The Optimal Taxation of Couples

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Research question

- About 70% of U.S. federal taxes are paid by married couples.
- How should earnings of couples be optimally taxed?

Framework

- Canonical static unitary household model.
  - No income effects.
  - Exogenous joint distribution of productivities.
  - Exogenous Pareto weights.
- Find optimal tax schedule.
- Cast as a mechanism design problem.

Our contributions

- Study validity of the first order approach (FOA).
- Develop techniques to analyze average taxes analytically.
- Obtain novel economic insights about optimal taxes.

Validity of FOA

- FOA in important benchmark — random matching and Pareto weights on couples separable in productivities:

$$\text{FOA holds in 2D} \quad \Longleftrightarrow \quad \text{FOA holds in 1D for each spouse.}$$

- Stark contrast to non-linear pricing literature.
  - FOA generically fails in 2D, even with iid types and separable preferences (Rochet & Chone [1998], Armstrong [1996]).
  - Key reason — no participation constraint in PF applications.
- Analytically: FOA holds away from benchmark when solution changes smoothly.
- Numerically: FOA holds unless Pareto weights change sharply.

Solution technique

- Under FOA, optimal taxes are described by non-linear pde.
  - No known methods to analytically solve it in general.
- Our approach.
  - Rewrite original pde as system of 2 pdes, one of them is linear.
  - Integrate linear pde over various subsets using Divergence Th.
  - Derive explicit formulas for conditional averages of taxes.
- Conditional averages are informative about econ tradeoffs.
- Use them to
  - derive expressions for top and bottom tax rates,
  - compare taxes on singles and taxes on couples,
  - study jointness and relative tax rates on spouses,
  - do comparative statics wrt matching and social objective.

Economic takeaways

- Trade-off — costs of distortions vs benefits of redistribution.
- Taxes are generally lower in 2D than in 1D.
- Assortativity of matching.
  - Taxes are positive if productivities are positively correlated.
  - Taxes are higher when correlation of productivities is larger.
- Jointness.
  - Negative jointness for high earners.
  - Positive jointness for low earners.
  - Quantitatively: taxes on primary earner are much less jointed than on secondary earner.

Literature review

- Multidimensional mechanism design:
  - Armstrong [1996], Rochet & Chone [1998], Basov [2005], Rothschild & Scheuer [2012], Boerma et al [2022].
- Taxation of couples using mechanism design approach:
  - Mirrlees [1986], Kleven et al [2009], Frankel [2014], Costa & Lima [2020], Alves et al. [2021].
- Taxation of couples using perturbational approach:
  - Golosov et al [2014], Spiritus et al [2022]

Canonical model of couples

- Continuum of couples with types $(w_1, w_2) \sim F$ and joint pdf f.
  - Domain $W \subseteq \mathbb{R}_+^2$ with lowest (highest) types $\underline{\boldsymbol{w}}$ (resp. $\underline{\boldsymbol{w}}$).
  - Marginals (G_1, G_2) and their densities (g_1, g_2).

- Each couples solves

$$v(\boldsymbol{w}) := \max_{\boldsymbol{y} \geq 0} \ c - \frac{1}{\gamma_1}\left(\frac{y_1}{w_1}\right)^{\gamma_1} - \frac{1}{\gamma_2}\left(\frac{y_2}{w_2}\right)^{\gamma_2} \ \text{ s.t. } \ c \leqslant y_1 + y_2 - T(\boldsymbol{y}).$$

- Government chooses to T to maximize welfare

$$\mathbb{E}[\alpha v] = \int_W \alpha(\boldsymbol{w}) v(\boldsymbol{w}) f(\boldsymbol{w}) d\boldsymbol{w},$$

where $\alpha \geq 0$, decreasing, continuous, bounded and $\mathbb{E}[\alpha] = 1$.

Mechanism design problem

By Revelation and Taxation principles, can equivalently solve

$$\max_{(v,c,\boldsymbol{y})\geqslant 0} \ \mathbb{E}[\alpha v] \ \text{ s.t. } \ \mathbb{E}[c] \leqslant \mathbb{E}[y_1 + y_2],$$

and

$$v(\boldsymbol{w}) = c(\boldsymbol{w}) - \frac{1}{\gamma_1}\left(\frac{y_1(\boldsymbol{w})}{w_1}\right)^{\gamma_1} - \frac{1}{\gamma_2}\left(\frac{y_2(\boldsymbol{w})}{w_2}\right)^{\gamma_2} \ \forall \boldsymbol{w} \in W,$$

$$\textbf{IC} \quad v(\boldsymbol{w}) \geqslant c(\widehat{\boldsymbol{w}}) - \frac{1}{\gamma_1}\left(\frac{y_1(\widehat{\boldsymbol{w}})}{w_1}\right)^{\gamma_1} - \frac{1}{\gamma_2}\left(\frac{y_2(\widehat{\boldsymbol{w}})}{w_2}\right)^{\gamma_2} \ \forall \boldsymbol{w}, \widehat{\boldsymbol{w}} \in W.$$

