# Minimum Wages, Efficiency and Welfare*

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**Abstract**

It has long been argued that a minimum wage could alleviate efficiency losses from monopsony power. In a general equilibrium framework that quantitatively replicates results from recent empirical studies, we find higher minimum wages can improve welfare, but most welfare gains stem from redistribution rather than efficiency. Our model features oligopsonistic labor markets with heterogeneous workers and firms and yields analytical expressions that characterize the mechanisms by which minimum wages can improve efficiency, and how these deteriorate at higher minimum wages. We provide a method to separate welfare gains into efficiency and redistribution. Under only the efficiency channel, the optimal minimum wage is narrowly around \$8, robust to social welfare weights, and generates welfare gains that recover only 2 percent of the efficiency losses from monopsony power. Under both channels and Utilitarian social welfare weights the optimal minimum wage is \$15, with 95 percent of the welfare gains from redistribution.

**JEL codes:** E2, J2, J42

**Keywords:** Labor markets, Market structure, Oligopsony, Minimum wages

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# 1 Introduction

The U.S. federal minimum wage has been roughly constant in real terms since the 1980s, and fallen almost 40 percent from its peak in the late 1960s (Figure 1A). As a result, the 2019 *Raise the Wage Act* proposed raising the Federal minimum wage to $15 an hour. If enacted, the minimum wage would exceed current wages for 41 percent of workers without a college education, 11 percent of college educated workers and 29 percent of workers overall (Figure 1B).[1] In this paper, we develop a tractable, general equilibrium oligopsony model of the labor market with heterogeneous workers and firms that is quantitatively consistent with the empirical minimum wage literature and use it to answer the following questions: *Is there a rationale for a non-zero minimum wage?*, if so, *how high should it be?*, and finally, *What would be the labor market consequences of raising minimum wages to a much higher level?*

A recent U.S. Treasury report on the *"State of Labor Market Competition"* makes clear two rationales for a positive minimum wage: efficiency and redistribution. First, if firms have market power in the labor market, wages are generically less than the marginal product of labor, and employment at each firm is inefficiently low. It was long been known that a well-targeted minimum wage could help alleviate the efficiency losses from monopsony power by inducing firms to hire more workers (Robinson, 1933).[2] Second, a higher minimum wage has the potential to benefit low income workers and reduce profits that tend to accrue to business owners and high income workers, redistributing economic output. Consistent with much of the debate around the utility of a higher minimum wage, arguments that start off from efficiency often shift to redistribution[3], highlighting the importance of isolating the contribution of each channel separately. Our analysis therefore returns to the beginning of the minimum wage debate and focuses on the ability of a national minimum wage to address inefficiencies due to labor market power.[4]

Our model, which includes both firm and worker heterogeneity, captures three empirically documented channels that could lead to higher efficiency. First, there is a *Direct effect* by which firms with market power increase their wages and expand employment when faced with a binding minimum wage. Evidence comes from increases in employment and wages following small minimum wage increases from initially low minimum wages (Clemens and Strain, 2021). Consistent with a large amount of other work, we also find employment decreases at higher minimum wages (Jardim, Long, Plotnick, Van Inwegen, Vigdor, and Wething, 2022). Second, there is a *Spillover effect* by which firms, whose competitors start paying higher wages, increase their wage in response and in doing so expand beyond their

[1] Raise the Wage Act (S. 53) was introduced to the Senate in January 2021, after being introduced a H.R. 582 in the House of Representatives in March 2019. The Congressional Budget Office reports on both bills can be found here for S. 53 and H.R. 582.

[2] From *The Economics of Imperfect Competition*: *"The amount of employment given by the monopsonist organization will be restricted to the amount at which the marginal cost of labour is equal to its demand price. The wage will be equal to the supply price of labour, and this, in each case, will be less than the value of the marginal physical product of labour. Thus exploitation will occur. Monopsonistic exploitation of this type can be removed by the imposition of a minimum wage."* (p.294)

[3] *"Raising the minimum wage is a straightforward approach to addressing lower wages under monopsony and can help increase employment."* (p.51, efficiency), and then *"Raising the federal minimum wage would give nearly 32 million Americans a raise and would boost the purchasing power of low-income families ..."* (p.52, redistribution)

[4] Furthermore, a focus on the redistributive properties of the minimum wage has the undesirable feature in that the answer depends significantly on what the social welfare weights are and, holding social welfare weights fixed, the availability and efficiency of other fiscal instruments.

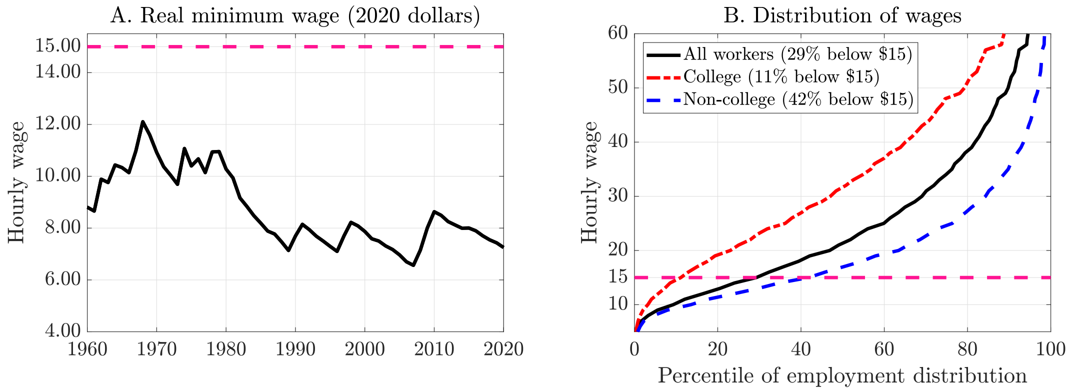


Figure 1: Minimum wages in the US, 1960-2020

Notes: **Panel A.** Real wages computed by deflating by the CPI-U. **Panel B.** CPS data constructed using MORG from 2019, and weighted using `hwtfinl`. Wages are computed as weekly earnings (`earnweek`) divided by usual weekly hours worked (`uhrsworkt`). We follow the Federal Reserve Bank of Atlanta Wage Growth Tracker, and remove individuals whose hourly pay is below the current federal minimum wage for tip-based workers ($2.13). We drop individuals who are coded as *hours vary* (`uhrsworkt=997`). We keep all other workers aged between 16 and 65.

inefficiently low levels of employment. Evidence comes from competitors' response to Amazon increasing its wage (Derenoncourt, Noelke, Weil, and Taska, 2021) and hospitals raising their wages in response to competitors (Staiger, Spetz, and Phibbs, 2010). Third, there is a *Reallocation effect* by which a minimum wage may destroy jobs at unproductive firms, with labor reallocated to more productive firms. Evidence comes from reallocation across firms in Germany (Dustmann, Lindner, Schoenberg, Umkehrer, and vom Berge, 2021).

Our model extends the textbook model of monopsony under a minimum wage to incorporate firm heterogeneity in productivity, worker heterogeneity in wealth and productivity, and a well-defined notion of labor market power: jobs are differentiated and firms compete strategically in many concentrated labor markets. Our general equilibrium framework captures the relevant trade-offs necessary to evaluate a higher minimum wage and capture the above channels. On the one hand, (i) monopsony power means that a higher minimum wage can raise wages and employment, (ii) oligopsony power allows for empirically plausible responses to competitors' wage policies, and (iii) firm heterogeneity allows for reallocation. Together, this creates the possibility that a higher minimum wage may be welfare improving. On the other hand, disemployment effects kick in if the minimum wage exceeds a firms' competitive wage, and, as wage differences between firms becomes compressed, some workers choose to move from high paying, high productivity firms to newly higher paying but less productive firms. This leads to misallocation. These imply that above a certain threshold, a higher minimum wage reduces welfare.

We quantify these forces by calibrating our model to the U.S. economy. The calibrated model reproduces the empirical distribution of markets in terms of the number of firms in each market, average firm employment and payroll, relative wages across worker types, distribution of consumption and non-wage income across worker types, average market concentration, labor and capital share, and the observed relationship between labor market share and wage and employment responses to shocks (Berger, Herkenhoff, and Mongey, 2022). Our calibrated model, quantitatively replicates the above studies, which both disciplines the channels through which minimum wages may improve efficiency and provides important overidentifying tests of our model as we do not target these replications in our calibration. We adapt our environment to the empirical setting studied in each paper under our baseline calibration, repeat the associated natural experiment and reproduce the authors' empirical analysis. Hence, both theoretically and empirically, the model captures the key margins through which minimum wages can lead to expanded output in the economy.[5]

We then use our model to answer: what is the minimum wage that maximizes efficiency, what are the welfare gains, and through which channels do these accrue? Because the minimum wage has both *efficiency* and *redistributive* implications, we develop a methodology for disentangling the two. This is necessary for two reasons. First, we do not know what the correct social welfare weights are and this choice matters quantitatively. Under Utilitarian weights a planner would choose a minimum wage of $15.12, while if it cared only about college educated workers it would choose $31.53, or down to $6.97 if it used the social welfare weights that rationalize the observed competitive equilibrium. Second, governments have access to additional tools for redistribution via the tax and transfer system, which may be changed along with the minimum wage. We cannot model all tools of the tax and transfer system, or compute how each tool should be reoptimized under each level of a minimum wage considered.

Our methodology addresses both issues via the following hypothetical accounting device, we ask: *what is the optimal minimum wage in the presence of budget-neutral, unrestricted lump-sum transfers across households?* This addresses the first issue, as taking labor market imperfections as given, the government can reoptimize transfers to meet the redistribution objectives encoded in any arbitrary social welfare weights. It addresses the second issue in that lump sum transfers encapsulate all possible tax and transfer schemes. With redistribution taken care of, an optimal minimum wage reflects only efficiency, which is the focus of this paper.[6] Now, rather than a range of $0 to $31 per hour—depending on social welfare weights, and keeping fiscal policies fixed—the answer narrows to $7.50 to $10 per hour—invariant to social welfare weights, and with flexible redistributive policy. Moreover, the associated efficiency gains are small—equivalent to a 0.1 percent increase in TFP—and retrieve only 2 percent of the efficiency losses due to market power in the labor market. From our model's perspective, a higher minimum wage is not justified based on efficiency grounds.

There are three reasons why efficiency gains are limited. First, the direct efficiency gains come from firms for whom the minimum wage is binding but still below the firm's competitive wage (the wage they pay with no markdown). These are lower productivity firms. The efficiency gains that can be squeezed from them are small because (i) they have little market power, limiting the scope of efficiency gains, (ii) they face highly elastic labor supply, so only a small increase in their wage shifts them to their efficient level of employment. Second, in our calibration, firms have a relatively flat marginal

[5]In Appendix B we show that our model also reproduces recent evidence (i) that the employment effects of minimum wages may be positive in concentrated markets and negative in less concentrated markets (Azar, Huet-Vaughn, Marinescu, Taska, and von Wachter, 2019), (ii) on the spillover effects of minimum wages across workers (Autor, Manning, and Smith (2016), in the US, and Engbom and Moser (2021) and Haanwickel (2020) in Brazil). Hence we make a separate contribution in providing a unifying quantitative framework for many existing empirical results.

[6]We provide aggregation theorems that allow us to compute optimal lump-sum taxes under arbitrary social welfare weights.

revenue product of labor schedule which implies that once firms start rationing employment, they do so very quickly. Third, the biggest efficiency gains would come from narrowing markdowns at highly productive, unconstrained firms who have the widest markdowns in each market.[7] However, because their low wage competitors that pay the minimum wage have small market shares, large firms largely ignore them and hence spillover effects to large firms are quantitatively small.

These results do not rule out the minimum wage as a tool for improving welfare. The potency of minimum wages to redistribute has been well documented (Derenoncourt and Montialoux, 2021; Cengiz, Dube, Lindner, and Zipperer, 2019), and we show our model generates spillovers up the wage distribution consistent with empirical evidence. Instead our exercises quantify the extent to which redistribution is the *primary channel* through which minimum wages improve welfare.[8] Our key result is that under the $15.12 per hour minimum wage that maximizes social welfare under Utilitarian weights, only 5 percent of the welfare gains come from improved efficiency, whereas 95 percent come from redistribution. To understand this result we first deliver aggregation theorems that allow us to compute the social welfare weights that would lead a planner to choose the observed competitive equilibrium allocation of consumption and labor. The weights we back out load disproportionately on college educated households. Importantly, their weight is higher than their population share, which is the weight assigned by a Utilitarian planner.[9] Hence, a minimum wage policy that maximizes a utilitarian objective will place a large emphasis on redistribution, which is exactly what we find.

These results also do not rule out the minimum wage as a tool for reducing income inequality or increasing labor's share of income, which are common empirical proxies for inequality and worker power. Indeed, we show that under a higher minimum wage, income inequality and the college wage premium fall, and labor's share of income increases.[10] Both indicators are *monotonic* as the minimum wage increases, despite the clear hump-shape of the welfare effects of the minimum wage. This result warns that observations such as a higher labor share of income or lower wage inequality caused by a higher minimum wage can be associated with *declining* welfare.

We provide a number of robustness checks of our main results. First, we calibrate the economy separately to low, medium and high income states. Welfare gains from three region specific minimum

[7]The cross-sectional relationship of larger market share and wider markdowns is documented for the U.S. in Hershbein, Macaluso, and Yeh (2022). As described in that paper, our model is consistent with their facts.

[8]This provides a quantitative answer to the first set of questions left open in Cahuc and Zylberberg (2004). The authors conclude their section on minimum wages by questioning whether a minimum wage primarily acts through efficiency or redistribution. The further set of questions, taken up by Hurst, Kehoe, Pastorino, and Winberry (2021), relates to whether alternative fiscal policies can do better at redistribution than a minimum wage.

[9]This is a common result in normative applications of quantitative heterogeneous agent models. For example, part of the exercise in Heathcote and Tsujiyama (2021) is to ask *"For what set of social welfare weights would the observed schedule of labor income taxes be the optimal schedule of labor income taxes?"*. They find that the answer is a set of welfare weights tilted toward high income households.

[10]Recently, Deb, Eeckhout, Patell, and Warren (2020) extends Berger, Herkenhoff, and Mongey (2022) to study inequality between skill types, but do not study minimum wages. In their model, built for positive analysis, both skill types live in the same household, share the same budget constraint, and consumption is joint. This removes the role of exposure to profits and wealth effects on labor supply from the analysis. Huneeus, Kroft, and Lim (2021) study a minimum wage policy in an environment where workers are all similarly exposed to profits, like in Deb, Eeckhout, Patell, and Warren (2020), but firms are monopsonistic and so do not respond to competitors' responses to the minimum wage.

wages are only marginally greater than the welfare gains from a federal minimum wage. Second, we compute the optimal minimum wage as the Frisch elasticity of labor supply varies, with little effect on our main results. Third, our main exercise is inherently long-run, so we consider a short-run exercise where capital is fixed type-by-type within the firm, leading to exit.[11] We theoretically characterize the effect of fixing capital and quantitatively find that it lowers the optimum by around one dollar.

**Literature.** Our paper analyzes concentrated markets with strategic interactions in the presence of price controls and firm heterogeneity. This problem of price controls has been studied in stylized cases with symmetric firms in the case of an oligopoly setting (Molho, 1995; Reynolds and Rietzke, 2018), while others have studied capacity constraints and rationing but no price setting (e.g. de Palma, Picard, and Waddell (2007) and Ching, Hayashi, and Wang (2015)). Our characterization is new, and handles heterogeneity in firms' marginal cost. We show that we can take the equilibrium conditions of the economy and express them in terms of *shadow wages* which are *shadow markdowns* relative to marginal revenue products. At the micro level, shadow markdowns simultaneously encode (i) multipliers on firm-specific constraints that ration equilibrium labor under a minimum wage, and (ii) deviations from efficiency due to market power. At the macro level, firm shadow markdowns aggregate to wedges that encode the deviations of the entire economy from an efficient benchmark in which there is no labor market power or minimum wages. These make precise the trade-off between increasing efficiency due to addressing market power, and decreasing efficiency due to employment losses.

The most closely related papers to our exercise of constructing a general equilibrium model with a minimum wage are complementary studies by Hurst, Kehoe, Pastorino, and Winberry (2021) and Ahlfeldt, Roth, and Seidel (2022). The emphasis of Hurst, Kehoe, Pastorino, and Winberry (2021) is redistribution and the positive implications of minimum wages, rather than the normative exercises regarding efficiency that we consider. In particular, Hurst, Kehoe, Pastorino, and Winberry (2021) have an expanded role for worker heterogeneity and homogeneous firms, which allows a richer discussion of redistribution, while our setting has an expanded role for firm heterogeneity as is required of a discussion of efficiency via the three channels discussed earlier. We focus on replicating studies in the empirical minimum wage literature that pertain to these channels. Hurst, Kehoe, Pastorino, and Winberry (2021) compare a $15 minimum wage to other existing transfer policies, and find that policies such as the EITC dominate the minimum wage for the majority of non-college workers. They also consider dynamic capital accumulation in a putty-clay setting, whereas we consider a simple short- and long-run exercise with fixed- and flexible- capital in order to demonstrate the robustness of our results.

