A p Theory of Government Debt and Taxes

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Motivation

- How much will US tax rates have to rise over time in order to finance the \$29.6 trillion dollar debt outstanding as of Q4 2021?
- What is a maximum sustainable debt-to-GDP ratio, the *equilibrium debt capacity*?
- How long will it take for the US to attain this debt capacity?
- How costly is it for a government to service its debt as a function of its debt-to-GDP ratio?
- Should a government borrow more when $r < g$ (Blanchard, 2019) as in the US today?

Model Key Features

- A minimalistic, pedagogical stochastic continuous-time model of government debt and taxes with

  AS complete financial spanning and risk premia
  - **Arrow securities**, Black and Scholes (1973), Merton (1971, 1973), Harrison and Kreps (1979), Lucas (1978), Shiller (1994), Bohn (1995), and Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019, 2022)

  DT **distortionary taxes**: deadweight costs
  - Barro (1979)

  LC **limited commitment** and option to default
  - Eaton and Gersovitz (1981), Thomas and Worrall (1988), Kehoe and Levine (1993), Kocherlakota (1996), Ai and Li (2015), Bolton, Wang, and Yang (2019), Rebelo, Wang, and Yang (2022)

  IP households' **impatience**
  - Aguiar and Amador (2021), dynamic corporate finance/contracting literature (DeMarzo and Sannikov, 2006)

Output and Financial Markets

- In the normal regime, output Y_t follows

$$\frac{dY_t}{Y_t} = g dt + \psi_h d\mathcal{Z}_t^h + \psi_m d\mathcal{Z}_t^m$$

  - $\mathcal{Z}_t^h$ and $\mathcal{Z}_t^m$: Idiosyncratic and systematic Brownian shocks

- Dynamic risk management:
  - Idiosyncratic risk exposure: $-\Pi_t^h \cdot d\mathcal{Z}_t^h$
  - Systematic risk exposure: $-\Pi_t^m \cdot (\eta dt + d\mathcal{Z}_t^m)$

- Bonds that pay interests at the risk-free rate r

- Unique stochastic discount factor (SDF) $\mathbb{M}_t$
  - **Arrow securities**, Harrison and Kreps (1979), Black and Scholes (1973), Merton (1973), Lucas (1978), Duffie and Huang (1985), and Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019, 2022)

Gov't Spending, Taxes, and Debt

- Exogenous government spending: $\Gamma_t = \gamma Y_t$
- Distortionary taxes: $C_t = c(\tau_t) Y_t$, where $\tau_t = \mathcal{T}_t / Y_t$
- Budget constraint:

$$B_0 \leq \underbrace{\mathbb{E} \int_0^{T^{\mathcal{D}}} \mathbb{M}_t [(\mathcal{T}_t - \Gamma_t)\, dt - dU_t]}_{\text{PV of primary surpluses}}, \tag{1}$$

  where dU_t is the (lumpy) transfer payment to households financed by debt issuance and B_0 is the initial debt level.

  - Debt dynamics:

    $$dB_t = (rB_t + (\Gamma_t - \mathcal{T}_t))\, dt + dU_t - \Pi_t^h d\mathcal{Z}_t^h - \Pi_t^m (\eta dt + d\mathcal{Z}_t^m)$$

- Tax-rate constraint (Keynes, 1923): $\tau_t \leq \overline{\tau}$
- Can allow for nominal debt and will discuss if time permits

Limited Commitment and Default Regime ($t \geq T^{\mathcal{D}}$)

- Default is costly: $\widehat{Y}_t = \alpha Y_t < Y_t$, as $\alpha < 1$
- Taxes are more distortionary in the default regime than in the normal regime:

$$\widehat{c}(\cdot) = \widehat{C}_t / \widehat{Y}_t = \kappa \, c(\cdot) > c(\cdot) = C_t / Y_t, \quad \text{as} \quad \kappa > 1$$

- **Zero primary deficit**: $\widehat{\mathcal{T}}_t = \Gamma_t = \gamma_t Y_t$
- Obtain the off-the-equilibrium-path value function in the default regime: $\widehat{P}(\widehat{Y}_t)$, which appears in the limited-commitment constraint in the normal regime
- Straightforward to extend the model to allow for stochastic exit from the default regime
- The same tax-rate constraint (Keynes, 1923): $\widehat{\tau}_t \leq \overline{\tau}$

