The real channel for nominal bond-stock puzzles[*]

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Abstract

We present evidence that the mix of transitory and permanent shocks to consumption is changing over time. We identify three regimes: two highly persistent regimes where either permanent or transitory shocks are relatively more dominant, and a disaster regime that is largely transitory. We study implications of this finding for asset prices. The transition from the second to the first regime in the mid-1990s makes the correlation between equities and bonds switch sign from positive to negative as in the data. The real bond and equity yield curves are approximately flat. The nominal bond curve is upward sloping. These results are achieved without relying on the nominal channel too much. That is, as in the data, the variation of inflation in the model is under 40% as a fraction of variation in nominal yields.

JEL Classification Codes: E21, E31, E43, E44, G12.

Keywords: permanent and transitory components of consumption, bond-stock comovement, bond yield curve, equity yield curve

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1 Introduction

The correlation between stock market and nominal Treasury bond returns changed sign around 1998, from positive to negative (Baele, Bekaert, and Inghelbrecht, 2010, Campbell, Shiller, and Viceira, 2009). Leading explanations of this phenomenon are based on the nominal channel: changing correlation between consumption growth and inflation (David and Veronesi, 2013, Campbell, Pfluger, and Viceira, 2020). In this paper we explore the real channel. Specifically, we argue that the changing nature of consumption dynamics goes a long way towards explaining the stock-bond correlation, real and nominal bond yield curves, as well as the equity yield curve.

We model the aggregate consumption level as the sum of two components that we label permanent and transitory, respectively. The novelty is that we allow for changes in the relative contribution of these components via regime-changing volatilities of their shocks. If the permanent component dominates, the economy behaves similar to workhorse asset-pricing models and features positive covariance between realized and expected consumption growth. In contrast, if the transitory component dominates, that covariance becomes negative.

We use consumption data only to address the "dark matter" concern (Chen, Dou, and Kogan, 2017). We identify three regimes. The changes between two of these regimes occur at a very low frequency. The "permanent" regime has appeared sporadically in the early sample, primarily at the peak of expansions, and prevails in the post-1995 period. The "transitory" regime has prevailed throughout the early part of our sample, up until about 1995. The third regime is even more transient than the "transitory" one, more rare, and features much higher volatility of innovations. We interpret it as a time-varying disaster state (Barro, 2006, Gabaix, 2012, Wachter, 2013).

We explore asset pricing implications of these dynamics in two steps. First, we assume a representative agent with log utility preferences and explore assets whose price can be computed from consumption alone: real bond yields and the consumption claim. In the later "permanent" regime, negative shocks to consumption tend to be followed by lower than average growth (positive conditional covariance between realized and expected consumption growth), making real bonds hedges. In the earlier "transitory" regime, however, negative shocks tend to be followed by higher than average growth (negative conditional covariance between realized and expected consumption growth), which makes real bonds risky. The disaster regime is even more strongly affected by such transitory shocks as crisis periods are followed by a recovery. Because real bonds transition from being risky to becoming hedges, the model implied correlation between consumption claim and real bond returns switches sign as well.

Second, we link the model's implications to observable asset prices, namely, aggregate equity and nominal bonds. Doing so requires more elaborate preferences and we choose to work with the Epstein and Zin (1989) ones. Further, one needs to specify and calibrate cash flows, dividends and inflation, respectively. Because our focus is on the role of consumption dynamics for asset pricing, we posit cash flows dynamics that are simple and in line with the literature. We show that the model is capable of generating realistic baseline asset-pricing moments, including risk premiums, return volatility, and predictability of both equity and bond excess returns. Also, the estimated consumption combined with calibrated cash flows are consistent with the nominal bond (e.g., Bansal and Shaliastovich, 2013, Piazzesi and Schneider, 2006) and equity yield curves (e.g., van Binsbergen, Brandt, and Koijen, 2012, Giglio, Kelly, and Kozak, 2020).

The results obtained using consumption dynamics alone help us capture evidence emphasized in the recent literature. Because the model generates sign-switching correlations between the consumption claim and real bond returns, it continues doing so for the equity claim and nominal bonds. That is, the model generates this pattern via the real channel, as emphasized by Campbell, Shiller, and Viceira (2009), Duffee (2018a), and Liu (2020).

Further, the model implies a flat real bond curve. That bonds are hedges in the permanent regime, pushes bond yields down as horizon increases. When bonds are risky the effect is the opposite. In the disaster regime long-term bond yields are pushed up even more strongly relative to short-term bond yields. Averaging across the different regimes delivers a flat curve. Because of that, the model does not require a lot of variation in inflation relative to overall variation in nominal yields to match the nominal bond curve. Thereby, our model is capable of addressing the Duffee (2018b) critique of the current mainstream asset-pricing models.

The nature of macroeconomic fluctuations has preoccupied economists since at least Hayek (1933) and, more recently, Lucas (1977). While the starting point was that departures from the trend are of a transitory nature, the literature has gradually shifted towards recognition that there are shocks to the trend itself, aka permanent shocks (Campbell and Mankew, 1987, Cochrane, 1988, among many others). Alvarez and Jermann (2005) demonstrate that asset prices are particularly sensitive to the differences in these perspectives on the macroeconomic dynamics.

The asset-pricing literature has primarily focused on the other end of the spectrum, that is, models where permanent shocks to consumption are the dominant driving force (e.g., Bansal and Yaron, 2004, Campbell and Cochrane, 1999). This permanent shock paradigm is experiencing difficulty matching some key asset-pricing facts, such as small positive slopes of the real yield curve (Backus, Chernov, and Zin, 2014) and the equity yield curve (van Binsbergen, Brandt, and Koijen, 2012), as well as the changing sign of the real equity-bond covariance.

Campbell (1986) studies theoretically joint modeling of bonds and stocks allowing for both transitory and permanent effects in consumption. Existing empirical work typically addresses a subset of the facts associated with bonds and stocks. Campbell, Pfluger, and Viceira (2020), David and Veronesi (2013), and Fang, Liu, and Roussanov (2021) focus on stock-bond correlations. Song (2017) additionally focuses on the nominal yield curve vs consumption-inflation correlation. These papers rely on the nominal component as the key channel, which Duffee (2018a,b) argues is counterfactual. Gomez-Cram and Yaron (2021) specifically target that critique. They achieve success in matching the nominal and real yield curves via preferences that combine Epstein-Zin utility with shocks to the time preference parameter. They do not discuss joint comovement between bonds and stocks.

Bansal, Kiku, and Yaron (2010) and Hassler and Marfe (2016) advocate permanent, transitory, and disaster components albeit in a functional form that is different from ours. In particular, the composition of permanent and transitory shocks is time-invariant. Research questions are different as well. The former paper focuses on the single-horizon equity premium and interest rate. The latter focuses exclusively on the term structure of dividend strips. Croce, Lettau, and Ludvigson (2015) study the effect of a bounded-rational agent who is filtering unobserved expected consumption and dividend growth rates. They extend the Bansal and Yaron (2004) model by adding additional shocks and allowing for time-invariant conditional covariances between them. The authors demonstrate, in the context of dividend strips, that filtering of the unobserved growth rates may switch the roles of persistent vs transitory shocks to consumption growth. Backus, Boyarchenko, and Chernov (2018) (section 6), motivated by the evidence about real and nominal bond and equity yield curves, advocate a combination of a less persistent expected consumption growth and disasters in an Epstein and Zin (1989) economy. Again, there is no time variation in the relative contribution of these components.

In contemporaneous work, Jones and Pyun (2021) extend the Bansal and Yaron

(2004) model with exogenously specified time-varying covariance between realized and expected consumption growth. The time-varying mix of the permanent and transitory components, which we advocate and explicitly model, is implicit in their specification and is referred to as time-varying consumption growth persistence. The authors primarily focus on the stock-bond covariance and on the leverage effect for equities. Implementation relies on calibration rather than on estimation of the consumption dynamics.

Nakamura, Steinsson, Barro, and Ursua (2013) and Nakamura, Sergeyev, and Steinsson (2017) estimate consumption dynamics using a cross-section of countries and a long sample. They focus on estimating consumption disasters, typically associated with wars, and time-variation in the conditional mean and its volatility. They do not allow for time-variation in the relative magnitude of permanent and transitory shocks as is our focus.

No-arbitrage models of stock-bond comovement are represented by Backus, Boyarchenko, and Chernov (2018), Campbell, Sunderam, and Viceira (2017), Koijen, Lustig, and Nieuwerburgh (2017), and Lettau and Wachter (2011). Laarits (2021) uses the latter model to study high-frequency variations in the stock-bond covariance with a focus on the role of precautionary savings. That effect is present in our equilibrium as well with different regimes generating different quantitative impact.

2 Model

We start by presenting initial regression-based evidence about changing properties of consumption growth. Next, we use the evidence as a motivation for the posited consumption dynamics. We describe how we estimate the model. We conclude by presenting and discussing the implications of the estimated dynamics.

2.1 Preliminary evidence

A real channel for the changing bond-stock correlation suggests that consumption dynamics are different across the two periods. We show that the autocorrelation of consumption growth indeed is significantly and economically different across these periods. We interpret this evidence as arising from a changing mix of permanent versus transitory shocks to the economy.

Specifically, consider the regression:

$$\Delta c_{t+1} = \alpha_0 + \alpha_1 \mathbb{1}_{t<1998} + \beta_0 \Delta c_t + \beta_1 \times (\mathbb{1}_{t<1998} \times \Delta c_t) + \epsilon_{t+1}, \tag{1}$$

where Δc_t is the change in log consumption growth, and $\mathbb{1}_{t<1998}$ equals one if observation t is in the pre-1998 sample and zero otherwise. If β_1 is significantly different from zero, there is different autocorrelation in the period before 1998 versus that after.

We use the real-time data series on real personal consumption expenditures (PCE) from 1947:Q2 to 2020:Q4, which are available from the Federal Reserve Bank of Philadelphia. We use real-time consumption data to best align them with investors' information set at the time they set asset prices. PCE offers the longest span of such data.[1]

In our first set of regressions, we use data up until 2019:Q4 so the Covid period is excluded from the sample. A simple regression is not well-suited to handle the extreme movements in consumption growth that occurred during that period. We also consider regressions where we exclude all NBER recession periods to check that the results are not due to the lower frequency of recessions in the later sample. When we do that, we ensure that the right hand side in the regression (observation t)

[1] As a robustness check, we also use revised real per capita nondurable and services consumption. The results are similar. See Appendix A.

is always the previous quarter relative to $t+1$ whether a NBER recession or not. Using NBER dates introduces a look-ahead bias, so the nature of these regressions is illustrative.

Table 1 shows the estimated coefficients. The results with and without NBER recessions are qualitatively similar. Our discussion focuses on the latter (right side of the Table). The column labeled (1) shows that $\beta_0 = 0.25$ and $\beta_1 = -0.39$, significant at the 5% and 1% level, respectively. Thus, the autocorrelation coefficient in the pre-1998 period is -0.14 ($\beta_0 + \beta_1 = 0.25 - 0.39$), while in the later period it is 0.25 (β_0). The significance of β_1 means this difference is statistically significant, and the economic magnitude is substantial.

The intercept is also statistically different across the two periods, as one would expect given that the slope is significantly different. In order to ensure that this intercept change does not reflect significantly different means of consumption growth, we also run a regression where we impose that the unconditional mean is the same across the two periods. In particular, we demean the consumption growth data we use in the regression across the whole sample and run the regression (1) setting $\alpha_1 = 0$. The results are reported in Table 1 in the column labeled (2), which shows that the effect is slightly stronger in this case.

Another manifestation of the increasing persistence of consumption growth is reflected in changing autocorrelations displayed in Figure 1. To show robustness to the choice of consumption data, we in this case use per capita and deseasonalized revised consumption from NIPA over the same sample. First, we observe that correlations at different lags are uniformly and significantly higher in the post-1997 sample relative to those in the early sample. Second, such increased persistence is consistent with higher variance in the long-run consumption response relative to the variance of the shock itself. Since the long-run variance of transitory shocks is zero, this evidence suggests this later period has seen a increase in the relative magnitude of permanent versus transitory components in consumption.

In this preliminary analysis, we made two short-cuts. First, we imposed the breakpoint as 1998, motivated by the evidence on the stock-bond correlation. Second, we excluded the Covid crisis and, in one specification, business cycle downturns in general. In the sequel, we present our model of consumption dynamics and estimate it, including the timing of different regimes and accounting for crisis periods. The conclusion remains the same.

2.2 Consumption dynamics

We model aggregate log consumption as the sum of a deterministic trend and two persistent components, $c_{p,t}$ and $c_{\tau,t}$, which we label permanent and transitory, respectively:

$$c_t = \mu_c t + c_{p,t} + c_{\tau,t}, \tag{2}$$

implying the following growth rate

$$\Delta c_{t+1} = \mu_c + \Delta c_{p,t+1} + \Delta c_{\tau,t+1}. \tag{3}$$

We model the permanent and transitory components as follows:

$$\begin{aligned} \Delta c_{p,t+1} &= \rho_p \Delta c_{p,t} + \epsilon_{p,t+1}, \\ c_{\tau,t+1} &= \rho_\tau c_{\tau,t} + \epsilon_{\tau,t+1}, \end{aligned} \tag{4}$$

where $\epsilon_{j,t+1}$ are mean-zero shocks and $j \in \{p, \tau\}$. Thus, the permanent component contains a unit root while the transitory component does not. In particular, if $\rho_p = 0$

then the permanent component, $c_{p,t}$, is a random walk. General equilibrium models with production, imply that a combination of permanent and transitory shocks to productivity leads to endogenous consumption dynamics similar to those specified above (e.g., Blanchard, L'Huillier, and Lorenzoni, 2013, Kaltenbrunner and Lochstoer, 2010).

Figure 2 demonstrates the different roles of the two components via impulse responses. A positive shock to the permanent component (the blue line) leads to a persistent increase in consumption since $\rho_p > 0$. Thus, the shock leads to an increase in future expected consumption growth. This is akin to the endogenous consumption response to a permanent technology shock in standard real business cycle models. A positive shock to the transitory component (the red line), however, is associated with a reversal. Thus, in this case a positive shock leads to a negative shock to future expected consumption growth. This is akin to the endogenous consumption dynamics that arises in real business cycle models when the technology shock is transitory.

