Tax Incidence and Optimal Taxation with General Equilibrium Effects and Transition

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Abstract

We study the incidence of redistributive income tax reforms in general equilibrium with transitional dynamics, using a variational approach to the nonlinear income tax system. In a standard incomplete market model with self insurance (Aiyagari (1994)), a tax reform changes the relative price of capital and labor income, which has important external welfare effects. Different from the standard trickle-down effects, the general equilibrium effects in Aiyagari economy can favor either more or less progressive tax reform, depending on the relative importance of the labor and savings responses and their transition path. We show that (i) with more redistribution in the initial tax schedule, the general equilibrium effects tend to favor more redistributive reform, (ii) introducing capital income tax tends to amplify the general equilibrium effects of labor income tax reform without changing directions, and (iii) accounting for local/global transition makes optimal tax schedule more progressive.

Keywords: Tax Incidence, Optimal Taxation, General Equilibrium, Transition, Trickle-donw Effects

JEL Classification: E62, H21, D52, D31, E21

1 Introduction

What is the welfare improving tax reform and the optimal tax schedule accounting for the general equilibrium effects and transition? Do the general equilibrium (hereafter GE) effects make the government prefer less progressive tax reform? How does the transitional dynamics affect the tax incidence and optimal tax schedule? This paper provides an answer to these questions in a standard incomplete market model with self insurance (Aiyagari (1994)).

In the optimal taxation literature, a growing number of studies have questioned the impact of endogenous wages on the optimal labor income tax schedule. The conventional result is that accounting for the endogenous prices makes the optimal tax schedule less progressive because of the so-called "trickle-down" forces. Stiglitz (1982) shows that in a model with two unobservable types and endogenous wages, we can achieve indirect redistribution by lowering tax rate at the high income, which closes the gap between the wages of the two types. Rothschild and Scheuer (2013) extends this to a Roy model where workers choose between different sectors and confirms less progressive optimal income taxes than the ones in a single-sector model. These results are entirely driven by the complementarity of different labor types. In this paper, we question the effects of endogenous prices on the tax incidence and optimal tax schedule in an economy with neoclassical technology where the two input factors—capital and labor—are complements. Does the conventional less progressive tax result (relative to the one in partial equilibrium) still apply?

We address this by analyzing the welfare effects of the tax reform in the so-called standard incomplete market model—the model with heterogeneous agents and uninsurable idiosyncratic productivity shock—with production (Bewley (1986), Aiyagari (1994)). We derive the formula for the tax reform incidence by using a variational approach in a nonlinear labor income tax system, and we say a tax schedule is optimal if there is no welfare improving tax reform. Although we allow a fully nonlinear tax system, the class of tax system is restricted to a time-invariant and history-independent tax system that only depends on the current annual income capturing the most common tax system in practice.[1] In an incomplete mar-

[1]This simple class of tax system, which was also adopted in Chang and Park (2021), allows us to clearly decompose the mechanisms of the welfare effects and show the role of the GE effects in a transparent way.

ket, precautionary savings as well as labor supply respond to the tax reform, which leads to a change in relative prices of the production inputs. The government can provide insurance directly through the tax reform and indirectly through its impact on the factor prices.

The tax incidence and optimal tax formula in the Aiyagari economy show that the general equilibrium forces affect the incidences of the tax reform on welfare through the pecuniary externalities and fiscal externalities. The pecuniary externalities are the welfare effects of the relative price change through the redistribution across the wealth dispersion and insurance against income shocks. The fiscal externalities are the government revenue effects of the price incidence. With these externalities, the formulas depend on substitution between production inputs, implying that the Tax-Formula result of Diamond and Mirrlees (1971) does not apply.[2] Since the Tax-Formula result requires enough tax instruments to affect every relative price, as long as we do not allow a history-dependent (capital) tax system, it is optimal to exploit the indirect redistribution through the price externalities.

Does a less progressive tax reform improve welfare as in the conventional trickle-down effects with complementarity in labor? The answer depends on whether a progressive reform increases or decreases interest rate because both the pecuniary and fiscal externalities imply that any tax reform which decreases interest rate (and increases wage rate) is welfare improving. Decrease in interest rate has positive redistribution by making the asset poor richer, and increase in wage rate will increase labor earnings and thus income tax revenue. Then the question boils down to which tax reform results in decrease in interest rate.

The tax incidence formula shows that the GE effects in an Aiyagari economy can favor either more or less progressive/redistributive tax reform depending on the relative strength of the labor and savings responses and their transition path. Consider a redistributive tax reform of increasing tax rate. This reform results in immediate drop in labor supply, while capital decreases slowly with sluggish adjustment in precautionary savings. As a result, in the short-run, interest rate decreases in response to a redistributive tax reform, and in the long run, interest rate is likely to increase as the response of savings dominates. Since lower

[2] The terminology of the Tax-Formula result was introduced by Saez (2004). It refers to the result of Diamond and Mirrlees (1971) that the optimal tax formulas are identical when prices of factors are fixed and when prices are variable and derived from a general production function.

interest rate has positive welfare implications through the price externalities, the GE effects will favor a more (less) redistributive tax reform if the short-run labor response dominates (if the long-run savings response dominates). Our formula shows that the time-average of the change in interest rate is the summary statistic of the GE effects and its sign determines the direction of the welfare incidence of a redistributive tax reform through the GE effects.

We then identify the key factors of the GE effects and investigate under which condition less (or more) progressive/redistributive tax reform is preferred by the government accounting for the GE effects relative to the one who does not. First, the level and progressivity of the initial tax schedule to which the reform is applied are crucial for the direction of the GE effects. With modest level of redistribution existing before the tax reform—e.g. under the current US tax—the GE effects tend to favor *less* redistributive reforms because the response of precautionary savings are sensitive to additional provision of public insurance, and more redistributive tax reforms is likely to result in significant decrease in savings, which has negative GE effects through increase in interest rate. On the other hand, when the optimal tax schedule is highly redistributive reflecting the equity-efficiency trade off in the standard optimal tax theory (especially when the government has strong redistributive preferences), considering the GE effect can favor even *more* redistributive reform as the savings become less sensitive to a redistributive tax reform while the response of labor gets stronger.

Production-efficiency theorem of Diamond and Mirrlees (1971) does not apply in this economy and it is generally optimal to tax/subsidize capital to exploit the indirect redistribution via change in relative prices. We investigate how the introduction of simple linear capital income taxes influences the GE effects of the labor income tax reform. We show that when introducing capital income tax, the direction of the welfare implication of the GE effects in general does not change. Moreover, it tends to amplify the GE effects in its original direction, as the steeper asset supply curve with capital income tax implies stronger price incidences.

The transition path becomes crucial for the tax incidence when the GE effects are considered. In a typical variational approach based on a small tax perturbation, the transition associated with this small tax reform is the *local* transition path. That is, the formulas of the tax incidence and optimal tax reflect the welfare incidences of a tax perturbation through the change in relative prices over the *local* transition. Accounting for the local transition makes the GE effects prefer a more progressive reform (and makes the optimal tax schedule more progressive), because such a reform has positive welfare effects in the short run.

We also investigate the welfare effects of the global transition—from the current US to the optimal steady state—and show that the global transition also makes the optimal tax schedule more progressive. Our analysis, however, raises a question whether accounting for the global transition is desirable when deriving the optimal tax. The typical concern about the optimal tax without considering global transition is that a reform to an optimal tax might generate welfare costs in the transition, especially in the short run. In this economy, however, such an optimal tax generates huge short-run welfare gains at the cost of long-run welfare loss because of the slow adjustment of capital. Moreover, the optimal tax considering the global transition becomes more progressive to exploit this sluggish adjustment, which generates even more asymmetric welfare effects over the transition. In this paper, we analyze the role of the global transition, but leave the question as an open debate.

There is a vast literature on how the endogenous prices affect the shape of the optimal labor income tax schedule, which was first emphasized by Stiglitz (1982). As we discussed above, Stiglitz (1982) found that in the presence of imperfect substitutability of different types of labor, the wage compression channel (or trickle-down channel) makes the optimal tax schedule less progressive—with only two types, the tax rate on the high skill agents is negative. Rothschild and Scheuer (2013), Scheuer (2014), Ales, Kurnaz, and Sleet (2015), and Sachs, Tsyvinski, and Werquin (2020) have extended the Stiglitz's effects to Roy model, economies with entrepreneurial sector, task-to-talent assignment model, and economies with continuum of endogenous wages, respectively. While these studies focus on the imperfect substitutability of different types of labor, we investigate a version of Stiglitz's effects in the presence of imperfect substitutability of capital and labor with neoclassical technology.[3]

The literature on optimal taxation using a variational approach was first pioneered by Piketty (1997) and Saez (2001). This approach is often called sufficient statistic approach, because the optimal tax formula obtained by the perturbation of the tax schedule is expressed

[3]Mayr (2021) also studies the trickle-down effects with neoclassical technology, but he investigates how the endogenous prices affect optimal capital income tax rates.

in terms of sufficient statistics (e.g., elasticity of the labor supply and the hazard rate of income). Among many studies that extend this approach to other contexts, the most closely related paper is Chang and Park (2021).[4] In Chang and Park (2021), we derive the optimal tax formula in the presence of private insurance market, and show that the formula inevitably depends on the market structure—obtaining the estimates of statistics regarding the response of private insurance market is not enough as we have to know whether the envelope theorem is applied to those statistics. Building on Chang and Park (2021), this paper exploits the structural sufficient statistic approach in an Aiyagari economy, but with a very different focus. The main focus of this paper is whether accounting for the GE effects and transition makes the redistributive tax reform more (or less) beneficial, with the identification of the key factors for the direction and size of the GE effects.[5]

The majority of studies on optimal fiscal/redistribution policy in the incomplete market model has focused on the steady state analysis. Recently, however, there has been increasing attention to the importance of the transition path. Several studies show that the transition path is crucial for policy evaluation by demonstrating that accounting for the transition path to the new steady state can lead to very different optimal tax schedule (Bakis, Kaymak, and Poschke (2015), Dyrda and Pedroni (2018), Krueger and Ludwig (2016)). These quantitative studies compute the optimal transition path, but they do not provide the clear-cut decomposition of mechanisms through which the transition path matters. This paper clearly shows that transition matters through the interaction with the GE effects for the tax incidence and optimal tax schedule by decomposing the mechanisms analytically.

The paper proceeds by setting up the benchmark model in Section 2. We analyze the role of GE effects in the tax incidence formula and optimal tax formula in Section 3 and Section 4, respectively. We then identify the key factors of the GE effects in Section 5, and the role of local and global transition is investigated in Section 6. Section 7 concludes.

[4]The extended context includes ulti-dimensional screening (Kleven, Kreiner, and Saez (2009)) and dynamic environments (Golosov, Tsyvinski, and Werquin (2014) and Saez and Stantcheva (2017)).

[5]We show that different from Huggett (1993) model with single production input—the benchmark in Chang and Park (2021), in an Aiyagari economy with capital and labor in production, accounting for the endogenous *relative* prices can make the government prefer either more or less redistributive tax reform.

2 Benchmark Model

The benchmark model is a standard incomplete market model with idiosyncratic productivity shock (Aiyagari (1994)) extended to include endogenous labor supply and tax policy.

2.1 Environment

Time is discrete and the economy consists of a continuum of measure one of heterogeneous consumers, a representative firm, and a government. Consumers are infinitely lived and ex ante identical but ex post heterogeneous consumers.

Preferences The preferences of the households are represented by

$$E_0 \sum_t \beta^t U(c_t, l_t),$$

where c is consumption and l is labor supply. For simplicity, we assume that the period utility U has the following form: $U(c,l) = u(c - v(l))$, where $u(\cdot)$ is concave and increasing and $v(\cdot)$ is convex and increasing in l. These are so-called GHH preferences of Greenwood, Hercowitz, and Huffman (1988) that have no wealth effect on labor supply. It is well known that the optimal tax formula can be greatly simplified under this assumption (e.g. Diamond (1998)). In the presence of the private market, the GHH preferences are even more helpful for simplification because we don't have to consider how the labor supply responds to the change in the private insurance market.

Productivity Process and Earnings Workers face uncertainty about their productivity shock. Individual productivity x_t follows a Markov process, with transition probability $f(x_{t+1}|x_t)$, that has an invariant stationary distribution $F(x)$, which has density $f(x)$. The history of productivity is denoted by $x^t \equiv (x_0, x_1, \cdots, x_t)$, and the probability of history of x^t given x_0 is denoted by $f(x^t|x_0)$. The earnings of a worker with productivity realization x_t are $z(x_t) = w_t x_t l(x_t)$ given wage rate w_t, which depends only on x_t under the GHH preferences. Thus, the earnings z_t and productivity x_t have one to one relationship, and the cumulative distribution of earnings is denoted by $F_z(z)$ that satisfies $F_z(z(x)) = F(x)$, whose density function is $f_z(z)$.

Technology The production technology is represented by a production function $F(K, L)$, where K and L are aggregate capital and labor. We assume that F has the constant returns to scale property (homogeneous of degree one), and F also satisfies standard properties: $F_K, F_L > 0$, $F_{KK}, F_{LL} < 0$, and $F_{KL} > 0$. In the benchmark economy, we use a standard Cobb-Douglas production function:

$$F(K, L) = A \cdot K^{\alpha} L^{1-\alpha},$$

and later we extend the analysis to the economy with a CES production function. The depreciation rate for capital is denoted by δ_k. In a competitive factor market, wages and interest rates are determined by $w = F_L(K, L)$ and $r = F_K(K, L) - \delta_k$.

Restriction on the Tax System We consider a fully nonlinear tax system without assuming a functional form, but focus on a restrictive class of tax system. We consider a time-invariant labor income tax function $T(z)$ which is fully nonlinear in labor income $z = wxl$, but the function T depends on current period's income only and does not depend on history. Although this restriction is subject to the criticism of the new dynamic public finance literature, it still captures the realism because in practice, most countries use a tax system that only depends on annual labor income. As in Chang and Park (2021), by imposing the restrictions, we can obtain the formulas that show the mechanisms of the GE effects and transition in a transparent way. For the benchmark, we abstract from the capital income tax for a simpler exposition. In Section 5.2, we will allow capital income taxation.

Equilibrium The problem of a consumer is to choose consumption, savings, and labor supply to maximize the expected present value of utility. We can state the consumer's problem recursively. The state variables of the individual household are $(a, x) \in S$. The probability measure $\mathbf{\Phi}$ is defined over the Borel sets of S. The law of motion of the distribution is written as $\mathbf{\Phi}' = T(\mathbf{\Phi})$. Then, the consumer's problem is

$$V(a, x; \mathbf{\Phi}) = \max_{c, a', l} (1-\beta) u(c - v(l)) + \beta E \left[V(a', x'; \mathbf{\Phi}') | x \right] \tag{1}$$

$$s.t. \quad c + a' = wxl - T(wxl) + (1+r)a \tag{2}$$

$$\mathbf{\Phi}' = T(\mathbf{\Phi}). \tag{3}$$

Given tax policy $T(z)$ and government expenditures $\{G_t\}$ and initial conditions K_0 and Φ_0, a competitive equilibrium of the economy consists of a value function, $V(a, x; \Phi)$, policy functions, $a'(a, x; \Phi)$ and $l(x; \Phi)$, a wage rate $w(\Phi)$, an interest rate $r(\Phi)$, and an evolution function $T(\Phi)$ such that

(i) Given prices, tax policies, and initial conditions, the value function $V(a, x; \Phi)$ solves the consumer's problem defined by (1) with the associated policy functions a' and l.

(ii) Prices $w(\Phi)$ and $r(\Phi)$ satisfy

$$r(\Phi) = \alpha A(K/N)^{\alpha-1} - \delta_k, \quad w(\Phi) = (1-\alpha)A(K/N)^{1-\alpha}.$$

(iii) The government budget is balanced:

$$G_t = \int T(w(\Phi_t)xl(x))d\Phi_t(a, x)$$

(iv) Markets clear:

$$K' = \int a'(a, x; \Phi)d\Phi(a, x), \quad \text{and} \quad N = \int w(\Phi)xl(x)d\Phi(a, x)$$

(v) Law of motion: $\Phi' = T(\Phi)$,

where the mapping T is defined as follows. Let $A = [\underline{a}, \bar{a}]$, $X = [\underline{x}, \bar{x}]$, and $S = A \times X$. For a Borel set $B = B_a \times B_x$ in σ-algebra Σ_S, we define $T(\Phi_t)(B) = \int_{A\times X} Q(\Phi_t, (a, x), (B_a \times B_x))d\Phi_t$ where the transition function is $Q(\Phi_t, (a_t, x_t), (B_a \times B_x)) = \int_{x_{t+1}\in B_x} f(x_{t+1}|x_t)\mathbb{1}_{a'(a_t,x_t)\in B_a}$, and $\mathbb{1}$ is the indicator function.

For the analysis below, we also note that the individual state can be expressed as (a_t, z_t) instead of (a_t, x_t).[6] In addition, as long as the mapping $x \mapsto y(x)$ is one to one, we can express $a'(a_t, x_t) = h^A_{t+1}(a_t, y_t(x_t))$, where $y_t(x_t) = w_t x_t l_t(x_t) - T(w_t x_t l_t(x_t))$.

[6]With GHH preferences, labor income z_t and productivity x_t have a one-to-one relationship so that we can use them interchangeably. Even under preferences with wealth effects on the labor supply, we can use state variables (a, x) and (a, z) interchangeably because x and $z(a, x) = wxl(a, x)$ have a one-to-one relationship given a.

3 Tax Reform Formulas: Implication of the GE Effects

In this section, we derive the tax incidence formulas to answer the main question of this paper—whether accounting for the GE effects and transition makes the government favor more or less progressive tax reform. The formulas are obtained by applying the variational approach (Piketty (1997) and Saez (2001)). Consider a perturbation from a given tax schedule. We first analyze the welfare effects of local tax reform at the status quo, where the status quo can be a stationary equilibrium under any tax schedule — the current US, optimal, or any other tax schedule we want to consider.

3.1 Local Tax Reform Incidence at the Status Quo

We start with the incidence of tax, the first-order effects of the local tax reforms at the status quo. We focus on the interpretation of the incidence. See Appendix A for the derivation.

Suppose that the economy is in a steady state under a given tax schedule $T(z)$, and the distribution of state variables $\Phi(x_t, a_t)$ is stationary. Consider an arbitrary tax reform, which can be represented by a continuously differentiable function $\tau(\cdot)$ on $\mathbb{R}_+$. Then, a perturbed tax schedule is $T(\cdot) + \mu\tau(\cdot)$, where $\mu \in \mathbb{R}$ parameterizes the size of the tax reform. As in Golosov, Tsyvinski, and Werquin (2014) and Sachs, Tsyvinski, and Werquin (2020), the first-order effects of this perturbation can be formally represented by the Gateaux derivative in the direction of τ. For example, the incidence on the labor supply is $dl_t(x) \equiv \lim_{\mu \to 0} \frac{1}{\mu}[l_t(x; T + \mu\tau) - l_t(x; T)]$.

We want to highlight that once we assume that the economy is in a steady state before the tax reform, the formulas (for both tax incidence and optimal tax) that we derive using a variational approach only take into account the *local* transition associated with the local tax reform. That is, when we compute the tax incidence starting from some status quo tax $T(\cdot)$ different from the current US tax schedule (e.g. optimal tax schedule), the incidence does not reflect the *global* transition, which is the transition starting from the current US economy to the steady state associated with the considered tax schedule $T(\cdot)$. This also implies that the optimal tax formula based on a variational approach does not take into account the welfare effects in the transition from the current US status quo to the optimal steady state, but only takes into account the local transition associated with the small tax perturbation at the optimal steady state. One might think that this is the limitation of the approach, but as we will show in Section 6, whether considering the global transition is desirable for the optimal tax derivation is not obvious at all because it exploits the short-run gain from the sluggish adjustment of capital.

For the benchmark analysis, we do not consider global transition by assuming that the economy is in a steady state before the local perturbation. Thus, in this and next section, we use the word "transition" to refer to the local transition. Later in Section 6, we analyze the role of local and global transition.

From now on, we focus on the *elementary tax reforms* represented by $\tau(z) = \frac{1}{1-F_z(z^*)}\mathbb{1}\{z \geq z^*\}$ for a given level of income z^*.[7] We can focus on this elementary tax reform without loss of generality, because any other perturbations can be expressed as a weighted sum of elementary tax reforms. See Sachs, Tsyvinski, and Werquin (2020) for further details.[8]

We derive the incidence on the labor supply, government revenue, individual utility and welfare *given* the incidence on the prices dw_t and dr_t and the incidence on the asset holding dh^A_{t+1}, without attempting to derive analytical expression of dw_t, dr_t, and dh^A_{t+1}. As we discuss in Chang and Park (2021), deriving analytical expressions for the incidences on the individual savings is extremely challenging because the incidences include all the dynamic effects—saving decisions at different times and histories are interlinked—and the general equilibrium effects. In turn, deriving the incidences on the prices are also extremely challenging because they depend on the incidences on the aggregate savings and labor. Despite this limitation, the incidence formulas given incidences on savings and prices are still very

[7]Under this tax reform, the tax payment of an individual with income above z^* increases by a constant amount $\frac{1}{1-F_z(z^*)}$, and the marginal tax rate at income level z^* is increased by $\frac{1}{1-F_z(z^*)}$ (which is obtained by the marginal perturbation: $\tau'(z) = \frac{1}{1-F_z(z^*)}\delta_{z^*}(z)$ where δ_{z^*} is the Dirac delta function at z^*). Note that with this tax reform, the increased government revenue due to a mechanical increase in tax payment is equal to \$1.

[8]This elementary tax reform is also consistent with the heuristic tax reform in Saez (2001), in which the marginal tax rate $T'(z)$ is increased by $\delta\tau$ on a small income bracket $[z^*, z^* + dz^*]$ and the tax payment $T(z)$ is increased by $\delta\tau \cdot dz^*$ $(= \frac{1}{1-F_z(z^*)})$ for income above z^*.

useful in analyzing the implications of the GE effects. In the quantitative investigation, we use the structural sufficient statistic approach to obtain these incidences quantitatively.