Debt, Taxes, and Risk Management Problem

- Impatience-adjusted SDF for households: $e^{-\zeta t}\mathbb{M}_t$
- Choose lumpy debt issuance (dU_t), tax rates (τ_t and $\widehat{\tau}_t$), and idiosyncratic and systematic risk hedging demands (Π_t^h and Π_t^m) to maximize households' value $P(B_0, Y_0)$:

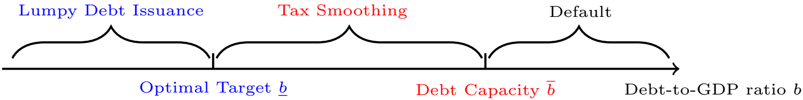
$$\mathbb{E}\int_0^\infty e^{-\zeta t}\mathbb{M}_t\left[(1-\mathbf{1}_t^{\mathcal{D}})[dU_t + (Y_t - (\mathcal{T}_t + C_t))\,dt] + \mathbf{1}_t^{\mathcal{D}}(\widehat{Y}_t - (\widehat{\mathcal{T}}_t + \widehat{C}_t))dt\right]$$

subject to

1. the budget constraint (1)
2. limited-commitment constraint for all $t \geq 0$: $P(B_t, Y_t) \geq \widehat{P}(\widehat{Y}_t)$, where $\widehat{P}(\widehat{Y}_t)$ is households' value function in the default regime.
3. tax-rate constraint: $\tau \leq \overline{\tau}$

Solution: 3 Regions

- **Three regions** of the debt-to-GDP ratio $b_t = B_t/Y_t$:



- In the tax-smoothing region
  - $$\dot{b}_t \equiv \mu^b(b_t) = \underbrace{\gamma - \tau(b_t)}_{\text{primary deficit}} + \underbrace{r \times b_t}_{\text{interest payment}} - \underbrace{g \times b_t}_{\text{growth}} + \underbrace{\xi \times b_t}_{\text{hedging cost}}$$

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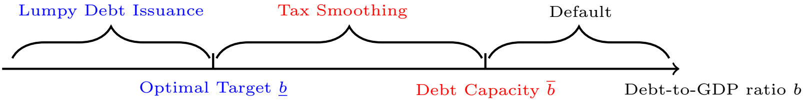

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  - scaled households' value, $p(b_t) = P(B_t, Y_t)/Y_t$, solves

$$\Big[\underbrace{(r+\xi+\zeta)}_{\text{discount rate}} - g\Big]p(b) = \underbrace{1 - \tau(b) - c(\tau(b))}_{\text{CF to households}} + \underbrace{[(r+\xi-g)b + \gamma - \tau(b)]}_{\text{drift of } b:\ \mu^b(b)} \cdot p'(b) \tag{2}$$

Solution: 3 Regions

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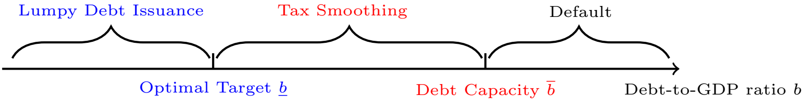

- In the tax-smoothing region
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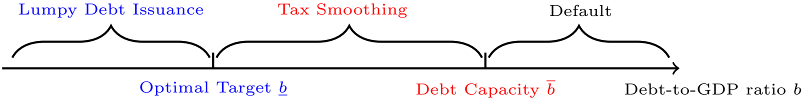
  - first-order condition for taxes: $1 + c'(\tau(b)) = -p'(b)$

Solution: 2 + 2 + 2 Equations

- **Three regions** of the debt-to-GDP ratio $b_t = B_t/Y_t$:



- Lumpy debt issuance region ($b < \underline{b}$) where 'debt is cheap'
  - $p'(\underline{b}) = -1$ and $p''(\underline{b}) = 0$ if $\underline{b} > 0$; otherwise, $\underline{b} = 0$

