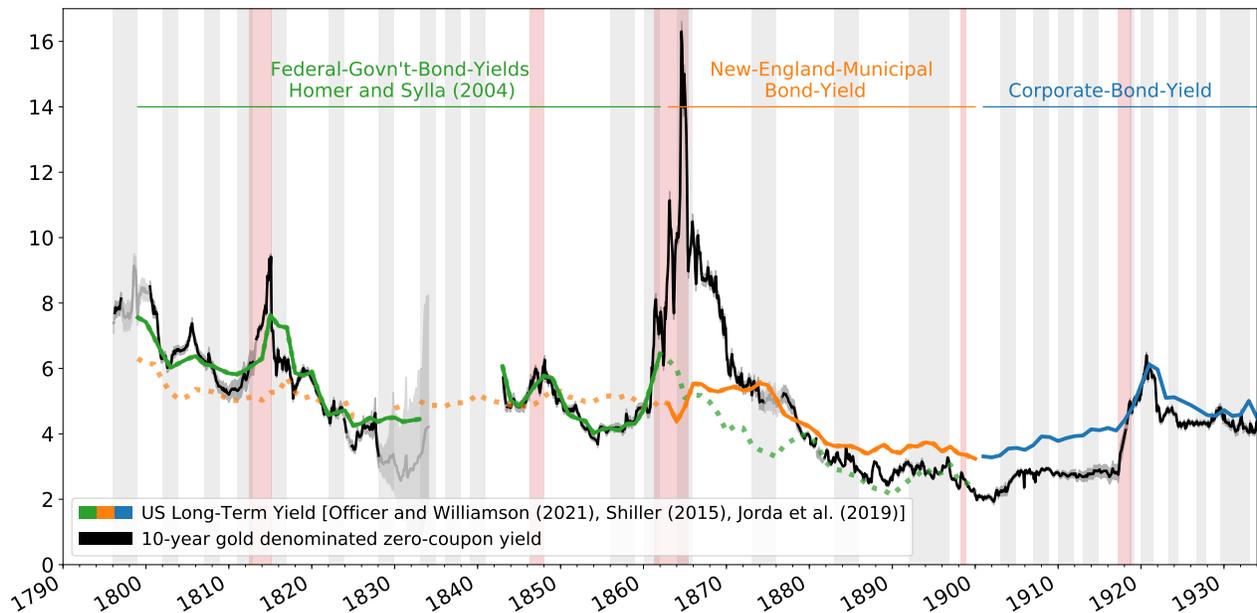


Figure 22 Alternative Long-Term Yield Estimates.

The solid black line depicts the mean of our posterior estimate for the 10-year, gold denominated, zero coupon yield. The dashed grey line depicts the mean of our posterior estimate for the 10-year, dollar denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquantile range. The green line (bold and dotted) depicts the ‘US Government Bond Yield’ series from [Homer and Sylla \(2004\)](#). The orange line (bold and dotted) depicts the New England Municipal Bond Yield reported by [Homer and Sylla \(2004\)](#). The blue line depicts the Corporate Bond Yield reported by [Homer and Sylla \(2004\)](#). The bold green-orange-blue line depicts the ‘composite’ bond series used by [Officer and Williamson \(2021\)](#). The light gray intervals depict recessions as dated by [Davis \(2006\)](#) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

F.2 Comparison to Other Short Term Yields

Figure 23 depicts our estimates for 1-year gold denominated zero-coupon yields alongside a short term yield series used by [Officer and Williamson \(2021\)](#) and [Jordà et al. \(2019\)](#).⁶⁴ We have more difficulty estimating the 1-year yields than the 10-year yields because some periods have very few price observations for bonds that are close to maturity. This is reflected in sizes of 95% interquantile ranges for 1-year zero-coupon yields in figure 23. We are most concerned about the period 1790-1815 when our only price observations are for the consol bonds that Alexander Hamilton issued to refinance the Revolutionary War debts.⁶⁵ By contract, the Hamilton consols had no maturity dates. Because the federal government ended up repurchasing and retiring all of these bonds, our perfect foresight assumption means that we treat them as finite maturity bonds.⁶⁶ This allows us to estimate a yield curve, but we are faced with two problems: investors may not have anticipated that the bonds

⁶⁴The figure depicts the series labeled as “Surplus Funds (Contemporary Series).” The Series involves the short-term lending or borrowing of surplus funds, that is, funds that are considered excess by the lending institution and are required for immediate temporary use by the borrowing entity.