Concurrent work by Ahlfeldt, Roth, and Seidel (2022) develops a general equilibrium spatial model of the German economy in order to compute welfare maximizing minimum wages. Relative to Ahlfeldt, Roth, and Seidel (2022), we incorporate (1) concentrated labor markets, (2) forward looking workers who accumulate capital (which is a factor of production), (3) worker-types (education levels) across which there is no consumption insurance, and (4) decompose the welfare effects of the minimum wage into the portions due to efficiency and redistribution.

[11]This is a simplified version of exercises in putty-clay models of Aaronson, French, Sorkin, and To (2018) and Sorkin (2015).

That monopsony can rationalize small, and positive, employment responses to minimum wages is in part responsible for the theory's historical development (Card and Krueger, 1994; Boal and Ransom, 1997; Manning, 2003). Whether minimum wages have positive or negative employment effects is a contentious topic. On the one hand, a lengthy review by Neumark and Wascher (2006) concludes that the balance of the empirical literature demonstrates negative employment effects. On the other hand, a summary by Allegretto, Dube, Reich, and Zipperer (2017) concludes that employment effects are small and positive. Our model provides a tent for all parties, by demonstrating circumstances under which employment effects are negative, and under which they are positive. A common theme in our results is that non-linearities warn against extrapolating from small increases in minimum wages to large changes.

Our paper studies a neoclassical labor market, similar to Cahuc and Laroque (2014), Lee and Saez (2012) among others, while the minimum wage has often been studied in frictional settings. Flinn (2010) and Flinn (2006) document the economic forces that shape the optimal minimum wage in a frictional setting. Flinn and Mullins (2021) study the choice of firms' optimal wage setting strategy in this environment, finding that higher minimum wages lead more firms to prefer renegotiation to wage-posting. Engbom and Moser (2021) equip a Burdett and Mortensen (1998) model for quantitative analysis and study the effects of a large increase in the minimum wage in Brazil on wage inequality, but do not compute or delineate the forces that would shape an optimal minimum wage in that framework.

As a starting point, we abstract from two effects of minimum wages that have been empirically documented. First, recent work has found conflicting results on whether higher minimum wages can be passed through to prices.[12] In our benchmark model, we assume price pass through is zero, however, our framework is fungible enough to include imperfect competition in the production market.[13] Second, Harasztosi and Lindner (2019) document that firms substitute away from labor and toward capital, increasing purchases of computers and other capital goods. Our model features a unit elasticity of substitution between labor and capital. In general both channels will weaken redistribution toward low wage workers. Since redistribution is not our focus, we leave adding these mechanisms to future work. Researchers or policy makers may also wish to compute optimal minimum wages as they vary by market concentration or some other observable. On the latter, some countries feature occupation specific minimum wages (e.g. Australia). Satisfactorily including such policies would require modeling occupational choice for different types of workers. We leave these interesting extensions to future work.

**Overview.** The rest of the paper proceeds as follows. Sections 2 and 3 lay out the model environment. Section 2 analyzes a simplified version of our model under neoclassical monopsony and oligopsony to fix ideas. Section 3 describes our quantitative model with worker heterogeneity, wealth effects on labor supply, capital, and a government. In Section 4 we calibrate the model. In Section 5 we replicate the design and estimates of five empirical studies. Section 6 documents the positive implications of the min-

[12] Ganapati and Weaver (2017) estimate zero pass-through in scanner data and cite several corroborating studies. Renkin, Siegenthaler, and Montialoux (2021) also use scanner data and estimate full pass-through.

[13] Our benchmark model incorporates a decreasing marginal revenue product of labor through decreasing returns in production, but that could be replaced by downward sloping demand under monopolistic competition. In such an environment, the pass-through (in logs) of minimum wages to prices is one for firms that ration employment.

imum wage for employment, wages and proxies for inequality. Section 7 separates out efficiency from redistribution and in doing so computes the optimal minimum wage. Section 8 provides our robustness exercises.

# 2 Model

Our general equilibrium framework with firm heterogeneity is not nested in existing treatments of the neoclassical theory of minimum wages.[14] We therefore carefully describe our environment and equilibrium. We first analyze a simplified version of our model under neoclassical monopsony in Section 2.1. In Section 2.2 we allow firms to strategically interact. Finally we describe our quantitative model in Section 3 which adds worker heterogeneity, wealth effects on labor supply, capital, and a government.

## 2.1 Simple monopsony economy

Assumptions in *blue* are relaxed in our quantitative model. In this simplified economy there is a *single market* and firms *do not strategically interact*. We define worker and firm problems, how they are linked, define an equilibrium, solve for optimal firm behavior, and characterize the solution in terms of *shadow wages*. We summarize our results in Figure 2. Key propositions used to characterize the monopsony economy readily handle oligopsony. The economy studied here is nested in our quantitative model, under parametric restrictions.

**Agents.** The economy consists of a *single household* and an infinite number of firms. Firms operate in a *single labor market*, are indexed $i \in [0, 1]$, and differ in total factor productivity $z_i \in (0, \infty)$.

**Goods and technology.** Each firm produces a homogeneous good which trades in a perfectly competitive market at price $P$, normalized to one. Goods are *only used for consumption*. A firm produces $y_i$ units of the good according to *production function*: $y_i = z_i n_i^{\alpha}$, $\alpha > 0$.

**Preferences.** Households have *linear preferences over consumption* and a convex disutility of labor. Labor disutility is a special case of the $CES$ functional form used by BHM. Consumption goods are perfect substitutes:

$$\mathcal{U} = C - \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \quad , \quad N := \left[\int n_i^{1+\frac{1}{\eta}} di\right]^{\frac{\eta}{\eta+1}} \quad , \quad C := \int c_i di. \tag{1}$$

Parameter $\varphi$ governs the household's overall willingness to supply labor, and $\eta$ governs the household's willingness to substitute workers across firms. A low $\eta$ implies sharply increasing disutility as labor is reallocated to firm $i$. A firm will perceive this as a more inelastic labor supply curve.[15]

**Budget constraint.** Profits ($\Pi$) are rebated to the household. Consumption is $C = \int_0^1 w_i n_i di + \Pi$.

[14] See for example Ehrenberg and Smith (1994) and Cahuc and Laroque (2014).

[15] In BHM we show how the labor supply curves obtained under nested $CES$ preferences (of which our current preferences are a special case) can also be obtained by individuals making discrete labor supply decisions. This is a straight-forward extension of techniques from the demand system literature: Anderson, De Palma, and Thisse (1987) (single nested) and Verboven (1996) (double nested).

**Minimum wages and rationing constraints.** Denote the minimum wage $\underline{w} \geq 0$. Since the labor market for a given firm may not necessarily clear for a given minimum wage, we allow each firm to specify a constraint $\overline{n}_i$. This is a sign on the firm's door telling the household the maximum amount of labor the firm is willing to hire, hence $n_i \leq \overline{n}_i$. We call this a *rationing constraint*.

**Markets and competition.** The household behaves competitively taking each firm's rationing constraint, wage and price as given. The goods market is perfectly competitive. Firms are atomistic, take aggregate quantities and wages as given, and compete under Cournot competition in the labor market.

**Household problem.** The household takes rationing constraints $\{\overline{n}_i\}_{i\in[0,1]}$, wages $\{w_i\}_{i\in[0,1]}$ and profits $\Pi$ as given. The household chooses employment $\{n_i\}_{i\in[0,1]}$ at each firm. Since it takes $\Pi$ as given, we can substitute the budget constraint into utility to obtain the following problem:

$$\max_{\{n_i\}_{i\in[0,1]}} \int_0^1 w_i n_i di - \frac{N^{1+1/\varphi}}{1+1/\varphi} \quad , \quad N = \left[\int_0^1 n_i^{\frac{1+\eta}{\eta}} di\right]^{\frac{\eta}{1+\eta}} \quad , \quad \text{subject to } n_i \leq \overline{n}_i \quad \text{for all } i \in [0,1] .$$

The first order necessary conditions and complementary slackness characterize labor supply:

$$w_i = \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} + \nu_i \quad , \qquad \nu_i\left(\overline{n}_i - n_i\right) = 0 \quad , \quad \text{for all } i \in [0,1] \tag{2}$$

Throughout we use *binding* to mean a strictly binding constraint ($\nu_i > 0$, $n_i = \overline{n}_i$), whereas *slack* indicates a weakly slack constraint ($\nu_i = 0$, $n_i \leq \overline{n}_i$). If the rationing constraint binds then the wage exceeds the marginal disutility of labor supply to firm $i$: $w_i > \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$. In such cases a firm could hire the same labor for a lower wage, and hence, in equilibrium, will only occur when the minimum wage binds.

These conditions deliver a well-defined function that is independent of $\nu_i$ and which we refer to as the *labor supply schedule* to firm $i$. Equivalently, we can obtain an *inverse labor supply schedule*, which takes the form of a correspondence:

$$\underbrace{n_i = n\left(w_i, \overline{n}_i, N\right) = \begin{cases} w_i^{\eta} N^{-\frac{\eta-\varphi}{\varphi}} & w_i < \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \\ \overline{n}_i & w_i \geq \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \end{cases}}_{\text{I. Labor supply schedule}} , \quad \underbrace{w_i = w\left(n_i, \overline{n}_i, N\right) = \begin{cases} \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} & , n_i \in \left[0, \overline{n}_i\right) \\ \in \left[\left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}, \infty\right) & , n_i = \overline{n}_i . \end{cases}}_{\text{II. Inverse labor supply schedule}} .$$

At $n_i = \overline{n}_i$, the household cannot supply more labor, producing a vertical inverse labor supply schedule. To employ $\overline{n}_i$ workers, the firm can pay any wage $w_i$ that is at least $(\overline{n}_i/N)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$.

**Firm problem.** Each firm $i$ takes as given the aggregate employment index $N$ and chooses its (i) wage $w_i$, (ii) employment $n_i$, and (iii) rationing constraint $\overline{n}_i$, to maximize profits. The firm is constrained by (a) the minimum wage $w_i \geq \underline{w}$, (b) its chosen rationing constraint $n_i \leq \overline{n}_i$, and (c) the *inverse labor supply schedule* of households. The firm has market power in the sense that they know their employment choices affect their wage through $w(n_i, \overline{n}_i, N)$. The firm problem is therefore,

$$\max_{\overline{n}_i, n_i, w_i} z_i n_i^{\alpha} - w_i n_i \quad , \quad \text{subject to} \quad w_i \geq \max\left\{ \underline{w}\, , w\left(n_i, \overline{n}_i, N\right) \right\} \quad , \quad n_i \leq \overline{n}_i$$

**Equilibrium.** Given a minimum wage $\underline{w}$, a *monopsonistic Cournot* equilibrium is (i) a household inverse labor supply schedule $w\left(n_i, \overline{n}_i, N\right)$, (ii) wages $\{w_i\}_{i\in[0,1]}$ , (iii) quantities of labor $\{n_i\}_{i\in[0,1]}$, (iv) rationing constraints $\{\overline{n}_i\}_{i\in[0,1]}$, (v) profits $\Pi$, and (vi) aggregate employment index $N$ such that (1) given wages $\{w_i\}_{i\in[0,1]}$, rationing constraints $\{\overline{n}_i\}_{i\in[0,1]}$, and profits $\Pi$, household optimization implies the inverse labor supply curve $w(n_i, \overline{n}_i, N)$, (2) given the aggregate employment index $N$, and the household inverse labor supply schedule, firm $i$'s optimization yields rationing constraint $\overline{n}_i$, wage $w_i$ and employment $n_i$, (3) firm employment is consistent with the aggregate employment index $N$ and profits $\Pi$, and (4) markets clear $w_i = w(n_i, \overline{n}_i, N)$ for all $i \in [0,1]$ .

**Characterization of monopsony equilibrium.** We first characterize the firm's optimal wage and rationing constraint. Second, we substitute these solutions into the firm problem and derive their optimal employment $n_i$. Third, we prove equivalence of the solution to with an auxiliary set of conditions that leverage household multipliers. We refer to multiplier-adjusted wages as shadow wages.

Define *legal wages* as all wages $w_i \geq \underline{w}$. Through lemmas in Appendix E, we (i) prove a firm would never pay a wage greater than the lowest legal wage necessary to obtain a given level of employment, and refer to the resulting mapping between employment and legal wages as the firm's *perceived inverse labor supply schedule*, (ii) show it is *weakly* optimal for a firm to impose a rationing constraint at an $\overline{n}_i$ such that the marginal product of labor at $\overline{n}_i$ is equal to the marginal cost of labor if paying the minimum wage, $\underline{w}$: $mrpl(\overline{n}_i) = \underline{w}$, and (iii) show that applying these results, the firm problem becomes,

$$\max_{n_i} z_i n_i^{\alpha} - w_i n_i \text{ , subject to } \quad n_i \leq \overline{n}_i \quad , \quad \underbrace{\overline{n}_i = \left(\frac{\alpha z_i}{\underline{w}}\right)^{\frac{1}{1-\alpha}}}_{mrpl(\underline{n}_i)=\underline{w}} \text{ , } w_i = \begin{cases} \underline{w} & \text{if } \underline{w} > \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \\ \max\left\{\underline{w}, \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}. \end{cases} \tag{3}$$

The last constraint is the *perceived inverse labor supply schedule*, which takes the form of a function defined on $[0, \overline{n}_i]$, rather than a correspondence.

We can split the solution of the firm problem into three regions. Let $\widetilde{n}_i$ denote the level of employment such that the minimum wage is binding: $\underline{w} = (\widetilde{n}_i/N)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$. Let $\widehat{w}(n_i)$ denote the *unconstrained* labor supply curve $\widehat{w}(n_i) = (n_i/N)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$. Appendix E.1, Lemma 3, establishes that the solution to (3) takes one of three forms:

$$mrpl(n_i^*) = \begin{cases} \underline{w} & n_i^* < \widetilde{n}_i & \text{Region III} \\ \in \left[\underline{w}, \widehat{w}'(n_i^*) n_i^* + \widehat{w}(n_i^*)\right] & n_i = \widetilde{n}_i & \text{Region II} \\ \widehat{w}'(n_i^*) n_i^* + \widehat{w}(n_i^*) & n_i^* > \widetilde{n}_i & \text{Region I} \end{cases} \tag{4}$$

In Region III, the firm faces a binding minimum wage, $w_i = \underline{w}$, and optimal employment sits at the rationing constraint. This equates the marginal product of labor to marginal cost ($\underline{w}$): $n_i^* = \overline{n}_i = mrpl^{-1}(\underline{w})$. From the household's perspective, the rationing constraint binds ($\nu_i > 0$) and the wage exceeds the household's marginal disutility of supplying $n_i^*$. In Region II, the firm faces a binding minimum wage, $w_i = \underline{w}$, the household is on its labor supply curve, and hence the rationing constraint is slack ($\nu_i = 0$). Employment $n_i$ is defined by $\underline{w} = \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$. In Region I, the firm is unconstrained, the household is on its labor supply curve (i.e., $w_i = \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$) and wages are a constant markdown on the marginal revenue product of labor, $w_i = \mu_i mrpl(n_i)$ where $\mu_i = \frac{\eta}{\eta+1}$. We summarize the firm's optimal employment, wage, and rationing choices below

$$\left(w_i, n_i, \overline{n}_i\right) = \begin{cases} \left( \underline{w} \quad , \left(\frac{\alpha z_i}{\underline{w}}\right)^{\frac{1}{1-\alpha}} , \left(\frac{\alpha z_i}{\underline{w}}\right)^{\frac{1}{1-\alpha}} \right) & n_i < \widetilde{n}_i \quad \text{Region III} \\ \left( \underline{w} \quad , \underline{w}^{\eta} N^{1-\frac{\eta}{\varphi}} , \left(\frac{\alpha z_i}{\underline{w}}\right)^{\frac{1}{1-\alpha}} \right) & n_i = \widetilde{n}_i \quad \text{Region II} \\ \left( \mu_i mrpl\left(n_i\right) , w_i^{\eta} N^{1-\frac{\eta}{\varphi}} , \left(\frac{\alpha z_i}{\underline{w}}\right)^{\frac{1}{1-\alpha}} \right) & n_i > \widetilde{n}_i \quad \text{Region I.} \end{cases} \tag{5}$$

**Shadow wages.** We can recast the equilibrium conditions for firms' optimal wages and employment in terms of *shadow wages*. Doing so allow us to (i) succinctly analyze firm behavior and, (ii) aggregate firm optimality to study general equilibrium, which (iii) allows us to pinpoint efficiency gains and losses due to minimum wages. Let $p_i$ be a normalization of the multiplier on the rationing constraint $\nu_i$:

$$\nu_i = w_i\left(1 - p_i\right) \quad , \quad p_i = \frac{w_i - \nu_i}{w_i}$$

Household optimality conditions (2) can be written

$$p_i w_i = \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \quad , \quad \left(1 - p_i\right)\left(\overline{n}_i - n_i\right) = 0 \quad , \quad \text{subject to } n_i \leq \overline{n}_i \quad \text{for all } i \in [0,1] .$$

Hence $p_i \in (0,1]$ with $p_i < 1$ if the constraint binds. Define the *shadow wage* $\widetilde{w}_i$ and *shadow markdown* $\widetilde{\mu}_i$:

$$\widetilde{w}_i := p_i w_i = \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \quad , \quad \widetilde{w}_i = \widetilde{\mu}_i mrpl\left(n_i\right) .$$

In Region I, $p_i = 1$, $\widetilde{w}_i = w_i$ and $\widetilde{\mu}_i$ is the monopsony markdown of $\widetilde{\mu}_i = \eta / \left(1 + \eta\right)$. In Region III, $p_i < 1$, $w_i = \underline{w} = mrpl_i$, and hence $\widetilde{\mu}_i = p_i$: the shadow markdown encodes the bindingness of the rationing constraint. In Region II, $p_i < 1$, $\widetilde{w}_i = \underline{w}$, and $n_i$ is on the household labor supply curve. In this region

$$\underline{w} = \left(\frac{n_i}{N}\right)^{1/\eta} N^{1/\varphi} \quad , \quad n_i = \underline{w}^{\eta} N^{1-\eta/\varphi} \quad , \quad \widetilde{\mu}_i = \frac{1}{\alpha z_i} \times \underline{w}^{1+\eta(1-\alpha)} N^{(1-\alpha)\left(1-\frac{\eta}{\varphi}\right)} .$$

As the minimum wage increases, $\widetilde{\mu}_i$ increases (narrows). At the border of Regions II and III, the wage and marginal revenue product are equalized, hence—at the firm level—the employment allocation is efficient. At this point $\widetilde{\mu}_i = 1$, as $\widetilde{w}_i = \underline{w} = mrpl\left(n_i\right)$. Hence the shadow markdown is determinative of deviations of employment allocations from efficiency due to either (i) market power, in Regions I and II, or (ii) the minimum wage, in Region III.