For simplicity, we set $\rho = \rho_p = \rho_\tau$ and note that

$$\rho \Delta c_{p,t+1} + (\rho - 1) c_{\tau,t+1} \equiv x_{t+1} = \rho x_t + \rho \epsilon_{p,t+1} + (\rho - 1)\epsilon_{\tau,t+1}. \tag{5}$$

We can then conveniently express consumption growth as

$$\Delta c_{t+1} = \mu_c + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}. \tag{6}$$

There are two approaches that would be consistent with the motivating evidence presented in the previous section, ex-ante. One could capture changing autocorrelation of consumption growth by making the coefficient ρ in Equation (5) time-varying. Alternatively, the variance of the shocks ϵ could be changing leading to time-variation in the relative magnitude of the two consumption components. Such variation would manifest itself in changing autocorrelation. We choose the latter approach as it allows us to connect directly to the literature on the nature of macroeconomic fluctuations.

Specifically, we assume $\epsilon_{j,t+1} \sim N(0, \sigma_j^2(S_{t+1}))$, where S_{t+1} is a discrete Markov state variable that takes on N values $S_{t+1} \in \{1, ..., N\}$. Agents observe the current regime S_{t+1} and make forecast of future regime based on the transition matrix below

$$\mathbb{P} = \begin{bmatrix} p_{11} & \dots & p_{1N} \\ \vdots & \ddots & \vdots \\ p_{N1} & \dots & p_{NN} \end{bmatrix} \tag{7}$$

where $\sum_{i=1}^{N} p_{ji} = 1$.

These changing volatilities drive time-variation in the relative importance of the long-run and short-run shocks that are depicted in Figure 1 via the relative magnitudes of the permanent and transitory components, as represented by $\sigma_p(S_{t+1})$ and $\sigma_\tau(S_{t+1})$, respectively. Further, Figure 2 implies that a higher proportion of the permanent component in consumption yields higher autocorrelations of consumption *growth*. This occurs as the correlation between shocks to realized versus future expected consumption growth has different signs across the permanent (positive) and transitory (negative) components. The model captures this effect via

$$Cov(\Delta c_{t+1}, x_{t+1} | x_t, S_{t+1}) = \rho \sigma_p^2(S_{t+1}) - (1 - \rho) \sigma_\tau^2(S_{t+1}). \tag{8}$$

The time variation in covariance is, again, driven by the relative magnitudes of $\sigma_p(S_{t+1})$ and $\sigma_\tau(S_{t+1})$. It could even switch signs because $\rho < 1$.

These observations prompt us to introduce an explicit measure of the relative contribution of the transitory, c_τ, and permanent, c_p, components to shocks to expected consumption growth via

$$\eta(S_{t+1}) = \frac{(1-\rho)\sigma_\tau^2(S_{t+1})}{\rho\sigma_p^2(S_{t+1})}. \tag{9}$$

When $\eta(S_{t+1}) = 1$ the covariance between realized and expected consumption growth is equal to zero. If $\eta(S_{t+1}) > 1$, the transitory component dominates, and the covariance is negative.

2.3 Estimation

Our preferred model features three different Markov states. That is the most parsimonious specification where the first two states capture economic phenomena that are more slowly moving than business cycles. The remaining third state captures business cycle downturns (combined, for parsimony, with potential disasters in the economy).

We estimate the model via maximum likelihood. See Appendix B for the regime-switching state-space representation as well as the evaluation of the likelihood function. Regimes are labeled by imposing that

$$\sigma_\tau(1) < \sigma_\tau(2) < \sigma_\tau(3). \tag{10}$$

Also, to ensure that the third regime is associated with the worst states, we assume

$$\sigma_p(3) > \max\{\sigma_p(1), \sigma_p(2)\}. \tag{11}$$

In order to reduce the number of parameters to estimate, we impose that

$$var(x_{t+1}|x_t, S_{t+1} = 1) = var(x_{t+1}|x_t, S_{t+1} = 2).$$

Restricting expected consumption growth to be homoscedastic across regimes 1 and 2 means that these regimes are identified by the relative fraction of permanent versus transitory shocks, which is the economic effect we are focusing on in this paper, as opposed to the Great Moderation or business cycle variation in overall volatility. The restriction implies that

$$\sigma_p(1)^2 = \sigma_p(2)^2 + \left[\frac{1-\rho}{\rho}\right]^2 \Big(\sigma_\tau(2)^2 - \sigma_\tau(1)^2\Big). \tag{12}$$

Restrictions (10)-(12) together imply that the 1st (3rd) regime has the smallest (largest) conditional variance for realized consumption growth.

2.4 Evidence

Table 2 reports estimated parameters of consumption dynamics and Figure 3 displays the probabilities of the regimes.[2] The first regime, when compared to the second regime, features lower volatility of the permanent shock (0.13% vs 0.30% per quarter) and lower volatility of the transitory shock (0.11% vs 0.77%). The third regime is a high volatility regime with the values of 0.31% for the permanent shock and 7.69% for the transitory one. Thus, our estimation implies that shocks to the transitory consumption component are the biggest drivers of short-run consumption volatility. This is very different from the long-run risk literature, which relies on shocks to the permanent consumption component.

Our estimates imply that the relative contribution of the transitory and permanent components, η, is 0.62, 5.74, and 72.12 for the three regimes, respectively. See Equation (9). This indicates that the first regime has the highest relative contribution

[2] We continue using per capita PCE data. We also estimate the model using quarterly revised real per capita nondurable and services consumption. The regimes and their intuition are very similar, though volatility is somewhat lower. See Appendix A.

of the permanent component, and that is why we label this regime as "permanent." After the mid-1990s, this regime dominates with the exception of the 2008 and the COVID crises. The second regime, which we label "transitory" as its $\eta > 1$ prevails up to the mid-1990s with a few exceptions occurring usually at expansion peaks.

The third regime occurs at various points throughout the sample and is capable of generating much more adverse events than the other two regimes. This regime captures bad states of the economy that are associated with particularly volatile transitory shocks, and we therefore label it as a disaster regime. In our model this regime is distinct from regular recessions, including the Great Recession, which are driven relatively more by the permanent component of c_t. That is reflected in x_t becoming low and remaining so for an extended period. See the last panel of Figure 3. In contrast, disasters are associated with a high conditional mean of consumption growth. That is because the disaster shocks are transitory and the level of consumption therefore reverts to the trend relatively quickly. These episodes occur during the Korean war, oil shocks, the 1981 monetary recession, and the COVID crisis.

The expected duration of regime j is roughly $1/(1-p_{jj})$ (e.g., Kim and Nelson, 1999). Thus, the expected duration of the first two regimes is 12 and 9 years, respectively, compared to less than a year for the third regime. These values highlight that the low-frequency shifts in the economy are captured in the first two regimes.

2.5 Discussion

Our model contributes to the existing literature by documenting that a mix of transitory and permanent shocks to aggregate consumption is changing over time. That potentially explains disagreement in the earlier literature that reached different conclusions using classical time-series methods: a specific sample could have tilted analyses towards one of the configurations. Next, this finding raises questions of how that matters and why that happens.

As regards the first question, Cochrane (1988) points out that "... the size or existence of a random walk component in GNP cannot directly distinguish broad classes of economic theories of the business cycle ...". Yet, Alvarez and Jermann (2004, 2005) forcefully demonstrate that asset prices are informative about the marginal rate of substitution and macroeconomic fluctuations. Therefore, it is natural to take exploration of the consumption dynamics that we have uncovered to asset-pricing data. Specifically, we evaluate if one can make progress on capturing the most recent puzzles that pertain to the interaction between stocks and bonds.

As regards the second question, Kaltenbrunner and Lochstoer (2010) show that the permanent or transitory components arise in general equilibrium with production depending on the nature of productivity shocks. Specifically, a positive permanent shock to the level of productivity leads to dynamics that are similar to the permanent consumption component we specify exogenously here with $\rho_p > 0$. Indeed, in response to this shock investors initially increase investment to bring the capital stock to its new optimal long-run level, which in turn temporarily depresses consumption before it converges to its new optimal level. In contrast a transitory shock to productivity leads to a transitory shock in consumption, similar to the transitory consumption component we specify exogenously here. Both specifications of productivity can account for the standard macroeconomic moments, but have markedly different effects on asset prices.

3 Asset-pricing implications

The key objective of this section is to evaluate implications of the estimated consumption dynamics for observed asset prices. However observable prices depend, besides consumption and preferences, on cash flows (dividends and inflation). Before we specify these additional cash flow dynamics, we develop intuition about the role of time-varying volatility of the transitory and permanent consumption components in a simplified setting of a representative agent with log preferences.

Next, we proceed with a more realistic implications in the context of recursive preferences, adding exogenous dynamics of dividends and inflation. While there are many plausible specifications of these two types of cash flows, the debate about the "best" specification continues unabated. Thus, instead of an extensive discussion, we assume something relatively simple in line with the literature (traditional conditional mean dynamics complemented with exposures to the consumption shocks).

Further, because of our main focus, we do not want to estimate cash flow growth and consumption growth jointly so that the inferred consumption dynamics are not affected by the choice or properties of the other series. Thus, we calibrate cash flow dynamics to illustrate the pricing implications.

3.1 Intuition

The regime-switching sign of serial covariance of consumption growth in Equation (8) has dramatic implications for the dynamics of asset prices within the model, such as the real term structure of interest rates. To see that we expand on the log utility preferences example considered by Piazzesi (2014). We evaluate a single-regime scenario, that is, constant volatility of shocks. All the derivations are in Appendix C.

An n-period bond risk premium is

$$-cov_t(m_{t+1}, r_{t+1}^{(n)}) = q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) = -q_{n-1,x} \rho \sigma_p^2 (\eta - 1),$$

where $r_{t+1}^{(n)}$ denotes a log return on a real bond maturing in n periods, and $q_{n-1,x} < 0$ is the exposure of that log bond price to x_t, e.g., $q_{1,x} = -1$. Further, let's use a consumption claim with a log returns r_{t+1}^c as a metaphor for a stock. Then the covariance between real bonds and stocks is

$$cov_t(r_{t+1}^{(n)}, r_{t+1}^c) = q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) = -q_{n-1,x} \rho \sigma_p^2 (\eta - 1). \tag{13}$$

Thus bond risk premiums and bond-stock covariance is literally the same object under log preferences.

It is evident that the conditional covariance between realized and expected consumption growth highlighted in Equation (8) is the central object here. If $\eta < 1$, then the permanent component dominates. In this case, consumption growth is positively serially correlated. Thus, states with low growth have low growth expectations resulting in a high bond price. Bonds act as hedges to equity, and the short rate is procyclical. If $\eta > 1$, then the transitory component dominates. Consumption growth is negatively serially correlated. Thus, states with low growth have high growth expectations resulting in a low bond price. Bonds are no longer hedges to equity, and the short rate is countercyclical.

In order to generate switching signs in the stock-bond covariance, $\eta(S_{t+1})$ has to vary over time with values both above and below 1. Existing models have a constant η. Put differently, this quantity does not take values both above and below 1 and thus does not generate switching signs in the stock-bond covariance. Examples include Bansal and Yaron (2004), which always has negative stock-bond covariance, and Blanchard, L'Huillier, and Lorenzoni (2013). In the latter paper the authors explicitly assume this channel away in a single-regime setting by requiring ρ, σ_p and σ_τ to be such that the covariance in Equation (8) is always equal to zero, or, equivalently, $\eta = 1$.

Figure 4 displays implications of our estimated consumption model combined with log utility. In order to relate the estimation to the intuition developed above, we consider only parameters corresponding to one of the three regimes. In other words, all the computations are based on the assumption that the current regime would prevail forever.

Consistent with the intuition, the "permanent" regime 1 features a correlation between the consumption claim and a 5-year real bond of -0.1, while the "transitory" regime 2 implies a correlation of 0.52. Regime 3 is dominated by the transitory component (the consumption claim-bond correlation is 0.96).

The real bond risk premiums in regime 1 is downward-sloping, since bonds are hedges in this state. That effect pushes the yield curve down. The reverse is occurring in regime 2. Regime 3 features steeply upward sloping bond premiums because of the strong negative correlation between shocks to realized and expected consumption growth.

3.2 Cash flows

In an exchange economy such as ours cash flows have to be specified exogenously. For our purposes, we have to specify two types: aggregate stock dividends and inflation. Inflation can be viewed as a cash flow to a nominal bond.

Following Bansal, Kiku, and Yaron (2012) and Schorfheide, Song, and Yaron (2018), dividend growth has levered exposures to both x_t and shocks as follows

$$\Delta d_{t+1} = \mu_d + \alpha x_t + \varphi_{d,p}\epsilon_{p,t+1} + \varphi_{d,\tau}\epsilon_{\tau,t+1} + \epsilon_{d,t+1}. \tag{14}$$

Idiosyncratic shocks to dividends are captured by $\epsilon_{d,t+1} \sim N(0, \sigma_d^2)$.

Inflation dynamics are assumed to follow

$$\pi_{t+1} = \mu_\pi + \rho_\pi(\pi_t - \mu_\pi) + \varphi_{\pi,p}\epsilon_{p,t+1} + \varphi_{\pi,\tau}\epsilon_{\tau,t+1} + \epsilon_{\pi,t+1}. \tag{15}$$

Idiosyncratic shocks to inflation are captured by $\epsilon_{\pi,t+1} \sim N(0, \sigma_\pi^2)$.

In order to appreciate the effect of inflation on the bond-stock covariance, we can extend the simple log-preference example in Equation (13) to the case of a nominal bond with log return $r_{t+1}^{\$,(n)}$:

$$sign\left[Cov_t(r_{t+1}^{\$,(n)}, r_{t+1}^c)\right] = sign[\Gamma\eta - 1], \tag{16}$$

where explicit expression for Γ is provided in Appendix C.2. Γ reflects the sign of consumption-inflation covariance. If that covariance is negative then $\Gamma > 1$, and vice versa. $\Gamma = 1$ corresponds to no inflation risk.

The previous literature emphasizes the importance of switching covariance between inflation and consumption growth for explaining the pattern in stock-bond covariance, that is, it entertains a fixed η and time-varying Γ (e.g., Campbell, Pfluger, and Viceira, 2020, David and Veronesi, 2013, Song, 2017). Equation (16) demonstrates that as long as η varies around the value of $1/\Gamma$, the sign of covariance between the nominal bond returns and consumption claim returns would be switching signs even if Γ itself varies around the value of 1. Thus, if we add time-varying Γ to our setup, our model has the potential of generating richer dynamics and implications.

