Global demand for financial assets, falling real interest rates and macroeconomic instability[*]

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July 1, 2022

Abstract

The sharp, secular decline in the world real interest rate of the past thirty years suggests that the observed surge in global demand for financial assets outpaced the growth in their supply. We argue that this phenomenon was driven by (i) faster growth in emerging markets, and (ii) changes in the financial structure of both emerging and advanced economies. We then show that the low-interest-rate environment made the world economy more vulnerable to financial crises. These findings are the quantitative predictions of a two-region model in which privately-issued financial assets (i.e., inside money) provide productive services and private debt can be defaulted on.

1   Introduction

Four key facts illustrated in Figure 1 highlight major changes in the world economy during the last three decades:

1. Emerging market economies (EMs) grew much faster than advanced economies. As shown in the first panel of the Figure, their GDP relative to that of advanced economies, measured in US dollars, rose from 28 to 68 percent between 1991 and 2020. Valuing GDP in PPP units, instead, yields an increase from 57 to 125 percent. Thus, the growth in the relative size of emerging economies is evident even setting aside real-exchange-rate movements.

2. The net foreign liabilities of advanced economies grew massively (a fact often labeled 'global imbalances'). As the second panel of the Figure shows, the net foreign assets (NFA) of advanced economies, as a share of their collective GDP, fell from close to zero at the beginning of the 1990s to about -20 percent in 2020.

3. The financial structure of both emerging and advanced economies changed so as to produce significant growth in credit to the private sector. The third panel of the Figure shows that private domestic credit as a percentage of GDP roughly tripled in EMs in the last 30 years and grew about half as much in advanced economies. Domestic credit as a share of GDP in EMs remains below that of advanced economies but the gap has narrowed markedly. This large expansion in worldwide financial intermediation could be driven by the growth in demand for financial assets and/or the growth in supply (i.e., issuance of liabilities). Whether demand or supply grew faster is important for determining the direction of the response of the equilibrium interest rate, which brings us to the last key fact.

4. The real interest rate fell sharply. The fourth panel of the Figure plots the ex-post real interest rate on U.S. long-term public debt, a proxy for the risk-free world interest rate. Starting from about 4 percent at the beginning of the 1990s, the real interest rate followed a declining trend reaching values close to zero at the end of 2020. Measures of expected real interest rates based on inflation expectations embedded in the pricing of inflation-indexed treasury bills also show significant

declines. The market yield on 10-year U.S. TIPS at constant maturity fell from 2.29 percent in January 2003 to -1 percent at the end of 2020.[1] This sharp drop in real interest rates suggests that the global demand for financial assets increased at a faster pace than the supply.
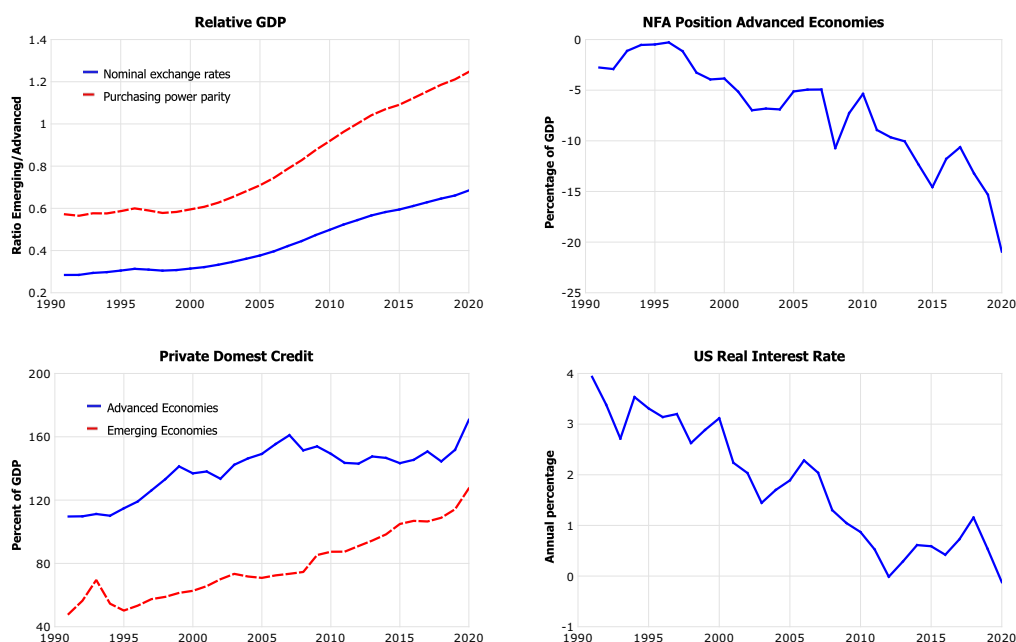

Figure 1: Real and Financial Trends in Advanced and Emerging Countries.

Note: **Emerging economies**: Argentina, Brazil, Bulgaria, Chile, China, Hong.Kong, Colombia, Estonia, Hungary, India, Indonesia, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Advanced economies**: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United.Kingdom, United.States. **Sources**: World Development Indicators (World Bank) and External Wealth of Nations database (Lane and Milesi-Ferretti (2018)).

The trends in the world economy documented in Figure 1 emerged during a period marked by financial globalization and a surge in the occurrence of financial crises. Well-established measures of de-jure and de-facto

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[1]The market yield is from FRED available at fred.stlouisfed.org/series/DFII10.

international capital mobility show the rapid progress of financial globalization as barriers to capital mobility were sharply reduced (see Chinn and Ito (2006)) and both gross external assets and liabilities grew in a large number of countries (see Lane and Milesi-Ferretti (2007)).[2] The increase in the frequency of financial crises is documented in well-known empirical studies (e.g. Reinhart and Rogoff (2009)). They show that there were no financial crises in advanced economies between 1940 and 1973 and only a handful between 1973 and 1990. Since then, between 15 and 20 crises have occurred, depending on the study one considers. Crises in emerging economies were also rare between 1940 and the onset of the sovereign debt crises of the 1980s, and the number of crises rose sharply after 1990 (see the survey by Sufi and Taylor (2021)).

This paper has two main goals. The first is to identify and measure the factors that caused the rise in net demand for financial assets—relatively to the growth in supply—and the drop in the world real interest rate. The second is to assess the implications of these changes for global financial and macroeconomic volatility. We do this through the lens of a quantitative model of two regions, one representative of emerging economies and the other representative of advanced economies.

In each region, there is a borrowing sector and a lending sector. Financial assets have features that make them akin to 'inside money.' They are issued by private agents—the debtors—and embody a 'convenience yield' to the holders—the creditors. The convenience yield emerges from the assumption that financial assets can be used in production. The debtors cannot commit to repay their liabilities and, as a result, private debt can be defaulted on.

A financial crisis occurs when the debt repayment is lower than the liquidation value of the debtors' real assets. This generates haircuts in credit recovery and, therefore, a financial crisis causes wealth redistribution from creditors to debtors. This redistribution is the central mechanism that causes real macroeconomic consequences. Importantly, the magnitude of these consequences depends on the changing structure of financial intermediation, which in the model is driven by exogenous structural changes, described below, as well as endogenous general equilibrium ad-

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[2]The latest update of the Chinn-Ito Index of financial openness is available at web.pdx.edu/ĩto/Chinn-Ito_website.htm and the latest update of the Lane-Milesi-Ferretti External Wealth of Nations database is available at www.brookings.edu/research/the-external-wealth-of-nations-database.

justments.

In each region we consider changes in three exogenous variables: (i) productivity, (ii) a structural parameter that affects the demand for financial assets, and (iii) a structural parameter that affects the supply of financial assets. We then use the model in conjunction with the data plotted in Figure 1 to identify and measure these changes over the same sample period. Finally, we conduct counterfactual simulations to assess their contribution to the observed trends as well as to macroeconomic and financial volatility.

We find that the measured exogenous changes in productivity and financial structure both contributed to increase macroeconomic and financial volatility over the 1991-2020 period. They raised the demand for financial assets relatively to the supply, causing the decline in the interest rate. The lower interest rate then encouraged higher effective levels of leverage which in turn increased financial and macroeconomic volatility.

The observed interest rate decline and NFA dynamics are key for the identification of the financial changes. As mentioned above, the reduction in the interest rate indicates that the worldwide growth in demand for financial assets outpaced the growth in their supply. NFA dynamics are important for determining in which countries the demand for financial assets grew more than the supply. In particular, the fact that the net liabilities of advanced economies widened over the sample period indicates that the net demand for assets in these countries increased less than in EMs.

Related literature. Our work is related to three important strands of literature: the literature on global imbalances, the literature on financial crises or Sudden Stops, and the literature on the growth of financial assets or corporate cash holdings. While the first two strands of literature are the field of international macroeconomics, the third includes many studies in the corporate finance field.

Research on global imbalances proposes several theories to explain the growth in NFA positions of emerging economies. One explanation is based on the idea that emerging economies have a lower ability to create viable saving instruments for inter-temporal smoothing (Caballero, Farhi, and Gourinchas (2008)). Another explanation argues that emerging economies have a higher demand for assets due to lower insurance, or lower financial development related to weaker enforcement (Mendoza, Quadrini, and

Ríos-Rull (2009)) or because of higher idiosyncratic uncertainty (Carroll and Jeanne (2009), Angeletos and Panousi (2011), Song, Storesletten, and Zilibotti (2011), Sandri (2014), Bacchetta and Benhima (2015), Fogli and Perri (2015)). The first theory highlights cross-country heterogeneity in the supply of assets while the second emphasizes heterogeneity in the demand. In both cases, emerging economies turn to advanced economies for the acquisition of saving instruments (financial assets).

Our model incorporates both types of heterogeneity between advanced and emerging economies. Importantly, the aim of our paper is not to examine why advanced economies are borrowing from emerging economies, which is the focus of the above referenced studies. Our paper, instead, has two objectives that are relatively new to this literature: The first objective is to 'measure' how the heterogeneity in both demand and supply has changed over time. The second objective is to explore how the change has affected macroeconomic and financial stability.

Various studies in the Sudden Stops literature examine the role of financial globalization, credit booms and high leverage as causing factors of financial crises. Examples include Calvo and Mendoza (1996), Caballero and Krishnamurthy (2001), Gertler, Gilchrist, and Natalucci (2007), Edwards (2004), Mendoza and Quadrini (2010), Mendoza and Smith (2014), Fornaro (2018).[3] Some of these studies emphasize mechanisms that cause financial crises because of equilibrium multiplicity due to self-fulfilling expectations[4]. Crises in our model also follow from periods of fast credit and leverage growth, and they are also the result of self-fulfilling expectations. However, the mechanism that operates in our model differs in that it relies on the interaction between the inside-money-like role of financial assets for creditors with the debtors' lack of commitment to repay, which could lead to debt renegotiation (financial crisis). The consequent redistribution of wealth could have important effects for the real economy.

Several studies in the corporate finance literature document and provide explanations for the raising demand of financial assets. An example is the literature on the growing cash holdings of nonfinancial businesses (e.g., Busso, Fernández, and Tamayo (2016) and Bebczuk and Cavallo (2016)). Our model has a similar feature in that entrepreneurs hold positive positions in financial assets that expand as a result of faster growth

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[3]See Bianchi and Mendoza (2020) for a survey of the literature.

[4]For example, Aghion, Bacchetta, and Banerjee (2001), Perri and Quadrini (2018)

of emerging economies and changes in financial structure in both emerging and advanced economies. Our focus, however, is on the macroeconomic implications. Through the lens of the structural model we show that the increase in net demand for financial assets depresses the interest rate which in turn increases the incentives to leverage. While the higher leverage allows for sustained levels of financial intermediation and economic activity, it also makes the economies of emerging and advanced economies more vulnerable to crises (global macroeconomic instability).

The remainder of the paper is organized as follows. Section 2 describes the model and characterizes the equilibrium. Section 3 uses the model in conjunction with the data plotted in Figure 1 to construct empirical series for productivity and exogenous variables that impact directly the demand and supply of assets. We then conduct counterfactual simulations to decompose the role played by changes in productivity and changes in financial structure for generating the observed trends. Section 4 analyzes the implications of the structural changes for macroeconomic and financial instability. Section 5 concludes.

2 Model

Consider a world economy that consists of two countries/regions indexed by $j \in \{1, 2\}$. Country 1 is representative of advanced economies and Country 2 is representative of emerging economies. In each country, there are two sectors: (i) an entrepreneurial sector that produces final output and (ii) a consolidated household/business sector that holds capital and supplies labor. By having two sectors we can generate borrowing and lending. This allows us to have in each country a clear distinction between the 'demand' from financial assets (from the sector that has a positive financial position, the creditors) and the 'supply' of financial assets (from the sector that has a negative financial position, the debtors).[5]

Countries are heterogeneous in three dimensions: (i) economic size formalized by differences in aggregate productivity, $z_{j,t}$; (ii) a financial parameter that affects directly the demand for financial assets, $\phi_{j,t}$; and (iii) a

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[5]We interpret the business sector that is consolidated with the household sector as composed of firms that hold physical capital with high collateral value. In that sense, these firms are similar to households holding real estate. High collateral value allows both households and firms to borrow. Keeping the household sector separate from the business sector would not change the key properties of the model.

financial parameter that affects directly the supply of financial assets, $\kappa_{j,t}$.

Although differences in economic size could be generated in the model by other factors besides productivity (for example, population, real exchange rates, etc.), for the questions addressed in this paper, the other factors are isomorphic to productivity differences. This will become clear in the quantitative section. Productivity $z_{j,t}$ and financial parameters $\phi_{j,t}$ and $\kappa_{j,t}$ are time varying but not stochastic. Their changes over time are fully anticipated. The only source of uncertainty in the model derives from shocks that will be described below.

2.1 Entrepreneurial sector

In each country, there is a unit mass of atomistic entrepreneurs that maximize the expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_{j,t}),$$

where $c_{j,t}$ is consumption in country j at time t.

