Constructing the Term Structure of Uncertainty from the Ragged Edge of SPF Forecasts

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Forecasting & Empirical Methods

RESEARCH AGENDA

Setup

We observe predictions from the SPF
(or similar sources)
in form of **point and/or density** forecasts
for fixed horizons and/or fixed events

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that are consistent with the SPF?

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Throughout we look at average SPF responses

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1) State-space model

Maps arbitrary sets of SPF point forecasts

(fixed-event & -horizon)

into term structures of expectations and uncertainty

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2) We match the SPF histograms with entropic tilting

We replicate the entire "bin" structure

instead of selected moments that were imputed

RELATED LITERATURE

Survey uncertainty based on past forecast errors

- Reifschneider & Tulip (2007/19), Clark, McCracken & Mertens (2020)
- Lahiri & Sheng (2010), Knüppel (2014), Jo & Sekkel (2019)

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Entropic tilting: recent applications

- Krüger, Clark & Ravazzolo (2017), Tallman & Zaman (2020)
- Galvao, Garratt, & Mitchell (2021), Ganics & Odendahl (2021) Banbura, Brenna, Parades & Ravazzolo (2021)

AGENDA

1. **SPF data**
2. **State space model for forecasts**
3. **Densities from SPF histograms and model**
4. **Effects of entropic tilting on predictive densities**
5. **Application: SEP-style fan charts**
6. **Conclusions**

OUR DATA: U.S. SPF

1) Point forecasts

2) Probabilistic forecasts (histograms)

to be discussed later

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- “Fixed horizons:” Quarters 0 to 4, since 1968Q4
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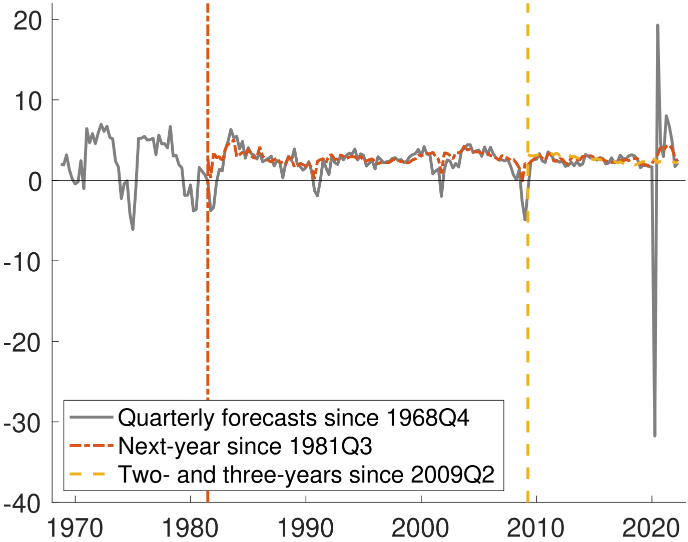
Throughout, we look at average forecasts

Today:
Focus on GDP growth results (RGDP)
w/others shown in paper

AVAILABILITY OF SPF POINT FORECASTS



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MODEL OVERVIEW

❶ **Map observed outcomes** and SPF point forecasts $\boldsymbol{Z_t}$ **into latent state vector** of fixed-horizon forecasts $\boldsymbol{Y_t}$

$$\boldsymbol{Z_t} = \boldsymbol{C_t}\ \boldsymbol{Y_t}$$

with $\boldsymbol{C_t}$ known (based on data definitions)

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2. **Use accounting identity** for forecast errors

$$\boldsymbol{Y_t = F \, Y_{t-1} + \eta_t}$$

with $\boldsymbol{F}$ known, and $\boldsymbol{\eta_t}$ a vector of forecast updates

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3. **Specify DGP for η_t, options:**
   a) Baseline: Martingale difference
   b) Alternative: Persistent process

   with SV or CONST shock variances

An accounting identity for fixed-horizon point forecasts $y_{t+h|t}$

Forecast errors (FE)

$$e_t \equiv y_t - y_{t|t}$$

$$e_{t\to t+h} \equiv y_{t+h} - y_{t+h|t}$$

FORECAST ERRORS AND FORECAST UPDATES

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$$\begin{aligned} e_t &\equiv y_t - y_{t|t} \\ e_{t\to t+h} &\equiv y_{t+h} - y_{t+h|t} \\ &= \left(y_{t+h} - y_{t+h|t+h}\right) + \sum_{i=1}^{h} \left(y_{t+h|t+i} - y_{t+h|t+i-1}\right) \end{aligned}$$

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Forecast updates

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Applied by CMM to fixed-horizon forecasts when observed

Next: state space for fixed-event forecasts

STATE SPACE FOR FORECASTS AND THEIR UPDATES

1) Accounting identity from CMM for H steps ahead:

$$\boldsymbol{y_{t+H} = e_{t+H} + \sum_{i=1}^{H} \mu_{t+H|t+i} + y_{t+H|t}}$$

Recall: $\boldsymbol{e_t = y_t - y_{t|t}}$ and $\boldsymbol{\mu_{t+h|t} = y_{t+h|t} - y_{t+h|t-1}}$

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$$Y_t = F\, Y_{t-1} + \eta_t$$

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Alternative: persistent forecast updates

- $E(\eta_t) = 0$: model's prior is centered on SPF
- $\eta_t \sim \text{VAR}(p)$
- Imputed bias: $b_{t+h|t} = y_{t+h|t} - E_t y_{t+h}$
- Entropic tilting not applied to this case

Martingale-difference case for forecast updates

Trend and gap shocks with SV

Decompose updates into long-run shifts and cyclical gaps

$$\eta_t = \tilde{\eta}_t + 1 \cdot \mu_t^*$$

$$\mu_t^* \sim N(0, \sigma_*^2)$$

$$\tilde{\eta}_t \sim N\left(0, \lambda_t \cdot \tilde{\Sigma}\right) \qquad \log \lambda_t \sim AR(1) \text{ (scalar)}$$

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- Combines slow-moving endpoint of term structure with time-varying volatility over near-/medium term
- Low-order factor structure suited for handling of missing observations
- Scale SV invariant to reordering variables in $\tilde{\eta}_t$ (Carriero, Clark & Marcellino, 2016; Chan, 2020)

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- C_t: known, reflects definition of forecast targets, e.g., growth in annual average level of GDP

$$\hat{y}_t = \frac{y_t + 2y_{t-1} + 3y_{t-2} + 4y_{t-3} + 3y_{t-4} + 2y_{t-5} + y_{t-6}}{16}$$

- As in Mariano & Murasawa (2003), Patton & Timmermann (2011), Aruoba (2020)

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- In Q4: next-year forecasts omitted (since spanned by quarterly forecasts)

