A Parsimonous Model of Idiosyncratic Income

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Motivation

[T]he key challenge for future work is to develop a specification for the ***wage process*** *that is both* ***parsimonious*** *enough to be used as an input to incomplete-markets models, and rich enough to* ***account empirically for the covariance structure of wages in both levels and differences****.*

Heathcote, Perri, and Violante (2010, p. 40)

Motivation

The standard model:

$$y_t = p_t + \varepsilon_t + \theta\varepsilon_{t-1}$$
$$p_t = p_{t-1} + \nu_t$$

can be estimated using level moments, $Cov(y_t, y_s)$, or difference moments, $Cov(\Delta y_t, \Delta y_s)$.

But choice of moments gives different results:

	Level moments	Difference moments
σ^2_{perm}	0.004	0.011
σ^2_{tran}	0.032	0.020

i.e., the standard model is *misspecified*.

What We Do

Propose minimal changes to the standard model:

1. Shocks are spread uniformly through the year (time-aggregation problem)
2. Two types of transitory shocks (“bonus” and “passing”)

Still parsimonious: only one extra parameter to estimate

Main Results

1. We estimate the standard model on simulated data from proposed model:

   Same misspecification ‘structure’ as in actual data

2. We estimate the proposed model in Norwegian data:

   Similar parameter estimates irrespective of level or difference

   Same for PSID

Literature

Misspecification in the standard model: Daly, Hryshko, and Manovskii (2022)

Time aggregation: Working (1960), Eika (2018), Crawley (2020).

Richer models of income dynamics: Arellano, Blundell, and Bonhomme (2017), Druedahl, Graber, and Jørgensen (2021), Guvenen, Karahan, Ozkan, and Song (2021), Guvenen, McKay, and Ryan (2022), ...

Outline

Data

Norwegian administrative data on annual income, 1971 – 2014

Main variable: log residualized pre-tax earnings (year, age, and education dummies)

Sample selection (Daly, Hryshko & Manovskii, 2022)

- males aged 35-50
- drop extreme income changes ($> 500\%$) and low income ($<$ approx USD 10,000)
- balanced panel

More than 500k individuals, in 27 cohorts, observed for 15 years each.

Outline

The Standard Model

Income process:

$$y_t = p_t + \varepsilon_t + \theta\varepsilon_{t-1}$$
$$p_t = p_{t-1} + \nu_t$$

p = permanent component, unit root

ε = transitory component

Parameters to be estimated: $\sigma^2_{\nu_t}$, $\sigma^2_{\varepsilon_t}$, θ, $\sigma^2_{p_0}$.

Estimation Procedure

Compute empirical level or difference moments:

$$L_{t,s} = \frac{1}{N}\sum_{i=1}^{N} y_{i,t} y_{i,s} \qquad \text{or} \qquad D_{t,s} = \frac{1}{N}\sum_{i=1}^{N} \Delta y_{i,t} \Delta y_{i,s}$$

where $y_{i,t}$ is residualized log income and minimize the loss function:

$$\mathcal{L} = \textit{vech}(D_{\text{data}} - D_{\text{model}})^T \Omega^{-1} \textit{vech}(D_{\text{data}} - D_{\text{model}})$$

Where Ω is either

- identity matrix (**Identity**)
- optimal minimum distance weighting matrix (**Optimal**)
- optimal matrix along the diagonal (**Diagonal**)

Results: Estimating the Standard Model with Norwegian Data

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
σ^2_{perm}	0.004	0.011	0.004	0.011	0.005	0.007
σ^2_{tran}	0.032	0.020	0.033	0.020	0.021	0.021
θ	0.570	0.070	0.574	0.071	0.163	0.145
σ^2_{init}	0.062	✗	0.062	✗	0.059	✗

1. Difference in permanent variance
2. Difference in transitory variance opposite of permanent variance
3. **Optimal**: 'similar' results irrespective of level or difference (Daly, Hryshko & Manovskii, 2022)