Entrepreneurs are business owners producing a single good with the production technology

$$y_{j,t} = z_{j,t}^{\gamma} m_{j,t}^{\alpha} l_{j,t}^{\gamma} k_{j,t}^{1-\alpha-\gamma}. \tag{1}$$

The variable $z_{j,t}$ denotes total factor productivity, $m_{j,t}$ is the financial wealth held by the entrepreneur, $l_{j,t}$ is the input of labor, and $k_{j,t}$ is the physical capital rented from households/firms. The long-run growth rate of productivity is $g-1$ in both countries. In the short-run, however, there can be significant deviations from the long-run growth rate.[6]

The assumption that financial assets enter the production function is not common in the literature but not new. For example, King and Plosser (1984) introduced privately-produced transaction services backed by bank

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[6]Although the model is presented as if final production is carried out by privately owned businesses, we should think of the entrepreneurial sector broadly, that is, including also publicly traded companies. Then, entrepreneurial consumption represents dividend payments and the concavity of the utility function reflects the risk aversion of managers and/or major shareholders. The concavity could also reflect, in reduced form, the cost associated with financial distress: even if shareholders and managers are risk-neutral, a convex cost of financial distress would make the objective of the business concave.

deposits as an input of production to link credit fluctuations to business cycles. Following a similar idea, we think that financial wealth provides working capital that is complementary to other production factors.

Usage of financial assets in production generates a cost $\phi_{j,t} m_{j,t}$. This cost captures expenses that increase with the production scale, as determined by the input $m_{j,t}$. One way to think about this cost is that it derives from the depreciation of capital: if the utilization of capital is complementary to the usage of financial assets, then the expenses required to replenish the depreciated capital increases in $m_{j,t}$. An alternative interpretation, based on King and Plosser (1984), is that $\phi_{j,t}$ represents the rental price of transaction services that financial assets provide. The time-varying parameter $\phi_{j,t}$ is exogenous and plays an important role in determining the demand for financial assets. As we will see, a lower $\phi_{j,t}$ strengthens the incentives for entrepreneurs to hold $m_{j,t}$.

Entrepreneurs have access to a market for bonds traded at price $q_{j,t}$. At equilibrium, the bonds held by entrepreneurs are liabilities issued by households/firms. Even if there is capital mobility, the prices of bonds issued by the two countries differ because they are characterized by repayment risks that are specific to each country.

The representative entrepreneur in country j enters period t with bonds issued by country 1, $b_{1j,t}$, and bonds issued by country 2, $b_{2j,t}$. The first subscript denotes the country that issued the bond while the second subscript denotes the residence of the entrepreneur. In the event of a financial crisis, the entrepreneur incurs financial losses proportional to the owned bonds.

Denote by $\delta_{1,t}$ and $\delta_{2,t}$ the repayment fractions realized at the beginning of the period on bonds issued, respectively, by country 1 and country 2. The residual values of the two bonds are then $\delta_{1,t} b_{1j,t}$ and $\delta_{2,t} b_{2j,t}^i$. The repayment fractions $\delta_{1,t}$ and $\delta_{2,t}$ are endogenous stochastic variables determined in general equilibrium. Given the realization of these two variables, the wealth of the entrepreneur becomes

$$m_{j,t} = \delta_{1,t} b_{1j,t} + \delta_{2,t} b_{2j,t}.$$

This is the financial wealth that enters the production function (1). In addition, the entrepreneur hires labor at the wage rate $w_{j,t}$ and rents physical capital from households/firms at the rental rate $r_{j,t}$. The end-of-period wealth, after production, is

$$a_{j,t} = (1 - \phi_{j,t}) m_{j,t} + z_{j,t}^\gamma m_{j,t}^\alpha l_{j,t}^\gamma k_{j,t}^{1-\alpha-\gamma} - w_{j,t} l_{j,t} - r_{j,t} k_{j,t}.$$

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Wealth is allocated to consumption, $c_{j,t}^i$, and new bonds, $q_{1,t} b_{1j,t+1}$ and $q_{2,t} b_{2j,t+1}$. The budget constraint is

$$c_{j,t} + q_{1,t} b_{1j,t+1} + q_{2,t} b_{2j,t+1} = a_{j,t}. \tag{2}$$

While the input of labor $l_{j,t}$ depends on $m_{j,t}$, the portfolio decisions, $b_{1j,t+1}$ and $b_{2j,t+1}$, are functions of $a_{j,t}$. To make the timing of the model precise, we can think of a period as divided in three subperiods:

1. **Subperiod 1**: Entrepreneurs enter with financial assets $b_{1j,t}$ and $b_{2j,t}$, and observe country-specific repayment fractions $\delta_{1,t}$ and $\delta_{2,t}$. Hence, their residual wealth is $m_{j,t} = \delta_{1,t} b_{1j,t} + \delta_{2,t} b_{2j,t}$.

2. **Subperiod 2**: Given the residual wealth $m_{j,t}$, entrepreneurs choose the inputs of labor $l_{j,t}$ and capital $k_{j,t}$. Market clearing determines the wage and rental rates $w_{j,t}$ and $r_{j,t}$.

3. **Subperiod 3**: The end-of-period wealth, $a_{j,t}$, is in part consumed, $c_{j,t}$, and in part saved in bonds issued by country 1, $q_{1,t} b_{1j,t+1}$, and country 2, $q_{2,t} b_{2j,t+1}$.

The following lemma characterizes the production decision (Subperiod 2) and the optimal portfolio decision (Subperiod 3).

Lemma 2.1 *The optimal entrepreneur's policies are*

$$l_{j,t} = z_{j,t}^{\frac{\gamma}{\alpha}} \left(\frac{\gamma}{w_{j,t}} \right)^{\frac{\alpha+\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{r_{j,t}} \right)^{\frac{1-\alpha-\gamma}{\alpha}} m_{j,t},$$

$$k_{j,t} = z_{j,t}^{\frac{\gamma}{\alpha}} \left(\frac{\gamma}{w_{j,t}} \right)^{\frac{\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{r_{j,t}} \right)^{\frac{1-\gamma}{\alpha}} m_{j,t},$$

$$c_{j,t} = (1-\beta) a_{j,t}^i,$$

$$q_{1,t} b_{1j,t+1} = \beta \theta_t a_{j,t},$$

$$q_{2,t} b_{2j,t+1} = \beta (1-\theta_t) a_{j,t},$$

where θ_t solves the first-order condition

$$\mathbb{E}_t \left\{ \frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_t \frac{\delta_{1,t+1}}{q_{1,t}} + (1-\theta_t) \frac{\delta_{2,t+1}}{q_{2,t}}} \right\} = 1.$$

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Proof 2.1 *See Appendix A.*

The demand functions for labor and capital are linear in residual financial wealth $m_{j,t}$. The proportionality factors depend positively on productivity, $z_{j,t}$, and negatively on both the wage rate, $w_{j,t}$, and the rental rate of capital, $r_{j,t}$. These demand functions illustrate how a financial distress propagates to the real economy: Lower repayments cause a drop in entrepreneurial wealth $m_{j,t}$ which in turn causes a decline in the demand of labor. In equilibrium this will be associated to lower employment and, therefore, a contraction in real economic activity.

Lemma 2.1 also indicates that entrepreneurs split their end-of-period wealth between consumption and saving according to the fixed factor β. This property derives from the log specification of the utility function. Finally, an endogenous fraction θ_t of savings is allocated to bonds issued by country 1 and the complement $1-\theta_t$ to bonds issued by country 2. The variable θ_t changes over time, as recovery rates and bond prices vary. However, it is the same for entrepreneurs of both countries. This is indicated by the fact that θ_t does not have the j subscript. This means that entrepreneurs choose the same portfolio composition independently of their residency. We would also like to point out that, because θ_t is the same for the two countries, the last two conditions in the lemma are not identities.

2.2 Consolidated households/firms sector

In each country, there is a consolidated sector with a unit mass of homogeneous households/firms. The reason we include some firms in this sector is to distinguish them from firms in the entrepreneurial sector. We think of these firms as large owners of collateralizable assets (capital). In this sense they are similar to households who also own large collateralizable assets in the form of residential assets. Entrepreneurial firms, instead, are more representative of businesses that own few collateralizable assets (zero for simplicity in the model). As we will see, the consolidated sector will borrow in equilibrium while the entrepreneurial sector will lend.

Households/firms maximize the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(e_{j,t} - z_{j,t} \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right),$$

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where $e_{j,t}$ is consumption (or dividends), $h_{j,t}$ is the supply of labor and ν is the elasticity of labor supply.

The assumption that households/firms have linear utility in consumption/dividends simplifies the characterization of the equilibrium. It allows us to derive analytic results without affecting the key properties of the model. The dependence of the dis-utility of labor on country-specific productivity $z_{j,t}$ supports balanced growth.

Households/firms make optimal plans to hold $k_{j,t}$ units of capital. To keep the model tractable we assume that the aggregate supply of capital grows exogenously at the same rate as the long-run growth rate of the economy, $g - 1$. Therefore, capital in both countries evolves over time according to $K_{j,t} = \bar{K}g^t$. We interpret capital broadly including real estate and land. An important assumption is that capital is held by consolidated households/firms, not entrepreneurs. However, households/firms rent the capital to domestic entrepreneurs at rate $r_{j,t}$. They can also trade the capital among households/firms at the market price $p_{j,t}$.

Borrowing and default. At the end of period $t - 1$, households/firms can borrow $d_{j,t}/R_{j,t-1}$ where $R_{j,t-1}$ is the gross interest rate and $d_{j,t}$ is the 'promised' repayment at time t. At the beginning of time t, however, when the debt $d_{j,t}$ is due, households/firms could default. In the event of default, creditors have the right to liquidate the capital $k_{j,t}$. However, the liquidation value at the beginning of period t, when the repayment is due, could be smaller than the loan.

Denote by $\tilde{p}_{j,t}$ the liquidation price of capital at the beginning of period t. If the debt is bigger than the liquidation value of capital, that is, $d_{j,t} > \tilde{p}_t k_{j,t}$, the debt is renegotiated. Under the assumption that borrowers have all the bargaining power, the post-renegotiation debt is

$$\tilde{d}(d_{j,t}, \tilde{p}_{j,t}k_{j,t}) = \min\left\{ d_{j,t} , \ \tilde{p}_{j,t}k_{j,t} \right\} \tag{3}$$

After renegotiation, the market for capital returns to normal at the end of the period. The assumption of an immediate fresh-start is a simplification that makes the model tractable.

We assume that there are states of nature in which the market for liquidated capital freezes and the liquidation price drops below its normal price $p_{j,t}$. More specifically, with probability λ the liquidation price becomes $\tilde{p}_{j,t} = \kappa_{j,t} < p_{j,t}$ while with probability $1 - \lambda$ remains at the nor-

mal price $\tilde{p}_{j,t} = p_{j,t}$. The variable $\kappa_{j,t}$ is time-varying but exogenous in the model.

Appendix C describes the mechanism that generates a freeze. The market structure described there allows for two self-fulfilling equilibria, one of which characterized by a market freeze where the liquidation price drops to $\kappa_{j,t}$. The probability λ is then the exogenous probability with which the market coordinates on the self-fulfilling equilibrium characterized by a market freeze. Readers who are not interested in the micro-foundation of the market freeze can skip this appendix without loss of continuity. All we have to remember is that the liquidation price $\tilde{p}_{j,t}$ is equal to $\kappa_{j,t}$ with probability λ and $p_{j,t}$ with probability $1 - \lambda$.

Borrowing also carries the convex cost

$$\varphi\left(d_{j,t+1}, \kappa_{j,t+1}k_{j,t+1}\right) = \eta \left[\frac{\max\{0, d_{j,t+1} - \kappa_{j,t+1}k_{j,t+1}\}}{d_{j,t+1}}\right]^2 d_{j,t+1}. \qquad (4)$$

As long as the the liquidation value with a freeze, $\kappa_{j,t+1}k_{j,t+1}$, exceeds the promised debt repayment, the cost is zero. Beyond that point, the cost rises at a quadratic rate. The budget constraint for consolidated households/firms, after renegotiation, is

$$\tilde{d}(d_{j,t}, \tilde{p}_{j,t}k_{j,t}) + p_{j,t}k_{j,t+1} + e_{j,t} + \varphi(d_{j,t+1}, \kappa_{j,t+1}k_{j,t+1}) =$$
$$w_{j,t}h_{j,t} + r_{j,t}k_{j,t} + p_{j,t}k_{j,t}g + \frac{d_{j,t+1}}{R_{j,t}}.$$

The value of capital is multiplied by g because it grows at the same rate as the long-run growth rate of productivity. The extra capital is a new endowment added to the budget constraint.

The gross interest rate $R_{j,t}$ depends on the individual borrowing decision. If the household/firm borrows more, relatively to the ownership of capital, the expected repayment rate could be lower in the next period. This will be reflected in a higher interest rate on the loan.

Denote by $\overline{R}_{j,t}$ the expected gross return from holding the debt issued in period t, and due at $t + 1$, by all households/firms in country j. This represents the aggregate expected market return from holding a diversified portfolio of debt. Since households/firms are atomistic and financial markets are competitive, the expected return on the debt issued by an 'individual' household/firm must be equal to the aggregate expected return

$\overline{R}_{j,t}$, that is,

$$\frac{d_{j,t+1}}{R_{j,t}} = \frac{1}{\overline{R}_{j,t}}\mathbb{E}_t\tilde{d}(d_{j,t+1}, \tilde{p}_{j,t+1}k_{j,t+1}). \tag{5}$$

The left-hand-side is the amount borrowed at time t while the right-hand-side is the expected repayment in period $t+1$, discounted by the market return $\overline{R}_{j,t}$. Since the household/firm renegotiates the debt when $d_{j,t+1} > \tilde{p}_{j,t+1}k_{j,t+1}$, the actual repayment $\tilde{d}(d_{j,t+1}, \tilde{p}_{j,t+1}k_{j,t+1})$ could differ from $d_{j,t+1}$. Competition in financial intermediation requires that the left-hand-side of (5) equals the right-hand-side.