3.3 Preferences

We consider an Epstein and Zin (1989) representative agent. The indirect utility, U_t, takes the form

$$U_t = \max_{C_t} \left[(1-\delta) C_t^{(1-\gamma)/\theta} + \delta \left(E_t U_{t+1}^{1-\gamma} \right)^{1/\theta} \right]^{\theta/(1-\gamma)},$$

where C_t denotes consumption, γ is the coefficient of relative risk aversion, $\theta = (1-\gamma)/(1-\psi^{-1})$ with ψ denoting the elasticity of intertemporal substitution, and δ is the time discount factor. The associated log stochastic discount factor (marginal rate of substitution) is

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1},$$

where $z_{c,t}$ is the log price to consumption ratio and $r_{c,t+1}$ is the log return on the consumption claim. Appendix D derives bond and consumption claim prices implied by the consumption dynamics and these preferences.

We set the time discount factor $\delta = 0.9958$ to match the low level of real bonds in the data (close to zero for the short-rate). We analyze the asset pricing moments implied from the consumption-only estimation, which in practice means less risk than in models that use financial asset data for estimation. As a result, we set risk aversion to $\gamma = 17$ to generate quantitatively meaningful asset pricing moments. Lastly, we assume that $\psi = 1.5$ implying that the agent prefers an early resolution of uncertainty.

Introduction of recursive preferences breaks the tight link between bond risk premiums and bond-stock correlation, which we observed in the log-preference case. Indeed, in the case of a single regime, the bond-consumption covariance is

$$cov_t(r_{t+1}^{(n)}, r_{t+1}^{c}) \;=\; q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) + q_{n-1,x} \kappa_{1,c} cov_t(pc_{t+1}, x_{t+1}),$$

and the real bond risk premium is

$$
\begin{aligned}
& - \quad cov_t(m_{t+1}, r_{t+1}^{(n)}) \\
& = \quad \gamma q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) - (\theta - 1) q_{n-1,x} \kappa_{1,c} cov_t(pc_{t+1}, x_{t+1}) \qquad (17) \\
& = \quad \gamma \left[cov_t(r_{t+1}^{(n)}, r_{t+1}^{c}) - \theta(\gamma\psi)^{-1} q_{n-1,x} \kappa_{1,c} cov_t(pc_{t+1}, x_{t+1}) \right],
\end{aligned}
$$

where pc_t is the log price to consumption ratio, and $\kappa_{1,c}$ is the constant in the Campbell-Shiller log linearization. See Appendix D.1. As a result, we obtain an extra term with $cov_t(pc_{t+1}, x_{t+1})$ as compared to the log-preference case. Moreover, the two covariance terms have different weights in bond risk premiums vs bond-stock covariance. One implication of that appears in the last line of Equation (17): with a preference for early resolution of uncertainty, $\theta < 0$, the bond risk premium is lower than the bond-stock return covariance. As a result, changes in sign do not have to coincide.

3.4 Data

We choose the following set of data to evaluate the pricing implications of our model. First, we rely on the value-weighted with- and without-dividend annual returns of the Center for Research in Security Prices (CRSP) stock market indexes (NYSE/AMEX/NASDAQ/ARCA) to construct nominal dividend growth series following Hodrick (1992). We take the CPI inflation rates from the Federal Reserve Bank of St. Louis to convert nominal dividend growth into real terms. We compute sample averages, standard deviations, and first-order autocorrelations of real dividend growth and inflation series and compare with the model-implied counterparts.

Second, we collect the zero-coupon Treasury (1971:Q1 to 2021:Q3) from Gurkaynak, Sack, and Wright (2007) and TIPS (1999:Q1 to 2021:Q3) from Gurkaynak, Sack, and

Wright (2010). Only maturities that are higher than two years are available for TIPS. We discard the initial four years and rely on the post-2003 TIPS data to alleviate any concern regarding the credibility of the TIPS data. We concatenate the TIPS data with the estimated real rates provided by Chernov and Mueller (2012) (1971:Q3 to 2002:Q4) for selective maturities to obtain long-sample averages of the real bond term structure. Also, we compute rolling correlation estimates between daily stock market returns and nominal bond returns of maturity five years over the rolling windows of five years (1963:Q1 to 2021:Q3).

Third, we consider two sets of equity strip yield data, one is the synthetic equity strip yield data provided by Giglio, Kelly, and Kozak (2020) available from 1975:Q4 to 2020:Q3 and the other is the traded equity strip yield data from Bansal, Miller, Song, and Yaron (2020) available from 2005:Q1 to 2016:Q4.[3] We take the sample average of the equity strip yield data.

Lastly, we take the inflation risk variation ratio estimates provided by Duffee (2018b), which are available for the maturities of 1, 5, and 10 years, respectively. We take the average of various different values of estimates in his Tables 1-3.

3.5 Calibration

We set the sensitivity of expected dividends to expected consumption growth to 4, as is common in the literature. The short-term dividend exposures to permanent and transitory shocks are both set to 6 to match the equity premium, while the mean dividend growth rate is set to be consistent with its empirical counterpart as well as a roughly flat dividend claim term structure, as in the data.

[3]We thank Serhiy Kozak for providing data.

In the case of inflation, a positive inflation risk premium obtains if the correlation between inflation and consumption is negative, as in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2006). In our setting with two shocks and changing volatilities, we can generate more sophisticated interrelationship between consumption and inflation. In fact, to match the evidence in Campbell, Pfluger, and Viceira (2020), David and Veronesi (2013), and Song (2017) we pick $\varphi_{\pi,p}$ and $\varphi_{\pi,\tau}$ to match the change in the time series of that correlation. In particular, the switch in sign of the correlation from negative to positive requires $\varphi_{\pi,p}$ to be positive and $\varphi_{\pi,\tau}$ to be negative. That is because the transitory (permanent) regime prevails when the consumption-inflation correlation is negative (positive). Another calibration requirement is that the slope of the nominal yield curve is sufficiently upward-sloping, while at the same time matching the 10-year Duffee (2018b) moment, that is contribution of inflation to the nominal yield variation, not to exceed 40%. The mean and autocorrelation parameters are set as close as possible to their sample counterparts, while again respecting the restriction implied by the Duffee (2018b) moment as well as the sample mean of the nominal short-term interest rate.

Table 3 summarizes the calibrated parameters. Table 4 provides how the calibrated cash flow processes (inflation and dividends) relate to the data. Generally speaking, we find that the sample averages, standard deviations, and first-order autocorrelations computed from the calibrated series are close to their data counterparts. That is not entirely surprising given that we also targeted these moments in the calibration. Still, it is reassuring that the model is capable of meeting the targets.

3.6 Evidence

Baseline asset-pricing moments

Table 5 shows that the model also does a good job accounting for the standard unconditional moments of equity returns, such as the equity risk premium and volatility, which are computed by averaging over a simulation of length 100,000. As we have seen, transitory shocks are responsible for the bulk of the consumption variance. Thus, a sizeable equity premium is obtained due to the disaster regime. Our disasters are akin to the ones discussed in Barro (2006) with the mean of a disaster size set to zero. Such a specification is capable of generating premiums in excess of the non-disaster benchmark albeit more modest ones as compared to the case of the negatively skewed disaster size, all else equal. In our case, we match equity premium because of large volatility of shocks in the disaster regime and high assumed risk aversion.

Equity returns are substantially more volatile than dividend growth. In fact, the price-dividend ratio is volatile and persistent, as in the data. The final rows of the table shows that the price-dividend ratio in the model indeed is negatively related to future excess market returns, as is widely documented in prior studies (e.g., Cochrane, 1994). Thus, the risk premium in the model is counter-cyclical as in the data.

Panel (2) of Table 5 shows that model also matches well the risk premium and return volatility of a 5-year nominal bond in excess of the 1-year nominal bond returns. The term spread (the difference between the 5- and 1-year yields) have a similar mean, volatility, and autocorrelation coefficient as in the data. Further, the model accounts for the failure of the expectations hypothesis for nominal bonds (Campbell and Shiller, 1991). In particular, Table 5 gives the slope coefficient and R^2 of regressions of annual excess bond returns on the lagged spread between the log yield of 5- and 1-yr nominal bonds. The coefficient is positive, as in the data, and the regression R^2 are similar in the data and in the model.

Correlations

Figure 5(A) displays the correlation between returns on the consumption claim and real bonds. We observe the dramatic switch in sign in the mid 1990s that occurs in the real economy. The correlation sign briefly switches sign back to positive during the GFC and COVID crises.

This correlation has no data counterpart. Therefore, Figure 5(B) displays the correlation between returns on the dividend claim and nominal bonds, both in the data and in the model. The two major takeaways are that (i) the model captures the correlation dynamics in the data; and (ii) the "nominal" correlation is very similar to the real one, indicating the importance of the real channel for stock-bond correlation. Consistent with the model, the empirical stock-bond correlation increases in regime 3 as well. That said, the realized correlation does not respond nearly as much as the conditional correlation in the model as it is constructed as a 90-day backward-looking sample correlation of daily returns, whereas the model numbers are the conditional forward-looking quarterly correlations. We do note, however, that this 90-day backward-looking realized correlation did become positive in the Covid-crisis, consistent with the model predictions.

Our calibration is reasonably successful in capturing the pattern in consumption-inflation correlation, as seen in Figure 5(C). The inflation-consumption correlation drifts from an average of -0.5 towards 0 in between 1990 and 2000, consistent with the evidence. Thus, the real asset-pricing channel associated with our model of consumption captures the pattern in inflation-consumption covariance as well. Further, unlike the earlier work on this subject, the effect of the increase in that correlation in mid-1990s on bond return dynamics is minor relative to that coming from the real side of the economy.

Market observers (e.g., AQR, 2021, Authers, 2021) attribute the spike in stock-bond correlation towards positive values in early 2021 to declining credibility of central banks and the weak anchoring of inflation expectations in the light of increasing worries about inflation. Our model offers alternative interpretation of the evidence. We demonstrate that a transition to regime 3 due to the COVID pandemic injects substantial transitory shocks making the stock-bond correlation behave as if we were in regime 2.

Yield curves

Figure 6 compares baseline summary statistics about yield curves associated bonds and equity in our model and in the data. Also, we use extant models as a reference. There are many asset-pricing models that focus on matching different dimensions of asset price data. Thus, a benchmark can be selected in a number of ways. We decided to focus on models that feature (i) long-run risk because their functional form is in the same family; and (ii) an Epstein and Zin (1989) representative agent as is the case for our computations. In this context, Bansal and Shaliastovich (2013) is a classic reference for bonds. However, the equity pricing implications of that model are not explored. Thus, we select Bansal, Kiku, and Yaron (2012) as a reference model for equity pricing. Neither of these models is capable of generating the changing relation between stocks and bonds.

The top left plot in Figure 6 shows the unconditional real yield curve, which in the data is slightly upward-sloping. In the model, the unconditional yield curve is approximately flat and comfortably within the two standard error bounds of that in the data. That is consistent with the bond risk premium curves in each regime shown in Figure 4 in combination with the relative rarity of regime 3, which has a strongly upward-sloping curve. In contrast, Bansal and Shaliastovich (2013) features a downward-sloping curve that is counter-factual relative to the data.

Importantly, a flat real curve makes it much easier to generate a realistic, upward-sloping nominal yield curve while satisfying the Duffee (2018b) moment, that is, the contribution of inflation risk to the risk of nominal yields measured as

$$\frac{Var\left((E_t - E_{t-1})n^{-1}\sum_{i=1}^{n}\pi_{t+i}\right)}{Var\left(y_t^{\$,(n)} - E_{t-1}y_t^{\$,(n)}\right)}. \tag{18}$$

Indeed, that is what we observe in the panels on the right of Figure 6. The top right plot shows that the model generates a nominal bond yield curve that is upward-sloping and consistent with the data, while the bottom right shows that the model's implication for nominal bond's inflation risk is in line with the data. In contrast, the reference model generates a steeper nominal curve than the one observed in the data and the contribution of inflation to yield variation is close to 100%, which is strongly counter-factual.

The bottom left plot of Figure 6 shows the model's implications for forward equity yields. The evidence about the shape of this curve has been the subject of some controversy in the literature, but the most recent evidence, both data and model-based, is pointing towards mildly upward-sloping curve (Bansal, Miller, Song, and Yaron, 2020 and Giglio, Kelly, and Kozak, 2020). In our model the curve is upward-sloping as well, but more flat. The relatively large magnitude of transitory shocks in our estimated consumption dynamics make longer-horizon equity relatively less risky than in the Bansal and Yaron (2004) model (see also Belo, Collin-Dufresne, and Goldstein, 2015 for a similar effect coming from leverage). In contrast, the long-run risk model of Bansal, Kiku, and Yaron (2012) we use as the reference model features a too strongly upward-sloping equity yield curve.

Having verified that the unconditional levels of the yield curves are consistent with those in the data, we turn our attention to yield curve slopes. As emphasized by Backus, Boyarchenko, and Chernov (2015), yield curve slopes are driven by the persistence of the cash flows and pricing kernel, which is at the heart of our model. For instance, with i.i.d. cash flows and pricing kernel, the yield curve would be flat, while negative autocorrelation yields and upward-sloping curve and positive autocorrelation vice versa.

Figure 7 shows the slopes of nominal bond and forward equity yield curves by regimes and compares to the data. There are two differences from the base intuition we have developed under log-preferences. First, the slopes are positive in both regimes 1 and 2. Second, despite the same sign of slopes, the bond-stock correlation does change sign as per Figure 5(B).

The log-preference intuition in Figure 4 was developed assuming the current regime prevails forever. In contrast, in the data yield curves reflect the possibility of different regimes in the future. Thus, the model counterpart to the data-based statistics has to allow for the different future regimes as well. Next, that the slopes are positive is a reflection of two effects: negative correlation between inflation and consumption growth, and the disaster regime, which generates a strongly upward sloping real curve. Lastly, Equation (17) explains how the bond-stock correlation switches sign, even if the bond risk premium does not, under recursive preferences.

The equity yield slopes, given in the lower plots, are slightly upward-sloping in regime 1 and flat in regime 2. The model-implied slopes match these patterns relatively well, within the two standard error bounds of the data, which indicates that the cash flow dynamics of our model have the right persistence properties. In regime 3 the slope of the nominal yield curve is more strongly upward-sloping in the model than in the data, while the equity yield curve is more strongly downward-sloping in the data than in the model. Thus, there is some tension in matching these slopes quantitatively, but qualitatively the model does a good job here as well.