⁶⁵[Bayley \(1882\)](#) calls these bonds: *The Six Percent Stock of 1790, The Deferred Six Percent Stock of 1790, and The Three Percent Stock of 1790.*

⁶⁶The time to maturity in figure 1 shows the time until the bonds were bought back by the government. The Act authorizing the issuance of the 1790 Stocks provided for a committee comprised of the president of the Senate, Chief Justice, Secretary of State, Secretary of the Treasury, and Attorney General to use surplus revenue to repurchase these stocks at market prices, if not exceeding par. Between 1791 and 1824, nearly all of the outstanding Six Percent and Deferred Six Percent Stocks were repurchased. By 1832, nearly all of the outstanding Three Percent Stock was repurchased. See [Bayley \(1882, pages 33, 110\)](#).

would be repurchased and when, and “times-to-repurchase” were typically greater than 10 years, providing us with little information about the short end of the yield curve. For these reasons, we drop data from 1790-95 and treat the short yield curve during 1790-1815 with caution.

Our short term yield series substantially departs from popular alternative series, especially during the Civil War when we estimate substantially higher yields, peaking at approximately 44% in July 1864. Anecdotal evidence indicates that Union short-term debt paid very high yields during the Civil War. For example, [Homer and Sylla \(2004, page 302\)](#) report that in 1860 the Treasury had issued one-year notes at rates of 10-12% and had rejected bids ranging from 15-36%. One-year yields are negative in the early 1880s and close to zero in the early 1890s. What parts of our data most influence our inferences about these negative yields? It is that these negative yields help price both the *Four Percent Loan of 1907* and the *Four and One-Half Percent Loan of 1891*.⁶⁷ Economic events that may or may not be sources of these low gold yields during the early 1880s are that financial markets were highly volatile, that the US government was using surpluses to repurchase bonds, and that the US had just returned the gold standard in January 1879 (see [Noyes, 1909, pp. 79-80](#)).

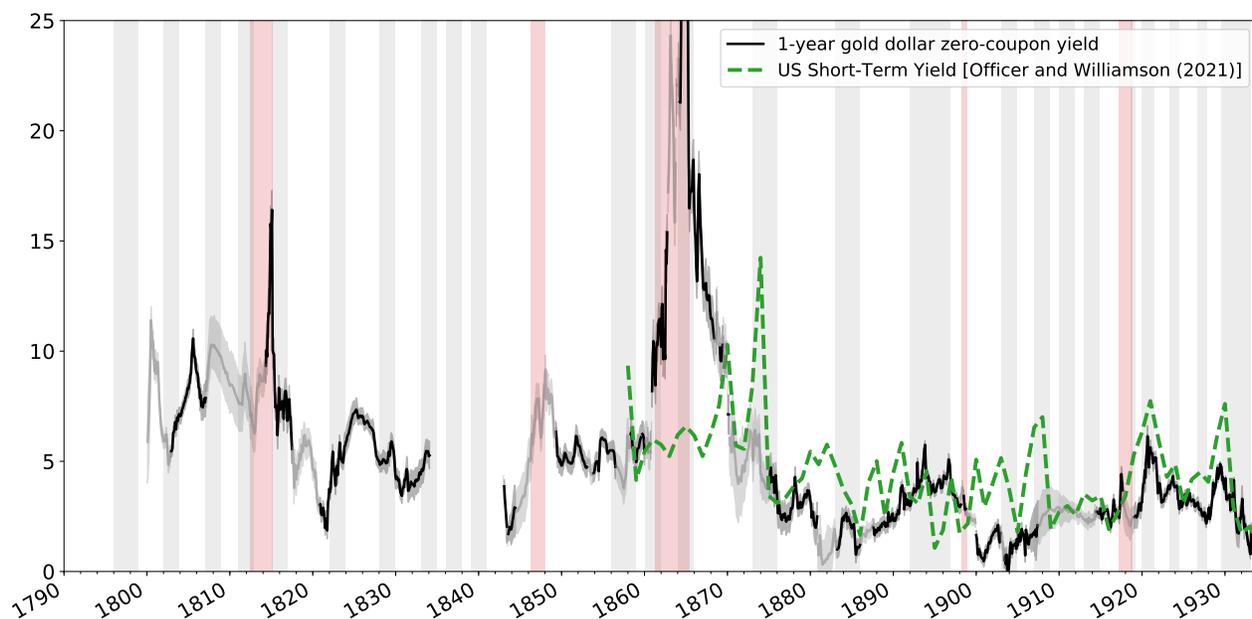


Figure 23 Short-Term Yields.

The solid black line depicts the mean of our posterior estimate for the 1-year, gold denominated, zero coupon yield. The dashed grey line depicts the mean of our posterior estimate for the 10-year, dollar denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquartile range. The green dotted line depicts the US short term yield series (surplus funds, contemporary) used by [Officer and Williamson \(2021\)](#) and [Jordà et al. \(2019\)](#). The light gray intervals depict recessions as dated by [Davis \(2006\)](#) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

⁶⁷These are the names used in [Bayley \(1882\)](#). We initially imposed non-negativity constraints in the estimate of the yield curve. This led to small pricing errors for the *Four Percent Loan of 1907* but large pricing errors for the *Four and One-Half Percent Loan of 1891* in the early years of the 1880s. Relaxing the non-negativity constraint significantly reduced the pricing errors on the *Four Percent Loan of 1907* without increasing other errors. We take this as suggestive statistical evidence that the yield curve went negative in the early 1880s, but further investigation is required.