**Graphical analysis.** To summarize the above, Figure 2 illustrates firms' optimality conditions. Panel A reproduces the firm's optimality condition in a neoclassical monopsony model without a minimum wage.[16] With monopsony power the firm contracts $n_i^0$ below the competitive benchmark $n_i^c$ and hence pays lower wages $w_i^0 < w_i^c$. Starting in Panel B, we study the firms' problem when a non-binding minimum wage is introduced: the shadow wage and shadow markdown coincide with Panel A. Note that the inverse labor supply schedule which the firm takes as given, emerges from household optimality and maps $n_i$ and $\overline{n}_i$ into $w_i$. The weakly optimal rationing constraint $\overline{n}_i$ truncates labor supply, and is slack.

[16] If the downward sloping marginal revenue product of labor reflected diminishing marginal revenue—as would be the case for a monopolistically competitive producer—this second component of profits would be due to a price markup.

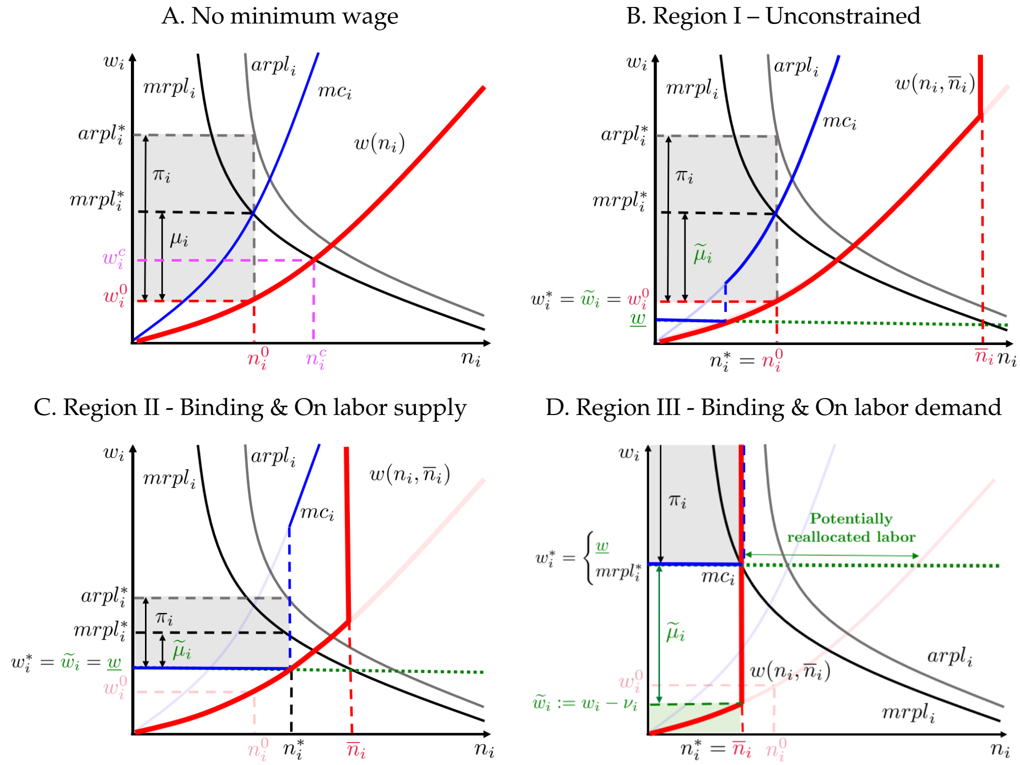


Figure 2: Increase in the minimum wage: Simple monopsony economy - Partial equilibrium

Notes: The dashed green line corresponds to the minimum wage $\underline{w}$. The red line gives the household's *inverse labor supply schedule* $w(n_i, \overline{n}_i, N)$, which depends on its labor supply and the rationing constraint $\overline{n}_i$. The blue line gives the firm's *marginal cost of labor* along its *perceived labor supply curve* $\max\{\underline{w}, w(n_i, \overline{n}_i, N)\}$ on $n_i \in (0, \overline{n}_i]$.

In Panel C, the higher minimum wage has pushed the firm into Region II: the minimum wage now binds, but employment is pinned down by household labor supply. Relative to Panel B, wages and employment increase, with the loss in profits born by the firm.[17] The rationing constraint remains slack ($p_i = 1$), and the shadow wage and minimum wage coincide. A further increase in the minimum wage would narrow the firm's equilibrium shadow markdown $\widetilde{\mu}_i$ with an elasticity of $1 + \eta(1 - \alpha)$, representing an erosion of monopsony power. This represents the *monopsony channel* through which a higher minimum wage can improve efficiency.

Increasing the minimum wage further pushes the firm past the efficient allocation ($\widetilde{\mu}_i = 1$) and into *Region III* (Panel D). At $(\underline{w}, \overline{n}_i)$, the marginal disutility of labor—read off the supply curve—, is below the wage, and hence absent the rationing constraint labor would be in excess supply. Here the shadow markdown measures this wedge, with the shadow wage falling below the wage. Note that output is redistributed: the firm has no market power and earns less profits, while the remaining workers earn higher wages. However, from an efficiency perspective, since $\overline{n}_i < n_i^0$ from Panel A, the minimum wage

[17] In Region II, the marginal cost curve is different from the benchmark economy. The new marginal cost curve is horizontal and equal to $\underline{w}$ until it reaches the labor supply curve. Up to this point workers are paid $\underline{w}$. Marginal cost then jumps, as above the minimum wage, additional hiring requires increasing pay for existing workers. Since marginal cost jumps above the marginal revenue product of labor, profit maximizing employment is on the labor supply curve at $\underline{w}$.

has created less efficient employment than the baseline with market power and no wage.

In an environment with *homogeneous firms*, one could label the gap between labor demanded at $\underline{w}$ and what would otherwise be supplied as non-employment generated by the minimum wage. In our environment, much of this slack can be reallocated to other firms in the economy. Since low productivity firms will be the first to enter Region III as $\underline{w}$ increases, this reallocation will be to more productive firms. This represents the *reallocation channel* through which a higher minimum wage can improve efficiency: jobs aren't necessary destroyed, they're partially reallocated.

In summary, at the microeconomic level of the firm, the introduction of rationing constraints delivers a clear picture of the wages and shadow wages that rationalize equilibrium employment. Shadow markdowns capture inefficiencies due to (i) market power in Region I, (ii) diminished market power in Region II, and (iii) binding rationing constraints in Region III. We now show how these objects characterize the efficiency effects of minimum wages at the market level.

**Search.** An alternative approach to handling non-market-clearing wages, would be via search frictions. We formulate, solve and discuss such a model in Appendix G. In a search model declining job finding probabilities effectively ration employment. In this way, the search model maps the *slack* in Panel D into frictional non-employment caused by the minimum wage. With homogeneous firms, this is innocuous. With heterogenous firms, equilibrium job finding probabilities decline most at Region III firms with low productivity, generating frictional non-employment that could be allocated to more productive firms. Rationing constraints, on the other hand, ensure no lines of workers hoping to get a job at a low productivity firm paying a high minimum wage. In the search model, such non-reallocated labor would itself be a substantial source of inefficiency. Furthermore, the rationing model has the benefit of introducing no additional parameters, whereas a search specification requires estimating parameters of search (i) costs (the utility cost of unobserved search effort) and (ii) benefits (matching functions at the firm-level).

**Aggregation.** Shadow wages allow an aggregation of the economy that locates the macroeconomic efficiency role of the minimum wage. First, we can define the *aggregate shadow wage* $\widetilde{W}$ and obtain:

$$\widetilde{W} := \left[\int \widetilde{w}_i^{1+\eta} di\right]^{\frac{1}{1+\eta}} \quad , \quad N = \widetilde{W}^{\varphi} \quad , \quad n_i = \left(\frac{\widetilde{w}_i}{\widetilde{W}}\right)^{\eta} N \quad , \quad \widetilde{W}N = \int \widetilde{w}_i n_i di. \tag{6}$$

Hence the *aggregate* allocation of labor $N$ is *as if*, the household has an aggregate labor supply curve determined by shadow wages. The market allocation of labor is *as if*, households respond to shadow wages, with their ratio pinning down the distribution of employment. While a minimum wage may increase *average wages*, the aggregate shadow wage—which is determinative of quantities—may be increasing when many firms are in Region II, or decreasing due to widening shadow markdowns in Region III.

Further applying these definitions, we can aggregate the firm optimality condition stated in terms of shadow wages, $\widetilde{w}_i = \widetilde{\mu}_i mrpl_i$, to express aggregate labor *demand* in terms of aggregated wedges:

$$\widetilde{W} = \widetilde{\mu}\alpha Z N^{\alpha-1} \quad , \quad Y = \omega Z N^{\alpha} \quad , \quad C = Y. \tag{7}$$

given *aggregate productivity* $Z$, *aggregate shadow markdown* $\widetilde{\mu}$, and *aggregate misallocation* $\omega$:

$$Z := \left[\int z_i^{1+\eta(1-\alpha)} di\right]^{\frac{1}{1+\eta(1-\alpha)}} \quad , \widetilde{\mu} := \left[\int \left(\frac{z_i}{Z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \widetilde{\mu}_i di\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}} \quad , \omega := \int \left(\frac{z_i}{Z}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\widetilde{\mu}_i}{\widetilde{\mu}}\right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}} di. \tag{8}$$

Allocations of $(C, N)$ which determine welfare are—with knowledge of $\{\widetilde{\mu}_i, z_i\}_{i\in[0,1]}$—pinned down by the above equations in blue for supply, demand, output, and resource constraint. Hence we have located the wedges $\{\widetilde{\mu}, \omega\}$, that determine any measured welfare gains or losses due to a minimum wage. These appropriately aggregate market power distortions and minimum wage distortions. In the simple monopsony model $\widetilde{\mu}_i$ is fixed at $\eta/(1+\eta)$ in Region I, and hence the *spillover* channels discussed in the introduction are absent. These require extending the model to oligopsony.

### 2.2 Simple oligopsony economy

We extend the previous environment to allow for oligopsonistic behavior, requiring changes to household and firm problems and a new equilibrium concept. The above descriptions remain for the household, goods, technology, budget constraints, minimum wage, rationing constraints, good market structure.

**Firms.** We split the continuum of firms $i \in [0, 1]$ into a continuum of markets $j \in [0, 1]$ with a finite number of firms in each market. The *only ex-ante difference* between markets is the exogenously given finite number of firms $M_j \in \{1, \ldots, \infty\}$. Denote a firm by $ij$ and its productivity $z_{ij}$.

**Preferences.** Preference over consumption are as before, with modified preferences over labor:

$$\mathcal{U} = C - \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \quad , \quad N := \left[\int_0^1 n_j^{\frac{\theta+1}{\theta}} dj\right]^{\frac{\theta}{\theta+1}} \quad , \quad n_j := \left[\sum_{i=1}^{M_j} n_{ij}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} \quad , \quad \eta \geq \theta. \tag{9}$$

We now take the nested-$CES$ labor supply disutility preferences directly from BHM. Elasticities of substitution $\eta$ and $\theta$ are such that the household finds jobs within a market to be closer substitutes than across markets. Labor supply to firms is more elastic within than across markets. The previous monopsony environment is nested under $\eta = \theta$.[18]

**Labor market competition.** With a finite number of firms in each local labor market, firms behave strategically. We assume Cournot competition: firms take as given the quantities of labor chosen by local competitors when taking their actions. As before, actions consist of choosing their own quantity of employment, wage and rationing constraint. Labor market $j$ is infinitesimal with respect to other labor markets in the economy, so firms take quantities and wages outside of their labor market as given. We refer to this as Cournot oligopsony.

[18]In Berger, Herkenhoff, and Mongey (2022) we show how the labor supply curves that obtain under these preferences can also be obtained by individuals making discrete labor supply decisions (i) across an employment / non-employment margin, (ii) across markets, (iii) across firms within markets. If preferences across these three are drawn from a correlated Gumbel distribution, then the parameter $\varphi$ maps into overall variance of draws, $\theta$ into the conditional variance across markets, and $\eta$ into the conditional variance within markets. This is a straight-forward extension of techniques from the demand system literature: Anderson, De Palma, and Thisse (1987) (single nested) and Verboven (1996) (double nested).

**Household problem.** The household takes rationing constraints $\{\overline{n}_{ij}\}$, wages $\{w_{ij}\}$ and profits $\Pi$ as given. The household chooses employment $\{n_{ij}\}$ at each firm $ij$ to maximize:

$$\max_{\{n_{ij}\}_{i\in\{0,M_j\},\, j\in[0,1]}} \int_0^1 \sum_{i=1}^{M_j} w_{ij}n_{ij}dj - \frac{N^{1+1/\varphi}}{1+1/\varphi} \quad , \quad N := \left[\int_0^1 n_j^{\frac{\theta+1}{\theta}}\, dj\right]^{\frac{\theta}{\theta+1}} \quad , \quad n_j := \left[\sum_{i=1}^{M_j} n_{ij}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} \quad , \quad \eta \geq \theta \tag{10}$$

subject to $n_{ij} \leq \overline{n}_{ij}$ for all $i \in \{1,\ldots,M_j\}$ and $j \in [0,1]$. Let $\nu_{ij}$ be the multiplier on the rationing constraint. The following optimality conditions characterize the inverse labor supply schedule of the household $w_{ij} = w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$:

$$w_{ij} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} + \nu_i \quad , \quad \nu_i(\overline{n}_i - n_i) = 0.$$

**Firm problem.** Firm $i$ in market $j$ takes as given local competitor employment levels $\{n_{-ij}\}$ the aggregate employment index $N$ and chooses its (i) wage $w_{ij}$, (ii) employment $n_{ij}$, and (iii) rationing constraint $\overline{n}_{ij}$ in order to maximize profits. The firm is constrained by (a) the minimum wage $w_{ij} \geq \underline{w}$, (b) its rationing constraint $n_{ij} \leq \overline{n}_{ij}$, and (c) the inverse labor supply schedule of households. Therefore the firm problem is given by,

$$\max_{\overline{n}_{ij}, n_{ij}, w_{ij}} z_{ij}n_{ij}^{\alpha} - w_{ij}n_{ij} \quad \text{subject to} \quad w_{ij} \geq \underline{w} \;,\; n_{ij} \leq \overline{n}_{ij} \;,\; w_{ij} = w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right). \tag{11}$$

Under our assumption of Cournot competition, the firm understands $\partial w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)/\partial n_{ij} \neq 0$ and that $\partial n_j/\partial n_{ij} \neq 0$, yielding *oligopsonistic* behavior. In particular, the firm understands that their hiring affects wages (i) directly and (ii) indirectly through the market level employment $n_j$:

$$n_j\left(n_{ij}, \{n_{-ij}\}\right) := \left[n_{ij}^{\frac{\eta}{\eta+1}} + \sum_{k\neq i}^{M_j} n_{kj}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}} \quad , \quad \left.\frac{\partial n_j\left(n_{ij}, \{n_{-ij}\}\right)}{\partial n_{ij}}\right|_{n_{-ij}} \neq 0. \tag{12}$$

**Equilibrium.** Given a minimum wage $\underline{w}$, an *oligopsonistic Nash-Cournot* equilibrium is (i) a household inverse labor supply curve $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$, (ii) wages $\{w_{ij}\}$, (iii) quantities of labor $\{n_{ij}\}$, (iv) rationing constraints $\{\overline{n}_{ij}\}$, (v) profits $\Pi$, and (vi) aggregate employment index $N$ and market level employment indexes $\{n_j\}$ such that (1) given wages $\{w_{ij}\}$, rationing constraints $\{\overline{n}_{ij}\}$, and profits $\Pi$, household optimization implies the inverse labor supply curve $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$, (2) for every firm $i$ in market $j$: given competitor employment $\{n_{-ij}\}$, the aggregate employment index $N$, and the household inverse labor supply curve, firm $ij$'s optimization yields rationing constraint $\overline{n}_{ij}$, wage $w_{ij}$ and employment $n_{ij}$, (3) firm employment decisions are consistent with the aggregate and market employment indices, $N$, $\{n_j\}$, as well as profits, $\Pi$, and (4) markets clear $w_i = w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)\ \forall i$.