- Off-the-equilibrium default region ($b > \overline{b}$):
  - zero drift: $\mu^b(\overline{b}) = 0$ or equivalently $\overline{b} = \frac{\tau(\overline{b})-\gamma}{r+\xi-g}$
  - $p(\overline{b}) = \alpha\widehat{p}$ or $\tau(\overline{b}) = \overline{\tau}$, whichever is tighter

Why p theory?

q theory of investment	p theory of debt and taxes
capital stock	government debt
capital adjustment costs	tax deadweight costs
marginal q = MC of investing	$-$ marginal p = MC of taxing
MM holds	limited commitment
marginal $q \geq 1$	marginal $p \geq 1$

- q theory of investment (Lucas and Prescott, 1971; Hayashi, 1982; Abel and Eberly, 1994)

Parameters

Parameter	Symbol	Value
A. Calibration inputs		
risk-free rate	r	1%
risk premium (Jiang et al., 2022)	ξ	4%
average output growth rate	g	3%
government spending-GDP ratio	γ	20%
output loss (Hebert and Schreger, 2017)	$1-\alpha$	5%
B. Calibration outputs		
impatience	ζ	0.1%
tax deadweight loss	φ	2.9
default deadweight loss	κ	1.2

- Quadratic deadweight loss function (Barro, 1979): $c(\tau) = \frac{\varphi}{2}\tau^2$
- Scaled total value: $v(b_t) = \frac{V(B_t,Y_t)}{Y_t} = p(b_t) + b_t$

Ricardian Equivalence (AS)

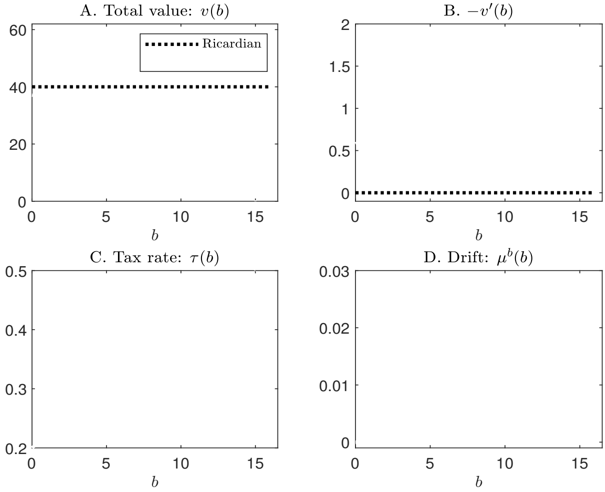


Figure: $r = 1\%$, $\xi = 4\%$, $g = 3\%$, and $\gamma = 20\%$.

Stochastic Barro (AS + DT)

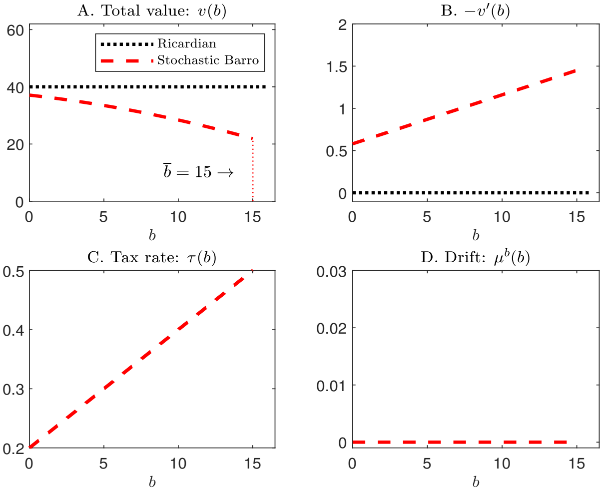


Figure: $r = 1\%$, $\xi = 4\%$, $g = 3\%$, $\gamma = 20\%$, and $\varphi = 2.9$.