G Additional Statistical Fits (For Online Publication)

We present additional information about model fit. In subsection G.1 we plot pricing errors over time and in subsection G.2 we show pricing errors across maturities.

G.1 Pricing Errors Over Time

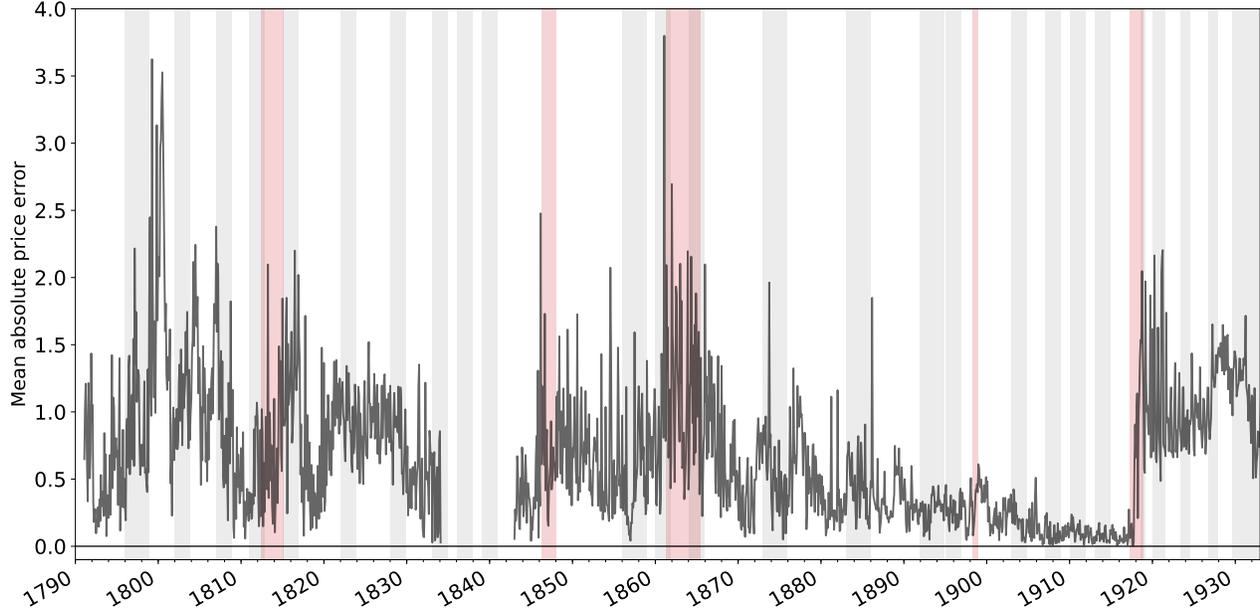


Figure 24 Time Series of Mean Absolute Pricing Errors

The black line depicts the cross-sectional average (over bonds for each month) of the absolute difference between observed prices and posterior median price forecasts. The light gray intervals depict recessions as dated by [Davis \(2006\)](#) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

G.2 Pricing Errors Across Maturities

Figure 25 replicates our “short rate disconnect” plot (see Figure 5) with a finer set of maturity bins. It shows that yield errors tend to average out for all maturity bins that we included in the estimation. The left panel in Figure 26 depicts distributions of errors for the specified maturity bins, illustrating the same point from a different angle. The left box represents maturities that we exclude from the estimation (0-1 years), the rest of the boxes correspond to the included maturity bins. Again, we see that on average our parametric yield curve specification fits observed prices well. The right panel in Figure 26 shows the mean absolute pricing errors at different maturities. We do not see systematic differences in the pricing errors at different maturities. We take this as evidence that our parametric specification captures the most important “local variations” across maturities.