First-order approach

- **IC** implies **local IC** via the Envelope theorem:

$$\textbf{local IC} \quad \nabla v(\boldsymbol{w}) = \left(\frac{y_1^{\gamma_1}(\boldsymbol{w})}{w_1^{\gamma_1+1}}, \frac{y_2^{\gamma_2}(\boldsymbol{w})}{w_2^{\gamma_2+1}} \right).$$

- First-order approach.
  - Consider relaxed problem with only **local IC**.
  - Let $(v^*, c^*, \boldsymbol{y}^*)$ be its solution.
  - $(v^*, c^*, \boldsymbol{y}^*)$ satisfies **IC** $\iff$ it solves original problem.

Validity of FOA

- Important benchmark.
  a) (w_1, w_2) are independent,
  b) α is separable, ie $\frac{\partial^2 \alpha}{\partial w_1 \partial w_2} = 0$.
- Under a) and b), planner's problem is additively separable.
- Benchmark directly corresponds to non-linear pricing with
  - independent types,
  - utility and cost functions separable across products.

Validity of FOA

Proposition

Under a), b), FOA works in 2D iff FOA is valid separately for each i.

In the paper, we

- discuss explicit conditions on primitives — FOA is valid unless $\widetilde{\alpha}_i(\boldsymbol{w})$ changes sharply at some $\boldsymbol{w}$;
- show that FOA works for settings $(f^{\varepsilon}, \alpha^{\varepsilon})$ converging to benchmark if
  - $v^{\varepsilon,*}$ is sufficiently smooth in ε,
  - FOA works "strictly" at $\varepsilon = 0$, ie certain transformation of v^* is strongly convex.

Validity of FOA

- Contrast to non-linear pricing literature.
  - Monopolist sells n goods to buyers with private valuations.
  - FOA often fails in nD, where $n > 1$ (Rochet & Chone [1988]).
  - There are examples when FOA works in 1D but fails in 2D with separable costs and iid types.
- Key distinction — no participation constraint in PF settings.
  - Participation constraint can bind only for the lowest type.
  - Equivalent to sharply decreasing (discontinuous) α.

Optimality conditions for FOA

- Define $\boldsymbol{\lambda} := (\lambda_1, \lambda_2)$ by

$$\lambda_i(\boldsymbol{w}) = \left(\frac{\partial v(\boldsymbol{w})}{\partial w_i}\right)^{(1-\gamma_i)/\gamma_i} w_i^{-1/\gamma_i} - 1.$$

- Optimality conditions — system of pdes for $\boldsymbol{\lambda}^*$ or pde for v^*:

$$\sum_i \frac{\partial}{\partial w_i}\left(\lambda_i^*(\boldsymbol{w})\frac{w_i}{\gamma_i}f(\boldsymbol{w})\right) = (\alpha(\boldsymbol{w})-1)f(\boldsymbol{w}) \qquad \boldsymbol{w} \in int(W),$$

$$\frac{\partial}{\partial w_2}\left((1+\lambda_1^*(\boldsymbol{w}))^{\gamma_1/1-\gamma_1} w_1^{1/(\gamma_1-1)}\right) = \frac{\partial}{\partial w_1}\left((1+\lambda_2^*(\boldsymbol{w}))^{\gamma_2/1-\gamma_2} w_2^{1/(\gamma_2-1)}\right) \quad \boldsymbol{w} \in int(W),$$

$$\sum_i \left(\lambda_i^*(\boldsymbol{w})\frac{w_i}{\gamma_i}f(\boldsymbol{w})\right) n_i(\boldsymbol{w}) = 0 \qquad \boldsymbol{w} \in \partial W.$$

Distortions and marginal taxes

- $\boldsymbol{\lambda}$ measures distortions.
- Tight link between $\boldsymbol{\lambda}$ and T:

$$\frac{\partial T(\boldsymbol{y}(\boldsymbol{w}))}{\partial y_i} = \frac{\lambda_i(\boldsymbol{w})}{1 - \lambda_i(\boldsymbol{w})}.$$

