A Fair Day's Pay for a Fair Day's Work:

Optimal Tax Design as Redistributional Arbitrage

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INTRODUCTION

Workhorse (static) theory of optimal taxation: Mirrlees (1971), Saez (2001)

$\hookrightarrow$ top income tax rate in terms of sufficient statistics: Pareto coeff. of earnings ρ_Y

First issue: multiple margins of inequality and redistributive policy instruments

$\hookrightarrow$ should we tax the rich by reducing their consumption, their leisure, their wealth?

Second issue: multiple data sources can be used to discipline the model: $\rho_Y = \rho_C$

$\hookrightarrow$ should we use income or consumption inequality data for optimal income taxes?

Empirically:* consumption has thinner tail than income: $\rho_Y = 1.6$, $\rho_C = 3.65$

$\hookrightarrow$ disclaimer: we *do* know that consumption inequality is difficult to measure!

THIS PAPER

Redistributional arbitrage: Novel, unified representation of optimal policy design

Theory: Optimal income (capital) tax equalizes the marginal resource benefits of redistributing via consumption vs. leisure (consumption vs. savings): B_C, B_Y, B_S

Identification: B_C, B_Y, B_S map to consumption, income, savings data: ρ_C, ρ_Y, ρ_S

Sufficient statistic formulas for top income & capital taxes (generalize Saez 2001)

Take-away: If consumption is more evenly distributed than income and wealth, i.e. $\rho_C > \rho_Y$, a large part of the burden of taxes should be shifted from labor to capital

RELATED LITERATURE

Labor income and capital taxation, sufficient statistics: Mirrlees (1971), Atkinson Stiglitz (1976), Saez (2001), Saez (2002)

Role of preference heterogeneity for nonlinear capital taxation: Golosov Troshkin Tsyvinski Weinzierl (2013), Gerritsen Jacobs Rusu Spiritus (2020), Schulz (2021), Scheuer Slemrod (2021), Ferey Lockwood Taubinsky (2021)

Role of consumption data in models of optimal risk-sharing: Townsend (1994), Ligon (1998), Kocherlakota Pistaferri (2009), Toda Walsh (2015)

Leisure inequality, work-life balance, and welfare: Aguiar Hurst (2007), Romer (2011), Schieman et al (2021)

Outline

1. Theory of Redistributional Arbitrage
2. Bringing the Formulas to the Data

INDIVIDUALS

Heterogeneous preferences indexed by $r \in [0,1]$: $U(C, Y; r) + \beta V(S)$

Single-crossing: higher r are more willing to work for a given consumption gain

$$\frac{\partial \ln\left(-U_Y/U_C\right)}{\partial r} = \frac{U_{Yr}}{U_Y} - \frac{U_{Cr}}{U_C} < 0$$

$\frac{U_{Yr}}{U_Y} < 0$: motive for redistributing leisure *"from each according to his ability"*

$\frac{U_{Cr}}{U_C} \lesseqgtr 0$: motive for redistributing consumption *"to each according to his needs"*

PLANNER'S PROBLEM

Rawlsian planner maximizes tax revenue (general SWF in the paper):

$$\max_{\{Y,C,S\}} \int_0^1 [Y(r) - C(r) - R^{-1}S(r)]dr$$

subject to incentive constraints (preferences r are private information):

$$U(C(r), Y(r); r) + \beta V(S(r)) \geq U(C(r'), Y(r'); r) + \beta V(S(r'))$$

Solution: Labor and savings wedges:

$$1 - \tau_Y(r) \equiv \frac{-U_Y(r)}{U_C(r)} \quad \text{and} \quad 1 + \tau_S(r) \equiv \frac{\beta R V'(S(r))}{U_C(r)}$$

OPTIMAL TAXES

Theorem 1: Redistributional arbitrage representation of optimal income taxes:

$$1-\tau_Y(r) = \frac{B_Y(r)}{B_C(r)} \equiv \frac{\mathbb{E}_{r'\geq r}\left[\frac{U_Y(r)}{U_Y(r')}\exp\left(\int_r^{r'}\frac{U_{Yr}}{U_Y}\right)\right]}{\mathbb{E}_{r'\geq r}\left[\frac{U_C(r)}{U_C(r')}\exp\left(\int_r^{r'}\frac{U_{Cr}}{U_C}\right)\right]}$$

and optimal capital taxes:

$$1+\tau_S(r) = \frac{B_S(r)}{B_C(r)} \equiv \frac{\mathbb{E}_{r'\geq r}\left[\frac{V'(S(r))}{V'(S(r'))}\right]}{\mathbb{E}_{r'\geq r}\left[\frac{U_C(r)}{U_C(r')}\exp\left(\int_r^{r'}\frac{U_{Cr}}{U_C}\right)\right]}$$

B_C, B_Y, B_S: mg resource benefits of reducing consumption, leisure, wealth above r

Interpretation: optimal income taxes equalize private trade-off between earnings and consumption to social trade-off in redistributing resources via leisure or consumption

PERTURBATION-BASED INTERPRETATION

Perturbation: raise consumption of $r' \geq r$ and their reduce leisure while preserving IC