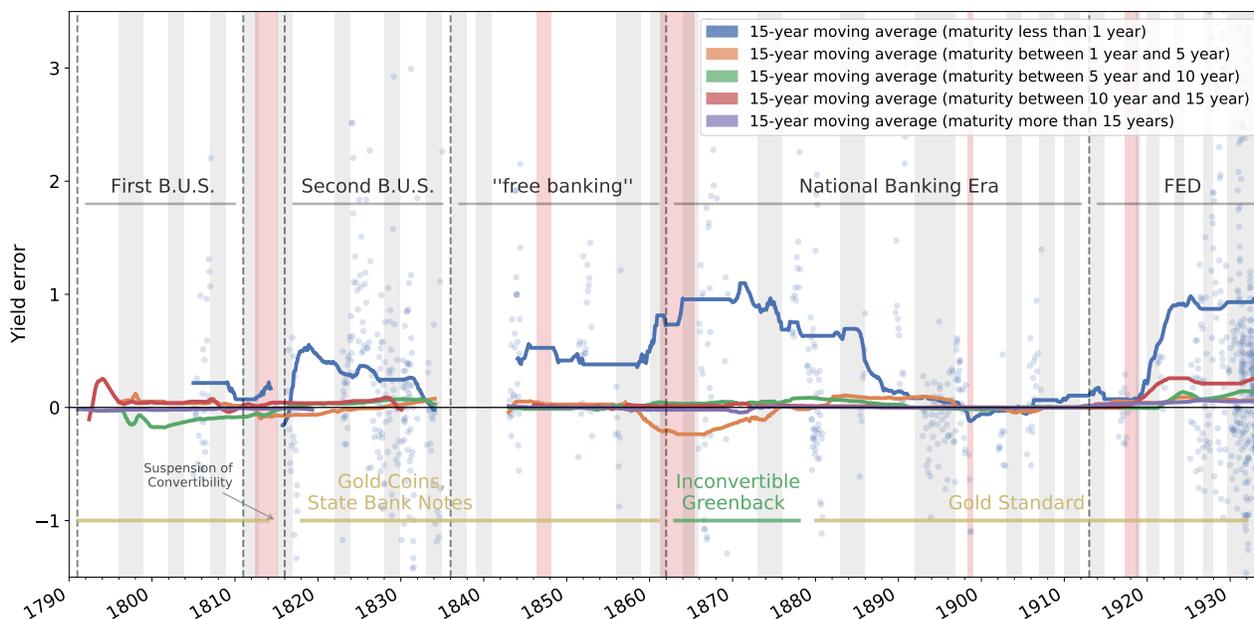


Figure 25

H Does the slope predict recessions? (For Online Publication)

Figure 27 depicts the yield on 5-year government bonds minus the yield on 1-year government bonds. We refer to this as a term spread. A positive term spread indicates an upward sloping yield curve (i.e., longer maturity bonds have higher rates), while a negative term spread indicates an inverted yield curve (i.e., shorter maturity bonds have higher rates). Yield curves were typically upward sloping throughout the nineteenth century, with notable inversions during the War of 1812, the early 1830s, the Mexican-American War, the Civil War, and in the late 1890s.

A large literature has used yields to help predict real GDP growth.⁶⁸ Our yield curve estimates open the way to extend such work back into the nineteenth century. As a preliminary step, our table 3 below emulates table 2 from [Ang et al. \(2006\)](#). It reports the coefficient $\beta_k^{(j)}$ and R^2 for the regression:

$$g_{t+k} = \alpha_k^{(j)} + \beta_k^{(j)} (y_t^{(10)} - y_t^{(j)}) + \varepsilon_{t+k,k}^{(j)}$$

where g_{t+k} is the annual percentage growth of real GDP over the next k years and $y_t^{(j)}$ denotes the annualized j -year zero coupon yield for $j \in \{1, 5\}$. Notice that an upward sloping yield curve appears to be positively correlated with future economic growth during the 19th century even though no central bank existed to engage in “active” monetary policy.⁶⁹

In table 3, we report the coefficients from the regression of the change in the spread on GDP growth and find additional suggestive evidence that nineteenth century spreads have some predictive ability.

⁶⁸See [Stock and Watson \(2003\)](#) for a critical literature review.

⁶⁹However, from 1897 until 1913, Republican Secretaries of the Treasury more and more violated the letter of the 1844 Independent Treasury Act by *de facto* conducting open market operations intended to lean against the wind.