ESTIMATION SETUP

1. Model applied **separately for each outcome variable** (RGDP, PGDP, UNRATE)
2. Estimated with **MCMC over growing samples** of **real-time data and SPF** that start in 1968Q3

   (FRB PHIL's Real-Time Data Set for Macroeconomics)
3. Generate **out-of-sample predictive densities** from 1992Q1 onwards
4. Predictions **evaluated against 2nd release** outcomes (for NIPA, and latest data for UNRATE)

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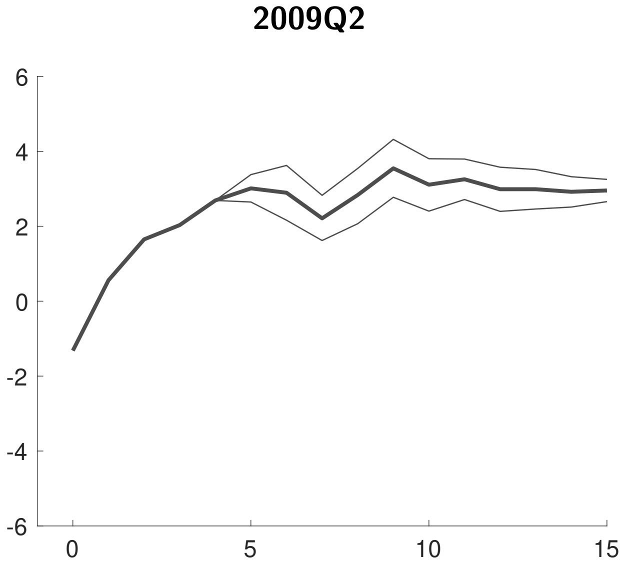
TERM STRUCTURE OF GDP GROWTH EXPECTATIONS

Quarterly real-time estimates w/68% bands for unobserved values



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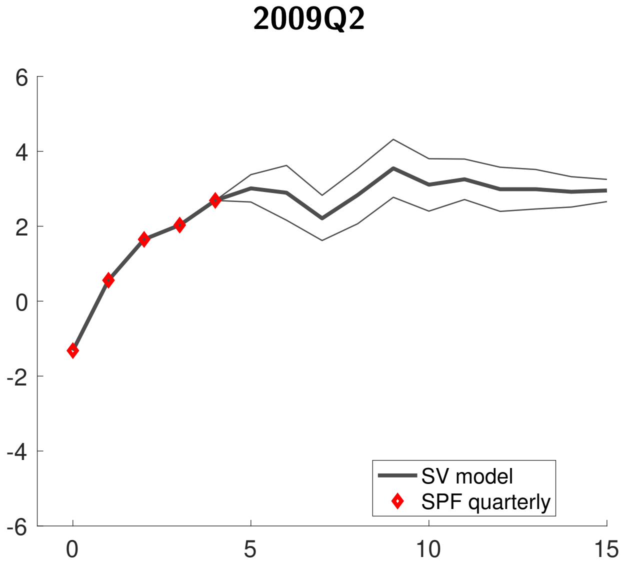

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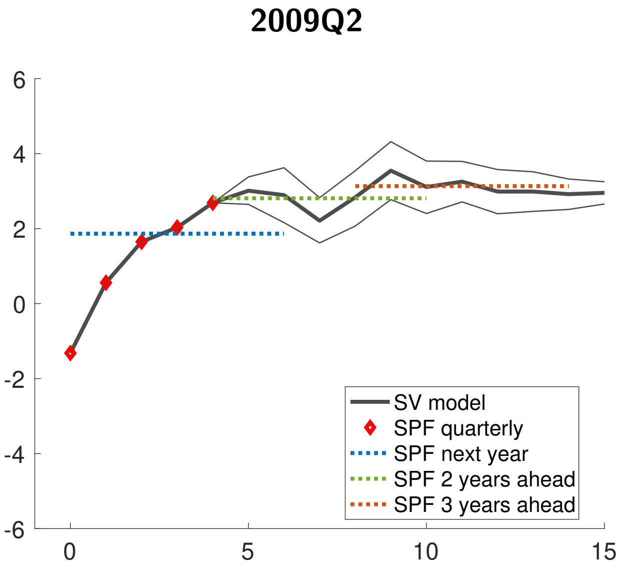

Dotted lines: quarters included in tent-shaped mapping from annual-average to quarterly changes

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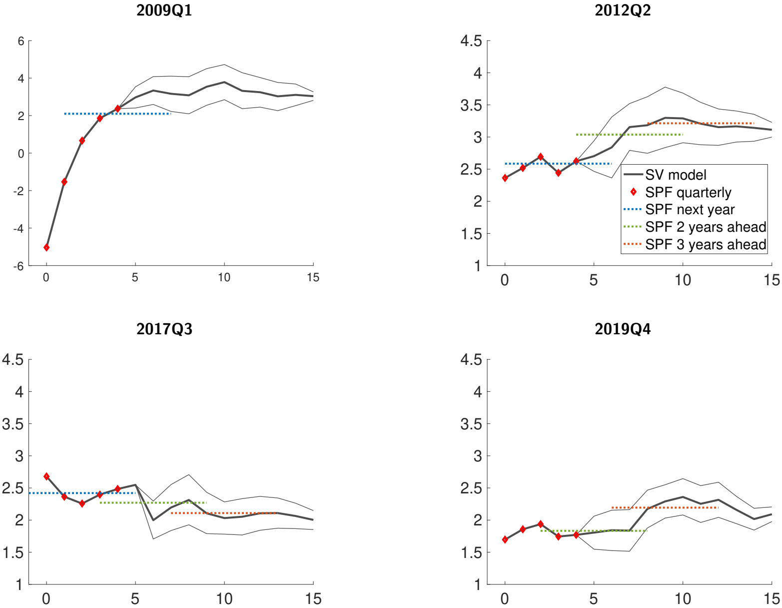

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FITTED TERM STRUCTURES OF EXPECTATIONS
Key feature

**We can perfectly match any shape
of the term structure of expectations
that could be seen in the data**

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Relaxation of MDS assumption

- Persistent forecast errors instead of $E_{t-1}\eta_t = 0$
- Transformation from Y_t to η_t still useful: motivates shrinkage to VAR(1)

$$
\begin{aligned}
Y_t &= FY_{t-1} + \eta_t \\
\eta_t &= G\eta_{t-1} + \varepsilon_t \,, \quad \varepsilon_t \sim N(0, \mathrm{Var}_{t-1}\, \varepsilon_t)
\end{aligned}
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Results:

- **Similar avg forecast performance** (relative to MDS)
- **Persistence in η_t matters most at turning points**
- . . . and is **hard to predict in real time**

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1) Point forecasts

- "Fixed horizons:" Quarters 0 to 4
- "Fixed events:" Calendar years 1 to 3

2) Probabilistic forecasts (histograms)