4 Conclusion

Financial assets exhibit high single-horizon risk premiums, and modest term structures of these premiums (e.g., Backus, Chernov, and Zin, 2014). The term structure evidence suggests that persistence of the conditional moments of consumption and dividend growth cannot be as high as usually calibrated in the literature, at least without offsetting effects that render the pricing kernel less autocorrelated. We use consumption data to estimate a model of consumption dynamics that allows for time-varying conditional volatility of permanent and transitory shocks to the economy. We find that disasters are driven mainly by transitory shocks, while there are low frequency regimes that determine whether permanent or transitory shocks drive expected consumption growth. The permanent component of consumption dominates in the sample from the late 1990s and on, with the exception of the global financial and Covid crises.

This model can account for flat or modestly upward-sloping term structures of real and nominal bonds, as well as zero-coupon dividend claims, as observed in the data, along with the change in the stock-bond correlation that occurred in the late 1990s. Risk premiums in this model are mainly driven by a bad state that we associate with disasters. The model differs from existing models in the literature in that there are no highly persistent components that generate substantial risk. Risks come mainly from the relatively short-lived regime 3. In this regime, real bonds are risky as transitory shocks dominate the economy, which allows us to account for the Duffee (2018b) moment. Existing models struggle with capturing this behavior of equity and bonds in one unifying framework.

While our model does surprisingly well at matching a wide set of moments existing models cannot jointly match, our calibration also exposes shortcomings. For instance, the unconditional real-term structure is not sufficiently upward-sloping relative to the data, and there is tension in jointly matching the slopes of the nominal and equity yield curves in the disaster regime. That suggests that further extensions of our framework to other forms of preferences or more sophisticated cash flow dynamics may be a fruitful avenue of research.

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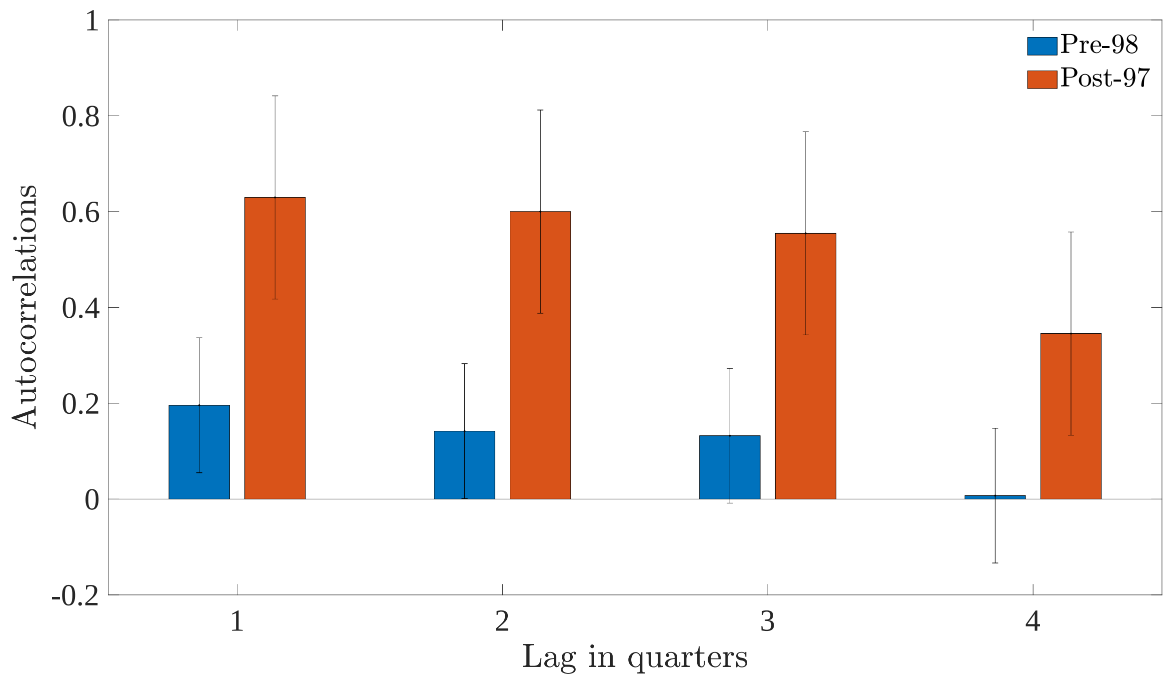
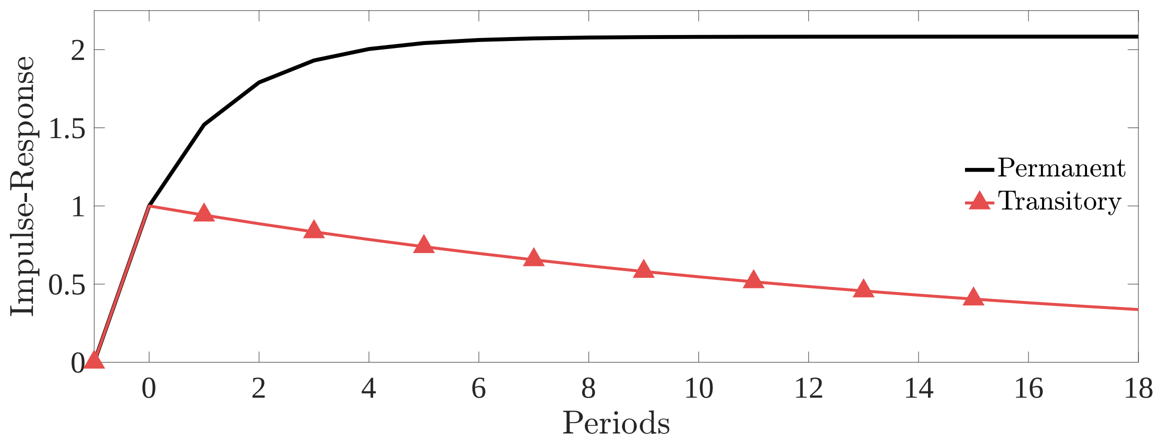
Figure 1: Autocorrelations of consumption growth



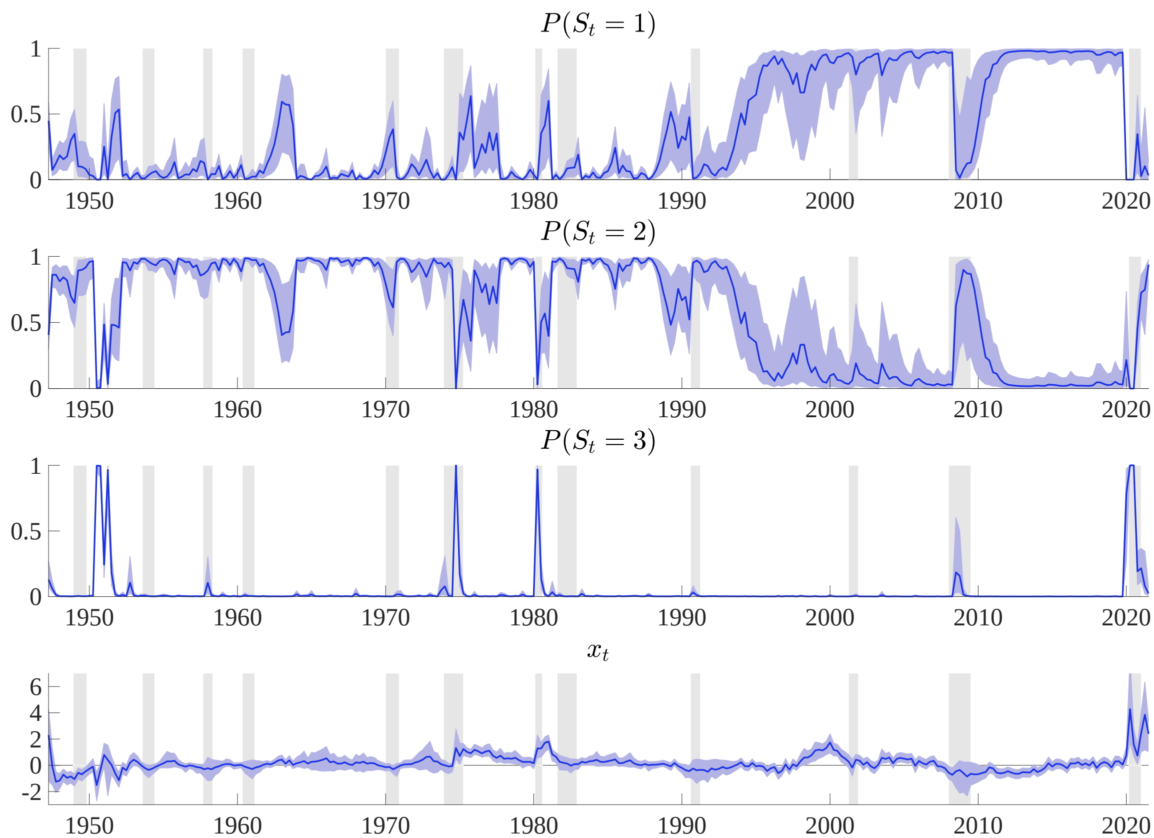
Notes: The plot shows autocorrelations of quarterly log consumption growth. The data is quarterly (deseasonalized) per capita nondurables plus services consumption from NIPA from 1947:Q2 to 2019:Q4. The blue bars shows the autocorrelations for lags 1 to 4 in the period up until 1997:Q4, while the red bars give the autocorrelations for the same lags in the period post 1997:Q4. The error bars give the +/- two standard error bounds of the estimated autocorrelation coefficients.

Figure 2: Transitory and permanent effects



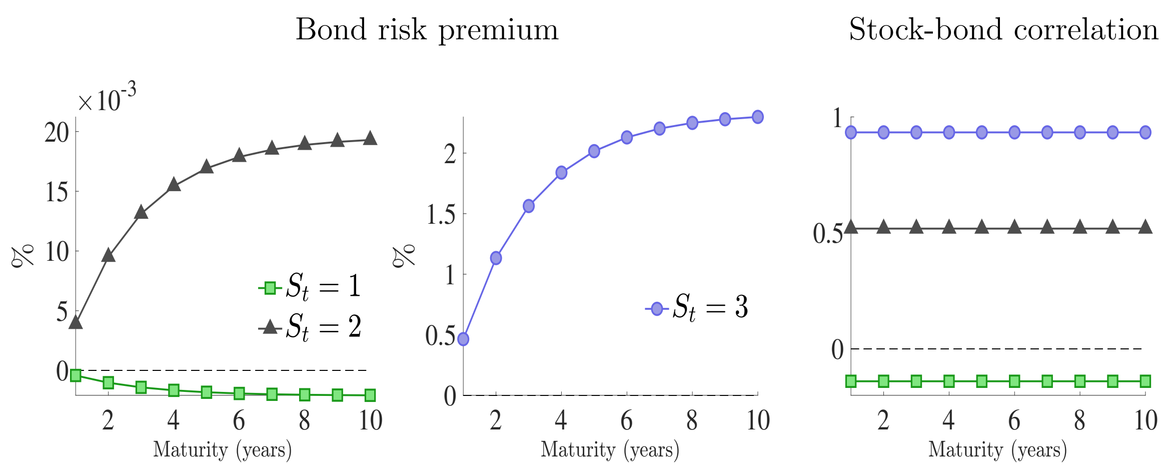
Notes: The plot shows the impulse-response of a positive shock to the consumption components we label as permanent (blue) and transitory (red). In the former case, a positive shock leads to a positive shock to future expected consumption *growth*, whereas in the latter case a positive shock leads to a negative shock to future expected consumption growth.

Figure 3: Estimated states



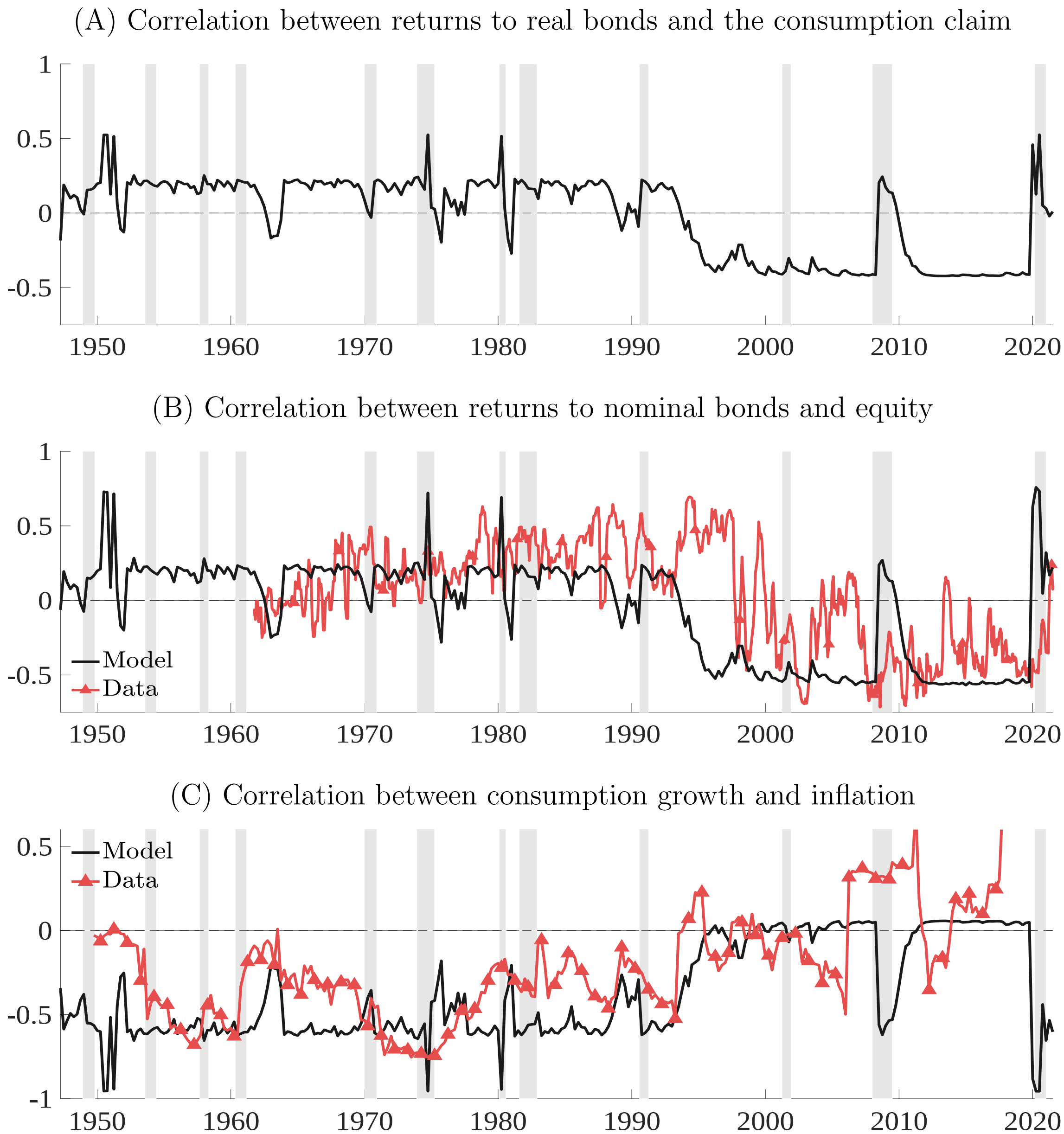
*N*otes: The plots show the filtering probabilities of each regime and expected consumption growth through the sample, along with 95% confidence bands. The grey bars represent NBER recessions. The first regime is the "permanent" regime, the second regime is the "transitory" regime, and the third regime is the "disaster" regime. The data frequency is quarterly and the sample is 1947:Q2-2021:Q3.