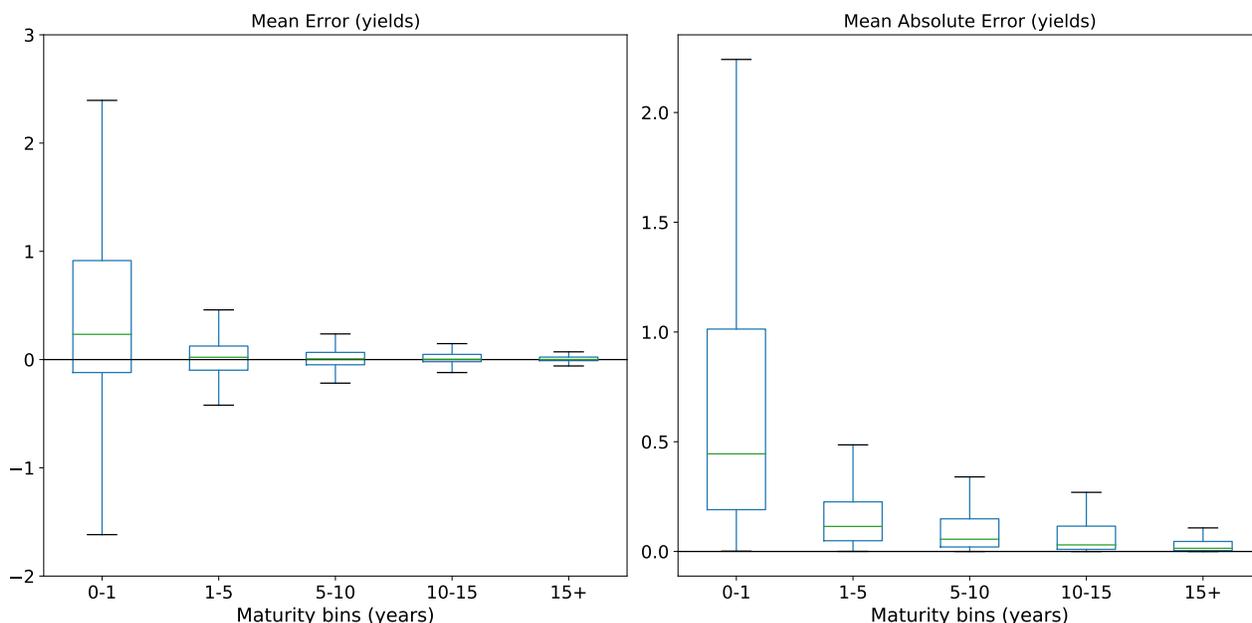


Figure 26 Mean Absolute Pricing Errors at Maturities.

	1797-1860				1866-1933				1950-2000			
	Term spread maturity											
Horizon	10y - 1y		10y - 5y		10y - 1y		10y - 5y		10y - 1y		10y - 5y	
k -years	β_k^1	R^2	β_k^5	R^2	β_k^1	R^2	β_k^5	R^2	β_k^1	R^2	β_k^5	R^2
1-year	0.23	0.040	0.42	0.018	0.07	0.000	0.46	0.002	1.28	0.230	3.54	0.159
	(0.12)		(0.37)		(0.32)		(0.80)		(0.46)		(1.66)	
3-year	0.79	0.084	1.63	0.051	0.31	0.002	1.00	0.003	1.56	0.103	2.71	0.028
	(0.31)		(0.99)		(0.81)		(1.83)		(0.72)		(1.13)	

Table 3 Forecasts of real GDP growth from term spreads

The table reports the coefficient $\beta_k^{(j)}$ and R^2 for the regression $g_{t+k} = \alpha_k^{(j)} + \beta_k^{(j)} (y_t^{(10)} - y_t^{(j)}) + \varepsilon_{t+k,k}^{(j)}$ where g_{t+k} is the annual percentage growth of real GDP over the next k years and $y_t^{(j)}$ denotes the annualized j -year zero coupon yield. We annualize the yields by taking the arithmetic average for each year. Newey and West heteroskedasticity- and autocorrelation-consistent standard errors with lag order one in parentheses.

Table 4 replicates table 3 but uses the change in the spread rather than the level of the spread.

