Market Power in Credit Markets*

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Abstract

We study, theoretically and in a quantitative model, the determinants of lender profits in the cross-section of households. We argue that the empirical pattern of high profit margins for high risk contracts calls for a departure from constant markups or ex-post perfect competition models. We propose an imperfectly competitive model in which credit pricing is pinned down not only by default probability but also by the household’s outside options, giving rise to profit margins distribution consistent with the data. In our economy, we study the effects of limiting lender market power via interest rate caps. We show that the parameterized economy features strong general equilibrium effects by which imposing a limit on a small number of contracts affects the whole interest rate distribution, implying significant welfare gains from the policy.

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1 Introduction

The scope and impact of lender market power in the unsecured credit market has long been a topic of intense debate in academic and policy circles\footnote{See, for example, Ausubel (1991), Evans and Schmalensee (2005), Galenianos and Gavazza (2021), Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015), Herkenhoff and Raveendranathan (2020).}, especially in the context of regulating financial products. On the one hand, regulating lender ability to extract profits can lead to credit rationing and welfare losses if the market is close to competitive. On the other hand, regulating high markup contracts can benefit borrowers. These considerations spurred a variety of regulatory actions affecting the unsecured credit market, most recently the 2009 CARD Act, which regulated financial re-pricing and fee structures, and the proposed Loan Shark Prevention Act, which advocates for universal interest rate caps on credit cards. Empirical evidence is consistent with departures from perfect competition in the unsecured credit market,\footnote{For an overview of empirical evidence, see Herkenhoff and Raveendranathan (2020).} and additionally documents a positive relationship between default risk and profits per dollar of debt (profitability).\footnote{Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015).} This suggests that accounting for these features of the market is important for any policy discussion about potential interventions in consumer credit markets. However, theoretical literature offering a discussion of the qualitative and quantitative implications of market power in credit markets, and in particular, its effect on policy evaluation, is still scant. This paper aims to fill this gap by providing a theoretical and quantitative analysis of the effect of market power, as well as the impact of limiting it via regulation.

Our analysis is divided into two parts. First, in a simplified analytical setup, we study how departing from the competitive paradigm affects model outcomes of important empirical counterparts. We find that higher lender market power is associated with higher debt and default on individual and aggregate levels. We also characterize the relation between profitability and borrower riskiness in the model. We show that in order to be consistent with the positive empirical relationship, the loan market must feature markups on loans that positively vary with households’ riskiness. This calls for a departure from standard assumptions of competitive pricing or constant markup pricing. Second, we build a quantitative model of unsecured borrowing in which the market is imperfectly competitive and contract pricing depends on households’ outside options. The
parameterized quantitative model confirms the qualitative results derived in the analytical setup, and generates a positive relationship between profitability and risk - consistent with the data. We use the model to study the consequences of limiting market power by introducing interest rate caps. We show that the parameterized economy features strong general equilibrium effects by which imposing a limit on a small number of contracts affects the whole interest rate distribution, implying significant welfare gains.

In order to derive our analytical results, we set up a stylized two-period borrowing problem with a credit market that is characterized by an exogenous arbitrary distribution of profits across household types. This distribution can be generated by a variety of modeling assumptions about frictions that give rise to imperfect competition, and encompasses perfect competition as a special case. Focusing on bilaterally efficient contracts, we provide a characterization of the effects of increasing markups in the credit market, i.e. increasing market power of the lenders. We show that contracts that deliver more surplus to the lender exhibit higher interest rates and higher borrowing, implying higher default rates. This result has immediate aggregate implications: the market with more market power of the lenders will be associated with higher bankruptcy rate, average interest rate, and debt/income. Next, we focus on the predictions of the model for the cross-section of profitability of credit contracts, i.e. profits per unit of debt. The empirical pattern against which we measure the predictions of the model is the finding that profitability of unsecured credit card accounts generally rises with bankruptcy risk. In our analytical setup, we show that this empirical pattern can never be reproduced by credit markets that are perfectly competitive, feature constant markups or upfront cost with ex-post competition – assumptions that are prominent in the literature. Our analysis implies that a positive profitability-risk relationship requires markups on loans to be variable across households and positively related to risk. In order to show the potential of imperfect competition to generate such markup variation, we consider a market in which market power is entirely on the side of the lenders, i.e. they are monopolists. In this case, the loan pricing

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4Our analytical results depend only on the resulting distribution of profits in the market and not on the specifics of the underlying model of competition.

5Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015) show that profitability (realized profits per unit of average balance) peak at low credit scores (Figure E in the paper), driven by interest charge income and fee income for high risk credit card accounts (Figures A and B). The paper documents a second, low peak in profitability for low risk credit card accounts which is driven by interchange income (Figure D). Since we focus on risky unsecured borrowing, the relevant empirical pattern relates to accounts used for borrowing, not for transaction purposes.

6These specific findings hold more generally and do not require simplifying the analysis to a two-period model.
is determined not only by default probability, but also by the household’s outside option, which pins down the amount of surplus the monopolist can extract. We show that the variation of the loan surplus across risk groups and the monopolist’s ability to extract it can shift the distribution of profitability closer to the empirical one. We interpret our findings as support for departures from standard competitive or near-competitive assumptions towards models in which loan pricing is partially determined by household’s outside options.

We fully explore the implications of market power of lenders in a parameterized quantitative dynamic incomplete markets model with income risk. We micro-found the distribution of markups in the economy by modeling search and matching frictions with high markup outside option for unmatched households. Specifically, in order to obtain a loan contract, the household searches in a competitive search environment. Market tightness (ratio of lenders to borrowers) determines the probability of obtaining a loan in the search market and the pricing for each household type, given by the household’s current debt and income. Crucially, if the household fails to match with a competitive search lender, it gets an option of borrowing from a lender who realizes the household was not able to find a match and makes a high markup, take-it-or-leave-it offer, i.e. acts as a monopolist. This option captures reduced borrowing opportunities faced by households who fail to obtain the competitive search contract, and can be generalized to involve other outside options. As a consequence, the search friction determines markups in the competitive search market and the monopolistic market, but does not imply credit rationing. That is, if there exist a credit contract for a household that generates positive surplus, then the household is able to borrow. Depending on how attractive the household is as a borrower, the markups on the contract vary endogenously in both markets. We parameterize the model to two groups of households using income processes for ‘High School’ and ‘College’ in Guvenen (2009), and set the empirical targets for debt and default for the two groups using the Survey of Consumer Finances, which contains information on education. Our main parameterization, called Benchmark, matches aggregate debt and default statistics well, as well as untargeted average interest rates, charge-off rates, and additionally gives a good account of the distribution of interest rates vis-a-vis the data. As predicted by our theoretical analysis, the Benchmark model also implies profitability which decreases with repayment probability - consistent with empirical evidence.

Next, we show that modeling market power has significant implications for the evaluation of
regulation and policies that impact markups and profitability of the lenders, even if the regulation directly applies to only a small number of contracts. Specifically, we consider the policy of introducing universal interest rate caps on credit contracts. Regulating unsecured lending interest rates is a mainstay in policy discussion on oversight of financial products, and introducing interest rate caps has been proposed as recently as 2019 as part of the Loan Shark Prevention Act which would impose a 15% limit on credit cards nationwide. In the baseline parameterization, the cap is set at 100%, which means that it is virtually not binding and we treat this as a reflection of the current state of regulation in the U.S.\footnote{Even though in principle each state has an interest rate cap policy (on credit cards), in practice the applicability of the caps is limited. The 1978 Supreme Court ruling in Marquette National Bank v. First of Omaha Corp. implies that lenders can apply the interest rate cap of the state where they headquarter rather than the state of the location of the borrower, which limited the ability to enforce strict state-level usury limits.} In our experiment, we tighten the cap on interest rates gradually from 100% to 10%.

We find that in the Benchmark model, limiting market power of lenders has a significant negative effect on the interest rates, but a significant positive effect on debt and fraction of households borrowing. Crucially, the pricing of all loans is more favorable: the entire distribution of interest rates shifts down as a result of the cap, even though only a small fraction of interest rates are actually against the cap. This suggests significant general equilibrium effects of regulating just the top interest rates. More favorable pricing of debt implies more households decide to borrow in equilibrium and borrow more while they do. Additionally, debt becomes safer in response to cheaper borrowing and roll-over costs – both the chargeoff rate and the bankruptcy rate go down under tighter interest rate caps. The response to the policy across the two income group differs substantially. The ‘High School’ group sees a moderate increase in debt and fraction borrowing (by 4% and 14%, respectively), and a 3-fold reduction in bankruptcy rate. For ‘College’, cheaper credit results in a large increase in debt (by 50%) and fraction borrowing (by 26%), but virtually no change in bankruptcy rate. For both groups, we find significant welfare gains from the policy, ranging from 4% of lifetime consumption equivalent for the poorest households to 0.5% and 0.7% for the median household in the ‘High School’ and ‘College’ groups, respectively.

We provide further evidence of strong general equilibrium effects of interest rate caps in the Benchmark model in three counterfactual exercises. First, we fix the matching probabilities in the competitive search market to baseline values, thus controlling for the extensive margin of switching
between the monopolistic and search market contracts. We find that the impact of changing search probabilities on debt outcomes is extremely small, implying the overwhelming importance of loan pricing. Second, we counterfactually allocate all borrowers to their relevant competitive search contracts, implying no household receives the monopoly outside option. We find that this counterfactual economy accounts for the entirety of the extensive margin increase of borrowing in response to the interest rate cap, and 56% of the increase in debt. This implies that the endogenous response of the competitive search market pricing accounts for the majority of the equilibrium effects of the interest rate caps. Finally, we document the fraction of interest rates in the search market and the monopoly market that decrease in response to the tightening cap despite being strictly below it to begin with. We find that between 56% and 84% of such interest rates are optimally adjusted downward. From these three experiments, we conclude that the majority of the effects in the model comes from equilibrium propagation from binding caps on a small number of high interest rate contracts to pricing of all contracts due to changing outside options of households.