Although we do not attempt to derive analytical expressions for the incidences on the prices, we make some remarks which are important for the analysis of the GE effects. The incidences on the prices are:

$$dr_t = F_{KK}dK_t + F_{KL}dL_t, \quad \text{and } dw_t = F_{LK}dK_t + F_{LL}dL_t,$$

which show that the relative responses of the aggregate capital and labor are crucial for the direction of price incidences. We also note that since a constant returns to scale production function has the following property: $F(K, L) = (r + \delta)K + wL$, by differentiating, we obtain the relationship between the incidence on the two factor prices:

$$L \cdot dw_t + K \cdot dr_t = 0. \tag{4}$$

That is, the incidences on w_t and r_t are always in the opposite direction. When the capital-labor ratio increases (decreases) the wage rate increases (decreases) and the interest rate decreases (increases).

We define the elasticity of labor supply with respect to retention rate and the elasticity with respect to wage rate *along the nonlinear budget constraint* as in Jacquet and Lehmann (2015) and Scheuer and Werning (2017):

$$\epsilon^l_{1-T'}(x) = \frac{e(x)}{1 + \rho(z(x))e(x)}, \quad \epsilon^l_w(x) = \frac{e(x)(1 - \rho(z(x)))}{1 + \rho(z(x))e(x)},$$

where $e(x) = \frac{v'(l(x))}{l(x)v''(l(x))}$ is the elasticity along the linear budget and $\rho(z(x)) = -\frac{\partial ln(1-T'(z(x)))}{\partial lnz(x)} = \frac{z(x)T''(z(x))}{1-T'(z(x))}$ denotes the local rate of progressivity of the tax schedule. Then the incidence on the labor supply given dw_t is $dl_t(x) = -\epsilon^l_{1-T'}(x)\frac{\tau'(wxl(x))}{1-T'(wxl(x))}l(x) + \epsilon^l_w(x)l(x)\frac{dw_t}{w}$.

The incidence on the government revenue $R = \int T(z(x))f(x)dx$ given dw_t is

$$\begin{aligned} dR_t \quad &= \quad \int_{x^*}^{\infty} \frac{f(x)}{1 - F(x^*)}\, dx - \frac{T'(z(x^*))}{1 - T'(z(x^*))}\epsilon^l_{1-T'}(x^*)\frac{z(x^*)}{z'(x^*)} \cdot \frac{f(x^*)}{1 - F(x^*)} \\ &\quad +dw_t \int (1 + \epsilon^l_w(x))xl(x)T'(z(x))f(x)dx, \quad \forall t. \end{aligned} \tag{5}$$

The first two terms are the standard revenue incidences in the partial equilibrium—mechanical increase of tax payment due to the tax reform and the decrease of government revenue due to decrease in labor supply. The third term captures the incidence through the change in wage rate. The change in wage rate affects the government revenue directly through the change in labor income given the labor supply and indirectly through the change in labor supply. Since we consider revenue-neutral tax reforms, any change in government revenue dR_t will be rebated back to households as a lump-sum transfer. Note that dR_t is time-varying due to the sluggish adjustment of wage rate after the tax reform. Moreover, as we investigate below, dw_t can switch its sign over time, thus transition path matters for the fiscal externalities.

The incidence on the consumer's utility whose state before the tax reform is (a_0, x_0) is

$$\begin{aligned} dV(a_0, x_0) \quad = \quad & (1-\beta)\sum_{t=0}^{\infty}\beta^t \int u'(a_0, x^t)\left[-\tau(z(x_t)) + dR_t\right] f(x^t|x_0)dx^t \\ & +(1-\beta)\sum_{t=0}^{\infty}\beta^t \int u'(a_0, x^t)\left[dr_t a_t(a_0, x^{t-1}) + dw_t x_t l(x_t)(1-T'(z(x_t)))\right] f(x^t|x_0)dx^t \\ & -(1-\beta)\sum_{t=0}^{\infty}\beta^t \left[u'(a_0, x^t) - \beta(1+r)E[u'(a_0, x^{t+1})|x^t]\right] dh^A_{t+1}(a_t(a_0, x^{t-1})), y(x_t)) f(x^t|x_0)dx^t. \end{aligned}$$

The first line shows the standard effects of tax reform through the extra payment of taxes and the lump sum transfer. It is well known that the incidence on the labor supply does not show up because of the envelope theorem. On the other hand, the incidence on the savings dh^A shows up in the third line for the consumers whose borrowing constraint is binding because the envelope theorem cannot be applied to those changes. The second line shows that the change in prices has first-order welfare effects because individuals take the prices as given and do not consider how their decisions affect prices.

Now, the tax incidences on the utilitarian social welfare can be obtained by integrating the incidences on the individual utility: $dW = \int dV(a_0, x_0)d\Phi(a_0, x_0)$. We denote the marginal value of public funds by λ, which is simply $\lambda = \int u'(a, x)d\Phi(a, x)$ with no income effects on labor supply. Next proposition shows the decomposition of the incidence by the mechanism.

Proposition 1. *The incidence of the elementary tax reform at z^* on welfare is given by*

$$
\begin{aligned}
dW &= R1 + R2 + R3 + R4, \quad \textit{where} \qquad (6)\\
R1 &= \lambda \left[\iint_{x^*}^{\infty} \left(1 - \frac{u'(a,x)}{\lambda}\right) \frac{\phi(a,x)}{1-F(x^*)} dxda - \frac{T'(z(x^*))}{1-T'(z(x^*))} \epsilon^l_{1-T'}(x^*) \frac{z(x^*)}{z'(x^*)} \frac{f(x^*)}{1-F(x^*)} \right]\\
R2 &= -(1-\beta) \sum_{t=0}^{\infty} \beta^t \iint \left[u'(a,x) - \beta(1+r)E[u'(a'(a,x),x')|x]\right] dh^A_{t+1}(a, y(x)) d\Phi(a,x)\\
R3 &= (1-\beta) \sum_{t=0}^{\infty} \beta^t \iint u'(a,x) \left[dr_t a + dw_t x l(x)(1 - T'(z(x)))\right] d\Phi(a,x)\\
R4 &= \lambda(1-\beta) \sum_{t=0}^{\infty} \beta^t \cdot dw_t \int (1 + \epsilon^l_w(x)) x l(x) T'(z(x)) f(x) dx.
\end{aligned}
$$

The $R1$ term captures standard redistribution and behavioral effects of the tax reform. The elementary tax reform levies extra taxes $\frac{1}{1-F(x^*)}$ for every households above $z(x^*)$ and the increased revenue, which is 1, is rebated back to the consumers. Thus, the welfare gain from the tax reform is decreasing in the average welfare of households above $z(x^*)$ (relative to the marginal value of public funds λ). The second term in $R1$ captures the negative revenue effects of decrease in labor supply at $z(x^*)$ where the marginal tax rate is raised.

The $R2$ term captures the effects of the borrowing constraints. As described above, we cannot apply the envelope theorem to the changes in savings/borrowings of the borrowing constrained households. In Aiyagari (1994) economy, however, this $R2$ term is quantitatively very small because the fraction of consumers released from the borrowing constraint after the small reform tends to be very small.[9]

The focus of this paper is the general equilibrium effects captured by the terms $R3$ and $R4$. We analyze the general equilibrium effects in detail in the next section.

3.2 Implication of the GE Effects

We now ask the main question of this paper: "Does accounting for the GE effects make the government favor more (or less) progressive tax reform?" To answer this question, we first rewrite $R3 + R4$ in the welfare incidence in terms of summary statistics. This expression

[9]In an economy with endogenous borrowing constraints, $R2$ term can be quantitatively important as the borrowing constraint itself responds to the tax reform.

shows 1. the mechanism of the GE effect and 2. the direction and size of the GE effects—whether they favor more/less progressive tax reform and how strongly they do.

Summary Statistic Representation The welfare incidence through the GE effects ($R3+R4$) can be rewritten as the product of the two factors—(i) the time average of the incidences on the interest rate and (ii) the externality effects of unit price change. Note that we use (4)—the relationship between the dr_t and dw_t—to rewrite the equation.

Corollary 2. *The incidence of elementary tax reform at z^* on the welfare through the general equilibrium effects (normalized by the marginal value of public funds λ) can be written by:*

$$\frac{1}{\lambda}(R3+R4) = (1-\beta)\sum_{t=0}^{\infty}\beta^t dr_t(z^*)\cdot K\times(\Delta_p-\Delta_f) \tag{7}$$

where

$$\Delta_p = \iint \frac{u'(a,x)}{\lambda}\left[\frac{a}{K}-\frac{xl(x)(1-T'(z(x)))}{L}\right]d\Phi(a,x) \tag{8}$$

$$\Delta_f = \int(1+\epsilon_w^l(x))\frac{xl(x)}{L}T'(z(x))f(x)dx. \tag{9}$$

We denote the time average of the incidences on the interest rates by $\overline{dr_t} = (1-\beta)\sum_{t=0}^{\infty}\beta^t dr_t$, then the GE effects are simply given by $\overline{dr_t}(z^*)K\times(\Delta_p-\Delta_f)$, where $\Delta_p-\Delta_f$ is the externality effect per unit price change.[10] Equivalently, the welfare incidence through the GE effects can be represented by $-\overline{dw_t}(z^*)L\times(\Delta_p-\Delta_f)$ as the factor prices always move in opposite directions. We also note that $\Delta_p-\Delta_f$ does not depend on z^*. Only the time average of change in interest rate ($\overline{dr_t}(z^*)$) depends on at which income the tax reform is applied.

Mechanisms of the GE effects We briefly summarize the mechanisms of the GE effects investigated in Chang and Park (2021) with Table 1. See Appendix B for more details. $\Delta_p-\Delta_f$ captures the two mechanisms through which increase in interest rate (and decrease in wage rate) changes welfare. The first channel captured by Δ_p is the pecuniary externalities in the incomplete market (Dávila, Hong, Krusell, and Ríos-Rull (2012)). Higher interest rate has negative redistribution effects by making the wealth-rich better off and the wealth-poor worse off. Despite the positive insurance effects of decreasing wage rate, the pecuniary

[10]More precisely, $\Delta_p-\Delta_f$ is the welfare effects when the interest rate increases by $\frac{1}{K}$ (or equivalently, when the wage rate decreases by $\frac{1}{L}$), and $K\cdot(\Delta_p-\Delta_f)$ is the welfare effects per unit increase of interest rate (or equivalently per $\frac{K}{L}$-unit decrease of wage rate).

externality tends to have negative welfare effects ($\Delta_p < 0$) with the high wealth inequality in reality. The second channel captured by $-\Delta_f$ is the fiscal externalities. When the wage rate decreases, the government revenue decreases due to both the direct effects given labor supply and the indirect effects via decrease in labor supply. In sum, both the pecuniary and fiscal externalities imply that any tax reform that leads to increase in interest rate has negative welfare effects: $\Delta_p - \Delta_f < 0$. Thus, accounting for the GE effects makes the government prefer a tax reform that decreases interest rate.

Table 1: $\Delta_p - \Delta_f$: Summary of Welfare Effects of $r \uparrow$ ($w \downarrow$)

	$\Delta_p = \Delta_K + \Delta_L$	$-\Delta_f$
Channel	pecuniary externality	fiscal externality
Equation	$\Delta_K = \int \frac{u'(a,x)}{\lambda}\left[\frac{a}{K} - 1\right] d\Phi < 0$ $\Delta_L = -\int \frac{u'(a,x)}{\lambda}\left[\frac{xl(x)(1-T'(z(x)))}{L} - 1\right] d\Phi > 0$	$-\Delta_f = -\int (1 + \epsilon^l_w(x)) \frac{xl(x)}{L} T'(z(x)) f(x) dx < 0$
Welfare Effects	$\Delta_K < 0$: $r \uparrow \Rightarrow$ negative redistribution over a $\Delta_L > 0$: $w \downarrow \Rightarrow$ positive insurance against x	$-\Delta_f < 0$: $w \downarrow \Rightarrow$ negative revenue effect
Overall	$\Delta_p - \Delta_f < 0$ as long as the asset inequality is high enough	

Direction and Size of $\overline{dr_t} K \times (\Delta_p - \Delta_f)$ We are interested in whether the conventional implication of the GE effects in the optimal taxation studies applies to our economy—whether accounting for the GE effects makes the government favor less progressive tax reform. From the summary statistic representation of the GE effects which is product of two factors $\overline{dr_t}(z^*) K \times (\Delta_p - \Delta_f)$, the direction and size of each factor and their schedule over z^* determine the implication of the GE effects. Especially, given the sign of $\Delta_p - \Delta_f < 0$—negative welfare effects of increasing interest rate (which is confirmed by every quantitative result below), the question boils down to whether a less progressive tax reform—decreasing tax rate especially at high income—leads to decrease in interest rate. For example, the following cases are possible.

(a) If the sign of $\overline{dr_t}(z^*) K$ is positive for the elementary tax reforms at all z^* income levels, any redistributive tax reform of increasing tax rates has negative GE effects. Considering the GE effects makes the government prefer less redistributive tax reform.

(b) In contrast, if the sign of $\overline{dr_t}(z^*) K$ is negative for all z^* income levels, considering the GE effects favor more redistributive tax reforms.

(c) If the sign of $\overline{dr_t}(z^*)K$ is negative for the tax reform at low income levels and positive for the reform at high income levels, the government considering the GE effects prefers less progressive tax reform relative to the government who ignores the GE effects.

The next step is then to figure out which tax reform leads to decrease in interest rate. We first note that the relative responses of labor and savings are the key. That is, whether the tax reform of increasing tax rate at z^* decreases (or increases) interest rate depends on the dominance of the labor supply response and the savings response to the tax reform at z^*. Increasing tax rate reduces labor supply and the same tax reform (except the reform at the very bottom income) also tends to decrease precautionary savings as higher tax rates imply more redistribution. Thus, depending on the strength of the labor supply response and savings response, the incidence on the interest rate can be either positive or negative. If the response of savings dominates, a government accounting for the GE effects favor less redistributive tax reform—which results in more precautionary savings and lower interest rate—compared to the government who ignores the GE effects. If the labor response dominates, however, the GE effects make the government favor more progressive (or more redistributive) reform, contrary to the conventional trickle-down effects.

The direction of the GE effects should be answered in the quantitative investigation. The result is not deterministic, because which response dominates depends on the economic conditions. In Section 5, we quantitatively identify key factors for the direction of the GE effects, and investigate under which conditions, the savings response (or the labor response) dominates. The factors we consider are the initial tax schedule at which the reform is taken—which affects the elasticities of labor supply and savings—and the key parameters of preferences and technologies such as risk aversion and elasticity of substitution as well as the additional sources of wealth inequality.

Role of Transition The transition path is crucial for the direction of the general equilibrium effects. This is because the response of labor supply is immediate after the tax reform, while the response of savings is sluggish. In the short run, the responses of labor tend to dominate—leading to decrease in interest rates that have positive welfare effects—and in the long run, the responses of savings tend to dominate—resulting in increase in interest rates and negative welfare effects. Thus, the implication of the GE effects on the welfare incidence of tax reform essentially depends on the transitional dynamics and the patience of the economy—how much weight is put on the short run outcome and the long run outcome. In Section 6, we investigate the role of the transition and discounting factor for the direction and size of the GE effects quantitatively.

We once more emphasize that the *transition* refers to the local transition associated with the local small tax reform, not the global transition from the current steady state to the optimal steady state. Since the economy starts from the steady state associated with the considered tax schedule and the variation approach exploits the local perturbation, the incidence formula and optimal tax formula do not consider the welfare gain/loss of the global transition. Later, in Section 6, we analyze the role of both local and global transition and discuss whether considering a global transition path is desirable.

Comparison with Chang and Park (2021) In Chang and Park (2021), we also analyzed the GE effects in the presence of the private insurance market, but the GE effects in Chang and Park (2021) made the optimal tax schedule less progressive, and the transition did not play much role. We now discuss why we obtain different implications of the GE effects in two economies—Huggett (the benchmark of Chang and Park (2021)) and Aiyagari.[11] The crucial difference between two economies is the production function. In a Huggett economy, labor is the only input of production, while in an Aiyagari economy, two inputs of production—capital and labor—are complements of each other ($F_{KL} > 0$). Thus, in a Huggett economy, the response of labor supply to tax reforms does not have a direct impact on the interest rate. Despite the sluggish shift of the asset supply curve, with no response of asset demand, the sign of dr_t is the same over the transition and in the steady state—interest rate always increases in response to a progressive tax reform. On the contrary, in an Aiyagari economy with two factor inputs, the sign of dr_t can change its sign over the transition because immediate and strong responses of labor shifts asset demand curve first. Thus the transition path can be critical for the tax reform analysis in an economy with production capital.

[11] Chang and Park (2021) also analyze the Aiyagari economy as an extension example, but that paper does not pay attention to the conditions under which the implication of the GE effects are determined.

4 GE Effects in Optimal Tax Formula

Optimal Tax Formula We say that a tax schedule $T(z)$ is optimal, if there is no welfare-improving perturbation within the class of tax system, the given tax schedule $T(z)$ is optimal. By imposing $dW = 0$, the optimal tax formula at z^* is given by

$$\begin{aligned}
\frac{T'(z^*)}{1-T'(z^*)} &= \frac{1}{\epsilon^l_{1-T'}(z^*)} \frac{1-F_z(z^*)}{z^* f_z(z^*)} \times [A(z^*) + B(z^*) + C(z^*) + D(z^*)] \qquad (10)\\
\textit{where} \quad & A(z^*) = \int \int_{z^*}^{\infty} \left(1 - \frac{u'(a,z)}{\lambda}\right) \frac{\phi(a,z)}{1-F_z(z^*)} dz da \\
& B(z^*) = -\frac{1-\beta}{\lambda} \sum_{t=0}^{\infty} \beta^t \int \left[u'(a,z) - \beta(1+r)E[u'(a'(a,z),z')|z]\right] dh^A_{t+1}(a, y(z)) d\Phi(a,z) \\
& C(z^*) = (1-\beta) \sum_{t=0}^{\infty} \beta^t dr_t K \int \frac{u'(a,z)}{\lambda} \left[\frac{a}{K} - \frac{xl(x)(1-T'(z))}{L}\right] d\Phi(a,z) \\
& D(z^*) = (1-\beta) \sum_{t=0}^{\infty} \beta^t dw_t \int (1+\epsilon^l_w(z)) \frac{z}{w} T'(z) f_z(z) dz.
\end{aligned}$$

The first term $A(z^*)$ is the standard term without private insurance as in Saez (2001). The additional terms $B(z^*), C(z^*)$, and $D(z^*)$ are exactly the counterpart of the terms $R2$, $R3$, and $R4$ in the welfare incidence at the status quo (6)—the borrowing constraint effects, pecuniary externalities, and fiscal externalities.[12]

We once more highlight that in the benchmark, the optimal tax formula (10) does not account for the global transition from the current US to the optimal steady state.[13] This can be seen from the fact that welfare incidences are evaluated using the optimal steady state distribution $\mathbf{\Phi}$. All the time varying terms in $B(z^*), C(z^*)$, and $D(z^*)$ reflect the local transition associated with the local tax reform at the optimal tax.

Failure of Diamond-Mirrlees If the optimal tax formulas are identical under the fixed prices and endogenous prices, one can ignore the substitution between production inputs

[12]The term $A(z^*)$ shows the amplification/mitigation of the Saez effects depending on the value of redistribution which is determined by the asset inequality as well as the income inequality. The second term $B(z^*)$ lowers tax rate in general, but the quantitative importance of this channel will be very small. To see the role of each term for the optimal tax schedule, see Chang and Park (2021).

[13]We emphasize that our optimal tax schedule is not the result of maximizing the steady state welfare. We do consider the transition of welfare incidence for every small tax reform.

when deriving optimal tax (Diamond and Mirrlees (1971)).[14] This Tax-Formula result requires the ability to tax trades of different goods at different rates, the government must have enough instruments to control every relative price. As Scheuer and Werning (2017) clarify, different levels of income can be interpreted as different goods in the Diamond-Mirrlees setup. Since our labor income tax schedule is nonlinear, different levels of labor income are taxed at different rates, but with some restriction on capital income tax, the government does not have enough instruments to affect every relative price in this Aiyagari economy. For example, with linear capital income tax rate, the government imposes the same tax rate on the agents with different asset position (different goods).[15] Given that direct redistribution based on asset income is limited, the labor income tax system exploits the GE effects and indirectly achieve redistribution across distribution of asset and earnings.

GE effects on the optimal tax formula: price effects and distribution effects How does the endogenous prices change the optimal tax schedule? In the tax incidence analysis, since the initial distribution before the tax reform is given (although we can consider any distribution), we could focus on the price incidence to analyze how the general equilibrium forces affect the tax reform incidences. On the other hand, evaluating the role of GE effects on the optimal tax schedule is more involved, as the optimal distribution endogenously responds to the price change. That is, the optimal distribution with GE effects is different from the optimal distribution without GE effects. In the optimal tax formula, not only the terms $C(z^*) + D(z^*)$ that are directly influenced by the GE effects but also other terms $A(z^*) + B(z^*)$ also depends on the GE effects through the endogenous optimal distribution.

Thus, to evaluate the role of the GE forces on the optimal tax schedule, we need to compare the optimal allocation to the alternative optimal allocation that would be obtained if the endogeneity of prices were neglected. This is done by comparing the optimal tax formula (10) with tax formula under the self-confirming policy equilibrium (SCPE).[16] A SCPE tax system is the one that would emerge if a government assumes that the prices are independent of the tax system.[17] See Appendix C for the formal definition and more detail. The natural

[14]Saez (2004) call this result the Tax-Formula result.

[15]In Appendix E, we show that we need a history-dependent tax to obtain the Tax-Formula result.

[16]The notion of SCPE was developed (in a somewhat different context) by Rothschild and Scheuer (2013).