Outline

The Proposed Model

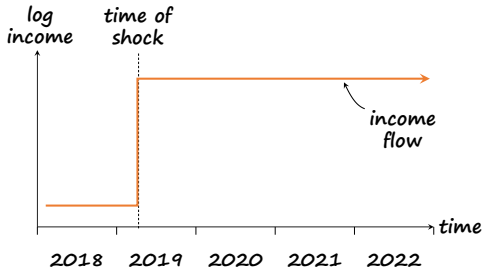
Two adjustments

1. We allow shocks to happen throughout the year (in continuous time)
2. We allow for three types of income shocks

1 Permanent Income Shocks
2 'Bonus' Income Shocks
3 'Passing' Income Shocks

} Transitory Shocks

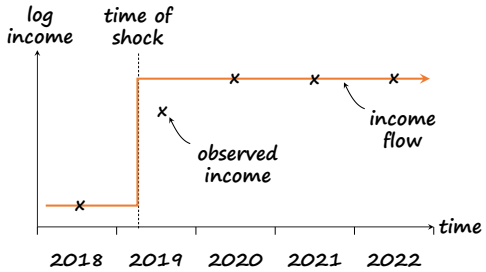
Shock Type 1: Permanent



(a) Income flow

(b) Covariance structure for difference moments

Shock Type 1: Permanent



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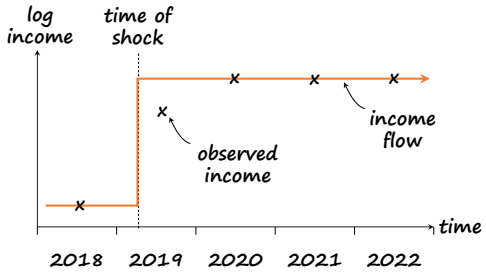
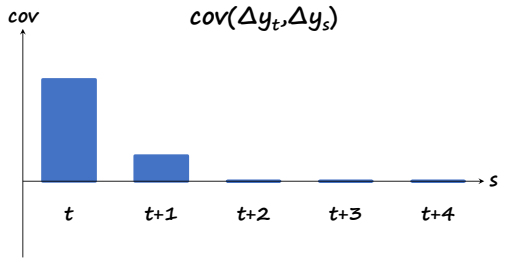
Shock Type 1: Permanent



(a) Income flow



(b) Covariance structure for difference moments

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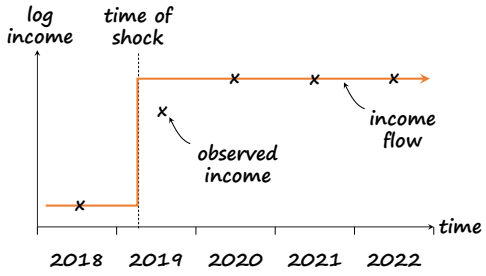

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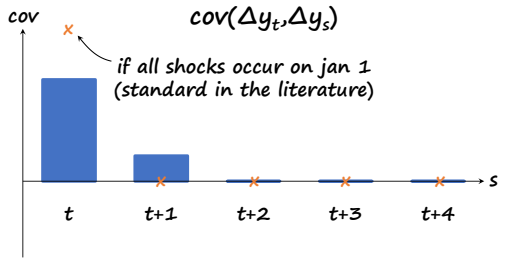
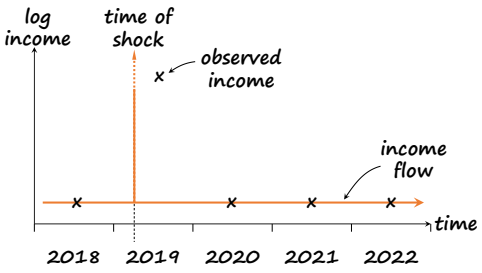


(b) Covariance structure for difference moments

Shock Type 2: Bonus



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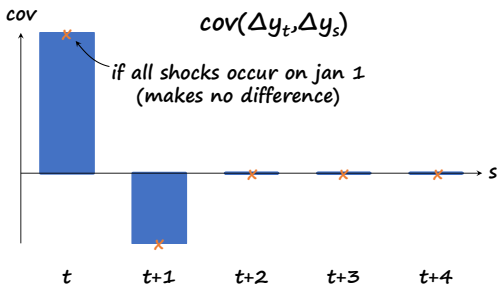
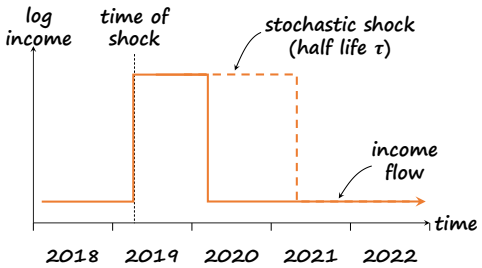