LC Model (AS+DT+LC)

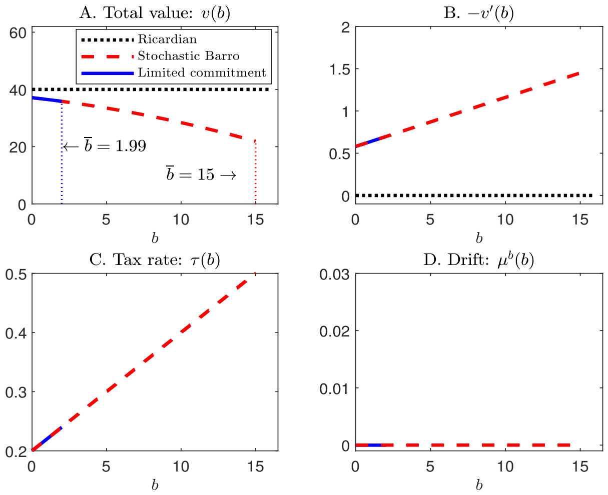


Figure: $r = 1\%$, $\xi = 4\%$, $g = 3\%$, $\gamma = 20\%$, $\varphi = 2.9$, $\alpha = 0.95$ and $\kappa = 1.2$.

Full Model (AS+DT+LC+IP)

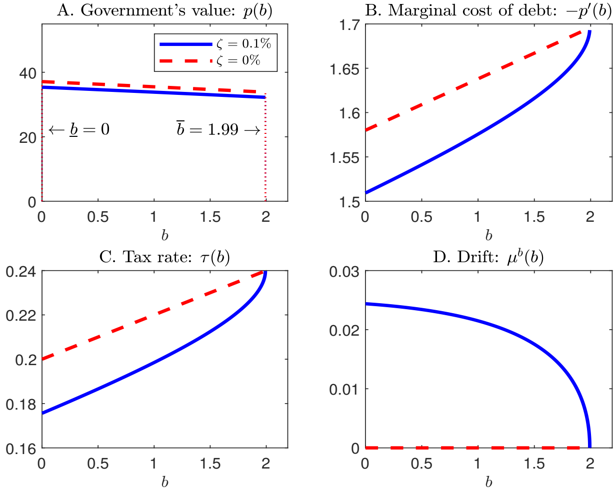


Figure: $r = 1\%$, $\xi = 4\%$, $g = 3\%$, $\gamma = 20\%$, $\varphi = 2.9$, $\alpha = 0.95$, $\kappa = 1.2$, and $\zeta = 0.1\%$.

Duality: Government's Value Maximization

- Government's value $F_t = F(W_t, Y_t)$:

$$F(W_t, Y_t) = \max \quad \mathbb{E}_t \int_t^{T^{\mathcal{D}}} \frac{\mathbb{M}_s}{\mathbb{M}_t} \underbrace{[(\mathcal{T}_s - \Gamma_s)\, ds - dJ_s]}_{\text{primary surplus}} .$$

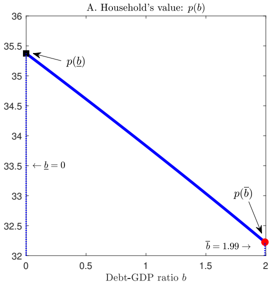
- Household's promised value W_t as in dynamic contracting models (e.g., Sannikov, 2008):

$$dW_t = [(\zeta + r)W_t - (Y_t - \mathcal{T}_t - C_t) - \eta \Phi_t^m]\, dt - dJ_t - \Phi_t^h d\mathcal{Z}_t^h - \Phi_t^m d\mathcal{Z}_t^m$$

- Scaled household's value $w_t = W_t / Y_t$ and scaled government's value:

$$f(w_t) = F(W_t, Y_t)/Y_t$$

EQUIVALENCE: $w = p(b)\,, b = f(w)$



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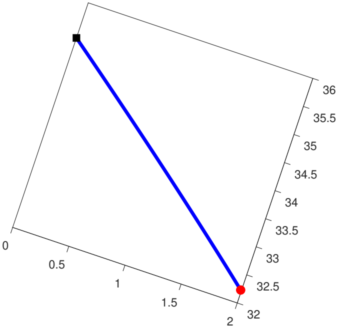
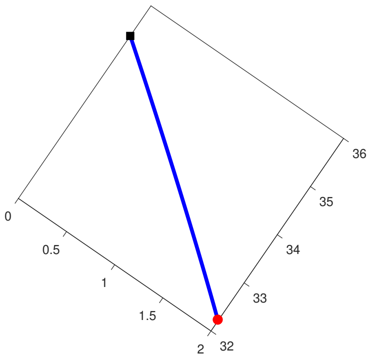