Figure 4: Model-implied asset pricing conditional moments: Log utility preference



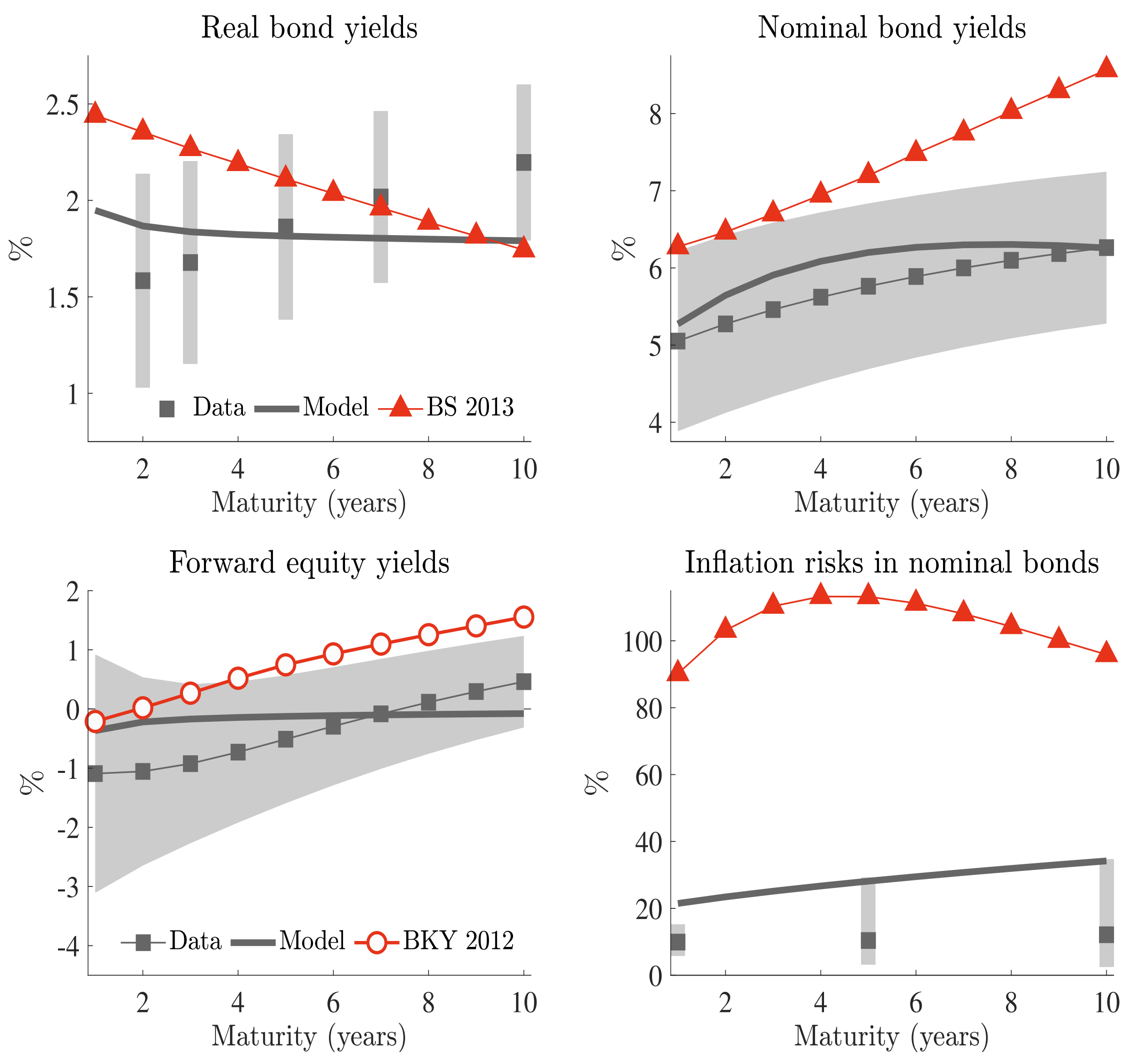
*N*otes: We provide the one-period bond risk premium and the conditional correlation between returns to real bonds and the consumption claim under log utility preference. We show the case of a fixed-regime case.

Figure 5: Model-implied correlations



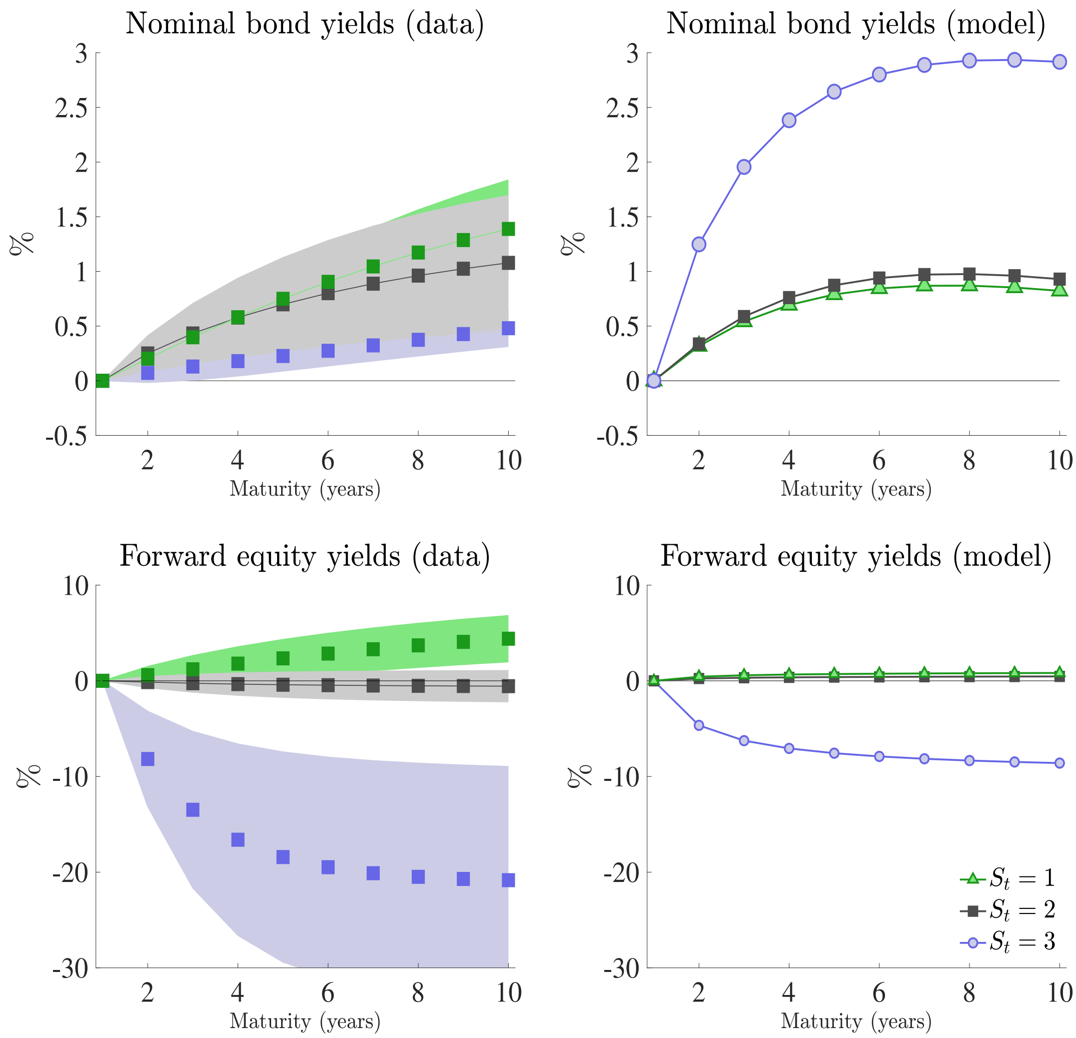
Notes: For panel (A), we provide our model-implied correlation between real returns to the consumption claim and to the 5-year maturity of real bond. For panel (B), we compute rolling correlation estimates between market returns and negative log changes of nominal bond yields of 5-year maturity over the rolling windows of 90 days (Data). We compare with our model-implied correlation between market returns and real returns of 5-year maturity for nominal bond (Model). For panel (C), we compute rolling correlation estimates between real personal consumption expenditure growth and cpi inflation over the rolling window of 5 years (Data). We compare with our model-implied correlation between consumption growth and inflation.

Figure 6: Model-implied asset pricing sample moments



Notes: The top plots show average real and nominal yield curves. We compute the sample averages of the zero-coupon Treasury (1971:Q1 to 2020:Q4) from Gurkaynak, Sack, and Wright (2007) and TIPS (1999:Q1 to 2020:Q4) from Gurkaynak, Sack, and Wright (2010), respectively. For the TIPS, maturities higher than two years are available. We discard the initial four years and rely on the post-2003 TIPS data to alleviate any concern regarding the credibility of the TIPS data. We concatenate the TIPS data with the estimated real rates provided by Chernov and Mueller (2012) (1971:Q3 to 2002:Q4) for selective maturities to obtain long-sample averages of the real bond term structure. Red triangles represent the average yield curves in the Bansal and Shaliastovich (2013) model. In the bottom left plot, we show sample average the synthetic equity strip yield data provided by Giglio, Kelly, and Kozak (2020) from 1975:Q4 to 2020:Q3. The pattern is similar if we use the traded equity strip yield data from Bansal, Miller, Song, and Yaron (2020) which are available from 2005:Q1 to 2016:Q4. Finally, in the bottom right plot we show the inflation risk variation ratio estimates provided by Duffee (2018b), which are available for the maturities of 1, 5, and 10 years, respectively. We take the average of various different values of estimates in his Tables 1-3. Inflation variation ratio is computed in (18). Red circles represent implications of the Bansal, Kiku, and Yaron (2012) model.

Figure 7: Model-implied asset pricing conditional moments with data comparison



*N*otes: The data for the left plots in this figure are described in the legend for Figure 6. The plots shows the slopes of the term structures at various maturities. The averages are calculated for each regime by taking the average of the product of the yield and the respective regime's filtering probabilities and plotted with 95% confidence bands. The model-based yields displayed in the right plots are given for each regime assuming $x_t = 0$.

Table 1: Consumption autocorrelation before and after 1998

	Full sample		Sample excluding recessions	
	(1)	(2)	(1)	(2)
α_0	0.0039***	0.0001	0.0058***	0.0002
	(0.0011)	(0.0005)	(0.0008)	(0.0005)
α_1	0.0040***		0.0053***	
	(0.0014)		(0.0014)	
β_0	0.4424***	0.4560***	0.2463**	0.3460***
	(0.1288)	(0.1317)	(0.1035)	(0.1014)
β_1	−0.4411***	−0.4541***	−0.3898***	−0.4848***
	(0.1605)	(0.1608)	(0.1585)	(0.1481)
R^2	0.0203	0.0191	0.0397	0.0286

*N*otes: The table shows the regression estimates from the regression in Equation (1). On the right side, business cycle downturns are excluded from the sample as explained in the main text. COVID period is excluded from all regressions. Robust standard errors are reported in parenthesis. The β_0 estimate gives the autocorrelation coefficient in the post-1998 sample, while the β_1 estimate gives the difference between the autocorrelation in the pre- vs. post-1998 samples. The specification in the column labeled (1) estimates the full regression, while the specification in the column labeled (2) is run using demeaned consumption growth and setting $\alpha_1 = 0$, as explained in the main text. One asterisk denotes significance at the 10% level, two at the 5% level, and three at the 1% level. The data is real-time quarterly real PCE data, from 1947:Q2 to 2019:Q4.

Table 2: Estimated parameters of consumption

	MLE	[5%,	95%]
Consumption growth, Δc			
μ_c	0.0051	[0.0039,	0.0062]
ρ	0.8951	[0.8242,	0.9662]
$\sigma_p(1)$	0.0013	[0.0005,	0.0022]
$\sigma_p(2)$	0.0011	[0.0004,	0.0017]
$\sigma_p(3)$	0.0031	[0.0013,	0.0090]
$\sigma_\tau(1)$	0.0030	[0.0027,	0.0038]
$\sigma_\tau(2)$	0.0077	[0.0067,	0.0098]
$\sigma_\tau(3)$	0.0769	[0.0572,	0.1092]

Transition probabilities, $\mathbb{P}$

0.9788 [0.9042,0.9923]	0.0167 [0.0018,0.0696]	0.0045 [0.0059,0.0262]
0.0055 [0.0030,0.0253]	0.9721 [0.9334,0.9831]	0.0224 [0.0139,0.0413]
0.0770 [0.0175,0.2282]	0.2301 [0.1439,0.2694]	0.6929 [0.5024,0.8386]

*N*otes: The table gives the maximum likelihood estimates (MLE) for the parameters governing the consumption process, along with the 5% and 95% confidence bands. The data frequency is quarterly. The estimated transition probability matrix gives the 5% and 95% confidence bands in brackets underneath the MLE. We order the regimes according to the transitory shock volatilities, and restrict the off-diagonal corner elements in the transition probability matrix to equal zero. The sample is 1947:Q2-2021:Q3.