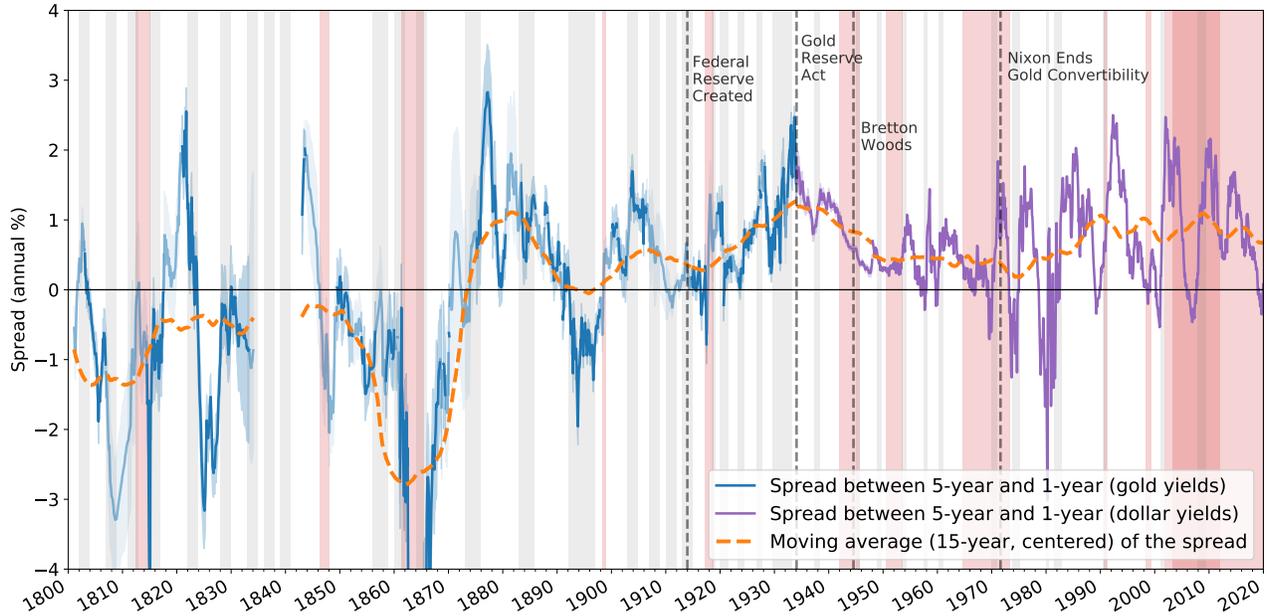


Figure 27 5 Year – 1 Year Yield Spread

The solid blue line depicts the yield on 5-year, gold denominated, zero coupon US government bonds minus the yield on 1-year, gold denominated, zero coupon US government bonds. The pale blue bands around the posterior mean depict the 95% interquartile range. The purple line depicts the same yield spread for dollar denominated bonds (after the US leaves the gold standard). The light gray intervals depict recessions as dated by Davis (2006) for the 1796-1914 period and NBER recessions thereafter. The dark gray intervals depict NBER recessions. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

	1797-1860				1866-1933				1950-2000			
	Term spread maturity											
Horizon	10y - 1y		10y - 5y		10y - 1y		10y - 5y		10y - 1y		10y - 5y	
<i>k</i> -years	β_k^1	R^2	β_k^5	R^2	β_k^1	R^2	β_k^5	R^2	β_k^1	R^2	β_k^5	R^2
1-year	-0.20	0.015	-0.42	0.011	-0.01	0.000	0.37	0.001	1.06	0.128	3.81	0.136
	(0.13)		(0.31)		(0.61)		(1.53)		(0.32)		(1.22)	
3-year	0.57	0.022	0.54	0.003	0.63	0.003	3.27	0.010	2.32	0.166	5.99	0.100
	(0.30)		(1.03)		(1.55)		(3.64)		(0.66)		(2.28)	

Table 4 Forecasts of real GDP growth from first differenced term spreads

The table reports the coefficient $\beta_k^{(j)}$ and R^2 for the regression $g_{t+k} = \alpha_k^{(j)} + \beta_k^{(j)} \left(\left(y_t^{(10)} - y_t^{(j)} \right) - \left(y_{t-1}^{(10)} - y_{t-1}^{(j)} \right) \right) + \varepsilon_{t+k,k}^{(j)}$ where g_{t+k} is the annual percentage growth of real GDP over the next k years and $y_t^{(j)}$ denotes the annualized j -year zero coupon yield. We annualize the yields by taking the arithmetic average for each year. Newey and West heteroskedasticity- and autocorrelation-consistent standard errors with lag order one in parentheses. *** 1%, ** 5%, and * 10% significance.

I Additional Figures (For Online Publication)