Equivalence: $w = p(b)\,, b = f(w)$



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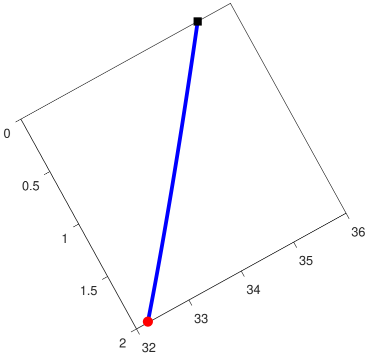

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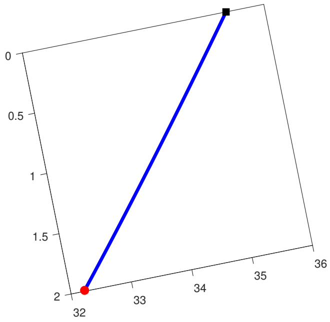

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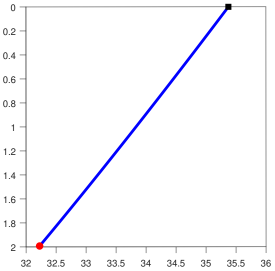

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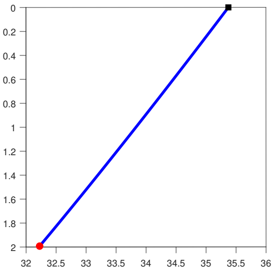

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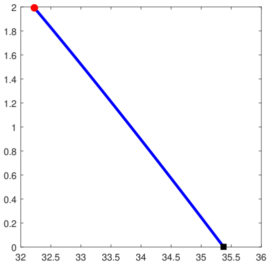

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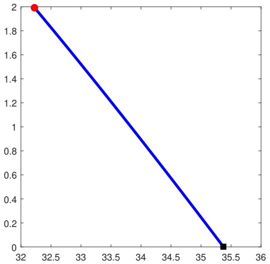

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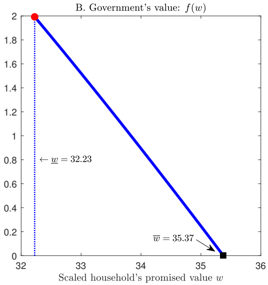

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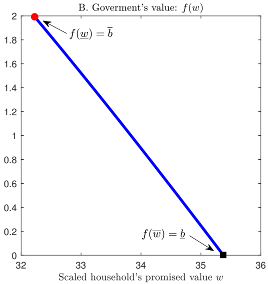
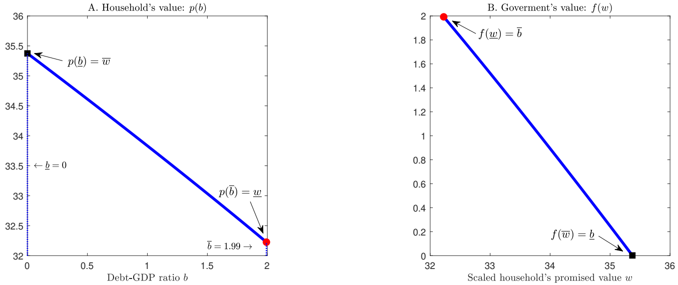
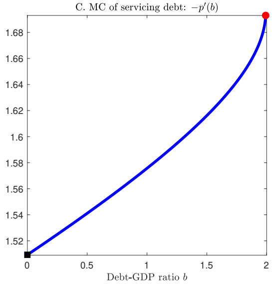
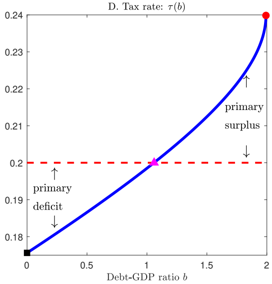


