

A Theory of Business Transfers

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Privately-owned firms

- Account for 1/2 of US business net income
- Dominate discussions on growth, wealth inequality, tax policy
- But pose challenge for
 - theory: technology of capital accumulation and transfer
 - measurement: no reliable data on private wealth

- Theory of private firm dynamics and capital reallocation
- Characterization of competitive equilibrium
- Administrative IRS data to discipline theory (*in progress*)
 - estimate share of transferable capital and investment technology
- Business transfers allows us to
 - back out model-based measure of business valuation
 - study effects of taxing business income, capital gains, wealth

- Buyers and sellers both report sale
 - seller has to pay capital gains
 - buyer has to report depreciable assets

- Price allocated across asset types
 - seller wants to allocate to long-term
 - buyer wants to allocate to short-term

⇒ Conflict of interest and thus consistent reporting

What do we know about Private Business Capital?

Form **8594** **Asset Acquisition Statement** OMB No. 1545-0074
Under Section 1060
 (Rev. November 2021) Department of the Treasury Internal Revenue Service **Attachment Sequence No. 109**
 ▶ Attach to your income tax return. ▶ Go to www.irs.gov/Form8594 for instructions and the latest information.

Name as shown on return _____ Identifying number as shown on return _____

Check the box that identifies you:
 Purchaser Seller

Part I General Information

1 Name of other party to the transaction _____ Other party's identifying number _____

Address (number, street, and room or suite no.) _____

City or town, state, and ZIP code _____

2 Date of sale _____ **3** Total sales price (consideration) _____

Part II Original Statement of Assets Transferred

4 Assets	Aggregate fair market value (actual amount for Class I)	Allocation of sales price
Class I	\$ _____	\$ _____
Class II	\$ _____	\$ _____
Class III	\$ _____	\$ _____
Class IV	\$ _____	\$ _____
Class V	\$ _____	\$ _____
Class VI and VII	\$ _____	\$ _____
Total	\$ _____	\$ _____

5 Did the purchaser and seller provide for an allocation of the sales price in the sales contract or in another written document signed by both parties? Yes No

If "Yes," are the aggregate fair market values (FMV) listed for each of asset Classes I, II, III, IV, V, VI, and VII the amounts agreed upon in your sales contract or in a separate written document? Yes No

6 In the purchase of the group of assets (or stock), did the purchaser also purchase a license or a covenant not to compete, or enter into a lease agreement, employment contract, management contract, or similar arrangement with the seller (or managers, directors, owners, or employees of the seller)? Yes No

If "Yes," attach a statement that specifies **(a)** the type of agreement and **(b)** the maximum amount of consideration (not including interest) paid or to be paid under the agreement. See instructions.

← Cash/securities
 ← Inventories
 ← Fixed assets
 ← Sec. 197 intangibles

What do we know about Private Business Capital?

- Transferred assets are primarily **intangible** (from form 8594 \approx 60%)
 - Customer bases and client lists, non-compete covenants
 - Licenses and permits, franchises, trademarks, tradenames
 - Workforce in place
 - Goodwill and on-going concern value
- Assets are **sold as a group**
- Sale **requires time** to find buyers/negotiate (from brokered data \approx 290 days)

⇒ Add intangible investment and transfers to Hopenhagen-style model

- Firm Dynamics
 - Hopenhayn (1992), Hsieh and Klenow (2009, 2014), Sterk et al. (2021)
- Capital Reallocation
 - Holmes and Schmitz (1990), Ottonello (2014), Guntin and Kochen (2020), Gaillard and Kankanamge (2020), David (2021)
- Entrepreneurship and Private Wealth
 - Cagetti and De Nardi (2006), Saez and Zucman (2016), Smith et al. (2019)
- Capital Gain Taxes and Wealth Taxes
 - Chari et al. (2003), Scheuer and Slemrod (2020), Guvenen et al. (2021), Agersnap and Zidar (2021)

- Infinite horizon, continuous time
- Demographics:
 - total population N : workers and business owners
 - newborns enter the economy, choose occupation, exit at rate δ
- Preferences: risk-neutral
- Workers supply labor inelastically

- Entry technology: entry cost $n_0 w$, draw $s \sim G(s)$, where $s = (z, k)$
- Productivity, z
 - non-transferable
 - evolves according to $dz = \mu z dt + \sigma z d\mathcal{B}$
- Business capital, k
 - built through investment: $dk = \theta - \delta_k$, convex cost $C(\theta)$
 - bilaterally traded
- Production technology:

$$y(s, n) = z(s)k(s)^\alpha n^\gamma$$

where n is a rentable input (today: labor)