[17]The SCPE formula has only two terms—the standard equity-efficiency term and the term associated

decomposition of the GE effects on the optimal tax formula is obtained by:

$$\frac{T'(z^*)}{1-T'(z^*)} - \frac{T'_{SCPE}(z^*)}{1-T'_{SCPE}(z^*)} = \frac{1}{\epsilon^l_{1-T'}(z^*)} \frac{1-F_z(z^*)}{z^* f_z(z^*)} \times \left[\Omega_{price}(z^*) + \Omega_{dist}(z^*)\right], \tag{11}$$

where

$$\Omega_{price}(z^*) = \overline{dr_t}(z^*)K \times (\Delta_p - \Delta_f) = C(z^*) + D(z^*)$$

$$\Omega_{dist}(z^*) = \left[\begin{array}{l} \iint_{z^*}^{\infty}\left(1-\frac{u'(a,z)}{\lambda}\right)\frac{\phi(a,z)}{1-F_z(z^*)}dzda - \iint_{z^*}^{\infty}\left(1-\frac{u'(a,z)}{\lambda_{SCPE}}\right)\frac{\phi_{SCPE}(a,z)}{1-F_z(z^*)}dzda \\ -\left\{\frac{1-\beta}{\lambda}\sum_{t=0}^{\infty}\beta^t\int[u'(a,z)-\beta(1+r)E[u'(a',z')|z]]dh^A_{t+1}(a,y(z))d\Phi \right. \\ \quad \left. -\frac{1}{\lambda_{SCPE}}\int[u'(a,z)-\beta(1+r)E[u'(a',z')|z]]dh^A_{SCPE}(a,y(z))d\Phi_{SCPE}\right\} \end{array}\right].$$

The gap between the optimal tax formulas with and without GE effects are decomposed into two terms. The first term Ω_{price} shows the direct price effects. The second term Ω_{dist} captures the distribution effects—(i) the standard term $A(z^*)$ is modified as the inequality is changed, and (ii) the effects of the borrowing constraint are also changed as the household who are released from the constraint will be changed. In Section 5 and Appendix C, we show that although the indirect channel through the change in distribution exists, it is strongly dominated by the direct price effect, and thus we mostly focus on the price effects.

Although the price effect in the optimal tax formula ($C(z^*)+D(z^*) = \overline{dr_t}(z^*)\cdot K\times(\Delta_p-\Delta_f)$) has the same expression as in the tax incidence formula, both the response of prices and the externalities per unit price change are now evaluated under the optimal tax. As long as the negative sign of $\Delta_p - \Delta_f$ is maintained—which is quantitatively proved in Section 5, it is still the case that a tax reform which decreases interest rate is welfare improving. Thus, the key question is whether $\overline{dr_t}(z^*)K$ changes its sign under the optimal tax, and this again should be answered quantitatively. In Section 5, we show that the sign of $\overline{dr_t}(z^*)K$ could be opposite under the optimal and the US tax schedules, as the relative response of labor and savings crucially depends on the tax schedule at which the reform is undertaken.

5 What determines the Direction and Size the GE effects?

In this section, we identify the key factors for the direction and size of the GE effects, and quantitatively investigate the conditions under which accounting for the GE effects favors a

with the borrowing constraint effects.

less progressive/redistributive tax reform.

Calibration For the quantitative analysis throughout this section, we use CRRA utility function for consumption and constant Frisch elasticity disutility function for labor: $u(c - v(l)) = \frac{1}{1-\sigma}\left(c - \frac{l^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}\right)^{1-\sigma}$. For the benchmark analysis, we set $\sigma = 1.5$ and $\gamma = 0.5$. Discount factor β and the borrowing limit $\underline{a}$ are calibrated to march the equilibrium interest rate $r = 0.015$ and 13% of households with negative asset holding.

We assume that individual productivity x can take values from a finite set of N grid points and follows a Markov process that has an invariant distribution. The transition probability matrix is approximated to match AR(1) process for log productivity x: $\ln x' = (1-\rho)\mu + \rho \cdot \ln x + \sigma_\epsilon \cdot \epsilon'$, where ϵ is distributed normally with mean zero and variance one. We set $\rho = 0.92$ and $(\mu, \sigma_x) = (2.757, 0.5611)$. We then modify the transition matrix so that the hazard rate of the stationary distribution is $\frac{xf(x)}{1-F(x)} = 1.6$ for the top 5% of productivities.[18]

We use Cobb-Douglas production function $Y = K^\alpha L^{1-\alpha}$ for the benchmark analysis with $\alpha = 0.33$ and the depreciation rate of capital δ is set to 0.1. The government expenditure is calibrated to match 18.9% of labor income in an economy without capital income.[19]

5.1 Shape of the Initial Tax Schedule

We first show that more redistribution of the initial tax with higher level and progressivity implies that the GE effects favor a more redistributive reform. With this knowledge, we understand very different tax incidences under the current US and optimal tax schedules.

5.1.1 Roles of the Level and Progressivity for the GE effects

To see the role of the level and progressivity of tax schedule for the implication of the GE effects, we consider a well-known parametric functional form of tax proposed by Heathcote, Storesletten, and Violante (2017) (HSV tax function hereafter) as the initial tax schedule:

$$T(z) = z - \lambda z^{1-\tau},$$

[18]This can be done by increasing persistence at the top. See Chang and Park (2021) for the details.
[19]With capital income tax, we calibrate the government expenditures to match 18.9% of the output.

where λ controls the level of tax rates and τ determines the progressivity of the tax schedule. We investigate how the welfare implication of the elementary tax reform incidence changes as the shape of the initial tax schedule is changed by varying λ and τ.Since the simple HSV tax function has some limitations in matching the actual tax schedule especially at the bottom income—$T(z)$ is decreasing in z at the bottom, we adjust the HSV function to guarantee nonnegative marginal tax rates at all income levels.[20] For computation, we approximate fully nonlinear $T(z)$ by a piecewise-linear tax function with N grid points. In the benchmark analysis, we use $N = 10$, and in Appendix D, we carry out sensitivity analysis with $N = 20$.

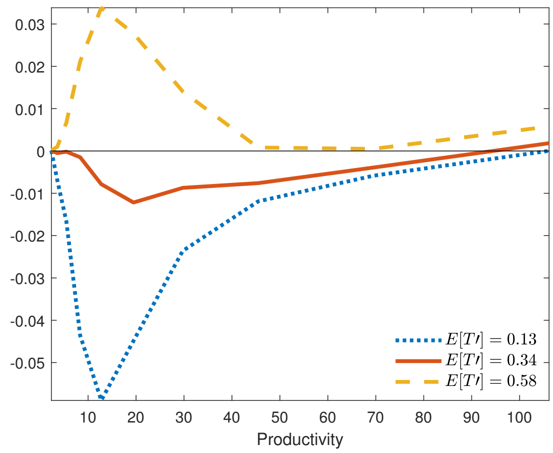
Role of the level of tax rates We investigate how the GE effect depends on the level of initial tax rates by carrying out tax incidence for various levels of λ of the HSV tax function. Lower λ leads to higher average tax rate associated with higher levels of public redistribution before the tax reform. Figure 1 plots the welfare incidence of elementary tax reforms at each of the N income levels through the GE effects ($R3 + R4$ in (6)) for various levels of λ in the initial tax schedule. We highlight the two key observations.[21] First, the GE effect changes its direction as the level of the initial tax rates increases. The GE effects of a redistributive tax reform—increasing tax rate at any income z^* to provide more transfers—has negative welfare incidence with low amount of public insurance initially, while it has positive welfare incidence with higher public insurance before the tax reform. Thus, accounting for the GE effects make the government favor *less redistributive* reform when initial tax rates are low, while it favors *more redistributive* reforms when tax rates are high. Second, the role of the initial tax schedule in the GE effects are more pronounced for the tax reforms at the lower-middle and top income levels. We explain these observations one by one.

First, preferences for more redistributive reform under a redistributive initial tax schedule is because with more existing public insurance, the savings response to additional insurance gets weaker, and thus a redistributive tax reform leads to decrease in interest rate. Figure 2 and 3 confirms that the directional change of the GE effects—from favoring less redistributive

[20] That is, we assume that $T'(z_i) = \max\{0, 1-(1-\tau)\lambda z_i^{-\tau}\}$ for all income levels z_i. Then, we approximate the tax schedule with piecewise-linear over the income grid.

[21] From now on, when we plot the welfare incidence or price incidence over productivity, we plot the incidences normalized by the tax reform size. For example, Figure 1 plots $\frac{R3(z^*)+R4(z^*)}{\lambda \cdot w \cdot \delta\tau \cdot dz^*}$.

Figure 1: dW through the GE effects (normalized $\frac{R3+R4}{\lambda}$)



to favoring more redistributive tax reforms—is caused by the change in sign of the price response ($\overline{dr_t}$). Recall that the welfare incidence through the GE effects is summarized by $\overline{dr_t}K \times (\Delta_p - \Delta_f)$. Given $\Delta_p - \Delta_f < 0$ for all the initial tax schedules we consider (Figure 3), decreasing interest rate has positive welfare effects.[22] With initially low levels of redistribution, the response of savings (decreased savings) to a more redistributive reform dominates the response of labor (decrease in labor supply), and thus the average incidence on the interest rate is positive, which has negative welfare effects. On the contrary, with initially high levels of redistribution, negative responses of labor supply to a redistributive reform dominate, and thus a decrease in interest rate has positive welfare incidence.

Second, the pronounced effects of the initial tax especially for the reforms at the lower-middle and top income levels are because these tax reforms are the most relevant reforms for the strength of the savings response and labor response respectively. Note that the response savings is the most sensitive to the tax reform of generating the most significant change in the amount of public redistribution. The amount of additional insurance depends on at which income level marginal tax rate is raised. On one hand, more progressive reform is likely to provide more redistribution, but at the same time, increasing marginal tax rate at higher income has dampening forces as the extra tax payment from the reform decreases

[22]Figure 3 also shows that $\Delta_p - \Delta_f$ is decreasing in average tax rate, but this is a relatively minor effect compared to the change in the price incidence. $-\Delta_f$ is decreasing in average tax rate, because with higher tax rate, decrease in wage rate will cause even more reduction in the government revenue.

Figure 2: (normalized) $\overline{dr_t}K$

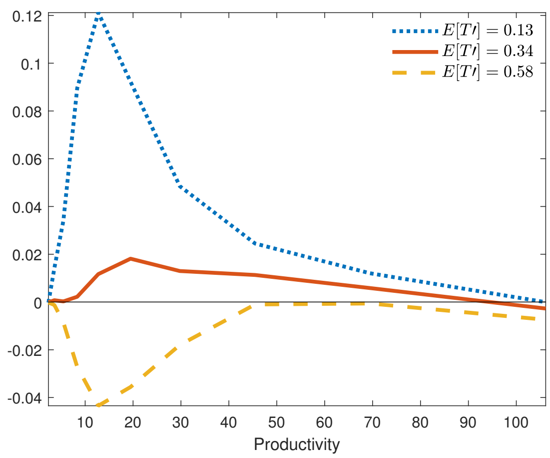
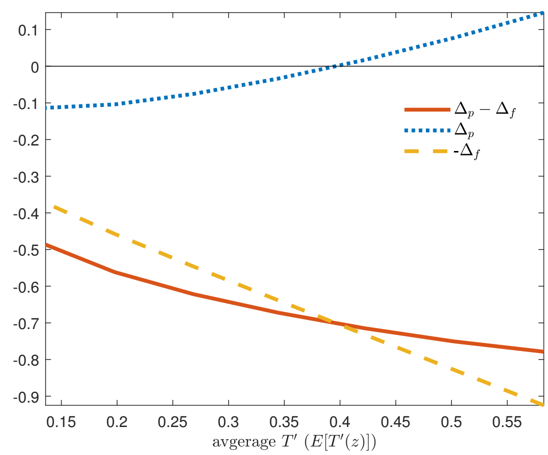


Figure 3: Externality per unit: $\Delta_p - \Delta_f$



with less fraction of people above that income level. With these two forces, the amount of redistribution increases the most when the marginal tax rate is increased at the lower-middle income. Thus, the mitigation of savings response caused by more existing redistribution is most pronounced when an elementary tax reform is performed at the lower-middle income. Next, the response of the labor supply is larger when the initial tax rate is higher. With the progressive HSV tax schedule ($\tau > 0$), the initial tax rate is the highest at the top income. Thus, the effect of higher initial tax rate which amplifies the response of labor is most noticeable when the marginal tax rate is increased at the top.

Role of the progressivity We now carry out the sensitivity analysis of the tax incidences by increasing progressivity τ of the initial tax. With higher progressivity in the initial tax, there will be more provision of public insurance before the tax reform. Thus, with the same analogy about the role of the level of tax rates, the GE effects can favor a more redistributive tax reform when the initial progressivity is high. Figure 4 shows that the directional change in welfare incidence through the GE effects is centered at the top income where the tax rate before the reform is very high with a highly progressive tax schedule. As we discussed above, the response of labor supply gets larger with higher initial tax rate, whose effect is again enlarged at the top. Figure 5 confirms that this stronger response of labor results in decrease in interest rate for the elementary reform at the high income. Figure 6 also verifies that the effect of progressivity on $\Delta_p - \Delta_f$ is relatively minor, and $\Delta_p - \Delta_f$ is always negative for all

Figure 4: dW through the GE effects (normalized $\frac{R3+R4}{\lambda}$)

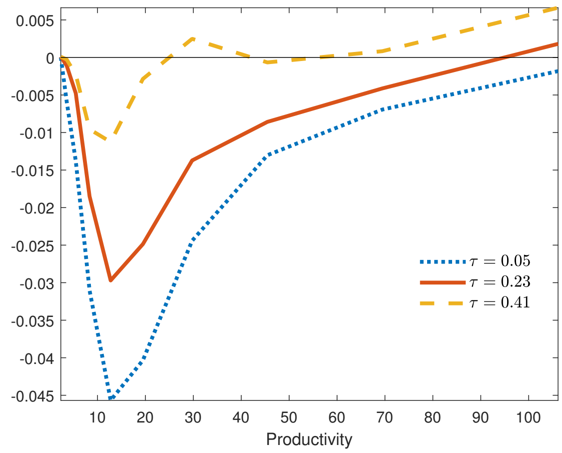


Figure 5: (normalized) $\overline{dr_t}K$

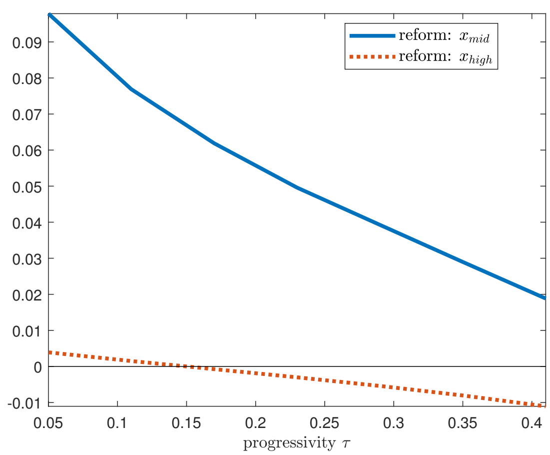
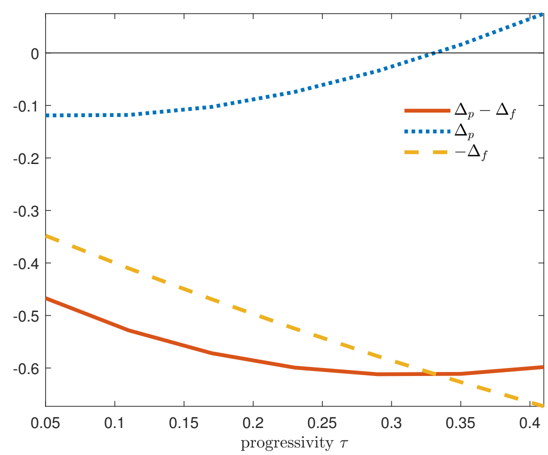


Figure 6: Externality per unit: $\Delta_p - \Delta_f$



ranges of τ we consider.

5.1.2 Tax Incidence under the US and Optimal Tax

With the knowledge we gained about how the GE effects are influenced by the initial level and progressivity, we can now understand the tax reform incidence under the US and optimal tax schedules. As Figure 7 shows, the marginal tax rate schedules of the current US and optimal tax are very different in two aspects: (i) optimal schedule is much more redistributive than the current US schedule with higher tax rates and higher transfer, and (ii) the local progressivity

Figure 7: Marginal Tax Rates

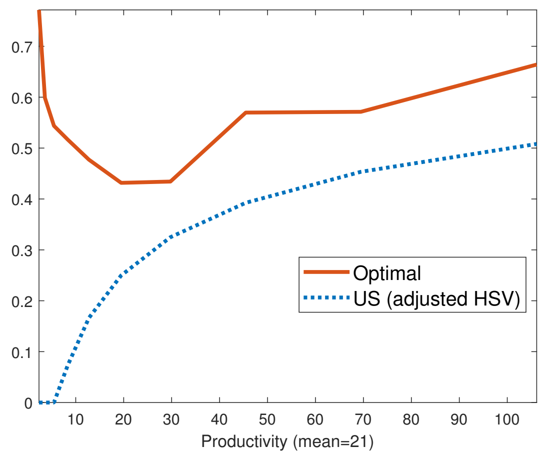
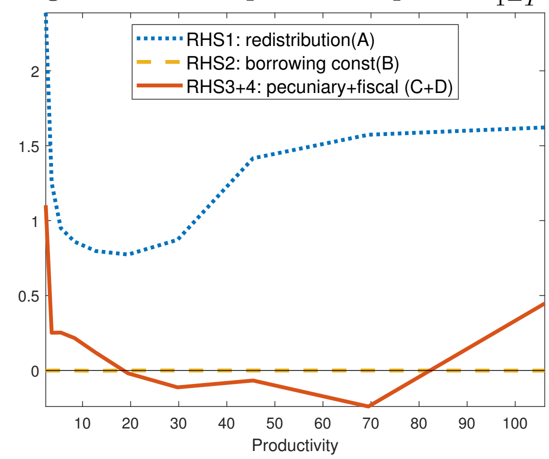


Figure 8: Decomposition: optimal $\frac{T'}{1-T'}$



of optimal schedule varies over income while the US tax shows stable progressivity.[23] The differences are mostly due to the standard equity-efficiency trade off captured by $A(z^*)$ in optimal tax formula (10) as we can see from Figure 8.

The difference between these two tax schedules driven by the standard equity-efficiency trade-off leads to very different welfare implications of the GE effects. Figure 9 plots the welfare incidence through the GE effects ($R3 + R4$ in (6)) under the US and optimal tax schedule, which shows that the GE effects can have opposite implications under the two different tax schedules. Under the current US tax schedule (with modest amount of redistribution), interest rate increases on average in response to a redistributive tax reform as the response of savings dominates the response of labor supply, and thus the GE effects make the government prefer a less redistributive tax reform. On the other hand, under the optimal tax schedule, the GE effects of *more redistributive* tax reforms have positive incidence for the tax reforms at the low and high income.

Since the tax reform at the lower-middle income increases the amount of redistribution the most (as we discussed above), the mitigated response of savings under large initial redistribution is the most pronounced there. Thus, around the lower-middle income, the incidence on the average interest rate shows different signs under the US and optimal tax

[23]By the definition of the HSV tax function, the progressivity of the HSV tax schedule is constant across income. Heathcote, Storesletten, and Violante (2017) shows that this tax function approximates the actual US tax schedule quite well.

Figure 9: (normalized) dW—GE effects

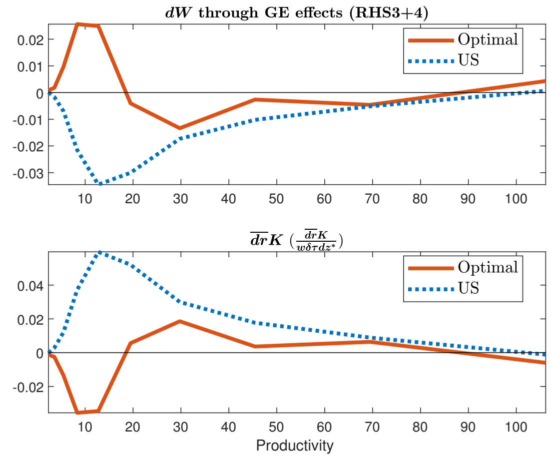
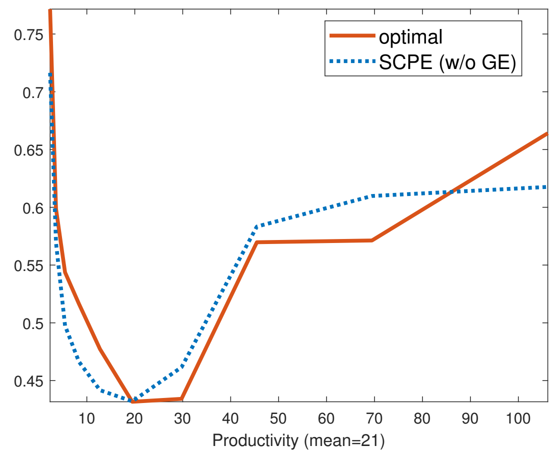


Figure 10: Optimal Marginal Tax Rates



schedules. In addition, higher tax rates at the top under the optimal tax schedule lead to the stronger labor responses, which also leads to negative incidence on the interest rates, thus the GE effect makes the government favor increasing tax rate at the top income.

Table 2 shows that although the welfare effects per unit interest rate change ($\Delta_p - \Delta_f$) also depends on the shape of initial tax schedules, the negative sign of $\Delta_p - \Delta_f$ does not change across the tax schedule.[24] Thus, the directional change in the implication of the GE effects are driven by the change in direction of the price incidences.

Table 2: $\Delta_p - \Delta_f$: US vs Opt

	$\Delta_p - \Delta_f$	Δ_p	$-\Delta_f$	L-Gini	K-Gini	$corr(\frac{a}{K}, \frac{xl}{L})$
US	-0.58	-0.10	-0.48	0.51	0.73	0.55
optimal	-0.72	0.03	-0.75	0.52	0.76	0.48

Figure 10 plots the optimal tax schedules with and without GE effects, and the gap between these two arises due to not only the direct price effects but also the indirect distribution effects as we discussed in Section 4.[25] In Appendix C, however, we confirm that the GE effect on

[24]The sign of Δ_p under the optimal tax is positive, which seems against the negative redistribution of increase in interest rate across wealth dispersion. Note that the pecuniary externalities can be decomposed into $\Delta_p = \Delta_{p1} + \Delta_{p2}$, where $\Delta_{p1} = \int \frac{u'(a,x)}{\lambda}\left[\frac{a}{K} - \frac{xl(x)}{L}\right] d\Phi$ and $\Delta_{p2} = \int \frac{u'(a,x)}{\lambda}\frac{T'(z(x))}{L} d\Phi$. Standard pecuniary externality Δ_{p1} is negative at every tax schedule we consider, but it is modified by Δ_{p2} in the presence of labor income tax. Increase in interest rate (decrease in wage rate) decreases the tax payment, implying $\Delta_{p2} > 0$. This counterbalancing force can be very strong with high tax rates under the optimal tax.