In order to provide a baseline for comparisons of our Benchmark economy results, we consider a version of our economy with ex-post competitive pricing (Competitive model economy). We then compare the predictions of the Benchmark model to the analogously parameterized Competitive economy. First, just as the analytical results imply, the Competitive model predicts that profitability is monotonically increasing with repayment probability – opposite to the data and Benchmark model. Second, relative to the Benchmark model, the Competitive model has fundamentally different responses to the interest rate cap policy in terms of debt, default and welfare. In the Competitive model, the interest rate cap works through mostly the extensive margin of credit. The interest rates there exhibit low dispersion, and hence the cap does not bind initially and has no effect. Eventually, however, it starts binding for a large fraction of contracts, rendering them unprofitable and shutting down those loans. Importantly, in the Competitive model, there is very little general equilibrium effect on the interest rate distribution: the interest rates that are below the cap are barely affected by the fact that other, high interest rate contracts are affected. This is in stark contrast to the Benchmark model, in which even contracts far below the cap are affected by the policy. This results in lower debt with tighter caps, eventually leading to no borrowing. The increasingly limited access to borrowing entails a significant welfare loss of about 1.5% of lifetime consumption for the median income household, and is significantly larger for the lowest incomes.
The prime feature of our quantitative model that differentiates it from other search-based credit market frictions modeled in the literature is the explicit introduction of the high markup outside option of unmatched households. We view this as a tractable way of capturing the fact that credit can be accessed by a household if there is positive surplus, but the exact pricing of that credit is pinned down by frictions. To capture the impact of allowing for the high markup outside option on model predictions, we analyze the quantitative predictions of a model with the competitive search market but no high markup option for unmatched households. What that means in equilibrium is that unmatched households with debt must repay or default. We find that this No High Markup Option version of the model, parameterized using the same targets as the Benchmark, has a hard time matching aggregate statistics for debt and charge-offs, and requires implausible values of the time discount factor of households. Additionally, we find that the response to interest rate caps is dramatically different than in the Benchmark model. In the No High Markup Option model, just like in the Competitive model, interest rate caps mostly affect contracts when they bind and do not have an equilibrium effect on the overall distribution of interest rates. As a result, caps on interest rates when binding, reduce the surplus from matches and lead to shutting down of the corresponding markets, leading to lower debt and fraction borrowing, and implying a welfare loss from the policy of 0.8% and 1.2% of lifetime consumption for the median College and High School household, respectively, driven mostly by the extensive margin of credit supply.

**Related Literature** Our study relates to the broader literature providing motivation for and evaluation of policies that can potentially improve the functioning of credit markets. In the context of behavioral biases, this literature includes Heidhues and Kőszegi (2010), Heidhues and Kőszegi (2017) and Exler, Livshits, MacGee, and Tertilt (2020) and in the context of frictions in credit markets that lead to lender market power it includes Herkenhoff (2019), Herkenhoff and Raveendranathan (2020), Raveendranathan (2020), Galenianos and Gavazza (2021), Nelson (2020), Stango (2002). Generally, relative to search-based credit market frictions modeled in the literature, we are the first to explicitly include the endogenously determined outside option to unmatched households, which as we show is quantitatively very important. We discuss the three most related papers in turn. Herkenhoff and Raveendranathan (2020) study the effects of improved competition through
entry in the unsecured credit market in an oligopolistic setup. They focus on equilibria with interest rate collusion and quantity competition, reflecting early evolution of the credit market. By contrast, we model a simpler contract structure, in which we allow contracts to vary by household state and focus on the general equilibrium effects of policy via the endogenous response of the price distribution. Galenianos and Gavazza (2021) study the aggregate response of the credit market to interest rate caps in an incomplete information environment in which the equilibrium response works through household search effort. Relative to that paper, we provide a fully dynamic consumption/saving model with full information in which the impact of interest rate caps comes from general equilibrium impact of households’ changing outside options. Raveendranathan (2020) studies the determinants of average profitability in an search environment with credit lines, focusing on the effects of information frictions. We focus on the impact of market power on the distribution of profitability and study its consequences for the effects of regulation. Finally, our Competitive model closely follows the perfectly competitive setups in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Livshits, MacGee, and Tertilt (2007) or Livshits, MacGee, and Tertilt (2010), where we allow for non-negative upfront cost of making loans.

The rest of the paper is organized as follows. Section 2 presents the 2-period analytical setup and derives theoretical results. Section 3 sets up the quantitative model, discusses parameterization and results. Section 4 concludes.

2 Theoretical analysis

This section develops a stylized two-period model which allows for a sharp characterization of the effects of competition on equilibrium outcomes such as access to credit, credit terms, debt and bankruptcy. We develop comparative statics results for debt, interest rates and bankruptcy across environments with different levels of market power. Additionally, we provide a sharp distinction between market environments in terms of the distribution of profitability across household risk, which we relate to findings in the empirical literature and show that it lends support for departures from the competitive assumption.
**Households** We consider a 2-period consumption smoothing problem with a household that has access to contracts characterized by different split of surplus between the borrower and the lender. Formally, the household chooses consumption for periods 1 and 2 and makes a default decision in period 2, to maximize the present value of expected utility:\(^8\)

\[
U(c_1, c_2) = u(c_1) + E[u(c_2)],
\]

where \(c_1\) and \(c_2\) is first-period and second-period consumption and the period utility \(u(c)\) satisfies \(u'(c) > 0, u''(c) < 0\) and the Inada conditions.

We assume that the household enters period 1 with cash-in-hand\(^9\) equal to \(y_1\), and receives income \(y_2\) in the second period, drawn from a distribution with cdf \(F(y)\) and support \([\underline{y}, \bar{y}]\), where \(0 < \underline{y} < \bar{y} < \infty\). Given that, the value of financial autarky is given by

\[
v^A(y_1) = u(y_1) + \int_{\underline{y}}^{\bar{y}} u(y_2) dF(y_2).
\]

**Loans and default** The household has access to a financial contract, summarized by the pair \((z_1, z_2)\). \(z_1\) is the amount transferred to the household in period 1 and \(z_2\) is the repayment amount, due in period 2. The household has an option of defaulting on the debt obligation \(z_2\), but in that case, suffers a proportional income loss \(\phi\). Given that, a household that takes a loan with a second-period repayment of \(z_2\) and experiences second-period income realization of \(y_2\) will choose to repay if and only if \(y_2 - z_2 \geq y_2(1 - \phi)\). Hence, the probability that a household with loan payment due \(z_2\) repays in the second period is:

\[
\rho(z_2) = 1 - F\left(\frac{z_2}{\phi}\right).
\]

Note that if \(z_2 \leq \phi\underline{y}\), the household repays with probability 1, and if \(z_2 > \phi\bar{y}\), the household defaults with probability 1.

**Profits and value of a loan** We define the value to the household of taking loan \((z_1, z_2)\) as the difference between the expected utility from taking the loan and the value of autarky. The value

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\(^8\)Without loss of generality, in the 2-period model, we assume no discounting

\(^9\)We interpret \(y_1\) as the household’s cash-in-hand, i.e. current income plus savings net of debt repayments. In the context of the dynamic model of Section 3, \(y_1\) would be endogenously determined by past consumption and savings decisions.
for the case where \( z_2 > \phi y \) (i.e. with possible default) and \( z_2 \leq \phi y \) (i.e. no default) is:

\[
v(z_1, z_2) = u(y_1 + z_1) + \int_{2 \phi}^{z_2} u((1 - \phi) y_2) dF(y_2) + \int_{2 \phi}^{y} u(y_2 - z_2) dF(y_2) - v^A(y_1) \text{ if } z_2 > \phi y
\]

\[
v(z_1, z_2) = u(y_1 + z_1) + \int_{2 \phi}^{y} u(y_2 - z_2) dF(y_2) - v^A(y_1) \text{ if } z_2 \leq \phi y
\]

We assume that lenders are risk neutral and normalize the risk free rate to zero. Then, the profit to a lender from making loan \((z_1, z_2)\) is:

\[
\pi(z_1, z_2) = -z_1 + z_2 \rho(z_2).
\]  (1)

**Exogenous-profit allocation** We define the allocation for an exogenous level of lender profits \( \tilde{\pi}(y_1) \geq 0 \) as the loan contract which maximizes household value, subject to a participation (non-negativity) constraint, and delivers profits of at least \( \tilde{\pi}(y_1) \) to lenders, i.e. is a solution to:

\[
\begin{align*}
\max_{z_1, z_2} & 
\quad v(z_1, z_2) \\
\text{s.t.} & 
\quad \pi(z_1, z_2) \geq \tilde{\pi}(y_1) \\
\quad & 
\quad v(z_1, z_2) \geq 0
\end{align*}
\]  (2)

Problem (2) defines *bilaterally efficient contracts*, i.e. gives contract terms that do not admit a Pareto improvement within the household-lender pair. The term \( \tilde{\pi} \) parametrizes lenders’ market power: when \( \tilde{\pi} = 0 \) lenders have zero market power, as in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007), and they are price-takers; when \( \tilde{\pi} > 0 \) lenders have positive market power. For now, we take lender market power to be exogenous and we perform comparative statics exercises with respect to \( \tilde{\pi} \). Section 3 endogenizes \( \tilde{\pi} \) in a quantitative model of competitive search and free entry by lenders.

Next, we show that, under Assumption 1 below, a bilaterally efficient contract which is a solution to (2) exists and provide comparative statics with respect to the level of \( \tilde{\pi} \).

