Procyclical Productivity in New Keynesian Models

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Abstract

We propose an easy-to-use search friction in the goods markets in medium-sized New Keynesian models. This friction allows increases in measured productivity in response to increases in expenditures via higher search effort from households. As a result markups can become procyclical and labor share countercyclical. Unlike in models that pose variable capital utilization and fixed costs to generate procyclical productivity, firms do not have to spend more to achieve it. We estimate the model matching impulse responses with Bayesian techniques and show superior performance of models with search frictions relative to the state of the art alternative models in the literature. Our estimates also display low fixed costs of production and lower Frisch elasticities.

Keywords: Procyclical Productivity, New Keynesian Models, Labor Share, Markups, Search

JEL Codes: E12, E32, E52

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1. Introduction

The impulse response functions of demand shocks such as monetary shocks display procyclical labor productivity and countercyclical labor share. New Keynesian models, which are close relatives of neoclassical models, face an uphill battle to deliver these properties. The most commonly used modeling device to get around these problems in medium-sized models (models that have capital and hence decreasing returns to labor) is variable capital utilization (yet, Christiano et al. (2005) with a nearly flat rate cost of capital utilization, only get less than one half of the movements in labor productivity). Such a feature postulates missmeasured capital inputs of production, inputs that have to be paid for, and that will not reduce marginal costs which makes it difficult to generate procyclical labor productivity or countercyclical labor share (in fact Cantore et al. (2021) show that matching countercyclical labor share without hurting other parts of the model is impossible with the commonly used features in medium-scale DSGE models).[1]

Motivated by this inconsistency between theory and evidence, we propose an extension of the standard New Keynesian model in which procyclical labor productivity, markups, wages and inflation are possible as well as countercyclical labor share even when the expansion is demand induced. Such an extension is based on the notion that consumers (or, in general, purchasers of goods and services) perform a productive role and that their contribution, which requires effort, goes up in demand-induced expansions when consumers squeeze more output out of the economy. The gist of our theory makes households responsible for transforming firms' activities into products that can be used for consumption in a way that requires effort (a disutility) to do so. When expenditures go up, consumers exert more effort, extracting more output from any given amount of firms activities which effectively looks like higher productivity. Central to this notion is that firms enjoy the productivity increase in the sense that they do not pay for it. Moreover, when firms set prices, they understand that their productivity is also affected, which results in an incentive to charge higher markups in expansions. As a result, the properties of economies with these features include not only those standard in New Keynesian models but also procyclical labor productivity and countercyclical labor share.

This paper has two main contributions. The first one is theoretical: we pose a directed search friction in the product markets on top of an otherwise standard New Keynesian model to obtain both endogenously procyclical TFP and desired markups (the markups that would occur in the absence of price adjustment frictions). Technically, we combine for the first time directed search in the product markets (Huo and Ríos-Rull (2015); Bai et al. (2019)) with multiple varieties (Huo and Ríos-Rull

[1] Choi and Ríos-Rull (2009), Choi and Ríos-Rull (2020) and Ríos-Rull and Santaeulàlia-Llopis (2010) have pointed to associated difficulties in neoclassical models.

(2014, 2020a)). In our environment households with taste for consuming different varieties of goods have to exert search effort to find them. Higher expenditures translate into more varieties and more consumption of each variety. More varieties translate into higher occupancy rates for firms resulting in higher productivity. Higher occupancy rates induce firms to increase their markups, not because of higher costs but because of less price competition. The endogenous change in TFP is neither a mismeasurement of inputs nor a costly choice of the firms, while the endogenous desired markup allows inflation to be procyclical even if the real marginal cost of production is not. The standard New Keynesian model becomes a special case of our search model when endogenous movements in TFP are absent so model comparison becomes straightforward.

Our second contribution is empirical. We estimate various alternative medium-scale New Keynesian models with and without our proposed extension and variable capital utilization. We find overwhelming support for our search friction in terms of both the goodness of fit and the plausibility of the parameter estimates.

We use a version of Christiano et al. (2016) and the Bayesian impulse response matching method developed in Christiano et al. (2010). Our estimation targets include the 9 standard variables under 3 structural shocks used in the baseline model of Christiano et al. (2016), and we also include labor productivity and labor share as explicit additional targets to highlight the main advantages of our extension.

Our benchmark economy, that includes both our proposed extension and variable capital utilization performs best with search frictions playing a more central role in shaping the outcomes. The two parameters that capture the endogeneity of TFP and of endogenous desired markups are well identified in the sense that their posterior distributions are much tighter than their priors. Conditional on the Federal Funds Rate shocks, labor productivity and labor share in the model are close to the target impulse responses in the structural VAR regression. The values of the parameters that we estimate are all reasonable.

But the true value of our theoretical extension shows up when comparing the estimation results across economies. Relative to the economy without the search frictions, this is, the standard New Keynesian model, the overall fit is much better (as measured by the log marginal likelihood) and we obtain parameter estimates that we think are much more reasonable: lower fixed costs of production (3% vs. 39%), lower Frisch elasticity of labor supply (1.6 vs. 9.1) and lower size of productivity shocks (0.22 vs. 0.32). A look at the impulse response (*i.e.* the eyeball metric) also confirms these results (if we may say that). Markups are decidedly procyclical. Further, if we impose a much lower size of fixed costs (13%) on the model without search frictions,[2] the difference in terms of fit becomes much larger.

The model with search frictions but not variable capacity utilization also does much better than the standard New Keynesian model. The log marginal likelihood is much larger. The parameter estimates are also more reasonable (lower fixed costs of production (0.14 vs 0.39)) lower Frisch elasticity of labor supply (1.9 vs. 9.1) and lower size of productivity shocks (0.19 vs. 0.32).

Two additional findings, documented in our robustness section, further support our views. First, qualitatively, the advantage of search frictions over variable capital utilization does not depend on whether we target labor productivity and labor share, or on how exactly we perform the accounting of the costs of variable capital utilization. Second, the implied correlation between markups and log GDP conditional on Federal Funds Rate shocks increases from nearly zero to above 0.7 in all of our estimation results when our search friction is turned on.

Why is search friction more powerful than variable capital utilization (Taubman and Wilkinson (1970), Greenwood et al. (1988))? The gist is who pays for the implied increase in productivity. Under variable capital utilization, firms pay for the costly additional capital services. Such a mechanism can at best remove all decreasing returns to scale induced by slow moving capital, yet firms do not have at all higher productivity. In contrast, the search friction directly increases the occupancy rate or production capacity at the cost of household disutility, inducing truly higher productivity from the point of view of the firm and hence higher profit margins. Moreover, higher desired markups help the lower marginal cost of production associated with higher productivity to avoid deflation.

Is there any direct evidence of our mechanism? Indeed there is. With respect to procyclical consumer search, Petrosky-Nadeau et al. (2016) document facts about procyclical shopping time and that the unemployed have larger drops of shopping time during the Great Recession and Bai et al. (2019) report that the fall in shopping effort during and after the Great Recession was due to the unusually large and sustained fall in consumption. With respect to procyclical number of varieties, Michelacci et al. (2021) document that about half of the changes in U.S. non-durable consumption is due to adding varieties to basket and Li (2021) also provides evidence of more consumption inducing more varieties. Stroebel and Vavra (2019) find explicit empirical evidence of a strong interaction between household shopping behavior and firm price setting that supports the mechanism in this paper. Moreover, recent empirical findings about productivity and markups using firm level data suggest that firms increase real output much faster than employment during demand expansions (Gottfries et al. (2021)) and that net profit margins are procyclical conditional on demand shocks (Anderson et al. (2020)) which we take as asking for a direct channel from increased expenditures.[3]

[2] Abraham et al. (2021) find that fixed costs amount to 23.4% of sales.

[3] There is a literature that emphasizes composition effects as a mechanism that affects productivity, labor share and

Are markups procyclical conditional on demand shocks? As we have stated, Nekarda and Ramey (2020), Stroebel and Vavra (2019) and Anderson et al. (2020) suggest that they are.[4] A potential criticism from Kudlyak (2014) and Basu and House (2016) emphasizes that the marginal cost of labor may be much more procyclical than its average cost, so that the markup measures based on labor cost would be strongly biased towards procyclical markups. Yet, more recent empirical and theoretical findings in Gertler et al. (2020), Grigsby et al. (2021) and Fukui (2020) are not in favor of this criticism. Our reading of the evidence is that on the whole, empirical evidence supports procyclical markups conditional on demand shocks.

The main mechanism that makes labor productivity procyclical and labor share countercyclical in this paper is the variable effort made by shoppers in squeezing output out of a given production capacity via search frictions. This idea was developed in Bai et al. (2019) using search for goods to show how demand shocks may look like productivity shocks. Among theory papers, Lehmann and Van der Linden (2010) have frictions in the goods market to study the steady state relation between inflation and output or employment. Michaillat and Saez (2015) also connect product market frictions to idle occupancy and unemployment and argue that under fixed prices, which leaves out any search equilibrium condition, tightness is the variable that clears the goods markets allowing for unemployment; Huo and Ríos-Rull (2015) in a quantitative model with heterogeneous agents show how increases in savings arising from a financial shock trigger a recession via a reduction in the consumption of goods that are subject to search frictions reducing productivity and output. This is exacerbated because poorer households are more adversely affected. They further model fixed prices in a competitive search environment and show how price rigidities amplify the recession. Huo and Ríos-Rull (2014) and Huo and Ríos-Rull (2020a) introduced search for varieties as a means to have negative wealth effects to generate recessions both in heterogeneous and in representative agent models. They use undirected search which implies that firms cannot affect their own productivity via pricing decisions.

There are other models in the literature that have endogenous productivity or markups. As for endogenous productivity, Fagnart et al. (1999) pose monopolistic firms with putty-clay technology that display fluctuations in capacity utilization and hence in productivity, Swanson (2006) shows that relative productivities across sectors can cause procyclical aggregate productivity, and Gottfries et al. (2021) use labor reallocation from production to non-production activities to generate a faster adjustment of output than that of employment. As for endogenous markups, Kaplan and Menzio (2016) model how

markups (e.g. Kaplan and Zoch (2020), Anderson et al. (2020), Meier and Reinelt (2021), and Baqaee et al. (2021)). We abstract from this margin.

[4] Bils et al. (2018), Hong (2017) and Burstein et al. (2020) find evidence supporting countercyclical markups in industry or firm level data. Their cyclicality results are unconditional, and hence do not reject procyclical markups conditional on demand shocks.

more price picking behavior of the unemployed in recessions drives down firms' markup, Bertoletti and Etro (2017) extend the standard Dixit-Stiglitz aggregator to make the elasticity of substitution between goods varieties and hence desired markups depend on income, and Hyun et al. (2021) study how a translog production function, a form of procyclical returns to scale, helps get procyclical markups. Jaimovich (2007) and Floetotto and Jaimovich (2008) have fluctuations in the number which causes movements in both markups and Solow residuals.

Section 2 has a one period model version of our ideas that include the core of the message. In fact, the main theoretical contribution is in Section 2.1. A dynamic version of the model is imposed on top of the static one in Section 3 and estimated in Section 4 to assess the empirical relevance of the theoretical possibilities described. We perform some robustness tests in Section 5. We discuss the role of the mechanisms in our estimation in Section 6. Appendix A provides various algebraic derivations while Appendix B provides optional technical materials for the interested reader to replicate our computations.

2. A Simple Model with Search Frictions Generating Procyclical TFP, Inflation, and Markups

We now pose a one-period model with exogenous expenditures but with all the necessary ingredients to discuss how directed search generates procyclical TFP and markups in response to a demand increase such as that induced by a monetary expansion in New Keynesian environments. The main features of this model are (1) preferences of the Dixit-Stiglitz love-for-varieties class but where finding varieties requires search effort, (2) pricing decisions that affect how many customers show up, *i.e.* the occupancy rate, (3) price-setting costs à la Rotemberg faced by firms, and (4) wage-setting frictions à la Calvo faced by unions.

We start with a very basic version of the model with neither search frictions nor wage movements to illustrate the difficulties of standard new Keynesian models to generate procyclical TFP and markups (Section 2.1). Next, we pose on top of this environment search frictions in the product markets that constitute the core of this paper to show how directed search overcomes these difficulties (Section 2.2). The endogenous determination of nominal wages given monetary policy (taken to be a nominal GDP target here) is in Section 2.3.

2.1. The Difficulties of Standard New Keynesian Models in Moving TFP or Markups

In standard New Keynesian models technology is either constant or decreasing returns to scale and movements in TFP can only occur due to mismeasurement in inputs (more on this later in Section 6).

We now turn to look formally at the relation between markups, real wages and inflation in a simple one-period New Keynesian model.

Consider standard Dixit-Stiglitz preferences:[5] goods produced come in differentiated varieties $i \in [0,1]$ with a constant elasticity of substitution. Each variety i is produced by a single firm with monopoly power that sets a price p_i and produces whatever is demanded in the market.

Observing prices $\{p_i\}_{i\in[0,1]}$, the representative household has nominal spending e to allocate optimally across all varieties $i \in [0,1]$ and solves:

$$v(e,\{p_i\}) = \max_{\{c_i\}} \quad U\left[\left(\int_0^1 c_i^{\frac{1}{\rho}}\, di\right)^{\rho}\right], \tag{1}$$

$$\text{s.t.} \qquad e \geq \int_0^1 p_i\ c_i\ \ di. \tag{2}$$

where $\rho > 1$. The solution is $c^i(e,P,p_i) = \left(\frac{p_i}{P}\right)^{-\frac{\rho}{\rho-1}} \frac{e}{P}$, where $P \equiv \left(\int_0^1 p_j^{\frac{1}{1-\rho}}\ dj\right)^{1-\rho}$.

Firm i sets price p_i optimally subject to the households' demand, linear technology $y_i = n_i$, a competitive labor market with nominal wage W, and a Rotemberg style price adjustment cost that we assume proportional to expenditures for simplicity,[6] $\chi(p_i/p^-)\, e$ (where $\chi(.)$ is non-negative, convex and satisfies $\chi(1) = \chi_p(1) = 0$, $\chi_{pp}(1) > 0$), to solve

$$\Omega(e,W,P,p^-) = \max_{p_i} \ \ (p_i - W)\ \left(\frac{p_i}{P}\right)^{-\frac{\rho}{\rho-1}} \frac{e}{P} - \chi\left(\frac{p_i}{p^-}\right) e, \tag{3}$$

with first order condition

$$\frac{1}{\rho-1}\left(\rho\frac{W}{p_i} - 1\right)\left(\frac{p_i}{P}\right)^{-\frac{1}{\rho-1}} = \frac{p_i}{p^-}\ \chi_p\left(\frac{p_i}{p^-}\right). \tag{4}$$

In the absence of adjustment costs, the solution, $p_i(e,W,P,p^-)$, satisfies $\frac{p_i}{W} = \rho$. An equilibrium given nominal wages W, nominal expenditures e, and inherited prices p^- is an allocation c^* and a price p^* such that households and firms choose optimally c^* and p^* when they face $P = p^*$.

By construction, the model has constant TFP. To see what happens to markups, we can look at the log-linearized equilibrium condition evaluated at the price that does not change in equilibrium

[5] Other types of preferences (see for instance the summary in Arkolakis and Morlacco (2017)) do not change our results qualitatively.

[6] Adjustment costs proportional to nominal expenditures guarantee that increases in nominal expenditures translate into increases in real output and not into increases in prices. Eliminating expenditures from the price adjustment costs changes this statement but not in a relevant manner.

$p^* = p_i(e, W, p^*, p^*)$, (analogous in this static model to the steady state in a dynamic model)

$$d\ln(p^*) = \frac{1}{1 + (\rho - 1)\chi_{pp}(1)}\, d\ln(W). \tag{5}$$

Here we can take the change in log nominal wages as either exogenous or as the labor market response to attract the labor needed to increase real output. Equation (5) shows that while prices are not affected directly by changes in nominal spending they still change via the increase of nominal wages. Prices move in the same direction but less than nominal wages. Consequently, an increase in nominal wages moves the gross markup p^*/W down, inflation p^*/p^- up, and the real wage W/p^* up.

To summarize, two limitations prevent procyclical markups: gross markups and real wages are the inverse of each other due to the lack of movement in labor productivity and inflation is completely cost driven, which means higher inflation must be associated with higher marginal cost of production and hence lower markup. As a result, getting procyclical real wages or procyclical inflation rules out the possibility of procyclical markups. The search friction in the goods market with households contributing to productivity removes these two limitations by allowing endogenous variations in TFP and in desired markups.

2.2. Directed Search: Endogenous TFP and Endogenous Desired Markup

We turn to add a shopping friction to the goods markets in the basic model described above. Now households need to exert effort to find varieties and they will choose not to consume all available varieties. Because there is a large number of households and varieties, all firms end up having the same occupancy rate which, as we will see, is the same as productivity. Suitably chosen preferences ensure that increases in spending imply both an increase in the quantity of varieties and in the consumption of each variety, or, in other words, both in productivity and in labor. Firms can attract consumers either with lower prices or with lower market tightness and an appropriately chosen matching function ensures that as the occupancy rate increases, firms have less incentives to engage in price competition, leading to higher desired markups.

Now the details. Assume that each firm or variety producer operates a continuum of locations, each one with its own preinstalled inputs and identical production technology. All locations of a firm go to one and only one submarket indexed by price and tightness $\{p, q\}$ and each location is matched with at most one household.[7] The total number of matches in each submarket is given by a constant returns to scale (CRS) matching function $\psi[J(p, q), D(p, q)]$, where market tightness is the ratio of $J(p, q)$

[7] We avoid the standard label of competitive search due to both monopolistic competition and the fact that what is searched for are not goods but varieties.

the measure of firms, and $D(p,q)$ total shopping effort in submarket $\{p,q\}$, $q = \frac{D(p,q)}{J(p,q)}$. Matching probabilities per firm and per unit of search are $\psi^f(q) \equiv \frac{\psi[J(p,q),D(p,q)]}{J(p,q)}$ and $\psi^h(q) \equiv \frac{\psi[J(p,q),D(p,q)]}{D(p,q)}$. Note that ψ^f is the probability that a location can be found and also the fraction of production capacity that is used. We can interpret ψ^f as TFP because total output equals total occupied labor, which is total labor times ψ^f. It is an endogenous variable that is increased by household effort without costing anything to the firm (a crucial difference with variable capital utilization models).

Households choose how much shopping effort d to exert in each submarket $\{p,q\} \in \Phi$ where Φ denotes the set of available markets (this is, markets where firms are willing to go to), and hence we say they choose $d(p,q)$. They value both the number of varieties and the quantity consumed of each variety according to utility function $U(c^A, d^A)$, with arguments

$$c^A \equiv \left(\int_\Phi d(p,q)\, \psi^h(q)\, c(p,q)^{\frac{1}{\rho}} \; dp\, dq \right)^\rho \tag{6}$$

$$d^A \equiv \int_\Phi d(p,q)\, dp\, dq, \tag{7}$$

where c^A is the utility-relevant aggregate of consumption of all varieties found and d^A is aggregate search effort.

Let's further specialize our environment to preferences à la Greenwood et al. (1988) (GHH preferences) in consumption and search effort (abusing notation in the process by referring to both the general utility function and the GHH utility function by U): $U(c^A, d^A) = U\left(c^A - \zeta \frac{(d^A)^{1+\nu}}{1+\nu}\right)$ for some strictly concave and increasing U. There are two advantages of these preferences: they imply that higher expenditures imply higher effort (proof in Appendix A.1), a necessary feature to have demand shocks increase productivity, and they are easy to characterize using a simple relation between the consumption and the search aggregates

$$c^A = \frac{\zeta}{\rho - 1} \cdot (d^A)^{1+\nu}, \tag{8}$$

allowing us to write $U(c^A, d^A) = U\left(\frac{2+\nu-\rho}{1+\nu} \cdot \frac{\zeta}{\rho-1} \cdot (d^A)^{1+\nu}\right)$ or $U(c^A, d^A) = U\left(\frac{2+\nu-\rho}{1+\nu} \cdot c^A\right)$.

We now pose the problem of the household given expenditures to show how to characterize the set of markets that guarantees a certain level of utility (Section 2.2.1). Firms maximize their profits restricted to go to one of those markets (Section 2.2.2). Given nominal expenditures and nominal wages equilibrium requires consistency with the utility achieved by households. At this stage we characterize equilibrium given exogenous changes in nominal expenditures and nominal wages and discuss what it takes to generate procyclical TFP and markups (Section 2.2.3).

2.2.1. Household Problem for a Given Set of Available Markets

Consider the problem of a household that can only search in a particular subset of markets, Φ, that specifies pairs of available prices and market tightnesses

$$v(e,\Phi) = \max_{\{c(p,q),d(p,q)\}\geq 0} \quad U\left(c^A, d^A\right), \tag{9}$$

$$\text{s.t.} \qquad e = \int_\Phi d(p,q)\ \psi^h(q)\ p\ c(p,q)\ \ dp\ dq, \tag{10}$$

and the definition of aggregates (6-7).

Taking first order conditions for $c(p,q)$ in any market where there is positive search, we get (a proof in Appendix A.2)

$$c(p,q) = \left(\frac{p}{P^A}\right)^{-\frac{\rho}{\rho-1}} c^A, \tag{11}$$

where we use price aggregate

$$P^A = \left(\int_\Phi d(p,q)\ \psi^h(q)\ p^{\frac{1}{1-\rho}}\ dp\ dq\right)^{1-\rho}. \tag{12}$$

Note that P^A depends on households' decisions. We denote $P^A(e,\Phi)$ as the value of P^A for optimal decisions. Note also that even though we cannot say which markets the household searches in, Equation (11) tells us how much is bought of any variety that is found. Furthermore, we can use the equation in the budget constraint to obtain

$$e = P^A \cdot c^A. \tag{13}$$

A very useful property of price aggregate in optimal decisions $P^A(e,\Phi)$ (a proof in Appendix A.3) is that it satisfies

$$P^A(e,\Phi) = \min_{\{p,q\}\in\Phi} \{P^A | d^A = d^A(e,\Phi)\} = d^A(e,\Phi)^{1-\rho} \cdot \min_{\{p,q\}\in\Phi} \left\{\psi^h(q)^{1-\rho}\ p\right\}. \tag{14}$$

which means that it can be substituted in Equations (8) and (13) to find optimal consumption and search aggregates.

Consequently, we can obtain the indirect utility function $v(e,\Phi)$ directly

$$v(e,\Phi) = U\left(\frac{2+\nu-\rho}{1+\nu}\cdot\left(\frac{\zeta}{\rho-1}\right)^{-\frac{\rho-1}{2+\nu-\rho}}\left(\frac{e}{\min_{\{p,q\}\in\Phi}\{\psi^h(q)^{1-\rho}\,p\}}\right)^{\frac{1+\nu}{2+\nu-\rho}}\right).$$

We can also obtain the set of markets $\Phi(e,\overline{v})$ that yields utility $\overline{v}$ by inverting $v(e,\Phi)=\overline{v}$ getting

$$\min_{\{p,q\}\in\Phi}\{\psi^h(q)^{1-\rho}\,p\} = \left(\frac{\zeta}{\rho-1}\right)^{-\frac{\rho-1}{1+\nu}}\left(\frac{1+\nu}{2+\nu-\rho}\,U^{-1}(\overline{v})\right)^{-\frac{2+\nu-\rho}{1+\nu}} e.$$

The market tightness q for each price p that solves the condition above and hence stays on the boundary of $\Phi(e,\overline{v})$ can be obtained using the inverse of the function that yields the probability of a location finding a customer

$$\widetilde{q}(e,\overline{v},p) = (\psi^h)^{-1}\left(\left(\frac{\zeta}{\rho-1}\right)^{\frac{1}{1+\nu}}\left(\frac{1+\nu}{2+\nu-\rho}U^{-1}(\overline{v})\right)^{\frac{2+\nu-\rho}{(1+\nu)(\rho-1)}}\left(\frac{p}{e}\right)^{\frac{1}{\rho-1}}\right). \tag{15}$$

Using Equations (11) and (13) derived before, and the same logic we can obtain the variety specific demand conditional on expenditures, a utility level and a price:

$$\widetilde{c}(e,\overline{v},p) = \left(\frac{1+\nu}{2+\nu-\rho}U^{-1}(\overline{v})\right)^{-\frac{1}{\rho-1}}\left(\frac{p}{e}\right)^{-\frac{\rho}{\rho-1}}. \tag{16}$$

Functions (15) and (16) are the objects that will enter the firm problem, the counterparts to the variety specific demand functions in the standard model. Note that throughout the derivation, the only functional assumption we impose is GHH utility.