Table 3: Calibrated parameters

Dividend growth		Inflation	
μ_d	0.0075	μ_π	0.0040
α	4.0	ρ_π	0.9975
$\varphi_{d,p}$	6.0	$\varphi_{\pi,p}$	0.40
$\varphi_{d,\tau}$	6.0	$\varphi_{\pi,\tau}$	-0.06
σ_d	0.0	σ_π	0.0

*N*otes: We report the calibrated quarterly frequency parameter values for dividend growth and inflation.

Table 4: Dividend growth and inflation moments

Dividend growth	Data	Model
Mean (%)	2.61	2.13
Standard deviation (%)	13.02	11.52
Autocorrelation	-0.26	-0.04
Inflation	**Data**	**Model**
Mean (%)	3.37	3.72
Standard deviation (%)	2.66	1.34
Autocorrelation	0.61	0.88

*N*otes: Both data and model-implied dividend growth and inflation are aggregated to annual frequency. Then, we compute the sample averages, standard deviations, and first-order autocorrelations of annual dividend growth and inflation, respectively. The data sample ranges from 1947 to 2020.

Table 5: Standard asset pricing moments

(1) Equity		Data	Model
Excess returns	Risk premium	7.48	6.63
	Volatility	15.85	21.30
	Sharpe ratio	0.47	0.31
Log pd ratio	Mean	3.49	3.61
	Volatility	0.43	0.15
	AR(1) coefficient	0.97	0.81
Predictability	1-year excess return on log pd ratio	-0.09	-0.14
	R^2 value	0.06	0.02

(2) Bond		Data	Model
Excess returns	Risk premium	1.36	1.43
	Volatility	2.55	3.38
	Sharpe ratio	0.46	0.38
Term spread (5y-1y)	Mean	0.95	0.94
	Volatility	1.00	0.96
	AR(1) coefficient	0.90	0.97
Predictability	5-year excess return on term spread	2.21	1.14
	R^2 value	0.12	0.10

*N*otes: Panel (1): Excess stock return is defined as the one-quarter holding period stock return in excess of one-quarter risk-free rate. All excess return moments are annualized. The risk premium is the average sample return in the data and the unconditional average excess equity return in the model. Volatility refers to the standard deviation of excess returns, while the Sharpe ratio is the ratio of the mean excess return to its standard deviation. The log price-dividend (pd) ratio is calculated by summing the dividends over the last year. In the predictability regression, we regress one-year excess stock return on the lagged log pd-ratio and report the slope coefficient and the R^2. Panel (2): Excess bond return is defined as the one-year holding period of the 5-year maturity nominal bond in excess of a one-year bond yield. Volatility refers to the standard deviation of excess returns, while the Sharpe ratio is the ratio of the mean excess return to its standard deviation. Term spread is defined as the difference nominal bond yields of maturities of 5- and 1-year. In the predictability regression, we regress one-year excess bond return on the lagged term spread between maturities of 5- and 1-year and report the slope coefficient and the $R2$. Model moments are averaged over a simulation of length 100,000.

A Robustness of estimation to alternative consumption data

Since our goal is to estimate the dynamics of consumption growth as perceived by investors, we use real-time consumption data. However, historically, a number of papers use the ex post revised real per capita nondurables + services consumption data from NIPA. In Table A-1 we show that there is a significant change in the autocorrelation of consumption growth in 1998 also using these data when estimating the regression in Equation (1). The results are similar to those obtained using the real-time PCE dataset.

In Table A-2 we show the maximum likelihood estimates that obtain when using these data. This alternative data yields similar overall dynamics. In particular, the first regime features relatively more of the permanent component, regime 2 reflects a greater share of the transitory component, and regime 3 is a disaster regime. Again, the shift between regime 1 and regime 2 happens in the later 1990s, while regime 3 picks up disasters like the Covid-crisis (the filtering probabilities are not shown). That said, the volatility coefficients are somewhat smaller relative to our main estimation and the parameter and regime uncertainty is larger.

B Estimation of the regime-switching model

Consider the state-space model

$$\begin{aligned} Y_t &= D(S_t) + Z(S_t)\alpha_t + v_t, \quad v_t \sim N(0, U) \\ \alpha_t &= T(S_t)\alpha_{t-1} + R(S_t)\epsilon_t, \quad \epsilon_t \sim N(0, \Sigma(S_t)), \end{aligned} \tag{A-1}$$

where α_t is the latent state. See Chapter 5 of Kim and Nelson (1999) for detailed descriptions. Given $\alpha^i_{t-1|t-1}, P^i_{t-1|t-1}$, for $i, j, \in \{1, ..., N\}$,

Forecasting

$$\begin{aligned} \alpha^{(i,j)}_{t|t-1} &= T(S_t = j)\alpha^i_{t-1|t-1} \\ P^{(i,j)}_{t|t-1} &= T(S_t = j)P^i_{t-1|t-1}T(S_t = j)' + R(S_t = j)\Sigma(S_t = j)R(S_t = j)' \\ e^{(i,j)}_{t|t-1} &= Y_t - D(S_t = j) - Z(S_t = j)\alpha^{(i,j)}_{t|t-1} \\ F^{(i,j)}_{t|t-1} &= Z(S_t = j)P^{(i,j)}_{t|t-1}Z(S_t = j)' + U. \end{aligned} \tag{A-2}$$

Updating

$$\begin{aligned} \alpha^{(i,j)}_{t|t} &= \alpha^{(i,j)}_{t|t-1} + \left(P^{(i,j)}_{t|t-1}Z(S_t = j)'\right)\left(F^{(i,j)}_{t|t-1}\right)^{-1} e^{(i,j)}_{t|t-1} \\ P^{(i,j)}_{t|t} &= P^{(i,j)}_{t|t-1} - \left(P^{(i,j)}_{t|t-1}Z(S_t = j)'\right)\left(F^{(i,j)}_{t|t-1}\right)^{-1}\left(Z(S_t = j)P^{(i,j)}_{t|t-1}\right). \end{aligned} \tag{A-3}$$

Each iteration of the Kalman filter produces an N-fold increase in the number of cases to consider. It is necessary to introduce some approximations to make the above Kalman filter operable. The key is to collapse terms in the right way at the right time. Therefore, it remains to reduce the $N \times N$ posteriors $\alpha_{t|t}^{(i,j)}, P_{t|t}^{(i,j)}$ into N posteriors $\alpha_{t|t}^{j}, P_{t|t}^{j}$. Note that

$$\begin{aligned}
E\big(\alpha_t|S_t=j,Y_t\big) &= \frac{\sum_{i=1}^{N} Pr(S_{t-1}=i,S_t=j|Y_t)E\big(\alpha_t|S_t=j,S_{t-1}=i,Y_t\big)}{Pr(S_t=j|Y_t)} \qquad \text{(A-4)}\\
&= \sum_{i=1}^{N}\Delta_t^{(i,j)}E\big(\alpha_t|S_t=j,S_{t-1}=i,Y_t\big), \quad \Delta_t^{(i,j)} = \frac{Pr(S_{t-1}=i,S_t=j|Y_t)}{Pr(S_t=j|Y_t)}\\
\alpha_{t|t}^{j} &= \sum_{i=1}^{N}\Delta_t^{(i,j)}\alpha_{t|t}^{(i,j)}.
\end{aligned}$$

The variance of α_t conditional on $S_t = j, Y_t$ could be derived in the following way:

$$E\Big((\alpha_t-\alpha_{t|t}^{j})(\alpha_t-\alpha_{t|t}^{j})'|S_t=j,Y_t\Big) = \sum_{i=1}^{N}\Delta_t^{(i,j)}E\Big((\alpha_t-\alpha_{t|t}^{j})(\alpha_t-\alpha_{t|t}^{j})'|S_t=j,S_{t-1}=i,Y_t\Big) \qquad \text{(A-5)}$$

Note that

$$\begin{aligned}
&E\Big((\alpha_t-\alpha_{t|t}^{j})(\alpha_t-\alpha_{t|t}^{j})'|S_t=j,S_{t-1}=i,Y_t\Big) \qquad \text{(A-6)}\\
&= E\Big((\alpha_t-\alpha_{t|t}^{(i,j)}+\alpha_{t|t}^{(i,j)}-\alpha_{t|t}^{j})(\alpha_t-\alpha_{t|t}^{(i,j)}+\alpha_{t|t}^{(i,j)}-\alpha_{t|t}^{j})'|S_t=j,S_{t-1}=i,Y_t\Big)\\
&= E\Big((\alpha_t-\alpha_{t|t}^{(i,j)})(\alpha_t-\alpha_{t|t}^{(i,j)})'|S_t=j,S_{t-1}=i,Y_t\Big) + \Big((\alpha_{t|t}^{j}-\alpha_{t|t}^{(i,j)})(\alpha_{t|t}^{j}-\alpha_{t|t}^{(i,j)})'\Big)\\
&\quad + 2E\Big((\alpha_t-\alpha_{t|t}^{(i,j)})|S_t=j,S_{t-1}=i,Y_t\Big)(\alpha_{t|t}^{(i,j)}-\alpha_{t|t}^{j})'.
\end{aligned}$$

Hence,

$$\begin{aligned}
P_{t|t}^{j} &= \sum_{i=1}^{N}\Delta_t^{(i,j)}E\Big((\alpha_t-\alpha_{t|t}^{j})(\alpha_t-\alpha_{t|t}^{j})'|S_t=j,S_{t-1}=i,Y_t\Big) \qquad \text{(A-7)}\\
&= \sum_{i=1}^{N}\Delta_t^{(i,j)}\Big[E\Big((\alpha_t-\alpha_{t|t}^{(i,j)})(\alpha_t-\alpha_{t|t}^{(i,j)})'|S_t=j,S_{t-1}=i,Y_t\Big) + \Big((\alpha_{t|t}^{j}-\alpha_{t|t}^{(i,j)})(\alpha_{t|t}^{j}-\alpha_{t|t}^{(i,j)})'\Big)\Big]\\
&\quad +2\sum_{i=1}^{N}\Delta_t^{(i,j)}\underbrace{E\Big((\alpha_t-\alpha_{t|t}^{(i,j)})|S_t=j,S_{t-1}=i,Y_t\Big)}_{=0}(\alpha_{t|t}^{(i,j)}-\alpha_{t|t}^{j})'\\
&= \sum_{i=1}^{N}\Delta_t^{(i,j)}\Big[\underbrace{E\Big((\alpha_t-\alpha_{t|t}^{(i,j)})(\alpha_t-\alpha_{t|t}^{(i,j)})'|S_t=j,S_{t-1}=i,Y_t\Big)}_{=P_{t|t}^{(i,j)}} + \Big((\alpha_{t|t}^{j}-\alpha_{t|t}^{(i,j)})(\alpha_{t|t}^{j}-\alpha_{t|t}^{(i,j)})'\Big)\Big]\\
&= \sum_{i=1}^{N}\Delta_t^{(i,j)}\Big[P_{t|t}^{(i,j)}+(\alpha_{t|t}^{j}-\alpha_{t|t}^{(i,j)})(\alpha_{t|t}^{j}-\alpha_{t|t}^{(i,j)})'\Big].
\end{aligned}$$

Merging

$$\alpha^j_{t|t} = \frac{\sum_{i=1}^N Pr(S_{t-1}=i, S_t=j|Y_t)}{Pr(S_t=j|Y_t)}\left(\alpha^{(i,j)}_{t|t}\right) \tag{A-8}$$
$$P^j_{t|t} = \frac{\sum_{i=1}^N Pr(S_{t-1}=i, S_t=j|Y_t)}{Pr(S_t=j|Y_t)}\left(P^{(i,j)}_{t|t} + \big(\alpha^j_{t|t} - \alpha^{(i,j)}_{t|t}\big)\big(\alpha^j_{t|t} - \alpha^{(i,j)}_{t|t}\big)'\right).$$

Finally, the likelihood density of observation Y_t is given by,

Likelihood

$$l(Y_t|Y_{1:t-1}) = \sum_{j=1}^N\sum_{i=1}^N f(Y_t|S_{t-1}=i, S_t=j, Y_{1:t-1})Pr(S_{t-1}=i, S_t=j|Y_{t-1}) \tag{A-9}$$
$$f(Y_t|S_{t-1}=i, S_t=j, Y_{1:t-1}) = (2\pi)^{-\frac{n}{2}}\det\left(F^{(i,j)}_{t|t-1}\right)^{-\frac{1}{2}}\exp\left[-\frac{1}{2}\big(e^{(i,j)}_{t|t-1}\big)'\left(F^{(i,j)}_{t|t-1}\right)^{-1}\big(e^{(i,j)}_{t|t-1}\big)\right].$$

C An illustration with log utility preferences

For ease of illustration, we assume homoscedasticity

$$\begin{aligned}\Delta c_{t+1} &= \mu_c + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}, \quad \epsilon_p \sim N(0,\sigma_p^2),\\ x_{t+1} &= \rho x_t + \rho\epsilon_{p,t+1} + (\rho-1)\epsilon_{\tau,t+1}, \quad \epsilon_\tau \sim N(0,\sigma_\tau^2),\\ \pi_{t+1} &= \mu_\pi + \rho_\pi(\pi_t - \mu_\pi) + \varphi_{\pi,p}\epsilon_{p,t+1} + \varphi_{\pi,\tau}\epsilon_{\tau,t+1},\end{aligned} \tag{A-10}$$

and consider log utility preference implying the following log stochastic discount factor

$$m_{t+1} = \ln\delta - \Delta c_{t+1}. \tag{A-11}$$

In this environment, the unexpected component of the return on consumption claim is

$$r^c_{t+1} - E_t r^c_{t+1} = \Delta c_{t+1} - E_t\Delta c_{t+1} \tag{A-12}$$

because the log price to consumption ratio is constant.