Equation (5) determines the interest rate $R_{j,t}$ for an individual household/firm. It can also be viewed as determining an individual borrowing spread $R_{j,t}/\overline{R}_{j,t} = d_{j,t+1}/\mathbb{E}_t\tilde{d}(d_{j,t+1}, \tilde{p}_{j,t+1}k_{j,t+1})$. For a household/firm expected to repay in full, the spread is zero ($R_{j,t}/\overline{R}_{j,t} = 1$). However, for a household/firm that could repay less, $R_{j,t}$ exceeds $\overline{R}_{j,t}$ by a factor that depends on how much the expected repayment after renegotiation ($\mathbb{E}_t\tilde{d}(d_{j,t+1}, \tilde{p}_{j,t+1}k_{j,t+1})$) is below the contracted repayment ($d_{j,t+1}$). At equilibrium, all households/firms make the same decisions and they all borrow at the same rate. However, in order to characterize the optimal decision, we need to allow an individual household/firm to deviate from others.

First-order conditions. Households/firms' decisions are also sequential. At the beginning of the period (Subperiod 1) they choose whether to default and renegotiate the debt. After that (Subperiod 2), they choose the supply of labor. Finally, at the end of the period (Subperiod 3), they choose the new debt. Appendix B describes the optimization problem and derives the following first-order conditions:

$$w_{j,t} = z_{j,t}h_{j,t}^{\frac{1}{\nu}}, \tag{6}$$

$$\frac{1}{\overline{R}_{j,t}} = \beta + \Phi\left(\frac{d_{j,t+1}}{\kappa_{j,t+1}k_{j,t+1}}\right), \tag{7}$$

$$p_{j,t} = \beta\mathbb{E}_t\left\{r_{j,t+1} + gp_{j,t+1}\right\} + \Psi\left(\frac{d_{j,t+1}}{\kappa_{j,t+1}k_{j,t+1}}\right). \tag{8}$$

The explicit functional forms for the functions $\Phi(.)$ and $\Psi(.)$ are derived in the appendix. The appendix also shows that the two functions are increasing in the ratio $d_{j,t+1}/\kappa_{j,t+1}k_{j,t+1}$, which is a measure of leverage (i.e., the ratio of contracted debt to the minimum liquidation value of capital).

Equation (6) equates the marginal disutility of labor to the real wage, which represents the labor supply. The wealth effect on the supply of labor is neutralized by the linearity of consumption in the utility function. Equation (7) is the Euler equation for debt. Since $\Phi(.)$ is increasing in leverage, the equation posits a *negative* relationship between the expected return on the debt (the interest rate) and leverage. Equation (8) is the Euler equation for capital. Since $\Psi(.)$ is increasing in leverage, the equation establishes a *positive* relationship between the price of capital and leverage. These two equations are important for the model's transmission mechanism: They imply that a decline in the interest rate increases leverage and generates an asset price boom.

2.3 General equilibrium

Using capital letters to denote aggregate variables, the aggregate states are given by bonds held by entrepreneurs, $B_{11,t}$, $B_{21,t}$, $B_{12,t}$, $B_{22,t}$, and sunspot shocks $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$. The aggregate debts issued by households/firms in the previous period are $D_{1,t} = B_{11,t} + B_{12,t}$ and $D_{2,t} = B_{21,t} + B_{22,t}$. In addition, the full infinite-horizon sequences of productivity, $z_{1,t}$ and $z_{2,t}$, and financial variables, $\phi_{1,t}, \phi_{2,t}, \kappa_{1,t}, \kappa_{2,t}$, starting from date t are also relevant for the equilibrium. Since these sequences are deterministic and perfectly anticipated, they are part of the state space. We denote the sequence of a variable starting at time t and running to ∞ by using t as a subscript and ∞ as a superscript. For example, $z_{j,t}^{\infty}$ represents the sequence of productivity for country j from time t to ∞. To use a compact notation we denote the complete state vector by

$$\mathbf{s}_t \equiv (z_{1,t}^{\infty}, z_{2,t}^{\infty}, \phi_{1,t}^{\infty}, \phi_{2,t}^{\infty}, \kappa_{1,t}^{\infty}, \kappa_{2,t}^{\infty}, B_{11,t}, B_{21,t}, B_{12,t}, B_{22,t}, \varepsilon_{1,t}, \varepsilon_{2,t}).$$

The equilibrium is determined sequentially in the three subperiods as shown in Figure 2.

1. **Subperiod 1**: Given the realization of the liquidation price $\tilde{p}_{j,t}$, households/firms choose whether to default. The renegotiated debt is

$$\tilde{D}_{j,t} = \begin{cases} \kappa_{j,t} K_{j,t}, & \text{if} \quad D_{j,t} \geq \kappa_{j,t} K_{j,t} \ \text{and} \ \varepsilon_{j,t} = 0 \\ D_{j,t}, & \text{otherwise} \end{cases}.$$

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s$_t$ **s**$_{t+1}$

Households/firms decide
whether to default.
**Equilibrium
in market for
liquidated assets**

Entrepreneurs
choose input demands.
Workers choose supplies.
**Equilibrium
in market for
production inputs**

Entrepreneurs
choose savings.
Households/firms choose
capital and debt.
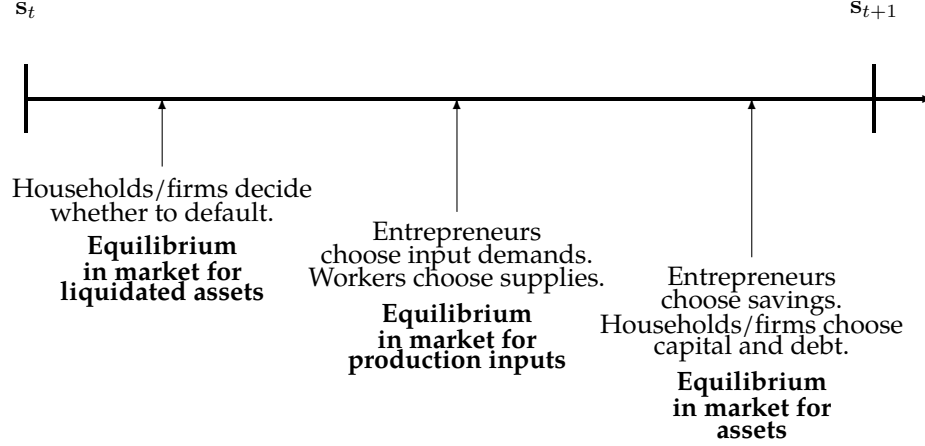**Equilibrium
in market for
assets**

Figure 2: Timing within period t.

The post-default wealth held by entrepreneurs in each country is pro-
portional to their holdings prior to default, that is,

$$M_{j,t} = \left(\frac{\tilde{D}_{1,t}}{D_{1,t}} \right) B_{1j,t} + \left(\frac{\tilde{D}_{2,t}}{D_{2,t}} \right) B_{2j,t}$$

2. **Subperiod 2**: Given post-default wealth $M_{j,t}$, entrepreneurs in coun-
try j choose the inputs of labor and capital, and households/firms
choose the supplies. The aggregate input demands in country j are
obtained from the individual demands derived in Lemma 2.1,

$$L_{j,t} = z_{j,t}^{\frac{\gamma}{\alpha}} \left(\frac{\gamma}{w_{j,t}} \right)^{\frac{\alpha+\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{r_{j,t}} \right)^{\frac{1-\alpha-\gamma}{\alpha}} M_{j,t},$$

$$K_{j,t} = z_{j,t}^{\frac{\gamma}{\alpha}} \left(\frac{\gamma}{w_{j,t}} \right)^{\frac{\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{r_{j,t}} \right)^{\frac{1-\gamma}{\alpha}} M_{j,t}.$$

The aggregate supply of labor is derived from the household's first
order condition (6). Imposing $h_{j,t} = H_{j,t}$ and inverting we obtain

$$H_{j,t} = \left(\frac{w_{j,t}}{z_{j,t}} \right)^{\nu}.$$

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The supply of capital is exogenous, $K_{j,t} = \bar{K}g^t$. Market-clearing in the labor and capital markets then determines the wage rate $w_{j,t}$, the rental rate $r_{j,t}$, and employment $L_{j,t} = H_{j,t}$ in each country.

3. **Subperiod 3**: The end-of-period wealth of entrepreneurs is

$$A_{j,t} = (1 - \phi_j)M_{j,t} + z_{j,t}^\gamma M_{j,t}^\alpha L_{j,t}^\gamma K_{j,t}^{1-\alpha-\gamma} - w_{j,t}L_{j,t} - r_{j,t}K_{j,t}.$$

A fraction $1-\beta$ is consumed while the remaining fraction β is saved in new bonds, $q_{1,j}B_{1j,t+1}$ and $q_{2,j}B_{2j,t+1}$. Households/firms choose new debt $D_{j,t+1}$ and new holdings of capital $K_{j,t+1}$.

Market-clearing in financial assets requires

$$B_{11,t+1} + B_{12,t+1} = D_{1,t+1}, \tag{9}$$
$$B_{21,t+1} + B_{22,t+1} = D_{2,t+1}. \tag{10}$$

Because of capital mobility and cross-country heterogeneity, the net foreign asset positions of the two countries could be different from zero, that is, $B_{1j,t+1} + B_{2j,t+1} \neq D_{j,t+1}$. Competition also implies that the price paid by entrepreneurs to purchase households/firms' debt is consistent with the interest rate, that is,

$$q_{j,t} = \frac{1}{\overline{R}_{j,t}}.$$

Since $\overline{R}_{j,t} = R_{j,t}\mathbb{E}_{t+1}\delta_{j,t+1}$, the above condition relates the price of bonds $q_{j,t}$ to their expected return.

Using the optimal savings of entrepreneurs derived in Lemma 2.1 and aggregating, we obtain the demand for bonds in country j,

$$q_{1,j}B_{1j,t+1} + q_{2,j}B_{2j,t+1} = \beta A_{j,t}. \tag{11}$$

The supply of bonds is derived from the borrowing decisions of households/firms. From the first-order condition (7) we have

$$\frac{1}{\overline{R}_{j,t}} = \beta + \Phi\left(\frac{D_{j,t+1}}{\kappa_{j,t+1}K_{j,t+1}}\right).$$

Because in equilibrium $\overline{R}_{j,t} = R_{j,t}\mathbb{E}\delta_{j,t+1}$ and $q_{j,t} = 1/R_{j,t}$, the first order condition can be rewritten as

$$q_{j,t} = \left[\beta + \Phi\left(\frac{D_{j,t+1}}{\kappa_{j,t+1}K_{j,t+1}}\right)\right]\mathbb{E}\delta_{j,t+1}. \tag{12}$$

The market for capital must also clear, that is, the demand $K_{j,t+1}$ must be equal to the exogenous supply $\bar{K}g^t$. The first-order condition (7) then determines the (end-of-period) price p_t.

Because $z_{j,t}, \phi_{t,t}, \kappa_{j,t}$ are time-varying and households/firms can default, the economy does not reach a steady state but displays stochastic dynamics driven by fluctuations in the liquidation price (sunspot shocks). In particular, a drop in the liquidation value of capital can lead to a financial crisis where bonds are only partially repaid. This redistributes wealth from lenders (entrepreneurs) to borrowers (households/firms). When entrepreneurs hold less wealth $M_{j,t}$, they demand less labor and in equilibrium there is lower employment and production. This is the main mechanism through which financial crises have real macroeconomic effects. A lower value of $M_{j,t}$ also decreases the demand for capital which reduces the rental rate $r_{j,t}$. The lower return on capital then causes a drop in its price p_t. Therefore, financial crises also have a negative impact on asset prices.

2.4   Sequential property of the equilibrium

The particular structure of the model allows us to solve for the equilibrium at time t independently of future equilibria, as if the model were static. The only exception is the price of capital $p_{j,t}$. More specifically, given the states s_t, we can find the values of all equilibrium variables at time t—with the exception of the price of capital $p_{j,t}$—by solving the system of nonlinear equations described in Appendix D. This implies that we can solve the model sequentially. For example, if we need to simulate the model from $t = 1991$ to $t = 2020$, we first solve for the equilibrium at time $t = 1991$. We then solve for the equilibrium at time $t = 1992$ and continue until $t = 2020$. The fact that this procedure does not allow us to solve for the price of capital $p_{j,t}$ is not a problem because the price of capital does not enter the system of equations listed in Appendix D.[7]

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[7]This property derives from the assumption that the liquidation price of capital under default is equal to the $\tilde{p}j, t = \kappa_{j,t}$, which is exogenous in the model. If the liquidation

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The particular structure of the model allows us also to reduce the sufficient set of state variables. In general, the full sequences of time-varying parameters $z_{j,t}$, $\phi_{j,t}$ and $\kappa_{j,t}$, from to to infinity, could affect the equilibrium. However, because of the sequential structure described above, equilibrium variables at time t are only affected by $z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$ and $\kappa_{j,t+1}$. Therefore, from now on, to characterize the equilibrium (except $p_{j,t}$), we can redefine the sufficient set of state variables

$$\mathbf{s}_t \equiv (z_{1,t}, z_{2,t}, \phi_{1,t}, \phi_{2,t}, \kappa_{1,t}^{t+1}, \kappa_{2,t}^{t+1}, B_{11,t}, B_{21,t}, B_{12,t}, B_{22,t}, \varepsilon_{1,t}, \varepsilon_{2,t}).$$

As we will see, this property will be very convenient for the quantitative application where we use the model to construct sequences for the time-varying variable $z_{j,t}$, $\phi_{j,t}$ and $\kappa_{j,t}$ from the data. More specifically, we can construct these sequences sequentially limiting the analysis to the sample period 1991-2020.