**Characterization of oligopsony equilibrium.** A series of results in Appendix E.2 parallel the simple monopsony model and characterize an oligopsonistic firm's choices. We derive the firm's *perceived inverse labor supply curve* and show the previously derived weakly optimal rationing constraint still holds: $\overline{n}_{ij} = mrpl^{-1}(\underline{w})$. An important property of firm $ij$'s problem is (i) given competitor labor demand $\{n_{-ij}\}$, competitor rationing constraints $\{\overline{n}_{-ij}\}$ are payoff irrelevant for firm $ij$, and (ii) the optimal rationing constraint is independent of $\{n_{-ij},\overline{n}_{-ij}\}$.[19] Applying the results in Appendix E.2, and taking competitor employment $\{n_{-ij}\}$ as given, the firm problem becomes,

$$\max_{n_{ij}} z_{ij}n_{ij}^{\alpha} - w_{ij}n_{ij} \ , \qquad \text{subject to}$$

$$n_{ij} \leq \overline{n}_{ij} \ , \ \underbrace{\overline{n}_{ij} = \left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}}_{mrpl(\underline{n}_{ij})=\underline{w}} \ , \qquad w_{ij} = \begin{cases} \underline{w} & \text{if } \underline{w} > \left(\frac{\overline{n}_{ij}}{n_j(\overline{n}_{ij})}\right)^{\frac{1}{\eta}}\left(\frac{n_j(\overline{n}_{ij})}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \\ \max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}}\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_{ij}}{n_j(\overline{n}_{ij})}\right)^{\frac{1}{\eta}}\left(\frac{n_j(\overline{n}_{ij})}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}. \end{cases}$$

where we have used $n_j(\overline{n}_{ij})$ to denote market level employment index at firm $ij$'s rationing constraint:

$$n_j\left(\overline{n}_{ij}\right) := \left[n_{1j}^{\frac{\eta}{1+\eta}} + \ldots + \overline{n}_{ij}^{\frac{\eta}{1+\eta}} + \ldots + n_{M_jj}^{\frac{\eta}{1+\eta}}\right]^{\frac{1+\eta}{\eta}}. \tag{13}$$

As before, define the value of employment $\widetilde{n}_{ij}$ such that the minimum wage is binding and the unconstrained labor supply curve $\widehat{w}\left(n_{ij}\right)$, suppressing dependence on aggregates and competitor employment, both of which are taken as given:

$$\underline{w} := \left(\frac{\widetilde{n}_{ij}}{n_j\left(\widetilde{n}_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_j\left(\widetilde{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \ , \ \widehat{w}\left(n_{ij}\right) = \left(\frac{n_{ij}}{n_j\left(n_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_j\left(n_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \tag{14}$$

Appendix E.2 shows that the solution to the firm problem remains described by equation (4) where $\widetilde{n}_{ij}$ in equation (14) defines Regions I, II, and III, and the markdowns depend on $\widehat{w}\left(n_{ij}\right)$ in equation (14). Firm behavior in Regions II and III is identical to the monopsony case. In Region I, however, oligopsonistic firms now set variable markdowns. Applying equation (4) to determine marginal cost in Region I under the supply schedule in (14) delivers wages that are a variable markdown $\mu_{ij}$ on the marginal revenue product of labor, $w_{ij} = \mu_{ij} mrpl\left(n_{ij}\right)$, where $\mu_{ij}$ is defined by

$$\mu_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{ij}+1} \quad , \quad \varepsilon_{ij} = \left[\left.\frac{\partial \log \widehat{w}_{ij}}{\partial \log n_{ij}}\right|_{n_{-ij}}\right]^{-1} \quad , \quad \underbrace{\left.\frac{\partial \log \widehat{w}_{ij}}{\partial \log n_{ij}}\right|_{n_{-ij}} = \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\left.\frac{\partial \log n_j}{\partial \log n_{ij}}\right|_{n_{-ij}}}_{\text{Differentiating (14)}} \tag{15}$$

**Shadow wage characterization.** Similar to the simple monopsony case, we can recast the firm's optimal wage and employment choices in terms of *shadow wages* ($\widetilde{w}_{ij} := p_{ij}w_{ij}$) and *shadow markdowns* ($\widetilde{w}_{ij} = \widetilde{\mu}_{ij} mrpl(n_{ij})$). Following identical steps to the monopsony case, the economy aggregates via a market level shadow wage index to express labor supply in terms of shadow wages:

$$\widetilde{w}_j := \left[\sum_{i\in j} \widetilde{w}_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{w}_j n_j = \sum_{i\in j} \widetilde{w}_{ij} n_{ij} \quad , \quad \underbrace{n_{ij} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{\eta} n_j}_{\text{Firm supply}} \quad , \quad \underbrace{n_j = \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N}_{\text{Market supply}} \tag{16}$$

$$\widetilde{W} := \left[\int \widetilde{w}_j^{1+\theta} dj\right]^{\frac{1}{1+\theta}} \quad , \quad \widetilde{W}N = \int \widetilde{w}_j n_j dj \quad , \quad \underbrace{N = \widetilde{W}^{\varphi}}_{\text{Aggregate supply}} \tag{17}$$

[19] In the general, quantitative model, this independence will be maintained, despite $\overline{n}_{ij}$ depending on aggregates, namely the rental rate of capital.

Lastly, to understand the *spillover* channel of minimum wages from constrained to unconstrained firms, we introduce the concept of a *shadow wage bill share*, $\widetilde{s}_{ij}$, which allows us to succinctly summarize markdowns in Region I. A firm's residual labor supply curve is determined by its competitors' employment, which can be expressed via their shadow wages. Intuitively, if firm $i$'s Region III competitor has a high wage, but their shadow wage and hence employment is low, then firm $i$'s labor supply curve will be less elastic as its choice of $n_{ij}$ has a larger effect on $n_j$. Analytically, $\widetilde{s}_{ij}$ captures this elasticity:

$$\widetilde{s}_{ij} := \frac{\widetilde{w}_{ij}n_{ij}}{\sum_{i\in j}\widetilde{w}_{ij}n_{ij}} \underbrace{= \frac{\widetilde{w}_{ij}n_{ij}}{\widetilde{w}_j n_j}}_{\text{From aggregation (16)}} \underbrace{= \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{1+\eta} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1+\eta}{\eta}}}_{\text{Using labor supply system (16)}} \underbrace{= \frac{\partial \log n_j}{\partial \log n_{ij}}\bigg|_{n_{-ij}}}_{\text{Differentiating preferences (9)}}$$

Substituting into equation (15) gives the perceived elasticity of labor supply $\varepsilon_{ij}$

$$\varepsilon_{ij} = \left[\frac{1}{\eta}\widetilde{s}_{ij} + \frac{1}{\theta}\left(1-\widetilde{s}_{ij}\right)\right]^{-1}.$$

Notably, equilibrium wages and employment for unconstrained firms in Region I are *as if* they set their markdowns based on competitors' shadow wages (the denominator of the shadow share). Thus, the shadow shares encode all relevant information about competitor employment.[20]

**Market level efficiency.** In the simple monopsony model, the only channel through which minimum wages improve efficiency was via the *direct channel* of moving firms toward their competitive wage. We now consider the same comparative statics as Figure 2 in *market equilibrium* which delivers two additional channels: *spillovers* and *reallocation*.

**Channel I - Direct.** Figure 3 considers the same comparative static but in a market equilibrium with three firms. All aggregates are held fixed. On the $x$-axis we plot the minimum wage relative to the unconstrained optimal wage of the low productivity firm: $\underline{w}/w^0_L$. The red line in Panel A describes the low productivity firm's movement through the three regions described in Figure 2. Its wage increases one-for-one across Region II and Region III as the minimum wage increases (Panel C). The positive *Direct* efficiency channel are seen in the grey shaded regions of Panel D as its employment increases in Region II, before shrinking along its labor demand curve, rationalized by a declining shadow wage (Panel B).

The behavior of the medium and high productivity firms, in blue and green, reflect the Nash equilibrium at the market level. Absent a minimum wage, these firms are larger, and pay higher wages. Since they have larger market shares, they face less elastic labor supply so their wages represent wider markdowns on their marginal product of labor.

**Channel II - Spillovers.** As the red firm's wage increases in Region II, its market share increases, which lowers the shares of the unconstrained firms. With lower market shares due to stiffer competition, the unconstrained firms' equilibrium markdowns narrow. Their wages consequently increase in the green shaded region in Panel C. This *Spillover* effect has positive implications for efficiency. Not only is the

[20]This observation also forms the basis for the algorithm used to solve for the Nash equilibrium in each market, explained in detail in Appendix C.

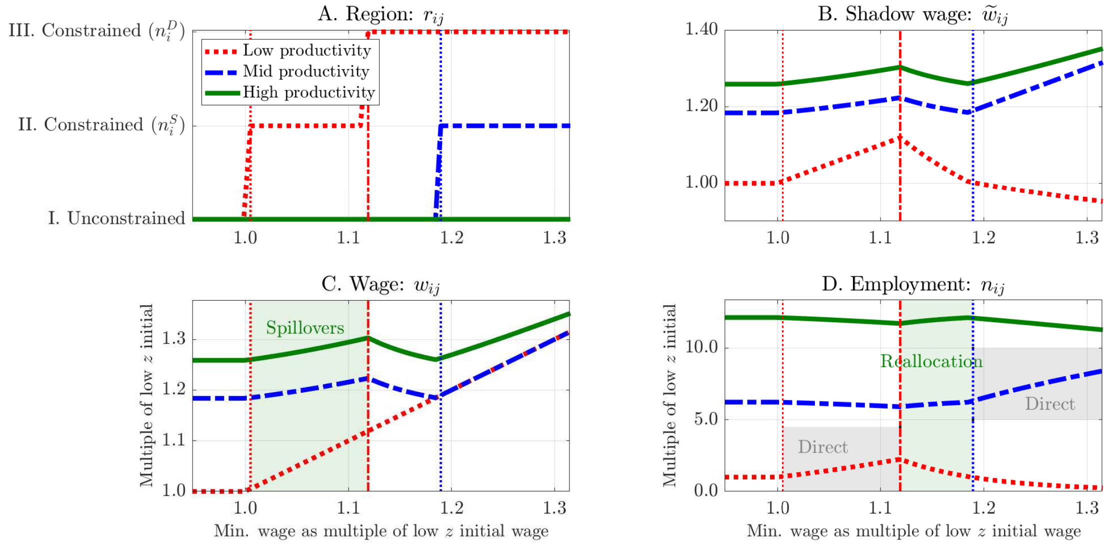


Figure 3: Increase in the minimum wage: Simple oligopsony economy - Market equilibrium

Notes: All aggregates are held fixed and we plot outcomes for a market with three firms as the minimum wage is increased. The $x$-axis plots the minimum wage relative to unconstrained optimal wage of the low productivity firm: $\underline{w}/w_L^*$. We increase the minimum wage from 10 percent below to 50 percent above this wage. Panel $A$ plots the regions corresponding to Figure 2. Panels $B$ and $C$ plots the shadow wage $\tilde{w}_{ij} = p_{ij}w_{ij}$, and actual wage $w_{ij}$. Panel $D$ plots employment relative to unconstrained employment at the low productivity firm.

markdown of the constrained firm narrowing in Region II, but the equilibrium markdowns of its competitors in Region I are also narrowing. The elasticity of firms' wages to competitors' is therefore a key determinant of the efficiency properties of minimum wages. In Section 5 we replicate observed spillovers in Derenoncourt, Noelke, Weil, and Taska (2021) and Staiger, Spetz, and Phibbs (2010).

**Channel III - Reallocation.** In the green shaded Region in Panel D, the red firm enters Region III and shrinks. As it does so, the high elasticity of substitution of labor within a market relative to across markets implies that its employment losses are largely reallocated to its more productive competitors. This leads to positive efficiency gains. This occurs despite the endogenous wage response of its competitors (Panel C), which cut their wages as their market power increases as the red firm's shadow wage declines (Panel B). In Section 7 we show that if $\theta = \eta$, such direct reallocation to more productive competitors is completely neutralized, as jobs cut at the low productivity firm are spread out across all markets.[21] The reallocation of employment from lower to higher productivity firms within markets is therefore also a key determinant of the efficiency properties of minimum wages. In Section 5, we replicate observed reallocation within markets in Dustmann, Lindner, Schoenberg, Umkehrer, and vom Berge (2021).

**Aggregation.** Figure 3 captures these three channels in the shaded regions, but outside of these regions shows that they can also operate in off-setting ways. The prior aggregation results from the monopsony model can be applied at the market-level to understand this better. The previous results in equations (7) and (8) naturally extend to market equilibrium given aggregates. Market-level output $y_j$, employment

[21] Even though the unaffected firms have higher productivity, because a finite measure of low productivity workers are reallocated across a continuum of markets the reallocation effect is infinitesimal.

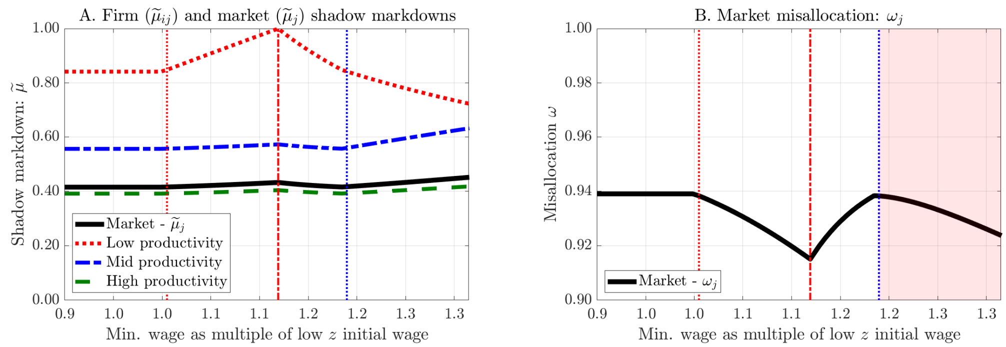


Figure 4: Market level outcomes: Shadow markdown and misallocation

Notes: All aggregates are held fixed and we plot outcomes for a market with three firms as the minimum wage is increased. The $x$-axis plots the minimum wage relative to unconstrained optimal wage of the low productivity firm: $\underline{w}/w_L^*$. We increase the minimum wage from 10 percent below to 50 percent above this wage. Panel $A$ plots the market markdown $\widetilde{\mu}_j$. Panel $B$ plots the market misallocation $\widetilde{\omega}_j$. Moving from left to right, the vertical dotted lines correspond to the low and then medium productivity firms move from Region I to Region II. The vertical dashed line corresponds to the low productivity firm moving from Region II to Region III.