- Fixed-event only, calendar years 1 to 3
- Using only predictions since 1992 (b/o data issues)
- To match SPF, we transform draws from log-linear model to actual annual-average changes

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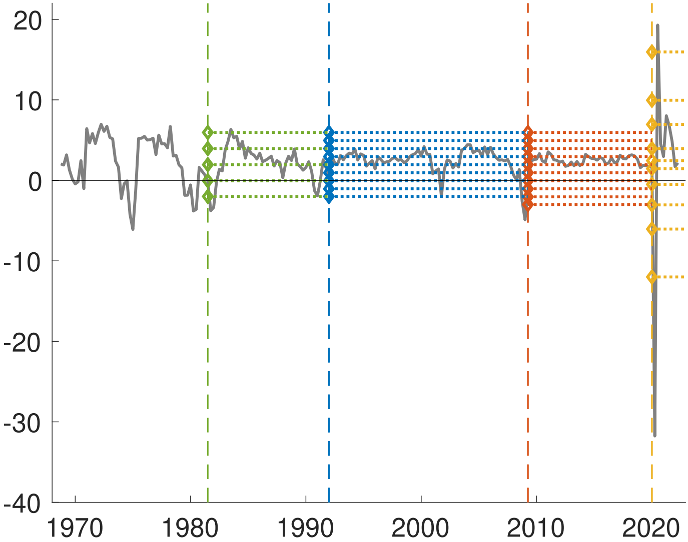
AVAILABILITY OF SPF DENSITY FORECASTS

Nowcast and widths of histogram bins



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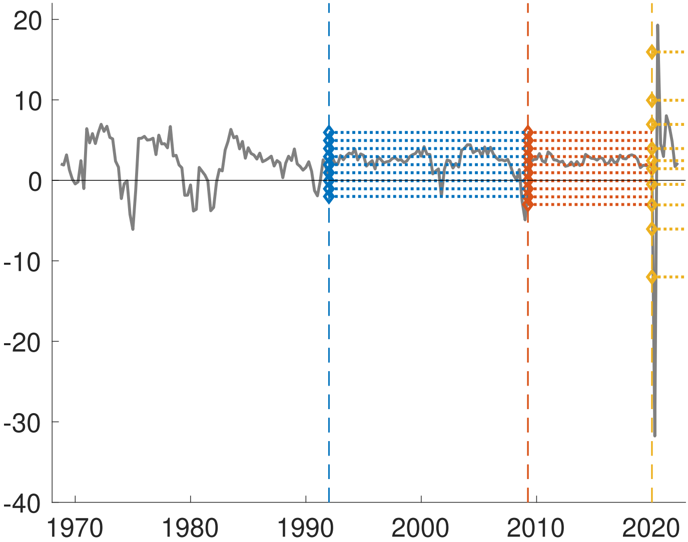
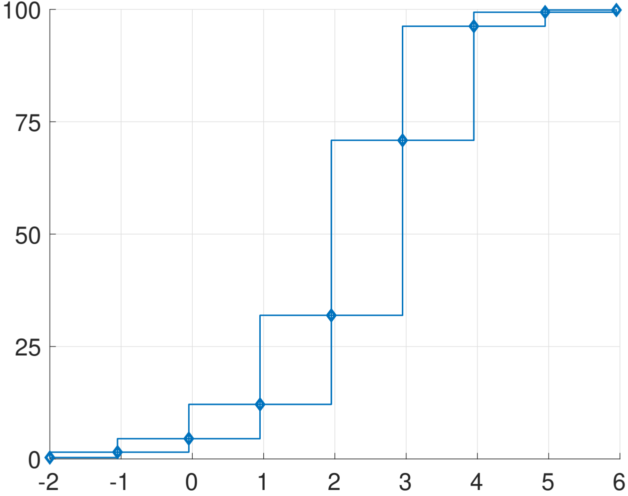


We consider only histograms as of 1992 (b/o data issues)

CDF'S IMPLIED BY SPF HISTOGRAMS

Next-year GDP growth

SPF histograms pin down selected CDF values:

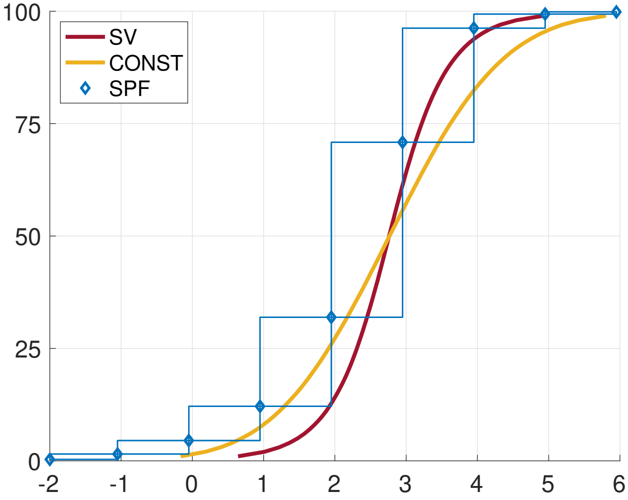


Growth rates (x-axis) and probabilities (y-axis) in percentage points

MODELS VS SPF

Cumulative densities for next-year GDP growth

By construction,
model-based densities have same median . . .



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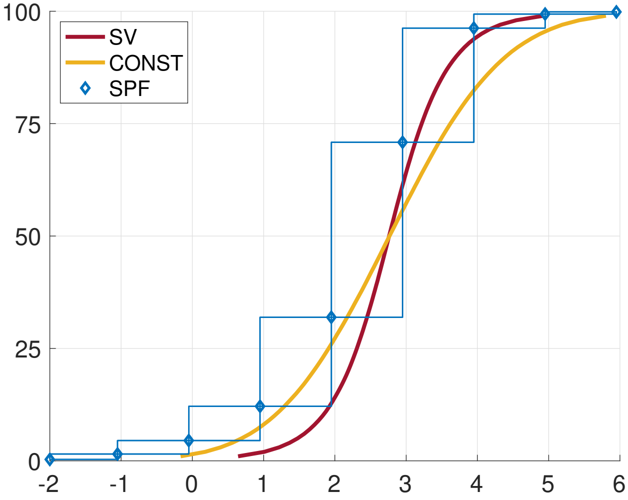
. . . but differ otherwise:



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SCORES FOR HISTOGRAM EVALUATIONS

Setup

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- Histogram provides discrete-valued CDF:

$$P_{j,t} = \textbf{Prob}_t(y_{t+h} \leq b_{j,t})$$

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Discrete Ranked Probability Scores

$$\textbf{DRPS}_t = \sum_j \left(P_{j,t} - \mathbb{1}\left(y^o_{t+h} \leq b_{j,t}\right)\right)^2$$

where y^o_{t+h} denotes the observed value

- Measures accuracy of predictions to fall into SPF bins
- Depends on specification of SPF bins ($b_{j,t}$)