2.5 Other properties and remarks

A property of the equilibrium is that the risk-free interest rate is lower than the inter-temporal discount rate (or, equivalently, the price of a risk-free bond is higher than the inter-temporal discount rate β). Despite the low interest rate, entrepreneurs hold the debt issued by households/firms. This is because bonds can be used as production inputs. Therefore, in addition to interest payments, bonds also generate profits for entrepreneurs.

The equilibrium property for which entrepreneurs are net savers and households/firms are borrowers might seem counterfactual at first. However, it is not inconsistent with the recent changes in the financial structure of US corporations. It is well known that during the last two-and-a-half decades, the corporate sector has increased its holdings of financial assets. This suggests that the proportion of financially dependent firms has declined significantly over time, which is consistent with the empirical findings of Shourideh and Zetlin-Jones (2012) and Eisfeldt and Muir (2016).

The large accumulation of financial assets by producers (often referred to 'cash') is related to the significance of business savings. Busso et al. (2016) document the share of savings done by firms both in advanced and emerging economies and present evidence that in Latin America this share

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value of capital under default was a function of $p_{j,t}$, we would not be able to solve the model sequentially since, to solve for the other variables we need to solve for $p_{j,t}$.

is even larger than in advanced economies. The importance of business savings is also documented in Bebczuk and Cavallo (2016). Using data for 47 countries over 1995–2013 they show that the contribution of businesses to national savings is on average more than 50%. Our entrepreneurial sector captures the growing importance of firms that are not very dependent on external financing.

At the same time, during the past three decades, we have witnessed significant increase in households' debt. Corporate debt has also increased during this period, indicating that the nonfinancial sector has issued more debt while at the same time it has accumulated more financial assets. However, we conjecture that there is significant heterogeneity among corporate firms and the increase in corporate debt was driven by a subset of firms, most likely those that own large amount of tangible assets. These firms are represented in the model by the consolidated households/firms sector.

Another remark is that the equilibrium property for which producers are net lenders does not rely on the risk neutrality of households/firms. What is crucial is that the overall return on bonds for entrepreneurs is greater than the interest rate. For borrowing households/firms, instead, bonds are valuable only because they pay interests. With risk averse households/firms, bonds could also provide an insurance benefit. However, as long as the extra return that entrepreneurs receive from bonds is sufficiently large, they would continue to be lenders while households/firms would continue to be borrowers.

3   Quantitative analysis

We now use the model to assess quantitatively how the faster growth of emerging economies and the changes in financial structure experienced by both emerging and advanced economies affected financial and macroeconomic volatility over the past three decades. The quantitative implementation follows three steps:

1. Calibration of basic parameters.

2. Construction of sequences for $z_{1,t}$, $z_{2,t}$, $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$, $\kappa_{2,t}$.

3. Counterfactual simulations given the constructed sequences of $z_{1,t}$, $z_{2,t}$, $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$, $\kappa_{2,t}$.

For the first two steps we use data for advanced economies (country 1) and emerging economies (country 2) for the period 1991-2020. The countries included in the groups of emerging and advanced economies are listed in Figure 1. The simulations conducted in the third step are over the periods 1991-2020.

3.1 Calibration of basic parameters

The model is calibrated annually and the discount factor is set to $\beta = 0.96$, implying an annual intertemporal discount rate of about 4%. We set the elasticity of labor supply to $\nu = 1$, a number often used in macroeconomics.

The probability that the liquidation price of capital drops to $\tilde{p}_{j,t} = \kappa_{j,t}$ (negative sunspot shock $\varepsilon = 0$) is $\lambda = 0.04$. This ensures that crises are low probability events, every twenty-five years on average. This is within the range of estimates of crisis probabilities provided in the literature (see, for example, Bianchi and Mendoza (2018)). Notice that, since sunspot shocks are country-specific, that is, they are independent across countries, a *global* financial crisis that arises contemporaneously in both countries is an even rarer event, happening with probability $0.04 \times 0.04 = 0.016$.

We calibrate next the share parameters of the production function. We set the labor share to $\gamma = 0.6$, which is a standard value. We interpret the cost $\phi_{j,t} M_{j,t}$ as depreciation of capital.[8] Then, to pin down the share of financial assets in the production function, α, we use the depreciation-output ratio as a calibration target. Specifically, we assume that the world-wide average, over the 1991-2020 period, of $\phi_{j,t} M_{j,t}$ over worldwide output is 0.2.[9] To determine α, however, we need to use an iterative procedure since its targeted value depends also on the values of the time-varying parameters $z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$. We start by assigning a value to α. After the determination of the sequences $z_{j,t}^{\infty}$, $\phi_{j,t}^{\infty}$, $\kappa_{j,t}^{\infty}$ (second step described in the next subsection), we check whether the average worldwide depreciation in the model is 20% the value of worldwide output. At this point we update the value of α until we reach the calibration target of 0.2. The full set of parameter values are listed in Table 1.

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[8]Higher input of financial assets increases production which leads to more intensive utilization of capital and, therefore, higher depreciation.

[9]If the average depreciation rate is 0.08 and the capital-output ratio is 2.5, then the depreciation-output ratio is 0.2.

Table 1: Parameter values.

Description	Parameter	Value
Discount factor	β	0.960
Share of financial wealth in production	α	0.294
Share of labor in production	γ	0.600
Elasticity of labor supply	ν	1.000
Probability of low sunspot shock	λ	0.040
Borrowing cost	η	0.100

3.2 Construction of sequences for $z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$

Differences in size and financial structure between the two regions are generated by the deterministic sequences $z_{j,1991}^{2020}$, $\phi_{j,1991}^{2020}$, $\kappa_{j,1991}^{2021}$, for $j \in \{1,2\}$. We construct these sequences to replicate the empirical time series showed in Figure 1 over the period 1991-2020. As observed in Section 2.4, the sequential structure of the model allows us to determine the sequences $z_{j,1991}^{2020}$, $\phi_{j,1991}^{2020}$, $\kappa_{j,1991}^{2021}$ without need to know the value of these time varying parameters beyond 2020 (2021 for $\kappa_{j,t}$).

A complication in using the model to construct specific sequences of $z_{j,t}$, $\phi_{j,t}$ and $\kappa_{j,t}$ is that the resulting values depend on the realizations of the (sunspot) shocks. Therefore, we need to pick a particular sequence of shocks over the 1991-2000 period. To this end we make the following assumption. We assume that $\varepsilon_{j,t} = 1$ (no crisis) in all years with only few exceptions. For emerging economies it takes the value of zero in 1997 and 2009 ($\varepsilon_{2,1997} = 0$ and $\varepsilon_{2,2009} = 0$). These two years correspond, respectively, to the 1997 Sudden Stops in South-East Asia and to the Global Financial Crisis that started in 2008 and extended to 2009. Both crises had an impact on emerging economies. For advanced economies, instead, it takes the value of zero only in 2009 ($\varepsilon_{1,2009} = 0$) reflecting, again, the Global Financial Crisis. It is important to point out that, even though we calibrate the model assuming a specific sequence of shocks, agents do not anticipate them and, therefore, they make decisions based on the random distribution of the shocks.

Productivity $z_{j,t}$. The productivity series $z_{1,1991}^{2020}$ and $z_{2,1991}^{2020}$ are constructed as Solow residuals from the production function. To do so, we need measurements of production inputs and outputs. For output, we use GDP measured at nominal exchange rates, not PPP. Since movements in nominal ex-

change rates affect the purchasing power of a country in the acquisition of foreign assets, our productivity measure should also reflect movements in exchange rates. Another factor that contributes to differences in aggregate GDP is population growth. Since population is not explicitly included in the model, however, the constructed sequences of productivity also capture changes in population.

Denote by $P_{j,t}$ the nominal price index for country j expressed in US dollars. The price is calculated by multiplying the price in local currency by the dollar exchange rate. We can then define the nominal (dollar) aggregate output of country j as

$$P_{j,t}Y_{j,t} \;\; = \;\; P_{j,t}\hat{z}_{j,t}^{\gamma}M_{j,t}^{\alpha}L_{j,t}^{\gamma}K_{j,t}^{1-\alpha-\gamma}N_{j,t},$$

where $\hat{z}_{j,t}$ is actual productivity, $M_{j,t}$ is per-capita financial assets, $L_{j,t}$ is per-capita employment, $K_{j,t}$ is per-capita capital, and $N_{j,t}$ is population. Notice that the above definition of output assumes that physical capital increases with population.

If we deflate the nominal GDP in both countries by the price index in country 1, we obtain

$$Y_{1,t} \;\; = \;\; \hat{z}_{1,t}^{\gamma}M_{1,t}^{\alpha}L_{1,t}^{\gamma}K_{1,t}^{1-\alpha-\gamma}N_{1,t},$$

$$\frac{P_{2,t}Y_{2,t}}{P_{1,t}} \;\; = \;\; \left(\frac{P_{2,t}\hat{z}_{2,t}^{\gamma}}{P_{1,t}}\right)M_{2,t}^{\alpha}L_{2,t}^{\gamma}K_{2,t}^{1-\alpha-\gamma}N_{2,t},$$

Therefore, aggregate productivity in the model corresponds to

$$z_{1,t} \;\; = \;\; \hat{z}_{1,t}N_{1,t}^{\frac{1}{\gamma}},$$

$$z_{2,t} \;\; = \;\; \hat{z}_{2,t}\left(\frac{P_{2,t}N_{2,t}}{P_{1,t}}\right)^{\frac{1}{\gamma}}.$$

Notice that, since $P_{2,t}$ is the dollar price of emerging-markets output, it includes movements in real exchange rate. Hence, the above expressions show that $z_{1,t}$ and $z_{2,t}$ also reflect cross-country differences in real exchange rates and population, in addition to true productivity. The productivity sequences that we use as inputs to the model can be calculated from the

data as

$$z_{1,t} = \left(\frac{Y_{1,t}}{M_{1,t}^{\alpha} L_{1,t}^{\gamma} K_{1,t}^{1-\alpha-\gamma}} \right)^{\frac{1}{\gamma}}, \tag{13}$$

$$z_{2,t} = \left(\frac{P_{2,t} Y_{2,t}/P_{1,t}}{M_{2,t}^{\alpha} L_{2,t}^{\gamma} K_{2,t}^{1-\alpha-\gamma}} \right)^{\frac{1}{\gamma}}. \tag{14}$$

The numerator is total real GDP, deflated by the nominal price in advanced economies. If the real exchange rate of emerging economies appreciates, it will be reflected in higher relative productivity in these economies. Although this does not increase actual productivity, it raises the ability of these countries to purchase assets in advanced economies, which is important for general equilibrium effects. Also notice that the change in relative price could simply be the result of movements in nominal exchange rates. Still, when the currencies of emerging economies appreciate, assets created in advanced economies become cheaper for emerging economies.

In order to use equations (13) and (14) to construct the productivity sequences, we need empirical counterparts for $Y_{1,t}$, $P_{2,t} Y_{2,t}/P_{1,t}$, $M_{1,t}$, $M_{2,t}$ $L_{1,t}$, $L_{2,t}$, $K_{1,t}$, and $K_{2,t}$.

The output variables $Y_{1,t}$ and $P_{2,t} Y_{2,t}/P_{1,t}$ are obtained by aggregating the GDP of advanced and emerging economies, both expressed at constant US dollars. To construct $M_{j,t}$ we use domestic private credit together with the net foreign asset positions. Both variables are expressed in constant US dollars, divided by population over 15 years of age. More specifically, denoting by $D_{j,t}$ domestic credit and $NFA_{j,t}$ the net foreign asset position of country j, financial assets used in production are $M_{j,t} = D_{j,t} + NFA_{j,t}$.

For the labor input $L_{j,t}$ we use employment-to-population ratio (population over 15 years of age). The variable $K_{j,t}$ grows in the model at the constant rate $g-1$. Therefore, we can express the stock of capital as $K_{j,t} = \bar{K} g^t$ with $\bar{K}$ normalized to 1. Notice that the constant growth rate of capital is the same in the two regions. We set this rate to the average growth rate of aggregate GDP in advanced economies which, over the period 1991-2020, is 1.89%. We take this number as the long-run growth rate for both advanced and emerging economies (after convergence). Data is from the World Development Indicators (WDI). The resulting productivity series are plotted in the top panel of Figure 3.

As expected, productivity has increased faster in emerging economies,

and became larger than that of advanced economies by 2010. This, however, does not mean that emerging economies have a more efficient technology since $z_{j,t}$ reflects also the size of the population which is much larger in emerging economies.
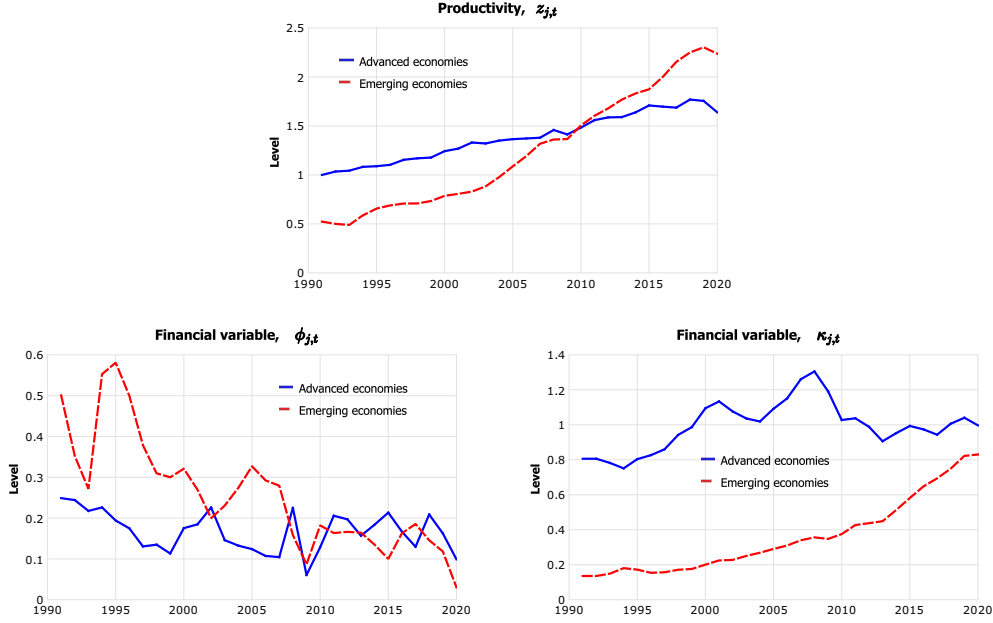

Figure 3: Computed productivity and financial variables series for advanced and emerging economies, 1991-2020.