**Assumption 1.** We assume the following inequalities hold:

(i) \( u'(y_1 + \phi y) > \int_{2 \phi}^{y} u'(y_2 - \phi y) dF(y_2) \)

(ii) \( y_1 \leq y(1 - 2\phi) \)
(iii) \( F'(y) + yF''(y) \geq 0, \; \forall \, y \in [\underline{y}, \overline{y}] \)

Parts (i) and (ii) of Assumption 1 guarantee that there is positive surplus from a lending contract between households and lenders and imply that the equilibrium loan features positive probability of default. Assumption 1 (iii) guarantees strictly concave iso-profit curves for lenders, in the relevant domain, which is important for existence and uniqueness.

**Proposition 1.** Fix a value for cash-in-hand, \( y_1 \) and denote the solution to (2) as \( z_1(\tilde{\pi}), z_2(\tilde{\pi}), \) with implied value \( \tilde{v}(\tilde{\pi}) \). Then, under Assumption 1:

(i) There is a unique \( \tilde{\pi}(y_1) \) such that if \( \tilde{\pi}(y_1) > \pi(y_1) \) there is no loan that can deliver profits \( \tilde{\pi}(y_1) \) to lenders and non-negative value to households. If \( \tilde{\pi}(y_1) \leq \pi(y_1) \) there is a unique loan \( (z_1(\tilde{\pi}), z_2(\tilde{\pi})) \) that maximizes household value, delivers profits \( \tilde{\pi}(y_1) \) to lenders and non-negative value to households.

(ii) For \( \tilde{\pi}(y_1) < \pi(y_1) \), we have value \( \tilde{v}'(\tilde{\pi}) < 0, \; z'_1(\tilde{\pi}) < 0 \) and \( z'_2(\tilde{\pi}) > 0 \). Therefore, loans that deliver higher profits feature a higher interest rate for households, higher level of debt and higher default probability.

**Proof.** See Appendix A.

Figure 1 illustrates the main results of Proposition 1. We plot households’ indifference curves and lenders’ iso-profit curves in the \( z_1 - z_2 \) plane. Households’ value increases in \( z_1 \) and decreases in \( z_2 \) and, therefore, loan value increases in the north-west direction. Lenders’ profits decrease in \( z_1 \) and increase in \( z_2 \) (for \( z_2 \) below a non-binding threshold) and, therefore, loan profits increase in the south-east direction.

Proposition 1 shows that there is a single point of tangency between the iso-profit curve that corresponds to \( \tilde{\pi} \) and the indifference curves. Varying \( \tilde{\pi} \) traces out a menu of equilibrium loans, denoted by the thick dotted line on the figure, where higher profits for lenders (greater market power of lenders) imply lower value for households (the tangent indifference curve is further in the south-east direction), lower \( z_1(\tilde{\pi}) \), higher \( z_2(\tilde{\pi}) \) and, hence, higher interest rate \( \frac{z_2}{z_1} \). The maximal household value contract is determined by the zero market power of the lenders and lies on the isoprofit curve corresponding to zero profits. The maximal profit contract, delivering \( \bar{\pi} \), corresponds to the indifference curve corresponding to the value of autarky, i.e. it is the profit that keeps the household at the participation constraint.
Proposition 1 also has immediate implications for aggregate statistics for debt, interest rates and bankruptcy as a function of the market power of lenders. Specifically, let an economy be populated by households characterized by their cash-in-hand $y_1$, distributed according to cdf $Y(y_1)$, and a common distribution of second period income with cdf $F(y)$ as before. Then, moving from a loan allocation characterized by a distribution $\tilde{\pi}(y_1)$ to a distribution $\hat{\pi}(y_1) < \tilde{\pi}(y_1)$ for all $y_1$, we have

**Corollary 1 (Aggregate Implications).** Fix the initial distribution of cash-in-hand $Y(y_1)$ and let Assumption 1 hold for all $y_1$. For two economies with different market power of lenders, $\tilde{\pi}(y_1)$ and $\hat{\pi}(y_1) < \tilde{\pi}(y_1)$ for all $y_1$, we have that the more competitive economy is characterized by lower aggregate debt, bankruptcy rate and lower interest rates.

### 2.1 Profitability and default risk

In this section we study how different assumptions about lender market power impact the distribution of profitability – defined as profits per unit of debt – with respect to risk. We relate the predictions of the model to the empirical findings in Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015) that the distribution of profitability has the highest peak for low credit scores and is low and relatively flat for high credit scores.

Specifically, we focus on a measure of profitability $P$ defined as profits per unit of outstanding
debt:
\[ P(z_1, z_2) = \frac{\pi(z_1, z_2)}{z_2} \]
and its relation to the repayment probability \( \rho(z_2) \), which we take as the model analog of the credit score.

In order for the model to be qualitatively consistent with the empirical patterns, profitability should be negatively related to the repayment probability. Below, we show that for a variety of model specifications this relation is weakly positive, and then argue that increasing market power of the lenders shifts the distribution towards more consistency with the data.

**Constant markup model** First, we focus on market structures implying constant markup \( \mu \geq 0 \) over the cost of funds, implying that interest rate \( z_2/z_1 \) is \( \mu \) above the zero profit level. This can be an outcome of competitive structure of the loan market or a technological constraint implying additional proportional transaction cost of making loans. In either case, it implies:

\[ z_1 = z_2 \rho(z_2) \frac{1}{1 + \mu}. \]

Profitability in this case is

\[ P(z_1, z_2) = \frac{-z_2 \rho(z_2)}{1 + \mu} + z_2 \rho(z_2) = \rho(z_2)(1 - \frac{1}{1 + \mu}) \]  
(3)

Equation (3) implies that profitability in the constant markup case is either flat at zero, as in the perfectly competitive case \( \mu = 0 \), or increasing in repayment probability in case of positive markup, opposite of what we see in the data.

**Upfront cost model** The second model we consider is one in which contracts are required to deliver a constant level of profits from a contract, i.e. they are solutions to (2) where \( \tilde{\pi}(y_1) = \kappa \). In this case, profitability is:

\[ P(z_1, z_2) = \frac{\kappa}{z_2}, \]

and is strictly decreasing with \( z_2 \). Since repayment probability \( \rho(z_2) \) is decreasing with \( z_2 \), again we obtain that profitability is increasing in repayment probability. An immediate implication of
this is that the same conclusion holds for any exogenous profit allocation in which profit does not vary across households.

**Variable markups/profits** The above discussion implies that in order for the model to imply a positive profitability-risk relationship, it must feature variable markups or profits, which are positively related to risk, high for high default probability contracts. In our analytical framework, because the second period income distribution is assumed identical across households, this requires that high debt \(z_2\) and hence high risk contracts be associated with high profits.\(^{10}\) Below, we show that imperfect competition introduces endogenously variable markups and profits in a way that has the potential to imply a positive profitability-risk relationship. In particular, we focus on the difference between a competitive allocation and a monopolistic allocation and show that moving from a competitive to a monopolistic allocation increases profitability of high risk contracts by more than low risk contracts. This is summarized in Proposition 2.

**Proposition 2 (Profitability).** Consider two equilibrium upfront cost model contracts with \(\kappa \geq 0\): \((q_1, b'_1)\) and \((q_2, b'_2)\), where \(b'_2 > b'_1\). Take two contracts that are monopolistic, with the same levels of debt: \((q_{M1}, b'_1)\) and \((q_{M2}, b'_2)\). Define profitability as profits per unit of debt, i.e. \(\pi(q, b')/b'\). Then \(\rho(b'_1) > \rho(b'_2)\) and:

1. For competitive contracts, profitability weakly decreases with default risk, i.e.
   \[\frac{\pi(q_1, b'_1)}{b'_1} \geq \frac{\pi(q_2, b'_2)}{b'_2} .\]

2. Moving to the monopolistic contract, profitability increases more for the more risky contracts:
   \[\frac{\pi(q_2, b'_2)}{b'_2} / \frac{\pi(q_1, b'_1)}{b'_1} < \frac{\pi(q_{M2}, b'_2)}{b'_2} / \frac{\pi(q_{M1}, b'_1)}{b'_1} .\]

**Proof.** In Appendix B.

Intuitively, high debt contracts \((b'_2\) above) are associated with households who value upfront payments \((z_1)\) more and have higher surplus from borrowing, and hence shifting that surplus to the lender implies a larger increase in profits and hence profitability. The implication of Proposition 2 is that moving from perfect competition to monopoly shifts the distribution of profitability towards

\(^{10}\)That does not generally imply a positive relationship between debt and risk in an environment with heterogeneous second period income distribution \(F(y)\), as would be the case for a persistent income process. However, in that case, all the intuition developed in this section for the relationship between riskiness and profitability goes through, as it is true for each household type, where type would be associated with current (income, debt) state instead of cash-in-hand. Since risk of default would cover the same \([0, 1]\) range for each household type, the monotonic relationship between risk and profitability for each household type translates into the same monotonic relationship on average.
high risk contracts, i.e. towards consistency with the data. Hence, we view this result as supporting departures from competitive outcomes in favor of imperfect competition. Whether such departure implies a positive relationship between profitability and risk is a quantitative question and depends on the specifics of the model and parameterization. In the next section, we provide one such micro-foundation and verify in a quantitative parameterized model that the qualitative result of Proposition 2 indeed implies a shift in the monotonicity of profitability relative to default risk.

3 Quantitative Analysis

In this section, we set up a quantitative model of unsecured borrowing with endogenous level of competition for borrowers. It is a fully fleshed out version of the economy we consider in the previous section, and quantifies the intuitions developed above. We parameterize the model and perform counterfactual exercises.