[25]Recall that the SCPE tax policy is the optimal policy of a government who assumes that the prices are independent of the tax system.

the optimal tax via distribution effects is quantitatively minor, which validates our focus on the major price effects throughout the paper.

5.2 Role of Capital Income Tax

So far, we have abstracted from the capital income tax, but capital income tax could be an important factor for the GE effects. With the price externalities, production-efficiency theorem of Diamond and Mirrlees (1971) does not apply, and it is generally optimal to tax capital income to affect the relative prices. But even with the capital income tax, the mechanism of the price externalities associated with the labor income tax reform does not disappear with realistic restrictions on the capital income tax system, since a very complicated history-dependent tax-transfer is required to fully internalize the externalities and to recover the Tax-Formula result (Appendix E).[26] We now show that introducing linear capital income tax tends to amplify the GE effects of the labor income tax reform without changing its direction.

GE Effects in the Presence of Linear Capital Income Tax We introduce linear capital income tax—with tax rate τ_k—and analyze how the GE effects of the labor income tax reform are affected by the (existing) capital income tax. With capital income tax, the incidence of the labor income tax reform has additional fiscal externalities through the change in revenue from the capital income tax: $d(\tau_k rK) = \tau_k \cdot (dr_t \cdot K + r \cdot dK_t)$. Then the welfare incidence through the fiscal externalities, $R4$ in the formula (7), is modified to include this additional revenue effect:

$$\frac{R4}{\lambda} = (1-\beta)\sum_t \beta^t dr_t \cdot K\left[-\Delta_{fL} + \Delta_{fK,t}\right], \quad \text{where} \quad \Delta_{fK,t} = \tau_k\left[1 + \frac{r \cdot dK_t}{dr_t \cdot K}\right],$$

and thus similar to the summary statistics representation (7), the welfare incidence through

[26] Tax-Formula result requires different tax rate on each trade of different goods. Since each level of asset income and each level of asset purchase for tomorrow in this economy can be interpreted as different commodities in Diamond-Mirrlees setup, we need tax instruments to control every relative price of these commodities to apply the Tax-Formula results.

the GE effects is expressed as

$$\frac{R3+R4}{\lambda} = \overline{dr_t}K \times (\Delta_p - \Delta_{fL} + \overline{\Delta_{fK,t}}), \quad \text{where} \quad \overline{\Delta_{fK,t}} = \frac{(1-\beta)\sum_t \beta^t dr_t \cdot \Delta_{fK,t}}{(1-\beta)\sum_t \beta^t dr_t}.$$

The only change in the formula is that the externality effects for the unit price change now include additional (average) fiscal externalities through the capital income tax $\overline{\Delta_{fK,t}}$.

Whether considering the GE effects makes the government prefer more/less progressive tax reform is still determined by the direction and size of $\overline{dr_t}K \times (\Delta_p - \Delta_{fL} + \overline{\Delta_{fK,t}})$. Thus the introduction of capital income tax can change the directional implication of the GE effects only if either $\overline{dr_t}$ or $\Delta_p - \Delta_{fL} + \overline{\Delta_{fK,t}}$ changes its sign.

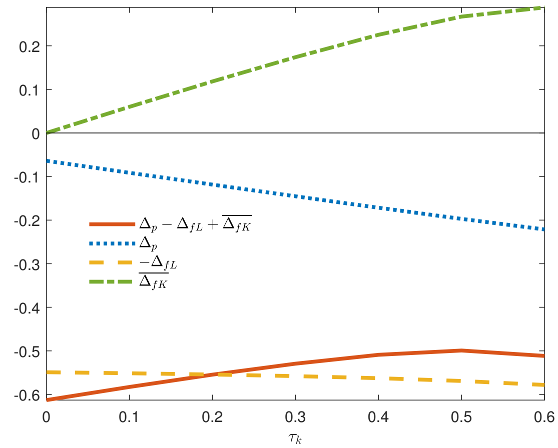
We start with the sign of the second factor (externalities per unit price change), $\Delta_p - \Delta_{fL} + \overline{\Delta_{fK,t}}$. As long as the negative welfare incidence of an increase in interest rate is maintained with $\Delta_p - \Delta_{fL} + \overline{\Delta_{fK,t}} < 0$, all the qualitative results without capital income tax apply to the economy with capital income tax. Note that the additional fiscal externalities through the capital income tax revenue can be decomposed into two terms, $\Delta_{fK,t} = \tau_k \left[1 + \frac{r \cdot dK_t^{GE}}{dr_t \cdot K}\right] + \tau_k \frac{r \cdot dK_t|^r}{dr_t \cdot K}$, where $dK_t|^r$ represents the change in capital supply given prices (shift of asset supply curve) which captures the change in precautionary savings due to the changed public insurance after the tax reform, and dK_t^{GE} is the incidence due to the change in prices (along the supply curve). Since the first term $\tau_k \left[1 + \frac{r \cdot dK_t^{GE}}{dr_t \cdot K}\right]$, the standard positive revenue effect of increasing interest rate, is positive as higher interest rate increases the capital income tax revenue (directly because asset return increases and indirectly because asset supply increases), the sign of $\Delta_p - \Delta_{fL} + \overline{\Delta_{fK,t}}$ can be either positive or negative in principle.[27]

Figure 11 plots the external effects per unit interest rate change by varying capital income tax rates, which is evaluated under the current US labor income tax schedule (approximated by the HSV tax function).[28] It shows that $\Delta_p - \Delta_{fL} + \overline{\Delta_{fK,t}}$ is negative for all capital income tax rates we consider, showing that introducing capital income tax does not change the

[27]On the other hand, the sign of the second term $\tau_k \cdot r \cdot dK_t|^r$ is likely to be negative for a redistributive tax reform as more public insurance decreases precautionary savings.

[28]Note that $\overline{\Delta_{fK,t}}(z^*)$ depends income level z^* at which the tax reform is applied. We plotted the average $\overline{\Delta_{fK,t}} = \int \overline{\Delta_{fK,t}}(z) f_z(z) dz$ in Figure 11.

Figure 11: Externalitly per unit price change



direction of the externality per unit price change.[29] Although the counterbalancing positive fiscal external effect through the capital income tax $\overline{\Delta_{fK,t}}$ gets stronger as τ_k increases, the negative pecuniary externality effect Δ_p also gets stronger with higher τ_k, and thus the original negative external effects $(\Delta_p - \Delta_{fL})$ dominate for all ranges of capital income tax rates we investigate.[30] In sum, introducing capital income tax slightly weakens the negative effects of increasing interest rate but the mitigation is quantitatively small.

What is more interesting is the effect of introducing capital income tax on the first factor of the GE effects, the time-average of change in interest rate $\overline{dr_t}$. It turns out that introduction of the capital income tax results in stronger response of prices $(\overline{dr_t})$, which amplifies the GE effects regardless of its direction. This is because with positive capital income tax rates, elasticity of asset supply with respect to interest rate decreases, and thus asset supply curve gets steeper. Thus a shift of asset supply curve results in larger response in interest rate compared to the response without capital income tax. The bottom graphs in Figure 12 and 13 show that introducing capital income tax $\tau_k = 0.36$ of the US (Trabandt and Uhlig (2011)) results in stronger incidences on the interest rate without changing the original directions

[29]Precisely speaking, the second term $\tau_k \frac{r \cdot dK_t|^r}{dr_t \cdot K}$ is not the price effect. As the term $\overline{\Delta_{fK,t}}$ is not a pure price effect, R3+R4 in the presence of capital income tax is not a pure price effect either. In Appendix E, we show that the qualitative results do not change even if we focus on the pure price effects.

[30]In the presence of taxes, the pecuniary externality term Δ_p also captures the welfare effects of extra tax payment due to the relative price changes as well as usual welfare effects from the redistribution and insurance. When interest rate increases, with higher asset return, capital income tax payment increases, that has additional negative welfare effects.

Figure 12: US: dW—GE effects

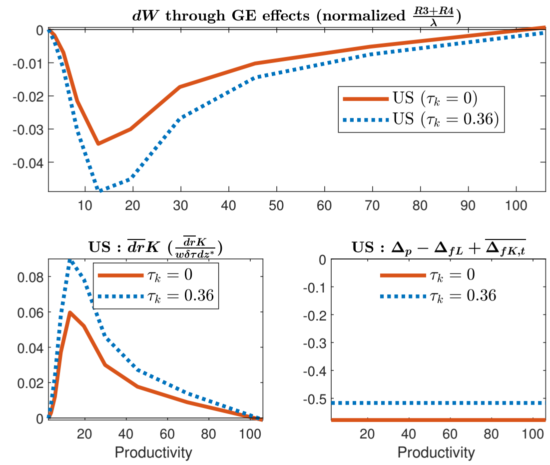
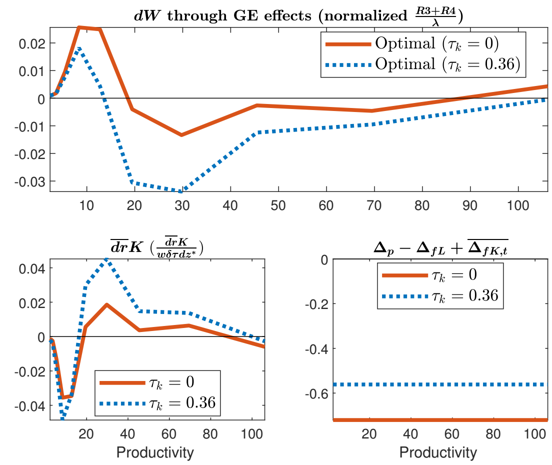


Figure 13: Optimal: dW—GE effects



of the incidences, although it mitigates the negative external effects per unit price change.[31] Top panels of Figure 12 and 13 show that introducing capital income tax rates tends to amplify the GE effects of labor income tax reform (except the reforms at the very bottom and top), as the amplification effects via $\overline{dr_t}K$ dominate the mitigation effects via $\Delta_{fK,t}$.[32] In sum, the qualitative results without capital income tax apply to the economy with capital income tax, and moreover, the strength of the GE effects tends to be stronger with capital income tax.[33]

GE Effects under the Joint Optimal Tax We also evaluate the role of capital income tax under the joint optimal tax system—jointly optimal labor income tax schedule $\{T'_{opt}(z)\}$ and (linear) capital income tax rate τ_k^*. Here, we just summarize the result. See Appendix E.3 for the detailed analysis.

It turns out that the GE effects of the labor income tax reform tend to be even more amplified when evaluated at the joint optimal tax schedule where capital income tax rate is very high with $\tau_k^* = 0.67$ (compared to those under the optimal labor income tax schedule

[31] More precisely, we impose capital income tax rate $\tau_k = 0.36$ for the positive asset income only.

[32] In Appendix E, we show that the response of interest rate gets stronger with higher τ_k making the GE effects even more amplified.

[33] Note that under the optimal tax, the GE effects for the tax reform at the top income does not show positive effects in the presence of capital income tax. This is because different from other reforms whose GE effects are mainly determined by the savings responses, for the tax reform at the top, the response of labor is the major determinant of the GE effects, and with the direct redistribution effect of capital income tax, lower value of redistribution leads to lower optimal tax rate at the top, which mitigates the response of labor supply.

given the US capital income tax rate $\tau_k^{US} = 0.36$.). Both the price incidences and externality per unit price change are amplified under $\tau_k^* = 0.67$ except for the tax reform at the very bottom and top, making the government prefer a less progressive tax schedule even more.

5.3 Other Determinants of the Relative Capital-Labor Response

Since the relative responses of labor and savings are crucial determinants of the GE effects, any factors which have influence on the relative responses could be potential factors that are important for the GE effects. We investigate some factors in this section.

5.3.1 Key Parameters for the Responses of Labor and Savings

The key parameters of the preferences and technology that play important roles in the relative response of labor and savings are risk aversion and elasticity of substitution. The pure effects of these parameters can be studied by analyzing how the price incidence changes by varying the parameter values *given* the tax schedule. On the other hand, when we evaluate the role of these parameters under the optimal tax schedule, varying parameters has influence on the GE effects indirectly through the change in optimal tax schedule.

Role of Risk Aversion Risk aversion is an important factor for savings. One might easily conjecture that higher risk aversion leads to the stronger response of savings to the tax reform, and thus higher risk aversion is likely to lead to more increase in average interest rate in response to a redistributive tax reform, as the long-run saving reduction becomes more dominant. This conjecture applies to the tax reform at the status quo tax schedule, and the first panel of Figure 14 confirms that with higher risk aversion, a redistributive tax reform under the US tax schedule has more negative welfare incidence through the GE effects. For the tax reform at the optimal tax schedule, however, higher risk aversion leads to more positive welfare incidence through the GE effects, as the second panel of Figure 14 shows. This is because with higher risk aversion, higher value of redistribution raises the optimal marginal tax rates through the standard equity-efficiency trade-off channel, which makes the price incidence of a redistributive tax reform have positive welfare effects—weaker savings response and stronger labor response to the reform lead to decrease in interest rate. See Appendix F for more details.

Role of Elasticity of Substitution Since the GE effects crucially depend on the complementarity between the two input factors in production function, elasticity of substitution between capital and labor could be potentially a very important parameter. In the benchmark we have used the Cobb-Douglas production function, whose elasticity of substitution is one. We now consider a constant elasticity of substitution (CES) production function: $Y = A\left(aK^{\frac{\sigma_{es}-1}{\sigma_{es}}} + (1-a)L^{\frac{\sigma_{es}-1}{\sigma_{es}}}\right)^{\frac{\sigma_{es}}{\sigma_{es}-1}}$, where σ_{es} is the elasticity of substitution between capital and labor, A is the efficiency parameter, and a is the distribution parameter. Here, we carry out sensitivity analysis with different levels of σ_{es}.

The incidences on the interest rate can be rewritten as:

$$dr_t \cdot K = -\frac{(1-\alpha)\alpha}{\sigma_{es}} Y \left(\frac{dK_t}{K} - \frac{dL_t}{L}\right), \quad \text{where } 1-\alpha = \frac{wL}{Y}, \tag{12}$$

which shows the role of σ_{es} explicitly. Lower elasticity of substitution makes the response of interest rate to the tax reform amplified given labor income share $1-\alpha$. Figure 15, however, shows that the elasticity of substitution has little impact on the GE effects of tax reform under the current US tax schedule because both the short-run incidence—decrease in interest rate—and the long-run incidence—increase in interest rate—are amplified, and they almost cancel out in the time average incidence. See Appendix F for more details.

On the other hand, the second panel of Figure 15 shows that under the optimal tax schedule, lower elasticity leads to more positive welfare incidence through the GE effects. This is again because with higher tax rates under the optimal tax schedule, labor responses become stronger, and lower elasticity amplifies these stronger responses (especially in the short run), which makes the price incidences have positive welfare effects for the reforms at the low and high income. We show this in more detail in Appendix F.

5.3.2 Role of the Additional Sources of Inequality

It is well known that a standard Aiyagari model cannot match realistic wealth dispersion because precautionary savings do not provide enough motivation for savings to the rich

Figure 14: Role of RRA for the GE effects

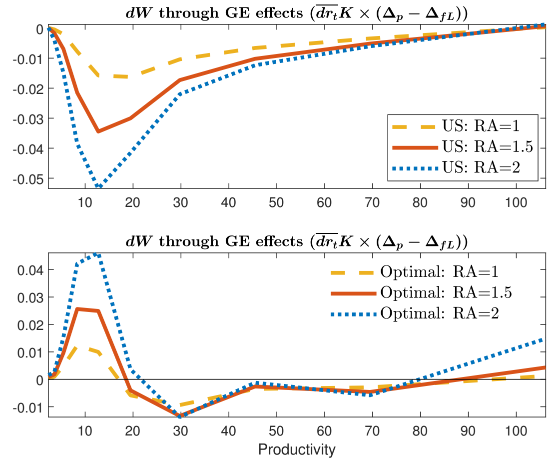
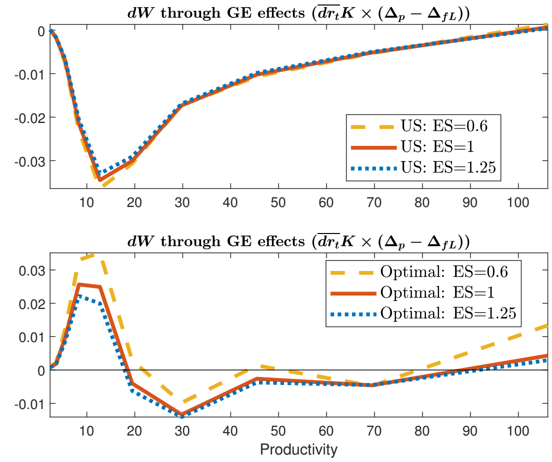


Figure 15: Role of ES for the GE effects



people. To generate a realistic wealth distribution, we need to account for additional sources of inequality.[34] We consider differential saving rates generated by the capitalist spirit as the additional source of generating wealth inequality (Carroll (2000), Straub (2019)) with non-homothetic preferences, where individuals gain utility from the relative wealth position ($\frac{a}{K}$). In Appendix F, we show that the negative welfare incidence through the GE effects are mitigated when capitalist spirit is introduced because with additional motivation for savings, the precautionary motive for savings is less important and the weaker response of savings dampens increase in interest rate. We show this in more detail in Appendix F.

6 Implication of the Transition

Our benchmark analysis so far has only accounted for the local transition associated with a small tax perturbation without considering the global transition from the current to the optimal steady state. In this section, we quantitatively investigate the role of both local and global transitions in the tax incidence and optimal tax schedule, and show that both local and global transitions tend to make the government prefer more progressive tax reform (and make the optimal tax schedule more progressive). We also discuss whether accounting for

[34]The candidate sources of generating wealth dispersion often discussed in the literature are: skewed and persistent distribution of stochastic earnings, differential saving rates across wealth levels, stochastic return to wealth, less progressive capital income tax.

the global transition is always desirable.

We start with investigating the role of the local transition for the implication of the GE effects, then we include the global transition. For the quantitative analysis, we use the same calibration as in Section 5.

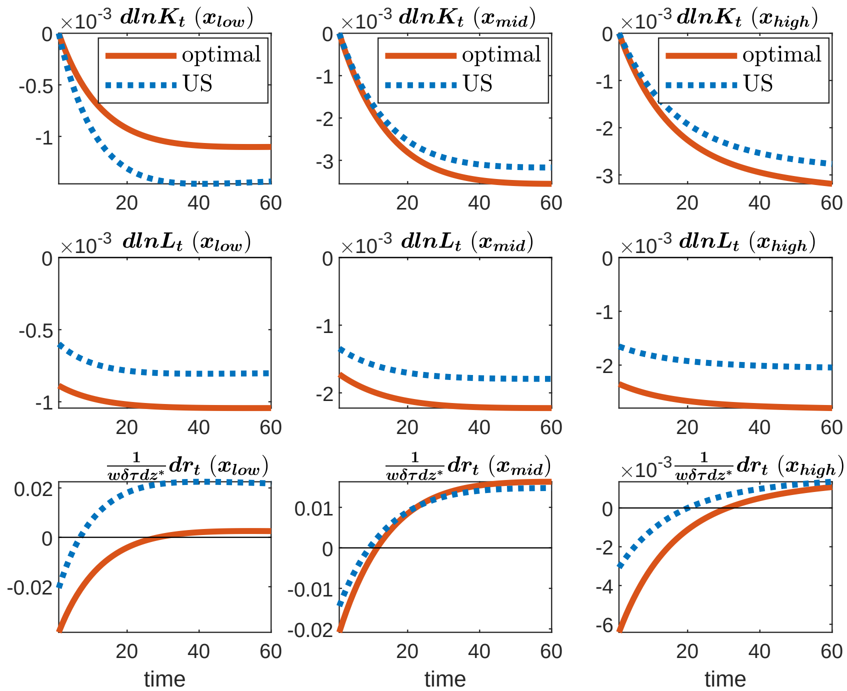
6.1 Local Transition Path

In Section 3 and 4, we briefly discussed the role of the local transition. Transition matters in the interaction with the GE effects because the incidences on prices tend to change their signs over the transition. A redistributive tax reform tends to decrease interest rate in the short run with an immediate decrease in labor supply, but in the long run, the same reform tends to increase interest rate as the sluggish decrease in savings becomes dominant. Figure 16 plots the transitional dynamics of capital, labor and interest rate associated with the tax reforms at low, middle, and high incomes both under the US and optimal tax schedules. They confirm the immediate responses of labor supply, sluggish adjustment of capital, and the sign change of the incidence on the interest rate over the local transition for every tax reform we consider.[35]

Role of the Local Transition in the GE effects If the government takes into account the GE effects but does not account for the transition associated with the local tax reform, the GE effects would make the government favor a too less progressive (or a less redistributive) tax reform. This is because the government would exaggerate the long-run benefit of less progressive tax reform which increases precautionary savings and thus decreases interest rate. Figure 17 and 18 confirm that when the local transition is ignored, the negative welfare effects of a redistributive tax reform through the change in prices are exaggerated both under the

[35] We also compare the incidences under the US and optimal tax schedule. Recall that in Section 5.1, for the tax reforms at the low and high incomes, the GE effects under the optimal tax schedule favor a more redistributive reform while a less redistributive reform is favored under the US tax schedule. We explained that this is because the response of savings is the most mitigated for the reforms around the lower-middle income level, and because the response of labor supply is the most enlarged for the reforms at the top income level. Figure 16 confirms that under the optimal tax, the tax reform at the low income leads to significantly weakened responses of savings, which is a major reason for the negative average incidence on the interest rates. On the other hand, the negative incidence on average interest rate associated with the reform at the high income under the optimal tax is mainly driven by the stronger response of the labor supply.

Figure 16: Transition over time



US and optimal tax schedule, and this pattern is more pronounced at the middle and top. As a result, accounting the local transition makes the GE effects favor more progressive tax reform (of increasing tax rate at the middle and top).