SCORES FOR HISTOGRAM EVALUATIONS

Setup

- Let $b_{j,t}$ denote the upper edge of SPF bin j (at t)
- Histogram provides discrete-valued CDF:

$$P_{j,t} = \textbf{Prob}_t(y_{t+h} \leq b_{j,t})$$

Discrete Ranked Probability Scores

$$\textbf{DRPS}_t = \sum_j \left(P_{j,t} - \mathbb{1}\left(y^o_{t+h} \leq b_{j,t}\right)\right)^2$$

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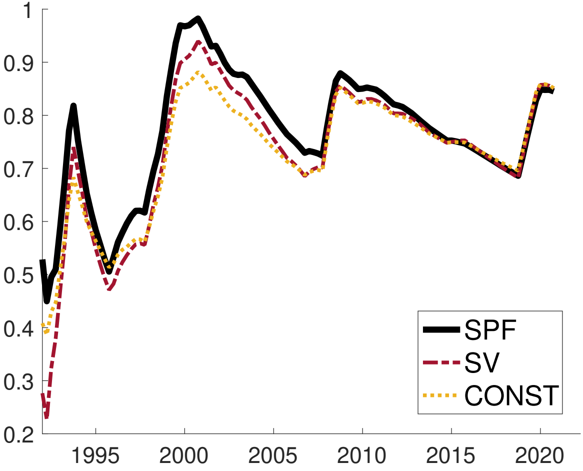
- Measures accuracy of predictions to fall into SPF bins
- Depends on specification of SPF bins ($b_{j,t}$)
- Bin-specific analogue to CRPS

ACCURACY OF PREDICTIONS FOR BIN EVENTS

Avg DRPS scores over growing samples, next-year GDP growth

Models better than SPF pre GFC and on par over full sample



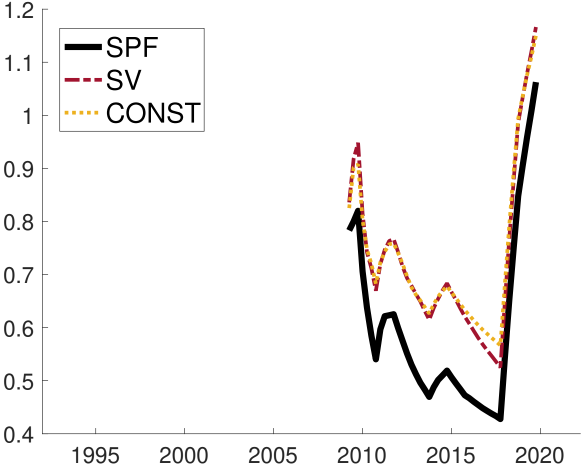
Lower score is better

ACCURACY OF PREDICTIONS FOR BIN EVENTS

Avg DRPS scores over growing samples, two-years ahead GDP growth

SPF better since GFC

also for predictions beyond the next year



Lower score is better

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5. Application: SEP-style fan charts
6. Conclusions

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ENTROPIC TILTING

Generic setup

- **Given:** predictive density draws $f := \{y_{t+h}^i\}_{i=1}^M$

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- Target: moment conditions $E[g(y_{t+h})] = \bar{g}$
- **Tilting problem:** Reweigh draws from f into $\tilde{f}$ to match $\bar{g}$ while minimizing KL divergence

$$\min_{\tilde{f} \in \mathbb{F}} \mathrm{KL}(\tilde{f}, f) \text{ subject to } E_{\tilde{f}}\left[g(y_{t+h})\right] = \bar{g}$$

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Key insight for our application

Bin probabilities are predictive moments

for example:

$$\mathrm{Prob}_t\left(2.5 < y_{t+h} \leq 3.0\right) = E_t\left(\mathbb{1}\left(2.5 < y_{t+h} \leq 3.0\right)\right)$$

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We target all bin probabilities

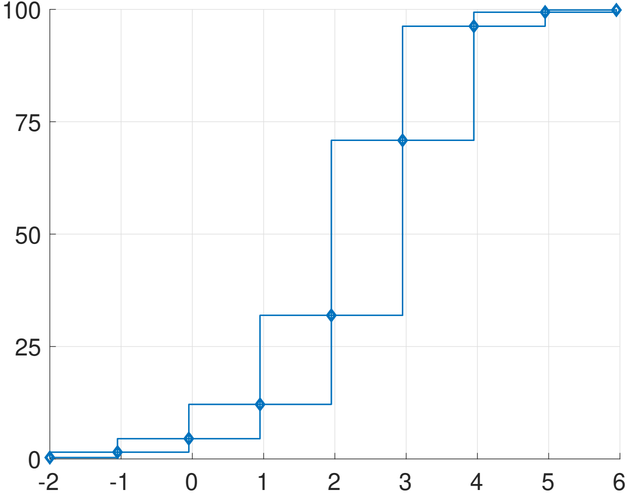
CDF'S IMPLIED BY SPF HISTOGRAMS

2007Q3

Next-year GDP growth

We use ET to target the SPF probabilities

as available for next year and beyond

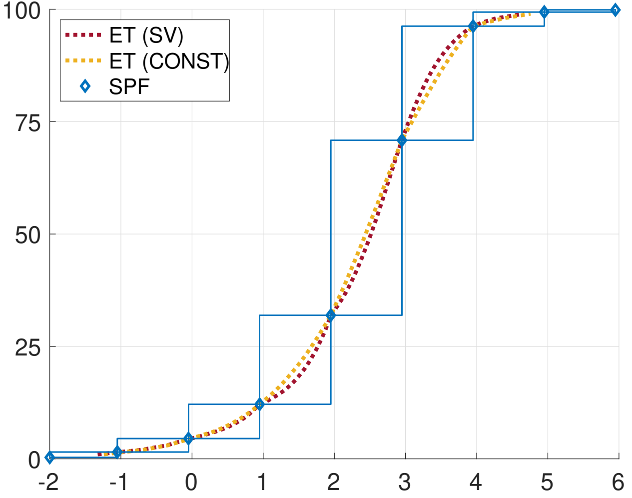


Growth rates (x-axis) and probabilities (y-axis) in percentage points

TILTED MODELS VS SPF

Cumulative densities for next-year GDP growth

After tilting, SV and CONST densities similar, but not identical:



Growth rates (x-axis) and probabilities (y-axis) in percentage points

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POINT FORECAST PERFORMANCE

RMSE relative to SV

	SV w/ET	CONST	CONST w/ET

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

POINT FORECAST PERFORMANCE

RMSE relative to SV

	SV w/ET		CONST		CONST w/ET	
h	92–22	92–16	92–22	92–16	92–22	92–16
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