- Jointness of tax schedule is sign of $\frac{\partial^2 T}{\partial y_1 \partial y_2}$:

$$\text{sign of } \frac{\partial^2 T(\boldsymbol{y}(\boldsymbol{w}))}{\partial y_1 \partial y_2} = -\text{sign of } \frac{\partial \lambda_2(\boldsymbol{w})}{\partial w_1} = -\text{sign of } \frac{\partial \lambda_1(\boldsymbol{w})}{\partial w_2}.$$

Taxation of singles

- In 1D, ode instead of pde (Mirlees, 1971 and Diamond, 1988):

$$\lambda^{single,*}(t) = \gamma \cdot \overline{a}(t) \cdot \mathbb{E}[(1-\alpha^{single})|w \geqslant t] =$$

$$= \underbrace{\frac{1}{\gamma^{-1} t g(t)}}_{\text{distortion of } w=t} \cdot \underbrace{\int_t^{\overline{w}} \left(1-\alpha^{single}(w)\right) g(w)\, dw}_{\text{redistribution from } [t,\overline{w}) \text{ to } [\underline{w},t)},$$

where $\overline{a}(w) := \frac{1-G(w)}{wg(w)}$ measures thickness of upper tail.

- In 2D, non-linear pde — hard solve for λ^* pointwise.
  - Can integrate over various regions to get conditional averages.

Separable Pareto weights

- Benchmark to compare taxes on singles and taxes on couples — symmetric model and separable $\alpha = \frac{\alpha^{single}(w_1)+\alpha^{single}(w_2)}{2}$.
  - Same welfare from giving 1$ to two individuals **irrespective** of their marital status.
- To further simplify exposition assume interdependence of $\boldsymbol{w}$ is specified by FGM copula, ie

$$F(\boldsymbol{w}) = G(w_1)G(w_2)+\rho \cdot G(w_1)(1-G(w_1))G(w_2)(1-G(w_2)).$$

  - Each w_i's marginal is exactly G.
  - Correlation of $\boldsymbol{w}$ is increasing in $\rho \in [-1,1]$, positive for $\rho > 0$.

Redistribution within couples

Proposition

If $\rho = 0$, then λ_i^* doesn't depend on w_{-i} and

$$\lambda_i^*(t) = \frac{1}{2} \cdot \lambda^{single,*}(t) \geq 0.$$

- More generally, λ_i^* is a complex function of w_{-i}.
- Find $\mathbb{E}[\lambda_i^* | w_i = t]$ by Divergence Th. applied to $\left\{ \boldsymbol{w} | w_i \geq t \right\}$.

Proposition

Optimal taxes satisfy

$$\mathbb{E}[\lambda_i^* | w_i = t] = \frac{1}{2} \cdot \lambda^{single,*}(t) + \rho \cdot \frac{\gamma G(t)(1-G(t))}{2tg(t)} \xi,$$

where ξ is known number such that $0 < \xi < 1 - \alpha^{single}(\overline{w})$.

Redistribution within couples

- Insights.
  - **(1)** Taxes on couples are lower than on singles if correlation is low.
  - **(2)** Top (bottom) tax on couples is always lower than on singles.
  - **(3)** Taxes are positive under positive correlation of productivites.
  - **(4)** Taxes on couples are increasing in degree of correlation.
- Intuition.
  - **(1)** Some redistribution occurs within couples already — inefficient to crowd it out through taxes.
  - **(2)** Such redistribution is highest when w_i is large, thus $w_{-i} \ll w_i$.
  - **(3)** Couple on average is richer when w_i is larger, positive tax then helps to redistribute from rich to poor.
  - **(4)** Such redistribution is smaller under more assortative matching.

Limit tax rates and jointness

Proposition

Optimal tax rates are given by

$$\lambda_i^*(t,t) = \frac{1}{2} \cdot \lambda_i^{single,*}(t) \quad \textit{for} \quad t \in \{\underline{w}, \overline{w}\}.$$

Proposition

If $\rho > 0$, then there exists threshold $\widehat{t} \in (0,\infty)$ such that

$$\mathbb{E}[\lambda_i^* | w_i = t] > \mathbb{E}[\lambda_i^* | w_i = t \leq w_{-i}] \iff t > \widehat{t}.$$

- Insights.
  - **(5)** Joitness is negative at the top.
  - **(6)** Joitness is positive at the bottom.
- Intuition.

(5)-(6) Fewer people who are jointly very rich (poor) than mass of people who are individually very rich (poor).