Financial structure $\phi_{j,t}$ and $\kappa_{j,t}$. Next we construct the sequences $\phi_{j,1991}^{2020}$ and $\kappa_{j,1991}^{2021}$. The first sequence is important for the 'demand' of financial assets (in the spirit of Mendoza et al. (2009)): Lower values of $\phi_{j,t}$ increase the demand for financial assets since their use in production is less costly. The second sequence is important for the 'supply' of financial assets (in the spirit of Caballero et al. (2008)): Higher values of $\kappa_{j,t}$ increase the incentive of households/firms to borrow.

We construct the sequences of $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$ so that the model replicates four empirical series over the period 1991-2020: (i) domestic credit-to-GDP ratio in advanced economies, (ii) domestic credit-to-GDP ratio in emerging economies, (iii) NFA position of advanced economies,

(iv) US risk-free real interest rate. These are the empirical series shown in the last three panels of Figure 1). The mapping of these four empirical targets to the corresponding variables in the model is as follows:

$$\text{Credit-to-GDP AEs} \;=\; \frac{q_{1,t}D_{1,t+1}}{Y_{1,t}}, \tag{15}$$

$$\text{Credit-to-GDP EEs} \;=\; \frac{q_{2,t}D_{2,t+1}}{Y_{2,t}}, \tag{16}$$

$$\text{NFA-to-GDP AEs} \;=\; \frac{q_{1,t}B_{11,t+1} + q_{2,t}B_{21,t+1} - q_{1,t}D_{1,t+1}}{Y_{1,t}}, \tag{17}$$

$$\text{US real interest rate} \;=\; \frac{\mathbb{E}_t\delta_{1,t+1}}{q_{1,t}} - 1. \tag{18}$$

The terms on the right-hand-side are equilibrium objects that we can compute after knowing $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$, $\kappa_{2,t}$, $\kappa_{1,t+1}$ and $\kappa_{2,t+1}$. Given the sequential structure of the model (see Section 2.4), we can find the relevant equilibrium variables in period t by solving the system of nonlinear equations described in Appendix D.[10] After initializing $\kappa_{1,1991}$ and $\kappa_{2,1991}$, we solve for $\phi_{1,1991}$, $\phi_{2,1991}$, $\kappa_{1,1992}$ and $\kappa_{2,1992}$ by applying two nested nonlinear solvers. The inner solver finds the equilibrium variables given the values of $\phi_{1,1991}$, $\phi_{2,1991}$, $\kappa_{1,1992}$ and $\kappa_{2,1992}$ (as described in Appendix D). The outer solver finds the values of $\phi_{1,1991}$, $\phi_{2,1991}$, $\kappa_{1,1992}$ and $\kappa_{2,1992}$ using conditions (15)-(18). We then move to the next period and find the values of $\phi_{1,1992}$, $\phi_{2,1992}$, $\kappa_{1,1993}$ and $\kappa_{2,1993}$. We continue until the end of the sample period $t = 2020$.

Figure 4 provides a graphical intuition for the identification of the four time-varying parameters. The graph depicts the financial market equilibrium where the interest rate is at the level for which the global demand of assets (sum of the demands from both countries) is equal to the global supply of assets (sum of the supplies from both countries). The parameters $\phi_{j,t}$ and $\kappa_{j,t+1}$ determine, respectively, the positions of the demand and supply curves in country j. An increase in $\phi_{j,t}$ shifts the demand of country j to the left while an increase in $\kappa_{j,t+1}$ shifts the supply of country j to the right. To identify these four parameters we use the four circled variables: (i) the debt in country 1; (ii) the debt in country 2; (iii) the Net For-

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[10]As pointed out in Section 2.4, we can solve for all equilibrium variables at any time t, except for the price of capital $p_{j,t}$. However, $p_{j,t}$ does not affect the equilibrium variables that are mapped to the four empirical targets listed in (15)-(18).

eign Asset position of country 1; (iv) the world interest rate. As indicated in equations (15)-(18), the empirical counterparts of these four variables are: (i) Private domestic credit in Advanced Economies; (ii) Private domestic credit in Emerging Economies; (iii) Net Foreign Asset position of Advanced Economies; (iv) US interest rate.
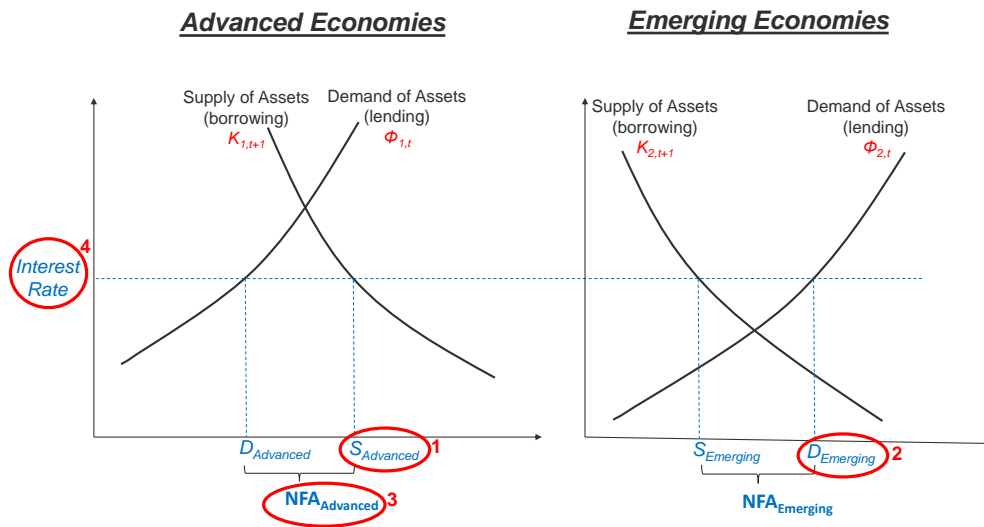

Figure 4: Counterfactual simulation with constant financial structure, 1991-2020.

The computed series for $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$ are plotted in the bottom panels of Figure 3. The first panel shows that ϕ does not display any significant trend for advanced economies while it trends downward for emerging economies. Since a reduction in ϕ increases the demand for financial assets, the computed series indicates that the higher growth of emerging economies was accompanied by a structural change that increased their demand for financial assets more than in advanced economies. The second panel shows that the variable κ has increased for both advanced and emerging economies. Since higher κ raises the supply of assets, the computed series indicate that financial constraints have been relaxed in both advanced and emerging economies.

3.3 Counterfactual simulations

In this section we explore how the changes in productivity and financial structure shown in Figure 3 contributed to real and financial dynamics. We do so by conducting counterfactual simulations in which we allow only one factor to change, while keeping the other factors fixed.

We start with productivity. We impose that $\phi_{j,t}$ and $\kappa_{j,t}$ remain constant at their 1991 values for the whole simulation period, while $z_{j,t}$ takes the values shown in Figure 3. The series generated by the counterfactual simulation are plotted in Figure 5.
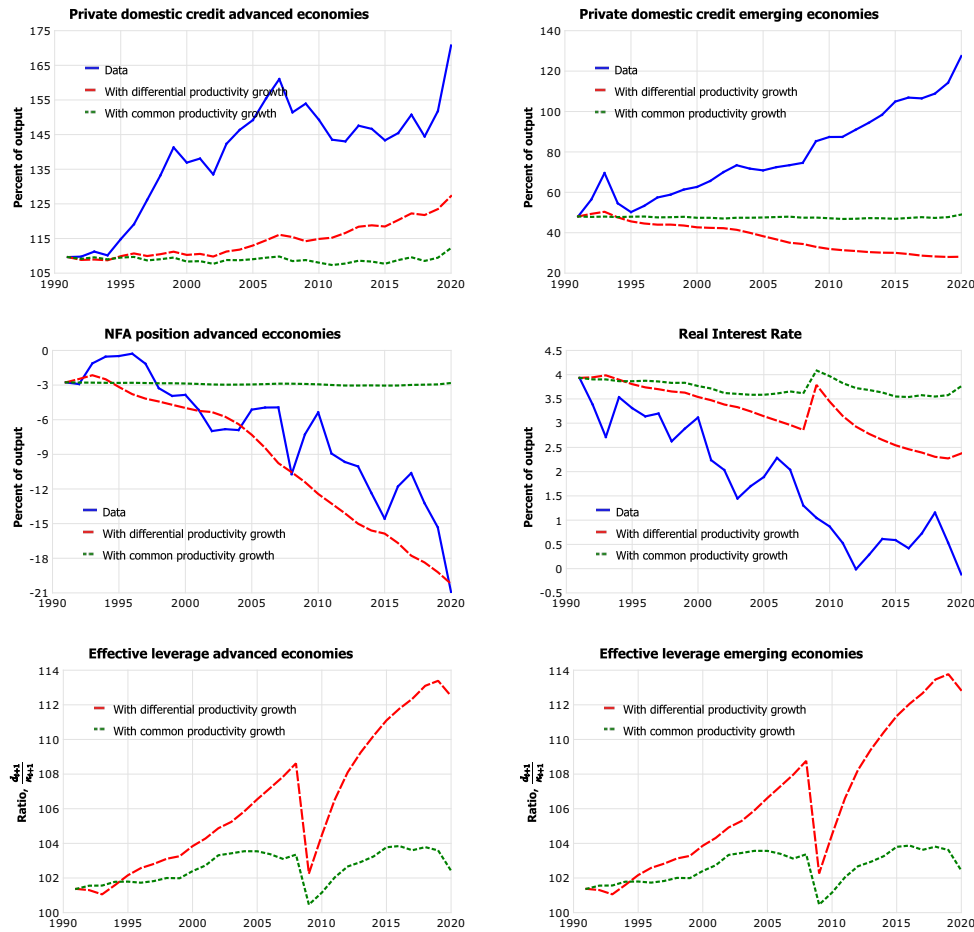

Figure 5: Counterfactual simulation with constant financial structure, 1991-2020.
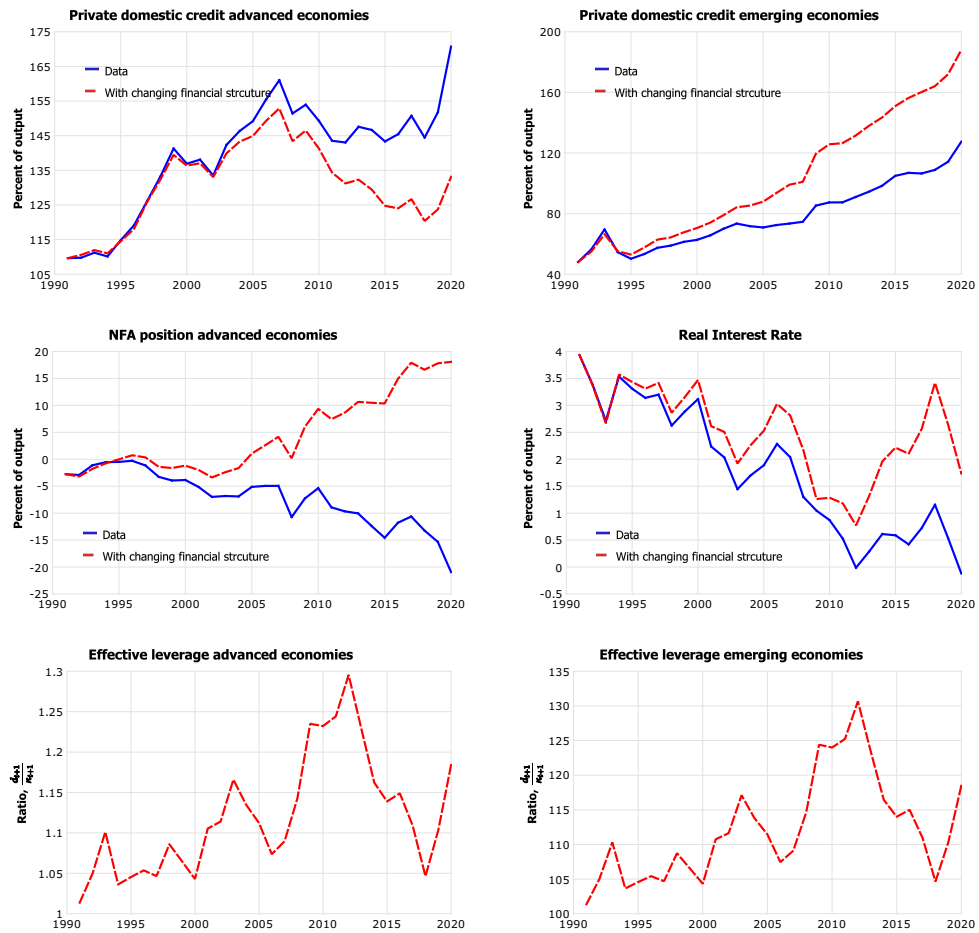
Figure 6: Counterfactual simulation with constant and common productivity growth, 1991-2020.