3.1 The model

Time is discrete, and the economy is populated by ex-ante identical, infinitely lived households, who receive income shocks and use unsecured credit for consumption smoothing. A household enters the period with default status \( d \in \{0, 1\} \) and state \( x \equiv (b, y) \). Default status \( d = 1 \) indicates that the household ended the previous period in default and \( d = 0 \) indicates that it did not. The state of a household consists of its default status \( d \), debt level \( b \) (negative if the household is a saver) and current income \( y \).

At the beginning of the period, households who are not in default (enter the period with \( d = 0 \)) choose whether to become savers, borrowers or default on their debt obligations. Household in default are restricted to save. After the saving, borrowing and default decisions are made, the household draws next period’s income \( y' \) from a distribution \( G(y', y) \) with support \( Y \times Y \). Then, households who enter the period in default transits out of default, i.e. enter next period with \( d = 0 \).\(^{11}\) Households die stochastically at the end of each period with probability \( \delta \).

The value function of a household with default status \( d \), debt \( b \) and income \( y \) at the beginning

\(^{11}\)This sets the period of exclusion from financial markets to the period of default plus 1 model period. Consistent with empirical literature finding that post-bankruptcy exclusion from borrowing is short (Jagtiani and Li (2013)). Longer, stochastic exclusion lengths do not change any of the conclusions from our quantitative analysis.
of the period is \( V_d(b, y) \). Denote the value function of saving for a household by \( V^S(b, y) \) and the value functions of borrowing and defaulting by \( V^B(b, y) \) and \( V^D(y) \), respectively. Then:

\[
V_0(b, y) = \max \left[ V^D(y), V^S_0(b, y), V^B(b, y) \right], \tag{4}
\]

\[
V_1(y) = V^S(0, y).
\]

Denote the next period’s debt (or savings, if negative) of the household by \( b' \). A household has access to a competitive saving market at the risk-free rate \( r \), which gives the value of saving:

\[
V^S(b, y) = \max_{c, b'} \left( u(c) + \beta(1 - \delta) \int_{y' \in Y} V_0(b', y') dG(y', y) \right) \tag{5}
\]

s.t. \( b' \leq 0 \)

and \( y - b = c - \frac{b'}{1 + r} \)

A household can only default if it is not currently in default. A defaulting household’s debt is erased \((b = 0)\), it transits to the default state \((d = 1)\) which precludes borrowing or saving during the period (i.e. consumption equals current income) and it incurs a one-off utility cost \( \Delta \). We denote the value of defaulting by \( V^D(y) \):

\[
V^D(y) = u(y) - \Delta + \beta(1 - \delta) \int_{y' \in Y} V_1(y') dG(y', y) \tag{6}
\]

A household can only borrow if it is not currently in default. We denote the value of borrowing by \( V^B(b, y) \), and it is pinned down by the equilibrium of the loan market, which we discuss in detail next.

**The loan market** Lenders are risk neutral, have access to borrowing at the risk free rate \( r \), and pay cost \( \kappa \) to make an offer of a one-period loan to a customer of a specific type \( x = (b, y) \). A loan specifies the amount that the household receives in the current period \( qb' \) and the amount that the household repays in the following period \( b' \). The loan market is subject to frictions and consists of two options for the household. The first one is modeled as a competitive search equilibrium.

---

\(^{12}\)Potentially, current income is also reduced by proportion \( \phi \), either as payment to creditors or as a pure utility cost. These two specifications are equivalent for log utility and can easily be added for different risk aversion values.
Lenders post loans for each type $x$ at cost $\kappa$ (if profitable) and households observe all posted loans and decide which loan to search for. The probability of getting a loan in the competitive search market depends on the matching functions and the household-lender ratio at that loan. Specifically, denote the measure of lenders posting loan $(q, b')$ targeted at type $x$ by $L_x(q, b')$, the measure of households of type $x$ searching for that loan by $H_x(q, b')$ and the resulting household-lender ratio (tightness) by $\theta_x(q, b') = \frac{H_x(q, b')}{L_x(q, b')}$. When the household-lender ratio is $\theta_x(q, b')$, a lender makes a loan with probability $\alpha_L(\theta_x(q, b'))$, the household receives a loan with probability $\alpha_H(\theta_x(q, b'))$ and, with the complementary probabilities, they are left unmatched. We make the following standard assumption about the trading probabilities:

**Assumption 2.** The trading probabilities satisfy $\alpha_L(\theta) = \theta \alpha_H(\theta)$, $\alpha_L(\theta)$ is increasing and concave in $\theta$, $\alpha_H(\theta)$ is decreasing and convex in $\theta$, $\lim_{\theta \to \infty} \alpha_L(\theta) = \alpha(H(0)) = 0$ and $\alpha_L(0) = \lim_{\theta \to \infty} \alpha_H(\theta) = 1$.

The second part of the lending market consist of an option of a high markup monopolistic contract, which the household can get with probability 1. Hence, this option is only attractive to households who are left unmatched in the competitive search market for loans. Specifically, when a household fails to match with a lender, we assume that they are matched to a monopoly lender, who makes a take-it-or-leave-it loan offer to the household. The household’s outside option in such case is the value of default or saving, whichever is highest. Hence, if the household’s search is unsuccessful, it receives the monopolistic contract’s value, which absent constraints on the market power of the monopolist is equal to the value of financial autarky (default or repayment, whichever is better) but without necessarily reducing debt levels to zero, since it receives a loan from the monopoly lender.\footnote{Under limits to market power, such as the ones considered in Section 3.3, the value delivered by the monopolistic contract may exceed the value of financial autarky.} This assumption is intended to capture the fact that households with low probability of obtaining a competitive offer may still be able to obtain a loan, just at less attractive terms, and can easily be relaxed to allow for more competitive outside option than a monopolistic offer. What is crucial is that it prevents the households who fail to match with a lender in the search market to be forced into bankruptcy by requiring that they repay their debt immediately. The existence of the monopolistic contract option means that the level of competition for a specific household type pins down the markup in the competitive search market and the probability of
getting the competitive search contract versus the monopolistic contract, which then determines average markups. Crucially, however, it does not lead to credit rationing, i.e. households whose surplus from a lending relationship is positive always get a contract offer.

Given the above, the value to a household of type \( x = (b, y) \) of receiving loan \((q, b')\) is:

\[
v_x(q, b') = u(y - b + qb') + \beta(1 - \delta) \int_{y' \in Y} V_0(b', y') dG(y', y) - V^M(b, y),
\]
where \( V^M \) is the value of monopolistic outside option, formally defined below. The value function of borrowing satisfies:

\[
V^B(b, y) = \max_{(q, b')} \left( \alpha_H(\theta_x(q, b')) v_x(q, b') + V^M(b, y) \right)
\]

subject to\( \kappa = \alpha_L(\theta_x(q, b')) \left( -qb' + \frac{b' \rho(b', y)}{1 + r} \right) \)

where \( \rho(b', y) \) is the repayment probability. The constraint determines market tightness and lender entry to satisfy the free entry condition which states that expected profits cover the up-front cost \( \kappa \).

Define the profits of delivering net value \( v \) to household type \( x \) as:

\[
\Pi_x(v) = \max_{q, b'} -qb' + \frac{\rho(b', y)b'}{1 + r}
\]

subject to

\( v = u(y - b + qb') + \beta(1 - \delta) \int_{y' \in Y} V_0(b', y') dG(y', y) - V^M(b, y) \)

In equilibrium, entry determines the optimal contract \( (q^*, b'^*) \) and corresponding market tightness \( \theta_x(q^*, b'^*) \) such that the free entry condition is satisfied:

\[
\alpha_L(\theta_x(q^*, b'^*)) \Pi_x(v_x(q^*, b'^*)) = \kappa.
\]
Finally, the loan offered to a type-\(x\) household by the monopoly lender solves:

\[
\begin{align*}
\max_{q,b} & \quad -qb' + \frac{\rho(b', y)b'}{1 + r} \\
\text{s.t.} & \quad u(y - b + qb') + \beta(1 - \delta) \int_{y' \in Y} V_0(b', y')dG(y', y) \geq \max[V^S(b, y), V^D(0, y)] \\
\end{align*}
\]

(8)

Notice that if monopoly profits are below \(\kappa\) then there is no lender entry in that market and the household can only choose between default and saving. Denote the solution to (8) as \((q^M, b'^M)\).

Then the value of the monopolistic contract to the household is:

\[
V^M(b, y) = u(y - b + q^M b'^M) + \beta(1 - \delta) \int_{y' \in Y} V_0(b'^M, y')dG(y', y).
\]

3.2 Parameterization

In this section, we detail the choices of our functional forms and parameter values. The utility function is CRRA, with relative risk aversion coefficient of \(\sigma = 2\):

\[
u(c) = \frac{c^{1-\sigma}}{1 - \sigma}.
\]

The matching function \(m(H, L)\) is

\[
m(H, L) = LH(L^\gamma + H^\gamma)^{-1/\gamma}
\]

which gives the matching probabilities as functions of the household-lender ratio \(\theta \equiv H/L\):

\[
\alpha_H = m(H, L)/H = (1 + \theta^\gamma)^{-1/\gamma}, \quad \alpha_L = m(H, L)/L = (\theta^{-\gamma} + 1)^{-1/\gamma}.
\]

In the benchmark parameterization, we set \(\gamma = 1\).

The period in the model is one year, and accordingly, we set the death hazard rate to \(\delta = 2\%\) (so that households live for 50 years, i.e. enter the economy at 21 and live up to 71 in expectation). We set the real risk free rate to \(r = 1.44\%\) which is the real yield on 3-month t-bills over the period
1990-2004. We focus our calibration on this period, as it covers the years before the 2005 BAPCPA overhaul of the bankruptcy filing regulation, and the Great Recession, hence avoiding any regime changes or unusually large shocks to the credit market.