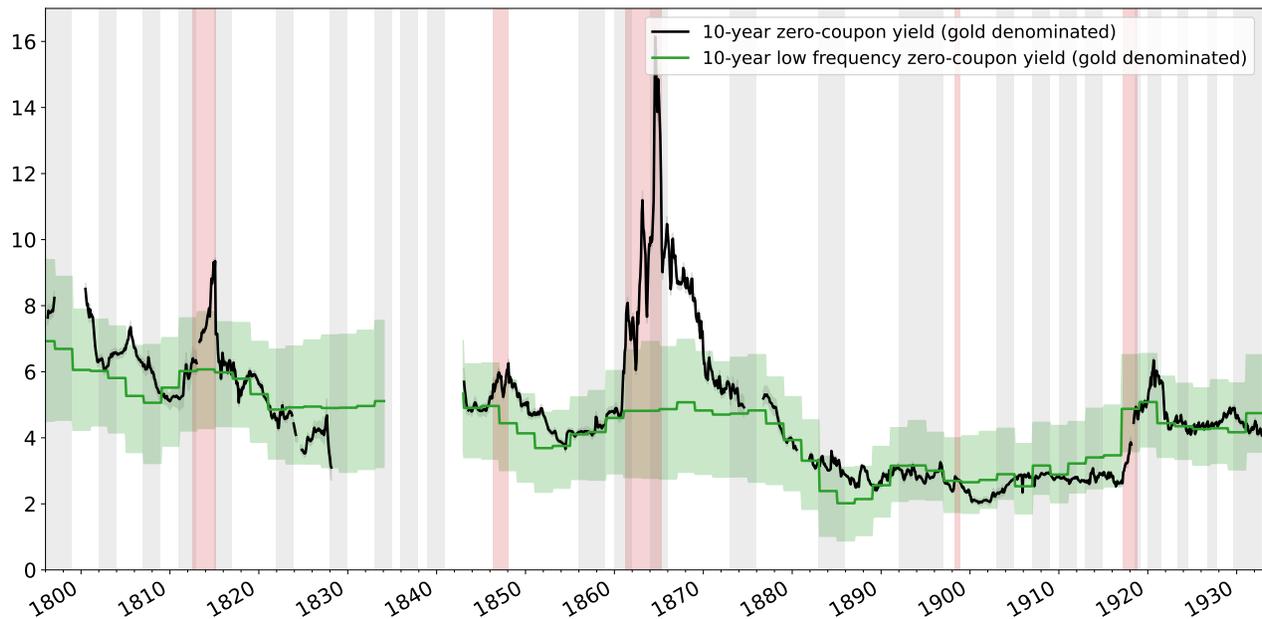


Figure 28 Low Frequency Zero Coupon Yield.

The solid black line depicts the mean of our posterior estimate for the 10-year, gold denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquartile range. The solid green line depicts the mean of our posterior estimate for the low frequency component of the 10-year, gold denominated, zero coupon yield. The light green bands around the posterior mean depict the 95% interquartile range. The light gray intervals depict recessions as dated by [Davis \(2006\)](#) for the 1796-1914 period and NBER recessions thereafter. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

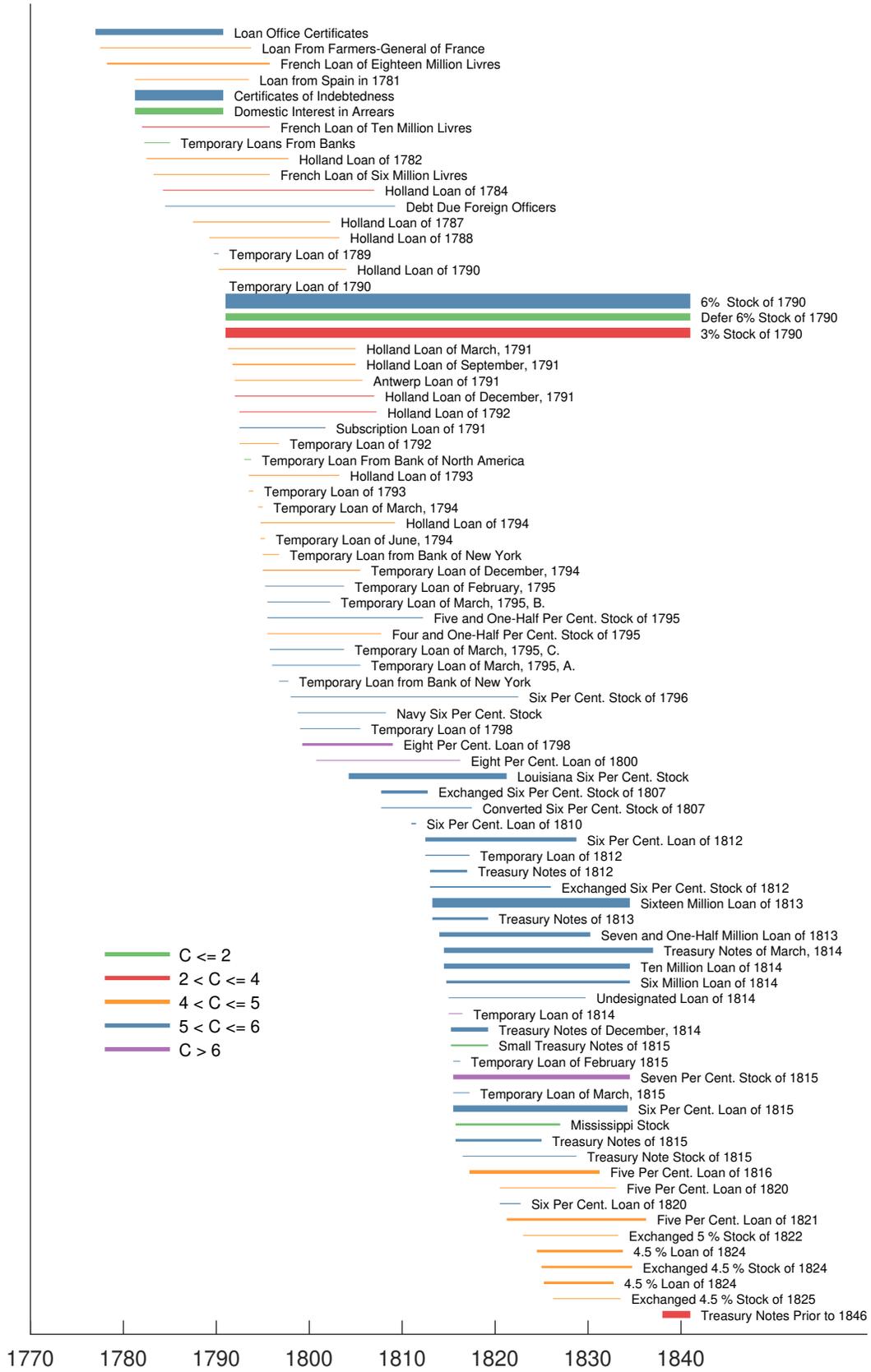


Figure 29 Treasury Bonds Issued from 1776 to 1840.