2.2.2. Firms Problem Conditional on Utility Level

A firm that knows it has to guarantee households a certain utility level $\overline{v}$ given expenditures, e, nominal wages, W, and previous period's price, p^-, and chooses price p understanding that the market that it has to go has tightness $q=\widetilde{q}(e,\overline{v},p)$ and quantity demanded $\widetilde{c}(e,\overline{v},p)$. We can write its problem as

$$\Omega(e,W,\overline{v},p^-) = \max_p \ \left(p\,\psi^f[\widetilde{q}(e,\overline{v},p)] - W\right)\ \widetilde{c}(e,\overline{v},p) - \chi\left(\frac{p}{p^-}\right)e.$$

Using shorthand notation (*i.e.* omitting the arguments in functions) the first order condition is

$$0 = \left(\psi^f + p\cdot\psi^f_q\cdot\widetilde{q}_p\right)\cdot\widetilde{c} + \left(p\cdot\psi^f - W\right)\cdot\widetilde{c}_p - \chi_p\,\frac{e}{p^-}.$$

We can rewrite this condition in terms of the elasticities of functions $\{\psi^f, \widetilde{q}, \widetilde{c}\}$[8]

$$0 = \left(\psi^f + \frac{\widetilde{q}\ \psi^f_q}{\psi^f} \cdot \frac{p\ \widetilde{q}_p}{\widetilde{q}} \cdot \psi^f + \frac{p\ \widetilde{c}_p}{\widetilde{c}} \cdot \psi^f - \frac{W}{p} \cdot \frac{p\ \widetilde{c}_p}{\widetilde{c}} \right) \widetilde{c} - \chi_p \frac{e}{p^-}.$$

We denote the elasticity of the probability of firms finding a customer (or TFP) by $\mathcal{E}(q) \equiv \frac{q\ \psi^f_q(q)}{\psi^f(q)} = \frac{d \ln(\psi^f(q))}{d \ln(q)}$, which because of the constant returns to scale in the matching technology is between 0 and 1 and satisfies $\frac{q\ \psi^h_q(q)}{\psi^h(q)} = \frac{q\ \psi^f_q(q)}{\psi^f(q)} - 1 = \mathcal{E}(q) - 1$.

If we evaluate function ψ^h at the value of $\widetilde{q}(\cdot)$ given by Equation (15) and take log partial derivatives for p on both sides we get

$$\frac{\partial \ln(\psi^h(\widetilde{q}(e, \overline{v}, p)))}{\partial \ln(p)} = \frac{\partial \ln(\psi^h)}{\partial \ln(\widetilde{q})} \frac{\partial \ln(\widetilde{q})}{\partial \ln(p)} = \frac{\widetilde{q}\ \psi^h_q}{\psi^h} \cdot \frac{p\ \widetilde{q}_p}{\widetilde{q}} = \frac{1}{\rho - 1} > 0.$$

Hence, the elasticity of function $\widetilde{q}(\cdot)$ with respect to p is

$$\frac{p\ \widetilde{q}_p}{\widetilde{q}} = -\frac{1}{(\rho - 1)[1 - \mathcal{E}(\widetilde{q})]}. \tag{17}$$

We can also use Equation (16) to get the elasticity of $\widetilde{c}(\cdot)$

$$\frac{p\ \widetilde{c}_p}{\widetilde{c}} = -\frac{\rho}{\rho - 1}. \tag{18}$$

Substituting these elasticities back to the first order condition yields

$$0 = \frac{1}{\rho - 1} \left[\rho \frac{W}{p} - \frac{\psi^f(\widetilde{q}(e, \overline{v}, p))}{1 - \mathcal{E}(\widetilde{q}(e, \overline{v}, p))} \right] \frac{p\ \widetilde{c}(e, \overline{v}, p)}{e} - \chi_p \left(\frac{p}{p^-} \right) \frac{p}{p^-}.$$

Note that absent price rigidity, the actual markup equals the desired markup

$$\frac{p\ \psi^f}{W} - 1 = \rho\,(1 - \mathcal{E}) - 1,$$

which is affected by the elasticity of TFP to market tightness which may not be constant. We denote $\Gamma \equiv \frac{p\ \psi^f}{W}$ as the gross markup for later use.

[8] In a model with random search instead of directed search as in Huo and Ríos-Rull (2020a), households do not know the price when shopping, and hence firms' pricing decisions will not affect the market tightness for them. As a result, $\widetilde{q}_p = 0$, in the firms' first order condition (not elsewhere). It is equivalent to have the term $\frac{\widetilde{q}\ \psi^f_q}{\psi^f}$ (not the ψ^f elsewhere) replaced by 0.

2.2.3. Equilibrium Given Nominal Wages and Expenditures

It is at this stage that we impose the equilibrium conditions given expenditures e and nominal wage W, which is just that the set of markets $\Phi(e, \overline{v})$ for utility level $\overline{v}$ will induce households' shopping effort and firms' pricing decisions consistent with $\Phi(e, \overline{v})$. We also impose the standard representative agent condition for households and firms so we use capital letters to refer to aggregate prices using P^- for the price inherited from the past that is common to all firms and using P^* and Q^* for equilibrium prices and tightness. We then summarize the equilibrium conditions by a pair of equations

$$\frac{1}{\rho - 1}\left[\rho \, \frac{W}{P^* \, \psi^f(Q^*)} - \frac{1}{1 - \mathcal{E}(Q^*)}\right] = \chi_p \left(\frac{P^*}{P^-}\right) \frac{P^*}{P^-}, \tag{19}$$

$$\frac{\zeta}{\rho - 1} \frac{(Q^*)^{1+\nu}}{\psi^f(Q^*)^{\rho-1}} = \frac{e}{P^*}. \tag{20}$$

Equation (19) is the first order condition of firms while Equation (20) imposes optimality of the household and the consistency of market tightness with total search effort.

Notice that we can use these two equilibrium objects $P^*(e, W, P^-)$ and $Q^*(e, W, P^-)$ directly in the problem of the household ignoring the cumbersome derivation of which markets are available, an approach that we will follow in Section 3. Formally,

$$\max_{\{c^A, d^A\} \geq 0} \; U\left(c^A, d^A\right)$$
$$\text{s.t.} \quad e = c^A \; \left(d^A \; \psi^h[Q^*(e, W, P^-)]\right)^{1-\rho} \; P^*(e, W, P^-).$$

To further the analysis we proceed by log-linearizing the equilibrium conditions, which requires us to choose a reference point. As before, the natural such point is when there is no need to adjust the price, i.e. $P^* = P^-$. We refer to this point as the steady state because in a dynamic version of this economy this is what it would be. We start by grouping parameters in each equation to highlight the role of directed search, and then characterize how large is the endogenous response of TFP that is needed to obtain procyclical markups. We also want to characterize what is needed for procyclical inflation, and real wages. While the more procyclical TFP is the more procyclical markups are, too strong a procyclicality of TFP will make inflation and real wages countercyclical. Consequently, we want to characterize the range of the elasticity of endogenous TFP that makes markups, inflation and real wages all procyclical.

Endogenous TFP and its elasticity $\widetilde{\Psi}$ Log-linearizing both sides of condition (20) around what we have called the steady state yields

$$(1+\nu)\ d\ln(Q^*) - (\rho-1)\ d\ln[\psi^f(Q^*)] = d\ln\left(e/P^*\right).$$

We now use the elasticity of TFP with respect to market tightness around the steady state Q_{ss}, $\mathcal{E}(Q_{ss}) \equiv \left.\frac{d\ln(\psi^f(q))}{d\ln(q)}\right|_{q=Q_{ss}}$, to obtain Equation (21) that describes how endogenous TFP is affected directly by exogenous changes in nominal spending e and indirectly by the equilibrium price P^*

$$d\ln(\psi^f(Q^*)) = \widetilde{\Psi}\cdot\left\{\underbrace{d\ln(e) - d\ln(P^*)}_{\text{real spending}}\right\}, \tag{21}$$

where we have denoted by $\widetilde{\Psi}$ the grouping of parameters that capture the corresponding elasticity:

$$\widetilde{\Psi} \equiv \left(\frac{1+\nu}{\mathcal{E}(Q_{ss})} + 1 - \rho\right)^{-1} = \left(\frac{1+\nu}{1-\frac{\Gamma_{ss}}{\rho}} + 1 - \rho\right)^{-1}.$$

The range of parameter values that we can use have to satisfy $1 < \rho < 2+\nu$ (so GHH preferences are increasing in its argument) and $\Gamma_{ss} \geq 1$ (non-negative steady state markups). These two properties by themselves imply that $0 < \widetilde{\Psi} < 1$ (note that $\widetilde{\Psi}^{-1} \geq \frac{1+\nu}{1-\frac{1}{\rho}} + 1 - \rho > \frac{1+\nu}{1-\frac{1}{2+\nu}} + 1 - (2+\nu) = 1$). In words, an increase in real spending translates always into a less than proportional increase in TFP.

Static Phillips Curve and its slope $\widetilde{\kappa}$ In the standard model the slope of the Phillips curve is the relation between the change in prices and the change in real wages. Here we can also obtain such relation, but the final effect in prices is also affected (and in exactly the same amount) by the endogenous changes in TFP and in the desired markup. To see this, differentiate both sides of the first order condition of firms (19) at the steady state to obtain

$$\frac{1}{(\rho-1)[1-\mathcal{E}(Q_{ss})]}\ d\ln\left[\frac{W}{P^*}\frac{1-\mathcal{E}(Q^*)}{\psi^f(Q^*)}\right] - \chi_{pp}(1)\ d\ln(P^*) = 0.$$

We rewrite this equation to separate the roles of changes in the real wage, productivity and the desired markup (equal to $\rho(1-\mathcal{E})-1$)

$$d\ln(P^*) = \widetilde{\kappa}\cdot\left\{\underbrace{d\ln(W) - d\ln(P^*)}_{\text{real wage}} - \underbrace{d\ln[\psi^f(Q^*)]}_{\text{TFP}} + \underbrace{d\ln[1-\mathcal{E}(Q^*)]}_{\text{desired markup}}\right\}.$$

where the grouped parameter $\widetilde{\kappa}$ satisfies

$$\widetilde{\kappa} \equiv \frac{1}{(\rho-1)(1-\mathcal{E}(Q_{ss}))\chi_{pp}(1)} = \frac{\frac{\rho}{\rho-1}}{\Gamma_{ss}\cdot\chi_{pp}(1)},$$

in which $\frac{\rho}{\rho-1}$ is the elasticity of substitution between varieties and $\chi_{pp}(1)$ is the curvature of the price adjustment cost. Since $\chi_{pp}(1)$ can take any positive value, so can $\widetilde{\kappa}$. Note that $\widetilde{\kappa}$ depends on the search frictions only through the steady-state gross markup Γ_{ss}.

Notice how the response of inflation is shaped by the response of the real wage net of the endogenous response of TFP and that the desired markup. In fact, search frictions could move inflation in either direction, depending on how much TFP and the desired markup move.

Endogenous desired markup and its elasticity $\widetilde{\gamma}$ To merge the contribution of TFP and the desired markup on inflation into a net effect of directed search it is useful to define one more grouped parameter, the elasticity of the desired markup with respect to endogenous TFP:

$$\widetilde{\gamma} \equiv \left.\frac{d\ln(1-\mathcal{E}(q))}{d\ln(\psi^f(q))}\right|_{q=Q_{ss}}.$$

Note that $\left[1+\frac{d\ln(1-\mathcal{E}(q))}{d\ln(\psi^f(q))}\right]^{-1}$ is the elasticity of substitution between the measure of production locations J and total shopping effort D in the constant returns to scale matching function $\psi(J,D)$ (proof in Appendix A.4). When $\widetilde{\gamma}$ is constant, the matching function has a constant elasticity of substitution (CES), and we obtain $\psi^f(q) = B(1-\varphi+\varphi q^{-\widetilde{\gamma}})^{-\frac{1}{\widetilde{\gamma}}}$, for some $B>0$, and some $\varphi \in (0,1)$. Moreover, for a Cobb-Douglas matching function $\widetilde{\gamma}=0$.

In principle, $\widetilde{\gamma}$ can take any value greater than or equal to -1, given a non-negative elasticity of substitution. In our environment, we only look for $\widetilde{\gamma}\geq 0$ such that for any $\widetilde{\kappa}\in(0,+\infty)$ and $\widetilde{\Psi}\in(0,1)$ TFP is strictly increasing in nominal spending e (proof in Appendix A.5).

Using this grouped parameter $\widetilde{\gamma}$, the relation between inflation and real wages (what we refer to as the Phillips Curve) has a more compact form that only depends on the real wage and the induced movement in TFP,

$$d\ln(P^*) = \widetilde{\kappa}\cdot\left\{\underbrace{d\ln(W)-d\ln(P^*)}_{\text{real wage}} - \underbrace{(1-\widetilde{\gamma})d\ln[\psi^f(Q^*)]}_{\text{directed search}}\right\}. \tag{22}$$

Necessary and Sufficient Conditions for Procyclical Markups, Inflation, and Real Wages The three grouped parameters $\{\widetilde{\Psi}, \widetilde{\kappa}, \widetilde{\gamma}\}$ in the log-linearized versions of the equilibrium conditions Equations (19) and (20) yield Equations (21) and (22) that allow us to solve for the partial derivatives of log prices and log TFP with respect to log nominal expenditures and wages, $\ln(e)$ and $\ln(W)$, which describes how equilibrium changes in response to changes in the nominal spending and nominal wage separately.

To discuss cyclicality in a straightforward way, we will treat nominal spending e still as an exogenous variable, but the nominal wage W as a function of e with certain elasticity $\frac{d\ln(W(e))}{d\ln(e)} > 0$. This allows us to use total derivatives with respect to $\ln(e)$ to describe the necessary and sufficient conditions to obtain procyclical markups, inflation, and real wages around the steady state. Equations (23) to (25) display the total responses of our variables of interest to a change in log nominal expenditures taken into account the effect of the latter on nominal wages:

$$\text{Markup:} \qquad \frac{d\ln\left(\frac{P^*(e,W(e),p^-)\,\psi^f(Q^*(e,W(e),p^-))}{W(e)}\right)}{d\ln(e)} = \frac{(1+\widetilde{\kappa}\widetilde{\gamma})\widetilde{\Psi} - (1+\widetilde{\kappa}\widetilde{\gamma}\widetilde{\Psi})\cdot\frac{d\ln(W(e))}{d\ln(e)}}{1+\widetilde{\kappa}[1+(\widetilde{\gamma}-1)\widetilde{\Psi}]} > 0, \qquad (23)$$

$$\text{Inflation:} \qquad \frac{d\ln(P^*(e,W(e),p^-))}{d\ln(e)} = \widetilde{\kappa}\cdot\frac{(\widetilde{\gamma}-1)\widetilde{\Psi} + \frac{d\ln(W(e))}{d\ln(e)}}{1+\widetilde{\kappa}[1+(\widetilde{\gamma}-1)\widetilde{\Psi}]} > 0, \qquad (24)$$

$$\text{Real Wage:} \qquad \frac{d\ln\left(\frac{W(e)}{P^*(e,W(e),p^-)}\right)}{d\ln(e)} = \frac{-\widetilde{\kappa}(\widetilde{\gamma}-1)\widetilde{\Psi} + [1+\widetilde{\kappa}(\widetilde{\gamma}-1)\widetilde{\Psi}]\cdot\frac{d\ln(W(e))}{d\ln(e)}}{1+\widetilde{\kappa}[1+(\widetilde{\gamma}-1)\widetilde{\Psi}]} > 0. \qquad (25)$$

Recall that we have established that we are looking at environments where $\widetilde{\kappa} > 0$, $0 < \widetilde{\Psi} < 1$, and $\widetilde{\gamma} \geq 0$ which in turn implies that $1+\widetilde{\kappa}[1+(\widetilde{\gamma}-1)\widetilde{\Psi}] > 0$ and $(1+\widetilde{\kappa}\widetilde{\gamma}\widetilde{\Psi}) > (1+\widetilde{\kappa}\widetilde{\gamma})\widetilde{\Psi} > 0$.

Then for markups to be procyclical, a necessary condition required for (23) to hold is that the percentage increase in nominal wages is smaller than that of nominal expenditures, $\frac{d\ln(W(e))}{d\ln(e)} < 1$. As we will see in Section 2.3 this is a reasonable property to have. In addition, a tighter lower bound for $\widetilde{\Psi}$ is needed. Isolating $\widetilde{\Psi}$ in Condition (23) we obtain

$$\widetilde{\Psi} = \left(\frac{1+\nu}{1-\frac{\Gamma_{ss}}{\rho}} + 1 - \rho\right)^{-1} > \frac{\frac{d\ln(W(e))}{d\ln(e)}}{1+\widetilde{\kappa}\cdot\widetilde{\gamma}\cdot\left(1-\frac{d\ln(W(e))}{d\ln(e)}\right)}.$$

The love for varieties parameter ρ must be sufficiently large relative to the search disutility parameter ν. Because $\widetilde{\kappa} > 0$ and $\widetilde{\gamma} > 0$, higher $\widetilde{\kappa}$ (e.g. less price rigidity) and higher $\widetilde{\gamma}$ (lower elasticity of substitution in the matching function) both relax the constraint for $\widetilde{\Psi}$. This constraint becomes tightest ($\widetilde{\Psi} > \frac{d\ln(W(e))}{d\ln(e)}$) when either $\widetilde{\kappa} \to 0$ (fully rigid prices) or $\widetilde{\gamma} = 0$ (Cobb-Douglas matching function), and disappears ($\widetilde{\Psi} > 0$) when $\widetilde{\kappa} \cdot \widetilde{\gamma} \to +\infty$ (either fully flexible prices or a Leontief matching function).

For procyclical inflation and procyclical real wage, we discuss conditions (24) and (25) jointly. When $\widetilde{\gamma} = 1$ (an elasticity of substitution in the matching function of 0.5), the effect of directed search in Equation (22) is exactly zero. Conditions (24) and (25) always hold regardless of $\{\widetilde{\kappa}, \widetilde{\Psi}\}$ as if there is no search like in Section 2.1.

When $\widetilde{\gamma} < 1$ (elasticity of substitution $> 1/2$ in matching function), Condition (25) (procyclical real wages) always holds, but Condition (24) (procyclical inflation) imposes an upper bound

$$\widetilde{\Psi} < \frac{\frac{d \ln(W(e))}{d \ln(e)}}{1 - \widetilde{\gamma}}.$$

This upper bound is tighter when $\widetilde{\gamma}$ gets smaller. It becomes tightest when $\widetilde{\gamma} = 0$ (Cobb-Douglas). Then, $\widetilde{\Psi} < \frac{d \ln(W(e))}{d \ln(e)}$ becomes the condition for procyclical inflation but $\widetilde{\Psi} > \frac{d \ln(W(e))}{d \ln(e)}$ for procyclical markups, which are incompatible.

When $\widetilde{\gamma} > 1$ (elasticity of substitution $< 1/2$ in matching function), Condition (24) (procyclical inflation) always holds, but Condition (25) (procyclical real wages) imposes an upper bound

$$\widetilde{\Psi} < \frac{\frac{d \ln(W(e))}{d \ln(e)}}{\widetilde{\kappa} \cdot (\widetilde{\gamma} - 1) \cdot \left(1 - \frac{d \ln(W(e))}{d \ln(e)}\right)}.$$

Higher $\widetilde{\kappa}$ (e.g. less price rigidity) and higher $\widetilde{\gamma}$ (lower elasticity of substitution in matching function) both make this constraint tighter. It becomes tightest when $\widetilde{\kappa} \cdot (\widetilde{\gamma} - 1) \to +\infty$ (fully flexible prices or Leontief matching function), which yields an impossible condition $\widetilde{\Psi} < 0$.

We can combine all three conditions in one constraint

$$\underbrace{\frac{\frac{d \ln(W(e))}{d \ln(e)}}{1 + \widetilde{\kappa}\widetilde{\gamma}\left(1 - \frac{d \ln(W(e))}{d \ln(e)}\right)}}_{\text{procyclical markups}} < \widetilde{\Psi} < \min \left\{ \underbrace{\frac{2 \cdot \frac{d \ln(W(e))}{d \ln(e)}}{1 - \widetilde{\gamma} + |1 - \widetilde{\gamma}|}, \frac{2 \cdot \frac{d \ln(W(e))}{d \ln(e)}}{\widetilde{\kappa}(\widetilde{\gamma} - 1 + |\widetilde{\gamma} - 1|) \cdot \left(1 - \frac{d \ln(W(e))}{d \ln(e)}\right)}}_{\text{procyclical inflation and procyclical real wages}} \right\}.$$

We can verify that the set of $\widetilde{\Psi} \in (0, 1)$ that satisfies this constraint is not empty as long as $\widetilde{\gamma} > 0$, i.e., desired markup is procyclical. The upper bound constraint is easier to satisfy when $\widetilde{\gamma}$ gets closer to 1 and when $\widetilde{\kappa}$ is smaller (e.g. more price rigidity) given $\widetilde{\gamma} > 1$, while the lower bound constraint is easier to satisfy when $\widetilde{\gamma}$ gets larger (more procyclical desired markup) and when $\widetilde{\kappa}$ gets larger (e.g. less price rigidity) conditional on $\widetilde{\gamma} \geq 0$.

To summarize, all three variables can be procyclical, but do not need to be. Estimation of the model in Section 4 will tell us what the data calls for.

2.3. Endogenous Nominal Spending and Nominal Wages

Obviously, both nominal expenditures and nominal wages are endogenous and we now provide a brief discussion of how a monetary expansion could translate into increases in nominal expenditures and in nominal wages in a way that the increase in nominal wages is only a fraction (and a constant fraction at that) of the increase in nominal expenditures.

Consider a version of the static economy described where in addition to having a price P^{-} inherited from the past, there is also a previous nominal wage W^{-}. In this economy a monetary injection will immediately increase nominal expenditures, which gives $d\ln(E)$ as the result of policy. One way of thinking about this is to have a monetary authority aiming for a certain level of nominal GDP. What matters at this stage, however, is not the origin of the increase in nominal expenditures but that when it happens it triggers a partial response of nominal wages. In Section 3 we use a noisy Taylor rule, the standard mechanism for monetary policy in New Keynesian models.

The details of the labor market are as follows. There is a set of labor varieties with elasticity of substitution, $\frac{\rho_w}{\rho_w - 1}$ each of which is represented by a union that posts nominal wages in a Calvo fashion this is, only a fraction of wages can actually change. Each household has a continuum of members, each one of them providing a different labor variety. Households have some disutility from the work of its members but they provide as much labor as told by the unions.[9] This makes their nominal income e predetermined and equal to aggregate income or expenditures E, inducing a problem identical to that in Section 2.2 (note that here we are starting to use the convention of capital letters for aggregate variables and small ones for individual variables, except for wages and rental prices in a dynamic model).

We use the notion of an aggregate state which here includes nominal expenditures, the inherited wages and the inherited prices $S = \{E, W^{-}, P^{-}\}$. Given S, and a yet to be determined aggregate wage index, $W(S)$, firms choose aggregate labor $\widetilde{\ell}(S, W(S), p^{-})$ and give back to the households aggregate profits $\widetilde{\Pi}^{f}(S, W(S), p^{-})$ as in Section 2.2. We also have the equilibrium product markets objects $\{P^{*}(S, W(S), p^{-}), Q^{*}(S, W(S), p^{-})\}$ exactly as we did in Section 2.2. For the purpose of determining the wage, firms also choose quantities of each labor variety j with wage W_j given $W(S)$,

[9] We are ignoring the possibility that households may not want to work at those wages, an issue raised by Huo and Ríos-Rull (2020b).

which can be shown to be $\ell_j^d(W_j, S, W(S), p^-) = \left(\frac{W_j}{W(S)}\right)^{-\frac{\rho_w}{\rho_w - 1}} \widetilde{\ell}(S, W(S), p^-)$.