C.1 Covariance between returns on a real bond and a stock

The n-maturity log real bond price is

$$q^{(n)}_t = q_{n,0} + q_{n,x}x_t \tag{A-13}$$

and the return on the n-maturity real bond is

$$r^{(n)}_{t+1} \equiv q^{(n-1)}_{t+1} - q^{(n)}_t. \tag{A-14}$$

We can deduce that its unexpected component is

$$r_{t+1}^{(n)} - E_t r_{t+1}^{(n)} = q_{n-1,x}\big(x_{t+1} - E_t x_{t+1}\big). \tag{A-15}$$

Introduce the relative contribution of permanent and transitory components via

$$\eta = \frac{1-\rho}{\rho} \cdot \frac{\sigma_\tau^2}{\sigma_p^2}. \tag{A-16}$$

We can express the risk premium on a real bond as:

$$-cov_t(m_{t+1}, r_{t+1}^{(n)}) = q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) = -q_{n-1,x}\rho\sigma_p^2(\eta - 1).$$

Then the sign of the bond risk premium is

$$\text{sign}(-cov_t(m_{t+1}, r_{t+1}^{(n)})) = \text{sign}(\eta - 1)$$

as $q_{n-1,x} < 0$ for $n \geq 2$.

Next, from (A-12) and (A-15), we can express the sign of the conditional covariance as

$$\begin{aligned} \text{sign}\big(cov_t(r_{t+1}^{(n)}, r_{t+1}^c)\big) &= \text{sign}\big(q_{n-1,x}\big) \cdot \text{sign}\big(cov_t(\Delta c_{t+1}, x_{t+1})\big), \\ &= -\text{sign}\big(cov_t(\Delta c_{t+1}, x_{t+1})\big), \\ &= \text{sign}(\eta - 1). \end{aligned} \tag{A-17}$$

C.2 Covariance between returns on a nominal bond and a stock

The n-maturity log nominal bond price is

$$q_t^{\$,(n)} = q_{n,0}^{\$} + q_{n,x}^{\$} x_t + q_{n,\pi}^{\$} \pi_t. \tag{A-18}$$

From the Euler equation

$$q_t^{\$,(n)} = \ln E_t\big[\exp(m_{t+1} - \pi_{t+1} + q_{t+1}^{\$,(n-1)})\big], \tag{A-19}$$

we can solve for

$$q_{n-1,x}^{\$} = -\left[\frac{1-\rho^{n-1}}{1-\rho}\right], \qquad q_{n-1,\pi}^{\$} = -\rho_\pi \left[\frac{1-\rho_\pi^{n-1}}{1-\rho_\pi}\right]. \tag{A-20}$$

The unexpected component of the return on the n-maturity nominal bond is

$$r_{t+1}^{\$,(n)} - E_t r_{t+1}^{\$,(n)} = q_{n-1,x}^{\$}\big(x_{t+1} - E_t x_{t+1}\big) + q_{n-1,\pi}^{\$}\big(\pi_{t+1} - E_t \pi_{t+1}\big). \tag{A-21}$$

From (A-12), (A-20), and (A-21), we can express the conditional covariance as

$$
\begin{aligned}
cov_t(r_{t+1}^{\$,(n)}, r_{t+1}^c) &= q_{n-1,x}^{\$} cov_t(\Delta c_{t+1}, x_{t+1}) + q_{n-1,\pi}^{\$} cov_t(\Delta c_{t+1}, \pi_{t+1}) \\
&\propto \left[\frac{1-\rho}{\rho} \cdot \frac{\sigma_\tau^2}{\sigma_p^2} - 1\right] - \left[\varphi_{\pi,p} + \varphi_{\pi,\tau} \cdot \frac{\sigma_\tau^2}{\sigma_p^2}\right] \cdot \left[\frac{\rho_\pi\left(\frac{1-\rho_\pi^{n-1}}{1-\rho_\pi}\right)}{\rho\left(\frac{1-\rho^{n-1}}{1-\rho}\right)}\right] \\
&\propto \left[\frac{1 - \varphi_{\pi,\tau} \cdot [\frac{\rho}{1-\rho}] \cdot \left[\frac{\rho_\pi\left(\frac{1-\rho_\pi^{n-1}}{1-\rho_\pi}\right)}{\rho\left(\frac{1-\rho^{n-1}}{1-\rho}\right)}\right]}{1 + \varphi_{\pi,p} \cdot \left[\frac{\rho_\pi\left(\frac{1-\rho_\pi^{n-1}}{1-\rho_\pi}\right)}{\rho\left(\frac{1-\rho^{n-1}}{1-\rho}\right)}\right]}\right] \cdot \eta - 1.
\end{aligned}
\tag{A-22}
$$

For ease of expression, we define

$$
\Gamma(\varphi_{\pi,\tau}, \varphi_{\pi,p}; \rho, \rho_\pi) = \frac{1 - \varphi_{\pi,\tau} \cdot [\frac{\rho}{1-\rho}] \cdot \Psi(\rho, \rho_\pi)}{1 + \varphi_{\pi,p} \cdot \Psi(\rho, \rho_\pi)}, \qquad \Psi(\rho, \rho_\pi) = \frac{\rho_\pi\left(\frac{1-\rho_\pi^{n-1}}{1-\rho_\pi}\right)}{\rho\left(\frac{1-\rho^{n-1}}{1-\rho}\right)}
\tag{A-23}
$$

to show that the sign of the conditional covariance is

$$
\operatorname{sign}\big(cov_t(r_{t+1}^{\$,(n)}, r_{t+1}^c)\big) = \operatorname{sign}\big(\Gamma(\varphi_{\pi,\tau}, \varphi_{\pi,p}; \rho, \rho_\pi) \cdot \eta - 1\big).
\tag{A-24}
$$

Two remarks can be made. First, when inflation has no exposures to consumption shocks, i.e., $\varphi_{\pi,p} = \varphi_{\pi,\tau} = 0$, then (A-24) is identical to (A-17) as $\Gamma(\varphi_{\pi,\tau} = 0, \varphi_{\pi,p} = 0; \rho, \rho_\pi) = 1$. Second, we find that $\forall j \in \{p, \tau\}$,

$$
\frac{\partial \Gamma(\varphi_{\pi,\tau}, \varphi_{\pi,p}; \rho, \rho_\pi)}{\partial \varphi_{\pi,j}} < 0.
\tag{A-25}
$$

Thus, holding η constant, (A-24) is a decreasing function of $\varphi_{\pi,j}$.

D Model solution with recursive preferences

This section provides solutions for the equilibrium asset prices.

D.1 Single-regime case

For ease of illustration, we assume homoscedasticity

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}, \quad \epsilon_p \sim N(0, \sigma_p^2), \\
x_{t+1} &= \rho x_t + \rho \epsilon_{p,t+1} + (\rho - 1)\epsilon_{\tau,t+1}, \quad \epsilon_\tau \sim N(0, \sigma_\tau^2), \\
\pi_{t+1} &= \mu_\pi + \rho_\pi(\pi_t - \mu_\pi) + \varphi_{\pi,p}\epsilon_{p,t+1} + \varphi_{\pi,\tau}\epsilon_{\tau,t+1},
\end{aligned}
\tag{A-26}
$$

and consider the recursive utility preference implying the following log stochastic discount factor

$$
m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{t+1}^c
\tag{A-27}
$$

where γ is the coefficient of relative risk aversion; $\theta = (1-\gamma)/(1-\psi^{-1})$ with ψ denoting the elasticity of intertemporal substitution; and r^c_{t+1} is the log return on the consumption claim.

We use the Campbell-Shiller log-linearization to express

$$r^c_{t+1} = \kappa_{0,c} + \kappa_{1,c} pc_{t+1} - pc_t + \Delta c_{t+1} \tag{A-28}$$

where the log price to consumption ratio is $pc_t = A_{0,c} + A_{1,c} x_t$. The unexpected component of returns is

$$r^c_{t+1} - E_t r^c_{t+1} = \kappa_{1,c}\big(pc_{t+1} - E_t pc_{t+1}\big) + \big(\Delta c_{t+1} - E_t \Delta c_{t+1}\big). \tag{A-29}$$

The real log SDF is

$$\begin{aligned} m_{t+1} = {} & \theta \ln \delta - \gamma\mu + (\theta - 1)\big\{\kappa_{0,c} + (k_{1,c} - 1)A_{0,c}\big\} - \frac{1}{\psi} x_t \\ & - \big\{\gamma - (\theta-1)\kappa_{1,c}A_{1,c}\rho\big\}\epsilon_{p,t+1} - \big\{\gamma - (\theta-1)\kappa_{1,c}A_{1,c}(\rho - 1)\big\}\epsilon_{\tau,t+1}. \end{aligned} \tag{A-30}$$

Assume that the n-maturity log real bond price is

$$q^{(n)}_t = q_{n,0} + q_{n,x} x_t \tag{A-31}$$

From the Euler equations $1 = E_t[\exp(m_{t+1} + r^c_{t+1})]$ and $q^{(n)}_t = \ln E_t[\exp(m_{t+1} + q^{(n-1)}_{t+1})]$, we can deduce that

$$A_{1,c} = \frac{1 - 1/\psi}{1 - \rho\kappa_{1,c}}, \quad q_{n-1,x} = -\frac{1}{\psi}\left(\frac{1-\rho^{n-1}}{1-\rho}\right). \tag{A-32}$$

The return on the n-maturity real bond is

$$r^{(n)}_{t+1} \equiv q^{(n-1)}_{t+1} - q^{(n)}_t. \tag{A-33}$$

We can deduce that its unexpected component is

$$r^{(n)}_{t+1} - E_t r^{(n)}_{t+1} = q_{n-1,x}\big(x_{t+1} - E_t x_{t+1}\big). \tag{A-34}$$

From (A-29) and (A-34), we can express the conditional covariance as

$$cov_t(r^{(n)}_{t+1}, r^c_{t+1}) = q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) + q_{n-1,x}\kappa_{1,c} cov_t(pc_{t+1}, x_{t+1}). \tag{A-35}$$

Moving to bond risk premiums, from (A-27) and (A-28), we can express the unexpected component of the log SDF as

$$m_{t+1} - E_t m_{t+1} = -\gamma\big(\Delta c_{t+1} - E_t \Delta c_{t+1}\big) + (\theta - 1)\kappa_{1,c}\big(pc_{t+1} - E_t pc_{t+1}\big). \tag{A-36}$$

From (A-34) and (A-36), we can express the bond risk premium as

$$
\begin{aligned}
-cov_t(m_{t+1}, r_{t+1}^{(n)}) &= \gamma q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) - (\theta-1) q_{n-1,x} \kappa_{1,c} cov_t(pc_{t+1}, x_{t+1}) \\
&= \gamma q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) - (\theta-1) q_{n-1,x} \kappa_{1,c} A_{1,c} var_t(x_{t+1}) \\
&= \gamma q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) + \gamma q_{n-1,x} \kappa_{1,c} A_{1,c} var_t(x_{t+1}) \\
&\quad - \big(\theta - 1 + \gamma\big) q_{n-1,x} \kappa_{1,c} A_{1,c} var_t(x_{t+1}) \\
&= \gamma \Big[cov_t(r_{t+1}^{(n)}, r_{t+1}^c) - \theta(\gamma\psi)^{-1} q_{n-1,x} \kappa_{1,c} A_{1,c} var_t(x_{t+1}) \Big].
\end{aligned} \tag{A-37}
$$

D.2 Exogenous dynamics with regime switching

The joint dynamics of consumption growth, dividend growth, and inflation are

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}, \\
\Delta d_{t+1} &= \mu_d + \alpha x_t + \varphi_d \epsilon_{p,t+1} + \varphi_d \epsilon_{\tau,t+1}, \\
\pi_{t+1} &= \mu_\pi (1-\rho_\pi) + \rho_\pi \pi_t + \varphi_\pi \epsilon_{p,t+1} + \varphi_\pi \epsilon_{\tau,t+1}, \\
x_t &= \rho x_{t-1} + \rho \epsilon_{p,t+1} + (\rho - 1)\epsilon_{\tau,t+1}, \quad \epsilon_{\tau,t+1} \sim N(0, \sigma_\tau(S_{t+1})^2), \quad \epsilon_{p,t+1} \sim N(0, \sigma_p(S_{t+1})^2)
\end{aligned} \tag{A-38}
$$

and the transition matrix is given by

$$
\mathbb{P} = \begin{bmatrix} p_{11} & p_{12} & 1 - p_{11} - p_{12} \\ p_{21} & p_{22} & 1 - p_{21} - p_{22} \\ p_{31} & p_{32} & 1 - p_{31} - p_{32} \end{bmatrix}. \tag{A-39}
$$

D.3 Consumption claim

The return on consumption claim is

$$
R_{t+1}^c = \frac{Q_{t+1}^c + C_{t+1}}{Q_t^c} = \left[\frac{C_{t+1}}{C_t}\right] \cdot \left[\frac{PC_{t+1} + 1}{PC_t}\right] \tag{A-40}
$$

where $PC_t = Q_t^c / C_t$. The SDF under EZ preference is

$$
\begin{aligned}
M_{t+1} &= \delta^\theta \left[\frac{C_{t+1}}{C_t}\right]^{-\frac{\theta}{\psi}} \cdot \left[\frac{C_{t+1}}{C_t}\right]^{(\theta-1)} \cdot \left[\frac{PC_{t+1}+1}{PC_t}\right]^{(\theta-1)} \\
&= \delta^\theta \left[\frac{C_{t+1}}{C_t}\right]^{-\gamma} \cdot \left[\frac{PC_{t+1}+1}{PC_t}\right]^{(\theta-1)}.
\end{aligned} \tag{A-41}
$$

We can deduce from (A-40) and (A-41) that the Euler equation can be expressed as

$$\begin{aligned} 1 &= E_t\Bigg[M_{t+1}\cdot R^c_{t+1}\Bigg] \\ &= E_t\Bigg[\delta^\theta\left[\frac{C_{t+1}}{C_t}\right]^{1-\gamma}\cdot\left[\frac{PC_{t+1}+1}{PC_t}\right]^\theta\Bigg] \\ PC_t^\theta &= E_t\Bigg[\delta^\theta\exp\left[(1-\gamma)\Delta c_{t+1}\right]\cdot\left[PC_{t+1}+1\right]^\theta\Bigg]. \end{aligned} \tag{A-42}$$