Figure 17: US: dW—GE Effects

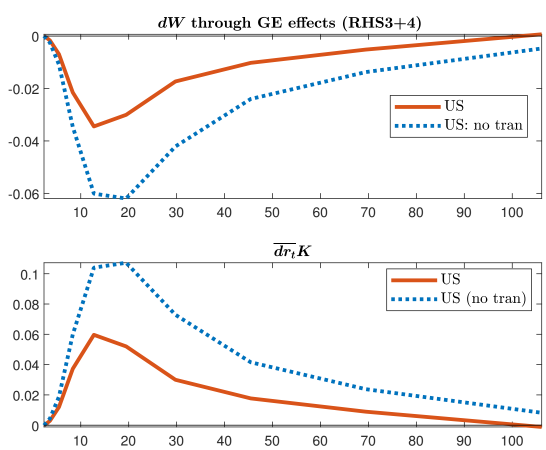
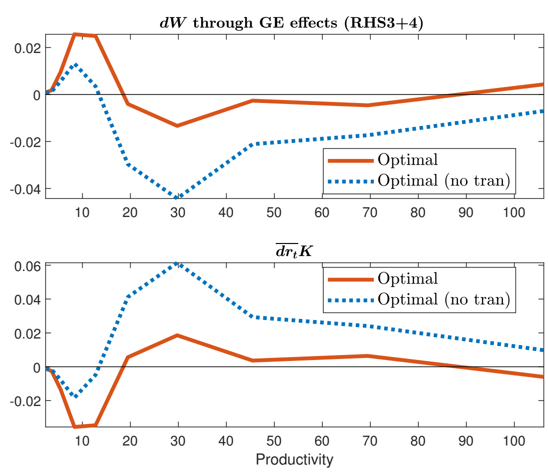


Figure 18: Optimal: dW—GE effects



Role of the Local Transition for the Optimal Tax Schedule Formally, the role of the local transition for the optimal tax schedule can be analyzed by comparing the optimal tax formulas of the government who does and does not account for the transition. Note that we still abstract from the global transition, thus the formulas in this comparison do not account for the global transition. In Appendix C, we formally define SCPE$_{nt}$ tax policy of the government who does not account for the local transition. Transition has its impacts on the optimal tax schedule through the indirect distribution effect as well as through the direct price effect over the transition, similarly as in the role of the GE effects for the optimal tax (Section 4). The gap between the two formulas is decomposed into:

$$\frac{T'(z^*)}{1-T'(z^*)} - \frac{T'_{nt}(z^*)}{1-T'_{nt}(z^*)} = \frac{1}{\epsilon^l_{1-T'}(z^*)}\frac{1-F_z(z^*)}{z^* f_z(z^*)} \times \left[\Omega^{nt}_{price}(z^*) + \Omega^{nt}_{dist}(z^*)\right] \tag{13}$$

where

$$\Omega^{nt}_{price}(z^*) = \left[\overline{dr_t}(z^*)K - dr_{nt}(z^*)K_{nt}\right] \times (\Delta_p - \Delta_f)$$

$$\Omega^{nt}_{dist}(z^*) = \left[\begin{array}{l} \iint_{z^*}^{\infty}\left(1-\frac{u'(a,z)}{\lambda}\right)\frac{\phi(a,z)}{1-F_z(z^*)}dzda - \iint_{z^*}^{\infty}\left(1-\frac{u'(a,z)}{\lambda_{nt}}\right)\frac{\phi_{nt}(a,z)}{1-F_z(z^*)}dzda \\ -\{\frac{1-\beta}{\lambda}\sum_t \beta^t \int [u'(a,z) - \beta(1+r)E[u'(a',z')|z]]dh^A_{t+1}(a,y(z))d\Phi \\ \quad -\frac{1-\beta}{\lambda_{nt}}\sum_t \beta^t \int [u'(a,z) - \beta(1+r)E[u'(a',z')|z]]dh^A_{t+1,nt}(a,y(z))d\Phi_{nt}\} \\ +dr_{nt}(z^*)K_{nt} \times [(\Delta_p - \Delta_f) - (\Delta_{p,nt} - \Delta_{f,nt})] \end{array}\right],$$

where the subscript nt represents "no transition."

Note that SCPE$_{nt}$ tax policy does account for the GE effects in the long run, although it ignores the GE effects during the transition. As a result, different from the overall price effect $\Omega_{price}(z^*)$ in the decomposition of the GE effects in the optimal tax schedule (11), the price effect $\Omega^{nt}_{price}(z^*)$ captures how the consideration of the short-run price effect (ignored by SCPE$_{nt}$) changes the optimal tax schedule. The distribution effect $\Omega^{nt}_{dist}(z^*)$ captures that the steady state distributions under the different optimal tax schedules—with and without transition—lead to different value of redistribution, the borrowing constraint effects, and the externalities effects per unit price change.

Figure 19 plots the decomposition (13), which shows that the price effects significantly dominate the distribution effects. Thus, the lesson from the tax incidence analysis given distribution applies again. If we do not take into account the local transition, the optimal tax schedule becomes too less progressive (than it should be) as we ignore the short-run response of the prices through the labor responses. Figure 20 confirms that accounting for

Figure 19: Decomposition of $\frac{T'}{1-T'} - \frac{T'_{nt}}{1-T'_{nt}}$

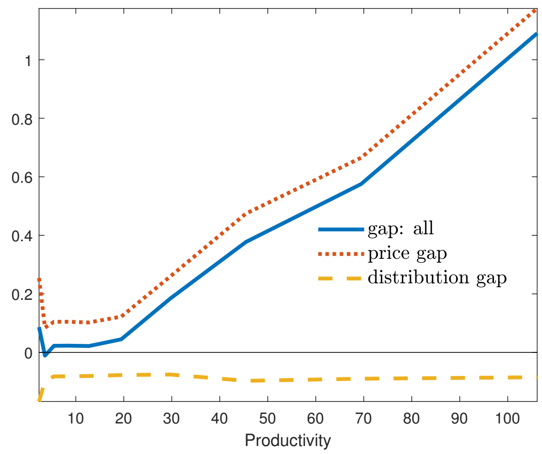
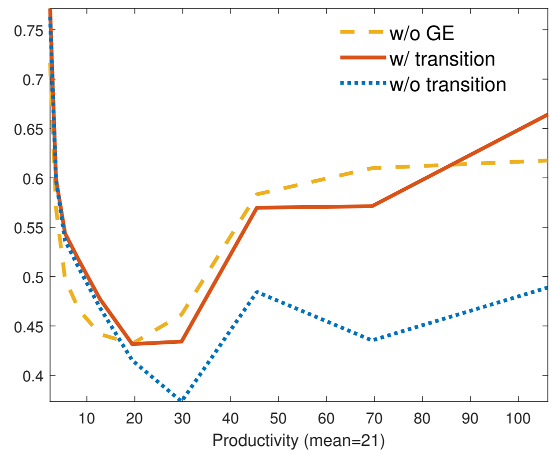


Figure 20: Optimal Marginal Tax Rates



the local transition leads to a more progressive optimal tax schedule (higher tax rates at high income levels), and also shows that the optimal tax schedules with and without transition are significantly different—the tax rate difference between the two schedules can be as much as 15 percentage points. The gap becomes larger especially at the top because the neglected short-run labor response is strongest at the top income.

Role of Discount Factor Patience of the economy is also very important for the welfare implications of the GE effects as the short-run response and the long-run response have opposite implications. Figure 21 shows the welfare incidence through the GE effects under the optimal tax schedule with two different levels of discount factor. The benchmark $\beta = 0.966$ is calibrated to match the interest rate target $r = 0.015$ under the US tax schedule. An alternative calibration sets $\beta = 0.940$ to match the alternative target interest rate $r = 0.04$. Figure 21 shows that with lower β, the role of the local transition which makes the government favor more progressive/redistributive tax reform becomes even more pronounced, as the long-run negative GE effects of a redistributive tax reform becomes less important. Thus, every tax reform of increasing tax rate at every income has indirect redistribution through the price change, generating additional positive welfare effects.

Figure 21: Optimal: dW—GE effects

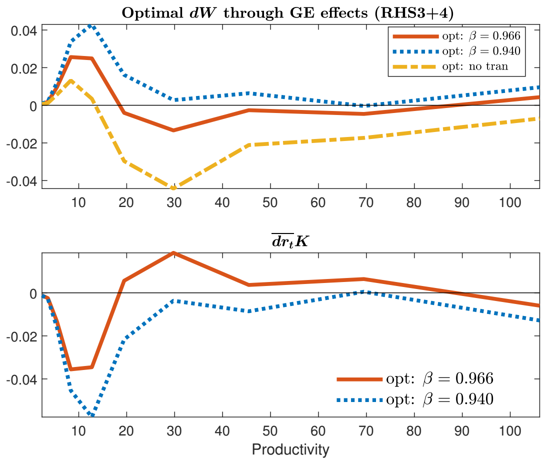
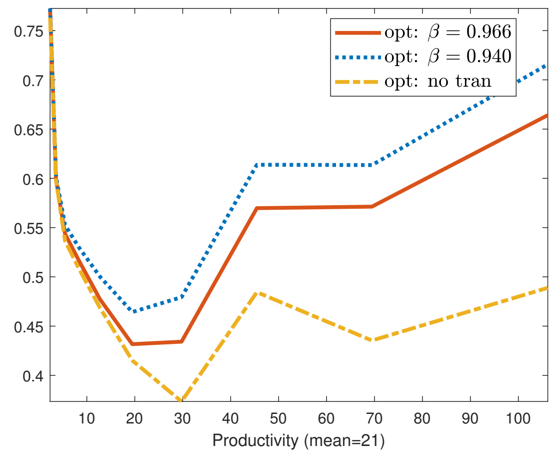


Figure 22: Optimal Marginal Tax Rates



6.2 Global Transition Path

We now consider the role of the global transition from the current US to the optimal steady state. First, we quantitatively investigate the welfare effects of a global tax reform, which is a large reform to the optimal tax schedule from the current US tax schedule. Different from the typical concern that the optimal tax schedule that does not consider the global transition might be costly in the short run (because the optimal tax schedule does not consider the status quo distribution), our analysis will show that a tax reform to the highly redistributive optimal tax schedule result in short-run welfare gain and long-run welfare loss. Then we investigate the optimal tax schedule accounting for the global transition, and show that the global transition makes the optimal tax schedule even more progressive to exploit the short-run welfare gain from the sluggish adjustment of capital.

6.2.1 Welfare Incidence of a Global Tax Reform

A typical concern about the optimal tax without considering the transition from the current status quo is that a global reform to such an optimal tax schedule might generate welfare costs during the transition (especially in the short-run). To see whether such a concern is valid one, we start with the welfare effects of a global tax reform, and investigate whether the reform generates welfare gain or loss during the global transition. Suppose that the economy is initially in the steady state under the current US tax schedule. Consider an unexpected, large tax reform to the optimal tax schedule in period 0, where the optimal tax schedule is obtained by (10) which does not account for the global transition.

The average discounted welfare gain over the global transition Δ of such a tax reform is measured by consumption equivalent variation (CEV). That is, Δ is the constant percentage increase to the status quo consumption that equates the discounted utilitarian welfare under the US tax schedule to that under the reformed tax system:

$$\int u((1+\Delta)c^{SQ}(a,x), l^{SQ}(a,x))d\Phi(a,x) = \int E_0 \left[(1-\beta)\sum_{t=0}^{\infty}\beta^t u(c_t^R, l_t^R)\right] d\Phi(a_0, x_0),$$

where (c^{SQ}, l^{SQ}) denote the steady state consumption and labor under the current US tax, Φ denotes the steady state distribution before the reform, and $\{c_t^R, l_t^R\}$ is the path of allocation after the tax reform.

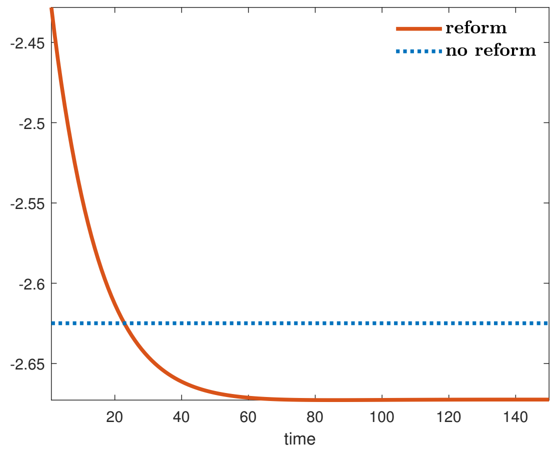
Table 3 shows the average welfare gain obtained from this global reform to the optimal tax system without τ_k, given τ_k^{US}, and with τ_k^{opt}, respectively. Since the welfare gain crucially depends on the discount factor, we compute Δ for both economies with high β (calibrated to match $r = 0.015$ under the current US tax) and low β (calibrated to match $r = 0.04$). Table 3 shows that the average welfare gain is positive for every global tax reform we consider.

Table 3: Average Welfare Gain

β	high β ($r^{US} = 0.015$)			low β ($r^{US} = 0.04$)		
reform to	optimal w/ $\tau_k = 0$	optimal w/ τ_k^{US}	joint optimal	optimal w/ $\tau_k = 0$	optimal w/ τ_k^{US}	joint optimal
Δ	0.019	0.004	0.013	0.028	0.012	0.023

A natural question is whether this welfare gain is obtained at the cost of welfare loss in the short run since the optimal tax schedule is computed without considering the welfare incidence during the global transition. But this is not the case in this economy. Figure 23 plots the average welfare over time after the global tax reform to the optimal tax schedule in the benchmark case (with high β and without τ_k), and it clearly shows that the average welfare gain Δ is positive due to a huge short-run welfare gain. Moreover, the average welfare gain is obtained rather at the cost of long-run welfare loss.

Figure 23: Average Welfare over Time



This is because the reform to the optimal tax schedule immediately provides much larger redistribution with higher tax rates and a larger transfer. As a result, Figure 25 shows that consumption inequality has significantly improved right after the tax reform. On the other hand, Figure 24 shows that since it takes time for the adjustment of capital, aggregate consumption does not drop that much in the short run despite the immediate drop of aggregate labor. Thus, the reform leads to an increase in welfare in the short run. In the long run, however, aggregate capital greatly decreases as the large redistributive tax reform significantly reduces precautionary savings, in turn, welfare gets lower with decrease in aggregate consumption despite the lower inequality in consumption relative to the inequality before the tax reform.

Another observation from Table 3 is the average welfare gain is higher in an economy with low β. This is partly because with low β, the government puts more weight to the short-run welfare gain while it puts less weight to the long-run welfare loss. Moreover, as we investigated in Section 6.1, the optimal tax schedule is more redistributive in an economy when β is low with more weight on the short-run gain from the price change associated with the local tax reform. Thus, with lower β, there will be even higher short-run gain over the global transition from the global tax reform. Table 3 also shows that the welfare gain in the presence of capital income tax is lower than that without capital income tax. This is also due to less redistributive tax schedule in an economy with τ_k (Figure 40), which leads to

Figure 24: Global Transition: Aggregates

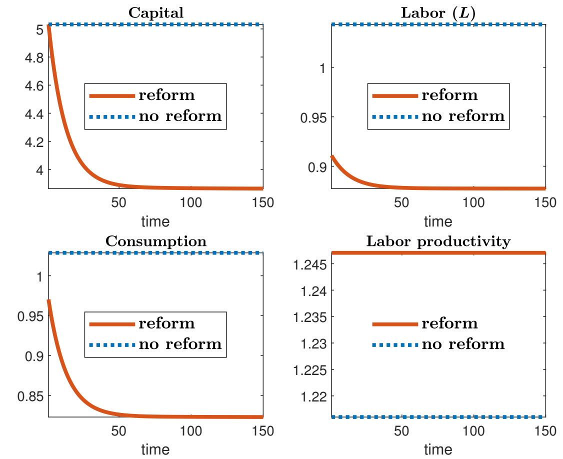


Figure 25: Global Transition: Distribution

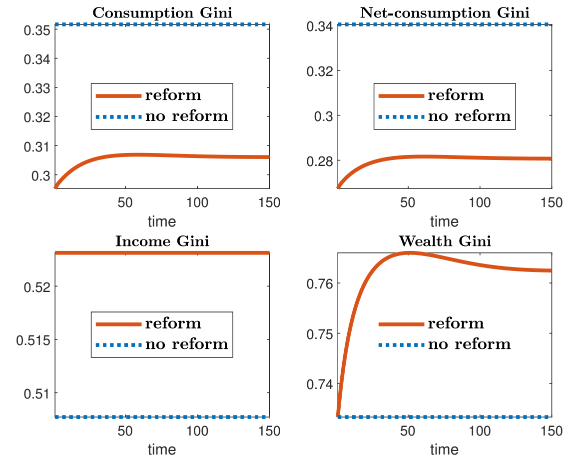


Figure 26: Global Transition to $T\prime^{opt}$: β-effect

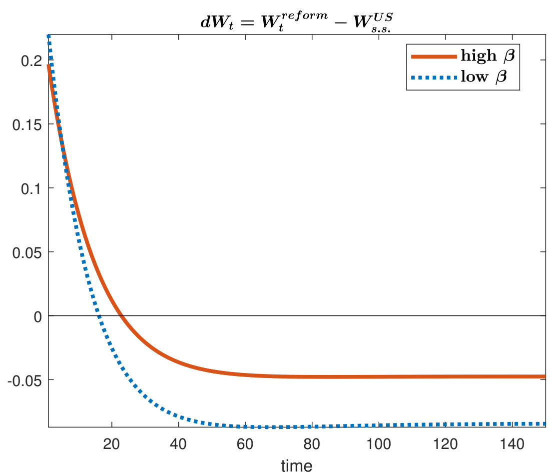
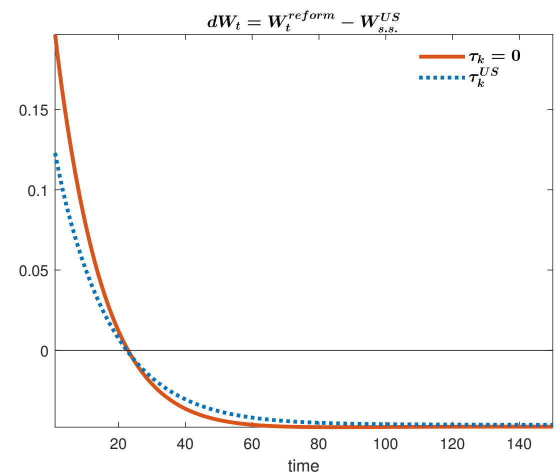


Figure 27: Global Transition to $T\prime^{opt}$: τ_k-effect



smaller short-run welfare gain. With capital income which provides redistribution itself, the value of redistribution is smaller, in addition, the amplified GE effects make the tax schedule less redistributive (as analyzed in Section 5.2).

In Appendix G, we also analyze the global transition path of the tax reform to the joint optimal tax system in detail. We show that higher average welfare gain with the joint optimal tax (Table 3) is obtained because of the higher short-run welfare gain.

6.2.2 Optimal Tax with Global Transition and the GE effects

Previous section shows that an optimal tax without considering the global transition already generates short-run welfare gain at the cost of long-run welfare loss. If we compute an optimal tax schedule accounting for the welfare effects during the global transition, this asymmetric (short-run vs long-run) welfare effects will be even more severe. This can raise the question whether it is always desirable to account for the global transition to obtain the optimal tax schedule. We do not provide an answer to this question, but analyze the effects of accounting for the transition on the optimal tax schedule, whose implication is thought-provoking, and this can open further debate on how to account for the global transition.

We can still use a variational approach to obtain the optimal tax schedule accounting for the global transition from the current status quo. Suppose that the economy is in the steady state under the US tax schedule at time 0. Consider a tax schedule $T(\cdot)$ implemented at time 0, which generates a global transition from the status quo to the steady state associated with $T(\cdot)$. This considered tax schedule $T(\cdot)$ is optimal if there is no small perturbation of this tax schedule that can improve welfare, but different from the analysis we have so far, the tax incidence on welfare from a small perturbation is evaluated considering the global transition from the US tax schedule to the optimal steady state before having a small perturbation.

We derive the tax incidence and optimal tax formulas in Appendix G. Since the economy was not in the steady state even before the tax reform, it is no longer possible to express the formulas with the steady state distribution and constant marginal value of public funds. The tax incidence takes into account the change in distribution due to the global transition path from the current US tax schedule.

Figure 28 plots the optimal tax schedules with and without global transition. The optimal tax schedule accounting for the global transition is slightly more progressive (with lower tax rates at low income levels and higher tax rates at high income levels), and the quantitative decomposition of the optimal tax formula in Figure 29 shows that this is driven by the gap in the GE effects. Roughly speaking, the optimal tax schedule with the global transition becomes more progressive because we can exploit the sluggish adjustment of capital to increase the short-run welfare gain.