Unions For each labor variety $j \in [0, 1]$, there is a labor union that sets the nominal wage W_j on behalf of all households. Union j requires household i to supply ℓ_{ij} units of labor and the households obey, yielding aggregate labor of variety j supplied $\ell_j = \int_0^1 \ell_{ij}\, di$. Unions that did not get to change wages (a fraction θ_w) keep the predetermined nominal wage $W_j = W^-$, while the rest of the unions set a new nominal wage W_j to maximize the utility of their members taking into account the labor disutility, that we assume linear. This implies maximizing marginal utility of the nominal labor income provided by variety j net of the disutility cost using their monopolistic power. Their implied first order condition is

$$0 = -\frac{1}{\rho_w - 1}\ell_j \cdot v_e + \frac{\rho_w}{\rho_w - 1}\frac{\ell_j}{W_j} \qquad \Longrightarrow \qquad W_j = \frac{\rho_w}{v_e}.$$

To simplify, we pose a logarithmic utility function that keeps the marginal indirect utility v_e quite simple. Then, we have $v_e(E, \Phi) = \frac{1+\nu}{2+\nu-\rho} \cdot \frac{1}{E}$. That is to say, the optimal adjusted wage $W^{\#}(S)$ of all labor varieties that can change wages should satisfy $W^{\#}(S) = \rho_w \cdot \frac{2+\nu-\rho}{1+\nu} \cdot E$, which implies that the wage aggregator is $W(S) = \left[\theta_w (W^-)^{-\frac{1}{\rho_w - 1}} + (1-\theta_w) W^{\#}(S)^{-\frac{1}{\rho_w - 1}}\right]^{-(\rho_w - 1)}$. Moreover, loglinearization yields

$$\frac{\partial \ln(W(S))}{\partial \ln(E)} = 1 - \theta_w.$$

In words, the response of log wages to an increase in log nominal GDP is equal to the fraction of firms that can adjust wages, which gives large possibilities for inflation, real wages and markups to respond positively to monetary impulses.

An issue that we have to deal with before we turn to the dynamic version is whether the structure that we have built based on love for varieties in consumption with search frictions extends to a dynamic environment with investment. Indeed, it does as shown in Appendix A.6 that displays an environment where households purchase the same varieties as for consumption and aggregate them with the same elasticity of substitution to generate an investment good. We show how in this environment households face the same type of problem as the one in Section 2.2 with a slightly different utility function. Obviously, the presence of habits just makes the problem identical provided we adjust the form of the objective function U.

3. A Dynamic Model with Search Frictions Suitable for Estimation

We now develop a dynamic environment on top of the one described in Section 2.3. It has all the bells and whistles commonly used in medium-scale New Keynesian models for the purpose of estimation, as described for instance in the "Calvo wage" version of Christiano et al. (2016) for the purposes of estimation. Of particular importance is endogenous variable capital utilization, a feature that could be confused with our endogenous productivity via a search friction. Endogenous variable capacity utilization means that the intensity with which factors of production are used is a choice resulting in mismeasurement of inputs, which allows for TFP to seemingly move. This is different from our product markets frictions. In endogenous variable capital utilization models firms have to pay for the increased capital utilization while with frictions in product markets firms get for free the increased TFP because it is due to the increased search effort of buyers.

The features that we pose are (internal) consumption habits, convex investment adjustment costs, variable capital utilization, nominal wages set à la Calvo, fixed costs of production, costly working capital loans, and a noisy Taylor Rule with interest rate smoothing. We have two minor differences from the canonical version in Christiano et al. (2016). We use Rotemberg pricing instead of Calvo pricing for the goods markets as it makes for an easier implementation of the search friction and we abstract from fiscal variables.

To pose all these relative standard features on top of the search frictions, we find it easier to use recursive language. We also abstract from the stochastic trend in the layout of this section as if all technology processes are stationary. We deal with non-stationarity at the estimation stage.

3.1. Production

There are a few items to deal with in the dynamic version of the model with respect to production relative to the version in Section 2. Issues related to variable capital utilization and adjustment costs are placed in the household problem where we pose the management of capital. Infinitely lived firms just hire labor and capital services each period and produce goods via a CRS Cobb-Douglas production function subject to fixed costs, shocks to productivity and the need to finance working capital. Their dynamic concerns are only about the costly price adjustment. Firms choose prices given a demand that has three components, consumption from households, investment also from households, both of which subject to search frictions, and purchases of intermediate goods from other firms not subject to search frictions in order to pay for price changing adjustment costs.

With this specification of the details, we can write the problem of the firm recursively with the arguments being the aggregate state S, yet to be determined, and the inherited price p^-. Because of the absence of a stock market we can write the dynamic programming problem of the firm excluding fixed costs even though we have to include them when calculating dividends. We skip the steps of deriving how demand and tightness respond to price for a firm, and have

$$\Omega(S, p^-) = \max_p \left\{\psi^f\left[\widetilde{q}(S,p)\right]p - MC(S)\right\}\ \widetilde{y}(S,p) + [p - MC(S)]\ \widetilde{\chi}(S,p) \\ - \chi(S,p,p^-) + \mathbb{E}\left\{\mathcal{M}(S,S')\ \Omega(S',p)\right\}. \quad (26)$$

Here $\widetilde{q}(S,p)$ is the equilibrium tightness associated to price p if the firm were to choose it, $MC(S)$ is the equilibrium marginal cost of inputs that, because of constant returns to scale, is independent of the quantity produced. The household produces for consumption and investment $\widetilde{y}(S,p)$ and for other firms price changing costs, $\widetilde{\chi}(S,p)$ quantities that depend on its choice of price. The firm also faces costs of changing prices $\chi(S,p,p^-)$. The term $\mathcal{M}(S,S')$ is the stochastic discount factor in nominal terms that depends on households' relative marginal utility of one dollar across time.

To get more analytical insights we specify the production technology as being a Cobb-Douglas function with fixed costs ϑ:

$$\widetilde{y} + \widetilde{\chi} = F(k^s, n) = A\ (k^s)^\alpha (z^n\ n)^{1-\alpha} - \vartheta,$$

in which A is a normalization constant, k^s are rented capital services, n is the labor composite, and z^n is a neutral technology level. Marginal cost of inputs is $MC(S) = \frac{1}{A}\left(\frac{R^k(S)}{\alpha}\right)^\alpha \left(\frac{R^B(S)W(S)/z^n(S)}{1-\alpha}\right)^{1-\alpha}$ where $R^B(S)$ is the nominal gross interest associated to having to prepay wages $W(S)$ and $R^k(S)$ is the rental price of capital services. The factor composition of production satisfies $\frac{k^s}{n} = \frac{\alpha}{1-\alpha}\frac{R^B(S)W(S)/z^n(S)}{R^k(S)}$. To compute the dividend in each period, those fixed costs have to be taken into account which amount $\vartheta \cdot MC(S)$. The output lost due to the fixed cost ϑ amounts to $\psi^f[\widetilde{q}(S,p)] \cdot \vartheta$. We refer to ψ^f as endogenous TFP because the quantities sold are $\psi^f\ F(k^s, n)$.

The first order condition of Equation (26) after defining $\omega(S,p,P^-) \equiv \chi(S,p,P^-) - [p - MC(S)]\ \widetilde{\chi}(S,p)$ to make the algebra more compact and taking into account the elasticity of TFP with respect to tightness becomes (eluding the arguments of the functions)

$$\frac{\omega_p}{\widetilde{y}} = \frac{1}{\rho - 1}\cdot\left(\rho\cdot\frac{MC}{p} - \frac{\psi^f}{1-\mathcal{E}}\right) + \mathbb{E}\left[\mathcal{M}'\cdot\left(-\frac{\omega'_{p^-}}{\widetilde{y}}\right)\middle| S\right].$$

which is the counterpart of Equation (19) in Section 2.2.2 when equilibrium conditions are imposed.

3.2. Households

Households choose how much to spend in consumption and investment (the details of the search are as above given total expenditures), plus the amount of capital services to extract from its stock of capital (where increasing capital utilization accelerates depreciation) and how many bonds to buy. The rates of return of these three forms of investment are not yet realized at the time when choices are made.

As stated, we pose internal habits and a constant Frisch elasticity of labor in preferences. We also assume a GHH structure between consumption and search effort but an additively separable one with labor supply. Formally, $U\left(c^A-\zeta\frac{(d^A)^{1+\nu}}{1+\nu}-\varsigma h\right)-\eta\frac{\ell_t^{1+\xi}}{1+\xi}$, where $h=c^{A-}-\zeta\frac{(d^{A-})^{1+\nu}}{1+\nu}$ is the stock of habits. Parameter ς captures the degree of habit persistence, ξ is the inverse of the Frisch elasticity of labor, ζ and η capture the slopes of the shopping and working disutilities.

Households purchase goods of various varieties and aggregate them into either a consumption good or an investment good with the same elasticity of substitution $\frac{\rho}{\rho-1}$. They add the investment good to their stock of capital subject to adjustment costs that depend on the investment of the previous period. The details of the shopping process are as before so we omit them. Because it is the union that chooses labor, we can eliminate the labor choice from the households' problem.

The individual state of a household is given by its habits, h, its stock of capital, k, its previous period investment net of the maintenance done because of non unitary capital utilization, x^{k-} and its stock of bonds in real terms, b. Its problem is

$$v(S,h,x^{k-},k,b)=\max_{\substack{c^A,\,d^A,\,b',\,k'\\ u,\,x^A,\,x^k}} U\left(c^A-\zeta\frac{(d^A)^{1+\nu}}{1+\nu}-\varsigma h\right)+\beta\,\mathbb{E}\left\{v(S',c^A-\zeta d^A,x^k,k',b')\right\} \quad (27)$$

$$\text{s.t.} \qquad x^k=x^A-\widetilde{\delta}(u)\;k, \quad (28)$$

$$k'=(1-\delta)k+\left[1-\mathcal{A}\left(x^{k-},x^k\right)\right]x^k, \quad (29)$$

$$[d^A\cdot\psi^h(q(S))]^{1-\rho}\left(c^A+\frac{x^A}{z^x}\right)+b'=w(S)L(S)+r^k(S)\;u\;k+\frac{R^-(S)}{\Pi(S)}\;b+T(S). \quad (30)$$

Here z^x is an investment-specific technology shock that determines how many investment units can be obtained from one composite unit of the good used for consumption.

Households' objective (27) is standard, and the constraints describe the three means to save into the future. Equation (28) shows that increasing capital utilization requires additional maintenance expenditures given by $\widetilde{\delta}(u)$. This particular form of maintenance does not affect the investment adjustment cost.[10] Function $\mathcal{A}$ poses the adjustment costs to changing the level of investment, so $\mathcal{A}(x,x) = 0$, and is convex in the investment growth. We write it as a function of two variables because it is easier to write its derivatives, $\mathcal{A}_1$ and $\mathcal{A}_2$, separately. Equation (29) shows how investment increases capital tomorrow taking into account adjustment costs. Finally, (30) is the budget constraint in real terms that shows how the purchase of bonds yields resources the following period. The first term of the budget constraint describes expenditures in consumption and investment (measured in consumption units). It uses the price aggregate defined in (12) and the fact that the budget constraint is defined in real terms so the price P is normalized to 1 to substitute prices for a function of aggregate search effort and market tightness. Funds are also used to purchase bonds measured in real terms. The sources of income include labor income, capital income at real rental price $r^k(S)$ with the household being able to extract more services from its stock by pumping up capital utilization, bonds that pay the promised interest rate plus the innovation of the interest rate policy, the government lump sum transfers that close the gap between the realized interest rate and the one promised by firms when borrowing, and the firms' transfer of profits.

3.2.1. Households First Order Conditions

To characterize the solution of this problem, it helps to define marginal utility of income or expenditures as $\Upsilon(S,s) = \left[d^A(S,s)\cdot\psi^h(q(S))\right]^{\rho-1}\left(U_{c^A} - \beta\varsigma\,\mathbb{E}\{U'_{c^A}\}\right)$, where we use the compact notation $s = \{h, x^{k-}, k, b\}$ to describe the individual state vector. Notice how marginal utility of expenditures involves the disutility generated by search[11] in addition to the more standard habit formation term.

Search versus Expenditures The simple static condition implied by the GHH structure between consumption and search effort is

$$d^A(S,s) = \frac{\rho-1}{\zeta}\left(c^A(S,s) + x^A(S,s)\right). \tag{31}$$

Optimality Condition for Bonds This is the standard dynamic Euler equation and it tells us that marginal utility of spending today equals expected discounted marginal utility of spending tomorrow times the expected real return of bonds:

$$\Upsilon(S,s) = \beta\,\mathbb{E}\left\{\frac{R(S,\epsilon^R)}{\Pi(S')}\,\Upsilon(S',s')\right\}. \tag{32}$$

[10] In Section 5 we consider two alternative types of specifications of capital utilization that differ in how to deal with NIPA accounting and investment adjustment cost. Here we follow Christiano et al. (2016).

[11] With a GHH utility function, the search disutility affects marginal utility only through the arguments of $U_{c^A}(\cdot)$.

Optimality Condition for Capital Utilization Increasing the intensity of capital utilization yields r^k units of the good per unit of capital, yet it costs $\frac{[d^A \cdot \psi^h]^{1-\rho}}{z^x}\ \widetilde{\delta}_u$ so

$$r^k(S) = \frac{[d^A(S,s) \cdot \psi^h(q(S))]^{1-\rho}}{z^x}\ \widetilde{\delta}_u(u(S,s)), \tag{33}$$

To simplify the algebra, we define $p^k(S,s) \equiv \frac{[d^A(S,s) \cdot \psi^h(q(S))]^{1-\rho}}{z^x}$.

Optimality Condition for investment The type of adjustment costs used here (on the change of investment) makes the investment first order condition a little bit contrived. It is best to proceed (as in the literature) by using Tobin's q or marginal cost of installing capital. Formally, it is the ratio of the Lagrange multipliers of Equation (29) and Equation (30). In this way we make Tobin's q, $q^k(S,s)$ a choice variable with Euler equation

$$q^k(S,s) = \beta\ \mathbb{E}\left\{\frac{\Upsilon(S',s')}{\Upsilon(S,s)}\left[(1-\delta)\ q^k(S',s) + r^k(S')\ u'(S',s) - p^k(S',s')\ \widetilde{\delta}(u(S',s'))\right]\right\}, \tag{34}$$

and use it to pose the first order condition for investment which becomes

$$\begin{aligned} p^k(S,s) = q^k(S,s) \cdot [1 - \mathcal{A}(x(S,s),x^-) - x(S,s)\ \mathcal{A}_1(x(S,s),x^-)] \\ - \beta\ \mathbb{E}\left\{\frac{\Upsilon(S',s')}{\Upsilon(S,s)} \cdot q^k(S',s) \cdot x(S',s')\ \mathcal{A}_2(x(S',s'),x(S,s))\right\}. \end{aligned} \tag{35}$$

Note that in the absence of investment adjustment cost, Tobin's q is just $p^k(S,s)$.

3.3. Wage Setting by Unions with a Calvo Friction

As in the static setting there are unions in each labor variety that set the wage monopolistically. Because the wage can only be reset with probability $1-\theta_w$ the union has to take the evolution of the economy into account. We write the value for a union of having real wage w when the aggregate state (that includes aggregate inherited aggregate nominal wage W^-) is S because the union is small relative to the labor force. For this let's denote by $\Upsilon(S)$ marginal utility of the numeraire (above we used $\Upsilon(S,s)$ to denote marginal utility of a household while here we are using marginal utility of a *representative* household where s equals the relevant parts of S). Because we keep track of real wages, the actual wage needs to be deflated by gross inflation $\Pi(S')$.

The value for the union of labor variety j of having today real wage w_j in state S, is

$$\mathcal{U}(S, w_j) = \Upsilon(S) \cdot w_j \; \ell_j^d(S, w_j) - \eta \cdot \frac{\ell_j^d(S, w_j)^{1+\xi}}{1+\xi} + \theta_w \cdot \beta \, \mathbb{E} \, \{\mathcal{U}(S', w_j/\Pi(S'))\} ,$$

$$\text{s.t.} \quad \ell_j^d(S, w_j) = \left(\frac{w_j}{w(S)}\right)^{-\frac{\rho_w}{\rho_w - 1}} L(S).$$

Note that the union values the household utility the way it can affect it, via the wage income it provides taking into account the disutility of the necessary work. When given the opportunity to set the wage, the union maximizes $\mathcal{U}(S, W_j)$ with respect to the wage. It is easier to solve this problem if we separate $\mathcal{U}$ into a marginal utility of real income component and a marginal disutility of working component:

$$H^1(S) = \Upsilon(S) \cdot L(S) \cdot w(S)^{\frac{\rho_w}{\rho_w - 1}} + \theta_w \cdot \beta \mathbb{E}\left[\Pi(S')^{\frac{1}{\rho_w - 1}} \cdot H^1(S')\right] = \Upsilon(S) \cdot \ell_j^d(S, w_j) \cdot w_j^{\frac{\rho_w}{\rho_w - 1}} + \theta_w \cdot \beta \mathbb{E}\left[\Pi(S')^{\frac{1}{\rho_w - 1}} \cdot H^1(S')\right]. \tag{36}$$

$$H^2(S) = \eta \cdot [L(S) \cdot w(S)^{\frac{\rho_w}{\rho_w - 1}}]^{1+\xi} + \theta_w \cdot \beta \, \mathbb{E}\left[\Pi(S')^{\frac{\rho_w}{\rho_w - 1}(1+\xi)} \cdot H^2(S')\right] = \eta \cdot [\ell_j^d(S, w_j) \cdot w_j^{\frac{\rho_w}{\rho_w - 1}}]^{1+\xi} + \theta_w \cdot \beta \, \mathbb{E}\left[\Pi(S')^{\frac{\rho_w}{\rho_w - 1}(1+\xi)} \cdot H^2(S')\right]. \tag{37}$$

So we obtain $\mathcal{U}(S, w_j) = w_j^{\frac{-1}{\rho_w - 1}} H^1(S) - w_j^{\frac{-\rho_w}{\rho_w - 1}(1+\xi)} H^2(S)$, that can be easily maximized and yields the value to which the newly updated wages are set

$$w^{\#}(S) = \left(\rho \cdot \frac{H^2(S)}{H^1(S)}\right)^{\frac{1}{1 + \frac{\rho_w}{\rho_w - 1}\xi}} . \tag{38}$$

The update of inherited real wage aggregate $w(S)$ depending on the newly set wages $w^{\#}(S)$ is given by a CES aggregator

$$w(S') = \left[\theta_w \cdot \left(\frac{w(S)}{\Pi(S')}\right)^{\frac{1}{1-\rho_w}} + (1 - \theta_w) \cdot w^{\#}(S')^{\frac{1}{1-\rho_w}}\right]^{1-\rho_w} . \tag{39}$$

These four equations fully characterize the wage setting block of our model.

3.4. Monetary Policy

Monetary policy follows a Taylor Rule that sets the nominal interest rate. As in Christiano et al. (2016) the shock in any period happens too late to affect that period's choices and it is implemented via an innovation in the interest rate paid to bond holders. Such innovation ϵ^R is paid to households the following period via the use of lump-sum taxes or subsidies. The interest rate that includes such innovation is what informs the new interest rate set by the central bank via a Taylor rule. Formally, we use $R(S, \epsilon^R)$ to denote the gross Federal Funds Rate that is the realized rate paid to bondholders and is given by

$$R(S, \epsilon^R) = \rho_R \cdot \ln\left(\frac{R^-(S)}{R_{ss}}\right) + (1-\rho_R) \cdot \left[\phi_\pi \cdot \ln\left(\frac{\Pi(S)}{\Pi_{ss}}\right) + \phi_y \cdot \ln\left(\frac{Y(S)}{Y_{ss}}\right)\right] + \epsilon^R, \tag{40}$$

where $R^-(S) = R\left(S^-, \epsilon^{R-}\right)$ is last period's Federal Funds rate, R_{ss} is the steady state gross Federal Funds rate, Π_{ss} is the steady state gross inflation and Y_{ss} is the steady-state GDP. The i.i.d. innovation is Gaussian, $\epsilon^R \sim \mathcal{N}(0, \sigma_R)$, with σ_R capturing the size of monetary policy shocks. The rate at which firms borrow from households is $R^B(S) = \mathbb{E}\{R(S, \epsilon^R)|S\}$. It is also the expected rate at which households lend.

The timing of the loan payments is such that firms retain the funds needed to pay the working capital loans including interest at rate $R^B(S)$ times loan positions when transferring profits to the households. They pay these funds at the beginning of the following period. Households receive $R(S, \epsilon^R)$ for their loans with the difference made up by lump-sum taxes or subsidies that in this model are part of monetary policy.

3.5. States

It is now the turn to specify the aggregate state vector S. It includes technology shocks z^n and z^x, inherited wages and prices, W^- and P^-, the realized interest rate of the previous period, R^-, this is, the interest rate that results from the previous period market including the innovation ϵ^{R-} (and hence we can write $R^-(S)$). Note that under this convention $R^-(S) = R^B(S^-) + \epsilon^{R-}$. The previous period innovation to monetary policy, ϵ^{R-}, does not need to be included as part of the state given its i.i.d nature. The aggregate state also includes the aggregate counterparts to the individual state vector, this is, the stock of habits, H, last period investment to determine adjustment costs X^{k-}, the stock of capital, K and bonds, B. Summarizing, $S = (z^n, z^x, W^-, P^-, R^-, H, X^{k-}, K, B)$.

3.6. Equilibrium

Equilibrium is standard. Agents maximize; the search friction is resolved as described in Section 2.2.3, capital services and bonds markets clear and the government balances its budget and follows the noisy Taylor rule.

3.7. National Income and Products Accounts

In this economy national accounting has a couple of differences with the standard measurements. Aggregate consumption as measured is not C^A, which is what matters for the agents, but $C \equiv \psi^f(q)\ \widetilde{c} = \psi^f(q)^{1-\rho}\ c^A$. The difference comes from the aggregation of varieties and the search. In the same fashion, investment becomes $I = \psi^f(q)^{1-\rho}\ x^k$. It is important to note that maintenance investment is NOT part of GDP as it is counted as an intermediate input.

To see the details it is best to consider the steady state, let's start with a notion of potential GDP that is what could be produced if all households used all varieties (recall that they are normalized to one)

$$\text{Potential Steady State GDP } = Y^P_{ss} \ = A\,(u_{ss}\ K_{ss})^\alpha\ \ L^{1-\alpha}_{ss} - \vartheta.$$

A few comments are in order. A is a units parameter. K_{ss} and L_{ss} are steady state observed quantities of inputs. We are normalizing the steady state utilization rate u_{ss} to 1, so there is no outside the books depreciation. Resources are used before getting any output (which we have referred to as fixed costs of production), with ϑ denoting the implied amount of potential output that is not generated. The costs of price adjustments are set to zero at the steady state inflation and this is why they do not show up above. It is useful to normalize steady state GDP, Y_{ss}, to 1 by means of a suitable choice of A.

The relation of GDP to potential GDP is given by search and is equal to potential GDP times the number of varieties that we have labeled as ψ^f_{ss}. We are now ready to define actual GDP not necessarily in steady state as

$$Y = \psi^f\ \left[A\,(u\ K)^\alpha\ (z^n L)^{1-\alpha} - \vartheta - \text{price adjustment terms}\right] - \frac{(\psi^f)^{1-\rho}}{z^x}\ \widetilde{\delta}(u)\ K.$$

Note the two new terms that appears relative to the steady state. The first one represents the resources used for price adjustments. The last term is the additional depreciation that comes from using variable capital utilization differing from the steady state value, where z^x is the relative price of investment and $\widetilde{\delta}(u)$ is a depreciation function with the property that $\widetilde{\delta}(1) = 0$.[12]

As is conventional in the literature we use the household relevant price index as the measure of inflation and not the indices actually used (CPI, GDP deflator), $\Pi \equiv \frac{P}{P^-}$. The same goes for wages.