We calibrate the income processes in the model to the early 2000s, allowing for two groups of households, identified in the data by their education attainment. Specifically we take the income processes estimated in Guvenen (2009) for ‘High School’ and ‘College’, and then we approximate the process for income of each of the groups, of the form

\[
\ln(Y_t) = \ln(z_t) + \varepsilon_t,
\]

where the persistent part is

\[
\ln(z_t) = \rho \ln(z_{t-1}) + \eta_t,
\]

and \( \eta_t \) and \( \varepsilon_t \) are i.i.d. with means zero and standard deviations \( \sigma_\varepsilon \) and \( \sigma_\eta \). Both the persistence parameter and the volatilities of innovations are education-group-specific. We map the above continuous log processes into a 25 state Markov chain using the method in Rouwenhorst (1995).

To compute the model, we use the income process in levels.\(^\text{14}\)

We calibrate the set of parameters which are at the heart of this paper: the discount factor by education group \( g, \beta_g \), the utility cost of default by education group, \( \Delta_g \), and the common entry cost of lenders, \( \kappa \).

**Details of data targets** For targets, we use the education group specific debt-to-income ratios, bankruptcy rates, and percent of households holding credit card debt. We use micro-data from the Survey of Consumer Finances (SCF) in 2001 and 2004 and aggregate data from the Flow of Funds for 2000-2004 and filing rates for Chapter 7 bankruptcy over 2000-2004. Using the SCF, we place a household in the ‘High School’ group if the household head has less than 16 years of education (i.e. this includes ‘some college’); this group accounts for 69.11% of all households in our sample. We place a household in the ‘College’ group if the household head has 16 years of education or more, which accounts for 30.89% of households in our sample. We use these group weights to calculate

\(^{14}\)Specifically, the persistence and the variances of the normal disturbances \( \eta \) and \( \varepsilon \) shocks are taken from Table 1 in Guvenen (2009).
the aggregates for the various variables. The data targets from the SCF are calculated using survey years 2001 and 2004, in order to avoid using the data after the introduction of the bankruptcy reform of 2005 (BAPCPA). As Albanesi and Nosal (2015) show, the reform had major effects on the bankruptcy and delinquency behavior of households, and it is not within the scope of the current paper to address these responses. For the group specific debt/income targets, we first calculate the average debt on major credit cards relative to average income for each of the 2 educational groups, averaged over survey years 2001 and 2004. Since unsecured debt is underreported in the SCF relative to aggregate data, we scale all groups debt/income by the same proportion so that the aggregate implied revolving credit/income over survey years 2001 and 2004 matches the aggregate revolving debt to income of 9.24%, which is the Flow of Funds 2000-2004 average. That gives 2 debt/income targets: 12.45% for High School and 6.7% for College. To calculate the group-specific proportion of households with unsecured debt, we compute the fraction of households in the SCF that carry a positive balance and pay interest on their credit cards, for each education group on average over the survey years 2001 and 2004.

The proportion of households with credit card debt is 39.77% for the High School group and 39.41% for the College group. Finally, for the default rate, we first compute the fraction of households in the SCF with a new bankruptcy over the last 3 years, for each education group on average over the survey years 2001 and 2004. We then rescale these fractions so that the aggregate bankruptcy rate equals the aggregate filing rate for Chapter 7 bankruptcy over 2000-2004 which is 5.82 per thousand. The default rate for the High School group and the College group is then 7.73 and 1.54 per thousand, respectively.

The above strategy gives us 5 calibrated parameters, \((\beta_{HS}, \beta_C, \Delta_{HS}, \Delta_C, \kappa)\) in total to match 6 moments in the data (debt/income, bankruptcy rate, fraction with debt for 2 education groups).

In addition to the Benchmark calibration of the model of Section 3, we calibrate the Competitive model, which is based on the standard model of default with competitive pricing of default risk, with entry cost for lenders (\(\kappa\)) and no matching frictions. This serves as a useful benchmark for our analysis, as this model is features ex-post perfect competition which is the basis of a large proportion of the papers used in the literature. Specifically, the loan size \((b')\) and price \((q)\) is

---

\(^{15}\)Some households with a positive credit card balance might repay their debts every months, i.e. use their credit card for transacting rather than borrowing purposes. Requiring that a household also makes interest payments on the credit card restricts attention to households that roll over at least some of their debt from month to month and, hence, are borrowing on their credit cards.
determined by the following zero profit condition

\[-q'b' + \frac{b'\rho(b', y)}{1+r} = \kappa\] (9)

For each of the two models, we pick the calibration which minimizes the sum of square deviations from the targets. Parameter values are reported in Table 1.

Table 1: Model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>risk aversion</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>elasticity of the matching function</td>
</tr>
<tr>
<td>$\delta$</td>
<td>death hazard rate</td>
</tr>
<tr>
<td>$r$</td>
<td>risk free rate</td>
</tr>
<tr>
<td>$\rho_{C}, \rho_{HS}$</td>
<td>persistence of income process</td>
</tr>
<tr>
<td>$\sigma^2_{\varepsilon}, \sigma^2_{\eta}$</td>
<td>volatility of the income innovations</td>
</tr>
<tr>
<td>$\sigma^2_{\varepsilon}, \sigma^2_{\eta}$</td>
<td>volatility of the income innovations</td>
</tr>
<tr>
<td><strong>Benchmark</strong></td>
<td></td>
</tr>
<tr>
<td>$\beta_C, \beta_{HS}$</td>
<td>time discount factors</td>
</tr>
<tr>
<td>$\Delta_C, \Delta_{HS}$</td>
<td>bankruptcy stigma</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>loan upfront cost</td>
</tr>
<tr>
<td><strong>Competitive</strong></td>
<td></td>
</tr>
<tr>
<td>$\beta_C, \beta_{HS}$</td>
<td>time discount factors</td>
</tr>
<tr>
<td>$\Delta_C, \Delta_{HS}$</td>
<td>bankruptcy stigma</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>loan upfront cost</td>
</tr>
</tbody>
</table>

**Model fit and aggregate statistics** Below, we present quantitative results from our two model versions: *Benchmark* and *Competitive*. Table 2 presents the aggregate statistics. On top of the targeted statistics, we additionally report the un-targeted charge-off rate and interest rate.\(^{16}\)

It is clear that both model versions are able to match the aggregate debt and bankruptcy statistics reasonably well. Additionally, both of them come close to also matching the un-targeted charge-off rates and interest rates. However, the differences between the models become apparent in

\(^{16}\)The model fit by education group is presented in Table 6 in the Appendix.
Table 2: Data and model outcomes.

<table>
<thead>
<tr>
<th></th>
<th>Targeted Moments</th>
<th>Data 2000-04</th>
<th>Benchmark</th>
<th>Competitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt/Income %</td>
<td></td>
<td>9.24</td>
<td>9.05</td>
<td>9.5</td>
</tr>
<tr>
<td>Bankruptcies per 1000</td>
<td></td>
<td>5.81</td>
<td>5.12</td>
<td>6.12</td>
</tr>
<tr>
<td>Fraction with debt %</td>
<td></td>
<td>39.7</td>
<td>42</td>
<td>40</td>
</tr>
<tr>
<td>Untargeted Moments</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rates %</td>
<td></td>
<td>13</td>
<td>14</td>
<td>13</td>
</tr>
<tr>
<td>ChargeOffs %</td>
<td></td>
<td>4.9</td>
<td>3.6</td>
<td>3.6</td>
</tr>
</tbody>
</table>

\[ ^a \text{Data point for interest rates is a 2001-2004 SCF average.} \]

the statistics related to the distribution of prices and profitability. Specifically, Figures 2-3 present average interest rates, debt balances and profitability (profit/debt) as a function of the expected repayment probability across the two models.

Figure 2, panel (a), shows that interest rates are higher for lower repayment (higher default) probabilities in the Benchmark model, falling monotonically as repayment probabilities go up. By contrast, in the Competitive model, interest rates vary much less with default probabilities and exhibit no clear monotonic pattern\(^{17}\). As for debt balances, presented in panel (b) of Figure 2, both models predictably imply a negative relationship between repayment probability and debt balances, with the Benchmark model implying higher debt balances per capita for higher risk individuals compared to the Competitive model.

The crucial distinction in the predictions of the two models is in the distribution of profitability as a function of default risk, as presented in Figure 3. Profitability in the Benchmark model is high for high risk borrowers and generally decreases with repayment probability, in stark contrast to the Competitive model, which implies that profitability increases with repayment probability. Intuitively, the difference in the two models’ predictions for the distribution of profitability is driven by the fact that in the Competitive model, pricing is exclusively determined by the risk of default, as in equation (9), while in the Benchmark model, it is additionally affected by the household’s outside options, as implied by problem defined by (7). This confirms the qualitative

\[ ^{17}\text{This is a feature of the competitive model with an upfront cost of making a loan } \kappa > 0. \text{ The low risk borrowers borrow less (panel (b) of Figure 2) and hence in order for the contracts to break even, the interest rates have to be relatively high. For the case of } \kappa = 0 \text{ in the Competitive model, interest rates qualitatively mimic those in the Benchmark model. However, profitability is just flat at zero, as discussed in Section 2.} \]
results from Proposition 2 in the full quantitative model, and implies that the Benchmark model is consistent with the empirical evidence in Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015) that profitability peaks at low credit scores. Additionally, the Benchmark model peaks at profitability of around 8%, which is within 5% and 12% the range reported for low FICO scores. Additionally, panel (b) of Figure 3 also presents profitability in the Benchmark model split between the competitive search and the monopolistic market. Both markets are characterized by a hump-shaped profitability distribution, peaking at relatively low repayment probability.

