Discussion of "Sparse Production Networks" by Bernard and Zi

Yuhei Miyauchi (Boston University)

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Paper Summary

- Balls-and-bins ("elementary") model can rationalize many stylized empirical findings of production network formation
- As data becomes more granular, we need to pay more attention to statistical uncertainty in data generating process (DGP)
  - cf Dingel-Tintelnot '21, Adão-Costinot-Donaldson '22

My Discussion

- Reformulate as an econometric problem
  - Are richer models identified from elementary model?
  - If so, can we statistically reject the null hypothesis of elementary model?
- Additional comments (if time allows):
  - Bilateral covariates
  - Response to shocks

The Elementary Model

- Buyers $j \in \mathcal{M}$ and suppliers $i \in \mathcal{N}$
- N transactions ("balls") in the economy
- (i, j) pair gets each "ball" with probability $s_i b_j$
  - $\{s_i\}$, $\{b_j\}$: parameters
  - $\sum_i s_i = 1$, $\sum_j b_j = 1$ (slight change from paper that takes buyer-side deterministic)
- Transaction volume (number of "balls") between i and j: Y_{ij}
- Joint likelihood of "network" $\mathbb{Y} \equiv \{Y_{ij}\}$:

$$\mathcal{L}(\mathbb{Y}) = \prod_{i,j} (s_i b_j)^{Y_{ij}}$$

"Richer" Models of (Sparse) Production Network Formation

1. Supplier selection by buyers

   e.g. Eaton-Kortum-Kramarz '22; Oberfield '18; Antras-de-Gortari '19; Boehm-Oberfield '20; Sugita-Teshima-Seira '21; Miyauchi '21; Panigrahi '21; Lenoir-Martin-Mejean '22

2. Endogenous search intensity

   e.g. Chaney '14; Demir-Fieler-Xu-Yang '21; Arkolakis-Huneeus-Miyauchi '22; Huang-Manova-Perello-Pisch '22

3. Relationship-specific fixed cost

   e.g. Bernard-Moxnes-Ulltveit-Moe '18; Lim '18; Huneeus '18; Dhyne-Kikkawa-Mogstad-Tintelnot '20; Zou '20; Bernard-Dhyne-Magerman-Manova-Moxnes '22

Question: Are these models identified from elementary model?

Model 1: Supplier selection by buyers (eg Eaton-Kortum-Kramarz '22)

- Buyer j draws a "task" with probability b_j
- Supplier i can undertake the task at marginal cost s_i
  - Can depend on wages, i's suppliers, ...
- i.i.d. Frechet shocks $\Rightarrow$ isomorphic to elementary model
- Potential identification from...
  - introducing covariates (eg spatial decay)
  - response to shocks that endogeneously changes s_i and b_j

Model 2: Endogenous search intensity (eg Demir-Fieler-Xu-Yang '21)

- Add suppliers' and buyers' "visibility"
  - Supplier i is recognized by buyers w.p. v_i^s; buyer j is recognized by suppliers w.p. v_j^b
- Likelihood:

$$\mathcal{L}(\mathbb{Y}) \propto \prod_{i,j} v_i^s v_j^b (s_i b_j)^{Y_{ij}}$$

- Elementary model: $v_i^s = v_j^b = 1 \ \forall i, j$
- Are $\{v_i^s, v_j^b\}$ identified?
  - Informative statistics: correlation between extensive margin (number of relationships) and intensive margin (transaction volume per relationship) across nodes
    cf. Bernard-Dhyne-Magerman-Manova-Moxnes '22, Arkolakis-Huneeus-Miyauchi '22

Model 3: Relationship-specific fixed cost (eg Bernard-Moxnes-Ulltveit-Moe '18)

- Model:

$$\mathcal{L}(\mathbb{Y}^*) = \prod_{i,j} (s_i b_j)^{Y_{ij}^*}, \quad Y_{ij} = Y_{ij}^* 1[Y_{ij}^* \geq F],$$

  i.e., link is formed if $Y_{ij}^* \geq F$

- Elementary model: $F = 1$

- Is F identified?
  - Yes, if we know transaction volume per "ball" ($F = \min\{Y_{ij} | Y_{ij} > 0\}$)
  - Even if not, magnitude of negative assortativity is likely informative

Hypothesis Testing

- Consider a statistic $\psi(\mathbb{Y})$: Is it likely to be generated by the elementary model?
- Challenge: need to test elementary model for any $\{s_i, b_j\}$ (composite null)
- Solution:
  - Inspired by network econometrics literature (Graham and Pelican '22)
  - Under elementary model, distribution of total transaction amount per supplier $\{Y_i^s = \sum_j Y_{ij}\}$ and per buyer $\{Y_j^b = \sum_i Y_{ij}\}$ are sufficient statistics for likelihood

$$\mathcal{L}(\mathbb{Y}) = \prod_{i,j} (s_i b_j)^{Y_{ij}} = \prod_i (s_i)^{Y_i^s} \prod_j (b_j)^{Y_j^b}$$