[12] This specification of GDP implies accounting rules that allow firms to account for the additional depreciation due to utilization as intermediate goods. In Section 5 we explore the implications of more rigid rules that imply that expenditures used to maintain the stock of capital when utilization differs from its steady state value are counted as investment.

When taking the model to data, we still map them to the GDP deflator and hourly wage rate in the data. Finally, labor productivity is just GDP divided by total hours and labor share is the ratio of labor income (defined as hourly wages times hours in a narrow scope) to GDP.

4. Mapping the Model to Data

While we have developed theory that allows procyclical TFP and markups conditional on demand shocks, we still have to see how it stands to macroeconomic data. Moreover, we want to know how it compares with state of the art New Keynesian Models with a variety of features designed to fare well when estimated. This is specially the case of capital utilization where the quantity of inputs, specifically capital, is allowed to vary in an unobservable way to increase output in periods of high demand which may look, as our theory does, as increased productivity. It does so by having costs increase when utilization is increased (otherwise why not push it to the max). We proceed then to estimate a variety of models where we compare the performance of our search friction and that of endogenous capacity utilization. In fact, we look at economies that have both features or only one of them.

To this end, we adopt the Bayesian impulse response matching approach for estimation as in Christiano et al. (2010) Christiano et al. (2016), which is particularly convenient for isolating the impact of Federal Funds Rate shocks. All the models that we estimate are calibrated to target the same steady-state composition of GDP and estimated to target the same SVAR impulse responses of standard observable aggregate variables.

Our findings are sharp: search outperforms variable capital utilization, when modeling separately and the best performance of the model is when both mechanisms are present. We start with explaining our procedure of structural VAR regression in Section 4.1. Section 4.2 turns first to how to accommodate secular growth and then specify the steady state before we proceed with the estimation in Section 4.3.

4.1. Structural VAR

Our structural VAR regression follows the procedure of Christiano et al. (2016) but with a different emphasis. We remove the 5 labor market variables in their regression that are not our focus. We pose labor productivity and labor share as additional targets, even though they are redundant (see below), to increase the weight of data information towards the properties that interest us.

We first collect time series over the period 1951Q1-2008Q4 for the following 9 variables: Nominal GDP (BEA: A191RC), Nominal Consumption (BEA: DNDGRC + DDURRC + DSERRC), Nominal Investment (BEA: A006RC), Nonfarm Weekly Hours (BLS: PRS85006023), Capacity Utilization (Fed Board: G.17, CAPUTL.B00004.S), Relative Price of Investment (BEA: B006RG), Nonfarm Nominal

Hourly Wage (BLS: PRS85006103), GDP Deflator (BEA: A191RD), Federal Funds Rate (Fed Board: H.15 (TB3MS for early sample)). We then transform the data to achieve stationarity through linear combinations and transformation of the the original time series obtaining

$$Y^{VAR} = \begin{bmatrix} \Delta \ln(\text{relative price of investment}) \\ \Delta \ln(\text{real GDP/hours}) \\ \Delta \ln(\text{GDP deflator}) \\ \ln(\text{capacity utilization}) \\ \ln(\text{hours}) \\ \ln(\text{real GDP/hours}) - \ln(\text{real wage}) \\ \ln(\text{nominal C/nominal GDP}) \\ \ln(\text{nominal I/nominal GDP}) \\ \text{Federal Funds Rate} \end{bmatrix}.$$

Denote by $\widehat{Y}_t^{VAR}$ the period t values of stationary vector Y^{VAR} after removing quadratic trends. We then run a reduced-form VAR regression with two lags $\widehat{Y}_t^{VAR} = A_1 \widehat{Y}_{t-1}^{VAR} + A_2 \widehat{Y}_{t-2}^{VAR} + e_t^{VAR}$. As in Christiano et al. (2010, 2016), the three structural shocks are defined as linear combinations of all elements in e_t^{VAR}, among which the Federal Funds rate shock induces no contemporaneous movements in the first 8 elements of $\widehat{Y}_t^{VAR}$ (short-run restriction). The neutral technology shock and the investment-specific technology shock induce nonzero accumulation of the second element in $\widehat{Y}_t^{VAR}$ in the long run (long-run restriction), and the investment-specific technology shock induces nonzero accumulation of the first element in $\widehat{Y}_t^{VAR}$ in the long run (long-run restriction). For convenience, we denote $IRF_h^{VAR}(j)$ as the h period ahead impulse response of the *jth* element in $\widehat{Y}_t^{VAR}$. We then pose the following impulse responses as the target for the estimation.

Note that the bottom two targets (labor productivity and labor share) are redundant in the sense that the information embodied in those series is already used in the previous 9 series. Its use emphasizes the two particular linear combinations of the other targets that we are interested in. We look at labor productivity because our theory of frictional goods markets provides a rationale for its endogenous movements. The reason to add labor share is the fact that it is closely related to markups[13] and its behavior presents a serious challenge to New Keynesian and neoclassic business cycle models. In Section 5 we also look at the set of variables that are conventionally used as well as various other specifications.

[13] Loecker et al. (2020) and Nekarda and Ramey (2020) measure markups based on inverse factor shares which is consistent with variations in TFP. Measures based on the price difference between finished goods and processed goods as in Kaplan and Zoch (2020) do not provide a direct link between labor share and markups.

Table 1: Constructing Estimation Targets from Stationary Variables

Targets	Sources from IRF_h^{VAR}
Real GDP	$\sum_{\tau=0}^{h} IRF_\tau^{VAR}(2) + IRF_h^{VAR}(5)$
Real consumption	$\sum_{\tau=0}^{h} IRF_\tau^{VAR}(2) + IRF_h^{VAR}(5) + IRF_h^{VAR}(7)$
Real investment	$\sum_{\tau=0}^{h} IRF_\tau^{VAR}(2) + IRF_h^{VAR}(5) + IRF_h^{VAR}(8) - \sum_{\tau=0}^{h} IRF_\tau^{VAR}(1)$
Hours	$IRF^{VAR}(5)$
Capacity utilization	$IRF^{VAR}(4)$
Relative price of investment	$\sum_{\tau=0}^{h} IRF_\tau^{VAR}(1)$
Real wage	$\sum_{\tau=0}^{h} IRF_\tau^{VAR}(2) - IRF_h^{VAR}(6)$
Inflation	$IRF_h^{VAR}(3)$
Federal Funds Rate	$IRF_h^{VAR}(9)$
Labor productivity	$\sum_{\tau=0}^{h} IRF_\tau^{VAR}(2)$
Labor share	$-IRF_h^{VAR}(6)$

4.2. Calibration

4.2.1. Functional Forms

In Sections 3.3 and 3.4 we have already posed the functional form for the working disutility, for the aggregators of varieties, for the production function and for the Taylor Rule.

Preferences We use log over a GHH structure of consumption and search effort with an internal habit, and an additively separable working disutility with constant Frisch elasticity $1/\xi$:

$$U(c^A, d^A, h) = \ln\left(c^A - \zeta\frac{(d^A)^{1+\nu}}{1+\nu} - \varsigma h\right) - \eta\frac{\ell^{1+\xi}}{1+\xi}, \qquad \text{with } h' = c^A - \zeta\frac{(d^A)^{1+\nu}}{1+\nu},$$

with c^A being the aggregate over varieties and d^A is an aggregate of search effort in all available markets. They are defined in (6) and (7).

Variable Capital Utilization The following function tells us how capital utilization affects the depreciation of existing capital in addition to that coming from a constant rate δ.

$$\widetilde{\delta}(u) \equiv \sigma_a\sigma_b/2 \cdot (u-1)^2 + \sigma_b \cdot (u-1).$$

Note that $\widetilde{\delta}(1) = 0$, that $\sigma_b = \widetilde{\delta}_u(1)$ which captures the marginal cost of utilization in steady state and that $\sigma_a = 1 \cdot \widetilde{\delta}_{uu}(1)/\widetilde{\delta}_u(1)$ captures the elasticity of the marginal utilization cost in steady state with respect to capital utilization rate u. Note also that variable capital utilization is not capacity utilization. Consistent with the definition of *capacity* provided by the Board of Governors of the Federal Reserve System, "the highest level of output a plant can sustain within the confines of its resources", we define capacity utilization in the model as $util \equiv \frac{\psi^f \cdot (u^\alpha \cdot AK^\alpha L^{1-\alpha} - \vartheta)}{AK^\alpha L^{1-\alpha} - \vartheta}$. Under log-linearization, $d\ln(util) = d\ln(\psi^f) + (1 + \vartheta\psi^f_{ss})\alpha \cdot d\ln(u)$.

Capital Accumulation Investment goods are an aggregate of varieties purchased by the household and allocated for investment purposes:

$$x^A = \left(\int_\Phi d(p, q)\ \psi^h(q) x(p, q)^{\frac{1}{\rho}} dp\ dq \right)^\rho,$$

All varieties are used for both consumption and investment. The same search effort applies to both consumption and investment.

Matching Function We pose a CRS–CES matching function. It can be specified directly in terms of varieties and locations or simply in terms of market tightness

$$M(J, D) = M(q) = B \cdot [(1 - \varphi)J^{-\gamma} + \varphi D^{-\gamma}]^{-\frac{1}{\gamma}} = B \cdot [(1 - \varphi) + \varphi q^{-\gamma}]^{-\frac{1}{\gamma}},$$

for some parameters $\{B, \varphi, \gamma\}$.

Technological Progress While we have described the model as if it were stationary to avoid cumbersome notation, there is both secular technological progress and shocks. Following Christiano et al. (2016) we pose stochastic processes for a labor neutral technology level denoted z^n, and for the investment-specific technology level denoted z^x. Their gross growth rates $\mu^n \equiv z^n/z^{n-}$ and $\mu^n = z^x/z^{x-}$ are assumed stationary and are described by processes

$$\begin{aligned}
&\ln(\mu^n) - \ln(\mu^n_{ss}) = \sigma_n \cdot \epsilon^n, \\
&\ln(\mu^x) - \ln(\mu^x_{ss}) = \rho_x \cdot [\ln(\mu^{x-}) - \ln(\mu^x_{ss})] + \sigma_x \cdot \epsilon^x.
\end{aligned}$$

Both innovations ϵ^n and ϵ^x are i.i.d. normalized Gaussian.

Adjustment Costs to Investment Christiano et al. (2016) pose

$$\mathcal{A}\left(\frac{x^k}{x^{k-}}\right) \equiv 1/2 \cdot \left\{ \exp\left[\sqrt{\widetilde{\mathcal{A}}}\ \left(\frac{x^k}{x^{k-}} - \mu^k_{ss}\right)\right] + \exp\left[-\sqrt{\widetilde{\mathcal{A}}}\ \left(\frac{x^k}{x^{k-}} - \mu^k_{ss}\right)\right]\right\} - 1,$$

for some parameter $\widetilde{\mathcal{A}}$. Note that the first and second derivatives of this function are

$$\mathcal{A}_1\left(\frac{x^k}{x^{k-}}\right) = 1/2 \cdot \sqrt{\widetilde{\mathcal{A}}}\left\{\exp\left[\sqrt{\widetilde{\mathcal{A}}}\left(\frac{x^k}{x^{k-}} - \mu^k_{ss}\right)\right] - \exp\left[-\sqrt{\widetilde{\mathcal{A}}}\left(\frac{x^k}{x^{k-}} - \mu^k_{ss}\right)\right]\right\},$$
$$\mathcal{A}_2\left(\frac{x^k}{x^{k-}}\right) = 1/2 \cdot \widetilde{\mathcal{A}}\left\{\exp\left[\sqrt{\widetilde{\mathcal{A}}}\left(\frac{x^k}{x^{k-}} - \mu^k_{ss}\right)\right] + \exp\left[-\sqrt{\widetilde{\mathcal{A}}}\left(\frac{x^k}{x^{k-}} - \mu^k_{ss}\right)\right]\right\}.$$

In steady-state, $\frac{x^k}{x^{k-}} = (\mu^x_{ss})^{\frac{1}{1-\alpha}}\mu^y_{ss}$, so we have $\mathcal{A}_1\left((\mu^x_{ss})^{\frac{1}{1-\alpha}}\mu^y_{ss}\right) = 0$ and $\mathcal{A}_2\left((\mu^x_{ss})^{\frac{1}{1-\alpha}}\mu^y_{ss}\right) = \widetilde{\mathcal{A}}$. Because of log-linearization, $\widetilde{\mathcal{A}}$ is the only parameter to be estimated.

Adjustment Costs of Goods Prices The cost of changing the nominal price of a good is

$$\chi(S, p, p^-) = \frac{\kappa}{2} \cdot \left(\frac{p}{p^-} - \Pi_{ss}\right)^2 \cdot P(S) \cdot \psi^f(Q(S)) \cdot \widetilde{y}(S, P(S)).$$

where $\widetilde{y}(S, P(S))$ is the quantity sold for consumption and investment purposes (not as intermediate goods for price changing adjustment costs), and $P(S) \cdot \psi^f[Q(S)] \cdot \widetilde{y}[S, P(S)]$ is a nominal expenditure term as in Section 2. A one percent of quarterly inflation above its steady state value implies that the costs of price adjusting is $\kappa/20000$ times GDP.

Of the parameters listed above, ν (curvature of search effort in the utility function) and φ (curvature of search effort in the matching function) cannot be separately identified since what matters is the marginal disutility in terms of additional varieties found. We pose $\nu = 0$ and make φ embody the curvature.

4.2.2. Steady State Calibration Targets

There are a total of 28 parameters to be specified. We proceed by setting 13 targets that the model has to satisfy exactly and we estimate by Bayesian methods the rest. Specifically, for any 15 parameters set by the estimation routines we solve a system of 13 equations and 13 unknown parameters that we require the model to satisfy. We choose the 13 parameters to be those that are traditionally linked to the calibration targets themselves. Some of those parameters are uniquely determined by the calibration target (*e.g.* the discount rate β and the steady state interest rate), but many others depend not only on the calibration targets but also on the 15 estimated parameters (*e.g.* the fixed cost ϑ). Table 2 reports the values of the calibration targets along with the parameter values typically associated to them (although as we have said such association may not always be meaningful).

Table 2: List and Values of Quarterly Calibrated Parameters in Benchmark Economy

Parameter	Symbol	Value	Moment	Value
Depreciation Mean of Utilization	σ_b	0.037	St–St Utilization Rate	1.0
Units in Production Function	A	1.17	St–St Output	1.0
Weight of labor in utility	η	0.69	Average Labor	1.0
Weight of Search in utility	ζ	0.55	St–St Search	1.0
Mean Growth of Neutral Technology Factor	μ^n_{ss}	1.004	Per Capita Output Growth	1.7%
Mean Growth of Relative Price of Investment	μ^x_{ss}	1.003	Per Capita Capital Growth	2.9%
Discount Rate	β	0.997	Interest Rate	3.0%
St St Depreciation Rate	δ	0.027	Depreciation Share	17.3%
Fixed Costs of Production	ϑ	0.042	Investment Share	21.9%
Coefficient of Capital in Production Function	α	0.264	Labor Share	66.7%
Mean Efficiency of Matching Function	B	0.78	Occupancy Rate	78.0%
Elasticity of Substitution for Labor	ρ_w	1.20	Wage Markup	20.0%
Probability of not Changing the Wage	θ_w	0.75	Duration Wages (years)	1.0

Normalization of Units: 4 We choose to measure some variables in units that yield steady state values of one. These are output, labor, units of search and capital utilization rate.

Secular Growth: 2 Annual average growth rates of real GDP and real investment per capita are 1.7% and 2.9%. To implement the balanced growth path note that the growth of investment (or capital) is the sum of the two rates of technological growth: $2.9\% = \ln(\mu^y_{ss}) + \ln(\mu^x_{ss})$, while the growth rate of output depends on the rates of technological improvement via the production function, $1.7\% = \frac{\alpha}{1-\alpha}\ln(\mu^x_{ss}) + \ln(\mu^n_{ss})$.

Interest Rate and Output Shares: 4 We set the real interest rate to 3%. We target the share of output that goes to depreciation (the share of the difference between gross and net output), the investment share of output and the labor share of GDP. We target values of 17.3%, 21.9%, and 66.7%, respectively. We have adjusted the sample means of 1951-Q1 to 2008-Q4 of depreciation and investment by a factor of 1.25 to accommodate the fact that we abstract from public expenditures.[14]

Average Occupancy Rate (Search Related Target): 1 We target a steady-state matching probability or average capacity of 78% based on an average notion of occupancy.[15]

Labor Market Targets: 2 We choose a gross wage markup of 20% and average wage changes of one year (see Christiano et al. (2016) for a discussion).

[14] In this partition we have treated durable goods as consumption to stay consistent with NIPA.
[15] See Bai et al. (2019) for a discussion.

Average Markup While we have targeted either observables (shares of output for instance) or features that we consider to be relatively uncontroversial (job market frictions), there are other steady state properties that are unobservable and that are jointly determined by the parameters that we estimate and the steady state targets that we impose. The most important among this is the steady state markup, which is determined by how the steady state parameters adapt to the other estimates. We will pay a strong attention to this variable. In our benchmark estimation, its value is 14%.

4.3. Estimation

There are 15 more parameters to be specified and we obtain their values by estimation, matching the impulse responses of the various time series to the three shocks that we have posed. The crucial feature of these shocks is that conditional on the model they can be observed and therefore we can construct their impulse responses in the model and compare them to their counterpart in the data.[16]

We estimate 11 parameters directly and 4 indirectly, this is, by using other regressors and computing afterwards the implied value of these four parameters. The 11 directly estimated parameters are standard and not directly related to the search friction, so we choose the same prior distributions as Christiano et al. (2016). The other four parameters are the effort share and the elasticity of substitution, $\{\varphi, \gamma\}$, of the matching function, the love for variety, ρ and the price adjustment cost, κ. It turns out that these four parameters jointly determine the 4 equilibrium elasticities related to search frictions, so we choose to estimate these elasticities directly and then we recover these 4 deep parameters using the elasticities. We find it easier to think of priors over those elasticities than over deep parameters due to the lack of prior evidence for these 4 deep parameters. Moreover, three of the elasticities that we estimate directly are the grouped parameters discussed in Section 2, while one is related to the fixed cost of production but also affected by our search friction.

We estimate the elasticity of the matching probability of the firm locations or TFP with respect to aggregate real spending, $\widetilde{\Psi}$, which is perhaps the key item in this paper. It depends on both the effort share parameters and the love for variety, φ and ρ. The first is a "new" parameter and the second, while standard, cannot be separately identified in our model from the curvature of search in the utility function, and also cannot be directly chosen to target the steady-state markup due to the "directed" nature of search. Therefore, we choose to estimate $\widetilde{\Psi}$ directly instead. In the models with no search, $\widetilde{\Psi} = 0$. In the models with search, we pose priors for $\widetilde{\Psi}$ with a uniform distribution between 0.0 and 0.5 making them identical, uninformative, and never binding.

[16] See Christiano et al. (2016) for additional details as what we estimate is essentially the "Calvo wage" model in Christiano et al. (2016) with slight differences in the steady-state calibration and in the facts that we use Rotemberg pricing, have no government purchases and have added the impulse response of labor productivity and labor share.

We also estimate directly the elasticity of the desired gross markup with respect to endogenous TFP, $\widetilde{\gamma}$. It turns out that with a CES matching function, $\widetilde{\gamma}$ equals γ, with $1/(1+\gamma)$ being the elasticity of substitution. Therefore, in this special case, γ is essentially estimated directly. Like for $\widetilde{\Psi}$, we impose a uniform prior distribution, this time between 0.0 and 2.0 for all models in which the endogenous desired markup is active.

The third elasticity that we estimate, $\widetilde{\kappa}$, is that of the inflation gap with respect to the real marginal cost of production, holding the effects of search constant. Recall that this is also the slope of the New Keynesian Phillips Curve. This elasticity or grouped parameter depends on three of the deep parameters $\{\varphi, \rho, \kappa\}$. We impose a uniform prior distribution for $\widetilde{\kappa}$ such that its 95% credible set coincides with the corresponding one induced by the prior of the Calvo pricing parameter in Christiano et al. (2016).

The fourth grouped parameter that we estimate is the steady state gross markup, $\widetilde{\Gamma} = \rho(1-\varphi)$. We impose a uniform distribution that is dispersed enough but still ensures that the lower bound implies non-negative fixed cost of production and the upper bound is consistent with the highest implied fixed cost of production from the point estimates among all estimated models in Christiano et al. (2016). We think of this parameter as an elasticity because it determines the returns to scale of the production function via its role in determining the calibrated parameter ϑ.[17]

We estimate four models for this specification of NIPA in the model and for the 3 sets of impulse responses over 11 variables. The "Benchmark" or "u and Search" economy is the one that we have described where both search via a CES matching function and capital utilization coexist. We then estimate a model with only search frictions and no variable capacity utilization ("Search Alone"). We also estimate two models without search frictions. One with the prior distribution of the steady state gross markup that allows all point estimates in Christiano et al. (2016) as we described above ("Standard u Alone"). Because this specification without search yields an implied fixed cost of production of 39% of GDP that we deem to be too large, we also add a capacity utilization alone economy estimated with a tighter prior (uniform on [1.1,1.25]) ("Tighter Prior u Alone").[18]

The estimation results are summarized in Table 3 and the plots of the impulse responses to the three

[17] Recall that hours and GDP are normalized to 1 while the occupancy rate is set to be 78%, making GDP satisfy $1 = .78(AK^{\alpha} - \vartheta)$, or $AK^{\alpha} = \frac{1}{.78} + \vartheta$. The markup is marginal revenue divided by marginal cost, which because of constant returns to scale after the fixed cost of production satisfies $wL + r^k K = \frac{0.78\ A\ K^{\alpha}}{\widetilde{\Gamma}}$. The coefficient of labor in the production function equals the ratio of labor costs to total costs, $1-\alpha = \frac{wL}{wL+r^kK}$. Using the definition of labor share and its target value of 2/3 we get $\frac{3}{2}(1-\alpha) = \frac{1}{wL+r^kK} = \frac{\widetilde{\Gamma}}{1+.78\ \vartheta}$, which can be readily solved for ϑ.

[18] We choose this threshold because is almost equal to that estimated for the steady state gross markup in the "Calvo wage model" of Christiano et al. (2016). We have also used the values estimated by Abraham et al. (2021). The relevant results for this alternative are a Log marginal likelihood of -97.3, a Frisch elasticity of 10.0 and a standard deviation of the neutral technology shocks of 0.33.