(b) Covariance structure for difference moments

Shock Type 3: Passing



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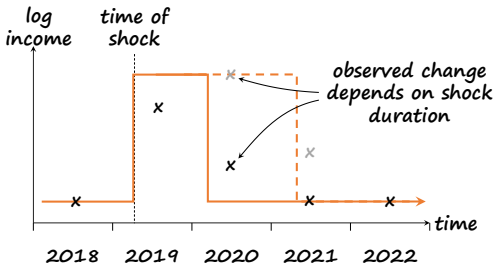

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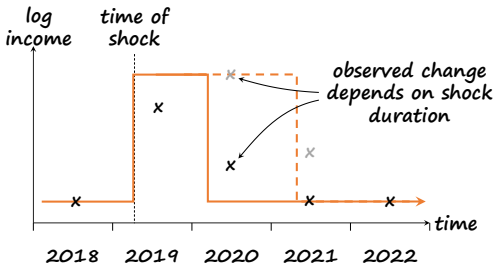

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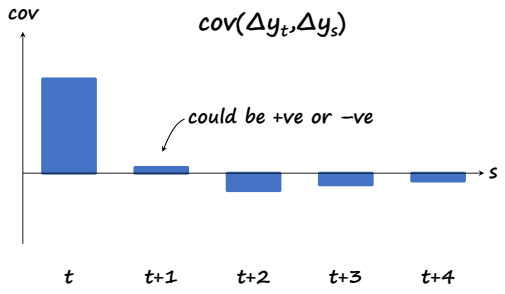

(b) Covariance structure for difference moments

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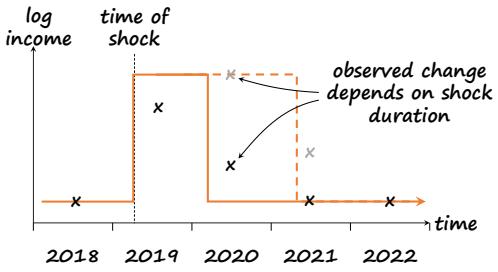

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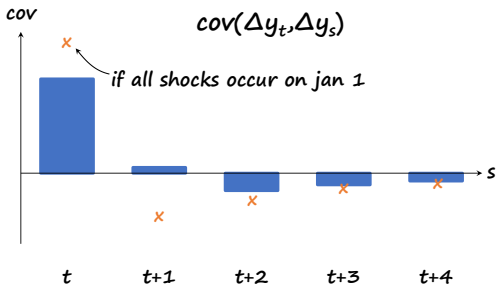

(b) Covariance structure for difference moments

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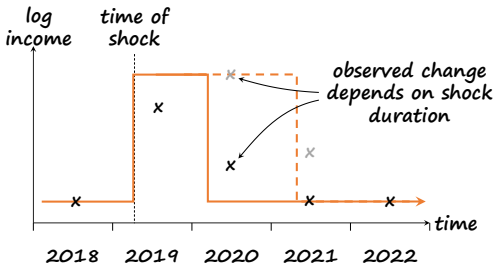

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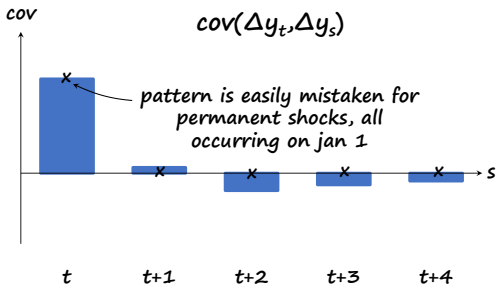


(b) Covariance structure for difference moments

Mapping between standard and proposed model parameters

Parameter Description	Proposed	Standard
Permanent income variance	σ^2_{perm}	$\text{Var}(\nu)$
Transitory income variance	σ^2_{tran}	$(1+\theta^2)\text{Var}(\varepsilon)$
Half life of passing shock	τ	✗
Bonus fraction of σ^2_{tran}	b	✗
MA(1) transitory persistence	✗	θ
Initial permanent income variance	σ^2_{init}	$\text{Var}(p_0)$

Outline

Simulation Exercise

Simulate 50k individuals using discretized (monthly) version of proposed model

Estimate standard model on the simulated data

Simulation Results

▸ Persistence

		Identity		Diagonal		Optimal	
Parameter	True Value	Level	Difference	Level	Difference	Level	Difference
σ^2_{perm}	0.005	0.005	0.013	0.005	0.013	0.008	0.009
σ^2_{tran}	0.038	0.031	0.017	0.032	0.017	0.021	0.020
τ	2.0 years	✗	✗	✗	✗	✗	✗
b	0.40	✗	✗	✗	✗	✗	✗
θ	✗	0.50	0.10	0.53	0.10	0.18	0.16
σ^2_{init}	0.065	0.065	✗	0.065	✗	0.064	✗

Main results:

1. Same 'structure' of misspecification as when using data ▸ Data Results
2. **Optimal** gives similar parameter estimates regardless of moments (DHM, 2022)
3.