Figure 28: Optimal Marginal Tax Rates

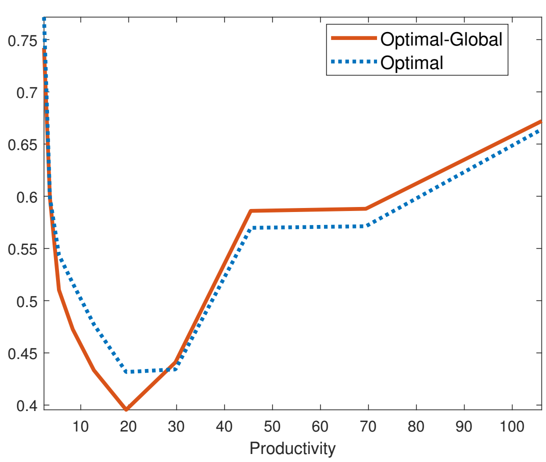
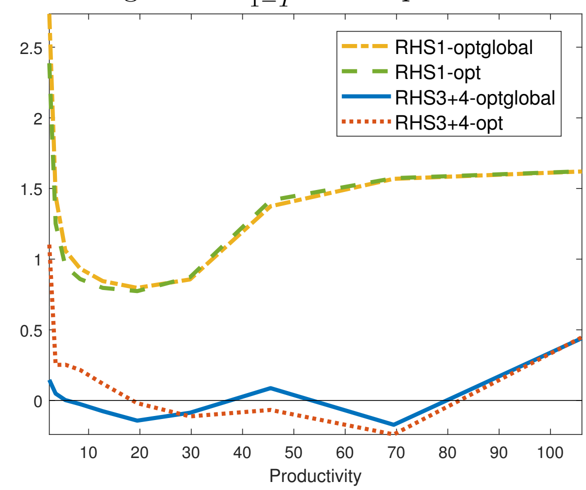


Figure 29: $\frac{T'}{1-T'}$ decomposition

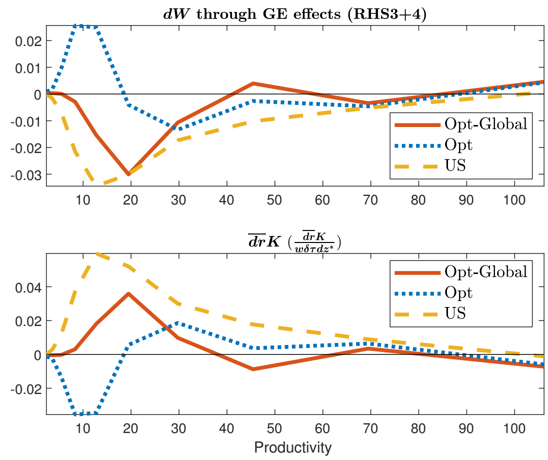


To understand this, Figure 30 compares the incidences of small tax perturbations (elementary tax reforms) on the welfare through the GE effects (normalized by the marginal value of public funds) under the current US steady state, optimal steady state without global transition, and global transition from the current status quo to the optimal steady state.[36] Note that when the global transition is considered, different from the analysis in Section 3, the welfare incidence through the GE effects $(1-\beta)\sum_{t=0}^{\infty}\beta^t dr_t(z^*)\cdot K\cdot(\Delta_{p,t}-\Delta_{f.t})$ cannot be decomposed into the summary statistics ($\overline{dr_t K}$ and $\Delta_p - \Delta_f$) because the externalities per unit price change ($\Delta_{p,t}-\Delta_{f,t}$) are now time varying with the change in distribution over the global transition.

Figure 30 shows that the main reason for the more progressive optimal schedule when considering the global transition is the sluggish adjustment of asset distribution. Recall that a tax reform of increasing at the lower middle income is the reform that can increase the public insurance the most. As the figure shows this reform has negative welfare incidence through the GE effects when the global transition is considered. This is because the savings response to a tax reform depends on the asset distribution which changes sluggishly over the global transition. That is, the provision of public insurance through the tax rate perturbation at the lower middle income results in stronger response of savings relative to the labor

[36]The optimal steady state without global transition refers to the steady state under the optimal tax schedule which is obtained without considering the global transition. The global transition from the current status quo to the optimal state state refers to the transition from the steady state under the current US tax schedule to the steady state under the optimal tax schedule which considers the global transition.

Figure 30: Optimal: dW—GE effects



responses as in the incidence under the US steady state in the initial periods of the global transition, and thus it has negative average welfare effects through GE effects.

Thus, accounting for the global transition makes the optimal tax schedule more progressive, and this is because the government wants to exploit the sluggish adjustment of savings over the global transition. As a result, there will be even more welfare gain in the short run and more welfare cost in the long run. This again opens the question whether exploiting the sluggish adjustment of assets associated with a large tax reform by accounting for the global transition is really desirable or not. We do not attempt to answer this and leave the question open for future research.

7 Conclusion

In this paper, we have investigated whether accounting for the general equilibrium effects and transition makes the government prefer more or less progressive/redistributive tax reform in an Aiyagari economy. Different from the conventional trickle down forces that favor less progressive tax reform, the answer depends on the relative responses of input factors of production to the income tax reform and their transitional dynamics. If the labor response dominates the savings responses, then the GE effect of a redistributive tax reform has positive welfare incidence through decrease in interest rate, and vice versa. We showed that the shape of tax schedule to which the reform is applied and capital income taxes could be important for these relative responses and their implication. Both the local and global transition make the government prefer a more progressive tax schedule when they are considered. But our analysis shows that accounting for the transition exploits the sluggish adjustment of asset distribution, whose desirability requires more discussion. We leave this question open for future studies.

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Appendix (Only for Online Publication)

A Proof of Equations in Section 3

Incidence on Labor Supply In Section 3, we defined the elasticity of labor supply with respect to retention rate and the elasticity with respect to wage rate along the nonlinear budget constraint. For the derivation of the elasticity expression, see the proofs in Sachs, Tsyvinski, and Werquin (2020) and Chang and Park (2021). Given these definitions of the elasticities, the derivation of the incidence on the labor supply given the incidence on the wage rate dw_t is straightforward.

The incidence of a tax reform on the labor supply is obtained from the following first-order condition under the perturbed tax function $T(z) + \mu\tau(z)$:

$$v'(l(x)+\mu dl(x)) = [1-T'((w+\mu dw_t)x(l(x)+\mu dl(x)))-\mu\tau'((w+\mu dw_t)x(l(x)+\mu dl(x)))]x(w+\mu dw_t).$$

A first-order Taylor expansion around the equilibrium implies:

$$\begin{aligned}v'(l(x)) + v''(l(x))\mu dl(x) &= [1 - T'(wxl(x))]wx - T''(wxl(x))[xl(x)\mu dw_t + wx\mu dl(x)]xw \\ &\quad -\mu\tau'(wxl(x))xw + [1 - T'(wxl(x))]x\mu dw_t.\end{aligned}$$

By solving for $dl(x)$, we obtain the expression of $dl(x)$ in the main text.

$$\begin{aligned}dl(x) &= \frac{-\tau'(wxl(x)) + [1 - T'(wxl(x)) - T''(wxl(x))wxl(x)]\frac{dw_t}{w}}{\left[\frac{l(x)v''(l(x))}{v'(l(x))} + \frac{T''(wxl(x))}{1-T'(wxl(x))}xwl(x)\right]\frac{1-T'(wxl(x))}{l(x)}} \\ &= -\frac{e(x)}{1+\rho(wz(x))e(x)}\frac{\tau'(wxl(x))}{1-T'(wxl(x))}l(x) + \frac{e(x)(1-\rho(wz(x)))}{1+\rho(wz(x))e(x)}\frac{dw_t}{w}l(x) \\ &= -\epsilon^l_{1-T'}(x)\frac{\tau'(wxl(x))}{1-T'(wxl(x))}l(x) + \epsilon^l_w(x)\frac{dw_t}{w}l(x).\end{aligned}$$

Incidence on Government Revenue The government revenue from the labor income tax is $R=\int T(z(x))f(x)dx$. Then, the incidence on the government revenue, dR_t, given dw_t is

$$\begin{aligned}
dR_t &= \int \tau(z(x))f(x)dx+\int T'(z(x))\left[wxdl_t(x)+dw_t xl(x)\right]f(x)dx \\
&= \int \tau(z(x))f(x)dx+\int T'(z(x))\left[-\epsilon^l_{1-T'}(x)\frac{\tau'(z(x))}{1-T'(z(x))}z(x)+dw_t\cdot\epsilon^l_w(x)xl(x)\right]f(x)dx \\
&\quad +\int T'(z(x))dw_t\cdot xl(x)f(x)dx \\
&= \int \tau(z(x))f(x)dx-\int T'(z(x))\epsilon^l_{1-T'}(x)\frac{\tau'(z(x))}{1-T'(z(x))}z(x)f(x)dx \\
&\quad +dw_t\int(1+\epsilon^l_w(x))xl(x)f(x)dx \quad \forall t.
\end{aligned} \tag{14}$$

As we discussed in the main text, the elementary tax perturbation $\tau(z)=\frac{\mathbb{1}_{z\geq z^*}}{1-F(z^*)}$ is not differentiable. To apply formula (14) to this non-differentiable perturbation, we use the construction technique discussed in Sachs, Tsyvinski, and Werquin (2020). That is, we can construct a sequence of smooth perturbation functions $\kappa_{z^*,\epsilon}(z)$ such that $\lim_{\epsilon\to 0}\kappa_{z^*,\epsilon}(z)=\delta_{z^*}(z)$, in the sense that for all continuous functions $h(\cdot)$ with a compact support,

$$\lim_{\epsilon\to 0}\int_{\mathbb{R}}\kappa_{z^*,\epsilon}(z)h(z)dz=h(z^*).$$

By changing variables in the integral, it implies:

$$\lim_{\epsilon\to 0}\int_X \kappa_{z^*,\epsilon}(z(x'))\left\{h(z(x'))\frac{dz(x')}{dx}\right\}dx'=h(z^*).$$

Let $\tau_{z^*,\epsilon}(\cdot)$ denote the function such that $\tau'_{z^*,\epsilon}=\frac{\kappa_{z^*,\epsilon}(\cdot)}{1-F_z(z^*)}$, the tax incidence of a tax reform $\tau_{z^*,\epsilon}$ on government revenue given wage rate ($dw_t=0$) is:

$$\int \tau_{z^*,\epsilon}(z(x))f(x)dx+\int T'(z(x))\Big[-\frac{\epsilon^l_{1-T'}(x)}{1-T'(z(x))}\cdot\frac{\kappa_{z^*,\epsilon}(z(x))}{1-F_z(z^*)}z(x)\Big]f(x)dx.$$

This yields dR from the elementary tax reform at z^*:

$$\begin{aligned}
\lim_{\epsilon\to 0} dR_t(\tau_{z^*,\epsilon}) = dR_t = & \int_{x^*}^{\infty} \frac{f(x)}{1-F(x^*)}dx - \frac{T'(z(x^*))}{1-T'(z(x^*))}\epsilon^l_{1-T'}(x^*)\frac{z(x^*)}{z'(x^*)} \cdot \frac{f(x^*)}{1-F(x^*)} \\
& + dw_t \int (1+\epsilon^l_w(x))xl(x)f(x)dx \quad \forall t \\
= & \int_{z^*}^{\infty} \frac{f_z(z)}{1-F_z(z^*)}\,dz - \frac{T'(z^*)}{1-T'(z^*)} \cdot \epsilon^l_{1-T'}(z^*) \cdot \frac{z^* f_z(z^*)}{1-F_z(z^*)} \\
& + dw_t \int (1+\epsilon^l_w(z))\frac{z}{w} f_z(z)dz \quad \forall t,
\end{aligned}$$

where the second third equality holds by applying the change in variable.

Incidences on Welfare The tax incidences on social welfare can be obtained by integrating the incidences on the individual utility. The updating operator for the sequence of distribution densities ϕ_t of savings a and productivity x is: $\phi(a_{t+1}, x_{t+1}) = \int f(x_{t+1} \mid x_t)\frac{\phi((a')^{-1}(a_{t+1},x_t),x_t)}{a'_a((a')^{-1}(a_{t+1},x_t),x_t)}\,dx_t$ at any period t, where $(a')^{-1}(\cdot, x)$ is the inverse of $a'(\cdot, x)$ given x. Therefore, for some function $\tilde{h}$ such that $\tilde{h}(a_0, x^t) = h(a_t(x^{t-1}), x_t)$, by applying the change of variables sequentially:

$$\iint \tilde{h}(a_0, x^t) f(x^t \mid x_0) dx^t \phi(a_0, x_0) da_0 dx_0 = \int h(a_t, x_t)\phi(a_t, x_t)\, da_t\, dx_t. \tag{15}$$

This yields:

$$\begin{aligned}
dW &= \int dV(a_0, x_0) d\Phi(a_0, x_0) \\
&= (1-\beta)\sum_{t=0}^{\infty}\beta \begin{bmatrix} \int_{x^*}^{\infty} \frac{f(x)}{1-F(x^*)}dx - \frac{T'(z(x^*))}{1-T'(z(x^{ast}))} \frac{\epsilon^l_{1-T'}(x^*)}{1-F(x^*)} \frac{z(x^*)}{z'(x^*)} f(x^*) \\ +dw_t \int (1+\epsilon^l_w(x))xl(x)T'(z(x))f(x)dx \end{bmatrix} \times \lambda \\
& \quad -(1-\beta)\sum_{t=0}^{\infty}\beta^t \int\int_{x^*}^{\infty} u'(a,x)\frac{\phi(a,x)}{1-F(x^*)}dxda \\
& \quad +(1-\beta)\sum_{t=0}^{\infty}\beta^t \int\int u'(a,x)\left[dr_t a + dw_t xl(x)(1-T'(z(x)))\right] d\Phi(a,x) \\
& \quad -(1-\beta)\sum_{t=0}^{\infty}\beta^t \int\int \left[u'(a,x) - \beta(1+r)E[u'(a,x')|x]\right] dh^A_{t+1} d\Phi(a,x),
\end{aligned}$$

where $\lambda = \int u'(a,x)d\Phi(a,x)$. Thus,

$$\begin{aligned} dW &= \lambda\left[\int\int_{x^*}^{\infty}\left(1-\frac{u'(a,x)}{\lambda}\right)\frac{\phi(a,x)}{1-F(x^*)}dxda - \frac{T'(z(x^*))}{1-T'(z(x^*))}\epsilon^l_{1-T'}(x^*)\frac{z(x^*)}{z'(x^*)}\frac{f(x^*)}{1-F(x^*)}\right] \\ &\quad -(1-\beta)\sum_{t=0}^{\infty}\beta^t\int\int\left[u'(a,x)-\beta(1+r)E[u'(a,x')|x]\right]dh^A_{t+1}(a,y(x))d\Phi(a,x) \\ &\quad +(1-\beta)\sum_{t=0}^{\infty}\beta^t\int\int u'(a,x)\left[dr_t a + dw_t xl(x)(1-T'(z(x)))\right]d\Phi(a,x) \\ &\quad +\lambda\times(1-\beta)\sum_{t=0}^{\infty}\beta^t\cdot dw_t\int(1+\epsilon^l_w(x))xl(x)T'(z(x))f(x)dx \end{aligned} \tag{16}$$

Note that the incidence formula we derive here does not consider the transition from the current US status quo if the considered tax schedule T is different from the current US tax schedule. This is because we assume that the economy is already in the steady state under the considered tax schedule $T(\cdot)$ and thus sequence of density ϕ_t does not change over time in the updating rule (15).

B Detailed Analysis of $\Delta_p - \Delta_f$

The externality effect per unit price change $\Delta_p - \Delta_f$ captures the two mechanisms through which increase in interest rate (and decrease in wage rate) changes welfare. The first channel captured by Δ_p is the pecuniary externalities in the incomplete market (Dávila, Hong, Krusell, and Ríos-Rull (2012))—the welfare effects of the change in relative price of labor income and capital income. When the relative price of labor income and capital income changes, there will be redistribution across the asset dispersion and insurance against the labor income uncertainty, which can be clearly seen in the following expression.

$$\Delta_p = \underbrace{\int\frac{u'(a,x)}{\lambda}\left[a/K-1\right]d\Phi(a,x)}_{\equiv\Delta_K} \qquad \underbrace{-\int\frac{u'(a,x)}{\lambda}\left[xl(x)(1-T'(z(x)))/L-1\right]d\Phi(a,x)}_{\equiv\Delta_L}$$

Δ_K shows the negative redistributive effects of increasing interest rates. Increase in interest rate makes the wealth-rich (with asset holding above the average: $a > K$) better off, while it makes the wealth-poor (with $a < K$) worse off. Since the wealth-poor are the consumers with low consumption and high marginal utility, under the utilitarian social welfare function, increasing interest rate has negative welfare effects: $\Delta_K < 0$. On the other hand, Δ_L shows the positive insurance effects of decreasing wage rate (equivalent to increasing interest rate). Decrease in the wage rate can decrease the risk of the consumers by scaling down the stochastic part of income—labor income, which has positive effects: $\Delta_L > 0$. Previous studies show that with reasonable calibration that generates realistic wealth distribution, the negative redistribution effects dominate, and thus $\Delta_p < 0$ (see Dávila, Hong, Krusell, and Ríos-Rull (2012) and Park (2018)).

The second channel of the price effects captured by $-\Delta_f < 0$ shows the negative fiscal externalities of decrease in wage rate (equivalently, of increase in wage rate). The government revenue increases (decreases) when the wage rate increases (decreases) because of both (i) the direct effects given labor supply and (ii) the indirect effects via increase (decrease) in labor supply.

In sum, both the pecuniary externalities channel and the fiscal externalities channel imply that any tax reform that leads to increase in interest rate has negative welfare effects through the general equilibrium: $\Delta_p - \Delta_f < 0$. Higher interest rate makes the asset-poor poorer and the associated lower wage rate reduces government revenue and transfer. Thus, accounting for the general equilibrium effects makes the government prefer a tax reform that decreases interest rate (relative to the case without general equilibrium effects). The next step is to figure out which tax reform leads to a decrease in interest rate.

C Self-Confirming Policy Equilibrium and Decomposition of GE Effects

In this section, we formally define the self-confirming policy equilibrium (SCPE). In the main text, we adopted two SCPE concepts which are similar but slightly different. The SCPE in Section 4 is introduced to evaluate effects of the general equilibrium on the optimal tax schedule, while the SCPE_{nt} in Section 6.1 is to analyze the effects of the local transition. Both the general equilibrium and transition affect the optimal tax schedule not only through the direct effect—change in prices and transitional dynamics of incidences respectively—but also through the indirect effect via change in endogenously optimal distribution. This is because the steady state distribution under the fully optimal tax schedule accounting for the GE effects and transition is different from that without GE effects (or without transition). The SCPEs are designed as a reference to evaluate both the direct and indirect effects of introducing the general equilibrium.

A SCPE tax system in Section 4 is defined as an optimal tax of a government which assumes that the prices are independent of the tax system. That is, a government chooses a tax system assuming that the observed prices (generated by the existing tax system) are given and remain unchanged. In a SCPE, the optimality of the tax system designed by the government is "confirmed" when a newly chosen tax schedule coincides with the existing one. Formally, this is achieved by requiring the fixed point of the mapping $E \mapsto \tilde{E}^*(E)$, where E is the original equilibrium allocation and $\tilde{E}^*(E)$ is the new equilibrium induced by the optimal tax policy given E.

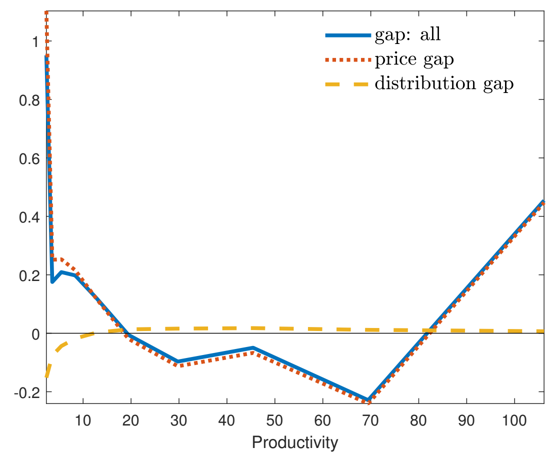
Since the government in the SCPE does not take into account the incidences on the prices, the SCPE formula has only two terms—the standard equity-efficiency term and the term associated with the borrowing constraint effects.

$$\begin{aligned}\frac{T'_{SCPE}(z^*)}{1-T'_{SCPE}(z^*)} &= \frac{1}{\epsilon^l_{1-T'}(z^*)}\frac{1-F_z(z^*)}{z^* f_z(z^*)} \times \\ &\left[\begin{array}{l} \iint_{z^*}^{\infty}\left(1-\frac{u'(a,z)}{\lambda_{SCPE}}\right)\frac{\phi_{SCPE}(a,z)}{1-F_z(z^*)}dzda \\ -\frac{1}{\lambda_{SCPE}}\int[u'(a,z)-\beta(1+r)E[u'(a',z')|z]]dh^A_{SCPE}(a,y(z))d\Phi_{SCPE}\end{array}\right] \end{aligned} \tag{17}$$

Then by comparing the optimal tax formula (10) with tax formula under the SCPE (17), we can obtain the gap between the two formulas as in (11) which are decomposed into price effects Ω_{price} and distribution effects Ω_{dist}.

Our quantitative analysis in the main text is mostly centered on the comparison of the tax reform incidence under the different initial tax schedule (e.g. current US tax and optimal tax) given the distribution before the tax reform. Here we carry out the quantitative analysis to show that for the role of the GE effects in the optimal tax schedule, the direct price effects are strongly dominating the indirect distribution effects, which supports our analysis focusing on the role of price effects throughout the paper. Figure 31 plots the quantitative decomposition of the gap between the formula with and without GE effects ($\frac{T'_{opt}}{1-T'_{opt}} - \frac{T'_{SCPE}}{1-T'_{SCPE}}$) in equation (11). It shows that the consideration of the general equilibrium changes the optimal tax schedule mostly through the price effects although the distribution effects have a minor role at the bottom income.

Figure 31: Decomposition of $\frac{T'_{opt}}{1-T'_{opt}} - \frac{T'_{SCPE}}{1-T'_{SCPE}}$



On the other hand, a SCPE$_{nt}$ tax system in Section 6.1 is defined as an optimal tax of a government which ignores the transition. That is, a government chooses a tax system accounting for the general equilibrium effects and borrowing constraint effects, but assumes that the economy jumps to the new steady state immediately. Thus the tax formula of a SCPE$_{nt}$ consists of all channels in the optimal tax formula (10), but it only accounts for long-run effects in those channels:

$$\frac{T'_{nt}(z^*)}{1-T'_{nt}(z^*)} = \frac{1}{\epsilon^l_{1-T'}(z^*)}\frac{1-F_z(z^*)}{z^* f_z(z^*)} \times \left[\begin{array}{l} \iint_{z^*}^{\infty}\left(1-\frac{u'(a,z)}{\lambda_{nt}}\right)\frac{\phi_{nt}(a,z)}{1-F_z(z^*)}dzda \\ -\frac{1-\beta}{\lambda_{nt}}\sum_t \beta^t \int [u'(a,z)-\beta(1+r)E[u'(a',z')|z]]dh^A_{t+1,nt}(a,y(z))d\Phi_{nt} \\ +dr_{nt}(z^*)K_{nt}\times(\Delta_{p,nt}-\Delta_{f,nt}) \end{array}\right],$$

where the subscript nt represents "no transition."

D Sensitivity Analysis with Finer Grid

In the main text, we approximated the nonlinear tax function with piecewise-linear function with grid point $N = 10$. Then all the tax reform incidence analysis was computed with the tax reforms of increasing marginal tax rate at each grid point. In this section, we do the sensitivity analysis by increasing grid points to $N = 20$ and repeat the tax reform analysis in Section 5.1.