The span of each line corresponds to the period the security was outstanding. The width is proportional to the size of the issue, and the color denotes the coupon rate.

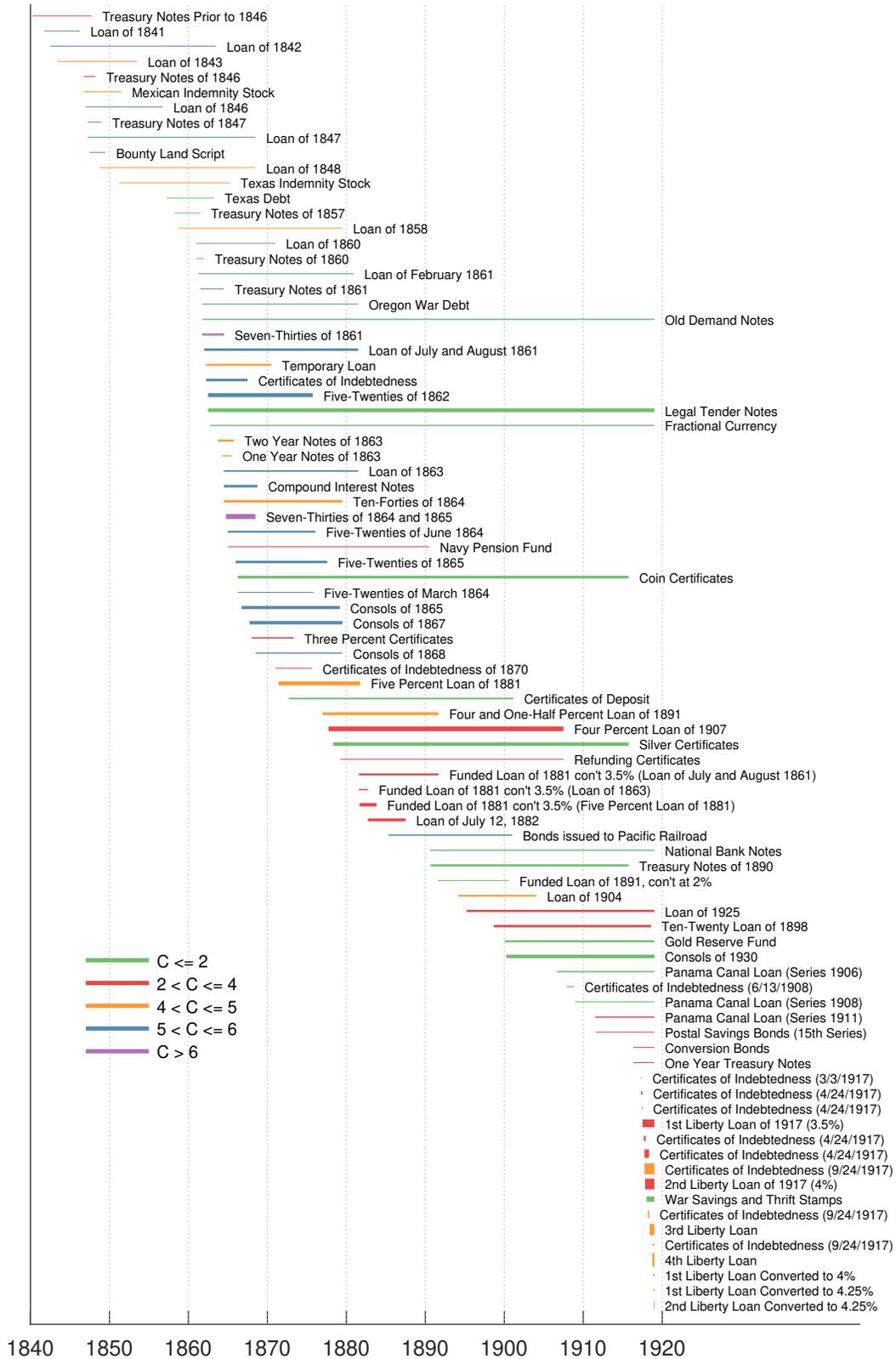


Figure 30 Treasury Bonds Issued from 1840 to 1918.

The span of each line corresponds to the period the security was outstanding. The width is proportional to the size of the issue, and the color denotes the coupon rate.