$n_j$ and shadow wage $\widetilde{w}_j$ satisfy:

$$\underbrace{y_j = \omega_j z_j n_j^{\alpha}}_{\text{1. Output}} \quad , \quad \underbrace{\widetilde{w}_j = \widetilde{\mu}_j \alpha z_j n_j^{\alpha-1}}_{\text{2. Shadow wage}} \quad , \quad \underbrace{\widetilde{n}_j = \left(\frac{\widetilde{w}_j}{\widetilde{w}}\right)^{\theta} n}_{\text{3. Labor supply}}$$

where $z_j$, $\widetilde{\mu}_j$ and $\omega_j$ are given by

$$\underbrace{z_j := \left[\sum_{i\in j} z_{ij}^{\frac{1+\eta}{1+\eta(1-\alpha)}}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}}_{\text{1. Market productivity}} \quad , \quad \underbrace{\widetilde{\mu}_j := \left[\sum_{i\in j}\left(\frac{z_{ij}}{z_j}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \widetilde{\mu}_{ij}^{\frac{1+\eta}{1+\eta(1-\alpha)}}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}}_{\text{2. Market shadow markdown}} \quad , \quad \underbrace{\omega_j := \sum_{i\in j}\left(\frac{z_{ij}}{z_j}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}}\left(\frac{\widetilde{\mu}_{ij}}{\widetilde{\mu}_j}\right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}}}_{\text{3. Market misallocation}} .$$

As in the monopsony case, the wedges encode deviations from an efficient market in which markdowns are equal to one and hence $(\widetilde{\mu}_j, \omega_j) = (1, 1)$, where deviations are due to market power and $\underline{w}$.

Figure 4 shows how $(\widetilde{\mu}_j, \omega_j)$ evolve in our example market from Figure 3, delivering two results. First, productivity weighting in $\widetilde{\mu}_j$ implies that the market shadow-markdown is shaped by the *Spillover* responses of unconstrained firms (Panel A), rather than the *Direct* effect via the narrowing of the red firm's markdown. The model has a potentially strong role for spillovers in shaping efficiency. Second, misallocation has ambiguous effects (Panel B). Misallocation does indeed improve while the red firm is in Region III and its competitors are unconstrained. However, it worsens as the medium productivity firm is bound by the minimum wage in Region II. In this segment, shaded in red, the endogenous response of the green firm to the increasing wage at the blue firm is less than one-for-one. With a high elasticity of substitution within markets, this leads to reallocation of employment down the productivity ladder, worsening $\omega_j$ which lowers output, conditional on $n_j$.

A key take-away from Figures 3 and 4 is that empirical evidence of each channel may not extend more generally. On the one hand, firm heterogeneity in productivity and strategic interactions in local labor markets are key ingredients to the presence of the three channels we have identified through which a minimum wage can improve efficiency, and have been documented empirically. On the other hand, when aggregated, these features also lead to a moderation of these channels. First, the *Direct* channel only operates in the narrow window in which a firm is in Region II. Second, the *Direct* channel is down-weighted due to operating through low productivity firms. Third, the *Spillover* channel is moderated by large firms responding little to small firms' wage increases. Fourth, the *Reallocation* channel can be undone via reallocation down the ladder, off-setting gains from reallocation up the ladder. We therefore study a general equilibrium framework that appropriately aggregates markets that are distributed across the spectrum of these effects.

# 3 Quantitative model

We now generalize the previous settings to our full quantitative model. Full derivations of each condition and full statement of equilibrium are contained in Appendix F. For brevity we skip details in the main text, leaning on the prior section.

## 3.1 Environment

**Agents.** The economy consists of $K$ households and a continuum of firms. As above, firms are heterogeneous in two dimensions, inhabiting a continuum of local labor markets $j \in [0,1]$, with $M_j$ firms in market $j$, and with total factor productivity $z_{ij} \in (0,\infty)$.[22] The *only ex-ante difference* between markets is the number of firms $M_j \in \{1,\ldots,\infty\}$. Households are heterogeneous in their measure $\pi_k$, disutility of labor supply $\overline{\varphi}_k$, factor-augmenting productivity $\xi_k$ and endowed share of capital and profit income $\kappa_k$.

**Goods and technology.** The good produced by firms may now be used for consumption and investment, and remains perfect substitutes, so trade in a competitive market. Firms operate a *value-added* production function that uses labor of each type $n_{ijk}$.[23] Let $\overline{Z}$ be a common component of productivity across firms. Firm-$ij$ produces $y_{ij}$ units of net-output according to the production function:

$$y_{ij} \quad = \quad \overline{Z} z_{ij} \sum_{k=1}^{K} \left( \left[\xi_k n_{ijk}\right]^{\gamma} k_{ijk}^{1-\gamma} \right)^{\alpha} \quad , \quad \gamma \in (0,1] \quad , \quad \alpha > 0$$

where $k_{ijk}$ is capital allocated to worker type $k$. Production has a unit substitution elasticity between capital and labor for each type, and is additively separable across types, which maintains tractability.

**Preferences.** Each household has a unit measure of workers, with concave preferences over per-capita consumption and the same disutility of total labor as above:

$$\mathcal{U}_k = \sum_{t=0}^{\infty} \beta^t u^k \left( \frac{c_{kt}}{\pi_{kt}}, n_{kt} \right) = \sum_{t=0}^{\infty} \beta^t \left[ \frac{\left(c_{kt}/\pi_k\right)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]. \tag{18}$$

The parameter $\widetilde{\varphi}_k$ expresses the disutility of labor supply on a per capita basis which we normalize by an aggregate measure $\overline{\varphi}$: $\widetilde{\varphi}_k = \left(\overline{\varphi}_k/\overline{\varphi}\right) \pi_k^{1+\varphi}$.[24] Labor disutility $n_{kt}$ for each household type is of the nested

[22] In our later *short-run* exercise we will allow $M_j$ to change with the minimum wage to encompass firm exit.

[23] Since aggregating firm-level value-added yields aggregate output (GDP), we abuse terminology and refer to the output of this production function interchangeably in terms of goods and value-added.

[24] Our preferences over consumption and labor supply for heterogeneous households with measure $\pi_k$ is similar to that used in Dyrda, Kaplan, and Rios-Rull (2019), who consider an economy with different ages of households.

form from Section 2.2, over markets $n_{jkt}$ and firms $n_{ijkt}$.

**Budget constraints and endowments.** As above, each household has its own budget constraint. In addition to labor income in previous sections, each household earns capital income and profits, and chooses how much to consume and invest. That is, within-household risk associated with labor being rationed due to the minimum wage is insured, but across-household risk is not. We discuss this further in Section 8.4. Endowments of initial capital $\{k_{k0}\}$ are a free-parameter of the competitive equilibrium. We assume each household's share $\kappa_k$ of $K_0$ is equal to its share of profits:

$$P_t c_{kt} + k_{kt+1} = \int \sum_{i=1}^{M_j} w_{ijkt} n_{ijkt} \, dj + R_t k_{kt} + (1-\delta) k_{kt} + \kappa_k \Pi_t \quad , \quad k_{k0} = \kappa_k K_0. \tag{19}$$

**Minimum wages, rationing constraints, markets and competition.** These are as in the simple oligopsony economy. Labor markets are local and type-specific, as are firms' choices of rationing constraints. Households are competitive, and additionally take the rental price of capital $R_t$ as given.

**Nesting.** The simple oligopsony economy of Section 2.2 is nested under: (i) linear consumption utility ($\sigma = 0$), (ii) no capital ($\gamma = 1$), and (iii) one household type $K = 1$. The simple monopsony economy of Section 2.1 is nested by further assuming either (a) $M_j = \infty$ in all markets, or (c) $\theta = \eta$.

### 3.2 Equilibrium

We briefly discuss deviations from the simple monopsony and oligopsony economies discussed earlier.

**Firm problem.** At a particular allocation and prices, a firm's profits are:

$$\pi_{ijt} = \overline{Z} z_{ij} \sum_{k=1}^{K} \left( \left[ \xi_k n_{ijkt} \right]^{\gamma} k_{ijkt}^{1-\gamma} \right)^{\alpha} - R \sum_{k=1}^{K} k_{ijkt} - \sum_{k=1}^{K} w_{ijkt} n_{ijkt}$$

Since these are additively separable across household types $k = 1, \ldots, K$, the firm solves each problem separately, choosing $(n_{ijkt}, \overline{n}_{ijkt}, w_{ijkt})$ as per the firm in the simple oligopsony model previously analyzed.

The weakly optimal rationing constraint is as before, $mrpl(\overline{n}_{ijkt}) = \underline{w}$, but now the marginal revenue product of labor depends also on the price of capital. Optimizing out the choice of type-$k$ capital from the above, the firm's profits for type-$k$ labor are

$$\pi_{ijkt} = \widetilde{Z} \widetilde{z}_{ijt} \widetilde{\xi}_k n_{ijkt}^{\widetilde{\alpha}} - w_{ijkt} n_{ijkt} \ , \ \widetilde{Z} := \overline{Z}^{\frac{1}{1-(1-\gamma)\alpha}} \ , \ \widetilde{\xi}_k := \xi_k^{\widetilde{\alpha}} \ , \ \widetilde{\alpha} := \frac{\gamma \alpha}{1-(1-\gamma)\alpha}.$$

Hence the weakly optimal rationing constraint satisfies

$$\underline{w} = \widetilde{\alpha} \widetilde{Z} \widetilde{z}_{ijt} \widetilde{\xi}_k \overline{n}_{ijkt}^{\widetilde{\alpha}-1} \quad , \quad \overline{n}_{ijkt} = \left( \frac{\widetilde{\alpha} \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_{ijt}}{\underline{w}} \right)^{\frac{1}{1-\widetilde{\alpha}}} \quad , \quad \widetilde{z}_{ijt} := \left[ 1 - (1-\gamma)\alpha \right] \left( \frac{(1-\gamma)\alpha}{R_t} \right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} z_{ij}^{\frac{1}{1-(1-\gamma)\alpha}}.$$

As before, a key property of the equilibrium is that the rationing constraint is independent of competitors' employment. Its dependence on $R_t$ presents no issue in the construction of the equilibrium.

**Household problem.** With non-linear utility in consumption, household labor supply will now feature income effects. Including these is important as the minimum wage will shift the distribution of non-labor income in the economy through capital and profits. First order conditions can be similarly rewritten in terms of shadow wages, with indexes defined at the household level. As before, the aggregate shadow wage $\widetilde{w}_{kt}$—now at the household level—proves determinative of the allocation of total labor $n_{kt}$:

$$\underbrace{w_{ijkt} = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}\left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}\left(-\frac{u_n^k(c_{kt}/\pi_k, n_{kt})}{u_c^k(c_{kt}/\pi_k, n_{kt})}\right) + \nu_{ijkt}}_{\text{First order conditions with rationing constraint multiplier } \nu_{ijkt}}, \quad \underbrace{n_{ijkt} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{\eta}\left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}}_{\text{Expressed in shadow wages, as in equation (16)}}, \quad \underbrace{n_{kt} = \pi_k \widetilde{\varphi}_k \widetilde{w}_{kt}^{\varphi}\left(\frac{c_{kt}}{\pi_{kt}}\right)^{-\sigma\varphi}}_{\text{Household labor supply curve}}$$

The implied household labor supply curve now shifts outward as consumption declines and marginal utility increases. As before, a key property of the equilibrium is that the labor supply curve is independent of competitors' rationing constraints. Each household has an euler equation for consumption. When evaluated in steady-state this yields $R = 1/\beta + (1-\delta)$, and at this price household capital $k_{kt}$ remains constant.

**Aggregation.** As before, the economy can be aggregated at the household level exploiting a household level shadow markdown $\widetilde{\mu}_k$ and misallocation $\omega_k$. Labor supply, demand and output are then expressed:

$$n_k = \pi_k \widetilde{\varphi}_k \widetilde{w}_k^{\varphi} c_k^{-\sigma\varphi} \quad , \quad \widetilde{w}_k = \widetilde{\mu}_k \widetilde{\alpha} \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_k n_k^{\widetilde{\alpha}-1} \quad , \quad y_k = \frac{1}{1-(1-\gamma)\alpha} \omega_k \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_k n_k^{\widetilde{\alpha}}. \tag{20}$$

We also require that consumption is consistent with the household budget constraint, which now includes non-labor income and replaces the prior condition that $C = Y$. This requires aggregating equilibrium profits and capital and distributing them to each household.[25] We provide full equilibrium conditions in Appendix F. Like the simpler economies, the set of wedges $\{\widetilde{\mu}_k, \omega_k\}_{k=1}^{K}$ summarize deviations from efficiency. This allows us to use equilibrium conditions and wedges to separate welfare effects of minimum wages into shadow markdowns and misallocation effects.

### 3.3 A fictitious government problem to separate out efficiency and redistribution effects

In the quantitative economy, the previously established *efficiency effects* of minimum wages will still be operative. In addition, there will be *redistributive effects* that were not present in the simple economies due to changes in profits and wages being offset in the budget constraint of the single household. Our introduction motivated the necessity to separate out efficiency and redistributive effects, which we discuss more below.

To separate these effects we conjecture the problem of a fictitious government with social welfare weights $\boldsymbol{\psi} = \{\psi_k\}_{k=1}^{k}$. The government faces prices determined by the imperfectly competitive labor market where firms are subject to the minimum wage, and firms' rationing constraints. The government is given access to lump-sum taxes $\{t_k\}_{k=1}^{K}$, with the restriction that total lump sum taxes add to zero. We take the standard approach of solving for the optimal allocation, and then solving for the transfers that implement it.

[25]For type-$k$, steady-state capital income is $\kappa_k((R-\delta)K + \Pi)$. Aggregate capital demand is $K = \alpha(1-\gamma)Y/R$, which clears at the initial capital stock under $1 = \beta(R + (1-\delta))$. Aggregate profits are $\Pi = Y - \sum_k \left[\int \sum_i w_{ijk} n_{ijk} dj\right] - RK$. Hence, aggregate capital income link households via wealth effects on labor supply.

**(Ramsey) Problem.** The government chooses allocations of consumption and labor to maximize social welfare

$$\mathcal{U} = \sum_k \psi_k \sum_{t=0}^{\infty} u^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right) = \sum_k \psi_k \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{kt}^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right]. \tag{21}$$

subject to an aggregate budget constraint, and firms' rationing constraints:

$$\sum_k c_{kt} + K_{t+1} = \sum_k \int \sum_{i\in j} w_{ijkt} n_{ijkt}\, dj + R_t K_t + (1-\delta)K_t + \Pi_t \quad , \quad n_{ijkt} \geq \overline{n}_{ijkt} \tag{22}$$

In Appendix F, we show that the government's allocation is implemented in a competitive equilibrium in which each household's budget constraint is augmented with lump-sum transfers $t_k$. In steady-state:

$$t_k = \int \sum_{i\in j} w_{ijk} n_{ijk}\, dj + (R+\delta)\kappa_k K + \kappa_k \Pi - c_k. \tag{23}$$

By equation (22), $\sum_k t_k$ is zero, and since they are lump-sum household optimality conditions are unaffected. With lump-sum transfers able to achieve any desired amount of redistribution, the minimum wage is left only to improve efficiency.

Importantly, we do not view the US economy as one in which the fiscal authority has access to optimal lump-sum transfers. Instead, modeling a government with unrestricted lump-sum transfers is an apparatus that allows us to separate the efficiency and redistributive effects of a minimum wage. Such accounting is important. First, our strategy implies that the efficiency component will be robust to social welfare weights, while the redistribution component will change depending on the social welfare weights the government has in mind. Second, suppose we knew the true $\boldsymbol{\psi}$, and that under the true weights welfare gains at the optimal minimum wage were entirely driven by redistribution while generating efficiency *losses*. This would lead one to consider alternative redistributive policies. Our results will reflect this case. The optimal minimum wage varies substantially depending on social welfare weights, reflecting redistribution, with small efficiency gains.

# 4 Calibration

We calibrate the economy to US data, using a combination of the Census Longitudinal Business Database (LBD, using moments released by Berger, Herkenhoff, and Mongey, 2022), BLS Current Population Survey (CPS), and Survey of Consumer Finances (SCF). LBD data is from 2014, the latest data available to us in the Census. We use pre-Covid 2019 data from the CPS and SCF. Parameters and moments are summarized in Tables 1 and 2. There are three sets of parameters we describe below.

**Households.** We consider four households types, $K = 4$, and note that our approach could be extended to much richer heterogeneity given suitable data. First, we split households by education, which is available in the CPS and SCF. The first two are workers that have not completed high school (NHS) and workers that have a high school diploma but not completed college (HS). The remainder represent workers that have completed college. Second, we use the SCF to split college households into two groups: college workers (C) which receive the majority of their income from labor income, and those that earn the majority of their income from capital income, which we call owners (O). We measure capital income as interest and dividend income, business and farm income, and realized capital gains.[26] By this metric, we allocate less than a fifth of college households to owners. When aggregated, non-college workers' capital income is not zero, but it is small, and hence our assumption that only college households are owners is reasonable.[27] Table 2B reports the implied population shares $\{\pi_k\}_{k=1}^{K}$; notably only 6 percent of households are owners.