POINT FORECAST PERFORMANCE

RMSE relative to SV

	SV w/ET		CONST		CONST w/ET	
h	92–22	92–16	92–22	92–16	92–22	92–16
0	1.01	1.01*	1.00	1.00	1.01**	1.01*
1	1.00	1.01	1.00	1.00	1.00	1.01
2	1.00	1.00	1.00	1.00	1.00	1.00
3	1.00	0.99	1.00	1.00	1.00	0.99
4	1.00	0.99	1.00	1.00	1.00	0.99
5	1.00	0.98	1.00	1.00	1.00	0.99
6	1.00	0.99	1.00	1.01	1.01	1.01
7	1.00	0.99	1.00	1.01	1.00	1.00
8	1.00	0.99	1.00	1.02	1.00	1.01
9	1.00	0.99	1.00	1.01	1.00	1.00
10	1.00	1.00	1.00	1.02	1.00	1.01
11	1.00	0.99	1.00	1.02*	1.00	1.01
12	1.00	1.00	1.00	1.02	1.00	1.01
13	1.00	1.01*	1.00	1.02	1.00	1.02
14	1.00	1.00	1.00	1.00	1.01	1.00
15	1.00	1.00	1.00	1.00	1.00	1.00

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

DENSITY FORECAST PERFORMANCE

CRPS relative to SV

	SV w/ET		CONST		CONST w/ET	
h	92–22	92–16	92–22	92–16	92–22	92–16
0	1.00	1.00				
1	1.00	1.00				
2	0.99	1.00				
3	0.99	0.99				
4	0.99	0.99				
5	0.99	0.98				
6	0.99	0.99				
7	1.00	0.99				
8	1.00	0.99				
9	1.00	0.99				
10	1.00	0.99				
11	0.99	0.98				
12	0.99	0.99				
13	1.00	1.00				
14	1.00	1.00				
15	0.99*	0.99				

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

DENSITY FORECAST PERFORMANCE

CRPS relative to SV

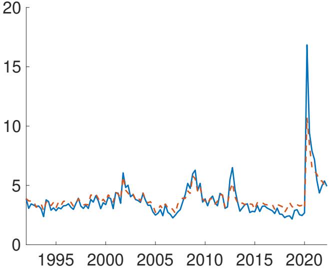
	SV w/ET		CONST		CONST w/ET	
h	92–22	92–16	92–22	92–16	92–22	92–16
0	1.00	1.00	0.99	0.99	1.00	0.99
1	1.00	1.00	1.04	1.01	1.04*	1.02
2	0.99	1.00	1.00	1.02	1.01	1.02
3	0.99	0.99	1.01	1.02	1.01	1.01
4	0.99	0.99	1.01	1.03*	1.01	1.01
5	0.99	0.98	1.01	1.04*	1.01	1.01
6	0.99	0.99	1.03	1.06**	1.03	1.04
7	1.00	0.99	1.03	1.06**	1.03	1.04
8	1.00	0.99	1.03	1.06**	1.03	1.04
9	1.00	0.99	1.03	1.06**	1.03	1.05
10	1.00	0.99	1.03**	1.07***	1.04**	1.06**
11	0.99	0.98	1.04**	1.07***	1.03	1.05*
12	0.99	0.99	1.04**	1.07***	1.03**	1.06**
13	1.00	1.00	1.04***	1.07***	1.04***	1.06***
14	1.00	1.00	1.04**	1.05***	1.04**	1.05**
15	0.99*	0.99	1.04**	1.04**	1.04**	1.04**

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

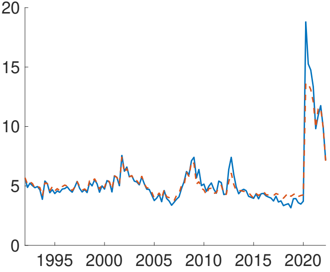
EFFECTS OF TILTING ON UNCERTAINTY

Real growth: SV model before (blue) and after ET (red)

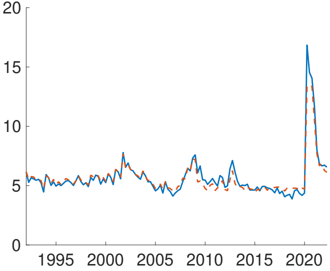
3 quarters ahead



7 quarters ahead



11 quarters ahead

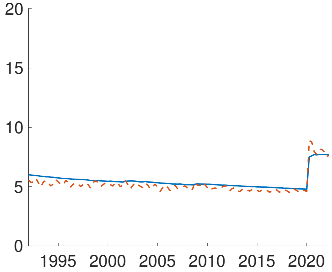


- Uncertainty measured by width of 68% bands
- Not much effect from ET
- Except for narrowing at onset of COVID-19
- Stronger effects on CONST (see next)

EFFECTS OF TILTING ON UNCERTAINTY

Real growth: CONST model before (blue) and after ET (red)

3 quarters ahead



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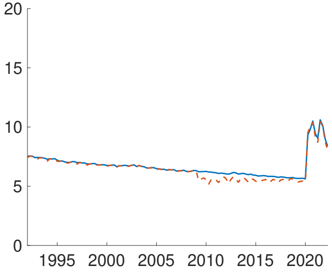

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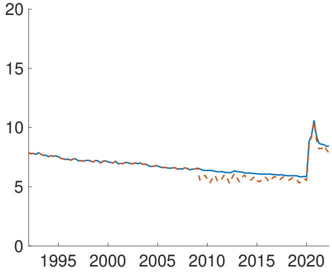

- **More visible effects of ET on CONST**
- Narrower until COVID
- Recall: Longer-run SPF histograms available only since 2009

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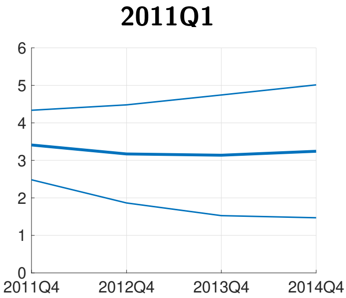
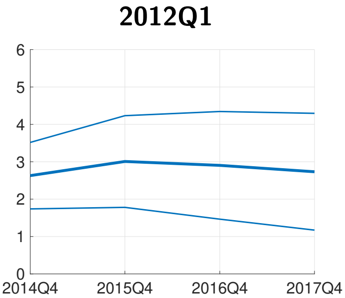
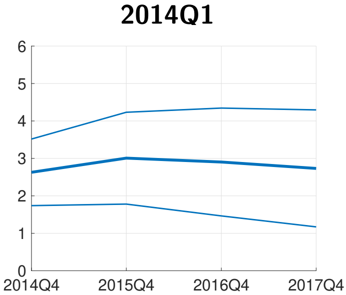
FAN CHARTS FOR Q4/Q4 GDP GROWTH