General approach

- Can show insights **(1)** - **(6)** (and more) for general economies.
- Key idea — apply Divergence Th. to various sets.
  - Rectangles, $\boldsymbol{w} \geq \boldsymbol{t}$:

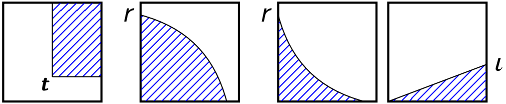
$$\sum_i \mathbb{E}[\lambda_i^* | \mathrm{w}_i = t_i, w_{-i} \geq t_{-i}] \cdot \frac{1}{\gamma_i} \cdot \frac{\partial \ln \mathbb{P}(\boldsymbol{w} \geq \boldsymbol{t})}{\partial \ln t_i} = \mathbb{E}[1 - \alpha | \boldsymbol{w} \geq \boldsymbol{t}].$$

  - Circles and hyperbolas, $R(\boldsymbol{w}) := \sqrt[\theta]{\frac{w_1^\theta + w_2^\theta}{2}} \leq r$:

$$\mathbb{E}\left[\frac{\lambda_1^*}{\gamma_1} \cdot \frac{w_1^\theta}{2r^\theta} + \frac{\lambda_2^*}{\gamma_2} \cdot \frac{w_2^\theta}{2r^\theta} | R(\boldsymbol{w}) = r\right] = \frac{d \ln r}{d \ln \mathbb{P}(R(\boldsymbol{w}) \leq r)} \cdot \mathbb{E}[\alpha - 1 | R(\boldsymbol{w}) \leq r].$$

  - Cones, $w_2 \leq w_1 \iota$:

$$\mathbb{E}\left[\frac{\lambda_2^*}{\gamma_2} - \frac{\lambda_1^*}{\gamma_1} | w_2 = w_1 \iota\right] = \frac{d \ln \iota}{d \ln \mathbb{P}(w_2 \leq w_1 \iota)} \cdot \mathbb{E}[\alpha - 1 | w_2 \leq w_1 \iota].$$



Analysis of conditional averages

- In the paper, we use these conditions to study
  - limit tax rates,
  - average jointness,
  - comparative statics wrt correlation in F,
  - comparative statics wrt complimentarity in α,
  - conditions for optimality of separable and joint income taxes,
  - relative taxes on two spouses and role of asymmetries,
  - ...
- This talk — optimal average jointness in symmetric model.

Top tax rates

Proposition

Optimal top taxes satisfy

$$\mathbb{E}[\lambda_i^*|w_i = \overline{w}] = \gamma \cdot \overline{a}(\overline{w}) \cdot \mathbb{E}[1-\alpha|w_i = \overline{w}], \text{ and}$$
$$\lambda_i^*(\overline{w},\overline{w}) = \gamma \cdot \overline{a}(\overline{w}) \cdot \overline{\chi} \cdot (1-\alpha(\overline{w},\overline{w})),$$

where $\overline{\chi}$ is coefficient of upper tail dependence defined through survival copula $\overline{C}$ as

$$\overline{\chi} := \lim_{u \to 1} \frac{\ln(1-u)}{\ln \overline{C}(1-u, 1-u)}.$$

- $\overline{\chi} \in (0,1]$ is well-known in statistics (Hefferman, [2000]).
  - It increases in correlation in upper quadrant.
  - $\overline{\chi}$ equals to $\frac{1}{2}$ for most copulas, and
  - $\overline{\chi} = 1$ under perfect assortative matching.

Jointness at the top

- Jointness at the top is negative:

$$\frac{\lambda_i^*(\underline{w},\underline{w})}{\mathbb{E}[\lambda_i^*|w_i=\overline{w}]} \leq 1 \quad \Longleftrightarrow \quad \underbrace{\frac{1-\alpha(\overline{w},\overline{w})}{\mathbb{E}[1-\alpha|w_i=\overline{w}]}}_{\geq 1} \leq \underbrace{1\Big/\overline{\chi}}_{\geq 1}.$$

- Key trade-off — positive jointness improves redistribution but worsens incentives.

Corollary

If either $\alpha(\cdot,\overline{w})$ or if α is separable and $\overline{\chi}=\frac{1}{2}$, then jointness is negative at the top.

- Similar result holds at the bottom.
- Can generalize it to α of CES form, ie $\alpha = \sqrt[\theta]{\frac{\widetilde{\alpha}^{\theta}(w_1)+\widetilde{\alpha}^{\theta}(w_2)}{2}}$.