Intuition I: Daly, Hryshko & Manovskii (2022) result

Result:

Optimal weighting gives similar parameter estimates regardless of level or difference moments.

Intuition:

Optimal estimation is invariant to *invertible* linear mappings of the moments used.

Level to Difference moments is *almost* invertible for long panels

Simulation Results

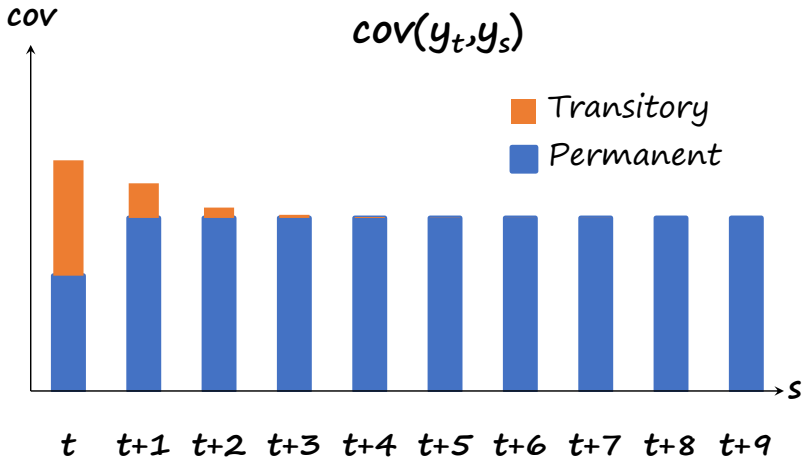
▸ Persistence

		Identity		Diagonal		Optimal	
Parameter	True Value	Level	Difference	Level	Difference	Level	Difference
σ^2_{perm}	0.005	0.005	0.013	0.005	0.013	0.008	0.009
σ^2_{tran}	0.038	0.031	0.017	0.032	0.017	0.021	0.020
τ	2.0 years	✗	✗	✗	✗	✗	✗
b	0.40	✗	✗	✗	✗	✗	✗
θ	✗	0.50	0.10	0.53	0.10	0.18	0.16
σ^2_{init}	0.065	0.065	✗	0.065	✗	0.064	✗

Main results:

1. Same 'structure' of misspecification as when using data ▸ Data Results
2. **Optimal** gives similar parameter estimates regardless of moments (DHM, 2022)
3. Level moments with **Identity**/**Diagonal** perform well

Intuition II: Level + Identity performs well



Outline

Data Results

Exercises:

1. Results using Norwegian data
2. Results using the PSID
3. Time-varying results using the PSID

Norwegian Results

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
Panel A: Proposed Model						
σ^2_{perm}	0.003	0.005	0.003	0.005	0.003	0.004
σ^2_{tran}	0.039	0.038	0.038	0.039	0.039	0.036
τ	1.547	2.045	1.770	2.058	2.234	1.862
b	0.360	0.478	0.341	0.473	0.443	0.480
σ^2_{init}	0.062	✗	0.062	✗	0.059	✗
Panel B: Standard Model						
σ^2_{perm}	0.004	0.011	0.004	0.011	0.005	0.007
σ^2_{tran}	0.032	0.020	0.033	0.020	0.021	0.021
θ	0.570	0.070	0.574	0.071	0.163	0.145
σ^2_{init}	0.062	✗	0.062	✗	0.059	✗

Norwegian Results (No Bonus Shock)

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
σ^2_{perm}	0.003	0.011	0.003	0.011	0.005	0.007
σ^2_{tran}	0.036	0.021	0.036	0.021	0.023	0.022
τ	0.982	0.065	1.202	0.080	0.231	0.157
σ^2_{init}	0.064	✗	0.063	✗	0.060	✗

Norwegian Results (Persistence)