- In format of FOMC's SEP
- Generated by SV model
- Next: comparison against SEP uncertainty bands

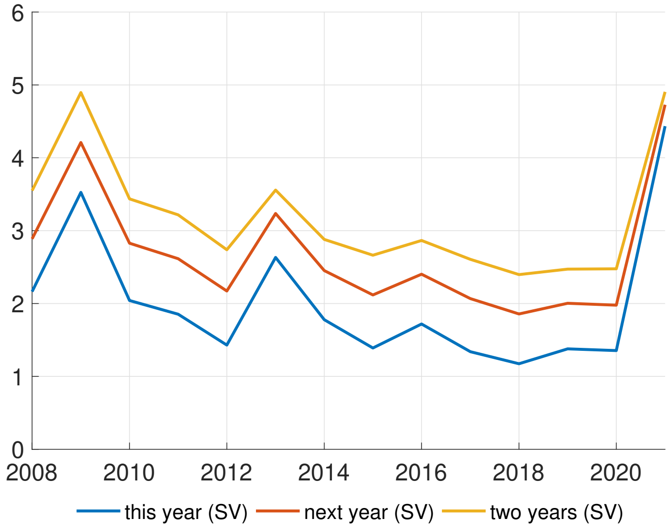
FAN CHART UNCERTAINTY: MODEL VS SEP

SEP setup

- **SEP** fan-chart bands based on historical forecast errors **assume constant variances over last 20-years**
- Uncertainty **bands reflect** $\pm$ **RMSE** around forecast
- ... and can differ from FOMC's subjective assessments

FAN CHART UNCERTAINTY OVER TIME

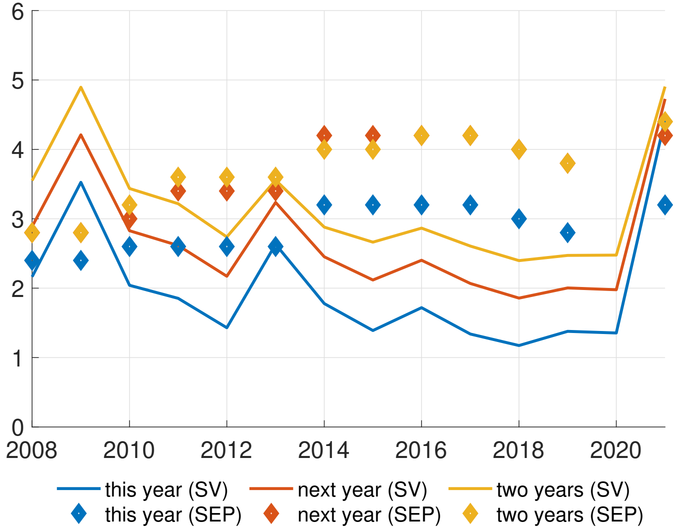
Width of 68% bands from SV model . . .



Width of 68% bands for Q4/Q4 forecasts

FAN CHART UNCERTAINTY OVER TIME

Width of 68% bands from SV model vs. SEP's RMSE-based bands



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Take aways

- **SV-model bands more nimble than SEP estimates**
- After GFC:
  - SV estimates returned to lower levels
  - while SEP remained elevated

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SUMMARY

Our contributions:

Model that transforms an arbitrary set
of fixed-event/-horizon SPF data
into a consistent term structure

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Our contributions:

Model that transforms an arbitrary set of fixed-event/-horizon SPF data into a consistent term structure

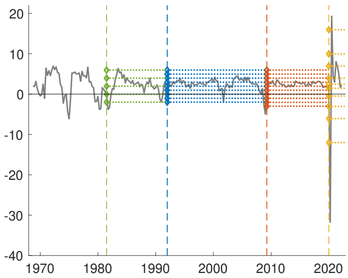
- Matches observed SPF
- Incorporates all SPF bins with entropic tilting

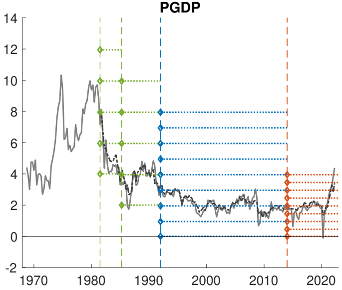
Findings

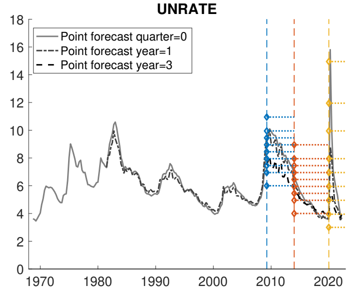
- Calendar-year **histograms add** some, but mostly **occasional value** . . .
- . . . *relative to model centered on SPF point forecasts*
- At onset of COVID-19, narrower uncertainty after tilting

AVAILABILITY OF SPF PREDICTIONS

Real growth (RGDP), inflation (PGDP), unemployment rate (UNRATE)







- Point forecasts since 1968
- Next-year bins since 1981
  (and since 2009 for UNRATE)
- . . . beyond next year since 2009
- But, w/data issues prior 1992
- . . . and bin changes throughout

Point forecasts, and widths of histograms

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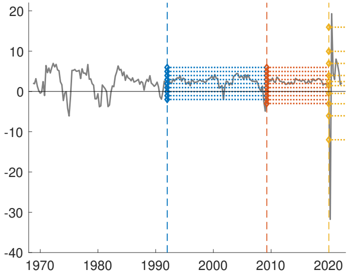
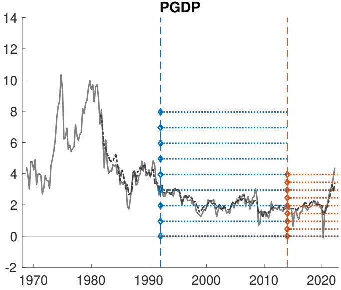
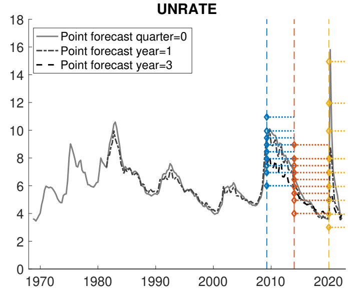






- Point forecasts since 1968
- Next-year bins since 1981 (and since 2009 for UNRATE)
- ... beyond next year since 2009
- But, w/data issues prior 1992
- ... and bin changes throughout
- Using only bin data since 1992