Duality: $p(b) = w$ and $f(w) = b$

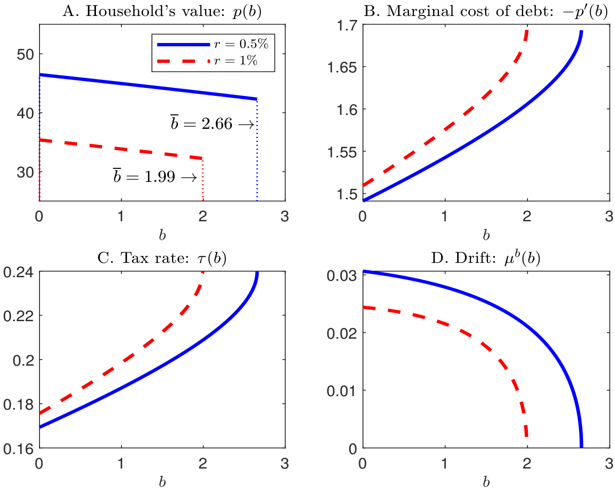


Marginal Cost of Servicing Debt $-p'(b)$ and Tax Rate $\tau(b)$

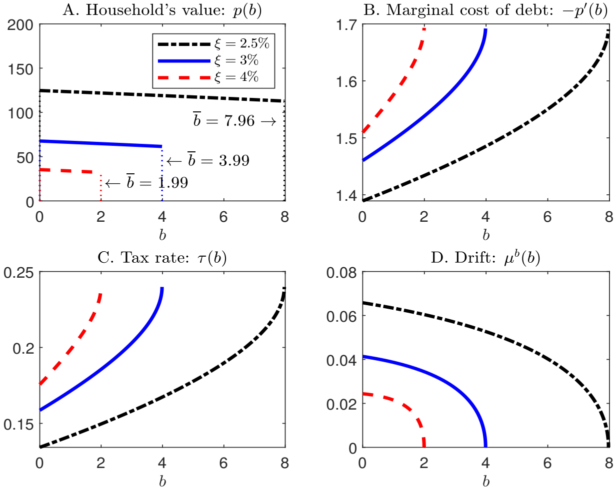




Effects of interest rate r



Effects of risk premium ξ



PREDICTING b_t

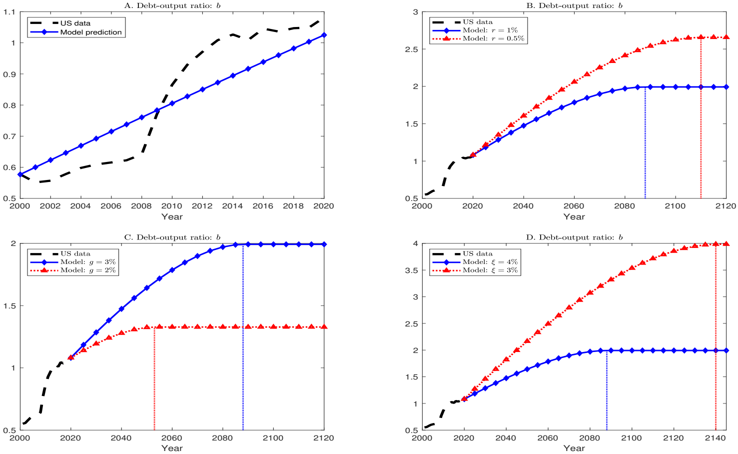


FIGURE: Steady-state debt capacity for our baseline calulation (blue lines in panels B, C, and D): $\bar{b} = 199\%$ within the plausible range of $150 - 300\%$. The US debt-GDP ratios in 2000 and 2020 are 57.5% and 108.1%, respectively.

Summary and On-going Work

- First pass at developing a pedagogical p theory to organize our thoughts about government debt and taxes: **SDF/intertemporal budget (with risk premium and no bubble) approach with endogenous debt capacity due to limited commitment and Barro's distortionary taxes**

- Quantify the effects of interest rate (r), risk premium (ξ), growth (g) on transition dynamics, equilibrium debt capacity, taxes, and **MC of servicing debt** $-p'(b)$

- On-going and future work

  - Interaction of fiscal and monetary policies (Sargent and Wallace, 1981): **inflation tax and nominal debt**

  - **Endogenous SDF** (Lucas and Stokey, 1983) and general equilibrium analysis