Bilateral trade of capital:

- Firms access market at rate η
- Allocation between s and \tilde{s} : $k^m(s, \tilde{s}) \in \{k(s) + k(\tilde{s}), 0\} \Rightarrow$ indivisibility (extension: costly divisibility)
- Price paid by s to \tilde{s} : $p^m(s, \tilde{s})$ price paid to \tilde{s} (negative if selling)

Spot market for labor:

- Labor demand: i) production: $n(s; w) = \left(\frac{\gamma z(s) k(s)^\alpha}{w} \right)^{\frac{1}{1-\gamma}}$; ii) entry: n_0 workers per firm

- The owner's value solves the following HJB

$$\begin{aligned}(r + \delta)V(s) = & \underbrace{\max_n y(s, n) - wn}_{\text{production}} + \underbrace{\max_{\theta} \partial_k V(s)(\theta - \delta_k) - C(\theta)}_{\text{investment}} \\ & + \underbrace{\mu z \partial_z V(s) + \frac{1}{2} \sigma^2 z^2 \partial_{zz} V(s)}_{\text{evolution of productivity}} + \underbrace{\max_{\lambda} \eta W(s; \lambda)}_{\text{trade}}\end{aligned}$$

where

$$W(s; \lambda) = \sum_{\tilde{s}} [V(z, k^m(s, \tilde{s})) - V(z, k) - p^m(s, \tilde{s})] \lambda(s, \tilde{s})$$

and

$$\sum_{\tilde{s}} \lambda(s, \tilde{s}) + \lambda(s, 0) = 1$$

- Occupational Choice ("free-entry")

$$\int V(s) dG(s) - n_0 w \leq \frac{w}{r + \delta}, \quad \phi_e \geq 0, \quad w/ \text{ c.s.}$$

- Labor supply: $N - \phi_e$
- Distribution over the state space ϕ evolves according to the Kolmogorov Forward (KF) equation

$$\dot{\phi} = \Gamma(\theta, \lambda; \phi) + \phi_e$$

- Evolution of ϕ induced by
 - ▶ investment
 - ▶ trade
 - ▶ individual productivity process
 - ▶ entry/exit

Definition of Recursive Equilibrium

A (stationary) equilibrium is a set of value functions $V(s)$, policy functions for investment $\theta(s)$ and trade $\lambda(s, \tilde{s})$, terms of trade $(k^m(s, \tilde{s}), p^m(s, \tilde{s}))$, wage w , and distribution over the state space $\phi(s)$ that satisfy

- business owners' optimality
- no-arbitrage in occupational choice
- market clearing
- consistency of measures

⇒ Trade of multiple differentiated goods

- Standard approach:
 - CES demand/monopolistic competition
 - frictional market with fixed point on matching set
- Our model:
 - frictionless matching: competitive equilibrium + stochastic trade opportunity
 - block structure, flexible trade module: $(\phi, V) \rightarrow (\lambda, p^m, k^m) \rightarrow (\phi', V')$

- With transferable utility, solution is linear programming problem
 - maximize (static) social surplus s.t. adding up constraints
- Delivers equilibrium allocation (λ, k^m) and prices (p^m)
- Gains from trade $W(s)$ from envelope theorem
- Easy to extend to non-transferable utility environment

details

- **Competitive prices** are independent of seller's z

$$p^m(s, \tilde{s}) = \mathcal{P}(\kappa(\tilde{s}))$$

Intuition: competitive nature of the equilibrium, same good sold at same price

- **Pairwise stability:** $\exists(s, \tilde{s})$ and feasible trade that makes the pair (strictly) better off
- **Competitive allocation** solves the planner's problem starting at $\phi(s, 0) = \phi^{ss}(s)$

- Calibration using data on
 - firm dynamics
 - business transfers
- Model deliverables
 - dispersion in mpk
 - business price and value
- Tax Policy Analysis

Parameter	Value
Discount rate	$r = 0.06$
Share of rentable input	$\gamma = 0.30$
Entry distribution, G	mass point at $z = z_0, k = 1$
Death rate, depreciation rate	$\delta = 0.1, \delta_k = 0.058$
Investment cost, $C(\theta) = A\theta^\rho$	$A = 20, \rho = 2$
Trading rate	$\eta = 1$
Returns to scale	$\alpha = 0.5$
Productivity process	$\mu = 0, \sigma = 0.15$