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
Panel A: Proposed Model						
σ^2_{perm}	0.004	0.006	0.004	0.006	0.003	0.005
σ^2_{tran}	0.038	0.034	0.041	0.035	0.039	0.033
τ	1.494	1.810	1.328	1.815	2.241	1.640
b	0.390	0.540	0.407	0.526	0.443	0.525
ρ	0.991	0.979	0.982	0.985	1.000	0.989
σ^2_{init}	0.063	✗	0.063	✗	0.059	✗
Panel B: Standard Model						
σ^2_{perm}	0.010	0.014	0.010	0.015	0.011	0.010
σ^2_{tran}	0.022	0.017	0.022	0.016	0.019	0.019
θ	0.216	0.026	0.233	0.000	0.088	0.092
ρ	0.923	0.894	0.921	0.809	0.926	0.952
σ^2_{init}	0.063	✗	0.062	✗	0.059	✗

PSID

Sample from Moffit & Zhang (2018): male heads, 1970-2014, 30-59 years old, not full-time students, positive weeks worked, wage and salary earnings, exclude non-sample men

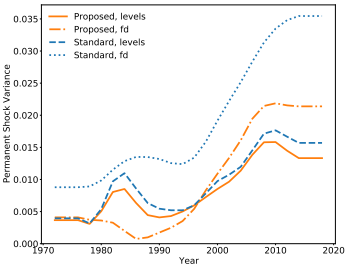
Only even-year observations: cannot identify MA(1) parameter θ

Balanced panels (Daly, Hryshko & Manovskii, 2022)

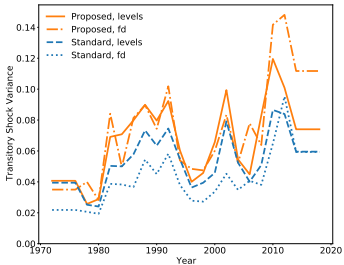
PSID Results

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
Panel A: Proposed Model						
σ^2_{perm}	0.010	0.008	0.008	0.010	0.006	0.007
σ^2_{tran}	0.065	0.080	0.066	0.074	0.053	0.051
τ	1.407	1.216	1.432	1.374	1.368	1.370
b	0.264	0.347	0.307	0.417	0.373	0.472
σ^2_{init}	0.078	✗	0.092	✗	0.105	✗
Panel B: Standard Model						
σ^2_{perm}	0.012	0.021	0.010	0.019	0.009	0.010
σ^2_{tran}	0.047	0.044	0.054	0.042	0.034	0.036
σ^2_{init}	0.069	✗	0.086	✗	0.105	✗

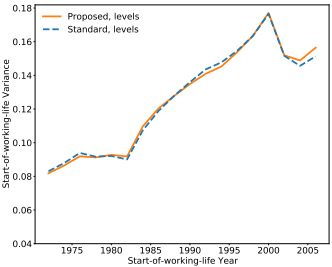
Time-varying Results, PSID



(a) Permanent income variance



(b) Transitory income variance



(c) Start-of-working-life variance

Outline

Conclusion

We propose a new model of idiosyncratic income. Two changes to standard model:

- **Continuous time** to solve time aggregation problem
- **Three types of income shocks** to match data.

The model is **parsimonious** and has **no obvious misspecification** issues.

- Only one extra parameter to be estimated.
- Parameter estimates are similar across moments and weighting matrix applied.

For practitioners:

- Use our model!
- If you for some reason have to use the standard model: use level moments!

Standard Model Estimated in Data

Back

	Identity		Diagonal		Optimal	
	Level	Difference	Level	Difference	Level	Difference
σ^2_{perm}	0.004	0.011	0.004	0.011	0.005	0.007
σ^2_{tran}	0.032	0.020	0.033	0.020	0.021	0.021
θ	0.570	0.070	0.574	0.071	0.163	0.145
σ^2_{init}	0.062	✗	0.062	✗	0.059	✗

Simulation Results (Persistence)

Back

		Identity		Diagonal		Optimal	
Parameter	True Value	Level	Difference	Level	Difference	Level	Difference
σ^2_{perm}	0.005	0.008	0.015	0.009	0.015	0.013	0.013
σ^2_{tran}	0.038	0.026	0.015	0.027	0.015	0.017	0.017
τ	2.0 years	✗	✗	✗	✗	✗	✗
b	0.40	✗	✗	✗	✗	✗	✗
θ	✗	0.38	0.07	0.39	0.07	0.12	0.11
ρ	1.00	0.97	0.93	0.97	0.93	0.94	0.95
σ^2_{init}	0.065	0.062	✗	0.063	✗	0.064	✗