The models’ predictions for the median, 10th, 90th and 95th percentile of the interest rate distribution are presented in Table 3, where we also present the empirical moments reported in Galenianos and Gavazza (2021). The interest rate distribution implied by the model does not imply large numbers of high interest contracts - which means that even the monopolistic contract interest rates are endogenously curtailed by the households’ participation constraint. At the same time it shows promise in the ability to account for interest rates of very high risk contracts, as implied by the 95th percentile of the distribution of 44%. These very high interest rates have been reported in Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015), who compute fee- and interest-inclusive rate *charges* of 45%. Overall, we view the predictions of the models in terms of interest rates as falling within the range reported in the empirical literature, with the Benchmark model better able to capture the range of interest rates of high risk groups.

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18See Table III in Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015).
Figure 3: Profitability (profit per unit of debt) by repayment probability.

(a) Benchmark and Competitive models

(b) Search Market and Monopoly Market

Table 3: Distribution of interest rates in the model and data (in %).

<table>
<thead>
<tr>
<th>Moments</th>
<th>p10</th>
<th>p50</th>
<th>p90</th>
<th>p95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>7.8</td>
<td>9.2</td>
<td>19.1</td>
<td>44</td>
</tr>
<tr>
<td>Competitive</td>
<td>8.9</td>
<td>11.2</td>
<td>18.9</td>
<td>20</td>
</tr>
<tr>
<td>Data Galenianos and Gavazza (2021) sub-prime</td>
<td>11.9</td>
<td>20.65</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Data Galenianos and Gavazza (2021) near-prime</td>
<td>10.5</td>
<td>18.24</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>Data Galenianos and Gavazza (2021) prime</td>
<td>9.9</td>
<td>16.7</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>Agarwal et al. (2015) fee-inclusive charges</td>
<td></td>
<td></td>
<td></td>
<td>45a</td>
</tr>
</tbody>
</table>

* This number comes from Agarwal et al. (2015), Table III and is the income (interest and fee charges) as a fraction of average daily balance for accounts below FICO score of 620.

3.3 Limiting Market Power: Interest Rate Caps

The analysis in the previous section indicates that the Benchmark model matches well the features of the distribution of pricing and profitability. The model implies pricing that is linked not only to default risk, but also to households’ outside options, that in turn are a function of the distribution of contracts in the market. Intuitively, this can have significant implications for the evaluation of regulation or policies that impact markups and profitability of the lenders, even if the regulation directly applies to only a small number of contracts. If the general equilibrium effects are strong, the whole distribution of offered rates can shift in response, implying significant effects on aggregates.
In this section, we illustrate this point by considering the effects of the introduction of universal interest rate caps. Regulating unsecured lending interest rates is a mainstay in policy discussion on oversight of financial products, and introducing interest rate caps has been proposed as recently as 2019 as part of the Loan Shark Prevention Act which would impose a 15% limit on credit cards nationwide. Below, we consider varying interest rate caps from the Benchmark value of 100%, i.e. virtually not binding, which we take as the current state of affairs in the US, to a much tighter level of 10%. We discuss the aggregate effects of introducing this policy, as well as the effect on the distribution of interest rates. Then, we conduct counterfactual exercises to shed more light on the the mechanism behind the model’s response, isolating in turn the general equilibrium effect of changing caps on the distribution of interest rates, the effects of changing matching probabilities, and the general equilibrium impact of the changing monopolistic outside option.

Figure 4, panel (a) and Figure 5 present the average interest rates, debt and fraction borrowing as a function of the interest rate cap. In the Benchmark model, limiting market power of lenders has a significant negative effect on the interest rates, but a significant positive effect on debt and fraction of household holding a balance. On the extensive margin, the fraction of households with a monopolistic and search market contracts both increase under the tighter cap (Figure 6). Both of these markets face more favorable pricing of debt (Figure 4 panel (b)), which explains why more households decide to borrow in equilibrium. Despite growing debt and fraction borrowing, debt becomes safer in the Benchmark model – both the chargeoff rate and the bankruptcy rate go down under tighter interest rate caps (Figure 7).

The interest rate cap applies to a small fraction of contracts in equilibrium, ranging from 2% to 11% up until the tightest cap of 10%, when 19% of contracts are at cap. However, in the Benchmark model it results in significant effects on the entire interest rate distribution, suggesting strong general equilibrium effects. Specifically, Figure 8 presents the percentiles of the interest rate distribution as a function of the interest rate cap. We can see that the whole distribution of offered interest rates shifts down in response to the cap in the Benchmark model, even though the cap does not directly bind for the majority of the contracts: the p90 of the interest rate distribution is below the cap up until it hits 19%. This illustrates the effect of pure outside option value of regulating a small number of high interest contracts: the fact that with positive probability in the...
Figure 4: Average interest rates under different interest rate caps.

(a) Benchmark and Competitive Models
(b) Search Market and Monopoly Market

Figure 5: Debt/income and fraction borrowing under different interest rate caps.
Figure 6: Fraction with Monopoly and Competitive Search contracts under different interest rate caps.

Figure 7: Bankruptcy rates and chargeoff rates under different interest rate caps.
future the household may be directly affected by the cap changes the contract received today. This is also reflected in Figure 4, panel (b): both the monopolistic and search market average interest rates go down significantly in response to the cap.

For comparison, Figures 4 – 7 also present the prediction of the Competitive model in response to interest rate caps. The differences between the predictions of the two models are stark. In the competitive model, both interest rates and debt barely respond to the interest rate caps initially, followed by a drop in both and then a shutdown of the debt market. In the Competitive model, the dispersion of interest rates is smaller than in the Benchmark model, and hence initially the tightening cap has little effect. However, when it starts binding for more contracts, this results in markets actually closing down, as more constraints renders more households unprofitable and there are no markups to cushion the cap tightening. The tightening of the upper bound of the interest rate distribution actually corresponds to an increase in the lower percentiles, contrary to the Benchmark model. Finally, because profits in the Competitive model are brought down to break-even by competition, a binding interest rate results in households being cut off from credit entirely.

The increased access to cheaper credit in the Benchmark model results in significant welfare gains, measured as lifetime consumption equivalent variation. Figure 9 presents the welfare gain/loss by income for each of the education groups in the model, evaluated at initial zero debt.\footnote{We use income relative to median for each of the groups in order to be able to express them on the same axis.}
Figure 9: Welfare gain/loss 10% interest cap relative to benchmark as a function of income (relative to group specific median).

(a) College
(b) High School

The welfare gain in the Benchmark model is up to 4% of lifetime consumption, and around 0.5% for median income households (0.5% and 0.7% on average for High School and College, respectively). By contrast, the credit rationing in the Competitive model results in a welfare loss of up to 20% of lifetime consumption for the poorest households, and around 1.5% for the median income household (-2.1% and -1.8% on average for High School and College, respectively). Intuitively, the contrasting predictions of the two models fit into two narratives of the proposed policy. One narrative claiming that regulating interest rates will make credit cards unprofitable and cut households off from credit is supported by the Competitive model. The other narrative that markups are high in the credit card market and regulating interest rates will make borrowing cheaper and increase access to credit, is supported by the Benchmark model.

The interest rate caps impact the two educational groups’ in different ways in terms of debt and bankruptcy, as presented in Figure 10. For the High School group, debt/income goes up by only 4% going from 100% to 10% cap, while fraction with debt goes up 14%. Together with these modest increases, cheaper credit is consistent with significantly reduced bankruptcy rate. By contrast, for the College group mostly gains from the reform by increased credit access, with a 50% increase in debt/income and 26% increase in fraction borrowing, while the bankruptcy rate for that group remains roughly constant.
Equilibrium effects of outside options on pricing In order to provide more evidence of general equilibrium effects of interest rate caps, below, we document how optimal policy functions on interest rates respond to the tightening interest rate caps. In particular, for the distribution of offered interest rates in the baseline calibration (i.e. cap of 100%), we calculate the fraction of those interest rates that fall strictly below each of the increasingly tightening caps that are nevertheless optimally adjusted down in the new equilibria. That is, what is the fraction of interest rates that do not have to be adjusted down in the new equilibrium but still do. Figure 11 presents the result for the Benchmark and Competitive models. In the Benchmark model, the majority of contracts that are not affected by the cap still adjust downwards, from 56% for the 90% cap to 85% for the 10% cap, suggesting strong general equilibrium effects. By contrast, in the Competitive model, almost none of the contracts under the cap are adjusted, which is consistent with the interpretation that outside options do not impact pricing in the Competitive model.

Effects of matching probabilities Clearly, the response of the Benchmark economy to the policy involves changes to the optimal matching probabilities, which implies varying fractions of borrowers with the search market versus monopoly contract. Below, we isolate this effect by simulating a counterfactual economy which is endowed with the same matching probability function $\alpha_H$ as the baseline calibration. Panel (a) of Figure 12 presents the result for debt and fraction borrowing. The aggregate response of the economy is almost identical, implying that changing the
probabilities of matching with the search market lender or monopoly lender (i.e. $1 - \alpha_H$) is not a significant driver of the results.

**Direct versus indirect effect of high markup option** Below, we quantify the indirect, equilibrium effect of changing monopolistic option pricing on search market pricing. To that end, we simulate the Benchmark economy with an counterfactual value of $\alpha_H \equiv 1$, i.e. no borrowers get the monopoly contract, but the search market contracts they receive are still impacted by the general equilibrium distribution of outside options in the economy. Panel (b) of Figure 12 presents the change in debt/income and fraction revolving relative to the baseline case for the counterfactual and actual response of the economy.\(^{21}\) We find that the pricing impact within the competitive search market accounts at the peak for 56% of the debt/income response and all of the response of fraction of population borrowing.\(^{22}\) We conclude that the general equilibrium effects of changing the outside options of households account for the majority of the aggregate response of the economy to the policy.