**1. Externally calibrated.** The first set of parameters are those we externally calibrate (Table 1A). The discount rate $\beta$ implies a risk free rate of 4 percent annually. The depreciation rate $\delta$ is 10 percent. We set the preference parameters governing curvature in marginal utility of consumption $\sigma$ to 1.05, so approximately log, and the Frisch elasticity of aggregate labor supply $\varphi$ to 0.62. In Appendix D, we contribute a simple method that combines recent evidence from Golosov et al. (2021) to infer a data-consistent $\varphi$ for any $\sigma$.[28] In Section 8.1 we repeat our main minimum wage counterfactuals under alternative values of these parameters, recalibrating all remaining parameters in each case.

The distribution of firms across markets matches LBD data. We treat markets as in BHM, and define a market as a combination of a NAICS 3-digit industry and a commuting zone. We define a firm in the data as the collection of all establishments with the same *firmid* in the commuting zone and compute total employment and average worker wage across these establishments. We specify the distribution of firms across markets $M_j \sim G(M)$ as comprised of a mass point of 0.09 at $M_j = 1$ and a generalized Pareto distribution for $M_j > 1$. The tail, shape and location parameters chosen to best match the mean (113.10), standard deviation (619.0) and skewness (26.1) of the empirical distribution of $M_j$, which we measure in the LBD. We solve the model with $J = 5,000$ markets.

Preference parameters $(\theta, \eta)$ are taken from BHM. With $M_j < \infty$, firms exercise market power in their local labor markets. If $\eta > \theta$, then labor supply is more elastic within- rather than across- markets, and firms with a larger market share will be less responsive to shocks. BHM uses the relative response of large and small market share firms to changes in state corporate taxes—which by distorting capital decisions, are shocks to the marginal revenue product of labor of only $C$-corp firms—to identify $\theta$ and $\eta$. The generalization to multiple types of workers does not interfere with this exercise and hence we

[26] We also consider an alternative approach, where we determine capital income as a residual in the household budget constraint. By this approach capital income is defined as total income minus labor income and transfers. This yields a very similar split of households.

[27] Two different cuts of the data support this. First, of the households that earn more than half of their income from capital income, 70 percent are college households, 25 percent are high-school, and only 5 percent are non-high school. Second, the share of college households that earn more than half of their income from capital income is 17.3 percent, while this is true for only 6.7 percent for high-school households and 3.7 percent for non-high-school households. As a robustness, we ran a calibration where we increased the number of business owners by 30 percent (pooling all business owners together), assuming for simplicity that these additional business owners come from college worker households so we do not need to add additional household types. This gives an upper bound to the possible effects for the optimal minimum wage (Section 7) because we are moving mass from the group with the highest desired minimum wage (college workers) to that with the lowest (business owners). Doing so lowers the optimal minimum wage under utilitarian social welfare weights by less than 1 percent from \$15.12 to \$14.99. We conclude that, quantitatively, including these non-college business owners makes little difference.

[28] In Appendix D we show how one can fix $\sigma$ and then use recent evidence to infer $\varphi$ by combining (i) estimates on marginal propensities to consume and earn from Golosov, Graber, Mogstad, and Novgorodsky (2021), (ii) data on the average propensity to consume from the BLS, and (iii) estimates of the progressivity of labor income taxes from Heathcote, Storesletten, and Violante (2020).

| Parameters | | Value | Moment and source | Value |
|---|---|---|---|---|
| **A. External** | | | | |
| Discount rate | $\beta$ | 0.962 | Risk free rate | 0.04 |
| Depreciation rate | $\delta$ | 0.10 | | |
| Coefficient of risk aversion | $\sigma$ | 1.05 | Fixed, approximately log | |
| Aggregate Frisch elasticity | $\varphi$ | 0.62 | Consistent with recent evidence given $\sigma$ (see Appendix D) | |
| Number of markets | $J$ | 5,000 | Normalization | |
| Distribution of number of firms | $G(M_j)$ | | Mean, variance, skewness of distribution of $M_j$ (LBD) | |
| Pareto with mass point at $M_j = 1$ | | | 9 percent of markets have 1 firm | |
| Across market substitutability | $\theta$ | 0.42 | Estimate from BHM (2021) | |
| Within market substitutability | $\eta$ | 10.85 | Estimate from BHM (2021) | |
| **B. Internally estimated** | | | | |
| Productivity dispersion | $\mathrm{Std}[\log z_{ij}]$ | 0.268 | Payroll weighted $\mathbb{E}[HHI^{wn}]$ (LBD) | 0.11 |
| Decreasing returns in production | $\alpha$ | 0.940 | Labor share | 0.57 |
| Labor exponent in production | $\gamma$ | 0.808 | Capital share | 0.18 |

Table 1: Calibration of common parameters

| *A. Aggregate parameters* | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Parameters** | | **Values** | | | | **Moments** | **Values** | | | |
| Labor disutility shifter | $\overline{\varphi}$ | $2.61 \times 10^6$ | | | | Average firm size (LBD) | 22.8* | | | |
| Productivity shifter | $\widetilde{Z}$ | 17.63 | | | | Binding at \$15 (CPS, %) | 29.3* | | | |
| ***B. Household parameters*** | | | | | | | | | | |
| **Parameters** | | **Non-HS** | **HS** | **Coll** | **Own** | **Moments** | **Non-HS** | **HS** | **Coll** | **Own** |
| Relative population (%) | $\pi_k / \sum \pi_k$ | 12.4 | 52.8 | 28.8 | 6.0 | Population shares (CPS, %) | 12.4* | 52.8* | 28.8* | 6.0* |
| Relative disutility labor supply | $(1/\widetilde{\varphi}_k)^{\varphi}$ | 3.09 | 0.71 | 1 | 0.45 | Share of agg. labor income (CPS, %) | 2.2* | 55.3* | 35.1* | 7.4* |
| Relative productivity | $\xi_k$ | 0.25 | 0.55 | 1 | 0.85 | Relative ave. earnings per hour (CPS, %)* | 41.7* | 60.1* | 100 | |
| Capital income share (%) | $\kappa_k$ | 0.09 | 3.88 | 4.14 | 91.89 | Ratio of h'hold capital/labor inc. (SCF) | 0.022* | 0.037* | 0.062* | 6.568 |
| ***C. Additional statistics*** | | | | | | | | | | |
| **Statistics** | | **Non-HS** | **HS** | **Coll** | **Own** | **Statistics** | **Non-HS** | **HS** | **Coll** | **Own** |
| Implied Negishi weights (%) | $\psi_k^* / \sum \psi_k^*$ | 1.3 | 36.0 | 23.7 | 39.0 | Ratio of h'hold capital/labor inc. (model) | 0.022 | 0.037 | 0.062 | 7.121 |
| Binding at \$15 (model, %) | | 82.7 | 35.6 | 9.1 | | Consumption share (model, %) | 1.5 | 37.6 | 61.0 | |
| Binding at \$15 (CPS, %) | | 68.7 | 38.1 | 11.1 | | Consumption share (BLS, %) | 2.7 | 38.2 | 59.1 | |

Table 2: Calibration of constants and additional statistics

Notes: Data with an $*$ indicates that the model matches the data exactly, by a direct inversion of data to model parameters. In both the CPS and BLS consumption data, we cannot split college households into owners and non-owners as we do in the SCF. Hence for *Binding at \$15*, *Consumption share*, *Relative average earnings* we consider all college educated households.

use the same parameter values: $(\theta, \eta) = (0.42, 10.85)$. In Section 7 we provide results for $\theta = \eta = 3.02$, which delivers the same labor share as the baseline economy, without oligopsony.

**2. Shifters.** The second group of parameters comprise a large set of constants. Common parameters $\widetilde{Z}$ and $\overline{\varphi}$ are identified by average firm size and any percentile of the wage distribution. In the LBD we compute an average size of a firm at the commuting zone level of 22.83, and in the CPS, 29 percent of workers earn below \$15 per hour. Given any other parameters, these pin down $\widetilde{Z}$ and $\overline{\varphi}$ exactly.

Parameters that are heterogeneous across households are relative shifters in productivity and labor supply disutility $\{\xi_k, \widetilde{\varphi}_k\}_{k=1}^K$, and shares of aggregate profits and capital income $\{\kappa_k\}_{k=1}^K$. We normalize $\xi_k = \widetilde{\varphi}_k = 1$ for college worker households. For any $\{\kappa_k\}_{k=1}^K$, the remaining productivity and labor disutility parameters can be inverted from data on average earnings per hour and each household's share of aggregate labor income, which we compute in the CPS. We assign college worker and owner

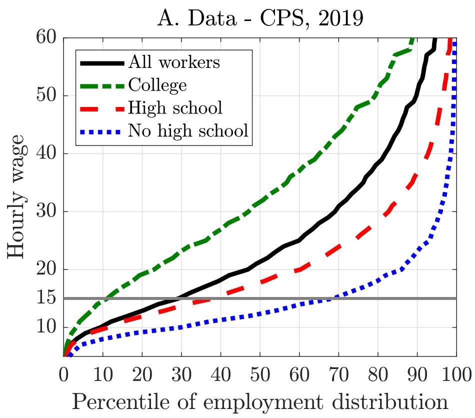


Figure 5: Distribution of wages by worker type and $15 minimum wage

Notes: CPS data constructed using combined MORG and March survey from 2019, using `hwtfinl`. Wages are computed as weekly earnings (`earnweek`) divided by usual weekly hours worked (`uhrsworkt`). We follow the Federal Reserve Bank of Atlanta Wage Growth Tracker, and remove individuals whose hourly pay is below the current federal minimum wage for tip-based workers ($2.13). We drop individuals who are coded as *hours vary* (`uhrsworkt=997`). We keep all other workers aged between 16 and 65.

households the same wage.[29] The average wage of non-high-school (high-school) workers is 42 percent (60 percent) of the average college wage. Productivity shifters are in line with these data.[30] Shares of aggregate labor income exactly pin down relative disutilities of labor supply.

Using the SCF we compute the ratio of total capital income to total labor income for each group of households, and choose $\kappa_k$ for each of the three non-owner households to match these ratios exactly. In the data, this ratio is below 0.10 for all non-owner households and 6.57 for owners. This provides further support for our approach of including owners as a separate group. Since the shares must sum to one, the share of owners is determined as a residual. As an over-identifying test, we verify that the ratio for owners is consistent with the data (Table 2C).

**3. Internally calibrated.** The final set of parameters are internally calibrated. We assume that productivity is log normally distributed. Its standard deviation $\sigma$ and decreasing returns $\alpha$ are identified by the average level of concentration in labor markets, and the labor share. The argument is as follows. First, more productivity dispersion increases the market power of the most productive firms. This increases concentration and decreases the labor share. Second, more linear technology also makes the most productive firms larger, but reduces profits. This increases concentration and increases the labor share. We infer a level of productivity dispersion $(0.268)$ consistent with direct empirical estimates (see Decker, Haltiwanger, Jarmin, and Miranda, 2020), and moderate decreasing returns to scale: $\alpha = 0.940$. The value of $\alpha$ implies a relatively elastic marginal revenue product of labor, hence firms will shrink quickly in Region III. Given other parameters, $\gamma$ determines the capital share, which we set to 0.18.

[29]This allows us to combine SCF and CPS data since we do not observe assets in the CPS. In the SCF, labor earnings are similar across the two college household types.

[30]Heterogeneity in capital income and wealth effects on labor supply require a different $\xi_k$ for owners to match the same wage as college workers.

### 4.1 Distribution of wages, consumption and implied Negishi weights

**Wages.** Figure 5 plots the distribution of wages in the benchmark economy and in the 2019 CPS data used to calibrate the model. By construction, the calibration matches 29 percent of workers earning wages less than \$15. The model also does well on the non-targeted fraction of college workers below \$15 (11% in data vs. 9% in model) and high school workers (38% in data vs. 36% in model). The model misses two features of the data. First, it slightly *overstates* non-high-school workers below \$15 (69% in data vs. 83% in model), which might bias us toward negative minimum wage effects. To address this, Appendix A.2 provides main results for an alternative calibration where $\xi_k$'s match the average level of wages of all worker types, ignoring the fraction below \$15. This calibration slightly *understates* non-high-school workers below \$15 (69% in data vs. 51% in model). We show that our main results are not substantially effected. Second, it misses the fat tails of the wage distribution. Additional worker heterogeneity would be needed to capture these. The tails are less consequential for analysis of minimum wages.

**Consumption.** Table 2C shows that the model accurately replicates data on consumption shares by household.[31] Non-high-school households accounting for around 2 percent of consumption despite being 12 percent of workers, and college households $(C + O)$ accounting for around 60 percent of consumption despite being only 35 percent of households.

**Negishi weights.** What kind of planner would choose the allocation of resources consistent with the US economy? Consider a planner solving a problem consistent with the previously described government problem (21). First order conditions give:

$$\psi_k = \frac{\pi_k (c_k/\pi_k)^\sigma}{\sum_{m=1}^K \pi_m (c_m/\pi_m)^\sigma}. \tag{24}$$

Using competitive equilibrium allocations on the right, the left gives *Negishi weights* $\boldsymbol{\psi}^*$.[32] These are a useful benchmark for welfare analysis. Table 2C shows the US economy is consistent with the distribution of consumption chosen by a planner with a combined 62 percent weight on college households $(\psi_C^* + \psi_O^*)$ and 38 percent on non-college households $(\psi_{NHS}^* + \psi_{HS}^*)$, while these groups comprise 35 percent $(\pi_C + \pi_O)$ and 65 percent of workers $(\pi_{NHS} + \pi_{HS})$. If we handed the economy to a planner with *Utilitarian weights*, $\psi_k = \pi_k$, the planner will seek to use the minimum wage to redistribute. This motivates our claimed need to separate out redistribution and efficiency effects.

## 5 Benchmarking to empirical studies

We have described two novel channels through which minimum wages may improve efficiency: (i) reallocation of employment to more productive firms, (ii) wage spillovers which undo markdown distortions at unconstrained firms. Two recent empirical studies speak directly to these. Dustmann, Lindner, Schoenberg, Umkehrer, and vom Berge (2021, henceforth DLSUB) provides evidence on reallocation of

[31] We do not use data for consumption by household from the BLS in calibration, since where household education is the education of the highest earner.

[32] In Appendix F we establish the results that, facing prices from imperfectly competitive labor markets, a planner under $\boldsymbol{\psi}^*$ would choose the competitive equilibrium allocation.

employment following a large minimum wage increase. Derenoncourt, Noelke, Weil, and Taska (2021, henceforth DNWT) provides evidence on the size of employer responses to a competitor's *voluntary* minimum wage. The model replicates both in sign and magnitude, giving confidence in our computation of efficiency implications of minimum wages. Appendix B contains additional replication exercises.

### 5.1 Reallocation effects of minimum wages

DLSUB, *"Reallocation Effects of the Minimum Wage,"* studies the effect of the introduction of a minimum wage in Germany and its impact on the cross-section of workers and firms. In January 2015, a national minimum wage of 8.50 euros was introduced into an environment with no pre-existing minimum wage. Moreover, the minimum wage introduced in Germany was large. Pre-reform, 15 percent of workers earned below 8.50, which was 48 percent of the median wage. The key finding is employment reallocation: small firms exit, and larger more productive firms expand, increasing average firm size.

**Empirical setting.** DLSUB consider a number of empirical approaches. The one we focus on computes the elasticity of firm characteristics with respect to minimum wage exposure. The authors compute a measure they call the minimum wage *Gap*: the percent increase in total earnings required to satisfy the new minimum wage, holding employment and hours fixed at their pre-reform level. Let workers be indexed by $\ell \in \{1, \ldots, n\}$. DLSUB define *Gap* using workers' pre-reform hours $h_\ell$ and wages $w_\ell$:

$$Gap := \frac{\sum_\ell \max\{\underline{w} - w_\ell, 0\} h_\ell}{\sum_\ell w_\ell h_\ell}$$

The authors group firms by geographic regions $r$, and regress changes in region outcomes on $Gap_r$. Since $Gap_r$ is in percent changes, the regressions yield elasticities. We focus on the following moments reported in their paper: (i) total employment $n = \sum_\ell \mathbf{1}_{[h_\ell > 0]}$, (ii) average wage: $\overline{w} = \sum_\ell w_\ell h_\ell / \sum_\ell h_\ell$, (iii) total number of operating firms, and (iv) average firm size. Their results are reported in Table 7, page 54.