Point forecasts, and widths of histograms

APPENDICES

COMMENTS ON STATE EQUATION

$$\boldsymbol{Y_t = F\, Y_{t-1} + \eta_t}$$

- All rows except last replicate CMM data for $\boldsymbol{\eta_t}$
- Transition matrix $\boldsymbol{F}$ is known
- All roots of $\boldsymbol{F}$ are zero except for one unit root
- $\boldsymbol{F}$ implies common trend in outcomes and forecasts (assuming stationary $\boldsymbol{\eta_t}$)
- $\mathbf{Var}\,(\boldsymbol{\mu_t^*}) \to \mathbf{0}$ captures (near) stationary $\boldsymbol{Y_t}$
- MDS assumption, $\boldsymbol{E_{t-1}\eta_t = 0}$, closes state space
- In extension, we consider VAR for $\boldsymbol{E_{t-1}\eta_t}$ (as in CMM)

Even if not literally true, MDS assumption provides useful shrinkage for VAR in $\boldsymbol{\eta_t}$

DYNAMICS OF y_t IN OUR MODEL

BACKUP

H-step Wold decomposition for y_t

$$y_t = e_t + \mu_{t|t} + \mu_{t|t-1} + \dots \mu_{t|t-H+1} + y_{t|t-H}$$

$$y_{t+H|t} = y_{t+H-1|t-1} + \mu_t^*$$

e_t, $\mu_{t+h|t}$, μ_t^* are MDS

Dynamics responses of y_t to shocks encoded in $\mathrm{Var}_t\, \eta_t$

For example, ignoring nowcast problem, let $y_t = \rho y_{t-1} + \varepsilon_t$:

$$(\mathrm{Var}_t\, \eta_t)^{1/2} = \begin{bmatrix} 1 \\ \rho \\ \rho^2 \\ \vdots \\ 0 \end{bmatrix} \mathrm{Var}_t\, \varepsilon_t$$

SAMPLING LATENT STATES

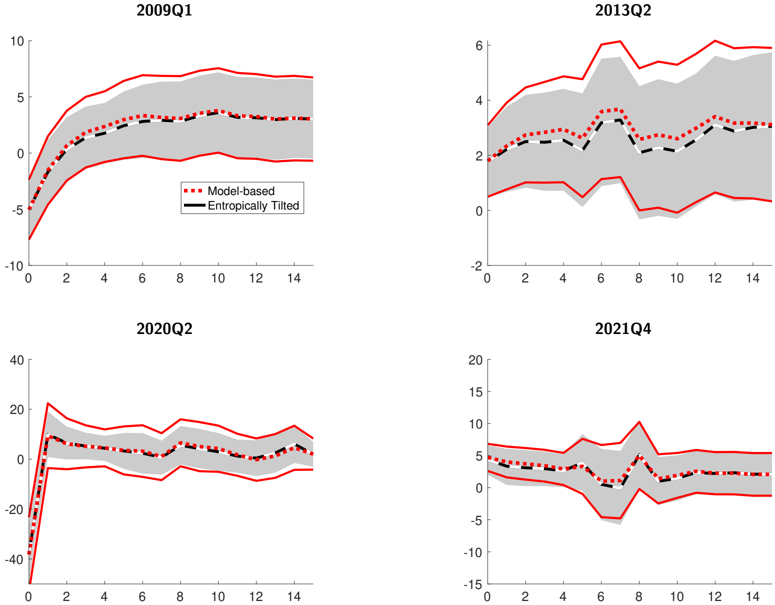
- We use **conditionally Gaussian models** for η_t (w/SV)
- Standard Gaussian signal extraction applies (conditional on SV states etc.)
- **Kalman filter/smoother would work,** but requires many loops
- **Precision-based samplers would be faster,** but does not apply (Chan & Jeliazkov, 2009)
- **Challenge: no noise in measurements** leads to ill-defined posterior precision

$$|\operatorname{Var}\left(\boldsymbol{Y_t}|\boldsymbol{Z^t}\right)| = \mathbf{0}$$

- **We use a new precision-based sampler** to efficiently draw $\boldsymbol{Y^t}|\boldsymbol{Z^t}$ in this case

FAN CHARTS FOR GDP GROWTH

SV model before (red) and after entropic tilting (black)

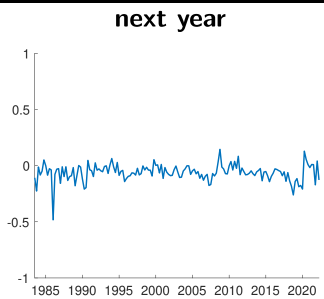


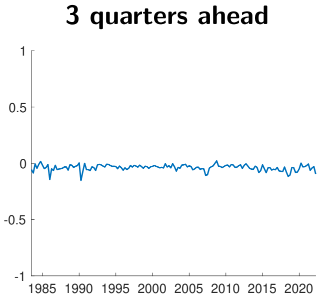
Predictive means and 68% uncertainty bands

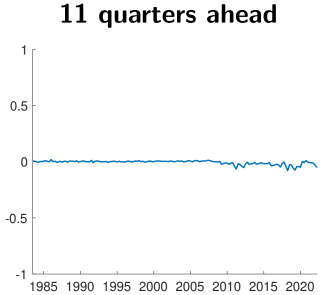
SKEW INDUCED BY TILTING

Bowley coefficient

- Our model has zero skew, only ET can induce skew
- *Some* skewness at targeted annual horizon
- But, w/o carrying over to quarterly term structure



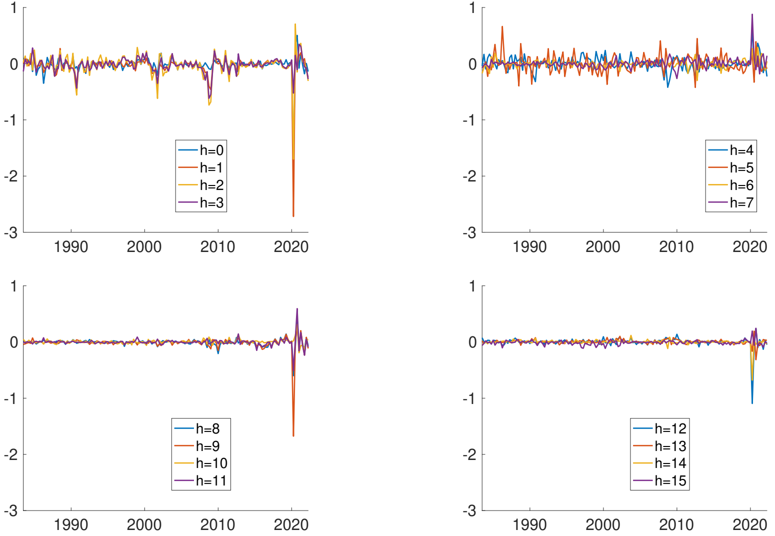




APPENDICES

BIAS IN SPF EXPECTATIONS OF GDP GROWTH

$\text{Bias}_t = E_t y_{t+h} - y_{t+h|t}$ from non-MDS model



MDS VS NON-MDS MODEL: FORECAST PERFORMANCE

Relative RMSE and CRPS (MDS in denominator)