  - Conditional on $\{Y_i^s,\ Y_j^b\}$, any $\mathbb{Y}$ realize at equal probability under the null $\Rightarrow$ null distribution of $\psi(\tilde{\mathbb{Y}})$: $\tilde{\mathbb{Y}} \sim$ random network given observed $\{Y_i^s,\ Y_j^b\}$

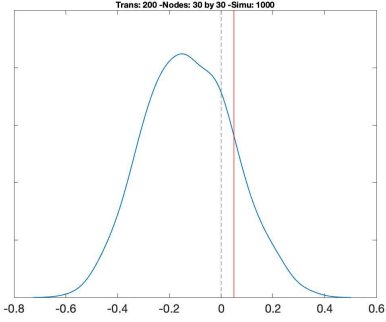
Hypothesis Testing: Comments

- Close but slightly different from the "falsification test" in paper
  - estimate $\{s_i\}$ instead of conditioning on $\{Y_i^s\}$
  - probably over-reject null
- No need to specify alternative model
  - unlike Bayes model selection in paper

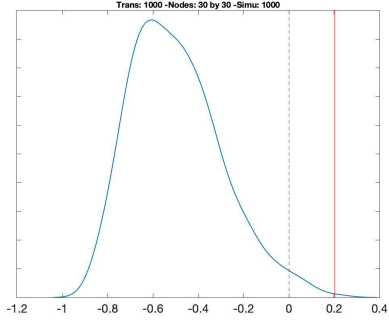
True DGP: Model 2 (Endogenous search intensity)

- $\mathcal{L}(\mathbb{Y}) \propto \prod_{i,j} v_i^s v_j^b (s_i b_j)^{Y_{ij}}$
- $\psi(\mathbb{Y})$: Correlation coef. b/w extensive and intensive margin $Corr(D_i^s, \overline{Y}_i^s)$

(a) $N = 200$ (number of "balls")



(b) $N = 1000$

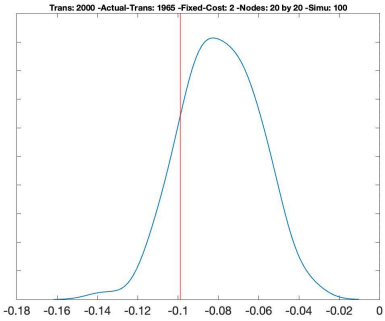


- Test statistics (red); null distribution (blue); zero (dotted)

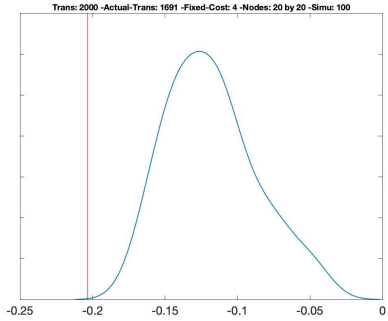
True DGP: Model 3 (Fixed Cost)

- $\mathcal{L}(\mathbb{Y}^*) = \prod_{i,j} (s_i b_j)^{Y_{ij}^*}$, $Y_{ij} = Y_{ij}^* 1[Y_{ij}^* \geq F]$
- $\psi(\mathbb{Y})$: negative degree assortativity $Corr(D_i^s, D_j^b | Y_{ij} > 0)$

(a) $F = 2$



(b) $F = 4$



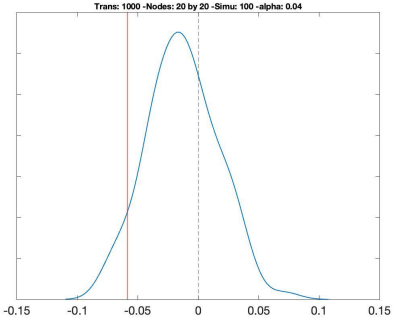
Additional Comments

- Bilateral covariates
  - Statistics involving exogenous covariates (e.g. spatial decay) are informative to reject elementary model
  - But granularity may still matter, see next slide
- Response to shocks
  - Many "richer" models emphasize "equilibrium effects" on $\{s_i, b_j\}$
    - Supplier's supplier, wages, competition, ...
  - "Panel" version of the test against the elementary model with fixed $\{s_i, b_j\}$?

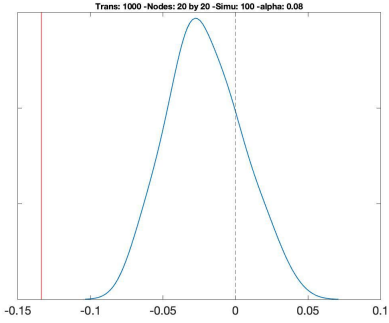
True DGP: Model 1 with Spatial Decay

- $\mathcal{L}(\mathbb{Y}^*) = \prod_{i,j} \left(\frac{s_i b_j |x_i - x_j|^{-\alpha}}{\sum_{i',j'} s_{i'} b_{j'} |x_{i'} - x_{j'}|^{-\alpha}} \right)^{Y_{ij}}$
- $\psi(\mathbb{Y})$: gravity coefficient $\log Y_{ij} = \beta \log |x_i - x_j| + \epsilon_{ij}$ for $Y_{ij} > 0$

(a) $\alpha = 0.04$



(b) $\alpha = 0.08$



- Slight spatial decay under the null because true DGP induces larger suppliers and buyers in geographic center