---

\(^{21}\)The normalization is necessary because at $\alpha_H = 1$, both the level of debt and the fraction revolving do not overlap with the baseline economy.

\(^{22}\)This last finding is not by construction, as the number of open markets on the simulation path depends on the now-different debt choices of households.
3.3.1 No High Markup Option Model

One of the fundamental features of our quantitative model that differentiates it from other search-based credit market frictions modeled in the literature is the explicit introduction of the high markup outside option of unmatched households. We view this as a tractable way of capturing the fact that credit can be accessed by a household if there is positive surplus, but the exact pricing of that credit is pinned down by frictions. In this section, we present results from the version of the Benchmark model without that outside option. What that means in equilibrium is that unmatched households with debt must repay or default. We parameterize this version of the model, labeled No High Markup Option, using the same targets as the Benchmark, and present the aggregate statistics and the policy experiment.

Overall, the No High Markup Option model offers a significantly worse aggregate fit compared to the Benchmark calibration (Table 5), significantly under-predicting aggregate debt/income and over-predicting fraction borrowing. In terms of untargeted moments, the model under-predicts charge-off rates by about 50%. It is important to note also that the No High Markup Option model’s calibration requires setting very low values of the time discount factors of the borrowers of around 0.6 (Table 4), which is very low relative to discount factors typically implied by the empirical rates of return savings. These predictions of the model are intuitive and follow directly
from the credit rationing implied by the search friction. Because unmatched households must repay the debt or default, this lowers aggregate debt in equilibrium, and the model requires high rates of impatience to match debt/income ratios to compensate.\textsuperscript{23}

The crucial impact of the high markup option also shows up in the results from the interest rate cap policy experiment. Contrary to the Benchmark model, the No High Markup Option model predicts that debt and fraction borrowing actually \textit{decline} in response to the tightening caps (Figure 13). The reason for this is that even though the model does link pricing to outside option of saving or borrowing, the general equilibrium effect of changing these values in response to interest rate caps is not strong enough to affect the aggregate distribution. This can be seen in Figure 14, which presents the interest rate distribution. We can see that the tightening cap has very little effect except at the very top, in contrast to the Benchmark model (i.e. compared to Figure 8). As a result, the tighter caps in this version of the model work mainly by limiting the interest rates within matches, and hence reducing the efficiency of the match, eventually making an increasing fraction of them unprofitable. That extensive margin effect results in reduced credit access, as well as reduced welfare: the No High Markup Option model predicts that the policy is associated with a welfare loss of 0.5\% and 0.8\% of lifetime consumption equivalent for College and High School groups, respectively.\textsuperscript{24}

Table 4: Model parameter values in the No High Markup Option model (if different from Benchmark).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_C$, $\beta_{HS}$</td>
<td>time discount factors</td>
</tr>
<tr>
<td>$\Delta_C$, $\Delta_{HS}$</td>
<td>bankruptcy stigma</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>loan upfront cost</td>
</tr>
</tbody>
</table>

\textsuperscript{23}As implied by the analytical results in Section 2, the No High Markup Option model also gives a positive relationship between profitability and risk, just like the Benchmark model as we show in Figure 16 in the Appendix. The reason for that since for the baseline specification of the interest rate cap, this model and the Benchmark both have the outside option of the household be the value of saving or default, giving a similar relationship between risk and surplus from a loan contract. What is different between the models is the equilibrium joint distribution of debt and income, since actual \textit{borrowing} options are different in the two models.

\textsuperscript{24}The figures for welfare gains/losses for this version of the model can be found in Appendix C.
Figure 13: Debt/income and fraction borrowing under different interest rate caps.

Figure 14: Percentiles of interest rate distribution under different interest rate caps.
Table 5: Data and model outcomes.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data 2000-04</th>
<th>No High Markup Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt/Income %</td>
<td>9.24</td>
<td>6.8</td>
</tr>
<tr>
<td>Bankruptcies per 1000</td>
<td>5.81</td>
<td>5.7</td>
</tr>
<tr>
<td>Fraction with debt %</td>
<td>39.7</td>
<td>48</td>
</tr>
<tr>
<td>ChargeOffs %</td>
<td>4.9</td>
<td>2.4</td>
</tr>
<tr>
<td>Interest rates %</td>
<td>13</td>
<td>20</td>
</tr>
<tr>
<td>Interest rate p10 %</td>
<td>10-12</td>
<td>12.8</td>
</tr>
<tr>
<td>Interest rate p50 %</td>
<td>16-20</td>
<td>16.4</td>
</tr>
<tr>
<td>Interest rate p90 %</td>
<td>26-30</td>
<td>32</td>
</tr>
</tbody>
</table>

* Data point for average interest rates is a 2001-2004 SCF average. Percentiles based on Galenianos and Gavazza (2021).

4 Conclusions

We provide a characterization of the impact of increased market power on the offered contracts in the credit market and derive predictions for the cross-sectional characteristics of profitability. Our analysis indicates that the data lends support to frameworks with lender market power and implied markups that are positively related to riskiness. We propose a mechanism in which such patterns emerge due to search and matching frictions in the credit market where a monopolistic outside option is available. When embedded in a quantitative dynamic model, the mechanism delivers aggregate statistics, interest rates and profitability distribution consistent with the data. We show that the Benchmark model outperforms the No High Markup Option model as well as the Competitive model. The parameterized model implies strong general equilibrium effects of households’ outside options driving the distribution of offered rates. In the context of a policy limiting market power of lenders in the form of interest rate caps, we show that in the Benchmark model, these general equilibrium effects are responsible for the majority of the aggregate response, and imply increased and cheaper access to credit for all households, as well as lower bankruptcy and chargeoffs. The regulation implies significant welfare gains, especially for the poorest households. Our conclusions are in stark contrast to ones implied by the Competitive and No High Markup Option frameworks, in which the same interest rate cap implies reduced access to credit and welfare losses. Our research offers a tool for capturing the aggregate effects of regulation aimed at a small
proportion of contracts, as it captures the equilibrium effects these contracts have on the entirety of the interest rate distribution.

References


Exler, Florian, Igor Livshits, Jim MacGee, and Michèle Tertilt, 2020, Consumer credit with over-optimistic borrowers, .


Jagtiani, Julapa, and Wenli Li, 2013, Credit access and credit performance after consumer bankruptcy filing: new evidence, .


Raveendranathan, Gajendran, 2020, Revolving credit lines and targeted search, *Journal of Economic Dynamics and Control* 118, 103964.


A Proof of Proposition 1

Proof. We first characterize the loan that maximizes household value and delivers profits \( \tilde{\pi} \) to lenders but without the restriction that it provides non-negative value to households. We reintroduce that restriction at the end.

If \( z_2 > \phi \bar{y} \) default is certain which implies \( \pi(z_1, z_2) \leq 0 \); therefore \( z_2 \leq \phi \bar{y} \) is necessary for delivering profits of at least \( \tilde{\pi} \). Profits are strictly decreasing in \( z_1 \) and strictly increasing in \( z_2 \) when \( z_2 \in [0, \phi \bar{y}] \), i.e. there is no default. For \( z_2 \geq \phi \bar{y} \) note that

\[
\frac{\partial \pi(z_1, z_2)}{\partial z_2} = 1 - F\left(\frac{z_2}{\phi}\right) - \frac{z_2}{\phi} F'(\frac{z_2}{\phi})
\]

\[
\frac{\partial^2 \pi(z_1, z_2)}{\partial z_2^2} = -\left(2 F'\left(\frac{z_2}{\phi}\right) + \frac{z_2}{\phi} F''\left(\frac{z_2}{\phi}\right)\right) < 0
\]

where the inequality follows from Assumption 1. Since \( \partial \pi(z_1, z_2)/\partial z_2 = 1 \) for \( z_2 < \phi \bar{y} \) and \( \partial \pi(z_1, z_2)/\partial z_2 \leq 0 \) for \( z_2 = \phi \bar{y} \) there is a unique \( \bar{z}_2 \in [\phi \bar{y}, \phi \bar{y}] \) such that \( \partial \pi(z_1, \bar{z}_2)/\partial z_2 = 0 \). Hence, loans with \( z_2 > \bar{z}_2 \) yield lower profits to lenders and lower value to households than loans with \( z_2 = \bar{z}_2 \) and they cannot be a solution to (2). For this reason, we restrict attention to \( z_2 \in [0, \bar{z}_2] \), where profits are strictly increasing and concave in \( z_2 \).

Since the objective function \( v(z_1, z_2) \) is strictly increasing in \( z_1 \) and strictly decreasing in \( z_2 \), we focus on loans where the constraint holds with equality. Define \( \tilde{z}_1(z_2; \tilde{\pi}) = z_2(1 - F(\frac{z_2}{\phi})) - \tilde{\pi} \) and \( G(z_2; \tilde{\pi}) = v(\tilde{z}_1(z_2; \tilde{\pi}), z_2) \). The maximization problem becomes:

\[
\max_{z_2 \in [0, \bar{z}_2]} G(z_2; \tilde{\pi}) = u(y_1 + z_2(1 - F(\frac{z_2}{\phi})) - \tilde{\pi}) + \int_{\max[y, \frac{\bar{z}_2}{\phi}]}^{\bar{y}} u(y_2(1 - \phi))dF(y_2)
\]

\[
+ \int_{\max[y, \frac{\bar{z}_2}{\phi}]}^{\bar{y}} u(y_2 - z_2)dF(y_2) - v_A
\]

This formulation incorporates the possibility that \( z_2 \) is lower than \( \phi \bar{y} \) (hence, no default) and higher (positive probability of default).