The first four panels plot domestic credit in advanced and emerging economies, NFA in advanced economies, and the risk-free real interest rate (which is common to the two regions). The continuous line is the original data shown in Figure 1. By construction, this is also the series generated by the baseline model with productivity and financial variables taking the values plotted in Figure 3. The dashed line is the model-generated data when only productivity changes. The dotted line also plots the model-generated data when only productivity changes but with the additional assumption that the productivity of emerging economies grows at the same rate as in

advanced economies. The comparison of the dashed and dotted lines illustrates the importance of faster growth experienced by emerging economies.

Comparing the data with the model shows that the faster growth of emerging economies accounts for most of the global imbalances (the sharp decline in NFA of advanced economies) and a sizable part of the decline in the interest rate. The spike in the interest rate in 2009 is caused by the financial crisis. The growing size of emerging economies also generates an increase in the domestic credit of advanced economies (as a percentage of GDP), while it falls in emerging economies. However, the latter is due to the fact that GDP (the denominator) grows faster than domestic credit (the numerator). Domestic credit itself also increases in emerging economies.

The last two panels plots the 'effective leverage' ratio. This is the ratio of the debt, $D_{j,t+1}$, to its recovery value in a financial crisis, $\kappa_{j,t+1}K_{j,t+1}$. Besides the temporary drop after the financial crisis, the model predicts a sharply increasing trend in effective leverage in response to the productivity changes experienced by the two regions (dashed line). This is directly related to the change in the interest rate: a lower interest rate is always associated with a higher effective leverage (see condition (7)).

As we will see, the increase in effective leverage plays an important role for aggregate volatility. It is important to note, however, that the upward trend in leverage would not have emerged if emerging economies had grown at the same (lower) rate experienced by advanced economies as indicated by the dotted line.

The main takeaway from the counterfactual exercise shown in Figure 5 is that the faster growth of emerging economies has been an important force for global imbalances and declining world real interest rate. Faster growth generates profits that increase entrepreneurial wealth and, therefore, the demand for financial assets. But when $\kappa_{j,t}$ does not change, the supply of financial assets remains the same. To clear the market, then, the interest rate has to drop. The faster growth of entrepreneurial wealth in emerging economies also implies that part of that wealth is invested abroad, which explains the imbalances.

We now conduct a second counterfactual exercise. This time the goal is to explore the importance of the changes in financial structure. We fix the productivity of the two countries and examine the implications of the measure changes in financial parameters $\phi_{j,t}$ and $\kappa_{j,t}$. Starting from the values in 1991, $z_{1,t}$ and $z_{2,t}$ both grow at the long-run rate $g - 1 = 0.0189$. This is the average GDP growth of advanced economies over the sample

period 1991-2020. The financial parameters $\phi_{j,t}$ and $\kappa_{j,t}$, however, take the values shown in Figure 3. The simulated variables are plotted in Figure 6.

This time the results show that the changes in financial structure are important for explaining the growing size of financial intermediation (higher credit-to-GDP ratios). They are also important for generating the large decline in the real interest rate. However, they cannot explain the observed global imbalances. In fact, in absence of differential productivity growth, advanced economies would accumulate positive NFA positions.

The changes in financial structure also led to an increase in effective leverage. As observed earlier, a lower interest rate is always associated with a higher effective leverage (condition (7)). This is important for understanding the impact of the structural changes for aggregate volatility.

4 Macroeconomic and financial instability

We now explore the main question addressed in this paper, that is, how the faster growth of emerging economies and the changes in financial structure impacted macroeconomic and financial volatility.

To compute measures of volatility, we simulate the model for 130 years in response to the random draws of sunspot shocks: $\varepsilon_{j,t} = 0$ with probability $\lambda = 0.04$ and $\varepsilon_{j,t} = 1$ with probability $1 - \lambda = 0.96$. When $\varepsilon_{j,t} = 0$ the liquidation price of capital drops to $\kappa_{j,t}$ and the outstanding debt is renegotiated.

During the first 100 years, the variables $\phi_{j,t}$ and $\kappa_{j,t}$ remain constant at their 1991 values, while the productivity of both countries grows at the same long-run rate $g - 1$. This is the average growth rate of GDP for advanced economies over the period 1991-2020. The first 100 simulated years are used to derive the invariant distribution. The remaining 30 years of simulation correspond to the sample period 1991-2020 where $z_{j,t}$, $\phi_{j,t}$ and $\kappa_{j,t}$ take the values plotted in Figure 3. The simulation is then repeated 10,000 times, each time with a new sequence of random draws of the sunspot shocks over the 130 periods.

The continuous line in Figure 7 plots the mean of aggregate output computed over the 10,000 repeated simulations (where each simulation is the response to the random draw of sunspot shocks over the 130 simulation periods). The repeated simulations give us 10,000 data points for every simulation year. The mean is computed as $\overline{Y}_t = \frac{1}{10,000} \sum_{i=1}^{10,000} Y_{i,t}$. The graph also

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plots the 5th and 95th percentiles of the 10,000 data points. The difference between the 95th and 5th percentiles provides a measure of volatility.

The top panels of Figure 7 are for the baseline model in which both productivity and financial structure parameters change over time (as shown in Figure 3). Both countries display a widening gap between the 95th and 5th percentiles, indicating an increase in volatility. As we will see, the increase in volatility is directly related to the increase in (effective) leverage.

The middle panels show the counterfactual simulation in which only productivity changes (i.e., keeping $\phi_{j,t}$ and $\kappa_{j,t}$ unchanged). The plots show that the distance between the two percentiles (the measure of volatility) also increases when productivity is the only source of change. However, the magnitude is significantly smaller. Roughly, productivity changes alone contribute about 30% to the increase in volatility.

The last panels of Figure 7 are based on the counterfactual simulation in which productivity in both regions grow at the same long-run rate $g - 1 = 0.0189$. What changes are the variables $\phi_{j,t}$ and $\kappa_{j,t}$ which we interpret as reflecting structural changes in the financial sector. These changes contribute about 70 percent to the increase in volatility.

Notice that the changes in financial structure affected not only volatility but also the level of output, as can be seen from the dynamics of the mean over time. The mean for advanced economies does not change significantly when the financial structure remains constant. This is because the expansion of the financial sector allowed by the changes in $\phi_{j,t}$ and $\kappa_{j,t}$ created more financial assets. Since financial assets enter the production function, the change in supply raised output.

To better illustrate the increase in volatility, for each year we compute the difference between the 95th and 5th percentiles of the 10,000 output points generated by the repeated simulations. We then express the difference in percentage of the mean of output. More specifically, denoting by $P_t(5)$ the threshold for the 5th percentile of the 10,000 points in year t, and $P_t(95)$ the threshold for the 95th percentile, output volatility at time t is computed as

$$VOL_t = \left(\frac{P_t(95) - P_t(5)}{\overline{Y}_t} \right) \times 100.$$

Figure 8 plots the volatility measure as well as the effective leverage $(d_{j,t}/\kappa_{j,t}k_{j,t})$. Four types of simulations are considered. The top panels are for the baseline model where both productivity and financial structure
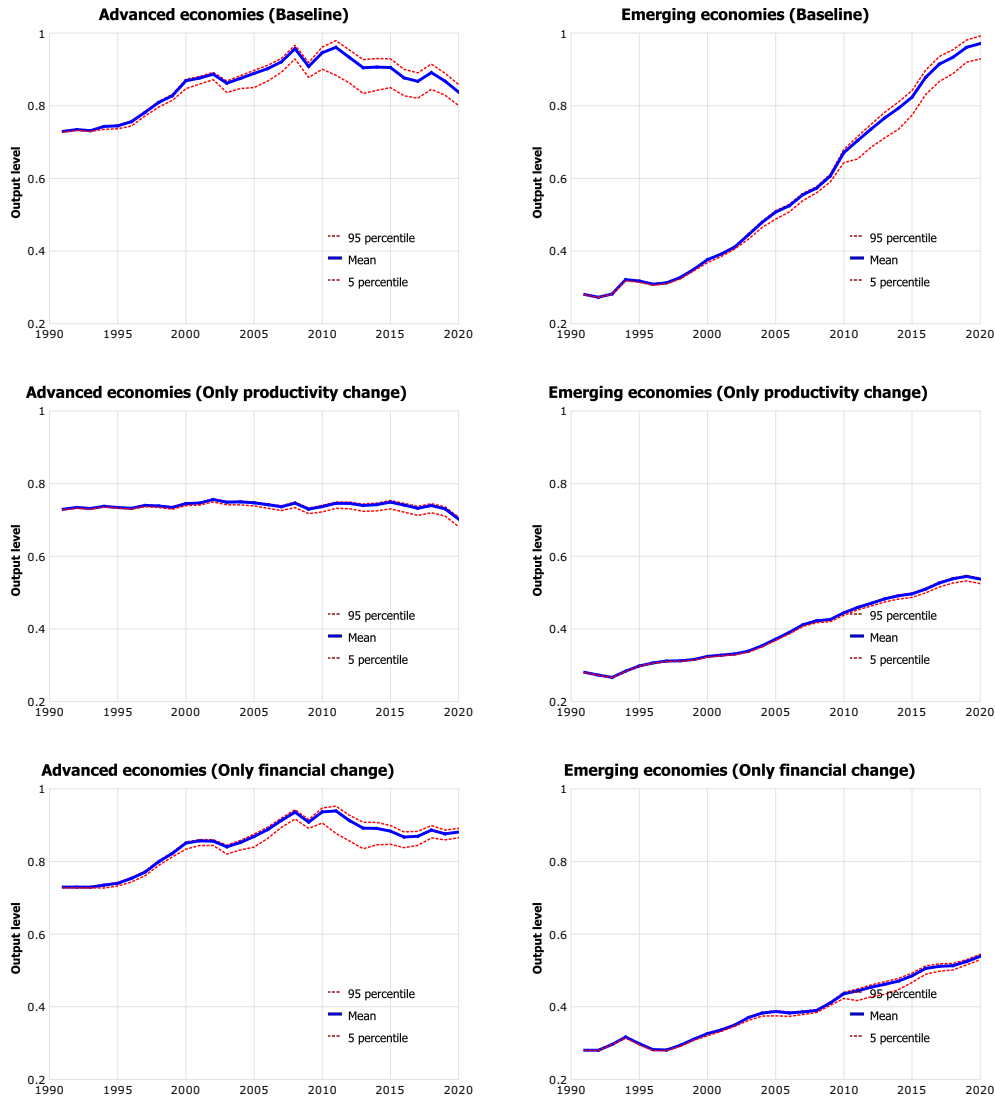
Figure 7: Output mean and percentiles of the 10,000 repeated simulations over the period 1991-2020. The mean is the average in every year over the 10,000 repeated simulations. The 5th and 95th percentiles are the thresholds for 5 and 95 percent of the sorted 10,000 data points.

change over time. As already illustrated in the previous figure, both countries experience an increase in volatility. The new figure enhances the visibility of the increase in volatility. The right panel plots the average value of the effective leverage. It shows that the increase in volatility is directly related to the increase in average (effective) leverage. As already mentioned, a financial crisis leads to debt restructuring which causes a redistribution of wealth from lenders (entrepreneurs) to borrowers (households/firms). The reduction in entrepreneurial wealth, then, reduces employment and production. Since the magnitude of the redistribution increases with leverage, the model generates an increase in volatility as a consequence of the increase in leverage.

The next panels illustrate the factors that contributed to the growth in volatility. The graphs in the middle rows show the importance of faster productivity growth in emerging economies. The growth in productivity, keeping $\phi_{j,t}$ and $\kappa_{j,t}$ unchanged, contributed about 30 percent to the increase in volatility. It is important to emphasize that the increase in volatility would not arise if emerging economies experienced the same productivity growth as advanced economies. This is shown in the third row of the figure. These plots are constructed under the counterfactual assumption that emerging economies experienced the same productivity growth as advanced economies. In this case, volatility does not change significantly over the simulated period. This is again related to the fact that effective leverage remains almost unchanged (see the right panel). This shows more clearly that the faster growth of emerging economies has been important for generating higher macroeconomic and financial instability.