D.4 Dividend claim

The return on dividend claim is

$$R^d_{t+1}=\frac{Q^d_{t+1}+D_{t+1}}{Q^d_t}=\left[\frac{D_{t+1}}{D_t}\right]\cdot\left[\frac{PD_{t+1}+1}{PD_t}\right] \tag{A-43}$$

where $PD_t = Q^d_t/D_t$. We can deduce from (A-41) and (A-43) that the Euler equation can be expressed as

$$\begin{aligned} 1 &= E_t\Bigg[M_{t+1}\cdot R^d_{t+1}\Bigg] \\ &= E_t\Bigg[\delta^\theta\left[\frac{C_{t+1}}{C_t}\right]^{-\gamma}\cdot\left[\frac{D_{t+1}}{D_t}\right]\cdot\left[\frac{PC_{t+1}+1}{PC_t}\right]^{(\theta-1)}\cdot\left[\frac{PD_{t+1}+1}{PD_t}\right]\Bigg] \\ PD_t &= E_t\Bigg[\delta^\theta\exp\left[-\gamma\Delta c_{t+1}+\Delta d_{t+1}\right]\cdot\left[\frac{PC_{t+1}+1}{PC_t}\right]^{(\theta-1)}\cdot\left[PD_{t+1}+1\right]\Bigg]. \end{aligned} \tag{A-44}$$

D.5 Equity premium

Denote the risk-free rate as

$$R_{f,t}=\frac{1}{E_t[M_{t+1}]}. \tag{A-45}$$

The expected return on dividend claim in excess of risk-free rate is

$$\begin{aligned} E_t\left[R^d_{t+1}\right]-R_{f,t} &= E_t\Bigg[\left[\frac{D_{t+1}}{D_t}\right]\cdot\left[\frac{PD_{t+1}+1}{PD_t}\right]\Bigg]-\frac{1}{E_t\left[M_{t+1}\right]} \\ &= E_t\Bigg[\exp\left[\Delta d_{t+1}\right]\cdot\left[\frac{PD_{t+1}+1}{PD_t}\right]\Bigg]-\frac{1}{E_t\left[\delta^\theta\exp\left[-\gamma\Delta c_{t+1}\right]\cdot\left[\frac{PC_{t+1}+1}{PC_t}\right]^{(\theta-1)}\right]}. \end{aligned} \tag{A-46}$$

D.6 Zero-coupon bond prices

The n-maturity zero-coupon bond price is determined by

$$Q_t^{(n)} = E_t\left[M_{t+1} \cdot Q_{t+1}^{(n-1)}\right] \tag{A-47}$$

with the initial condition $Q_t^{(0)} = 1$. Based on (A-41) we can re-express (A-47) as

$$Q_t^{(n)} = E_t\left[\delta^\theta \exp\left[-\gamma \Delta c_{t+1}\right] \cdot \left[\frac{PC_{t+1}+1}{PC_t}\right]^{(\theta-1)} \cdot Q_{t+1}^{(n-1)}\right]. \tag{A-48}$$

The respective log bond yield is $y_t^{(n)} = -\frac{1}{n}\log Q_t^{(n)}$.

D.7 Zero-coupon dividend equity prices

The n-maturity zero-coupon equity price

$$Q_t^{d,(n)} = E_t\left[M_{t+1} \cdot Q_{t+1}^{d,(n-1)}\right]. \tag{A-49}$$

Denote $Z_t^{(n)} = Q_t^{d,(n)}/D_t$. Instead of (A-49), we solve for

$$Z_t^{(n)} = E_t\left[M_{t+1} \cdot \left[\frac{D_{t+1}}{D_t}\right] \cdot Z_{t+1}^{(n-1)}\right]. \tag{A-50}$$

Based on (A-41) we can re-express (A-49) as

$$Z_t^{(n)} = E_t\left[\delta^\theta \exp\left[-\gamma \Delta c_{t+1} + \Delta d_{t+1}\right] \cdot \left[\frac{PC_{t+1}+1}{PC_t}\right]^{(\theta-1)} \cdot Z_{t+1}^{(n-1)}\right]. \tag{A-51}$$

The respective log equity yield is $e_t^{(n)} = -\frac{1}{n}\log Z_t^{(n)}$.

D.8 Zero-coupon nominal bond prices

The n-maturity zero-coupon nominal bond price is determined by

$$Q_t^{\$,(n)} = E_t\left[\left[\frac{M_{t+1}}{\Pi_{t+1}}\right] \cdot Q_{t+1}^{\$,(n-1)}\right] \tag{A-52}$$

with the initial condition $Q_t^{(0)\$} = 1$. Based on (A-41) we can re-express (A-52) as

$$Q_t^{\$,(n)} = E_t\left[\delta^\theta \exp\left[-\gamma \Delta c_{t+1} - \pi_{t+1}\right] \cdot \left[\frac{PC_{t+1}+1}{PC_t}\right]^{(\theta-1)} \cdot Q_t^{\$,(n)}\right]. \tag{A-53}$$

The respective log bond yield is $y_t^{\$,(n)} = -\frac{1}{n}\log Q_t^{\$,(n)}$.

D.9 Equity and bond return correlation

The one-period return on equity is $R^d_{t+1} = \left[\frac{D_{t+1}}{D_t}\right] \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]$.

D.9.1 With real return on a one-period real bond

The one-period return on zero-coupon bond of maturity n is $R^{(n)}_{t+1} = \frac{Q^{(n-1)}_{t+1}}{Q^{(n)}_t}$. The conditional covariance between the two is

$$\begin{aligned} cov_t\left[R^d_{t+1}, R^{(n)}_{t+1}\right] &= E_t\left[R^d_{t+1}R^{(n)}_{t+1}\right] - E_t\left[R^d_{t+1}\right]E_t\left[R^{(n)}_{t+1}\right] \\ &= E_t\left[\exp(\Delta d_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right] \cdot \left[\frac{Q^{(n-1)}_{t+1}}{Q^{(n)}_t}\right]\right] \\ &\quad - E_t\left[\exp(\Delta d_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]\right] \cdot E_t\left[\frac{Q^{(n-1)}_{t+1}}{Q^{(n)}_t}\right]. \end{aligned} \tag{A-54}$$

The conditional variances of equity return and bond return of maturity n are

$$var_t\left[R^d_{t+1}\right] = E_t\left[\exp(2\Delta d_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]^2\right] - \left(E_t\left[\exp(\Delta d_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]\right]\right)^2, \tag{A-55}$$

$$var_t\left[R^{(n)}_{t+1}\right] = E_t\left[\left[\frac{Q^{(n-1)}_{t+1}}{Q^{(n)}_t}\right]^2\right] - \left(E_t\left[\frac{Q^{(n-1)}_{t+1}}{Q^{(n)}_t}\right]\right)^2.$$

The equity and bond return correlation is

$$corr_t\left[R^d_{t+1}, R^{(n)}_{t+1}\right] = \frac{cov_t\left[R^d_{t+1}, R^{(n)}_{t+1}\right]}{\sqrt{var_t\left[R^d_{t+1}\right] \cdot var_t\left[R^{(n)}_{t+1}\right]}}. \tag{A-56}$$

D.9.2 With real return on a one-period nominal bond

The one-period return on zero-coupon nominal bond of maturity n is $R^{\$,(n)}_{t+1} = \left[\frac{Q^{\$,(n-1)}_{t+1}}{Q^{\$,(n)}_t}\right]$. The real return on a one-period nominal bond of the same maturity is $\frac{R^{\$,(n)}_{t+1}}{\Pi_{t+1}}$. The conditional covariance between the one-period real returns on equity and nominal bond is

$$\begin{aligned} cov_t\left[R^d_{t+1}, \frac{R^{\$,(n)}_{t+1}}{\Pi_{t+1}}\right] &= E_t\left[\exp(\Delta d_{t+1} - \pi_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right] \cdot \left[\frac{Q^{\$,(n-1)}_{t+1}}{Q^{\$,(n)}_t}\right]\right] \\ &\quad - E_t\left[\exp(\Delta d_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]\right] \cdot E_t\left[\exp(-\pi_{t+1}) \cdot \left[\frac{Q^{\$,(n-1)}_{t+1}}{Q^{\$,(n)}_t}\right]\right]. \end{aligned} \tag{A-57}$$

The conditional variances of real return of a one-period nominal bond of maturity n is

$$var_t\left[\frac{R_{t+1}^{\$,(n)}}{\Pi_{t+1}}\right]=E_t\left[\exp(-2\pi_{t+1})\cdot\left[\frac{Q_{t+1}^{\$,(n-1)}}{Q_t^{\$,(n)}}\right]^2\right]-\left(E_t\left[\exp(-\pi_{t+1})\cdot\frac{Q_{t+1}^{\$,(n-1)}}{Q_t^{\$,(n)}}\right]\right)^2. \quad \text{(A-58)}$$

The equity and bond return correlation is

$$corr_t\left[R_{t+1}^d,\frac{R_{t+1}^{\$,(n)}}{\Pi_{t+1}}\right]=\frac{cov_t\left[R_{t+1}^d,\frac{R_{t+1}^{\$,(n)}}{\Pi_{t+1}}\right]}{\sqrt{var_t\left[R_{t+1}^d\right]\cdot var_t\left[\frac{R_{t+1}^{\$,(n)}}{\Pi_{t+1}}\right]}}. \quad \text{(A-59)}$$

D.10 Numerical solution methodology for endogenous quantities

The log price-consumption ratio, pc_t, is the endogenous state-variable that, along with the exogenously given consumption dynamics, drive the stochastic discount factor and real bond yields. In particular, from (A-42) we have that:

$$pc\left(S_t,x_t\right)=\frac{1}{\theta}\ln E_t\left(\delta^\theta e^{(1-\gamma)\Delta c_{t+1}}\left(e^{pc(S_{t+1},x_{t+1})}+1\right)^\theta\right), \quad \text{(A-60)}$$

where we have made explicit dependence of the price-consumption ratio on the current state, S_t, and expected consumption growth, x_t. For simplicity, we assume a conditionally linear representation of $pc\left(S_t,x_t\right)$ for each regime S_t. We discretize the state space by considering grid points for x_t that are evenly spaced along the real line between the upper (0.04) and lower (-0.04) bounds. In annual percentage terms, these bounds translate into expected growth rate of $\pm$ 16% that can accommodate disasters in the economy. We solve for $pc\left(S_t,x_t\right)$ for a grid of values for x_t using an initial guess for $pc\left(S_{t+1},x_{t+1}\right)$, linear interpolation, and standard numerical integration methods. We iterate on this function starting from the initial guess of a constant equal to $\delta/(1-\delta)$ until a convergence criteria has been met. Details can be referred to the Online Appendix of Kaltenbrunner and Lochstoer (2010) on policy function iteration to solve models with Epstein-Zin preferences. The solution to the log price-dividend ratio (A-44) is found using similar methods iterating on:

$$pd\left(S_t,x_t\right)=\ln E_t\left(\delta^\theta e^{\Delta d_{t+1}-\gamma\Delta c_{t+1}+(1-\theta)pc(S_t,x_t)}\left(e^{pc(S_{t+1},x_{t+1})}+1\right)^{\theta-1}\left(e^{pd(S_{t+1},x_{t+1})}+1\right)\right). \quad \text{(A-61)}$$

Real bond yields are found using numerical integration, iterating recursively backwards using (A-48). Similarly, we assume a conditionally linear structure for the log nominal bond price of the maturity n, i.e., $q^{(n)}\left(S_t,x_t,\pi_t\right)$. We consider grid points for π_t that are evenly spaced along the real line between the upper (0.05) and lower (-0.03) bounds. Note that we place more grid points in areas that imply high inflation in the economy. Nominal bond yields are derived based on (A-53). Finally, conditional risk premiums and covariances are found through numerical integration and the corresponding equations given previously in the Appendix.

Table A-1: Robustness: Consumption autocorrelations with revised data

	Full sample		Sample excluding recessions	
	(1)	(2)	(1)	(2)
α_0	0.0012***	0.0002	0.0022***	0.0003
	(0.0004)	(0.0003)	(0.0004)	(0.0003)
α_1	0.0030***		0.0039***	
	(0.0008)		(0.0010)	
β_0	0.6308***	0.6877***	0.4884***	0.6126***
	(0.0821)	(0.0868)	(0.0825)	(0.0918)
β_1	−0.4337***	−0.4855***	−0.4461***	−0.5573***
	(0.1212)	(0.1209)	(0.1415)	(0.1548)
R^2	0.1240	0.1171	0.1004	0.0809

*N*otes: The table shows the regression estimates from the regression in Equation (1). On the right side, business cycle downturns are excluded from the sample as explained in the main text. COVID period is excluded from all regressions. Robust standard errors are reported in parenthesis. The β_0 estimate gives the autocorrelation coefficient in the post-1998 sample, while the β_1 estimate gives the difference between the autocorrelation in the pre- vs. post-1998 samples. The specification in the column labeled (1) estimates the full regression, while the specification in the column labeled (2) is run using demeaned consumption growth and setting $\alpha_1 = 0$, as explained in the main text. One asterisk denotes significance at the 10% level, two at the 5% level, and three at the 1% level. The data is the revised real, per capita, nondurables+services consumption data from 1947:Q2 to 2019:Q4.

Table A-2: Estimated parameters of NIPA revised nondurables+services consumption

	50%	[5%,	95%]
Consumption growth, Δc			
μ_c	0.0048	[0.0038,	0.0060]
ρ	0.8609	[0.8105,	0.9254]
$\sigma_p(1)$	0.0016	[0.0011,	0.0021]
$\sigma_p(2)$	0.0013	[0.0008,	0.0018]
$\sigma_p(3)$	0.0053	[0.0021,	0.0130]
$\sigma_\tau(1)$	0.0020	[0.0016,	0.0024]
$\sigma_\tau(2)$	0.0059	[0.0049,	0.0074]
$\sigma_\tau(3)$	0.0895	[0.0526,	0.2657]

Transition probabilities, $\mathbb{P}$

$$\begin{bmatrix} \underset{[0.9380,0.9839]}{0.9639} & \underset{[0.0077,0.0484]}{0.0248} & \underset{[0.0020,0.0268]}{0.0096} \\ \underset{[0.0141,0.0466]}{0.0319} & \underset{[0.9260,0.9776]}{0.9564} & \underset{[0.0010,0.0369]}{0.0089} \\ \underset{[0.0125,0.2677]}{0.1290} & \underset{[0.0438,0.2510]}{0.1975} & \underset{[0.5992,0.8135]}{0.6952} \end{bmatrix}$$

*N*otes: The table gives the maximum likelihood estimates (MLE) for the parameters governing the consumption process using the NIPA revised, per capita, real nondurables+services consumption data. The data frequency is quarterly. We order the regimes according to the transitory shock volatilities, and restrict the off-diagonal corner elements in the transition probability matrix to equal zero. The sample is 1947:Q2-2021:Q3.