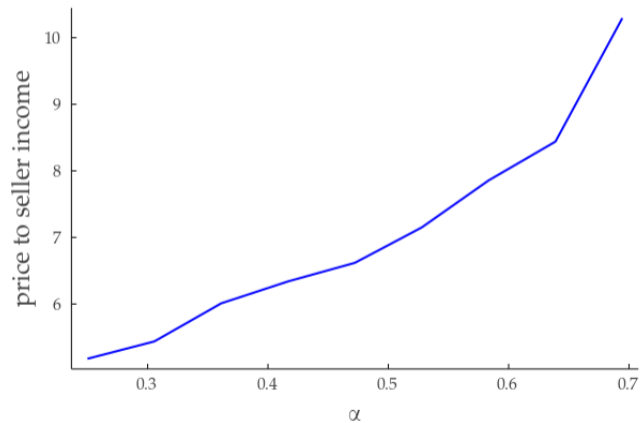
Key parameters

- meeting rate η
- investment cost $C(\theta) = A\theta^p$
- output elasticity wrt k , $y = zk^\alpha$

Key moments from data

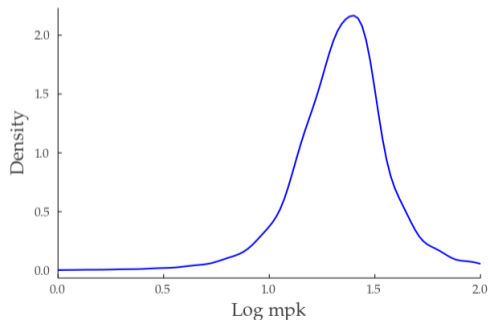
- external data on time to sell and/or trade volume (brokered sales)
- relationship between prices and income (IRS filings)

- $\alpha \uparrow \Rightarrow k$ share of seller's output $\uparrow \Rightarrow p/y_s \uparrow$



Dispersion in MPK

- Idiosyncratic change in productivity \rightarrow input reallocation toward higher MPK
- Dispersion in marginal product of capital induced by
 - decentralized trading
 - indivisibility of asset sold



- Finance textbook: Present value of owner's dividend
 - Model counterpart: $V(s)$

- SCF respondent: Answer to the survey question—"What could you sell it for?"
 - Model counterpart: $\mathcal{P}(k(s))$

Model Predictions for Business Wealth

- Heterogeneity in transferable share and returns
- Inputs to analysis of capital and wealth taxation

Distribution Pctile	Transferable Share	Income Yield
	$\frac{\mathcal{P}(k(s))}{V(s)}$	$\frac{y(s) - C(\theta(s))}{V(s)}$
5	0.00	-0.18
25	0.24	-0.01
50	0.36	0.04
75	0.51	0.08
95	0.68	0.13
99	0.80	0.15

- Recent debate on business taxation
- What to tax
 - flows: business income
 - stocks: business capital or wealth
 - transfers: capital gains
- Our model is well-suited to address this question

[details](#)

Comparison of

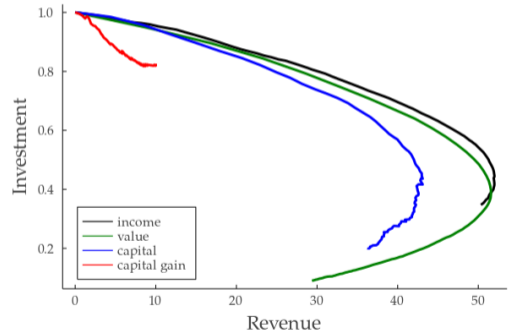
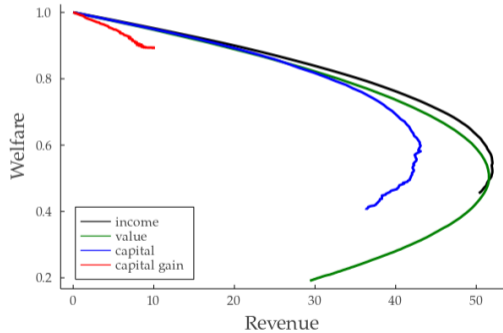
- **capital gains:** $\tau_c \mathcal{P}(k(s))$
- **business income:** $\tau_b y(s)$
- **business capital:** $\tau_k \mathcal{P}(k(s))$
- **wealth:** $\tau_v V(s)$

Welfare measure: steady-state value at entry conditional on raising revenue R

- by indifference at entry, all agents' ex-ante value is proportional to w

Main Results

- For most levels of R , exclusively use τ_b



⇒ 3 margins: entry, investment, reallocation

Compared with tax on income,

- tax on capital gains
 - distorts capital reallocation across firms
 - decreases investment to sell
- tax on business capital
 - increases taxes on low z , high k ⇒ investment ↓
 - lowers taxes on high z , low k (but z is inelastic!)
- tax on wealth taxes option value of selling capital