	RMSE				CRPS			
	SV		CONST		SV		CONST	
h	92–22	92–16	92–22	92–16	92–22	92–16	92–22	92–16
0	1.00	1.00	1.12	1.01	1.01	1.01	1.08	1.01
1	1.02	1.00	1.06	1.01	1.02	1.01	1.04	1.01
2	1.00	0.98*	1.00	0.97**	1.01	0.99*	1.00	0.99
3	1.00	1.00	1.00	0.99	1.02	1.01	1.00	1.00
4	1.00	1.00	1.00	0.99	1.02	1.01	1.01	1.01
5	1.00	1.00	1.00	1.01	1.01	1.00	1.01	1.00
6	1.00	1.00	1.00	1.02	1.00	0.99	1.01	1.01
7	1.00	1.00	1.00	0.99	1.00	0.99	0.99	0.99
8	1.00	1.01	1.00	1.00	1.00	0.99	0.99	0.99
9	1.00	1.01	1.01	1.01	1.00	0.99	1.01	1.01
10	1.00	1.01	1.01	1.02	0.99	0.98	1.01	1.01
11	1.00	1.01	1.00	1.02	1.00	0.98	1.01	1.02
12	1.00	1.00	1.00	1.02	0.99	0.97**	1.01	1.01
13	1.00	1.00	1.01	1.01	0.99	0.97**	1.00	1.00
14	1.00	1.00	1.00	1.00	0.99	0.98	1.00	1.01
15	1.00	1.00	1.00	1.00	0.98*	0.98**	1.00	1.00

Stars indicate Diebold-Mariano significance. Green/red colors indicate gains/losses.

Persistence in forecast updates
matters mostly at turning points

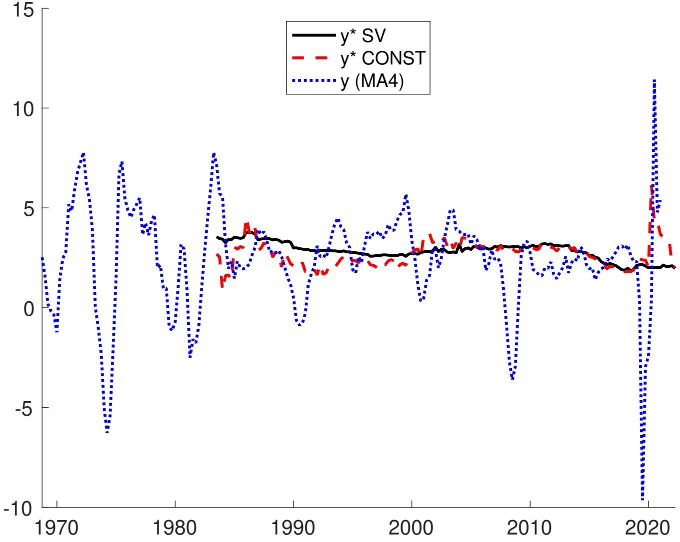
. . . and is hard to predict in real time

Croushore (2010), Mertens & Nason (2020),
Matthes & Foerster (2021), Hajdini and Kurmann (2022),
Farmer, Nakamura & Steinsson (2022),
Bianchi, Ludvigson & Ma (2022)

APPENDICES

ESTIMATED END POINTS OF TERM STRUCTURE

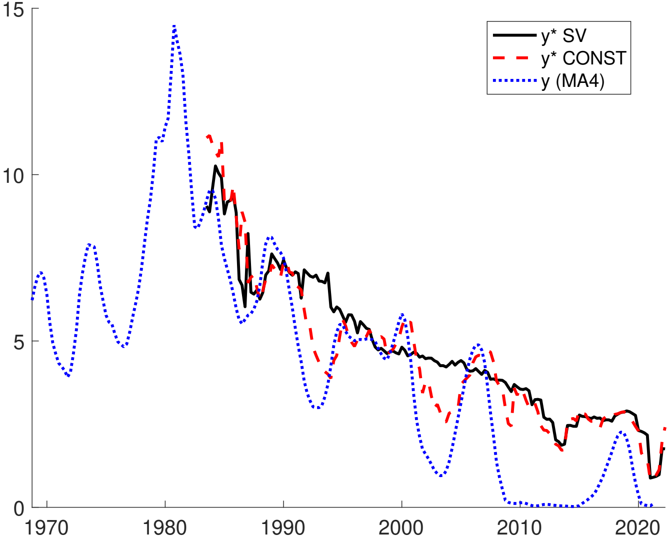
$y_{t+H|t}$ from SV and CONST model for GDP growth ($H = 15$)



Real-time estimates

ESTIMATED END POINTS OF TERM STRUCTURE

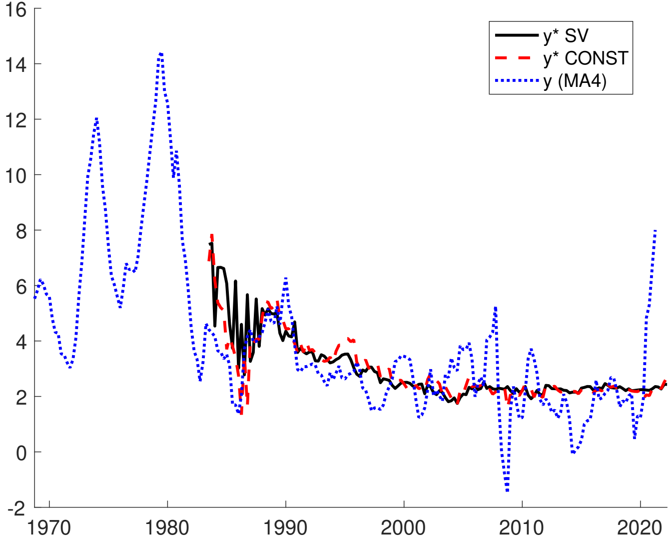
$y_{t+H|t}$ from SV and CONST model for T-bill rate ($H = 15$)



Real-time estimates

ESTIMATED END POINTS OF TERM STRUCTURE

$y_{t+H|t}$ from SV and CONST model for CPI inflation ($H = 15$)



Real-time estimates

SUMMARY

Our contributions:

Model that transforms an arbitrary set
of fixed-event/-horizon SPF data
into a consistent term structure

- Matches observed SPF
- Incorporates all SPF bins with entropic tilting

Findings

- Calendar-year **histograms add** some, but mostly **occasional value** . . .
- . . . *relative to model centered on SPF point forecasts*
- At onset of COVID-19, narrower uncertainty after tilting