Differentiating with respect to \( z_2 \) yields

\[
G'(z_2; \tilde{\pi}) = u'(y_1 + z_2(1 - F(\frac{z_2}{\phi})) - \tilde{\pi})(1 - F(\frac{z_2}{\phi}) - \frac{z_2}{\phi} F'(\frac{z_2}{\phi})) - \int_{\max[y, \frac{\bar{z}_2}{\phi}]}^{\bar{y}} u'(y_2 - z_2)dF(y_2)
\]
Notice that $G'(\phi y; \tilde{\pi}) > 0$ by Assumption 1 and $G'(\tilde{\pi}_2; \tilde{\pi}) < 0$.

The second derivative for cases $z_2 \leq \phi y$ and $z_2 \geq \phi y$, respectively, is:

\[
G''(z_2; \tilde{\pi}) = u''(y_1 + z_2(1 - F(\frac{z_2}{\phi}) - \tilde{\pi})) \left(1 - F(\frac{z_2}{\phi}) - \frac{z_2}{\phi} F'(\frac{z_2}{\phi})\right)^2 + \int_{y}^{\tilde{\pi}} u''(y_2 - z_2)dF(y_2)
\]

\[
- \frac{1}{\phi} \ u'(y_1 + z_2(1 - F(\frac{z_2}{\phi}) - \tilde{\pi}))(2F'(\frac{z_2}{\phi}) + \frac{z_2}{\phi} F''(\frac{z_2}{\phi}))
\]

\[
G''(z_2; \tilde{\pi}) = u''(y_1 + z_2(1 - F(\frac{z_2}{\phi}) - \tilde{\pi})) \left(1 - F(\frac{z_2}{\phi}) - \frac{z_2}{\phi} F'(\frac{z_2}{\phi})\right)^2 + \int_{z_2}^{\tilde{\pi}} u''(y_2 - z_2)dF(y_2)
\]

\[
- \frac{u'(y_1 + z_2(1 - F(\frac{z_2}{\phi}) - \tilde{\pi}))(2F'(\frac{z_2}{\phi}) + \frac{z_2}{\phi} F''(\frac{z_2}{\phi}))}{\phi} + \frac{u'(\frac{z_2}{\phi} - z_2)F'(\frac{z_2}{\phi})}{\phi}
\]

(11)

We have $G''(z_2; \tilde{\pi}) < 0$ for $z_2 \leq \phi y$ and therefore $G'(z_2; \tilde{\pi}) < 0$ for $z_2 \leq \phi y$.

If $z_2 \in [\phi y, \tilde{\pi}_2]$, then the first three terms of (11) are negative and the fourth is positive.

Comparing the third and fourth terms:

\[
u'(y_1 + z_2(1 - F(\frac{z_2}{\phi}) - \tilde{\pi}))(2F'(\frac{z_2}{\phi}) + \frac{z_2}{\phi} F''(\frac{z_2}{\phi}) > u'(\frac{z_2}{\phi} - z_2)F'(\frac{z_2}{\phi})
\]

which follows from Assumption 1 (ii) and (iii). Therefore $G''(z_2; \tilde{\pi}) < 0$ when $z_2 \leq \tilde{\pi}_2$. As result, there is unique $z_2(\tilde{\pi}) \in (\phi y, \tilde{\pi}_2)$ such that $G'(z_2(\tilde{\pi}); \tilde{\pi}) = 0$, which is the solution to maximization problem (10).

The implicit function theorem yields

\[
z'_2(\tilde{\pi}) = -\frac{\partial G'(z_2; \tilde{\pi})}{\partial \tilde{\pi}} \frac{1}{G''(z_2; \tilde{\pi})}
\]

and we have $z'_2(\tilde{\pi}) > 0$ because $G''(z_2; \tilde{\pi}) < 0$ and

\[
\frac{\partial G'(z_2; \tilde{\pi})}{\partial \tilde{\pi}} = -u''(y_1 + z_2(1 - F(\frac{z_2}{\phi}) - \tilde{\pi}))(1 - F(\frac{z_2}{\phi}) - \frac{z_2}{\phi} F'(\frac{z_2}{\phi})) > 0
\]

Let $z_1(\tilde{\pi}) = z_2(\tilde{\pi})(1 - F(\frac{z_2(\tilde{\pi})}{\phi})) - \tilde{\pi}$. We have:

\[
z'_1(\tilde{\pi}) = \left(1 - F(\frac{z_2(\tilde{\pi})}{\phi}) - \frac{z_2(\tilde{\pi})}{\phi} F'(\frac{z_2(\tilde{\pi})}{\phi})\right)^2 \frac{u''(y_1 + z_2(\tilde{\pi})(1 - F(\frac{z_2(\tilde{\pi})}{\phi})) - \tilde{\pi})}{G''(z_2(\tilde{\pi}); \tilde{\pi})} - 1
\]
Using equation (11), we have that $z_1'(<\tilde{\pi}) < 0$ if

$$
\frac{u'(y_1 + z_2(<\tilde{\pi}))(1 - F(z_2(<\tilde{\phi}))) - \tilde{\pi})(2F'(z_2(<\tilde{\phi})) + \frac{z_2(<\tilde{\phi})F''(z_2(<\tilde{\phi})))}{\phi} > \int_{\tilde{z}_2(<\phi)}^{y} \frac{u''(y_2 - z_2(<\tilde{\pi}))dF(y_2)}{\phi} + \frac{u'(<\tilde{\phi} - z_2)F'(z_2(<\tilde{\phi}))}{\phi}
$$

which holds, as shown earlier.

We have $\tilde{v}'(<\tilde{\pi}) < 0$ since $z_1'(<\tilde{\pi}) < 0$ and $z_2'(<\tilde{\pi}) > 0$. Since repayment probability is decreasing in $z_2$, default probabilities are higher for higher $<\tilde{\pi}$. Assumption 1 shows that $v(0) > 0$ and it is easy to see that $v(<z_2) < 0$. Therefore, there exists $<\pi > 0$ such that $v(<\pi) = 0$. There is a unique loan that solves (2) if $<\pi \leq <\pi$ and there is no such loan if $<\pi > <\pi$. Finally, $v = v(0)$ is the maximum value that a loan can deliver to households when lenders make zero profits.

\[\square\]

**B Proof of proposition 2**

Result 1 is just a consequence of the analysis in the upfront cost section: that both contracts have to satisfy (2) with $<\pi = \kappa$, and hence competitive profit is equal to $\kappa/b'$, which is decreasing with $b'$ for positive $\kappa$, hence increasing in default probability $1 - \rho(b')$.

For 2, notice that the monopolistic contracts extract all of the surplus, which means that each of them lies on a different indifference curve that each passes through the origin. Given the indifference curves are defined by the conditions:

$$
\bar{v} = u(y_1 + z_1) + \int_{y}^{z_2} u((1 - \phi)y_2)dF(y_2) + \int_{\tilde{z}_2}^{y} u(y_2 - x_2)dF(y_2) \text{ if } z_2 > \phi y,
$$

$$
\bar{v} = u(y_1 + z_1) + \int_{y}^{y} u(y_2 - z_2)dF(y_2) \text{ if } z_2 \leq \phi y,
$$

totally differentiating with respect to $x_2$ gives the slope

$$
\frac{dz_1}{dz_2} = \frac{\int_{y}^{y} u'(y_2 - z_2)dF(y_2)}{u'(y_1 + z_1)} \text{ if } z_2 > \phi y,
$$

$$
\frac{dz_1}{dz_2} = \frac{\int_{y}^{y} u'(y_2 - z_2)dF(y_2)}{u'(y_1 + z_1)} \text{ if } z_2 \leq \phi y.
$$
Hence, the flatter indifference curve at the origin corresponds to lower value (lower $y_1$), lower $z_1$ for the same $z_2$ and is everywhere flatter.

Proof of Proposition 1 established that iso-profit curves are strictly concave. Since iso-profit curves are given by

$$z_1 = z_2 - z_2 F(z_2/\phi) - \kappa - \bar{\pi} \text{ if } z_2 > \phi y,$$

$$z_1 = z_2 - \kappa - \bar{\pi} \text{ if } z_2 \leq \phi y,$$

they are also parallel in the $z_1 - z_2$ space. Since the higher debt contract ($b'_2$) lies on the flatter part of the iso-profit than the lower debt contract ($b'_1$), the higher debt monopolistic contract will lie on an indifference curve that is everywhere flatter than the lower debt contract, and hence the higher debt monopolistic contract will lie on a lower iso-profit line (in the $z_1$ dimension), which corresponds to higher profits. Therefore, monopolistic profits are $\pi(q_{M1}, b'_1) < \pi(q_{M2}, b'_2)$ and the result follows.

C Additional Tables and Figures
Figure 15: Fraction of interest rates at cap in the Benchmark model as a function of cap.

![Fraction of interest rates at cap in the Benchmark model as a function of cap.]

Table 6: Data and model outcomes by education group.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data 2000-04</th>
<th>Benchmark</th>
<th>Competitive</th>
<th>No High markup Option</th>
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<tbody>
<tr>
<td></td>
<td>High School</td>
<td>College</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt/Income %</td>
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<td>8.6</td>
<td>7.4</td>
</tr>
<tr>
<td>Bankruptcies per 1000</td>
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<td>1.46</td>
<td>1.56</td>
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<td>Fraction with Debt %</td>
<td>39.77</td>
<td>39.4</td>
<td>31.37</td>
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Figure 16: Profitability as function of repayment probability in the No High Markup Option model.

Figure 17: Welfare gain/loss 10% interest cap relative to benchmark as a function of income (relative to group specific median).

(a) College  
(b) High School