Table 3: Benchmark Estimation Targeting 11 observable series with Three Shocks

	Prior Distribution. $\mathcal{D}$,**Mode**,[2.5-97.5%]	Benchmark u and Search	Search Alone	Standard u Alone	Tighter Prior u Alone***
		Posterior Distribution* **Mode**,[2.5-97.5%]			
Elast. of TFP wrt real spending, $\widetilde{\Psi}$	$\mathcal{U}$,**0.25**,[0.01-0.49]	**0.39**,[0.34-0.42]	**0.42**,[0.40-0.47]	-	-
Elast. of desired markup wrt TFP, $\widetilde{\gamma}$	$\mathcal{U}$,**1.00**,[0.05-1.95]	**0.79**,[0.70-0.88]	**0.27**,[0.21-0.40]	-	-
Steady state gross markup, $\widetilde{\Gamma}$	$\mathcal{U}$,**1.32**,[1.12-1.52]	**1.14**,[1.11-1.19]	**1.25**,[1.20-1.28]	**1.53**,[1.52-1.53]	**1.25**,[1.24-1.25]
Slope of NK Phillips Curve $\widetilde{\kappa}$,	$\mathcal{U}$,**0.15**,[0.03-0.27]	**0.15**,[0.09-0.21]	**0.08**,[0.06-0.11]	**0.17**,[0.08-0.21]	**0.10**,[0.05-0.15]
Implied search share in match func., φ		**0.31**	**0.31**	**0.00**	**0.00**
Implied preference for varieties, ρ		**1.65**	**1.83**	**1.54**	**1.25**
Implied price adjustment cost, $\kappa/200$**		**0.08**	**0.11**	**0.06**	**0.20**
Implied fixed cost of production, $B\vartheta$		**0.03**	**0.14**	**0.39**	**0.13**
Implied weight of search in utility, ζ		**0.55**	**0.67**	-	-
Implied weight of labor in utility, η		**0.69**	**0.73**	**0.40**	**0.27**
Implied units in production function, A		**1.17**	**1.29**	**1.23**	**1.00**
Capital utilization cost, σ_a	$\mathcal{G}$,**0.32**,[0.09-1.23]	**0.36**,[0.17-0.65]	-	**0.10**,[0.08-0.21]	**0.32**,[0.20-0.42]
Internal consumption habit, ς	$\mathcal{B}$,**0.50**,[0.21-0.79]	**0.82**,[0.71-0.87]	**0.22**,[0.08-0.42]	**0.79**,[0.73-0.84]	**0.77**,[0.72-0.81]
Inverse Frisch elasticity, ξ	$\mathcal{G}$,**0.94**,[0.57-1.55]	**0.64**,[0.43-0.75]	**0.99**,[0.79-1.07]	**0.11**,[0.08-0.21]	**0.11**,[0.06-0.14]
Investment adj. cost, $\widetilde{\mathcal{A}}$	$\mathcal{G}$,**7.50**,[4.57-12.4]	**8.64**,[7.19-12.0]	**5.63**,[4.54-6.53]	**6.64**,[5.57-8.72]	**5.21**,[4.07-6.90]
Taylor rule: inflation, ϕ_π	$\mathcal{G}$,**1.69**,[1.42-2.00]	**2.21**,[1.90-2.47]	**2.80**,[2.50-3.07]	**2.09**,[1.82-2.34]	**1.97**,[1.67-2.18]
Taylor rule: GDP, ϕ_y	$\mathcal{G}$,**0.08**,[0.03-0.22]	**0.03**,[0.02-0.04]	**0.07**,[0.06-0.07]	**0.14**,[0.11-0.20]	**0.28**,[0.18-0.36]
Taylor rule: smoothing, ρ_R	$\mathcal{B}$,**0.76**,[0.37-0.94]	**0.76**,[0.74-0.81]	**0.67**,[0.61-0.72]	**0.86**,[0.83-0.88]	**0.86**,[0.83-0.89]
Std. monetary policy, $400\sigma_R$	$\mathcal{G}$,**0.65**,[0.56-0.75]	**0.71**,[0.60-0.75]	**0.70**,[0.64-0.79]	**0.68**,[0.61-0.75]	**0.66**,[0.60-0.74]
Std. neutral tech., $100\sigma_n$	$\mathcal{G}$,**0.08**,[0.03-0.22]	**0.22**,[0.20-0.27]	**0.19**,[0.15-0.24]	**0.32**,[0.29-0.35]	**0.35**,[0.32-0.39]
Std. investment tech., $100\sigma_x$	$\mathcal{G}$,**0.08**,[0.03-0.22]	**0.12**,[0.08-0.15]	**0.06**,[0.05-0.07]	**0.14**,[0.12-0.18]	**0.15**,[0.13-0.19]
AR(1) investment tech, ρ_x	$\mathcal{B}$,**0.75**,[0.53-0.92]	**0.74**,[0.66-0.84]	**0.93**,[0.91-0.95]	**0.74**,[0.65-0.78]	**0.71**,[0.63-0.77]
Log marginal likelihood	-	**167.1**	**117.1**	**3.8**	**-162.6**
corr($markup, logGDP \| \epsilon^R$)	-	**0.96**	**0.76**	**0.06**	**-0.01**

* The posterior distribution is estimated using totally 1,000,000 draws for 10 MCMC chains with the first 50% of each chain burned.

** The cost is evaluated when annualized inflation is 4% away from the steady-state inflation rate. The unit is % of steady state GDP.

*** The prior for the steady state gross markup $\widetilde{\Gamma}$ instead of being a Uniform distribution over $[1.11-1.53]$ is now a Uniform distribution over $[1.11-1.25]$.

fundamental shocks are in Figures 1 to 3. The Table reports the priors and posteriors, the estimates, the implied estimates for the deep parameters and for those calibrated parameters that change in order to get the steady state targets, a measure of goodness of fit (the log marginal likelihood) and the correlation between the markup and GDP conditional on the shocks being monetary to highlight that we eliminate the difficulties that the recent work of Nekarda and Ramey (2020) has pointed to. Let's first comment on the rankings of fit. The model that does best is by far the one that has both search and variable capital utilization, better than the one with only search and far better than the one with search alone, especially if we reduce the fixed costs of production to a reasonable level (from 39% to 13% of GDP). Of All the priors with uniform distribution, the only one that binds in the estimation is the steady state gross markup that goes to an already very high upper bound in the "u Alone" economy. To see how much the model relies on the fixed cost of production to fit the data we also we re-estimate it with a tighter prior.

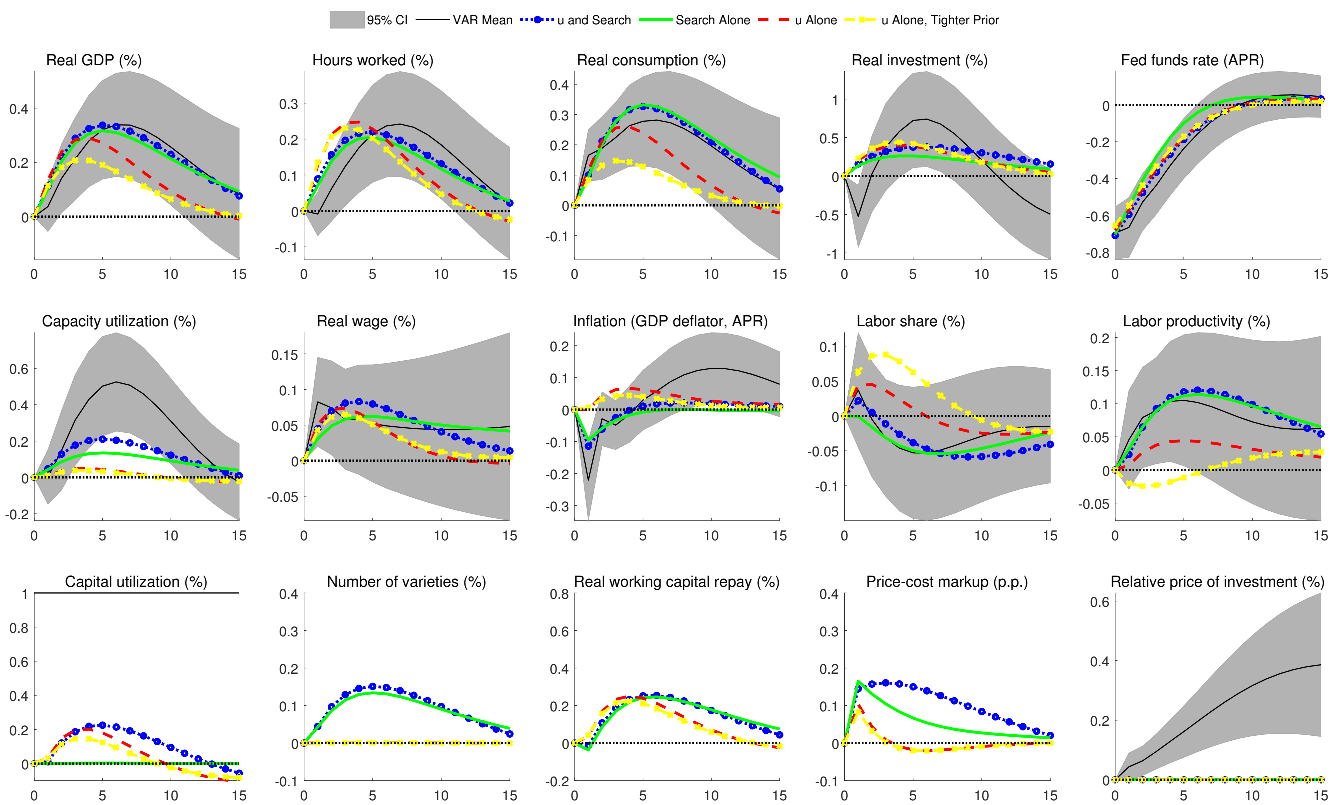

Figure 1: Response to 1 ST.DEV of Federal Funds Rate Shocks (Benchmark)

The actual estimates of our benchmark economy strongly support our search friction. Recall that our benchmark model that has both variable capital utilization and search nests the standard model with only variable capital utilization but no search as a special case. The grouped parameter $\widetilde{\Psi}$, which is the elasticity of TFP with respect to real spending, controls whether the search friction is turned on

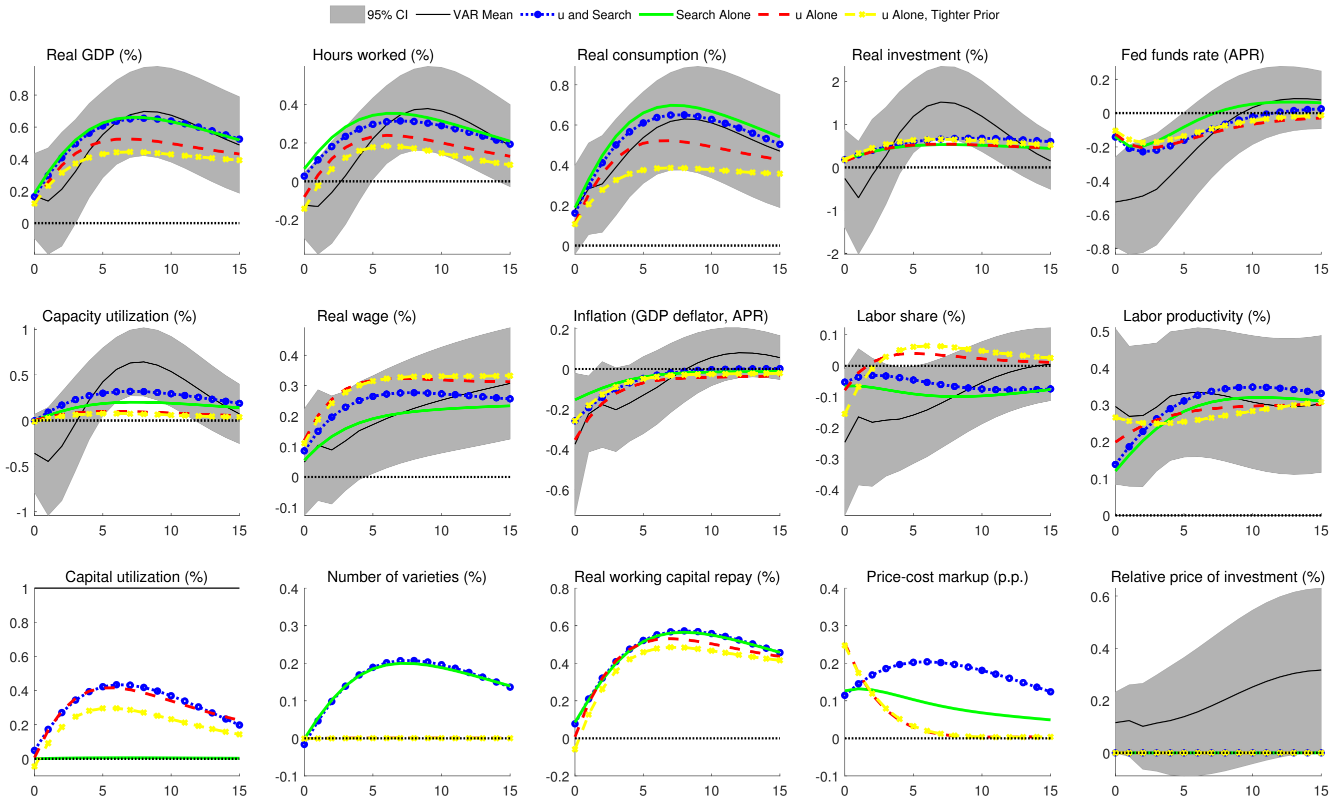

Figure 2: Response to 1 ST.DEV of Neutral Technology Shocks (Benchmark)

($\widetilde{\Psi} > 0$) or off ($\widetilde{\Psi} = 0$). The grouped parameter $\widetilde{\gamma}$ controls whether the variable desired markup is turned on ($\widetilde{\gamma} > 0$) or off ($\widetilde{\gamma} = 0$) conditional on $\widetilde{\Psi} > 0$. Our estimation results are strongly in favor of $\widetilde{\Psi} > 0$ and $\widetilde{\gamma} > 0$, in the sense that even though the priors of $\{\widetilde{\Psi}, \widetilde{\gamma}\}$ are uniform distributions with $\widetilde{\Psi} = \widetilde{\gamma} = 0$ in their supports, the posterior distributions are far away from zero. The point estimates $\widetilde{\Psi} = 0.39$ and $\widetilde{\gamma} = 0.79$ show that the two elasticities of the search friction are sizable economically, and hence we should expect the benchmark model with search to perform very differently from the standard model.

The true value of our theoretical extension shows up when we compare our benchmark model with the standard one that has only variable capital utilization but no search. First, our benchmark model is far superior in terms of model fit, with a log Bayes factor (difference of log marginal likelihoods) of more than 160 (Kass and Raftery (1995)). Second, our benchmark model has more *reasonable*[19] estimated parameters and associated implied moments. These include a much lower steady state markup (14% vs 53%), a much smaller Frisch elasticity of labor supply (1.6 vs 9.1), a much smaller reliance on neutral technology shocks (0.22 vs 0.32), and a much stronger procyclicality of markups conditional on Federal Funds Rate shocks (0.96 vs 0.06). Third, our benchmark model can fully match the procyclical labor

[19] Or at least more frequently seen in the literature.

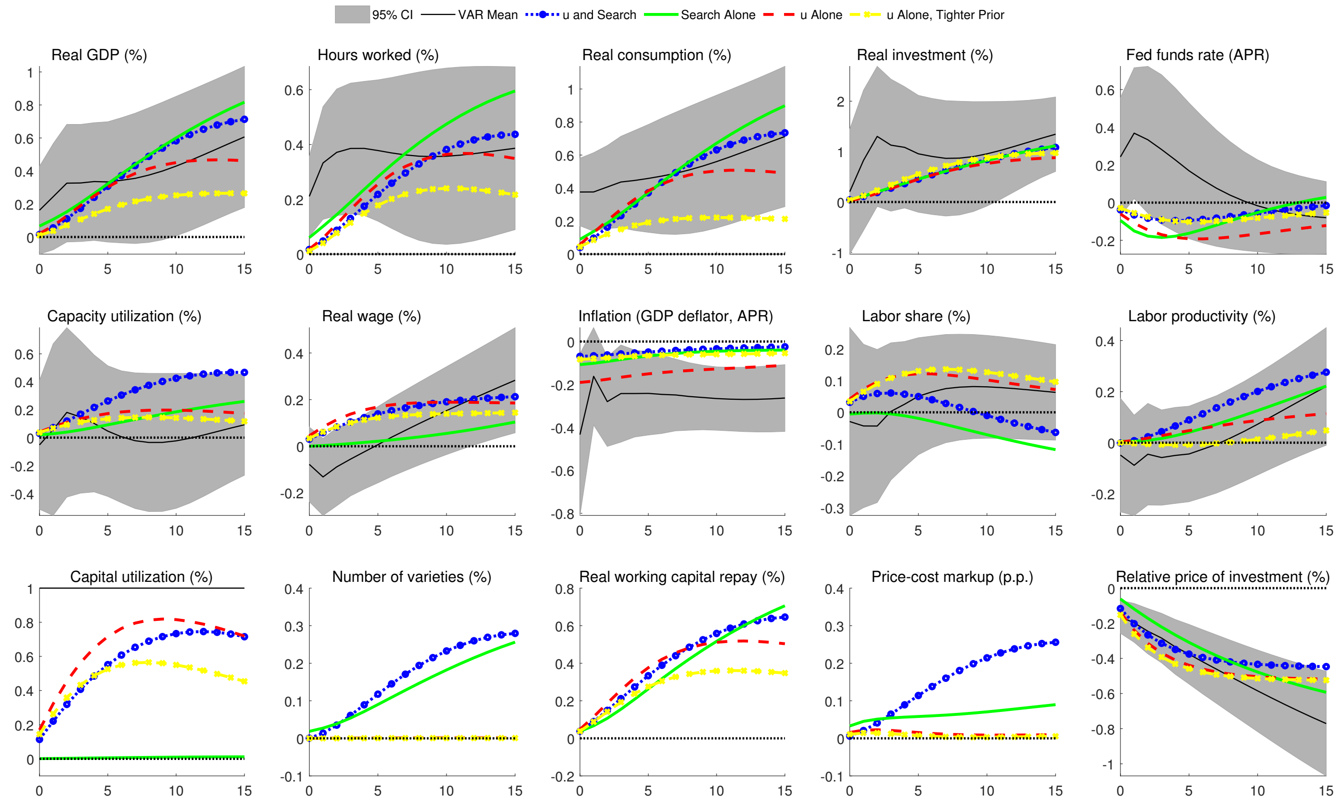

Figure 3: Response to 1 ST.DEV of Investment-Specific Technology Shocks (Benchmark)

productivity and countercyclical labor share conditional on Federal Funds Rate shocks (Figure 1), while the model with only variable capital utilization fails to do so. The advantage of our benchmark model becomes even larger when we impose a tighter prior on the steady state markup. These results suggest that our search friction can be a very useful modeling device for medium-scale DSGE models.

When comparing the model with only search friction but no variable capital utilization with the standard one, all the advantages of benchmark remain. The log Bayes factor (110) is still large. The steady state markups (0.25 vs 0.53), Frisch elasticity of labor supply (1.0 vs 9.1), and size of neutral technology shocks (0.19 vs 0.32) are all still much smaller. The markups (0.76 vs 0.06) are still more procyclical. The labor productivity and labor share can still match data very well. We can therefore conclude that our search friction is a more useful modeling device for medium-scale New Keynesian models than variable capital utilization. Further, because the size of the Bayes factor between the benchmark and the variable capital utilization economies is much larger than that between the benchmark and the search only economies, search frictions play a more central role in the benchmark economy than variable capital utilization.

We can find where the model performance is improved from Figures 1 to 3. For the Federal Funds

Rate and the neutral technology shocks, the two economies with search outperform the two without it in almost every aspect, especially in replicating the impulse response of labor productivity and labor share. For investment-specific technology shocks, the advantages are less clear.

Overall, we find that our search model does a much better job than the standard model, especially when we use a tighter prior over the steady state markup and hence the size of the fixed costs. The standard model wants huge increasing returns to scale so that higher productivity can be created along with a reduction in labor share. We obtain these features naturally via the increased search effort.

5. Robustness

We now turn to look at alternative circumstances to ensure that our findings are not a fluke.

5.1. Targeted Impulse Responses

The first thing to check is that our explicit targeting of labor productivity and labor share does not bias our findings. So we drop the impulse response functions of these variables and we reestimate the parameters.

Table 4: Robustness Estimation Targeting 9 Observable Series with Three Shocks

		Benchmark u and Search	Search Alone	Standard u Alone	Tighter Prior u Alone
	Prior Distribution $\mathcal{D}$,**Mode**,[2.5-97.5%]	Posterior Distribution **Mode**,[2.5-97.5%]			
Elast. of TFP wrt real spending, $\widetilde{\Psi}$	$\mathcal{U}$,**0.25**,[0.01-0.49]	**0.42**,[0.31-0.44]	**0.49**,[0.32-0.49]	-	-
Elast. of desired markup wrt TFP, $\widetilde{\gamma}$	$\mathcal{U}$,**1.00**,[0.05-1.95]	**0.72**,[0.40-0.84]	**0.42**,[0.05-0.54]	-	-
Steady state gross markup, $\widetilde{\Gamma}$	$\mathcal{U}$,**1.32**,[1.12-1.52]	**1.12**,[1.11-1.24]	**1.12**,[1.12-1.33]	**1.53**,[1.52-1.53]	**1.25**,[1.24-1.25]
Slope of NK Phillips Curve $\widetilde{\kappa}$,	$\mathcal{U}$,**0.15**,[0.03-0.27]	**0.08**,[0.03-0.20]	**0.03**,[0.03-0.21]	**0.09**,[0.05-0.14]	**0.07**,[0.05-0.13]
Implied search share in match func., φ		**0.33**	**0.36**	**0.00**	**0.00**
Implied preference for varieties, ρ		**1.67**	**1.75**	**1.54**	**1.25**
Implied price adjustment cost, $\kappa/200^{**}$		**0.14**	**0.34**	**0.10**	**0.27**
Implied fixed cost of production, $B\vartheta$		**0.02**	**0.01**	**0.39**	**0.13**
Capital utilization cost, σ_a	$\mathcal{G}$,**0.32**,[0.09-1.23]	**1.13**,[0.59-1.74]		**0.30**,[0.17-0.41]	**0.34**,[0.24-0.42]
Internal consumption habit, ς	$\mathcal{B}$,**0.50**,[0.21-0.79]	**0.82**,[0.74-0.89]	**0.77**,[0.19-0.87]	**0.79**,[0.73-0.83]	**0.75**,[0.70-0.79]
Inverse Frisch elasticity, ξ	$\mathcal{G}$,**0.94**,[0.57-1.55]	**0.32**,[0.24-0.43]	**0.23**,[0.19-0.49]	**0.23**,[0.17-0.27]	**0.09**,[0.05-0.12]
Investment adj. cost, $\widetilde{\mathcal{A}}$	$\mathcal{G}$,**7.50**,[4.57-12.4]	**10.6**,[8.81-14.3]	**10.4**,[9.94-15.8]	**10.5**,[7.93-12.9]	**8.59**,[6.06-10.1]
Log marginal likelihood	-	**108.3**	**103.8**	**30.9**	**-59.6**
corr($markup, logGDP \mid \epsilon^R$)	-	**0.95**	**0.90**	**-0.05**	**-0.31**

Table 4 shows the estimates of the main parameters. It indicates that none of the previously highlighted good properties of the search friction is qualitatively different. The two models with search still perform much better than the two without it in therms of the log marginal likelihood even if the differences across economies shrink somewhat. The estimated size of the fixed costs of production is much lower than in the no search economies. There are no large differences in the rest of the parameter estimates: the implied price adjustment costs and investment adjustment costs are uniformly a bit larger in all economies; the Frisch elasticity is larger than before in the search economies. Therefore, we can safely conclude that our search friction improves a lot the fit of the model either alone or in tandem with variable capital utilization that now plays a much more diminished role even if the focus of study is not labor productivity or labor share.

5.2. Alternative Measures of the Cost of Capital Utilization

Following Christiano et al. (2010), Christiano et al. (2016) and Beraja et al. (2019) we have treated capital utilization costs as intermediate goods (e.g. replacing a tire for a truck) and hence those costs are neither in investment nor in GDP. This is not the only way to think of these costs. They can be explicitly treated as investment in NIPA and they can or cannot affect the investment adjustment costs.

Capital Utilization Costs are Part of Investment and Do Not Affect Adjustment Costs We can think of this as replacing machines and increasing (or decreasing) the production of machines is subject to adjustment costs (a truck breaks up and a new truck is purchased, with seamless incorporation into the company).[20]

The estimates are in the top panel of Table 5. We have used the same 11 variables that in Table 3. Again the search economies do much better even though the differences in terms of the log marginal likelihood are smaller. The estimates are similar with some subtle differences. The elasticities of TFP with respect to expenditures, $\widetilde{\Psi}$, and of the desired markup with respect to TFP, $\widetilde{\gamma}$, are now smaller than before in the Benchmark economy (both variable capital utilization and search) while in the search alone this is not the case, in fact the second elasticity is larger. The capital utilization costs are now smaller for the no search economies, the Frisch elasticities are larger except for the large fixed costs variable capital utilization, and the investment adjustment costs are uniformly larger. This last feature is to be expected.

Capital Utilization Costs are Part of Investment and Do Affect Adjustment Costs We can think of this as replacing machines but convex adjustment costs occur when the number of machines

[20] Smets and Wouters (2007) and Christiano et al. (2014) have included them in GDP but not as part of investment or consumption. This is close to our treatment, but inconsistent with NIPA.

used differ across periods (a truck breaks up and a new truck is purchased, there are large adjustment costs to change the number of trucks produced year to year).[21] The estimates are in the bottom panel of Table 5. Some differences arise with respect to the results in Table 3: in the Benchmark economy the elasticity of the desired markup with respect to TFP is now smaller and the capital utilization cost is mostly smaller (as more resources affect adjustment costs the actual costs per unit are naturally smaller). But overall, again the search economies perform much better than the no search economies and the estimates are close to those with the Benchmark accounting method.