The top panels of Figure 32 and 33 plot the normalized welfare incidence through the GE effects ($\frac{1}{\lambda}(R3 + R4)$) under the US tax schedule and optimal tax schedule respectively. And the bottom panels plot the normalized incidence on the interest rate associated with each tax reform. Both figures show that increasing the number of grid points does not change the direction of the GE effects for every tax reform we consider, although the size of the GE effects is somewhat changed. Thus our findings on whether the GE effects make the government favor more or less progressive (or redistributive) tax reform are not sensitive to the number of grid points.

Figure 32: (normalized) dW through GE

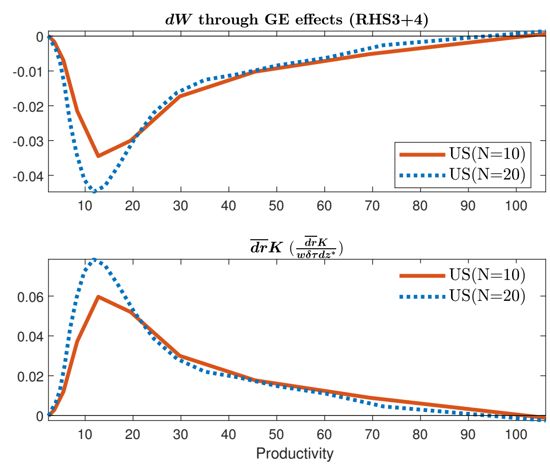


Figure 33: (normalized) dW through GE

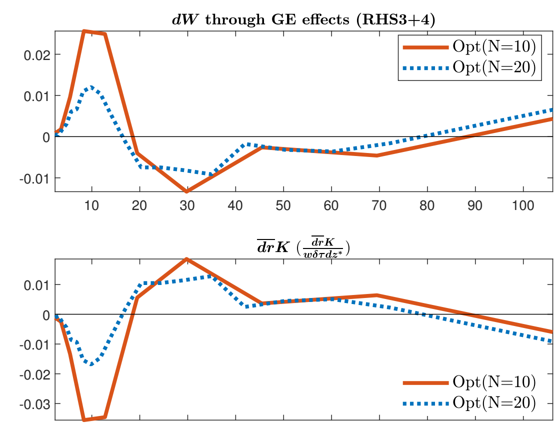


Figure 34 plots the optimal marginal tax rate schedules in the presence of the GE effects with the number of grid points $N = 10$ and $N = 20$. They also confirm that the effects of the GE effects on the optimal tax schedule are not much sensitive to the number of grid points. The quantitative gap between the tax schedules with different numbers of grid points is mostly due to the quantitative gap in the GE effects in Figure 33. With a finer grid, optimal tax rates show a smoother schedule. Figure 35 plots the optimal marginal tax rate schedules with and without GE effects. This figure shows the the role of the GE effects on optimal tax schedule with finer grid points, and the comparison with Figure 10—the same plot with the number of grid points $N = 10$—also confirms that the role of GE effects on the optimal tax schedule is not sensitive to the the number of grid points.

Figure 34: Optimal Marginal Tax Rates

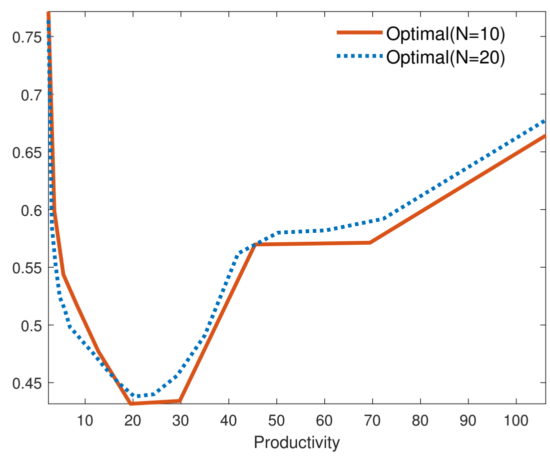
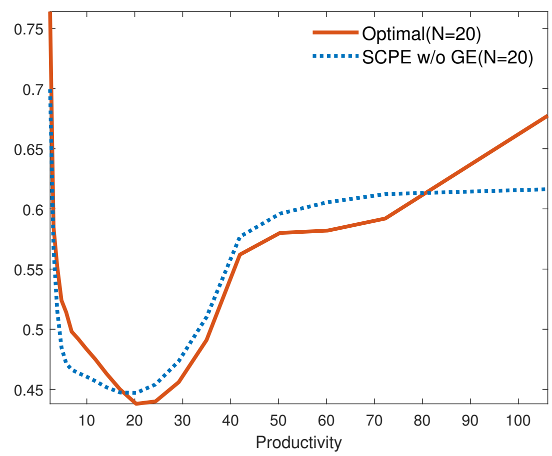


Figure 35: Optimal Marginal Tax Rates



E Further Analysis with Capital Income Tax

E.1 Capital Income Tax Internalizing the Externalities

We first analyze with which tax system the pecuniary externalities can be fully internalized. Suppose that given the labor income tax schedule $T(z)$, the government does not want to use the capital income tax for the direct redistribution, but just wants to internalize the GE effects with the capital income tax. Which tax system can achieve this goal? To answer this, we consider the constrained planner's problem that internalizes both the pecuniary externalities and fiscal externalities without completing the market. That is, the government solves the following (recursive form of) problem of the constrained planner who internalizes the effects of change in prices given $T(z)$:

$$\begin{aligned}
\Omega(\Phi) \quad &= \quad \max \int u\left(wxl - T(wxl) + R(w) + (1+r)a - a'(a,x) - v(l)\right) d\Phi(a,x) + \beta\Omega(\Phi') \\
s.t. \quad & \quad \Phi' = H(\Phi, Q(\cdot; a')) \\
& \quad R(w) = \int T(wxl) f(x) dx - E \\
& \quad w = F_L(K,L), \quad r = F_K(K,L) - \delta,
\end{aligned}$$

where H is the mapping from today's distribution to the distribution in the next period and Q is the transition function given the policy function for saving a'. Note that the transfer function $R(w) = \int T(wxl)f(x)dx - E$ is added in the household's budget to capture the fiscal externalities.[37]

The first order condition with respect to a' is then:

$$u'(c - v(l)) = \beta(1+r) \int f(x'|x) u'\left(c(a',x') - v(l')\right) dx' + \beta\Delta, \tag{18}$$

where $\Delta \equiv F_{KK}K \cdot (\Delta_p - \Delta_f)$ is an additional marginal benefit of savings due to the price externalities—both the pecuniary and fiscal externalities that we discussed above. That is, (18) is the planner's Euler equation which internalizes both the pecuniary and fiscal externalities. Note that we focus on the constrained-optimal steady states—long-run outcomes of an infinite horizon planning problem, and thus $\frac{dr_{t+1}}{dK_{t+1}} = F_{KK}(K_{t+1}, L_{t+1}) = F_{KK}$ is constant.

The government can fully internalize the externalities Δ in (18) with the following history-dependent tax-transfer scheme. Consider a tax rate imposed on asset income in period $t+1$, $\tau_{k,t+1}(a_t, y_t) = -\frac{\Delta}{\beta r E_{y_{t+1}}[u'(\tilde{c}(a'(a_t,y_t),y_{t+1})))|a_t,y_t]}$ and lump-sum transfers $Tr_{t+1}(a_t, y_t) = \tau_k(a_t, y_t) \cdot r \cdot a_{t+1}(a_t, y_t)$, where $y = wxl - T(wxl)$ and $\tilde{c} = c - v(l)$. This tax system implements the constrained-efficient allocation that internalizes the externalities given $T(z)$. With this tax system, the tax incidence formula does not involve the price externalities any more. Note that we need history-dependent tax rates to fully internalize the externalities. Intuitively, the external marginal benefit of additional savings is the same for every household

[37]Note that $R(w) = 0$ because the tax schedule $T(z)$ is set to satisfy the government budget constraint.

while the marginal benefit and cost of savings at the individual level are different across households depending on their history of shocks, thus different tax rates are required for different histories to internalize the constant benefit.

Lastly, we show that with this tax system, in the tax reform incidence formula (incidence on welfare), the GE effects terms do not show up any more. Note that when we introduce the capital income tax rate and the transfer above, the individual budget does not change. Thus, the incidence on the individual utility is not changed with the introduction of $(\tau_{k,t+1}, Tr_{t+1})$. On the other hand, the GE effects in the incidence on welfare (6) are essentially $R3 + R4 = \sum_{t=0}^{\infty} \beta^t \Delta \iint dh_{t+1}^A(a, y(x)) d\Phi(a, x)$, which are now combined into $R2$:

$$\begin{aligned} dW &= \lambda \left[\int \int_{x^*}^{\infty} \left(1 - \frac{u'(a,x)}{\lambda}\right) \frac{\phi(a,x)}{1 - F(x^*)} dx da - \frac{T'(z(x^*))}{1 - T'(z(x^*))} \epsilon_{1-T'}^{l}(x^*) \frac{z(x^*)}{z'(x^*)} \frac{f(x^*)}{1 - F(x^*)} \right] \\ &\quad -(1 - \beta) \sum_{t=0}^{\infty} \beta^t \int \int \left[u'(a,x) - \beta(1+r)E[u'(a,x')|x] - \beta\Delta\right] dh_{t+1}^A(a, y(x)) d\Phi(a, x). \end{aligned} \tag{19}$$

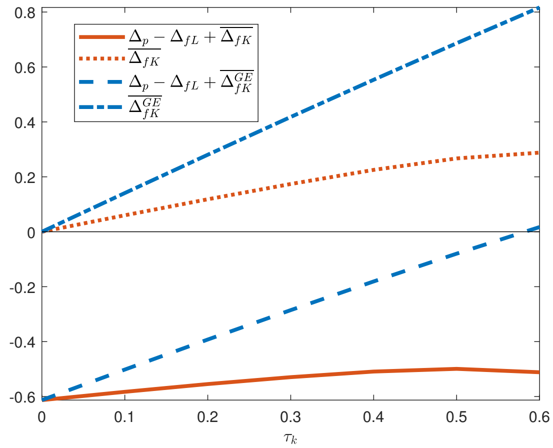
The new $R2$-term with additional $\beta\Delta$ is exactly the $R2$-term in dW under the tax system $(\tau_{k,t+1}, Tr_{t+1})$. Thus, the externality terms disappear in the incidence formula.

E.2 Linear Capital Income Tax

In this section, we investigate how the GE effects are affected by the introduction of capital income tax in more detail. In Section 5.2, we show that the GE effects can still be represented by the summary statistic $\overline{dr_t} K \times (\Delta_p - \Delta_{fL} + \overline{\Delta_{fK,t}})$, and introducing linear capital income tax does not change the direction of the GE effects. As we already discussed in Section 5.2, $\overline{\Delta_{fK,t}}$ is not a pure price effect, as it also includes the revenue effects via change in asset supply for fixed price. We still interpret this term as the GE effects, because the shift of the asset supply curve is the cause of price change. When we want to focus on the welfare incidence caused purely by the price change, however, the incidence on aggregate capital needs to be decomposed into dK_t^{GE} and $dK_t|^r$.

Figure 36 shows such a decomposition of $\overline{\Delta_{fK,t}}$—revenue effects of capital income tax for one unit increase of interest rate—for various capital income tax rates τ_k, where the tax incidence is computed under the current US labor income tax schedule. The revenue effects

Figure 36: Externalitly per unit price change



purely due to change in interest rate $\overline{\Delta_{fK,t}^{GE}}$ is always positive and is larger than $\overline{\Delta_{fK,t}}$ as the negative revenue effect due to the shift of asset supply curve is not taken into account. As a result, the negative external effect of increase in interest rate $\Delta_p - \Delta_{fL}$ is significantly mitigated by the additional $\overline{\Delta_{fK,t}^{GE}}$, but for most of τ_k, $\Delta_p - \Delta_{fL} + \overline{\Delta_{fK,t}^{GE}}$ is still negative.

As we already investigated in Section 5.2, introducing capital income tax leads to stronger response of interest rate, which amplifies the GE effects. Figure 37 and 38 show that this amplification gets stronger as capital income tax rate τ_k increases. With higher τ_k, elasticity of asset supply with respect to interest rate further decreases, leading to a much steeper asset supply curve and much stronger change of interest rate for the shift of asset supply curve.

E.3 Joint Optimal Tax

In this section, we evaluate the role of capital income tax under the joint optimal tax system—jointly optimal labor income tax schedule $\{T'_{opt}(z)\}$ and (linear) capital income tax rate τ_k^*. To derive optimal capital income tax rate, the same perturbation approach is used. When the welfare incidences of small tax reforms on both labor income and capital income taxes are zero, we get the joint optimal tax schedule.

From the numerical simulation, the optimal capital income tax rate τ_k^* is 0.67, which is much higher than the current US capital income tax rate. To see what causes this high tax

Figure 37: (normalized) $\overline{dr_t}K$

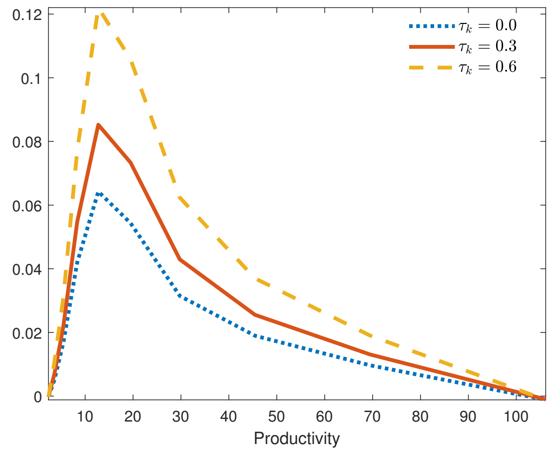
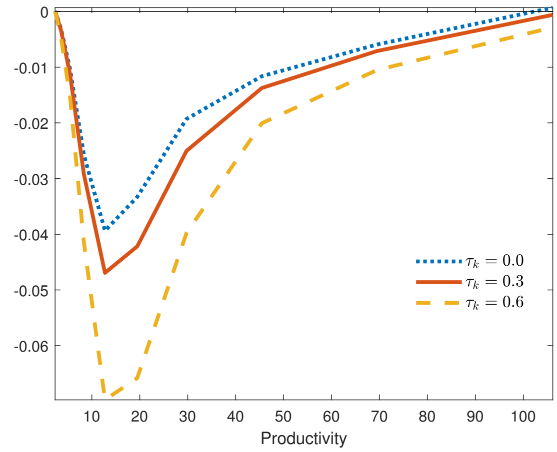


Figure 38: dW through the GE effects



rate, we can decompose the optimal capital income tax formula:

$$
\begin{aligned}
\frac{\tau_k}{1-\tau_k} \quad &= \quad \frac{1}{(1-\beta)\sum_{t=0}^{\infty}\beta^t \epsilon^S_{K,1-\tau_k,t}} \times [A_{\tau_k} + B_{\tau_k} + C_{\tau_k} + D_{\tau_k}] \\
where \quad & A_{\tau_k} = \int \left(1 - \frac{u'(a,x)}{\lambda}\frac{a}{K}\right) d\Phi(a,x) \\
& B_{\tau_k} = -\frac{1-\beta}{\lambda}\sum_{t=0}^{\infty}\beta^t \int [u'(a,x) - \beta(1+r)E[u'(a'(a,x),x')|x]] \frac{dh^A_{t+1}(a,y(x))}{\delta\tau_k r K} d\Phi(a,x) \\
& C_{\tau_k} = (1-\beta)\sum_{t=0}^{\infty}\beta^t \frac{dr_t K}{\delta\tau_k r K}\Delta_p, \quad \Delta_p = \int \frac{u'(a,x)}{\lambda}\left[\frac{a(1-\tau_k))}{K} - \frac{xl(x)(1-T'(z(x)))}{L}\right] d\Phi(a,x) \\
& D_{\tau_k} = (1-\beta)\sum_{t=0}^{\infty}\beta^t \frac{dr_t K}{\delta\tau_k r K}\left[-\Delta_{fL} + \Delta_{fK,t}\right], \\
& \text{with} \quad \Delta_{fL} = \int (1+\epsilon^l_w(x))\frac{xl(x)}{L}T'(z(x))f(x)dx, \quad \Delta_{fK,t} = \tau_k \times \left(1 + \frac{dK^{GE}_t}{dr_t}\frac{r}{K}\right).
\end{aligned}
$$

Note that $\epsilon^S_{K,1-\tau_k,t} = -\frac{dK|^r}{\delta\tau_k}\frac{1-\tau_k}{K}$ is the partial equilibrium elasticity of aggregate capital with respect to capital income tax rate (given interest rate), and the price elasticity of aggregate capital is represented by $\frac{dK^{GE}_t}{dr_t}\frac{r}{K}$. The quantitative decomposition of the formula (E.3) is as follows.

	1st	2nd	3rd	4th
$\frac{\tau_k}{1-\tau_k}$	1.9306	-0.0001	-0.7989	0.9734

This decomposition result shows that the standard redistribution effect of increasing capital income tax rate is the major reason for the high level of optimal τ_k^*. Although the pecuniary

Figure 39: Optimal: dW—GE effects

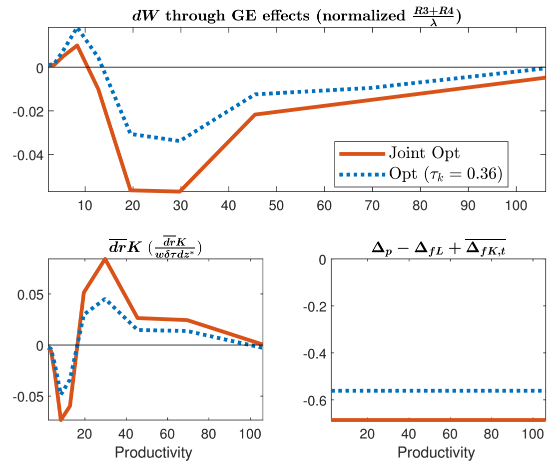
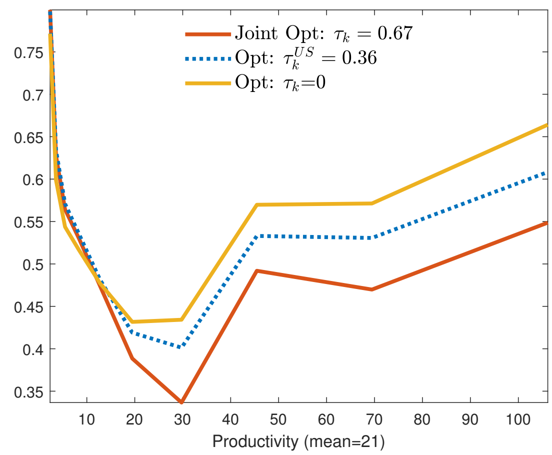


Figure 40: Optimal Marginal Tax Rates



externality effect (3rd term) lowers the optimal τ_k^* taking into account the negative welfare effects of increase in interest rate, the direct positive revenue effects $\Delta_{fK,t}$ of increasing interest rate (in the 4th term) is stronger than the pecuniary externality effect and thus the total GE effect makes the optimal τ_k^* even higher.

We then show that the GE effects of the labor income tax reform tend to be even more amplified when evaluated at the joint optimal tax schedule where capital income tax rate is very high with $\tau_k^* = 0.67$ (compared to those under the optimal labor income tax schedule given the US capital income tax rate $\tau_k^{US} = 0.36$.). Figure 39 shows that higher τ_k amplifies both factors of the GE effects ($\overline{dr_t}K$ and $(\Delta_p - \Delta_{fL} + \overline{\Delta_{fK,t}})$) except the very bottom and top, making the government prefer a less progressive tax schedule even more. Figure 40 shows that the optimal labor income tax schedule under the joint optimal tax system is less progressive (and less redistributive) relative to the optimal tax schedule given τ_k^{US}. Two forces drive this less progressive tax schedule. The amplified GE effects we just discussed contribute to this schedule, and the value of redistribution gets lower as the high optimal τ_k^* already provides significant amount of redistribution.

F Detailed analysis on the Role of Other Factors

In Section 5.3, we investigated the role of risk aversion, elasticity of substitution, and capitalist spirit in the welfare incidence through the GE effects. In this section, we analyze their roles in more detail.

F.1 Risk Aversion and Elasticity of Substitution

In Section 5.3, we showed that the implication of higher risk aversion on the welfare incidence through the GE effects is not consistent under the current US tax and optimal tax schedule. Under the US tax schedule, considering GE effects make the government favor less redistributive tax reform, and this effect is stronger when risk aversion is higher, as the stronger savings response to a redistributive tax reform has more negative welfare effects through more increase in interest rates. Under the optimal tax schedule, however, higher risk aversion implies positive welfare incidence through the GE effects for the tax reform at the low and high income. This is because risk aversion affects the GE effects indirectly through the change in optimal tax schedule. With higher risk aversion, higher value of redistribution raises the optimal marginal tax rates through the standard equity-efficiency trade-off channel (Figure 41). Significant public insurance with higher tax rates under the initial (optimal) tax schedule leads to mitigated response of savings to a tax reform, but the reform has a stronger impact on labor supply as the elasticity of labor supply gets higher with higher initial tax rates. Thus, higher risk aversion makes the labor response to tax reforms more dominant, decreasing interest rates on average (Figure 42) when tax rates are increasing. As a result, higher risk aversion can have a positive welfare incidence to a redistributive tax reform through the GE effects. This applies especially to the tax reform at the low and high income where the optimal tax rates are especially higher.