Practical implementation: k and V are not observed

- Better estimation using IRS data:
 - life-cycle dynamics
 - production and investment technology
- Full study of tax policy
 - undiversifiable risk
 - financial constraints
 - alternative instruments

Lemma

- $W(s) = \frac{\partial Q}{\partial \phi(s)} = \frac{\mu^a(s) + \mu^b(s)}{2} \equiv \mu(s) \Rightarrow$ HJB
- $\lambda(s, \tilde{s}) = \frac{2\pi(s, \tilde{s})}{\phi(s)} \Rightarrow$ KF
- $k^m(s, \tilde{s}) = \arg \max X(s, \tilde{s}) \quad p^m(s, \tilde{s}) = V(z, k^m(s, \tilde{s})) - V(z, k) - W(s) \Rightarrow$ Data

$$X(s, \tilde{s}) = \max_{k^m \in \{k(s) + k(\tilde{s}), 0\}} \{V(z(s), k^m) + V(z(\tilde{s}), k(s) + k(\tilde{s}) - k^m)\} - (V(s) + V(\tilde{s}))$$

$$Q(\phi) = \max_{\pi \geq 0} \sum_{s, \tilde{s}} X(s, \tilde{s}) \pi(s, \tilde{s})$$

$$s.t. \quad \sum_{\tilde{s}} \pi(s, \tilde{s}) + \pi(s, 0) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^a(s)]$$

$$\sum_{\tilde{s}} \pi(\tilde{s}, s) + \pi(0, s) = \frac{\phi(s)}{2} \quad \forall s \quad [\mu^b(s)]$$

- From the minimax thm, the solution of the primal problem is equal to the solution of the dual
- The multipliers in the primal are equal to the choice variable in the dual, and vice versa

$$Q(\phi) = \min_{\mu^a \geq 0, \mu^b \geq 0} \sum_s (\mu^a(s) + \mu^b(s)) \frac{\phi(s)}{2}$$
$$s.t. \quad \mu^a(s) + \mu^b(\tilde{s}) \geq X(s, \tilde{s}) \quad \forall s, \tilde{s} \quad [\pi(s, \tilde{s})]$$

- After-trade values for buyers (v_b) and sellers (v_s)
 - $v_b(s, \hat{k}; p)$: value from buying \hat{k}
 - $v_s(s, 0; p)$: value from selling $k(s)$
- Matching probability

$$\lambda(s, \hat{k}; p) = \exp\left(\frac{v_b(s, \hat{k}; p) - W(s)}{\sigma}\right)$$

$$\lambda(s, 0; p) = \exp\left(\frac{v_s(s, 0; p) - W(s)}{\sigma}\right)$$

where $W(s) = \mathbb{E} \max\{v_b(s, \hat{k}; p), v_s(s, 0; p)\}$

- Find $\{p(s)\}$ such that $\forall \hat{k}$

$$\underbrace{\int \lambda(s, \hat{k}; p)}_{\text{demand}} = \underbrace{\int \lambda(s, 0; p) \mathbb{I}\{k(s) = \hat{k}\}}_{\text{supply}}$$

- Under capital gain tax τ ,

$$\begin{aligned}v_b(s; \hat{k}) &= V(z, k(s) + \hat{k}) - p(\hat{k}) \\v_s(s) &= V(\tilde{s}, 0) + (1 - \tau)p(k(s))\end{aligned}$$

- Under cap on paid price equal to $\psi y(s)$

$$v_b(s; \hat{k}) = \begin{cases} V(z, k(s) + \hat{k}) - p(\hat{k}) & \text{if } p(\hat{k}) \leq \psi y(s) \\ -\infty & \text{o/w} \end{cases}$$
$$v_s(s) = V(\tilde{s}, 0) + p(k(s))$$

Feasibility and Pair-wise stability

Terms of trade $\{p^m, k^m\}$ satisfy

- feasibility

$$k^m(s, \tilde{s}) \in \{k(s) + k(\tilde{s}), 0\}$$

$$k^m(s, \tilde{s}) + k^m(\tilde{s}, s) \leq k(s) + k(\tilde{s})$$

$$p(s, \tilde{s}) + p(\tilde{s}, s) \geq 0$$

- pair-wise stability: $\nexists(s, \tilde{s})$ and feasible trade that makes the pair (strictly) better off