If $U_{Cr} = 0$, incentives require raising utility above r *uniformly*: $\Delta C\left(r'\right) \propto 1/U_C\left(r'\right)$

If $U_{Cr} \neq 0$, incentives require raising utility above r in proportion to $\exp\left(\int_r^{r'} \frac{U_{Cr}}{U_C}\right)$

Resource cost of raising consumption above r is $\mathbb{E}_{r' \geq r}\left[\Delta C\left(r'\right)\right] = B_C\left(r\right)$

Similarly, the resource benefit of reducing leisure (raising output) above r is $B_Y\left(r\right)$

Joint perturbation keeps agent r's utility unchanged: $\frac{-U_Y(r)}{U_C(r)} = \frac{B_Y(r)}{B_C(r)}$

WHEN SHOULD CAPITAL BE TAXED?

Corollary: Optimal $\tau_S > 0$ for all r if and only if $U_{Cr} < 0$ for all r

Atkinson Stiglitz (1976): $\tau_S = 0$ if $MRS_{C,S}$ is independent of r, i.e. $U_{Cr} = 0$

Taxing capital is optimal if tastes for work and savings are positively correlated

Key question: How to identify U_{Cr} and U_{Yr} in data?

RELATIONSHIP TO THE "ABC" FORMULAS

Optimal taxes as solution to "ABC" formulas: $A \equiv \frac{U_{Cr}}{U_C} - \frac{U_{Yr}}{U_Y}$

$$\frac{\tau_Y}{1-\tau_Y} = A \cdot B_C, \qquad \tau_Y = A \cdot B_Y, \qquad \frac{\tau_Y}{1-\tau_Y}(1+\tau_S) = A \cdot B_S$$

Interpretation: equalize efficiency cost of tax distortions w/ mg benefits B_C, B_Y, B_S

B_Y/B_C, AB_C and AB_Y are equivalent but emphasize different sufficient statistics

Redistributional arbitrage also provides a unified representation of τ_Y and τ_S

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2. Bringing the Formulas to the Data

SUFFICIENT STATISTICS

ρ_Y, ρ_C, ρ_S: Pareto coefficients of the earnings, consumption, wealth distributions

s_C: share of consumption in after-tax earnings

$\zeta_C = -\frac{CU_{CC}}{U_C}$: relative risk-aversion parameter in period 1

$\zeta_S = -\frac{SV''}{V'}$: relative risk-aversion parameter in period 2

$\zeta_Y = \frac{YU_{YY}}{U_Y}$: inverse elasticity of labor supply (simple map w/ Hicksian elasticity ζ_Y^H)

$\zeta_{CY} = \frac{YU_{CY}}{U_C}$: coefficient of complementarity between consumption and leisure

IDENTIFICATION RESULT

Lemma 1: For top earners ($r \to 1$), U_{Cr}, U_{Yr} can be estimated in the data as:

$$\frac{U_{Cr}}{U_C} = \frac{\zeta_C}{\rho_C} - \frac{\zeta_S}{\rho_S} - \frac{\zeta_{CY}}{\rho_Y} \qquad \text{and} \qquad \frac{U_{Yr}}{U_Y} = -\frac{\zeta_Y}{\rho_Y} - \frac{\zeta_S}{\rho_S} + \frac{s_C \zeta_{CY}}{\rho_C}$$

Comparing the tails of C and S reveals underlying preference structure, e.g. $U_{Cr} \lesseqgtr 0$

In particular, $\rho_C > \rho_S$ means top earners consume vanishing fraction of their income

Note: dynamic structure of the model is critical: a static model cannot separately identify U_{Cr}, U_{Yr} even though both matter independently for optimal taxes

OPTIMAL TOP TAX RATES

Theorem 2: Optimal top tax rates in terms of sufficient statistics:

$$1-\overline{\tau}_Y = \frac{1-\frac{\zeta_C}{\rho_C}+\frac{\zeta_{CY}}{\rho_Y}}{1+\frac{\zeta_Y}{\rho_Y}-\frac{s_C\zeta_{CY}}{\rho_C}} \equiv \frac{1/B_C}{1/B_Y} \quad \text{and} \quad 1+\overline{\tau}_S = \frac{1-\frac{\zeta_C}{\rho_C}+\frac{\zeta_{CY}}{\rho_Y}}{1-\frac{\zeta_S}{\rho_S}} \equiv \frac{1/B_C}{1/B_S}$$

$\overline{\tau}_Y$ generalizes Saez (2001) to dynamic model, $\overline{\tau}_S$ provides analogue for capital tax

$B_C \uparrow$ in consumption inequality, $B_Y \downarrow$ in earnings inequality, $B_S \uparrow$ in wealth ineq.

$B_Y \uparrow$ and $B_C \downarrow$ in complementarity between consumption and labor: Corlett-Hague

INCOME AND CONSUMPTION INEQUALITY

Key: $\overline{\tau}_Y$ depends on Pareto tails of income and consumption ρ_Y, ρ_C (via B_Y, B_C)

This is already the case in static model, but the budget constraint imposes $\rho_C = \rho_Y$
$\hookrightarrow$ no guidance about which measure is most appropriate for optimal income taxes!