6. Understanding the Mechanisms

We ask three further question to shed light on how our model works. First, why is the increase of labor productivity during demand-induced expansions in search models more powerful than in costly variable capital utilization models? (Section 6.1). Second, how is labor productivity related to labor share, profit share, and markups? (Section 6.2). Third, are variable desired markups necessary to obtain countercyclical labor shares? (Section 6.3).

6.1. Labor Productivity in Search and Capital Utilization Models

To compare search and variable capital utilization models, we look at a block of static equilibrium conditions that take as given the capital stock K and the real wage w. This makes clear the power of the search friction in affecting labor productivity. We perform our analysis treating the utilization cost as a part of the investment that does not induce investment adjustment costs so as to maximize the power of variable capital utilization in accounting for the data (see Section 5.2).

Using $\mathcal{I}$ to denote the number of varieties sold, which is equal to $\psi^f(Q)$, real expenditures by households $\mathcal{Y}$ satisfy

$$\mathcal{Y} = \mathcal{I}\,[A\,(u\,K)^{\alpha}\,L^{1-\alpha} - \vartheta].$$

Taking capital K as fixed, output $\mathcal{Y}$ can be increased by either a higher matching probability $\mathcal{I}$, more intensive capital utilization u, or more employment L. Labor productivity, $\ell p \equiv \mathcal{Y}/L$, can be higher if L increases less than $\mathcal{Y}$ due to either $\mathcal{I}$ or u.

To understand how much $\mathcal{I}$ and u move endogenously when firms supply output $\mathcal{Y}$, we start noting that under GHH preferences, $\mathcal{I} = \psi^f(Q) = \psi^f\left(\frac{\rho}{\zeta-1}\cdot\mathcal{I}^{\rho-1}\,\mathcal{Y}\right)$, which determines the endogenous movement of $\mathcal{I}$ in response to any movement in $\mathcal{Y}$. Given that change in $\mathcal{I}$, the pair of $\{u, L\}$ that

[21] This approach is common in models without a monetary policy shock (Fernández-Villaverde et al., 2015; Basu and Bundick, 2017).

Table 5: Additional Depreciation is Part of Investment

Treating Utilization Cost as Investment without Adjustment Cost

	Prior Distribution $\mathcal{D}$,**Mode**,[2.5-97.5%]	Benchmark u and Search	Search Alone	Standard u Alone	Tighter Prior u Alone
		Posterior Distribution **Mode**,[2.5-97.5%]			
Elast. of TFP wrt real spending, $\widetilde{\Psi}$	$\mathcal{U}$,**0.25**,[0.01-0.49]	**0.34**,[0.24-0.38]	**0.42**,[0.39-0.45]	-	-
Elast. of desired markup wrt TFP, $\widetilde{\gamma}$	$\mathcal{U}$,**1.00**,[0.05-1.95]	**0.63**,[0.46-0.81]	**0.32**,[0.18-0.39]	-	-
Steady state gross markup, $\widetilde{\Gamma}$	$\mathcal{U}$,**1.32**,[1.12-1.52]	**1.20**,[1.12-1.31]	**1.22**,[1.19-1.30]	**1.53**,[1.51-1.53]	**1.25**,[1.24-1.25]
Slope of NK Phillips Curve $\widetilde{\kappa}$,	$\mathcal{U}$,**0.15**,[0.03-0.27]	**0.13**,[0.09-0.22]	**0.08**,[0.05-0.13]	**0.10**,[0.05-0.15]	**0.05**,[0.04-0.10]
Implied search share in match func., φ		**0.28**	**0.32**	**0.00**	**0.00**
Implied preference for varieties, ρ		**1.66**	**1.78**	**1.54**	**1.25**
Implied price adjustment cost, $\kappa/200^{**}$		**0.08**	**0.12**	**0.09**	**0.39**
Implied fixed cost of production, $B\vartheta$		**0.09**	**0.10**	**0.39**	**0.13**
Capital utilization cost, σ_a	$\mathcal{G}$,**0.32**,[0.09-1.23]	**0.33**,[0.05-0.51]		**0.02**,[0.00-0.05]	**0.01**,[0.00-0.02]
Internal consumption habit, ς	$\mathcal{B}$,**0.50**,[0.21-0.79]	**0.76**,[0.65-0.84]	**0.54**,[0.07-0.67]	**0.76**,[0.70-0.79]	**0.74**,[0.71-0.78]
Inverse Frisch elasticity, ξ	$\mathcal{G}$,**0.94**,[0.57-1.55]	**0.87**,[0.54-1.01]	**0.55**,[0.39-1.10]	**1.00**,[0.76-1.03]	**1.08**,[0.88-1.26]
Investment adj. cost, $\widetilde{\mathcal{A}}$	$\mathcal{G}$,**7.50**,[4.57-12.4]	**11.5**,[9.93-17.1]	**9.86**,[4.35-9.95]	**13.7**,[10.8-17.8]	**13.8**,[10.5-18.7]
Log marginal likelihood	-	**178.2**	**133.7**	**121.6**	**75.6**
corr($markup$, $logGDP\|\epsilon^R$)	-	**0.92**	**0.72**	**0.12**	**0.09**

Additional Depreciation Contributes to Investment Adjustment Costs

	Prior Distribution $\mathcal{D}$,**Mode**,[2.5-97.5%]	Benchmark u and Search	Search Alone	Standard u Alone	Tighter Prior u Alone
		Posterior Distribution **Mode**,[2.5-97.5%]			
Elast. of TFP wrt real spending, $\widetilde{\Psi}$	$\mathcal{U}$,**0.25**,[0.01-0.49]	**0.39**,[0.32-0.43]	**0.44**,[0.40-0.46]	-	-
Elast. of desired markup wrt TFP, $\widetilde{\gamma}$	$\mathcal{U}$,**1.00**,[0.05-1.95]	**0.34**,[0.08-0.48]	**0.27**,[0.20-0.34]	-	-
Steady state gross markup, $\widetilde{\Gamma}$	$\mathcal{U}$,**1.32**,[1.12-1.52]	**1.19**,[1.14-1.28]	**1.25**,[1.22-1.28]	**1.53**,[1.51-1.53]	**1.25**,[1.24-1.25]
Slope of NK Phillips Curve $\widetilde{\kappa}$,	$\mathcal{U}$,**0.15**,[0.03-0.27]	**0.11**,[0.03-0.24]	**0.07**,[0.06-0.09]	**0.05**,[0.03-0.10]	**0.05**,[0.03-0.07]
Implied search share in match func., φ		**0.30**	**0.32**	**0.00**	**0.00**
Implied preference for varieties, ρ		**1.71**	**1.83**	**1.54**	**1.25**
Implied price adjustment cost, $\kappa/200^{**}$		**0.09**	**0.13**	**0.17**	**0.42**
Implied fixed cost of production, $B\vartheta$		**0.08**	**0.13**	**0.39**	**0.13**
Capital utilization cost, σ_a	$\mathcal{G}$,**0.32**,[0.09-1.23]	**2.97**,[1.93-4.25]		**0.03**,[0.01-0.10]	**0.02**,[0.00-0.04]
Internal consumption habit, ς	$\mathcal{B}$,**0.50**,[0.21-0.79]	**0.78**,[0.64-0.86]	**0.21**,[0.09-0.46]	**0.61**,[0.54-0.71]	**0.67**,[0.57-0.70]
Inverse Frisch elasticity, ξ	$\mathcal{G}$,**0.94**,[0.57-1.55]	**0.39**,[0.28-0.55]	**1.02**,[0.86-1.06]	**1.38**,[0.84-1.52]	**1.50**,[1.00-1.65]
Investment adj. cost, $\widetilde{\mathcal{A}}$	$\mathcal{G}$,**7.50**,[4.57-12.4]	**9.27**,[7.35-11.8]	**4.95**,[4.11-5.08]	**3.45**,[3.08-5.24]	**3.68**,[2.73-5.26]
Log marginal likelihood	-	**121.9**	**114.6**	**1.4**	**-62.4**
corr($markup$, $logGDP\|\epsilon^R$)	-	**0.92**	**0.76**	**-0.94**	**-0.94**

is chosen must satisfy the optimal input allocation (recall that we are abstracting from movements in either the wage or capital)

$$\frac{R^B w\ L}{1-\alpha} = \frac{r^k\ u\ K}{\alpha} = \frac{\mathcal{I}^{1-\rho}\ \widetilde{\delta}_u(u)\ u\ K}{\alpha}.$$

As a result, an approximation under constant capital stock K and real wages w yields that an increase in output $\mathcal{Y}$ induces changes in $\{\mathcal{I}, u, L\}$, and hence in labor productivity that under log-linearization satisfy

$$d\ln(\ell p) = \left[1 - \frac{1+\sigma_a}{1+(1-\alpha)\sigma_a}\frac{1-\widetilde{\Psi}}{1+\vartheta\mathcal{I}_{ss}} + \frac{\alpha(\rho-1)\widetilde{\Psi}}{1+(1-\alpha)\sigma_a}\right] d\ln(\mathcal{Y}) + \frac{\alpha}{1+(1-\alpha)\sigma_a}\, d\ln(R^B). \quad (41)$$

We now look at the three economies that we have estimated, the Benchmark ("u and Search"), and those with only one mechanism at play, "Search Alone", and "u Alone", all with a fixed cost of production.

Variable Capital Utilization Alone (u Alone) Here, we have $\widetilde{\Psi} = 0$, and $\mathcal{I}_{ss} = 1$ to turn off directed search, and $\sigma_a \to 0$ to maximize the effect of variable capital utilization. As a result, Equation (41) becomes

$$d\ln(\ell p) = \frac{\vartheta}{1+\vartheta} \cdot d\ln(\mathcal{Y}) + \alpha \cdot d\ln(R^B).$$

Note that ignoring the change in the real wage, variable capital utilization allows for the same capital services to labor ratio, and, as a result, the economy is not subject to decreasing returns to scale from slow moving capital. Then procyclical labor productivity can only come from the fixed cost of production ϑ or from procyclical nominal interest rate that increases the cost of working capital (but this latter effect goes in the wrong direction under an expansionary monetary policy), as the marginal cost of labor $R^B w$ induces substitution between labor and capital. Consequently, large fixed costs are needed for any meaningful increase in labor productivity. For instance, a fixed cost of production equal to 43% of GDP (ϑ) is needed to increase labor productivity by 0.3%, when real GDP goes up by 1%.

Search Alone Economy In this case, we have $\widetilde{\Psi} > 0$, $\mathcal{I}_{ss} < 1$ to turn on directed search and $\sigma_a \to +\infty$ to turn off variable capital utilization. As a result, Equation (41) becomes

$$d\ln(\ell p) = \frac{\frac{\widetilde{\Psi}+\vartheta\mathcal{I}_{ss}}{1+\vartheta\mathcal{I}_{ss}} - \alpha}{1-\alpha} \cdot d\ln(\mathcal{Y}).$$

Procyclical labor productivity comes from the search friction $\widetilde{\Psi}$ and the fixed cost of production ϑ. There is a negative term $(-\alpha)$ due to the decreasing returns to scale and the lack of variable capital utilization. This implies that a minimum size of the search friction or fixed costs of production is required to ensure procyclical labor productivity. Working capital costs due to R^B no longer play a role because there is no trade-off between capital and labor given that capital services cannot be adjusted. The extent to which the search friction alone can generate a procyclical labor productivity is a quantitative issue. For instance, to get an increase of labor productivity of 0.3% out of a 1% increase in real GDP, a value of the elasticity of TFP with respect to expenditures (grouped parameter $\widetilde{\Psi}$) equal to 0.48 is needed in the absence of fixed costs of production and a value of 0.38 is needed under 20% fixed costs to GDP ratio ($\vartheta\mathcal{I}_{ss} = 0.2$).

Both Search and Variable Capacity Utilization We have $\widetilde{\Psi} > 0$, $\mathcal{I}_{ss} < 1$ and to maximize the role of variable capacity utilization we pose $\sigma_a \to 0$. Equation (41) becomes

$$d\ln(\ell p) = \left[\frac{\widetilde{\Psi} + \vartheta\mathcal{I}_{ss}}{1 + \vartheta\mathcal{I}_{ss}} + \alpha(\rho - 1)\widetilde{\Psi}\right] \cdot d\ln(\mathcal{Y}) + \alpha \cdot d\ln(R^B).$$

In this economy both mechanisms are at work, which makes it a lot easier to achieve procyclical labor productivity. The negative term $-\alpha$ that showed up before is no longer there due to the variable capacity utilization and a new term appears that further increases the possibility of labor productivity being positively affected by total expenditures. But even ignoring this new term, to get the 0.3 value discussed above, we need $\widetilde{\Psi} = 0.30$ with no fixed costs or $\widetilde{\Psi} = 0.16$ with fixed costs being 20% of GDP, which is a lot easier than before.

6.2. Labor Productivity, Labor Share, Profit Share, and Markups

What about the other variables that we are interested in? Under constant $\{K, w\}$ as before and log-linearization, labor share, profit share, and markup satisfy

$$\begin{aligned} d\ln(\ell s) &= d\ln(w) - \ln(\ell p), \\ d\ln(\Pi^f/\mathcal{Y}) &= -\frac{1}{\frac{\Gamma_{ss}}{1+\vartheta\mathcal{I}_{ss}} - 1} \cdot \left[d\ln(\ell s) + d\ln(R^B)\right], \\ d\ln(\Gamma) &= -\frac{\vartheta\mathcal{I}_{ss}}{1 + \vartheta\mathcal{I}_{ss}}(1 - \widetilde{\Psi}) \cdot d\ln(\mathcal{Y}) - \left[d\ln(\ell s) + d\ln(R^B)\right]. \end{aligned}$$

There are a few takeaways from these results. First, labor share is just the mirror image of labor

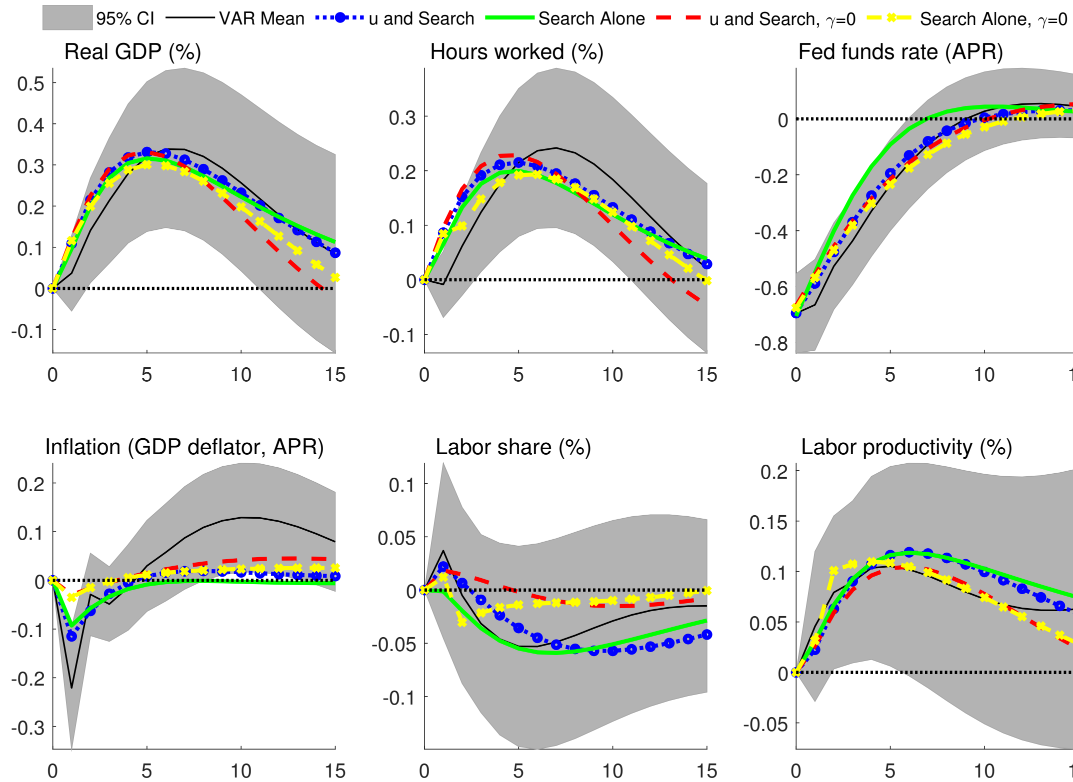

Figure 4: Response to 1 ST.DEV of Federal Funds Rate Shocks (the Role of $\widetilde{\gamma}$)

productivity when the real wage is given, hence labor productivity and labor share must be explained jointly. Second, under countercyclical costs of working capital, the profit share must be procyclical when labor share is countercyclical. Third, a procyclical markup is sufficient but not necessary for a procyclical profit share, because the fixed cost of production can make profit share procyclical without a procyclical markup. We cannot measure profit shares or markups easily but the behavior of the labor share informs us about the behavior of the other variables through the lens of the model. Since profit shares and markups are closely related to labor share, and labor share is closely related to labor productivity, we can expect that search models are likely to generate countercyclical labor share, procyclical profit share, and procyclical markups more easily because they can easily generate procyclical labor productivity.

6.3. Procyclical Desired Markups for Countercyclical Labor Share

One of the key features of the directed search friction is the endogenous desired markup captured by parameter $\widetilde{\gamma}$ ($\widetilde{\gamma} = \gamma$ in the CES matching function). In the necessary and sufficient conditions derived in Section 2.2.3, $\widetilde{\gamma} > 0$ plays a crucial role in making procyclical markups compatible with procyclical inflation. In quantitative models, more mechanisms may pollute the results but we would like to see if $\widetilde{\gamma} > 0$ is still necessary. Therefore, we re-estimate both the "u and Search" model and the "Search Alone" model with $\widetilde{\gamma} = 0$ and compare them to those with $\widetilde{\gamma} > 0$ in Figure 4. We can see that despite the similar performance of real GDP, hours worked, Federal Funds Rate, and labor productivity, labor share moves very little and the initial response of inflation is muted when $\gamma = 0$. These results are consistent with our theoretical findings about the tension between procyclical markups (closely related to labor share) and procyclical inflation if desired markup is constant ($\widetilde{\gamma} = 0$).

7. Conclusion

We have proposed a new mechanism to add to medium-sized New Keynesian models suitable for estimation. It is based on the notion that expenditures increase productivity temporarily due to increased search effort of households. We have shown how to implement it, using directed search in the space of varieties. It is very simple to use and can be added with very low cost to any model currently in play. We have used it to estimate a version of the canonical Christiano et al. (2016) model and shown how it is far superior in terms of fit to standard modeling approaches (e.g. variable capital utilization) and it is best when used in combination of those approaches. This is especially the case for the performance of labor productivity and labor share, both variables with a behavior that has proved to be hard to replicate. Hence, we think search frictions in the goods markets should be considered as part of the standard ingredients in medium-scale DSGE models.

References

Abraham, F., Y. Bormans, J. Konings, and W. Roeger (2021): "Price-cost margins and fixed costs," Working papers, London, Centre for Economic Policy Research.

Anderson, E., S. Rebelo, and A. Wong (2020): "Markups Across Space and Time," *Working Paper*.

Arkolakis, C. and M. Morlacco (2017): "Variable Demand Elasticity, Markup, and Pass-Through," *Working Paper*.

Bai, Y., J.-V. Ríos-Rull, and K. Storesletten (2019): "Demand Shocks as Productivity Shocks," *Working Paper*.

Baqaee, D., E. Farhi, and K. Sangani (2021): "The Supply-Side Effects of Monetary Policy," *NBER Working Paper No 28435*.

Basu, S. and B. Bundick (2017): "Uncertainty Shocks in a Model of Effective Demand," *Econometrica*, 85, 937–958.

Basu, S. and C. L. House (2016): "Chapter 6 - Allocative and Remitted Wages: New Facts and Chanllenges for Keynesian Models," in *Handbook of Macroeconomics*, ed. by J. B. Taylor and H. Uhlig, Elsevier, vol. 2, 297–354.

Beraja, M., E. Hurst, and J. Ospina (2019): "The Aggregate Implications of Regional Business Cycles," *Econometrica*, 87, 1789–1833.

Bertoletti, P. and F. Etro (2017): "Monopolitic Competition when Income Matters," *Economic Journal*, 127, 1217–1243.

Bils, M., P. J. Klenow, and B. A. Malin (2018): "Resurrecting the Role of the Product Market Wedge in Recessions," *American Economic Review*, 108, 1118–1146.

Burstein, A., V. M. Carvalhoz, and B. Grassi (2020): "Bottom-up Markup Fluctuations," *Working Paper*.

Cantore, C., F. Ferroni, and M. A. León-Ledesma (2021): "The Missing Link: Labor Share and Monetary Policy," *Journal of the European Economic Association*, Accepted.

Choi, S. and J.-V. Ríos-Rull (2009): "Understanding the Dynamics of Labor Share: the Role of Noncompetitive Factor Prices," *Annals of Economics and Statistics*, 1, 251–277.

——— (2020): "Labour Share and Productivity Dynamics," *The Economic Journal*, 131, 2856–2886.

Christiano, L., R. Motto, and M. Rostagno (2014): "Risk Shocks," *American Economic Review*, 104, 37–65.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45.

Christiano, L. J., M. S. Eichenbaum, and M. Trabandt (2016): "Unemployment and Business Cycles," *Econometrica*, 84, 1523–1569.

Christiano, L. J., M. Trabandt, and K. Walentin (2010): "Chapter 7 - DSGE Models for Monetary Policy Analysis," *Handbook of Monetary Economics*, 3, 285–367.

Fagnart, J. F., O. Licandro, and F. Portier (1999): "Firm heterogeneity, capacity utilization, and the business cycle," *Review of Economic Dynamics*, 2, 433–455.

Fernández-Villaverde, J., P. Guerrón-Quintana, K. Kuester, and J. Rubio-Ramírez (2015): "Fiscal Volatility Shocks and Economic Activity," *American Economic Review*, 7, 3352–3384.

Floetotto, M. and N. Jaimovich (2008): "Firm Dynamics, Market Variations, and the Business Cycle," *Journal of Monetary Economics*, 55, 1238–1252.

Fukui, M. (2020): "A Theory of Wage Rigidity and Unemployment Fluctuations with On-the-Job Search," *Working Paper*.

Gertler, M., C. Huckfeldt, and A. Trigari (2020): "Unemployment Fluctuations, Match Quality and the Cyclical Behavior of New Hires Wages," *Review of Economic Studies*, 87, 1876–1914.

Gottfries, N., G. Michelsson, and K. Stadin (2021): "Deep Dynamics," *Working Paper*.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988): "Investment, Capacity Utilization and the Real Business Cycle," *American Economic Review*, 78, 402–417.

Grigsby, J., E. Hurst, and A. Yildirmaz (2021): "Aggregate Nominal Wage Adjustments: New Evidence from Administrative Payroll Data," *American Economic Review*, 111, 428–471.

Hong, S. (2017): "Customer Capital, Markup Cyclicality, and Amplification," *Working Paper*.

Huo, Z. and J.-V. Ríos-Rull (2014): "Financial Frictions, Asset Prices, and the Great Recession," Manuscript, Federal Reserve Bank of Minneapolis.

——— (2015): "Tightening Financial Frictions on Households, Recessions, and Price Reallocations," *Review of Economic Dynamics*, 18, 118–139.

——— (2020a): "Demand induced fluctuations," *Review of Economic Dynamics*, 37, S99 – S117, the twenty-fifth anniversary of "Frontiers of Business Cycle Research".

——— (2020b): "Sticky Wage Models and Labor Supply Constraints," *American Economic Journal: Macroeconomics*, 12, 284–318.

Hyun, J., R. Kim, and B. Lee (2021): "Business Cycles with Cyclical Returns to Scale," *Working Paper*.