Next, we analyze the role of the elasticity of substitution for the GE effects. As we discussed in Section 5.3, we use the CES production function, $Y = A\left(aK^{\frac{\sigma_{es}-1}{\sigma_{es}}} + (1-a)L^{\frac{\sigma_{es}-1}{\sigma_{es}}}\right)^{\frac{\sigma_{es}}{\sigma_{es}-1}}$ for this analysis. There is little agreement in macroeconomics about the value of the elasticity of substitution, σ_{es}. Many empirical studies have estimated σ_{es} and suggest that capital and

Figure 41: Optimal Marginal Tax Rates

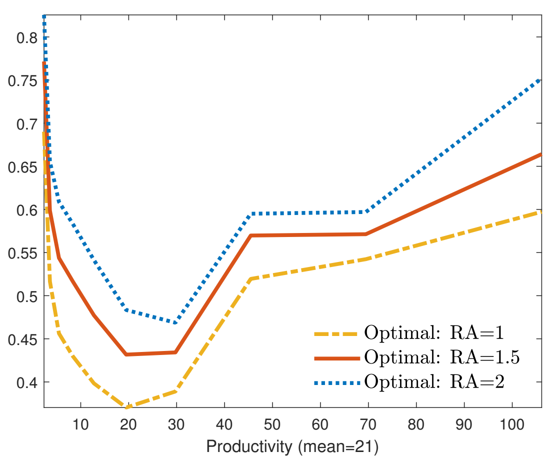
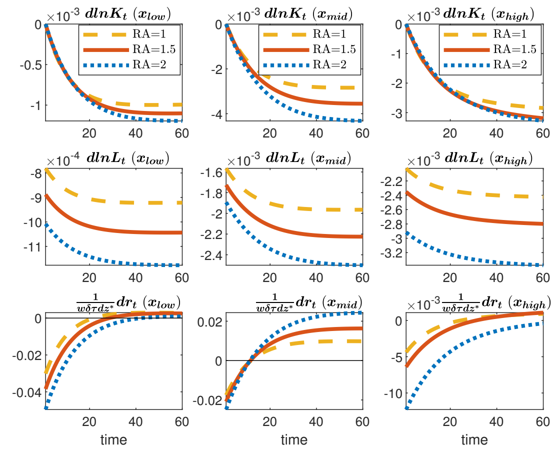


Figure 42: Transition under Optimal Tax



labor are gross complements ($\sigma_{es} < 1$), with σ_{es} between 0.4 and 1 (e.g. Antras (2004)). On the other hand, as Karabarbounis and Neiman (2014) show, the observed declining labor income share is consistent with σ_{es} above 1, and they estimate an elasticity of 1.25. Here, we carry out sensitivity analysis with various levels of σ_{es}. For each σ_{es}, we recalibrate A and a so that the steady state is identical to the benchmark with Cobb-Douglas.

In Section 5.3 of the main text, we showed that under the current US tax schedule, σ_{es} has little impact on the welfare incidence through the GE effects despite the amplification/mitigation effects of lower/higher σ_{es} on price incidences. Figure 43 shows that this is because with lower σ_{es}, both the short-run incidence and the long-run incidence are amplified, but they are amplified in opposite directions. As a result, the bottom right panel of Figure 43 shows that the time average incidences on the interest rate is not much sensitive to the change in σ_{es}.

In the main text, we also showed that under the optimal tax schedule, lower elasticity leads to more positive welfare incidence through the GE effects, which can be explained by the following. First, when $\sigma_{es} < 1$, smaller optimal capital-income ratio due to the more redistribution with optimal tax schedule (driven by the standard-equity efficiency trade-off) results in higher capital income share α, then as the equation (12) shows, the price incidence is amplified not only by smaller σ_{es} but also by higher $(1-\alpha)\alpha$.[38] Second, the short-run price

[38]Note that α is typically smaller than 0.5, and thus $(1-\alpha)\alpha$ is increasing in α.

Figure 43: Transition under the US

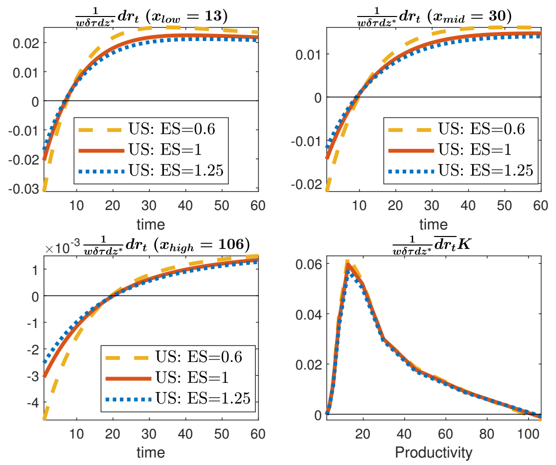
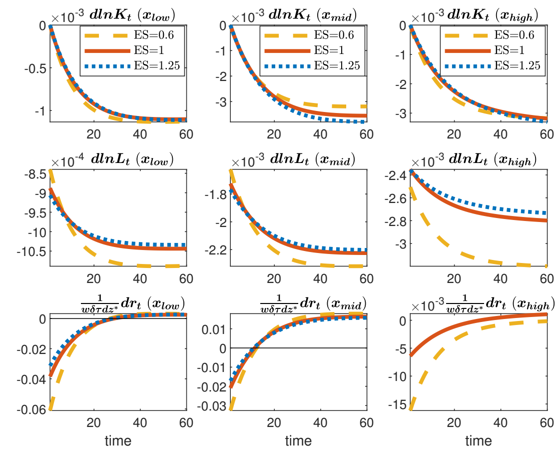


Figure 44: Transition under Optimal Tax



incidences are significantly amplified by this channel, but in the long run, the incidences on capital and labor almost cancel out (Figure 44). This is again because the response of savings is mitigated when the agents have enough public insurance with initially high tax rates. As a result, lower elasticity ($\sigma_{es} < 1$) leads to larger decrease in interest rate in response to a redistributive tax reform, which has more positive welfare effects.

F.2 Models with Capitalist Spirit

It is well known that standard incomplete market models with precautionary savings cannot generate realistic wealth dispersion in the data. To generate wealth inequality, we consider a model with capitalist spirit as the additional source of generating wealth inequality (Carroll (2000), Straub (2019)). Consider the following non-homothetic preferences:

$$u(c - v(l)) + U\left(\frac{a}{K}\right) = \frac{1}{1-\sigma}\left(c - \frac{l^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}\right)^{1-\sigma} + \kappa\frac{(\frac{a}{K} + \underline{k})^{1-\phi}}{1-\phi},$$

where $U\left(\frac{a}{K}\right)$ is the utility over the wealth position a relative to average wealth K. We jointly calibrate ϕ, $\underline{k}$, and κ to match the wealth Gini coefficient, fraction of households with negative asset holding.

We first point out that with this relative status model, there is extra term in the welfare incidence of the tax reform, which captures the positional externalities. That is, the incidence on welfare (6) has additional term

$$R5 = -(1-\beta)\sum_{t=0}^{\infty}\beta^t \frac{dK_t}{K}\iint U'\left(\frac{a}{K}\right)\frac{a}{K} f(x^t|x_0)dx^t d\Phi(a_0,x_0),$$

which arises due to the incidence of average capital that has welfare effects by adjusting individuals' wealth position. Figure 45, however, shows that this term is quantitatively small. Thus, we focus on the price externalities terms ($R3 + R4$) to analyze the role of the additional sources of wealth inequality on the welfare incidence through the GE effects.

Since the purpose of introducing the additional sources of inequality is to match the wealth distribution, one might conjecture that incorporating the capitalist spirit influences the GE effects mostly through the change in distribution. However, it turns out that the GE effects are sensitive to the capitalist spirit mostly through the change in the price incidences as the additional sources of inequality plays an important role in the price incidences. Figure 45 plots the decomposition of the welfare incidence of the tax reform at the current US tax under the benchmark model and the model with capitalist spirit. The bottom panel shows that the negative welfare incidence through the GE effects is mitigated when the capitalist spirit is introduced except at the top. This is because with additional motivation for savings, the precautionary motive for savings is less important, and as a result, the response of savings to a tax reform tends to be mitigated. Figure 46 shows that the decrease in savings to the redistributive tax reforms gets smaller, which dampens the increase in interest rate in the long run.[39]

Of course, the negative welfare effects of increasing interest rate by one unit through the pecuniary externalities (Δ_p) are amplified with the capitalist spirit in the model, as more severe wealth inequality generated by the additional sources of inequality implies that increase in interest rate has further negative redistribution across the wealth dispersion. Table 4 confirms that in the model with the capitalist spirit, with higher wealth inequality, the negative value of Δ_p becomes stronger relative to that in the benchmark.[40] The amplification

[39]For the tax reform at the top income, however, aggregate savings decrease mainly because the rich decrease their savings as their disposable income reduces with the tax reform. This income effects would be stronger in the model with capitalist spirit where the saving rate is higher.

[40]Table 4 shows that the wealth Gini index generated by the capitalist spirit model is slightly lower than the wealth Gini in the data. Note that we jointly calibrate the parameters ϕ, $\underline{k}$, and κ to match the wealth

Figure 45: Decomposition of dW

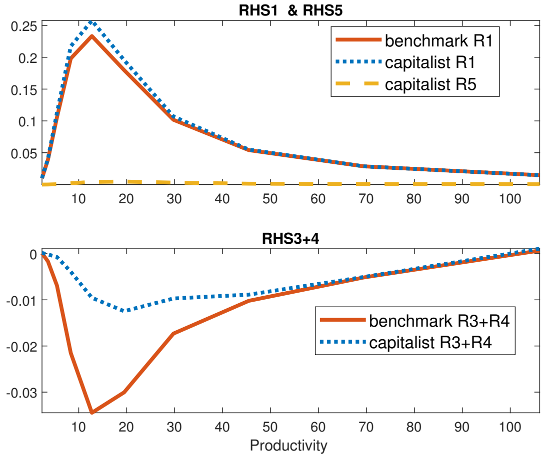
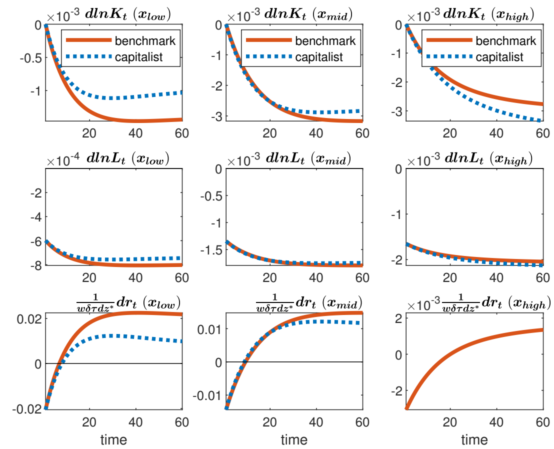


Figure 46: Transition under the US



of the GE effects due to the stronger pecuniary externalities, however, are dominated by the significant mitigation in the price incidences, resulting in dampened welfare incidence through the GE effects ($R3 + R4$) in Figure 45 .

Table 4: $\Delta_p - \Delta_f$: US vs Opt

	$\Delta_p - \Delta_f$	Δ_p	$-\Delta_f$	L-Gini	K-Gini	$corr(\frac{a}{K}, \frac{xl}{L})$
benchmark	-0.58	-0.10	-0.48	0.51	0.73	0.55
capitalist	-0.65	-0.17	-0.48	0.51	0.82	0.46

We also remark that this mitigation of the GE effects is not specific to the capitalist spirit model, but it is a more general feature with the models with additional sources of wealth inequality. For example, we found that with the introduction of the capital income risk (either additive or multiplicative risk), the model has the similar implication for the GE effects as the additional sources of inequality dampen the response of savings with relatively less important precautionary motive of savings.

Gini coefficient, fraction of households with negative asset holding. We can better match the wealth Gini if we do not try to match the fraction of households with negative asset together, but this calibration generates a very high fraction of negative asset holding households, which is counterfactual.

Figure 47: Average Welfare over Time

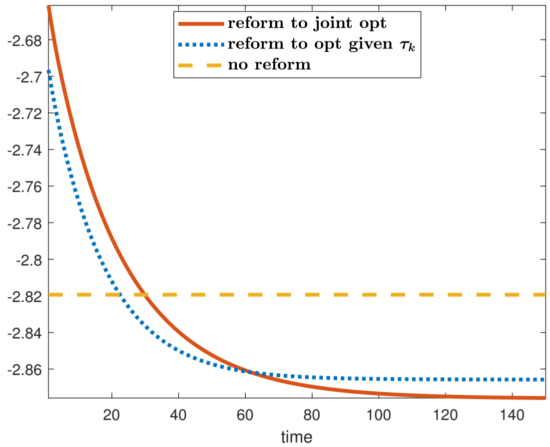
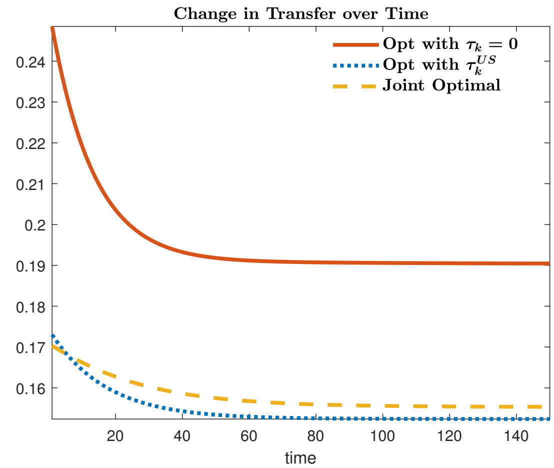


Figure 48: Global Transition: Transfer



G Further Analysis of Global Transition

G.1 Welfare Effects of Joint Optimal with Global Transition

In this section, we analyze the welfare effects of a global tax reform over the global transition path from the current US status quo to the steady state associated with the *joint optimal* tax schedule. In Section 6.2.1, we investigate the global transition to optimal tax schedule given capital income tax (either zero or $\tau_k^{US} = 0.36$) and show that the average welfare gain is positive because of the short-run welfare gain at the cost of the long-run welfare cost even if the optimal tax schedule is set without considering the global transition. Here, we do the same welfare analysis for the reform to the joint optimal tax system to show the same qualitative results with the global transition given τ_k^{US} and explain the quantitative difference.

Figure 47 shows that the result of the short-run welfare gain and long-run welfare loss still applies to the reform to the joint optimal tax. Moreover, the asymmetric welfare effects in the short run and long run are even more pronounced under the joint optimal tax reform compared to the optimal tax reform given τ_k^{US}.

This even higher short-run welfare gain under the joint optimal tax reform can be explained by the lower labor income tax rates and more redistribution by the capital income tax. Figure 49 compares the marginal (labor income) tax schedule of the optimal tax system without

Figure 49: Optimal Marginal Tax Rates

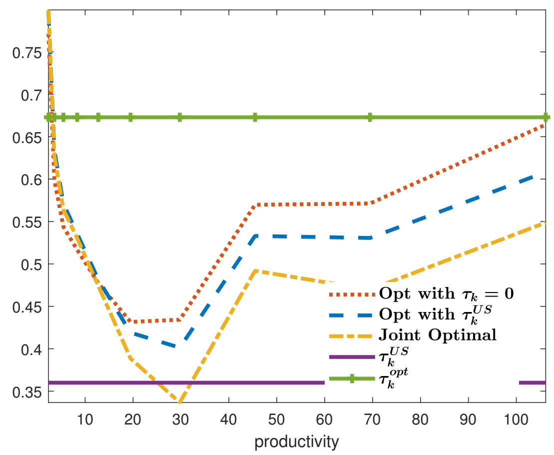
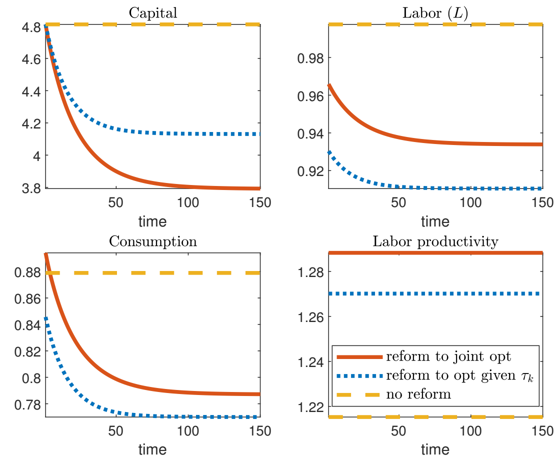


Figure 50: Global Transition: Aggregate



τ_k, with τ_k^{US}, and with jointly optimal τ_k^{opt}. Under the joint optimal tax system, labor income tax schedule is less redistributive, because of the lower value of redistribution with additional redistribution from the high capital income tax and the amplified GE effects which also favors less redistribution. Due to the lower labor income tax rates relative to those with τ_k^{US}, aggregate labor drops less over the transition under the joint optimal tax (relative to that under the optimal with τ_k^{US}), and as a result, aggregate consumption drops less under the joint optimal reform (Figure 50). Since the capital adjustment takes time, welfare gain is higher in the short run, but the long-run welfare decreases further due to more decrease in aggregate capital with much higher capital income tax.

G.2 Formulas with Global Transition

In Section 6.2.2, we investigate the optimal tax schedule with global transition and investigate the GE effects of the tax incidence. In this section, we formally derive the tax incidence formula and optimal tax formula accounting for the global transition—from the current US tax schedule to the optimal tax schedule. Suppose that the economy starts from the steady state under the US tax schedule, and at time 0, a new (time-invariant) optimal tax schedule is implemented. During the global transition which converges to the optimal steady state, the distribution $\Phi_t(a, x)$ slowly changes from the stationary distribution under the current US tax schedule $\Phi_0(a, x)$ to the stationary distribution under the optimal tax schedule $\Phi_T(a, x)$

for some period T when the economy converges. Given this global transition, we consider a small perturbation of the (time-invariant) optimal tax schedule—an elementary tax reform $\tau(z) = \frac{1}{1-F_z(z^*)}\mathbb{1}\{z \geq z^*\}$ for a given level of income z^*—to get the tax incidence formula. The tax incidence is obtained in regards to the small perturbation, but different from the tax incidence formulas starting from the steady state distribution associated with the tax schedule before the small tax reform (in Section 3.1), the incidence of the small perturbation is now evaluated using the time-varying distribution and elasticities over the global transition.

Despite the time-invariant optimal tax schedule, income level for a given productivity x, $z_t(x) = w_t x l_t(x)$, is time varying as wage rate w_t and thus l_t change over the global transition. Thus, an elementary tax perturbation at a constant income z^* is the reform at productivity level x_t^* in period t such that $z_t(x_t^*) = z^*$. Then the tax incidence on social welfare is written by

$$\begin{aligned}
dW &= (1-\beta)\sum_{t=0}^{\infty}\beta^t \cdot \lambda_t \cdot \frac{1-F(x_t^*)}{1-F(x^*)}\iint_{x_t^*}^{\infty}\left(1-\frac{u'(\tilde{c}_t(a,x))}{\lambda_t}\right)\frac{\phi_t(a,x)}{1-F(x_t^*)}dxda \\
&\quad -\frac{T'(z^*)}{1-T'(z^*)}(1-\beta)\sum_{t=0}^{\infty}\beta^t \cdot \lambda_t \cdot \epsilon^l_{1-T',t}(x_t^*)\frac{z_t(x_t^*)}{z_t'(x_t^*)}\frac{f(x_t^*)}{1-F(x^*)} \\
&\quad -(1-\beta)\sum_{t=0}^{\infty}\beta^t\iint\left[u'(\tilde{c}_t(a,x))-\beta(1+r_{t+1})E[u'(\tilde{c}_t(a',x'))|x]\right]dh^A_{t+1}(a,y_t(x))d\Phi_t(a,x) \\
&\quad +(1-\beta)\sum_{t=0}^{\infty}\beta^t \cdot dr_t K_t\iint u'(\tilde{c}_t(a,x))\left[\frac{a}{K_t}-\frac{xl(x)(1-T'(z(x)))}{L_t}\right]d\Phi_t(a,x) \\
&\quad +(1-\beta)\sum_{t=0}^{\infty}\beta^t \cdot \lambda_t \cdot dw_t L_t\int(1+\epsilon^l_{w,t}(x))\frac{xl_t(x)}{L_t}T'(z_t(x))f(x)dx,
\end{aligned}$$

where $\lambda_t = \int u'(\tilde{c}_t(a,x))d\Phi_t(a,x)$ and $\tilde{c}_t(a,x) = c_t(a,x) - v(l_t(x))$. From this expression, we can see that the (normalized) incidence on welfare through the GE effects can have a similar representation as the one starting from the steady state,

$$(1-\beta)\sum_{t=0}^{\infty}\beta^t dr_t K_t \times (\Delta_{p,t} - \Delta_{f,t}),$$

where $\Delta_{p,t} = \frac{\iint u'(\tilde{c}_t(a,x))\left[\frac{a}{K_t}-\frac{xl(x)(1-T'(z(x)))}{L_t}\right]d\Phi_t(a,x)}{(1-\beta)\sum_{s=0}^{\infty}\beta^s\lambda_s}$ and $\Delta_{f,t} = \frac{\lambda_t\int(1+\epsilon^l_{w,t}(x))\frac{xl_t(x)}{L_t}T'(z_t(x))f(x)dx}{(1-\beta)\sum_{s=0}^{\infty}\beta^s\lambda_s}$. The decomposition in terms of summary statistics, however, does not work here because of the time varying externalities per unit price changes ($\Delta_{p,t}$ and $\Delta_{f,t}$).

Then the optimal tax formula can be obtained by $dW = 0$:

$$\frac{T'(z^*)}{1-T'(z^*)} = \frac{1+e}{e} \times \frac{1}{(1-\beta)\sum_{t=0}^{\infty}\beta^t \cdot \frac{x_t^* f(x_t^*)}{1-F(x^*)} \cdot \lambda_t} \times$$
$$\left[\begin{array}{l} (1-\beta)\sum_{t=0}^{\infty}\beta^t \cdot \lambda_t \cdot \frac{1-F(x_t^*)}{1-F(x^*)} \iint_{x_t^*}^{\infty} \left(1 - \frac{u'(\tilde{c}_t(a,x))}{\lambda_t}\right) \frac{\phi_t(a,x)}{1-F(x_t^*)} dxda \\ -(1-\beta)\sum_{t=0}^{\infty}\beta^t \cdot \iint \left[u'(\tilde{c}_t(a,x)) - \beta(1+r_{t+1})E[u'(\tilde{c}_t(a',x'))|x]\right] dh_{t+1}^A(a, y_t(x)) d\Phi_t(a,x) \\ +(1-\beta)\sum_{t=0}^{\infty}\beta^t dr_t K_t \iint u'(\tilde{c}_t(a,x)) \left[\frac{a}{K_t} - \frac{xl(x)(1-T'(z(x)))}{L_t}\right] d\Phi_t(a,x) \\ +(1-\beta)\sum_{t=0}^{\infty}\beta^t \cdot \lambda_t \cdot dw_t L_t \int (1+\epsilon_{w,t}^l(x)) \frac{xl_t(x)}{L_t} T'(z_t(x)) f(x) dx \end{array}\right].$$