**Replication.** To an economy with no minimum wage, we introduce a minimum wage of \$8.95/hr. This is relatively low, but also equals 48 percent of the pre-reform median wage. The empirical setting is a national reform, so we solve the pre- and post-reform economy in general equilibrium. The regions considered in DLSUB, comprise all industries in multiple commuting zones and rural areas. These are much larger than markets $j$ in our model. We therefore treat our whole economy as one region, which generates a single *Gap* measure directly comparable to theirs:

$$Gap = \frac{\sum_k \int \sum_i \max\{\underline{w} - w_{ijk}, 0\} n_{ijk}\, dj}{\sum_k \int \sum_i w_{ijk} n_{ijk}\, dj} \tag{25}$$

To compute the elasticity of variable $x$ with respect to *Gap*, we divide economy-wide $\Delta \log x$ by *Gap*.

**Results.** Figure 6 gives the results. There are two sets of the authors' results: 'Data 1' and 'Data 2'. Both feature controls that account for observable regional differences (e.g. average age) and region specific trends in the moments. 'Data 2' additionally interacts these trends with year fixed effects. We plot results for a range of minimum wages $\underline{w}_1$, indexed by the ratio of $\underline{w}_1$ to the pre-reform median wage $w_0^{p50}$. The vertical line marks the $\underline{w}_1 / w_0^{p50} = 0.48$ corresponding to DLSUB.

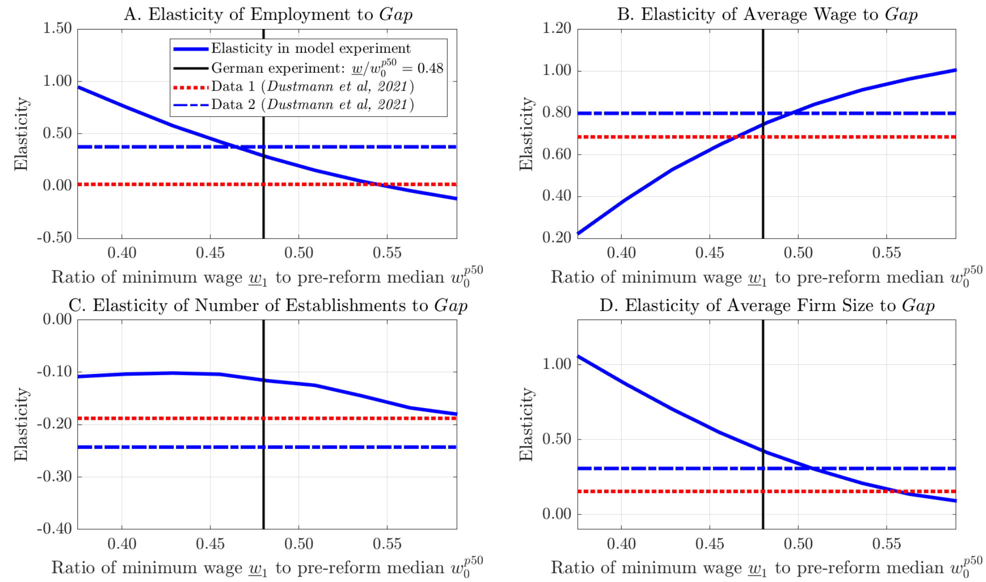


Figure 6: Replication of DLSUB (2021) - Reallocation effects of minimum wages

Notes: Corresponding data estimates for "Data 1" and "Data 2" are respectively taken from p.54 of DLSUB, Table 7, Columns (2) [regional controls and region specific linear trend] and (4) [regional controls interacted with year fixed effects]. The solid blue line plots the elasticity of the relevant moment to the minimum wage *Gap*, computed as in equation (25). The horizontal axis plots the minimum wage in the policy experiment simulated in the model as a fraction of the pre-reform median wage in the model.

First, consistent with other empirical studies of minimum wage effects, Figure 6A shows small or no disemployment effects, with employment increasing marginally in 'Data 1'. Employment increases in Region II, dominate reductions in Region III. At higher minimum wages, however, this flips, and the effect becomes negative. Market wage increases significantly in response to the minimum wage change (Figure 6B). Through the lens of the model, both constrained and unconstrained firms pay higher wages, regardless if they cut or expand employment.

Second, consistent with the new reallocation facts in DLSUB, Panel C shows that small firms exit and Panel D shows that reallocation causes average firm size to grow. In the model all firms still operate due to decreasing returns and since $n_{ijk}$ is continuous it can go below one (recall Figure 2D). To compare our model to DLSUB, we classify a firm as 'operating' when their employment is above one worker. The elasticity of the number of operating firms with respect to *Gap* is negative and thus correctly signed, but moderately less responsive compared to the data. The model's elasticity of average firm size with respect to minimum wage exposure is positive and moderately higher than the data, and consistent with the German data for slightly larger minimum wage increases. The increase in average firm size represents reallocation, and moderated at larger minimum wage increases due to firms shrinking in Region III.

**Interpretation.** One of the key take-aways of DLSUB is that minimum wage increases have heterogeneous effects across firms. Low productivity firms exit, but their workers do not move out of the labor market. Jobs which existed due to the small amount of market power at these low productivity firms are destroyed, but workers are reallocated to larger, more productive firms. This can improve allocative efficiency, and our model generates dynamics consistent with these observations.

## 5.2 Spillovers from competitors' minimum wages

DNWT, *"Spillover effects from voluntary employer minimum wages"*, studies how *voluntary minimum wages* as part of large firms' policies affect the wages and employment of firms within the large firms' market. In the context of a \$15/hr minimum wage instituted nationally by Amazon in 2018, which increases Amazon wages on average by 18.1 percent, DNWT estimate competitors increase their wages by 4.7 percent. The authors' headline result is that this constitutes a *cross-employer wage elasticity* of 0.26.[33]

**Replication.** To replicate the exercise, we need to identify firms in markets that we can call 'Amazon', and institute a policy that increases their observed wage by 18.1 percent. We do this by exogenously narrowing markdowns. We solve the baseline model, then take a firm $i$ in market $j$ and narrow the firms' markdown for all types by a fraction $\zeta$ toward the efficient markdown $\mu^*_{ijk} = 1$:

$$\mu'_{ijk} = (1-\zeta) \times \mu_{ijk} + \zeta \times 1 \quad , \quad \zeta \in (0,1).$$

We then solve the Nash equilibrium among the remaining firms in each market. We run this experiment in every market, keeping aggregates fixed, since this is a partial equilibrium exercise.

In order to proceed we need to choose a firm among the firms in each market that corresponds to 'Amazon'. DNWT do not provide summary statistics on (i) the average size of Amazon relative to competitors, or (ii) the number of competitors that Amazon faces in each market. Absent (i) we consider two cases, one we label the most productive firm in each market 'Amazon', and one where it is the second most productive firm. Absent (ii) we conduct our experiment in all markets $j$, and then drop markets based on a cut-off for the number of firms operating in a market, starting at $\underline{M} = 2$ and going up to $\underline{M} = 30$. If $\zeta$ were left fixed, we would find that the average change in 'Amazon's wage is smaller when $\underline{M}$ is larger, due to tighter competition, and larger when 'Amazon' is smaller relative to the market. We therefore recalibrate $\zeta$ to keep the average wage change of 'Amazon' consistent with the data.

**Results.** Figure 7 gives the results of this exercise. Panel A shows that our strategy for recalibrating $\zeta$ generates data consistent increases in 'Amazon' wages as we vary $\underline{M}$ and in our two specifications of 'Amazon's identity. Qualitatively, as in the data, Panel B shows that the model generates an increase in competitors' wages. The increase in the leaders' wage increases its market share and reduces that of its competitors. This tightening of competition leads competitors' markdowns narrow and their wages to increase. Quantitatively, the effect is in the range of the data.[34] In cases where Amazon is the largest firm and in markets with at least 36 firms, or is the second largest firm and in markets with at least 12 firms, the outcome is exactly as estimated by DNWT.

[33] These results are summarized on page 2 of DNWT (2021): *"In the case of Amazon, we estimate an increase in average hourly wages [of competitors] as a result of the policy of 4.7%, controlling for unrelated trends in wages at the occupation and commuting zone level. Given the size of the increase for Amazon's wages, roughly 20%, our results imply a cross-employer wage elasticity of 0.26.* Note that 4.7%/20% would imply a cross employer wage elasticity of 0.235, not 0.26. The authors refer to the 0.26 and 0.047 numbers, while only here mention the "roughly 20%" Amazon wage increase. Therefore we target an Amazon wage increase of 0.181 such that $0.047/0.181 = 0.26$.

[34] As in DNWT we compute the average log change in competitors' wages market by market, and then take an unweighted average across markets.

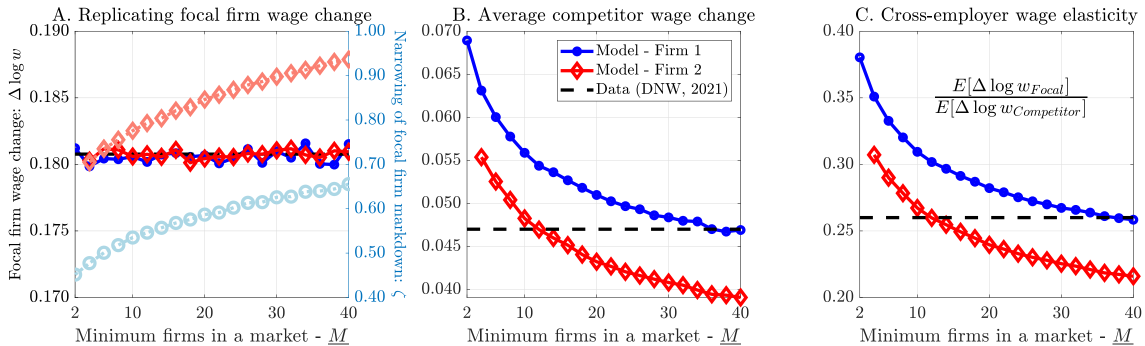


Figure 7: Replication of DNWT (2021) - Competitor responses to voluntary minimum wages

Notes: **Panel A.** Faint, increasing, lines are plotted on the righthandside vertical axis and give the value of $\zeta$ required to deliver the observed average increase in focal firm wages. Solid lines are plotted on the lefthandside vertical axis and give the model implied average increase in focal firm wages. These are compared to the data, with a value of 0.181 (= 0.047/0.26) (page 2 of DNWT). **Panel B.** Plots the average non-focal firm wage change. Blue-circle line corresponds to the case where the focal firm is the most productive firm in each market. Red-diamond line corresponds to the case where the focal firm is the second most productive firm in each market. Data value is 0.047. **Panel C.** Plots the *cross-employer wage elasticity*, which is the ratio of panel B to panel A. Data value is 0.26.

Apart from replicating the empirical evidence, these results suggest an additional margin of cross-sectional variation. In markets that are more competitive, or when the focal firm is relatively small, the cross-employer elasticity is lower. It may therefore be challenging to extrapolate from this evidence to the effects of minimum wages more broadly. Within a market, minimum wages will first affect small firms, and most employment is in competitive markets. Both facts suggest spillovers within markets may be small, the second fact suggests these might not matter when aggregated.

### 5.3 Summary and additional quantitative replications

The model successfully replicates and gives a natural interpretation to key reduced form results from the empirical literature on minimum wages. These are necessary features of the data for a model to replicate. We view this as positioning the model well for our main quantitative exercises.

Appendix B includes three additional tests. First, we document that the model generates empirically relevant spillovers across the distribution of workers. We show this by replicating the Brazilian minimum wage increase studied in Engbom and Moser (2021) and Haanwickel (2020).[35] Second, our model generates similar systematic heterogeneity in employment responses to the minimum wage by market concentration to that documented in Azar, Huet-Vaughn, Marinescu, Taska, and von Wachter (2019), with larger disemployment effects in competitive markets. Third, we replicate the Seattle minimum wage increase studied in Jardim, Long, Plotnick, Van Inwegen, Vigdor, and Wething (2022). The model matches the small disemployment effect in low wage jobs (less than $19/hr) after an initial increase in the minimum wage from $9.47 to $11, and larger negative effects after a consequent rise from $11 to $13. If anything, the model slightly *understates* the negative disemployment effects.

[35] Both studies' empirical approaches extend the approach used in the US of Autor, Manning, and Smith (2016). The lack of large minimum wage changes in the US leads us to consider the Brazilian evidence.

## 6 Positive implications of the minimum wage

Before turning to our normative exercises we briefly describe the positive implications of the minimum wage for (i) aggregate and cross-sectional outcomes, and (ii) inequality metrics used as proxies for welfare absent a structural model: the college premium, overall wage inequality and the labor share.

**Aggregates.** Qualitatively, a main result is the non-linear effects of minimum wages on many aggregate variables. The top row of Figure 8 shows the increasing and then decreasing effect of the minimum wage on consumption, capital, output (Panel A), and employment (Panel C). Consumption, output and employment increase due to a more efficient allocation of resources across firms, with small firms shrinking and employment and capital reallocated to more productive firms. These aggregates rapidly deteriorate at higher minimum wages as misallocation via (i) more productive firms entering Region III, and (ii) employment being reallocated from Region I to Region II firms.[36] Panel B shows that despite monotonically increasing wages, the welfare relevant aggregate shadow wage is also hump-shaped. Initially, narrowing shadow markdowns in Region II increase the wage and shadow wage in tandem, but as rationing constraints bind the shadow wage declines. Panel C shows that the shadow wage is indeed allocative, determining aggregate employment rather than the increasing average wage. Profits monotonically decrease, reallocating payments from owners to non-owners.

**Employment non-linearities.** Quantitatively, the minimum wage has small positive effects on aggregates, with less than one percent increases in output, consumption, capital and employment. Importantly, Panel C shows the model can rationalize either positive or negative employment effects of the minimum wage. This is consistent with the broad empirical literature that finds conflicting signs of the employment effect. In particular the recent, broad study of state minimum wage increases by Clemens and Strain (2021), finds positive minimum wage effects following relatively smaller minimum wage increases from relatively lower initial minimum wages, and negative effects for large changes and initially high minimum wages. The model is consistent with this. Crucially, our model implies that while a higher minimum wage can increase employment levels, once the minimum wage is greater than \$10, aggregate employment effects become quickly negative, with job losses concentrated at workers with less education (Panel D). This is consistent with the Seattle study we replicate in Appendix B. Thus, the effect of a higher minimum wage is highly non-linear and caution must be used if extrapolating based on evidence developed from minimum wage increases at lower minimum wage levels.

**Cross-sectional outcomes.** The bottom row of Figure 8 plots employment, wage and shadow wages for each worker type. For sake of exposition, we suppress owners from the figures. As a summary, each panel is consistent with negative effects emerging more swiftly for non-high school workers. Again, the allocative wage for each type of worker is the hump-shaped shadow wage (Panel C), rather than the sharply increasing average wage (Panel B).

Figure 8 shows that accounting properly for heterogeneity in income from labor and profits is necessary to understanding aggregate employment effects of minimum wages in general equilibrium. If

[36]Appendix A.1 plots the share of jobs and employment in each region for each type of worker.