Jaimovich, N. (2007): "Firm Dynamics and Markup Variations: Implications for Sunspot Equilibria and Endogenous Economic Fluctuations," *Journal of Economic Theory*, 137, 300–325.

Kaplan, G. and G. Menzio (2016): "Shopping Externalities and Self-Fulfilling Unemployment Fluctuations," *Journal of Political Economy*, 124, 771–825.

Kaplan, G. and P. Zoch (2020): "Markups, Labor Market Inequality and the Nature of Work," *Working Paper*.

Kass, R. E. and A. E. Raftery (1995): "Bayes Factors," *Journal of the American Statistical Association*, 90, 773–795.

Kudlyak, M. (2014): "The Cyclicality of the User Cost of Labor," *Journal of Monetary Economics*, 1, 53–67.

Lehmann, E. and B. Van der Linden (2010): "Search Frictions on Product and Labor Markets: Money in the Matching Function," *Macroeconomic Dynamics*, 14, 56–92.

Li, N. (2021): "An Engel Curve for Variety," *Review of Economic Statistics*, 103, 72–87.

Loecker, J. D., J. Eeckhout, and G. Unger (2020): "The Rise of Market Power and the Macroeconomic Implications," *Quarterly Journal of Economics*, 135, 561–644.

Meier, M. and T. Reinelt (2021): "Monetary Policy, Markup Dispersion, and Aggregate TFP," *Working Paper*.

Michaillat, P. and E. Saez (2015): "Aggregate Demand, Idle Time, and Unemployment," *Quarterly Journal of Economics*, 130, 507–569.

Michelacci, C., L. Paciello, and A. Pozzi (2021): "The Extensive Margin of Aggregate Consumption Demand," *Review of Economic Studies*, Accepted.

Nekarda, C. J. and V. A. Ramey (2020): "The Cyclicality of the Price-Cost Markup," *Journal of Money, Credit, and Banking: 50th Anniversary Issue*, 52, 319–353.

Petrosky-Nadeau, N., E. Wasmer, and S. Zeng (2016): "Shopping Time," *Economics Letters*, 143, 52–60.

Ríos-Rull, J.-V. and R. Santaeulàlia-Llopis (2010): "Redistributive Shocks and Productivity Shocks," *Journal of Monetary Economics*, 57, 931–948.

Smets, F. and R. Wouters (2007): "Shocks and Frictions in US Business Cycle; A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.

Stroebel, J. and J. Vavra (2019): "House Prices, Local Demand, and Retail Prices," *Journal of Political Economy*, 127, 1391–1436.

Swanson, E. T. (2006): "The relative price and relative productivity channels for aggregate fluctuations," *Contributions in Macroeconomics*, 6, 1–39.

Taubman, P. and M. Wilkinson (1970): "User Cost, Capital Utilization and Investment Theory," *International Economic Review*, 11, 209–15.

Appendix

A. Various Proofs

A.1. GHH Preferences Imply that More Expenditures Yield Both More Search and More Consumption of Each Variety under the Same Set of Available Markets.

We start with deriving the F.O.C. of the consumers' optimization problem in Section 2.2.1.

$$0 = \frac{\partial \mathcal{L}}{\partial c(p,q)} = U_{c^A} \cdot \rho \cdot (c^A)^{\frac{\rho-1}{\rho}} \cdot d(p,q)\psi^h(q) \cdot \frac{1}{\rho} \cdot c(p,q)^{\frac{1-\rho}{\rho}} - \lambda \cdot d(p,q)\psi^h(q)\ p,$$
$$0 = \frac{\partial \mathcal{L}}{\partial d(p,q)} = U_{c^A} \cdot \rho \cdot (c^A)^{\frac{\rho-1}{\rho}} \cdot \psi^h(q) c(p,q)^{\frac{1}{\rho}} + U_{d^A} - \lambda \cdot \psi^h(q)\ p\ c(p,q),$$

in which λ is the Lagrange multiplier. Rearranging the algebra to simplify the expressions above yields

$$0 = U_{c^A} \cdot \left(\frac{c^A}{c(p,q)}\right)^{\frac{\rho-1}{\rho}} - \lambda \cdot p,$$
$$0 = \rho \cdot U_{c^A} \cdot \left(\frac{c^A}{c(p,q)}\right)^{\frac{\rho-1}{\rho}} + \frac{U_{d^A}}{\psi^h(q)c(p,q)} - \lambda \cdot p.$$

Combining the two conditions above to eliminate λ and rearranging the expression yields

$$0 = (\rho - 1) \cdot d(p,q)\psi^h(q) \left(\frac{c(p,q)}{c^A}\right)^{\frac{1}{\rho}} + \frac{U_{d^A}}{U_{c^A}} \cdot \frac{d(p,q)}{c^A}.$$

Taking integral $\int_\Phi \cdot dp\ dq$ on both sides of the condition above yields

$$0 = \rho - 1 + \frac{U_{d^A}}{U_{c^A}} \cdot \frac{d^A}{c^A}.$$

To proceed with analytical results, use GHH utility function $U(c^A, d^A) = U\left(c^A - \zeta\frac{(d^A)^{1+\nu}}{1+\nu}\right)$ to get

$$\frac{U_{d^A}}{U_{c^A}} = -\zeta \cdot (d^A)^\nu.$$

The two conditions above combined yield

$$c^A = \frac{\zeta}{\rho - 1} \cdot (d^A)^{1+\nu}.$$

A.2. A Useful Price Aggregate P^A for Directed Search

Note that without directed search, the useful price aggregate should be

$$P = \left(\int_0^1 p_j^{\frac{1}{1-\rho}} dj \right)^{1-\rho}.$$

With directed search, we start with a first order condition derived in Appendix A.1

$$0 = U_{c^A} \cdot \left(\frac{c^A}{c(p,q)} \right)^{\frac{\rho-1}{\rho}} - \lambda \cdot p.$$

We would like to have a price aggregate P^A such that

$$0 = U_{c^A} - \lambda \cdot P^A.$$

This will allow us to have a variety demand analogous to the version of model with no search

$$c(p,q) = \left(\frac{p}{P^A} \right)^{-\frac{\rho}{\rho-1}} c^A.$$

To see the exact formula of P^A, rearrange the condition above to get

$$\left(\frac{p}{P^A} \right)^{\frac{1}{1-\rho}} = \left(\frac{c(p,q)}{c^A} \right)^{\frac{1}{\rho}}.$$

Multiplying both sides by $d(p,q)\psi^h(q)$ yields

$$d(p,q)\psi^h(q) \left(\frac{p}{P^A} \right)^{\frac{1}{1-\rho}} = d(p,q)\psi^h(q) \left(\frac{c(p,q)}{c^A} \right)^{\frac{1}{\rho}}.$$

Taking integral $\int_\Phi \cdot dp\ dq$ on both sides of the condition above yields

$$\int_\Phi d(p,q)\psi^h(q) \left(\frac{p}{P^A} \right)^{\frac{1}{1-\rho}} dpdq = 1.$$

Rearranging it to solve out p^A yields

$$P^A = \left(\int_\Phi d(p,q)\ \psi^h(q) p^{\frac{1}{1-\rho}}\ dp\ dq \right)^{1-\rho}.$$

A.3. Using $P^A(e,\Phi)$ to Summarize Optimal Shopping Effort Allocation

Recall Equations (8) and (13) derived in Appendix A.1 and Appendix A.2

$$
\begin{aligned}
c^A &= \frac{\zeta}{\rho-1}\cdot (d^A)^{1+\nu},\\
e &= P^A \cdot c^A.
\end{aligned}
$$

The households' problem can be simplified to

$$
\begin{aligned}
v(e,\Phi) &= \max_{\{d(p,q)\}\geq 0} U\left(\frac{2+\nu-\rho}{1+\nu}\cdot\frac{\zeta}{\rho-1}\cdot\left(d^A\right)^{1+\nu}\right),\\
\text{s.t.} \quad e &= \left(\int_\Phi d(p,q)\,\psi^h(q)\,p^{\frac{1}{1-\rho}}\,dp\,dq\right)^{1-\rho}\cdot\frac{\zeta}{\rho-1}\cdot\left(d^A\right)^{1+\nu},\\
\text{in which} \quad d^A &= \int_\Phi d(p,q)\,dp\,dq.
\end{aligned}
$$

This problem essentially maximizes the total shopping effort d^A subject to the budget constraint, which requires $d(p,q)=0$ for $\forall\{p,q\}\notin \max_{\{p,q\}\in\Phi}\left\{\psi^h(q)\,p^{\frac{1}{1-\rho}}\right\}$, which hence requires

$$
P^A = \left(\int_\Phi d(p,q)\max_{\{p,q\}\in\Phi}\left\{\psi^h(q)\,p^{\frac{1}{1-\rho}}\right\}\,dp\,dq\right)^{1-\rho} = (d^A)^{1-\rho}\cdot\min_{\{p,q\}\in\Phi}\left\{\psi^h(q)^{1-\rho}\,p\right\}.
$$

Therefore, under optimal shopping effort allocation, we have

$$
P^A = \min_{\{p,q\}\in\Phi}\left\{P^A|d^A=d^A(e,\Phi)\right\} = d^A(e,\Phi)^{1-\rho}\cdot\min_{\{p,q\}\in\Phi}\left\{\psi^h(q)^{1-\rho}\,p\right\}\equiv P^A(e,\Phi).
$$

This allows us to solve $d^A(e,\Phi)$ from the budget

$$
e = P^A(e,\Phi)\cdot c^A(e,\Phi) = \frac{\zeta}{\rho-1}\cdot d^A(e,\Phi)^{2+\nu-\rho}\cdot\min_{\{p,q\}\in\Phi}\left\{\psi^h(q)^{1-\rho}\,p\right\}.
$$

Now, we can get

$$
\begin{aligned}
d^A(e,\Phi) &= \left(\frac{\zeta}{\rho-1}\right)^{-\frac{1}{2+\nu-\rho}}\left(\frac{e}{\min_{\{p,q\}\in\Phi}\{\psi^h(q)^{1-\rho}\,p\}}\right)^{\frac{1}{2+\nu-\rho}},\\
c^A(e,\Phi) &= \left(\frac{\zeta}{\rho-1}\right)^{-\frac{\rho-1}{2+\nu-\rho}}\left(\frac{e}{\min_{\{p,q\}\in\Phi}\{\psi^h(q)^{1-\rho}\,p\}}\right)^{\frac{1+\nu}{2+\nu-\rho}},\\
v(e,\Phi) &= U(c^A(e,\Phi),d^A(e,\Phi)).
\end{aligned}
$$

A.4. The Elasticity of Substitution Between Locations and Search Effort in the Matching Function is $\frac{1}{1+\widetilde{\gamma}}$

Consider a **CRS** matching function $\psi(J,D)$ with $\psi^f \equiv \psi/J$ and $q \equiv D/J$,

$$J\psi_J + D\psi_D = \psi \quad \Longrightarrow \quad \psi_J = \frac{\psi - D\psi_D}{J} = \psi^f - q\,\psi_D \quad \Longrightarrow \quad \psi_J/\psi_D = \frac{\psi^f}{\psi_D} - q.$$

Also note that

$$\psi_D = \frac{\partial \psi}{\partial D} = \frac{\partial(\psi/J)}{\partial(D/J)} = \frac{\partial \psi^f}{\partial q} = \psi^f_q.$$

Denote the elasticity of substitution between D and J as ε_{DJ}, we have

$$\begin{aligned}\frac{1-\varepsilon_{DJ}}{\varepsilon_{DJ}} &= \frac{d\ln(\psi_J/\psi_D)}{d\ln(D/J)} - 1 = \frac{d\ln(\psi^f/\psi^f_q - q)}{d\ln(q)} - 1 = \frac{d\ln(\psi^f/(q\psi^f_q)-1)}{d\ln(q)}\\ &= \frac{d\ln(1/\mathcal{E}(q)-1)}{d\ln(q)} = \frac{\mathcal{E}(q)d\ln(1/\mathcal{E}(q)-1)}{\mathcal{E}(q)d\ln(q)} = \frac{\mathcal{E}(q)[d\ln(1-\mathcal{E}(q)) - d\ln(\mathcal{E}(q))]}{d\ln(\psi^f(q))}\\ &= \frac{\mathcal{E}(q)\left[-\frac{d\mathcal{E}(q)}{1-\mathcal{E}(q)} - \frac{d\mathcal{E}(q)}{\mathcal{E}(q)}\right]}{d\ln(\psi^f(q))} = \frac{\mathcal{E}(q)\left[\frac{d(1-\mathcal{E}(q))}{1-\mathcal{E}(q)} + \frac{d(1-\mathcal{E}(q))}{\mathcal{E}(q)}\right]}{d\ln(\psi^f(q))} = \frac{\mathcal{E}(q)\left[\frac{d(1-\mathcal{E}(q))}{(1-\mathcal{E}(q))\mathcal{E}(q)}\right]}{d\ln(\psi^f(q))} = \frac{d\ln(1-\mathcal{E}(q))}{d\ln(\psi^f(q))}.\end{aligned}$$

Note that $\widetilde{\gamma} = \left.\frac{d\ln(1-\mathcal{E}(q))}{d\ln(\psi^f(q))}\right|_{q=Q_{ss}}$. Hence, $\frac{1}{1+\widetilde{\gamma}}$ is the elasticity of substitution between J and D in the matching function ψ evaluated at the steady state. We restrict our attention to matching functions in which the complementarity between J and D is no less than that in Cobb-Douglas, which implies

$$0 < \frac{1}{1+\widetilde{\gamma}} \le 1 \quad \Longleftrightarrow \quad \widetilde{\gamma} \ge 0.$$

CES Example Consider the special case of CES matching function $\psi^f(q) = B\cdot(1-\varphi+\varphi q^{-\gamma})^{-\frac{1}{\gamma}}$.

$$\begin{aligned}\mathcal{E}(q) &= \frac{d\ln(\psi^f(q))}{d\ln(q)} = \frac{-\frac{1}{\gamma}\cdot d\ln(1-\varphi+\varphi q^{-\gamma})}{d\ln(q)} = \frac{-\frac{1}{\gamma}\cdot\frac{-\gamma\cdot\varphi q^{-\gamma-1}}{1-\varphi+\varphi q^{-\gamma}}dq}{\frac{1}{q}dq} = \frac{\varphi\cdot q^{-\gamma}}{1-\varphi+\varphi\cdot q^{-\gamma}}\\ &= 1 - \frac{1-\varphi}{1-\varphi+\varphi\cdot q^{-\gamma}} = 1-(1-\varphi)[\psi^f(q)/B]^{\gamma}.\end{aligned}$$

Now it is clear that

$$\frac{d\ln(1-\mathcal{E}(\widetilde{q}))}{d\ln(\psi^f(\widetilde{q}))} = \gamma.$$

A.5. Conditional on $\frac{d\ln(W)}{d\ln(e)} < 1$, $\widetilde{\gamma} \geq 0$ is the Necessary and Sufficient for Endogenous TFP to Be Always Procyclical Given Any $\widetilde{\kappa} \in (0, +\infty)$ and $\widetilde{\Psi} \in (0, 1)$.

The log-linearized solution of the endogenous TFP is

$$\frac{d\ln(\psi^f(Q^*))}{d\ln(e)} = \frac{(1+\widetilde{\kappa})\widetilde{\Psi} - \widetilde{\kappa}\widetilde{\Psi} \cdot \frac{d\ln(W)}{d\ln(e)}}{1+\widetilde{\kappa}[1+(\widetilde{\gamma}-1)\widetilde{\Psi}]}.$$

As we require $\frac{d\ln(W)}{d\ln(e)} < 1$, the numerator satisfies

$$(1+\widetilde{\kappa})\widetilde{\Psi} - \widetilde{\kappa}\widetilde{\Psi} \cdot \frac{d\ln(W)}{d\ln(e)} > 1 > 0.$$

Sufficiency: When $\widetilde{\gamma} \geq 0$, the denominator satisfies

$$1+\widetilde{\kappa}[1+(\widetilde{\gamma}-1)\widetilde{\Psi}] > 1+\widetilde{\kappa}(1-\widetilde{\Psi}) > 1 > 0.$$

Necessity: For $-1 < \widetilde{\gamma} < 0$, let $\widetilde{\Psi} = \frac{1-\widetilde{\gamma}/2}{1-\widetilde{\gamma}}$ and $\widetilde{\kappa} = \frac{4}{\widetilde{\gamma}}$, then the denominator satisfies

$$1+\widetilde{\kappa}[1+(\widetilde{\gamma}-1)\widetilde{\Psi}] = -1 < 0.$$

A.6. Households' Search Problem with Investment and Saving

This appendix extends Section 2.2.1 by allowing the households to invest and save. Our goal is to show that the elasticities of the tightness and demand functions taken as given in the firms' pricing problem remains unchanged, and hence can be applied in the dynamic model directly.

The representative household has an extra investment choice $x(p, q)$ and a saving choice e^b. It solves a more general search problem with objective function $U(\cdot)$.

$$
\begin{aligned}
v(e,\Phi) &= \max_{\{c(p,q),x(p,q),d(p,q)\}\geq 0,\ e^b} U(c^A, x^A, d^A, e^b),\\
\text{s.t.}\quad e &\geq \int_\Phi d(p,q)\ \psi^h(q)\ p\ c(p,q)\ dp\ dq + \int_\Phi d(p,q)\ \psi^h(q)\ p\ x(p,q)\ dp\ dq + e^b,\\
\text{in which}\quad c^A &\equiv \left(\int_\Phi\ d(p,q)\ \psi^h(q)\ c(p,q)^{\frac{1}{\rho}}\ dp\ dq\right)^\rho,\\
x^A &\equiv \left(\int_\Phi\ d(p,q)\ \psi^h(q)\ x(p,q)^{\frac{1}{\rho}}\ dp\ dq\right)^\rho,\\
d^A &\equiv \int_\Phi\ d(p,q)\ dp\ dq.
\end{aligned}
$$

The F.O.C.s w.r.t. $\{c(p,q), x(p,q), d(p,q), e^b\}$ are

$$
\begin{aligned}
0 &= \frac{\partial \mathcal{L}}{\partial c(p,q)} = U_{c^A}\cdot\rho\cdot(c^A)^{\frac{\rho-1}{\rho}}\cdot d(p,q)\psi^h(q)\cdot\frac{1}{\rho}\cdot c(p,q)^{\frac{1-\rho}{\rho}} - \lambda\cdot d(p,q)\psi^h(q)\ p,\\
0 &= \frac{\partial \mathcal{L}}{\partial x(p,q)} = U_{x^A}\cdot\rho\cdot(x^A)^{\frac{\rho-1}{\rho}}\cdot d(p,q)\psi^h(q)\cdot\frac{1}{\rho}\cdot x(p,q)^{\frac{1-\rho}{\rho}} - \lambda\cdot d(p,q)\psi^h(q)\ p,\\
0 &= \frac{\partial \mathcal{L}}{\partial d(p,q)} = U_{c^A}\cdot\rho\cdot(c^A)^{\frac{\rho-1}{\rho}}\cdot\psi^h(q)\ c(p,q)^{\frac{1}{\rho}} + U_{x^A}\cdot\rho\cdot(x^A)^{\frac{\rho-1}{\rho}}\cdot\psi^h(q)\ x(p,q)^{\frac{1}{\rho}} + U_{d^A}\\
&\qquad\qquad - \lambda\cdot\psi^h(q)\ p\ c(p,q) - \lambda\cdot\psi^h(q)\ p\ x(p,q),\\
0 &= \frac{\partial \mathcal{L}}{\partial e^b} = U_{e^b} - \lambda.
\end{aligned}
$$

Simplifying the expressions and eliminating λ yield

$$
0 = U_{c^A}\cdot\left(\frac{c^A}{c(p,q)}\right)^{\frac{\rho-1}{\rho}} - U_{e^b}\cdot p, \tag{42}
$$

$$
0 = U_{x^A}\cdot\left(\frac{x^A}{x(p,q)}\right)^{\frac{\rho-1}{\rho}} - U_{e^b}\cdot p, \tag{43}
$$

$$
0 = \rho\cdot U_{c^A}\cdot\left(\frac{c^A}{c(p,q)}\right)^{\frac{\rho-1}{\rho}} c(p,q) + \rho\cdot U_{x^A}\cdot\left(\frac{x^A}{x(p,q)}\right)^{\frac{\rho-1}{\rho}} x(p,q) + \frac{U_{d^A}}{\psi^h(q)} - U_{e^b}\cdot p\cdot[c(p,q)+x(p,q)].
$$

Combining these conditions above to eliminate U_{e^b} yields

$$
0 = (\rho-1)\cdot U_{c^A}\cdot\left(\frac{c(p,q)}{c^A}\right)^{\frac{1}{\rho}} c^A + (\rho-1)\cdot U_{x^A}\cdot\left(\frac{x(p,q)}{x^A}\right)^{\frac{1}{\rho}} x^A + \frac{U_{d^A}}{\psi^h(q)}.
$$

Taking integral $\int_\Phi d(p,q)\psi^h(q)\cdot\ dp\ dq$ on both sides of the condition above yields

$$0 = (\rho - 1)\cdot(U_{c^A}\cdot c^A + U_{x^A}\cdot x^A) + U_{d^A}\cdot d^A. \tag{44}$$

Define a price aggregate as in Section 2.2.1

$$p^A \equiv \left(\int_\Phi d(p,q)\ \psi^h(q)\ p^{\frac{1}{1-\rho}}\ dp\ dq\right)^{1-\rho}.$$

Taking integral $\int_\Phi d(p,q)\psi^h(q)\cdot\ dp\ dq$ on Equations (42) and (43) (to the power of $\frac{1}{1-\rho}$) yields

$$U_{c^A} = U_{x^A} = U_{e^b}\cdot p^A. \tag{45}$$

Like Equation (11) in Section 2.2.1, putting Equation (45) back to Equations (42) and (43) yields

$$\frac{c(p,q)}{c^A} = \frac{x(p,q)}{x^A} = \left(\frac{p}{p^A}\right)^{-\frac{\rho}{\rho-1}}. \tag{46}$$

Like Equation (13) in Section 2.2.1, putting Equation (46) back to the budget (binding) yields

$$e = p^A\cdot(c^A + x^A) + e^b. \tag{47}$$

Sharper characterization of d^A calls for a GHH utility on $\{c^A, d^A\}$. Let

$$U(c^A, x^A, d^A, e^b) \equiv \widetilde{U}\left(c^A - \zeta\frac{(d^A)^{1+\nu}}{1+\nu}\right) + \widetilde{V}\left(c^A - \zeta\frac{(d^A)^{1+\nu}}{1+\nu}, x^A, e^b\right),$$

in which $\widetilde{U}(\cdot)$ captures the direct utility for consumption aggregate c^A and shopping aggregate d^A, while $\widetilde{V}(\cdot)$ captures the effects of internal habit $c^A - \zeta\frac{(d^A)^{1+\nu}}{1+\nu}$, investment aggregate x^A, and bond position e^b in continuation values. Let $\widetilde{V}$ be general enough to accommodate the dynamic problem, including also investment adjustment cost captured by $\widetilde{V}(\cdot, x^A, \cdot)$. This GHH utility allows us to simplify Equation (44) for a counterpart of Equation (8) in Section 2.2.1.

$$c^A + x^A = \frac{\zeta}{\rho - 1}\cdot(d^A)^{1+\nu}. \tag{48}$$

Using Equations (47) and (48), we can simplify the households' problem to

$$
\begin{aligned}
v(e,\Phi) = &\max_{c^A,x^A,p^A,d^A,\{d(p,q)\}\geq 0} U\left(c^A, x^A, d^A, e - p^A(c^A+x^A)\right) \\
\text{s.t.}\ \ & p^A = \left(\int_\Phi d(p,q)\,\psi^h(q)\,p^{\frac{1}{1-\rho}}\,dp\,dq\right)^{1-\rho}, \\
& d^A = \int_\Phi d(p,q)\,dp\,dq, \\
& d^A = \left[\frac{\rho-1}{\zeta}(c^A+x^A)\right]^{\frac{1}{1+\nu}}.
\end{aligned}
$$