Calibration

- Assume symmetric Pareto-lognormal marginal distribution, joint distribution with FGM copula.
- Choose parameters so that under stylized U.S. tax code we match:
  - Distribution of individual earnings (for married individuals when both spouses work) — mean, Pareto and Gini coefficients.
  - Correlation of earnings within couples.
- Study optimal taxes under assortative neutral weights $\alpha(\mathrm{w}) = e^{-w_1} + e^{-w_2}$.
- Set $\gamma = 4$ (so that elasticity of labor supply is $\frac{1}{3}$).

Goodness of fit

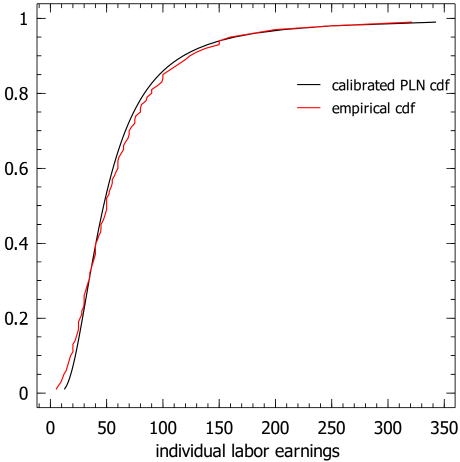


Figure: Goodness-of-fit of marginal distribution.

Optimal marginal taxes

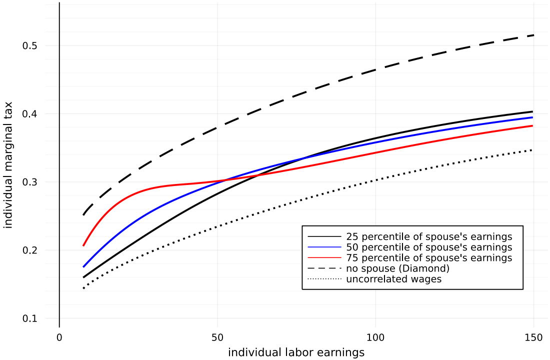


Figure: Optimal marginal tax rates, $\frac{\partial T^*}{\partial y_i}$.

Gender-neutral implementation

- Gender-specific: $T(y_1, y_2)$ where y_i are earnings of spouse i.
- Can also construct gender-neutral taxes

$$T\Big(\underbrace{\max\{y_1, y_2\}}_{=y_{primary}}, \underbrace{\min\{y_1, y_2\}}_{=y_{secondary}}\Big).$$

Optimal top marginal taxes: primary

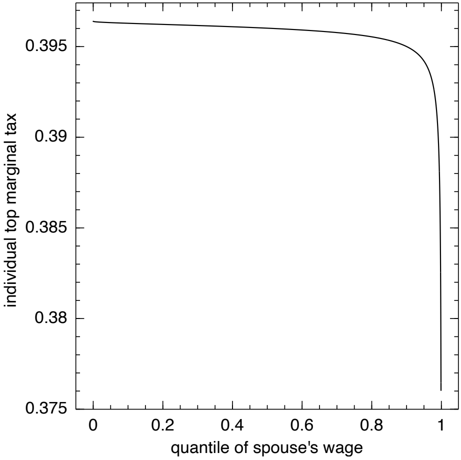


Figure: Optimal top marginal tax rates, $\lim_{y_{primary}\to\infty} \frac{\partial T^*}{\partial y_{primary}}$.

Optimal marginal taxes: primary

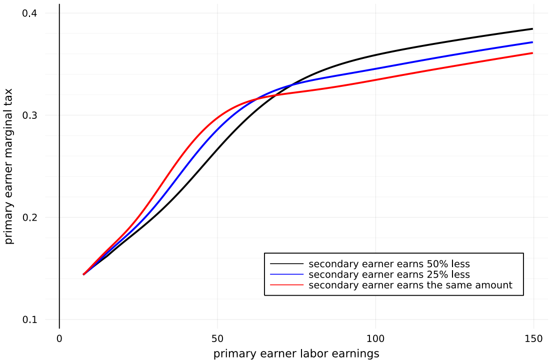


Figure: Optimal marginal tax rates, $\frac{\partial T^*}{\partial y_{primary}}$.

Optimal marginal taxes: secondary

Figure: Optimal marginal tax rates, $\frac{\partial T^*}{\partial y_{seconary}}$.

Conclusion

- Methodologically: a way to analyze multidimensional screening problems in public finance.
- Economically: a lot of insights about optimal taxation of couples.
  - Lower taxes on couples than on individuals
  - Non-negative under positive correlation and $\uparrow$ in correlation.
  - Negative/positive jointness for high/low earners.