The last row of Figure 8 is based on the counterfactual simulation in which productivity in both regions grows at the same long-run rate of $g - 1 = 0.0189$. What changes are the variables $\phi_{j,t}$ and $\kappa_{j,t}$. These changes contributed about 70 percent to the increase in volatility. Also in this case the increase in volatility is related to the increase in effective leverage (see right panel). The changes in financial structure led to a worldwide increase in net demand for financial assets. This caused a decline in the interest rate, which in turn increased effective leverage.
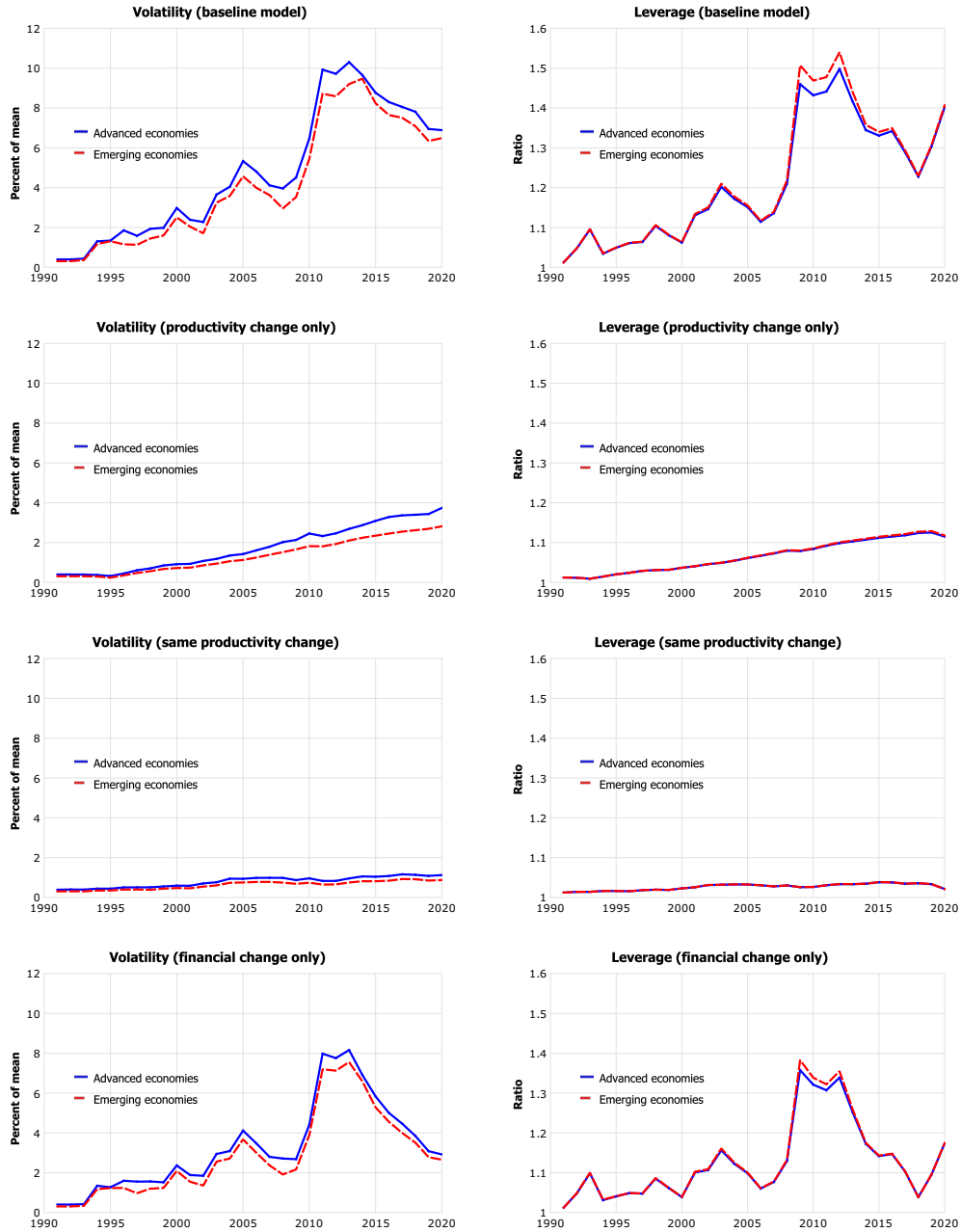
Figure 8: Output volatility and mean of effective leverage over the period 1991-2020. The volatility is the difference between the 5th and 95th percentiles as a percentage of the mean. Effective leverage is the ratio of debt over the liquidation value of capital in a crisis.

5 Discussion and conclusion

An implication of the increased size of emerging economies is that they are more influential in the world economy. The view that countries in emerging markets are a collection of small open economies with negligible impact on advanced economies is no longer a valid approximation.

There are many channels through which emerging markets can affect the rest of the world. In this paper we emphasized one of these channels: the increased demand for financial assets traded in globalized capital markets. In particular, we showed that the worldwide increase in the demand for financial assets raises the incentives to leverage. On the one hand, this allows for the expansion of the financial sector with positive effects on real macroeconomic activities. On the other, it increases the fragility of the financial system, raising the probability and/or the consequences of a crisis. From a policy perspective there is a trade-off: the benefit of an expanded financial system versus the potential cost of more severe crises. A similar mechanism also arises in models with asset price bubbles and borrowing constraints as in Miao and Wang (2011).

Appendix

A Proof of Lemma 2.1

The optimization problem of an entrepreneur in country j is

$$\max_{\{l_{j,t},k_{j,t},c_{j,t},b_{1j,t+1},b_{2j,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_{j,t})$$

subject to

$$
\begin{aligned}
m_{j,t} &= \delta_{1,t}b_{1j,t} + \delta_{2,t}b_{2j,t}, \\
a_{j,t} &= (1 - \phi_{j,t})m_{j,t} + z_{j,t}^{\gamma}m_{j,t}^{\alpha}l_{j,t}^{\gamma}k_{j,t}^{1-\alpha-\gamma} - w_{j,t}l_{j,t} - r_{j,t}k_{j,t}, \\
c_{j,t} &= a_{j,t} - q_{1,t}b_{1j,t+1} - q_{j,t}b_{2j,t+1}.
\end{aligned}
$$

The first order conditions for $l_{j,t}$ and $k_{j,t}$ are

$$\gamma z_{j,t}^{\gamma}m_{j,t}^{\alpha}l_{j,t}^{\gamma-1}k_{j,t}^{1-\alpha-\gamma} = w_{j,t}$$

$$(1 - \alpha - \gamma)z_{j,t}^{\gamma}m_{j,t}^{\alpha}l_{j,t}^{\gamma}k_{j,t}^{-\alpha-\gamma} = r_{j,t}$$

These two conditions give us the first two equations in Lemma 2.1. Since the inputs of labor and capital are linear functions of $m_{j,t}$, the end of period wealth is also linear in $m_{j,t}$, that is, $a_{j,t} = \pi_{j,t}m_{j,t}$. Here the term $\pi_{j,t}$ is a function of parameters and aggregate prices that are taken as given by an individual entrepreneur. Since $m_{j,t} = \delta_{1,t}b_{1j,t} + \delta_{2,t}b_{2j,t}$, we can write the end-of-period wealth at time t and at $t+1$ as

$$
\begin{aligned}
a_{j,t} &= \pi_{j,t}(\delta_{1,t}b_{1j,t} + \delta_{2,t}b_{2j,t}), \\
a_{j,t+1} &= \pi_{j,t+1}(\delta_{1,t+1}b_{1j,t+1} + \delta_{2,t+1}b_{2j,t+1}).
\end{aligned}
$$

We can now derive the first order conditions with respect to $b_{1j,t+1}$ and $b_{2j,t+1}$,

$$\frac{q_{1,t}}{c_{j,t}} = \beta\mathbb{E}_t\left(\frac{\pi_{j,t+1}\delta_{1,t+1}}{c_{j,t+1}}\right), \tag{19}$$

$$\frac{q_{2,t}}{c_{j,t}} = \beta\mathbb{E}_t\left(\frac{\pi_{j,t+1}\delta_{2,t+1}}{c_{j,t+1}}\right). \tag{20}$$

In the next step we guess that optimal consumption is a fraction $1-\beta$ of wealth,

$$c_{j,t} = (1 - \beta)a_{j,t}.$$

The saved wealth is allocated bonds issued by country 1 and to bonds issued by country 2. Denoting by $\theta_{j,t}$ the share allocated to country 1, we have

$$q_{1,t} b_{1j,t+1} = \theta_{j,t} \beta a_{j,t} \qquad \text{and} \qquad q_{2,t} b_{2j,t+1} = (1 - \theta_{j,t}) \beta a_{j,t}. \qquad (21)$$

Multiplying equation (19) by $b_{1j,t+1}$ and equation (20) by $b_{2j,t+1}$, adding the resulting expressions, and using the equations that define consumption and next period wealth, we obtain

$$q_{1,t} b_{1j,t+1} + q_{2,t} b_{2j,t+1} = \beta a_{j,t}.$$

This is obviously satisfied given (21). Thus, the Euler equation is satisfied if consumption is a fraction $1 - \beta$ of wealth, which verifies our guess.

We now replace the guess for $c_{j,t}$ into equation (20), to obtain

$$\mathbb{E}_t \left\{ \frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{j,t} \frac{\delta_{1,t+1}}{q_{1,t}} + (1 - \theta_{j,t}) \frac{\delta_{2,t+1}}{q_{2,t}}} \right\} = 1. \qquad (22)$$

This condition determines the share of savings invested in the bonds of the two countries. Since the condition is the same for entrepreneurs in country 1 and in country 2, it must be that $\theta_{1,t} = \theta_{2,t} = \theta_t$. ∎

B First order conditions for households/firms

The optimization problem of households/firms can be written recursively as

$$V(d,k) = \max_{l,c,d'} \left\{ e - z \frac{h^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}} + \beta \mathbb{E} V(d', k') \right\},$$
$$\text{subject to}$$

$$\tilde{d}(d, \tilde{p}k) + pk' + c + \varphi(d', \kappa'k') = wh + rk + pkg + \frac{1}{R} \mathbb{E} \tilde{d}(d', \tilde{p}'k'),$$

where the function $\tilde{d}(d, \tilde{p}k)$ is defined in (3) and the function $\varphi(d', \kappa'k')$ in (4).

The first order conditions with respect to h, d', k' are, respectively,

$$z h^{\frac{1}{\nu}} = w,$$

$$\frac{1}{R} \mathbb{E} \left\{ \frac{\partial \tilde{d}(d', \tilde{p}'k')}{\partial d'} \right\} - \frac{\partial \varphi(d', \kappa'k')}{\partial d'} + \beta \mathbb{E} \left\{ \frac{\partial V(d', k')}{\partial d'} \right\} = 0,$$

$$\frac{1}{R} \mathbb{E} \left\{ \frac{\partial \tilde{d}(d', \tilde{p}'k')}{\partial k'} \right\} - \frac{\partial \varphi(d', \kappa'k')}{\partial k'} + \beta \mathbb{E} \left\{ \frac{\partial V(d', k')}{\partial k'} \right\} = p_{j,t}.$$

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The envelope conditions are

$$\frac{\partial V(d,k)}{\partial d} = -\frac{\partial \tilde{d}(d,\tilde{p}k)}{\partial d},$$

$$\frac{\partial V(d,k)}{\partial k} = r + pg - \frac{\partial \tilde{d}(d,\tilde{p}k)}{\partial k}.$$

Updating by one period and substituting in the first order conditions for debt and capital we obtain

$$\frac{1}{R} = \beta + \frac{\frac{\partial \varphi(d',\kappa'k')}{\partial d'}}{\mathbb{E}\left\{\frac{\partial \tilde{d}(d',\tilde{p}'k')}{\partial d'}\right\}}, \tag{23}$$

$$p_{j,t} = \frac{1}{R}\mathbb{E}\left\{\frac{\partial \tilde{d}(d',\tilde{p}'k')}{\partial k'}\right\} + \beta\mathbb{E}\left\{r' + gp' - \frac{\partial \tilde{d}(d',\tilde{p}'k')}{\partial k'}\right\} - \frac{\partial \varphi(d',\kappa'k')}{\partial k'}. \tag{24}$$

We now derive the analytical expressions for the for derivative expressions using the functional forms for the functions $\tilde{d}(d,\tilde{p}k)$ and $\varphi(d',\kappa'k')$ defined, respectively, in (3) and (4):

$$\frac{\partial \tilde{d}(d,\tilde{p}k)}{\partial d} = \begin{cases} 0, & \text{if} \quad d \geq \tilde{p}k \\ \\ 1, & \text{otherwise} \end{cases}$$

$$\frac{\partial \tilde{d}(d,\tilde{p}k)}{\partial k} = \begin{cases} \tilde{p}, & \text{if} \quad d \geq \tilde{p}k \\ \\ 0, & \text{otherwise} \end{cases}$$

$$\frac{\partial \varphi(d',\kappa'k')}{\partial d'} = \begin{cases} 2\eta\left(1 - \frac{\kappa'k'}{d'}\right)\frac{\kappa'k'}{d'} + \eta\left(1 - \frac{\kappa'k'}{d'}\right)^2, & \text{if} \quad d' \geq \kappa'k' \\ \\ 0, & \text{otherwise} \end{cases}$$

$$\frac{\partial \varphi(d',\kappa'k')}{\partial k'} = \begin{cases} -2\eta\left(1 - \frac{\kappa'k'}{d'}\right)\kappa', & \text{if} \quad d' \geq \kappa'k' \\ \\ 0, & \text{otherwise} \end{cases}$$

We now assume that the equilibrium is always characterized by $d' \geq \kappa'k'$ and $d' > p'k'$. This will be the case in the parameterized model. Under this assumption we always have default only if $\tilde{p} = \kappa$, which arises with probability λ. The

expected values of the above derivatives can then be written as

$$\mathbb{E}\left\{\frac{\partial \tilde{d}(d, \tilde{p}k)}{\partial d}\right\} = 1 - \lambda$$

$$\mathbb{E}\left\{\frac{\partial \tilde{d}(d, \tilde{p}k)}{\partial k}\right\} = \lambda \kappa$$

$$\frac{\partial \varphi(d', \kappa' k')}{\partial d'} = 2\eta \left(1 - \frac{\kappa' k'}{d'}\right) \frac{\kappa' k'}{d'} + \eta \left(1 - \frac{\kappa' k'}{d'}\right)^2$$

$$\frac{\partial \varphi(d', \kappa' k')}{\partial k'} = -2\eta \left(1 - \frac{\kappa' k'}{d'}\right) \kappa$$

Using these expressions in the first order conditions (23) and (24) we obtain

$$\frac{1}{R} = \beta + \Phi\left(\frac{d'}{\kappa' k'}\right), \tag{25}$$

$$p = \beta \mathbb{E}(r' + gp') + \Psi\left(\frac{d'}{\kappa' k'}\right), \tag{26}$$

where

$$\Phi\left(\frac{d'}{\kappa' k'}\right) = \left(\frac{1}{1-\lambda}\right) \left[2\eta\left(1 - \frac{\kappa' k'}{d'}\right)\frac{\kappa' k'}{d'} + \eta\left(1 - \frac{\kappa' k'}{d'}\right)^2\right],$$

$$= \left(\frac{1}{1-\lambda}\right)\eta\left(1 - \left(\frac{\kappa' k'}{d'}\right)^2\right)$$

$$\Psi\left(\frac{d'}{\kappa' k'}\right) = \left[\lambda \Phi\left(\frac{d'}{\kappa' k'}\right) + 2\eta\left(1 - \frac{\kappa' k'}{d'}\right)\right] \kappa'.$$

It is evident from these expressions that both functions are increasing in $\frac{d'}{\kappa' k'}$. In addition, taking derivatives we can verify also that they are increasing in d' and decreasing in both k' and κ'. ∎

C Market for liquidated capital and equilibrium multiplicity

In the main body of the paper we have assumed that the liquidation price $\tilde{p}_{j,t}$ can fluctuate between $\kappa_{j,t}$ and $p_{j,t}$ with constant probabilities λ and $1 - \lambda$. In this section we describe the market structure that provides the micro-foundation for the determination of $\tilde{p}_t$. In this specification there are multiple equilibria and

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λ represents the probability of a sunspot shock that selects one of the two self-fulfilling equilibria.