As long as $U_{e^b} > 0$, which is naturally true, households will choose the markets that minimize p^A for any given (c^A, x^A, d^A). Hence, this problem can be further simplified to

$$
\begin{aligned}
v(e,\Phi) = &\max_{c^A,x^A,d^A\geq 0} U\left(c^A, x^A, d^A, e - (d^A)^{1-\rho}\min_{\{p,q\}\in\Phi}\left\{\psi^h(q)^{1-\rho}\,p\right\}\cdot(c^A+x^A)\right) \\
\text{in which}\ \ & d^A = \left[\frac{\rho-1}{\zeta}(c^A+x^A)\right]^{\frac{1}{1+\nu}}.
\end{aligned}
$$

Here, $v(e,\Phi)$ depends on Φ only through $\min_{\{p,q\}\in\Phi}\left\{\psi^h(q)^{1-\rho}\,p\right\}$. As $U_{e^b} > 0$, $v(e,\Phi)$ is strictly decreasing in $\min_{\{p,q\}\in\Phi}\left\{\psi^h(q)^{1-\rho}\,p\right\}$. As a result, for $\forall\overline{v}$ such that $v(e,\Phi)=\overline{v}$ can hold under some Φ, we can always invert it to solve for $\min_{\{p,q\}\in\Phi}\left\{\psi^h(q)^{1-\rho}\,p\right\}$ as a function $G(e,\overline{v})$.

$$
\min_{\{p,q\}\in\Phi}\left\{\psi^h(q)^{1-\rho}\,p\right\} = G(e,\overline{v}).
$$

Although we no longer have the analytical solution of $G(e,\overline{v})$ like before, $\Phi(e,\overline{v})$ can be described by a function $\widetilde{q}(e,\overline{v},p)$ solved by inverting the equation above like Equation (15) in Section 2.2.1.

$$
\widetilde{q}(e,\overline{v},p) = (\psi^h)^{-1}\left[\left(\frac{p}{G(e,\overline{v})}\right)^{\frac{1}{\rho-1}}\right] \quad\Longrightarrow\quad \frac{p\,\widetilde{q}_p}{\widetilde{q}} = -\frac{1}{(\rho-1)[1-\mathcal{E}(\widetilde{q})]}.
$$

When $\Phi=\Phi(e,\overline{v})$, the variety demand like Equation (16) in Section 2.2.1 from Equation (46) becomes

$$
\begin{aligned}
\frac{\widetilde{c}(e,\overline{v},p)}{c^A(e,\Phi(e,\overline{v}))} = \frac{\widetilde{x}(e,\overline{v},p)}{x^A(e,\Phi(e,\overline{v}))} = \left(\frac{p}{p^A(e,\Phi(e,\overline{v}))}\right)^{-\frac{\rho}{\rho-1}}, \\
\widetilde{y}(e,\overline{v},p) \equiv \widetilde{c}(e,\overline{v},p) + \widetilde{x}(e,\overline{v},p) \quad\Longrightarrow\quad \frac{p\,\widetilde{y}_p}{\widetilde{y}} = -\frac{\rho}{\rho-1}.
\end{aligned}
$$

B. A Toolkit of Using Our Model

B.1. All 35 Variables Needed to Solve the Model (Search Variables in Red)

Variable	Symbol	Trend
Aggregate hours	L	-
Real rental price of capital services	r^k	$(\mu_{ss}^x)^{-t}$
Real marginal cost of composite inputs	mc	$(\mu_{ss}^y)^t$
Gross inflation rate	Π	-
Aggregate market tightness	Q	-
Aggregate real consumption	C	$(\mu_{ss}^y)^t$
Variable capital utilization	u	-
Aggregate real investment (no utilization cost)	X	$(\mu_{ss}^k)^t$
Aggregate capital stock	K	$(\mu_{ss}^k)^t$
Tobin's q	q^k	$(\mu_{ss}^x)^{-t}$
Recursive component for marginal utility of income	H^1	$(\mu_{ss}^y)^{\frac{1}{\rho-1}\cdot t}$
Recursive component for marginal disutility of working	H^2	$(\mu_{ss}^y)^{\frac{\rho}{\rho-1}(1+\xi)\cdot t}$
Optimal reset wage rate	$w^{\#}$	$(\mu_{ss}^y)^t$
Real wage index	w	$(\mu_{ss}^y)^t$
Real GDP	Y	$(\mu_{ss}^y)^t$
Real gross output per match	$\widetilde{Y}$	$(\mu_{ss}^y)^t$
Federal Funds Rate	R	-
Ex ante Federal Funds Rate	R^B	-
Growing factor of neutral technology	μ^n	-
Growing factor of investment-specific technology	μ^x	-
Growing factor of output	μ^y	-
Growing factor of capital	μ^k	-
Number of matched varieties	$\mathcal{I}$	-
Internal habit	h	$(\mu_{ss}^y)^t$
Marginal utility of real consumption	Υ	$(\mu_{ss}^y)^{-t}$
Stochastic discount factor	$\mathcal{M}$	-
Growing factor of investment aggregate	G^x	-
Investment adjustment cost	$\mathcal{A}$	-
Marginal investment adjustment cost	$\mathcal{A}_{G^x}$	-
Capital utilization cost	$\widetilde{\delta}$	-
Marginal capital utilization cost	$\widetilde{\delta}_u$	-
Price adjustment cost	$\widetilde{\chi}$	$(\mu_{ss}^y)^t$
Labor productivity	ℓp	$(\mu_{ss}^y)^t$
Labor share	ℓs	-
Gross markup	Γ	-

Note: The last 13 of the 35 are auxiliary variables used to simplify the algebra.

B.2. All 35 Detrended and Denominated Equations (Search Friction in Red)

1. Production technology (current capital is K^-):

$$\widetilde{Y} + \widetilde{\chi} = A \left(\frac{u\ K^-}{\mu^k} \right)^{\alpha} L^{1-\alpha} - \vartheta. \tag{49}$$

2. Optimality condition for capital-labor allocation:

$$\frac{r^k\ u\ K^-}{\alpha\ \mu^k} = \frac{R^B\ w\ L}{1-\alpha}. \tag{50}$$

3. Real marginal cost of composite inputs:

$$mc = \left(\frac{r^k}{\alpha} \right)^{\alpha} \left(\frac{R^B\ w}{1-\alpha} \right)^{1-\alpha}. \tag{51}$$

4. New Keynesian Philips Curve with Directed Search:

$$\Pi\ (\Pi - 1) = \widetilde{\kappa} \cdot \frac{\Gamma_{ss}}{\rho} \left[\frac{\rho \cdot mc}{\mathcal{I}} - \frac{(\mathcal{I}/B)^{-\gamma}}{1-\varphi} + \frac{\widetilde{\chi}}{\mathcal{I}\widetilde{Y}} \cdot (\rho \cdot mc - 1) \right] + \frac{\mathcal{I}'\widetilde{Y}'}{\mathcal{I}\widetilde{Y}}\ \mathcal{M}' \cdot \Pi'\ (\Pi' - 1). \tag{52}$$

5. Optimality condition for shopping effort:

$$Q = \frac{\rho - 1}{\zeta} \cdot \mathcal{I}^{\rho} \widetilde{Y}. \tag{53}$$

6. Optimality condition for bonds:

$$1 = \frac{\mathcal{M}'}{\mu^{y\prime}} \cdot \frac{R}{\Pi'}. \tag{54}$$

7. Optimality condition capital utilization:

$$r^k = \mathcal{I}^{1-\rho} \cdot \widetilde{\delta}_u. \tag{55}$$

8. Optimality condition for investment:

$$\mathcal{I}^{1-\rho} = q^k \cdot (1 - \mathcal{A} - \mathcal{A}_{G^x} \cdot G^x) + \frac{\mathcal{M}'}{\mu^{y\prime}} \cdot \frac{q^{k\prime}}{\mu^{x\prime}} \cdot \mathcal{A}'_{G^x} \cdot (G^{x\prime})^2. \tag{56}$$

9. Optimality condition capital:

$$1 = \frac{\mathcal{M}'}{\mu^{y\prime}} \cdot \frac{(1-\delta)\cdot q^{k\prime} + r^{k\prime}\cdot u' - \mathcal{I}'^{1-\rho}\cdot \widetilde{\delta}'}{q^k \cdot \mu^{x\prime}}. \tag{57}$$

10. Capital law of motion:

$$K = (1-\delta)\cdot K^-/\mu^k + (1-\mathcal{A})\cdot \mathcal{I}^{\rho-1} X. \tag{58}$$

11. Recursive component for Marginal utility of income:

$$H^1 = \Upsilon \cdot L \cdot w^{\frac{\rho_w}{\rho_w - 1}} + \theta_w \cdot \beta(\mu^{y\prime}\Pi')^{\frac{1}{\rho_w-1}} \cdot H^{1\prime}. \tag{59}$$

12. Recursive component for marginal disutility of working:

$$H^2 = \eta \cdot (L \cdot w^{\frac{\rho_w}{\rho_w-1}})^{1+\xi} + \theta_w \cdot \beta(\mu^{y\prime}\Pi')^{\frac{\rho_w}{\rho_w-1}(1+\xi)} \cdot H^{2\prime}. \tag{60}$$

13. Optimality condition for reset real wage:

$$(w^{\#})^{1+\frac{\rho_w}{\rho_w-1}\xi} = \rho \cdot H^2/H^1. \tag{61}$$

14. Wage aggregator:

$$w^{\frac{1}{1-\rho_w}} = \theta_w \cdot [w^-/(\mu^y \Pi)]^{\frac{1}{1-\rho_w}} + (1-\theta_w)\cdot (w^{\#})^{\frac{1}{1-\rho_w}}. \tag{62}$$

15. Definition of real GDP (capital utilization cost as intermediate inputs):

$$Y = C + X. \tag{63}$$

16. Real gross output (capital utilization cost needs to be produced):

$$\mathcal{I}\widetilde{Y} = C + X + \mathcal{I}^{1-\rho}\, \widetilde{\delta}\, K^-/\mu^k. \tag{64}$$

17. Taylor Rule with no noise:

$$\ln(R^B/R_{ss}) = \rho_R \cdot \ln(R^-/R_{ss}) + (1-\rho_R)\cdot[\phi_\pi \cdot \ln(\Pi/\Pi_{ss}) + \phi_\pi \cdot \ln(Y/Y_{ss})]. \tag{65}$$

18. Realized Federal Funds Rate (current Federal Funds Rate is R' and current shock is $\epsilon^{R'}$):

$$R = R^{B-} - \sigma_R \cdot \epsilon^R. \tag{66}$$

19. Neutral technology process:

$$\ln(\mu^n / \mu^n_{ss}) = \sigma_n \cdot \epsilon^n. \tag{67}$$

20. Investment-specific technology process:

$$\ln(\mu^x / \mu^x_{ss}) = \rho_x \cdot \ln(\mu^{x-} / \mu^x_{ss}) + \sigma_x \cdot \epsilon^x. \tag{68}$$

21. Composite technology for output:

$$\ln \mu^y = \frac{\alpha}{1-\alpha} \ln \mu^x + \ln \mu^n. \tag{69}$$

22. Composite technology for capital:

$$\ln \mu^k = \ln \mu^x + \ln \mu^y. \tag{70}$$

23. Number of matched varieties:

$$\mathcal{I} \equiv B \cdot (1 - \varphi + \varphi \; Q^{-\gamma})^{-\frac{1}{\gamma}}. \tag{71}$$

24. Internal habit (current habit is h^-):

$$h \equiv \mathcal{I}^{\rho-1} C - \zeta \; Z \; Q. \tag{72}$$

25. Marginal utility of real consumption

$$\Upsilon \equiv \mathcal{I}^{\rho-1} \cdot \left[(h - \varsigma h^- / \mu^y)^{-1} - \beta \varsigma \cdot (h' \mu^{y\prime} - \varsigma h)^{-1} \right]. \tag{73}$$

26. Stochastic discount factor:

$$\mathcal{M}' \equiv \beta \, \Upsilon' / \Upsilon. \tag{74}$$

27. Growing factor of investment aggregate

$$G^x \equiv (\mathcal{I}/\mathcal{I}^-)^\rho \cdot (X/X^-) \cdot \mu^k. \tag{75}$$

28. Investment adjustment cost:

$$\mathcal{A} \equiv 1/2 \cdot \left\{\exp\left[\sqrt{\widetilde{\mathcal{A}}}\left(G^x - \mu^k_{ss}\right)\right] + \exp\left[-\sqrt{\widetilde{\mathcal{A}}}\left(G^x - \mu^k_{ss}\right)\right]\right\} - 1. \tag{76}$$

29. Marginal investment adjustment cost:

$$\mathcal{A}_{G^k} \equiv 1/2 \cdot \sqrt{\widetilde{\mathcal{A}}}\left\{\exp\left[\sqrt{\widetilde{\mathcal{A}}}\left(G^x - \mu^k_{ss}\right)\right] - \exp\left[-\sqrt{\widetilde{\mathcal{A}}}\left(G^x - \mu^k_{ss}\right)\right]\right\}. \tag{77}$$

30. Capital utilization cost:

$$\widetilde{\delta} \equiv \sigma_a\sigma_b/2 \cdot (u-1)^2 + \sigma_b \cdot (u-1). \tag{78}$$

31. Marginal capital utilization cost:

$$\widetilde{\delta}_u \equiv \sigma_a\sigma_b \cdot (u-1) + \sigma_b. \tag{79}$$

32. Price adjustment cost:

$$\widetilde{\chi} \equiv \frac{\rho}{2(\rho-1)\Gamma_{ss} \cdot \widetilde{\kappa}} \cdot (\Pi/\Pi_{ss} - 1)^2 \cdot \mathcal{I}\ \widetilde{Y}. \tag{80}$$

33. Labor productivity:

$$\ell p \equiv Y/L. \tag{81}$$

34. Labor share:

$$\ell s \equiv w\ L/Y. \tag{82}$$

35. Gross markup:

$$\Gamma \equiv \mathcal{I}/mc. \tag{83}$$

B.3. 35 Objects to Know in the Calibration and Solution of Steady State

Solve the Parameters for Targets and Some Variables as Byproduct The predetermined targets include the steady state gross inflation Π_{ss} and another 13 objects in Table 2 below[22]

$$\left(u_{ss}, Y_{ss}, L_{ss}, Q_{ss}, \mu^y_{ss}, \mu^k_{ss}, \frac{R_{ss}}{\Pi_{ss}}, \delta\frac{\mathcal{I}^{\rho-1}_{ss}K_{ss}}{Y_{ss}}, \frac{X_{ss}}{Y_{ss}}, \ell s_{ss}, \mathcal{I}_{ss}, \text{wage markup}, \text{wage duration}\right).$$

The first 11 objects above are parts of or functions of the 35 variables needed to solve the model, while the last two do not have such correspondence. We use the first 11 steady state objects above as targets to solve for 11 parameters (marked in red below). As a byproduct, another 11 steady state variables are also solved. Wage markup and duration directly determine the values of parameters $\{\rho_w, \theta_w\}$, which we will use together with the draw of our 15 estimated parameters as in Table 3 in the calibration and steady state solving procedure. Now we have the 22 equations below as solutions for the 11 parameters and 11 byproducts. Given Π_{ss} already known, we have now pinned down the values of 23 objects.

1. Given $\mathcal{I}_{ss}$, solve for the mean efficiency of matching function

$$B = \mathcal{I}_{ss}.$$

2. Given $\{\mu^y_{ss}, \mu^k_{ss}\}$, solve for growing factor of investment-specific technology

$$\mu^x_{ss} = \mu^k_{ss}/\mu^y_{ss}.$$

3. Given $\{\mu^y_{ss}, \mu^x_{ss}\}$, solve for growing factor of investment-specific technology

$$\mu^n_{ss} = \mu^y_{ss} \cdot (\mu^x_{ss})^{-\frac{\alpha}{1-\alpha}}.$$

4. Given $\{\mu^y_{ss}, R_{ss}/\Pi_{ss}\}$, solve for the discount factor β

$$\beta = \mu^y_{ss}\Pi_{ss}/R_{ss}.$$

[22] In our paper, we have imposed $u_{ss} = Y_{ss} = L_{ss} = Q_{ss} = 1$ as is in Table 2. In fact, the steady state solution procedure we use relies on $u_{ss} = Q_{ss} = 1$ but not on how we normalize Y_{ss} and L_{ss}. These 14 objects have incorporated the full information of the two variables $\{R_{ss}, X_{ss}\}$, so that we can use them directly in our calibration and steady state solving.

5. Given $\{\mu_{ss}^k, X_{ss}/Y_{ss}, \delta\mathcal{I}_{ss}^{1-\rho}K_{ss}/Y_{ss}\}$, solve for the depreciation rate δ

$$\delta = \frac{\mu_{ss}^k - 1}{\frac{X_{ss}/Y_{ss}}{\delta\mathcal{I}_{ss}^{1-\rho}K_{ss}/Y_{ss}} - 1}.$$

6. Given $\{\beta, \delta, \mu_{ss}^k\}$, solve for the marginal cost of utilization cost σ_b

$$\sigma_b = \mu_{ss}^k/\beta - 1 + \delta.$$

7. Given $\{\beta, \delta, \mu_{ss}^k, \mu_{ss}^y, \ell s_{ss}, \delta\mathcal{I}_{ss}^{1-\rho}K_{ss}/Y_{ss}\}$, solve for the capital share α

$$\alpha = \left(\frac{\ell s_{ss}}{\delta\mathcal{I}_{ss}^{1-\rho}K_{ss}/Y_{ss}} \cdot \frac{\delta}{\sigma_b} + 1\right)^{-1}.$$

8. Given $\{\alpha, B, \ell s_{ss}, Y_{ss}\}$, solve for the fixed cost parameter ϑ

$$\vartheta = \left(\frac{\ell s_{ss}}{1-\alpha} \cdot \widetilde{\Gamma} - 1\right) \cdot \frac{Y_{ss}}{\mathcal{I}_{ss}}.$$

9. Given $\{\widetilde{\Psi}, \widetilde{\Gamma}\}$, solve for the love for varieties ρ

$$\rho = \frac{\sqrt{(2+\widetilde{\Gamma}-\widetilde{\Psi}^{-1})^2 + 4\left(\widetilde{\Psi}^{-1}-1\right)} + (2+\widetilde{\Gamma}-\widetilde{\Psi}^{-1})}{2}.$$

10. Given $\{\rho, \widetilde{\Gamma}\}$, solve for the search share in matching function φ

$$\varphi = 1 - \widetilde{\Gamma}/\rho.$$

11. Given $\{\sigma_b, \rho, \mathcal{I}_{ss}\}$, solve for the real rental price of capital r_{ss}^k

$$r_{ss}^k = \mathcal{I}_{ss}^{1-\rho} \cdot \sigma_b.$$

12. Given $\{\ell s_{ss}, L_{ss}, Y_{ss}\}$, solve for the real wage index w_{ss}

$$w_{ss} = \ell s_{ss} \cdot (Y_{ss}/L_{ss}).$$

13. Given $\{\mathcal{I}_{ss}, \widetilde{\Gamma}\}$, solve for the real marginal cost of inputs mc_{ss}

$$mc_{ss} = \mathcal{I}_{ss}/\widetilde{\Gamma}.$$

14. Given $\{\alpha, r^k_{ss}, R_{ss}, w_{ss}, mc_{ss}\}$, solve for the units in production function A

$$A = \left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha} \left(\frac{R_{ss} w_{ss}}{1-\alpha}\right)^{1-\alpha} \frac{1}{mc_{ss}}.$$

15. Given $\{X_{ss}, Y_{ss}\}$, solve for the real consumption C_{ss}

$$C_{ss} = Y_{ss} - X_{ss}.$$

16. Given $\{\rho, \mathcal{I}_{ss}, Y_{ss}, Q_{ss} = 1\}$, solve for the weight of search in utility ζ

$$\zeta = (\rho - 1)\, \mathcal{I}_{ss}^{\rho-1} \cdot Y_{ss}.$$

17. Given $\{\zeta, C_{ss}, \mathcal{I}_{ss}, Q_{ss} = 1\}$, get the interval habit term H_{ss}

$$h_{ss} = \mathcal{I}_{ss}^{\rho-1} C_{ss} - \zeta.$$

18. Given $\{\beta, \varsigma, \mu^y_{ss}, \mathcal{I}_{ss}, h_{ss}\}$, get the marginal utility of real consumption Υ_{ss}

$$\Upsilon_{ss} \equiv \mathcal{I}_{ss}^{\rho-1} \cdot \left(\frac{\mu^y_{ss} - \varsigma}{\mu^y_{ss} - \beta\varsigma} h_{ss}\right)^{-1}.$$

19. Given $\{asdf\}$, solve for the optimal reset wage rate $w^{\#}_{ss}$

$$w^{\#}_{ss} = \left[\frac{1-\theta_w}{1-\theta_w(\mu^y_{ss}\Pi_{ss})^{\frac{1}{\rho_w-1}}}\right]^{\rho_w-1} w_{ss}.$$

20. Given $\{\theta_w, \rho_w, \beta, \mu^y_{ss}, w_{ss}, \Upsilon_{ss}, L_{ss}, \Pi_{ss}\}$, solve for the recursive component H^1

$$H^1_{ss} = \frac{\Upsilon_{ss} \cdot L_{ss} \cdot w_{ss}^{\frac{\rho_w}{\rho_w-1}}}{1-\theta_w \cdot \beta(\mu^y_{ss}\Pi_{ss})^{\frac{1}{\rho_w-1}}}.$$

21. Given $\{\xi, \rho_w, w_{ss}^{\#}, H_{ss}^1\}$, solve for the recursive component H^2

$$H_{ss}^2 = \rho_w^{-1} \cdot (w_{ss}^{\#})^{1+\frac{\rho_w}{\rho_w - 1}\xi} \cdot H_{ss}^1.$$

22. Given $\{\xi, \theta_w, \rho_w, \beta, \mu_{ss}^y, w_{ss}, L_{ss}, H_{ss}^2, \Pi_{ss}\}$, solve for the weight of labor in utility η

$$\eta = \frac{\left[1 - \theta_w \cdot \beta(\mu_{ss}^y \Pi_{ss})^{\frac{\rho_w}{\rho_w - 1}(1+\xi)}\right] \cdot H_{ss}^2}{\left(L_{ss} \cdot w_{ss}^{\frac{\rho_w}{\rho_w - 1}}\right)^{1+\xi}}.$$

Note that the calibration of parameters $\{\vartheta, A\}$ depends on the draw of $\widetilde{\Gamma}$, the calibration of ζ depends on the draw of $\{\widetilde{\Gamma}, \widetilde{\Psi}\}$, and the calibration of η depends on the draw of $\{\widetilde{\Psi}, \xi\}$.

Other Steady State Variables Other 12 remaining steady-state variables (35 objects to know in total - 23 already known) can be obtained straightforwardly as below.

1. Obtain the capital stock from optimal input allocation

$$K_{ss} = \frac{\alpha}{1-\alpha} \cdot \frac{R_{ss} w_{ss}}{r_{ss}^k} \cdot L_{ss} \cdot \mu_{ss}^k.$$

2. Obtain the Tobin's q by imposing zero investment adjustment cost

$$q_{ss}^k = \mathcal{I}_{ss}^{1-\rho}.$$

3. Real gross output per match = output / matches

$$\widetilde{Y}_{ss} = Y_{ss}/\mathcal{I}_{ss}.$$

4. Ex ante Federal Funds Rate = the ex post one

$$R_{ss}^B = R_{ss}.$$

5. Stochastic discount factor = discount factor

$$\mathcal{M}_{ss} = \beta.$$

6. Growing factor of investment aggregate = growth factor of investment

$$G^x_{ss} = \mu^k_{ss}.$$

7. Zero investment adjustment cost

$$\mathcal{A}_{ss} = 0.$$

8. Zero marginal investment adjustment cost

$$\mathcal{A}_{G^x,ss} = 0.$$

9. Zero capital utilization cost

$$\widetilde{\delta}_{ss} = 0.$$

10. Constant market capital utilization cost

$$\widetilde{\delta}_{u,ss} = \sigma_b.$$

11. Zero price adjustment cost

$$\widetilde{\chi}_{ss} = 0.$$

12. Definition of labor productivity

$$\ell p = Y_{ss}/L_{ss}.$$