Gains from further redistribution linked to consumption, not (just) income, inequality

Townsend (1994), Ligon (1998), Kocherlakota Pistaferri (2009)

Our model also has a budget constraint, which imposes $\rho_Y = \min\{\rho_C, \rho_S\}$

A TALE OF THREE TAILS

Case 1: $\rho_Y = \rho_C < \rho_S$: top earners consume all of their income. Then $\overline{\tau}_Y = \overline{\tau}_Y^{Saez}$

Case 2: $\rho_Y = \rho_S < \rho_C$: top earners save all of their income. Then $\overline{\tau}_Y^{Saez} = \frac{1-\overline{\tau}_Y}{1+\overline{\tau}_S}$

Intuitively: model is static in Y, S at the top, so $\overline{\tau}_Y^{Saez}$ gives the total wedge on Y, S

Hence: $\overline{\tau}_Y < \overline{\tau}_Y^{Saez}$ if and only if $\overline{\tau}_S > 0$ (recall Atkinson-Stiglitz)

Case 3: $\rho_Y = \rho_C = \rho_S$: top earners consume & save positive shares, anything goes

CALIBRATION

Pareto coefficients: $\rho_Y = \rho_S = 1.5 < \rho_C \in (2.15, 3.65)$ (Toda Walsh 2015, Straub 2019)

Hicks elasticity, income effect: $\zeta_Y^H = \frac{1}{3}, \zeta_Y^I = \frac{1}{4}$ pin down $\zeta_S = \frac{3}{4}$ and $\frac{B_Y}{B_S} = \frac{1-\overline{\tau}_Y}{1+\overline{\tau}_S}$

First-period risk-aversion of top earners: $\zeta_C = \zeta_S = \frac{3}{4}$

Complementarity between consumption & labor supply: $\frac{\zeta_{CY}}{\zeta_C} \in (0, 0.15)$ (Chetty 2006)

Alternative calibration for Case 3 in the paper

QUANTITATIVE RESULTS

	$\rho_Y/\rho_C = 0.4$		$\rho_Y/\rho_C = 0.6$		$\rho_Y/\rho_C = 0.75$		
	$\overline{\tau}_Y$	$\overline{\tau}_S$	$\overline{\tau}_Y$	$\overline{\tau}_S$	$\overline{\tau}_Y$	$\overline{\tau}_S$	$\overline{\tau}_Y^{Saez}$
Baseline	68%	38%	72%	29%	75%	20%	80%
$\zeta_Y^H = 0.5$	60%	17%	65%	5%	69%	−7%	67%
$\zeta_Y^I = 1/3$	66%	58%	70%	52%	73%	47%	86%
$\zeta_C = 1.25$	73%	25%	80%	0%	85%	−33%	80%
$\zeta_{CY}/\zeta_C = 0.15$	64%	44%	68%	38%	71%	31%	80%

$\overline{\tau}_S = 40\%$: top earners receive p.v. of \$0.71 of additional pension payments for each additional \$1 of s.s. contributions (or 1.8% annual wealth tax with 5% annual return)

CONCLUSION

Theoretically: novel representation of optimal taxes as redistributional arbitrage

$\hookrightarrow$ straightforward generalization to arbitrary preferences and commodities

Bringing the formulas to the data: sufficient-statistic representation

$\hookrightarrow$ key role of consumption data to discipline the model

Take-away: consumption is substantially more evenly distributed than income

$\Rightarrow$ optimal to shift a substantial share of the tax burden from labor to capital

"*A Fair Day's Pay for a Fair Day's Work*"

ARBITRARY PREFERENCES AND COMMODITIES

Assume $\frac{U_{nr}}{U_n}$ is increasing in n, so $\frac{U_m}{U_n}$ is increasing in r whenever $m > n$

$$\max \int_0^1 \omega(r)\, U(\boldsymbol{X}(r); r)\, dr - \int_0^1 p \cdot \boldsymbol{X}(r)\, dr \quad \text{s.t. IC constraints}$$

Redistributive arbitrage: $\frac{U_m(r)}{U_n(r)} \frac{p_n}{p_m} \equiv 1 - \tau_{m,n}(r) = \frac{B_m(r)}{B_n(r)}$ for all m, n, with

$$B_n(r) = \mathbb{E}_{r' \geq r}\left[\frac{U_n(r)}{U_n(r')} \mu_n(r')\right] \left(1 - \frac{\mathbb{E}_{r' \geq r}[\mu_n(r')\, \omega(r')]}{p_n\, \mathbb{E}_{r' \geq r}[\mu_n(r') / U_n(r')]}\right)$$

Non-uniform commodity taxes when $\mu_n(r') \equiv \exp\left(\int_r^{r'} \frac{U_{nr}}{U_n}\right)$ differ. Sufficient stats:

$$1 - \tau_{m,n}(r) = \frac{B_m(r)}{B_n(r)} = \frac{\left[1 - \sum_{k=1}^N \frac{\zeta_{mk}}{\rho_k}\right]^{-1}}{\left[1 - \sum_{k=1}^N \frac{\zeta_{nk}}{\rho_k}\right]^{-1}}$$