The market for liquidated capital meets at the beginning of the period. We make two important assumptions about how this market operates.

Assumption 1 *Capital can be sold to domestic households/firms or entrepreneurs. If sold to entrepreneurs, capital loses its functionality as a productive asset and it is converted to consumption goods at rate $\kappa_{j,t}$.*

This assumption formalizes the idea that capital may lose value when reallocated to non-specialized owners, provided that $\kappa_{j,t}$ is sufficiently low. In order for capital to keep its functionality as a productive asset, it needs to be purchased by domestic households/firms, not foreign households/firms. With this assumption a crisis could be local, that is, it could take place in one country without spreading to the other country. However, even if a crisis takes place only in one country, it has real economic effects also in the other country due to the cross-country diversification of bond portfolios.

Assumption 2 *Households/firms can purchase liquidated capital only if the liquidation value of their capital exceeds the debt obligations, $d_{j,t} < \tilde{p}_{j,t}k_{j,t}$.*

If a household/firm starts with liabilities bigger than the liquidation value of the owned assets, that is, $d_{j,t} > \tilde{p}_{j,t}k_{j,t}$, it will be unable to raise additional funds to purchase the liquidated capital. Potential investors know that the new liabilities (as well as the outstanding liabilities) are not collateralized, and the debt will be renegotiated immediately by households/firms after taking the new debt. We refer to a household/firm with $d_{j,t} < \tilde{p}_{j,t}k_{j,t}$ as 'liquid' since it can raise extra funds at the beginning of the period. Instead, a household/firm with $d_{j,t} > \tilde{p}_{j,t}k_{j,t}$ is 'illiquid'.

To better understand Assumptions 1 and 2, consider the condition for not renegotiating, $d_{j,t} \leq \tilde{p}_{j,t}k_{j,t}$. Furthermore, assume that $p_{j,t} > \kappa_{j,t}$, that is, the price at the end of the period is bigger than the liquidation price when the market freezes. If this condition is satisfied, households/firms have the ability to raise funds to purchase additional capital. In turn, this ensures that the liquidation price is $\tilde{p}_{j,t} = p_{j,t}$. If $d_{j,t} > \kappa_{j,t}k_{j,t}$ for all households/firms, however, there will be no households/firms capable of buying the liquidated capital. Then, liquidated capital can only be purchased by entrepreneurs at price $\tilde{p}_{j,t} = \kappa_{j,t}$.

This shows that the market price for liquidated capital depends on the financial decision of households/firms, which in turn depends on the liquidation price. This interdependence is critical for our argument because it can lead to self-fulfilling equilibria (i.e, it is what triggers financial crises in the model).

Proposition C.1 *There exists multiple equilibria only if $d_{j,t} > \kappa_{j,t} k_{j,t}$.*

Proof C.1 *At the beginning of the period households/firms choose whether to renegotiate the debt. Given the initial states d_t and k_t, the renegotiation decision boils down to a take-it or leave-it offer made to creditors for the repayment of the debt.*

Denote by $\tilde{d}_t = \psi(d_t, k_t, \tilde{p}_t)$ the offered repayment. This depends on the individual liabilities, d_t, individual capital, k_t, and the price for liquidated capital, $\tilde{p}_t$. The price of the liquidated capital is the price at which the lender could sell the capital after rejecting the offer from the borrower. The best offer made by the household/firm is

$$
\psi(d_t, k_t, \tilde{p}_t) = \begin{cases} d_t, & \text{if} \quad d_t \leq \tilde{p}_t k_t \\[2mm] \tilde{p}_t^e k_t, & \text{if} \quad d_t > \tilde{p}_t k_t \end{cases}, \tag{27}
$$

which is accepted by creditors if they cannot sell at a price higher than $\tilde{p}_t$.

For the moment we assume that the equilibrium is symmetric, that is, all households/firms start with the same ratio d_t/k_t. At this stage this is only an assumption. However, we will show below that households/firms do not have an incentive to deviate from the ratio chosen by other households/firms.

Given the assumption that the equilibrium is symmetric (all households/firms choose the same ratio d_t/k_t), multiple equilibria arise if $d_t/k_t \in [\kappa_t, p_t]$. If the market expects that the liquidation price is $\tilde{p}_t = \kappa_t$, all households/firms are illiquid and they choose to renege their liabilities (given the renegotiation policy (27)). As a result, there will be no households/firms that can purchase the liquidated capital of other households/firms. The only possible liquidation price that is consistent with the expected price is $\tilde{p}_t = \kappa_t$. On the other hand, if the market expects $\tilde{p}_t = p_t$, households/firms are liquid and, if one household/firm reneges, creditors can sell the liquidated assets to other households/firms at the liquidation price $\tilde{p}_t = p_t$. Therefore, it is optimal for households/firms not to renegotiate.

We now address the issue of whether individual households/firms have an incentive to deviate from the symmetric equilibrium and choose a different ratio d_t/k_t in the previous period $t-1$. In particular, we need to show that, in the anticipation that the liquidation price could be $\tilde{p}_t = \kappa_t$, a household/firm does not find convenient to borrow less at time $t-1$ so that it could purchase the liquidated capital if the price drops to κ_t.

The first point to consider is that, in equilibrium, capital is never liquidated. The low liquidation price κ_t simply represents the threat value for creditors. However, in equilibrium all creditors accept the renegotiation offer and no capital is ever liquidated.

What would happen if there is a household/firm that is liquid and, therefore, has the ability to purchase the liquidated capital at a higher price than κ_t? This would arise if a household/firm deviates from the symmetric equilibrium. In this case debtors know that their creditors could liquidate the capital and sell it at a higher price than κ_t. Knowing

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this, debtors will offer a higher repayment and, as a result, capital is not liquidated. Poten-
tially, this could drive the liquidation price to p_t. This shows that a household/firm cannot
make any profit by remaining liquid. Therefore, there is no incentive to deviate from the
symmetric equilibrium. ∎

Assume that the equilibrium is symmetric. Then, all households/firms choose the same ratio d_t/k_t and multiple equilibria determined by self-fulfilling expectations of the liquidation price can exist. The proof above have shown that this requires $d_t/k_t \in [\kappa_t, p_t)$. On the one hand, if the market expects a liquidation price $\tilde{p}_t = \kappa_t$, all households/firms are illiquid and choose to renege on their liabilities. As a result, there are no households/firms that can purchase the liquidated capital and, therefore, the only liquidation price consistent with the expected price is $\tilde{p}_t = \kappa_t$. On the other hand, when the market expects $\tilde{p}_t = p_t$, households/firms are liquid and, if one household/firm reneges, creditors can sell the liquidated capital to other households/firms at price $\tilde{p}_t = p_t$, which makes it optimal not to renege.

When multiple equilibria are possible, that is, we have $d_{j,t} > \kappa_{j,t} k_{j,t}$, the equilibrium is selected by a random draw of sunspot shocks. Let $\varepsilon_{j,t}$ be a variable that takes the value of 0 with probability λ and 1 with probability $1 - \lambda$. If the condition for multiplicity is satisfied, agents coordinate their expectations on the low liquidation price $\kappa_{j,t}$ if $\varepsilon_{j,t} = 0$. This implies that the probability distribution of the low liquidation price is

$$
f_{j,t}\left(\tilde{p}_{j,t} = \kappa_{j,t}\right) = \begin{cases} 0, & \text{if} \quad d_{j,t} \leq \kappa_{j,t} k_{j,t} \\ \\ \lambda, & \text{if} \quad d_{j,t} > \kappa_{j,t} k_{j,t} \end{cases}
$$

The ratio $d_{j,t}/\kappa_{j,t} k_{j,t}$ is the relevant measure of leverage. When sufficiently small, households/firms remain liquid even if the (expected) liquidation price is $\kappa_{j,t}$. But then the liquidation price cannot be low and the realization of the sunspot shock is irrelevant for the equilibrium. Instead, when leverage is high, households/firms' liquidity depends on the liquidation price. The realization of the sunspot shock $\varepsilon_{j,t}$ then becomes important for selecting one of the two equilibria. When $\varepsilon_{j,t} = 0$—which happens with probability λ—the market expects that the liquidation price is $\kappa_{j,t}$, making the household's sector illiquid. On the other hand, when $\varepsilon_{j,t} = 1$—which happens with probability $1 - \lambda$—the market expects that households/firms are capable of participating in the liquidation market, validating the expectation of a higher liquidation price.

Notice that this argument is based on the assumption that $\kappa_{j,t}$ is sufficiently low (implying a low liquidation price if the capital freezes). Also, the equilibrium value of capital without a freeze, $p_{j,t} k_{j,t}$, is always bigger than the debt $d_{j,t}$.

Otherwise, households/firms would be illiquid with probability 1 and the equilibrium price is always $\kappa_{j,t}$. Condition (5) guarantees that this does not happen at equilibrium: if the probability of default is 1, the anticipation of the renegotiation cost increases the interest rate, which deters households/firms from borrowing too much.

D Equilibrium system of equations at time t

Given the values of $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$, $\kappa_{2,t}$, $\kappa_{1,t+1}$, $\kappa_{2,t+1}$, and the stochastic states s_t, we can find the values of $\delta_{j,t}$, $M_{j,t}$, $L_{j,t}$, $K_{j,t}$, $w_{j,t}$, $r_{j,t}$, $q_{j,t}$, $A_{j,t}$, $B_{j1,t+1}$, $B_{j2,t+1}$, $D_{j,t+1}$ and θ_t, by solving the following system of equations:

$$\delta_{j,t} = \begin{cases} \min\left\{1, \frac{\kappa_{j,t}K_{j,t}}{D_{j,t}}\right\}, & \text{if} \quad \varepsilon_{j,t} = 0 \\ \\ 1, & \text{if} \quad \varepsilon_{j,t} = 1 \end{cases} \tag{28}$$

$$M_{j,t} = \delta_{1,t}B_{1,t} + \delta_{2,t}B_{2,t} \tag{29}$$

$$L_{j,t} = z_{j,t}^{\frac{2}{\alpha}}\left(\frac{\gamma}{w_{j,t}}\right)^{\frac{\alpha+\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{r_{j,t}}\right)^{\frac{1-\alpha-\gamma}{\alpha}}M_{j,t}, \tag{30}$$

$$K_{j,t} = z_{j,t}^{\frac{2}{\alpha}}\left(\frac{\gamma}{w_{j,t}}\right)^{\frac{\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{r_{j,t}}\right)^{\frac{1-\gamma}{\alpha}}M_{j,t}, \tag{31}$$

$$L_{j,t} = \left(\frac{w_{j,t}}{z_{j,t}}\right)^{\nu}, \tag{32}$$

$$K_{j,t} = \bar{K}g^t, \tag{33}$$

$$A_{j,t} = (1-\phi_j)M_{j,t} + z_{j,t}^{\gamma}M_{j,t}^{\alpha}L_{j,t}^{\gamma}K_{j,t}^{1-\alpha-\gamma} - w_{j,t}L_{j,t} - r_{j,t}K_{j,t}, \tag{34}$$

$$B_{1j,t+1} = \frac{\theta_t\beta A_{j,t}}{q_{1,t}}, \tag{35}$$

$$B_{2j,t+1} = \frac{(1-\theta_t)\beta A_{j,t}}{q_{2,t}}, \tag{36}$$

$$1 = \mathbb{E}_t\left\{\frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_t\frac{\delta_{1,t+1}}{q_{1,t}} + (1-\theta_t)\frac{\delta_{2,t+1}}{q_{2,t}}}\right\}, \tag{37}$$

$$D_{j,t+1} = B_{j1,t+1} + B_{j2,t+1}, \tag{38}$$

$$q_{j,t} = \left[\beta + \Phi\left(\frac{D_{j,t+1}}{\kappa_{j,t+1}K_{j,t+1}}\right)\right]\mathbb{E}_t\delta_{j,t+1}. \tag{39}$$

Equation (28) defines the optimal renegotiation strategy (the fraction of the debt repaid). Equation (29) defines entrepreneurial wealth after default. Equa-

tions (30) and (31) are the demand for labor and capital from entrepreneurs, given the prices $w_{j,t}$ and $r_{j,t}$, and their wealth $M_{j,t}$. Equations (32) and (33) are the supplies of labor and capital from households/workers. Equation (34) defines the end-of-period wealth of entrepreneurs after production. This is allocated to bonds issued by the two countries as indicated in equations (35) and (36). Equation (37) is the condition that determines the investment share θ_t. This is the Euler equation derived from the optimization problem of entrepreneurs. Equation (38) is equilibrium in the bond market. The final equation (39) is the Euler equation for the households/firms determining the price of bonds.

The above system determines all equilibrium variables except the price of capital $p_{j,t}$. To solve for the price of capital we need to use condition (26) where the current price $p_{j,t}$ depends on the future price $p_{j,t+1}$. This implies that we cannot solve for the equilibrium price in the current period without solving for the equilibrium in the future. Therefore, we need to use an iterative procedure. However, since the current price $p_{j,t}$ does not affect other variables in the current period, we can use the above system to solve for the equilibrium in period t ignoring the price. Notice that this would not be the case if the liquidation value of capital was a function of $p_{j,t}$.

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