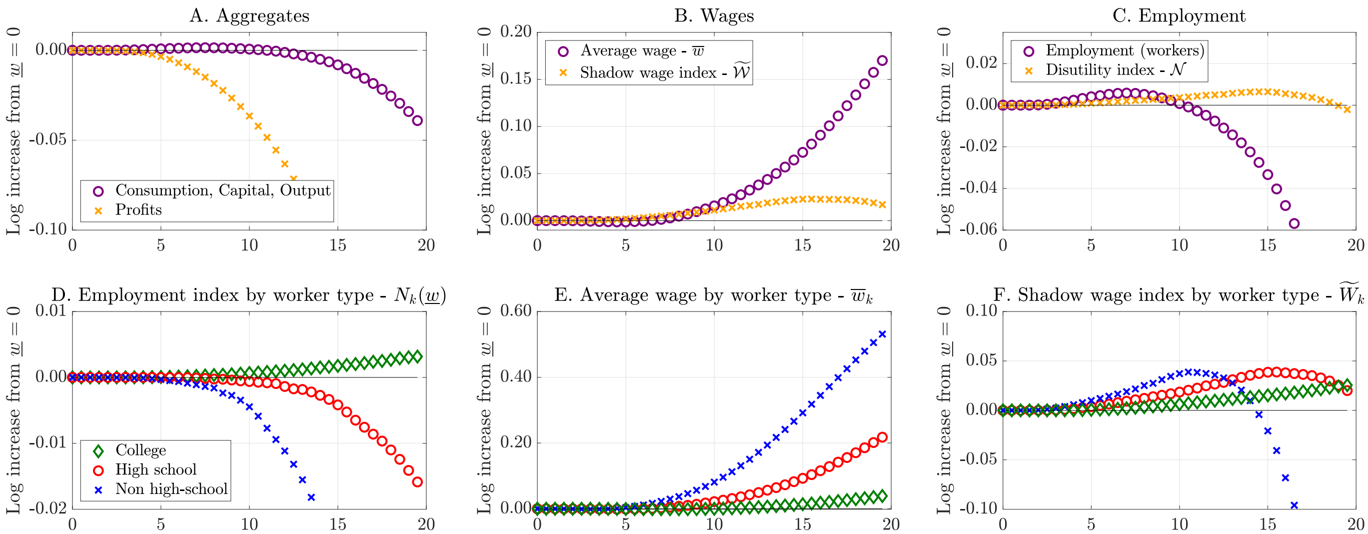


Figure 8: Aggregate and worker type outcomes

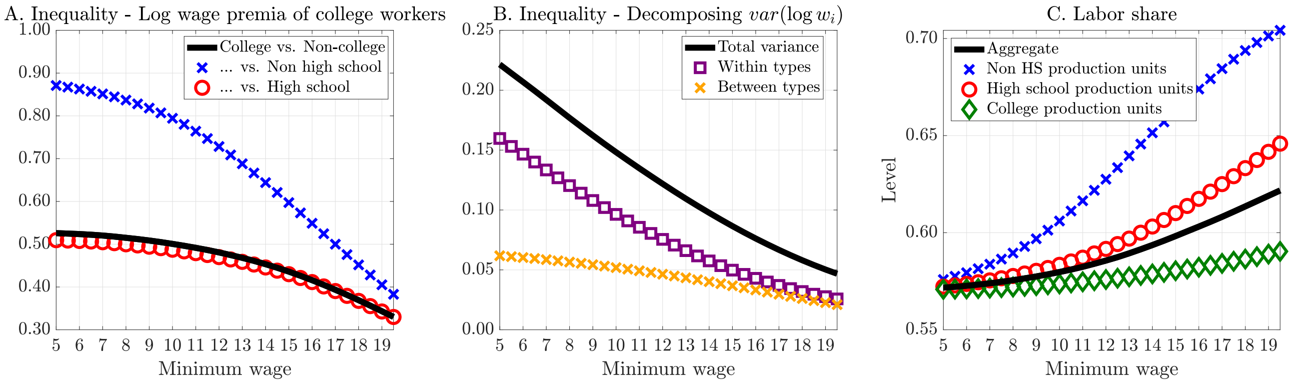


Figure 9: Minimum wages and commonly used empirical proxies for welfare

households had equal shares of profits, wealth effects would dampen the negative employment effects in Panel D. However, owners earn 92 percent of dividends, and account for only 7 percent of labor income (Table 2B). The skewed distribution of capital and profit income silos these wealth effects in a narrow slice of the work-force, and hence implies wealth effects are small in aggregate.

**Empirical proxies for welfare.** Figure 9 shows that frequently used empirical proxies for welfare are monotonically increasing in the minimum wage. Consider the range of $\underline{w}$ from \$7.50 to \$15. The log wage premia between college and non-college workers declines by one fifth (Panel A). The total variance of log wages declines by nearly a half, driven equally by declining in within- and between-type inequality (Panel B). The aggregate share of payments to labor increases by about 3 ppt, as profits decline (Figure 8A), with an even larger increase in low wage jobs (Panel C). Next we show that economically founded measures of aggregate welfare inherit the non-monotonicities found in Figure 8, and hence these empirical proxies are misleading measures.

## 7 Normative implications of the minimum wage

We (i) describe how we measure welfare, (ii) compare welfare maximizing minimum wages under alternative social welfare weights, (iii) use our fictitious government problem to separate out efficiency and redistribution, and (iv) explain the quantitative mechanisms behind our results. For convenience, we benchmark welfare gains relative to an economy with a zero minimum wage.[37]

### 7.1 Measurement and social welfare weights

For each household, under a particular minimum wage $\underline{w}$, the *consumption equivalent welfare gain relative to a no minimum wage economy* (henceforth, *welfare gains*) is the proportional increase in consumption $\lambda_k(\underline{w})$ that delivers the same utility as the minimum wage economy. The aggregate welfare gain, $\Lambda(\underline{w})$, is defined similarly and requires taking a stand on *social welfare weights* $\{\psi_k\}_{k=1}^K$:

$$\underbrace{u^k\left(\left(1+\lambda_k(\underline{w})\right)\frac{c_k(0)}{\pi_k}, n_k(0)\right) = u^k\left(\frac{c_k(\underline{w})}{\pi_k}, n_k(\underline{w})\right)}_{\text{Worker type welfare gains, } \lambda_k(\underline{w})} \quad , \quad \underbrace{\sum_k \psi_k u^k\left(\left(1+\Lambda(\underline{w})\right)\frac{c_k(0)}{\pi_k}, n_k(0)\right) = \sum_k \psi_k u^k\left(\frac{c_k(\underline{w})}{\pi_k}, n_k(\underline{w})\right)}_{\text{Aggregate welfare gains, } \Lambda(\underline{w})}.$$

With separable, power utility, aggregate welfare gains are a weighted harmonic mean of gains by type:

$$1+\Lambda(\underline{w}) = \left[\sum_k \widetilde{\psi}_k(0)\left(1+\lambda_k(\underline{w})\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \quad , \quad \widetilde{\psi}_k(0) := \frac{\psi_k\left(\frac{c_k(0)}{\pi_k}\right)^{1-\sigma}}{\sum_k \psi_k\left(\frac{c_k(0)}{\pi_k}\right)^{1-\sigma}}. \tag{26}$$

Since the choice of social welfare weights is arbitrary, we provide results for two informative sets of weights: (i) Utilitarian weights, under which $\psi_k = \pi_k$, and (ii) Negishi weights associated with a zero minimum wage economy $\psi_k(0) = \psi_k^*(0)$, measured from equation (24).[38] Recall from Table 2 that these weights are as follows:

| | | |
|---|---|---|
| Utilitarian weights: | $(\psi_k = \pi_k)$ | $(NHS, HS, C, Own) = (0.12, 0.53, 0.29, 0.06)$ |
| Negishi weights: | $(\psi_k = \psi_k^*)$ | $(NHS, HS, C, Own) = (0.01, 0.36, 0.24, 0.39)$. |

### 7.2 Social welfare maximizing minimum wage, absent transfers

**Optimal minimum wage.** Figure 10 depicts our first set of welfare results. Panel A plots consumption equivalent welfare gains for each type of worker under minimum wages in the range of zero to \$20.00 per hour. Consistent with the effects of minimum wages described in the previous section, welfare gains are shortest lived for the least productive groups of workers. Owners of capital face welfare losses due to the erosion of profits.

Panel B plots aggregate welfare gains. As per equation (26), aggregate welfare gains are averages over the worker level gains in Panel A, where the weights reflect social welfare weights. Under Utilitarian

[37] This choice is easy to amend and has little implications for our results. See also Appendix A.3, referenced below.

[38] Under the Negishi weights, using the governments's optimality conditions (24) in (26) imply that $\widetilde{\psi}_k(0)$ are equal to consumption shares: $\widetilde{\psi}_k(0) = c_k(0)/C(0)$. If we benchmark welfare gains to an alternative minimum wage $\underline{w}_0$, then the effective weights $\widetilde{\psi}_k(\underline{w}_0)$ will be different. However Appendix A.3 shows that under both Negishi and Utilitarian weights, the associated effective weights $\widetilde{\psi}_k(\underline{w}_0)$ vary very little in $\underline{w}_0$.

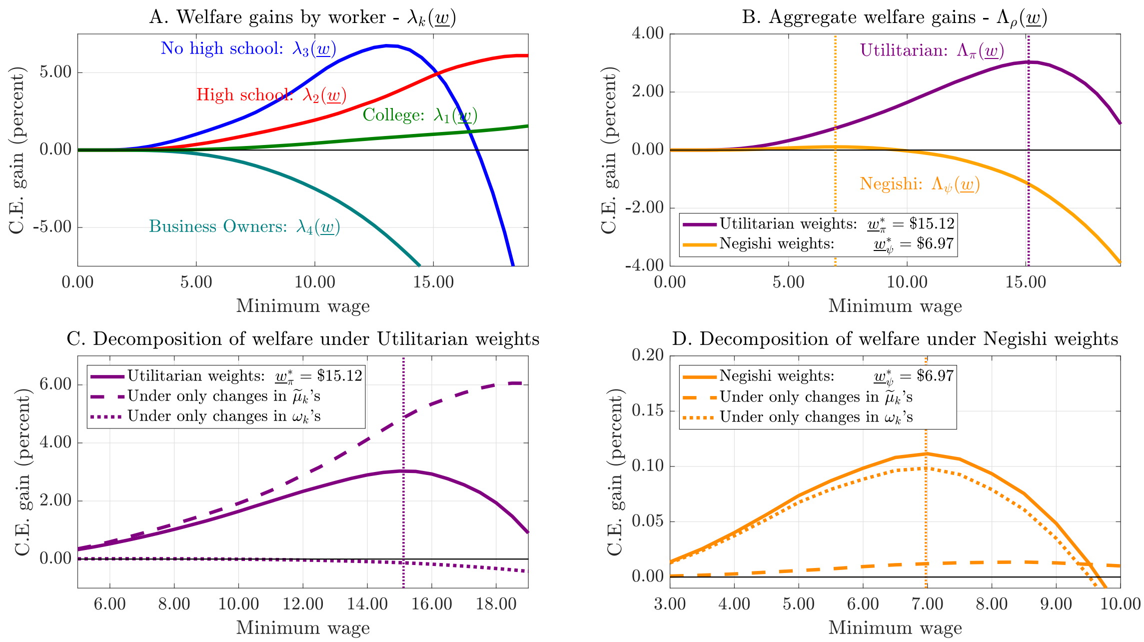


Figure 10: Minimum wages and welfare

Notes: In all cases we plot objects from the equilibrium under various values of the minimum wage $\underline{w}$, on the horizontal axis. In all cases the vertical axis plots consumption equivalent welfare gains relative to an economy with a zero minimum wage. **Panel A.** Plots the consumption equivalent welfare gains of each household: $\lambda_k(\underline{w})$. **Panel B.** Plots the aggregate consumption equivalent welfare gains, under alternative sets of social welfare weights $\psi_k$. **Panel C.** Plots consumption equivalent welfare gains under utilitarian welfare weights under changes in shadow markdown wedges only, $\tilde{\mu}_k(\underline{w})$, and under misallocation wedges only, $\omega_k(\underline{w})$. In each case a competitive equilibrium is solved under only the specified wedges changing, while the remaining wedges are kept at their value under $\underline{w} = 0$. **Panel D.** Repeats panel C, but with welfare evaluated under Negishi weights.

weights, the optimal minimum wage is \$15.12, at which point welfare gains of non-college workers are high. Welfare of college workers increases at higher minimum wages, but by this point welfare losses among non-college workers are steep. Under Negishi weights, the optimal minimum wage is much lower at \$6.97. The Negishi weights put a weight of about 39 percent on owners, whereas they represent only 6 percent of the population. With more weight on firm owners and less on non-college workers, these weights imply a much lower optimal minimum wage.

These results make clear that the optimal minimum wage, holding other fiscal instruments fixed, is in the eye of the beholder in the sense that it depends largely on social welfare weights. This is because the minimum wage redistributes significantly. As an additional example, a weight of 0.97 on High-school households produces a $\underline{w}^*$ of \$18.31, at which point employment losses of non-high school households are minus 9 percent. This leads us, in the following section, to separate out the efficiency and redistributive components.

**Welfare gains.** The *magnitude* of welfare gains also depend on social welfare weights, with large gains under Utilitarian weights and relatively small gains under Negishi weights. In both cases we compute that these gains are small with respect to the potential welfare gains available in the economy. Consider the efficient allocation which obtains when all firms' markdowns are one, and hence wages are equal to marginal revenue products at all firms. In the case of Utilitarian weights, the welfare gains from the efficient allocation are 28.61 percent, and the optimal minimum wage gains deliver about one ninth of this (3.04 percent).

**Decomposition.** The decomposition of welfare gains also depends on the perspective of the analysts' social welfare weights. We can use our construction of the general equilibrium to decompose welfare gains into those due to changes in shadow markdowns, and those due to changes in misallocation. In Panel C, at $\underline{w}^*$ under Utilitarian weights, misallocation due to $\{\omega_k\}_{k=1}^K$ is worse than in the zero minimum wage economy, but the planner is happy to trade this off against large gains from narrowing shadow markdowns in Region II $\{\widetilde{\mu}_k\}_{k=1}^K$, which redistribute income to low wage workers. The presence of labor market power provides the possibility of these gains from redistribution. In Panel D, at $\underline{w}^*$ under Negishi weights, more than 80 percent of the welfare gains are instead driven by improved reallocation measures $\{\omega_k\}_{k=1}^K$. As we showed earlier, these gains stem from reallocating employment from low $z$ to medium $z$ firms. Quantitatively, gains from misallocation are limited, as reallocation begins to occur from high $z$ to medium $z$ firms. We return to these channels in Section 7.6.

In summary, redistribution generated by the minimum wage implies that the optimal minimum wage depends crucially on the social welfare weights of the policy maker. The focus of our second set of results, therefore, will be efficiency, which our model—and the associated empirical exercises that we have validated it against—are well purposed to discuss.

### 7.3 Efficiency maximizing minimum wage

The above results are unsatisfactory, in that they depend strongly on social welfare weights. We now allow the planner to choose the minimum wage and unrestricted, balanced-budget, lump sum transfers $\{t_k\}_{k=0}^K$, in order to maximize welfare. If the government considers conducting policy under different sets of social welfare weights, then lump sum transfers can adjust to soak up the different redistributive motives encoded in the different weights. With redistribution taken care of, the optimal minimum wage now reflects efficiency. We call this the *efficiency maximizing minimum wage*.

Table 3 provides our second set of results, with five key results in Panel B. First, as anticipated, with flexible lump sum transfers, the efficiency maximizing minimum wage is robust to social welfare weights. Second, we find that it is in the range of \$7.50 to \$8.50. For example, under Utilitarian weights, the government can meet its redistributive objectives by transfers (columns 9 to 12), leading to a much lower optimal minimum wage. Third, the welfare gains associated with the efficiency maximizing minimum wage are small, and also consistent across welfare weights. Column 4 shows that the welfare gains are robustly around 0.16 percent. These represent only one percent $(0.16/15.26)$ of the welfare gains associated with the efficient allocation with no labor market power.[39] Fourth, a back of the envelope calculation suggests that under Utilitarian weights, around 94 percent of the welfare gains come from

[39]The welfare gains associated with the efficient benchmark are computed with optimal lump sum transfers in the efficient economy, and hence are constant with respect to social welfare weights.

| Policy | Weights | Min. wage | Welfare gain | Welfare gains by type, $\lambda_k(\underline{w}^*)$ | | | | Transfers ($t_k/GDP$, %) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\psi_k$ | $\underline{w}^*$ | $\Lambda(\underline{w}^*)$ vs. $\Lambda^{Eff}$ | Non-HS | HS | Coll. | Own | Non-HS | HS | Coll. | Own |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11). | (12) |
| **A. No transfers** | Utilitarian | \$15.12 | 3.04% vs. 28.61% | 5.04% | 4.91% | 1.03% | -8.60% | - | - | - | - |
| | Negishi | \$6.97 | 0.11% vs. -2.43% | 2.10% | 0.90% | 0.14% | -0.79% | - | - | - | - |
| | 97% HS | \$18.31 | 5.69% vs. 34.37% | -7.13% | 6.09% | 1.43% | -14.53% | - | - | - | - |
| **B. Transfers** | Utilitarian | \$8.27 | 0.17% vs. 15.26% | 0.31% | 0.03% | 0.34% | 0.19% | 10.4% | 18.2% | 5.4% | -34.0% |
| | Negishi | \$7.76 | 0.16% vs. 15.26% | -0.81% | -0.11% | 0.37% | 0.35% | -0.0% | -0.4% | 0.0% | 0.4% |
| | 97% HS | \$9.95 | 0.20% vs. 15.26% | -0.56% | 0.35% | -8.66% | -2.68% | 0.4% | 76.7% | -39.9% | -37.2% |

Table 3: Optimal minimum wage

Notes: 'Utilitarian' corresponds to population share weighted consumption equivalent welfare. 'Negishi' corresponds to the weights that rationalize current U.S. data. '97% HS' places weights of 0.01 on all other types, and 0.97 on high-school graduates. Panel A restricts transfers to be zero across household types, consistent with Figure 10. Panel B allows for optimal lump sum transfers across household types. Column (3) reports the optimal minimum wage. Column (4) reports the aggregate consumption equivalent welfare gain $\Lambda(\underline{w}^*)$ and compares this to the aggregate consumption equivalent welfare gain associated with the efficient allocation $\Lambda^{Eff}$ in which firms have no market power. Columns (5) through (8) report the type specific consumption equivalent welfare gains. Columns (9) through (12) report the transfers relevant for exercises in Panel B.

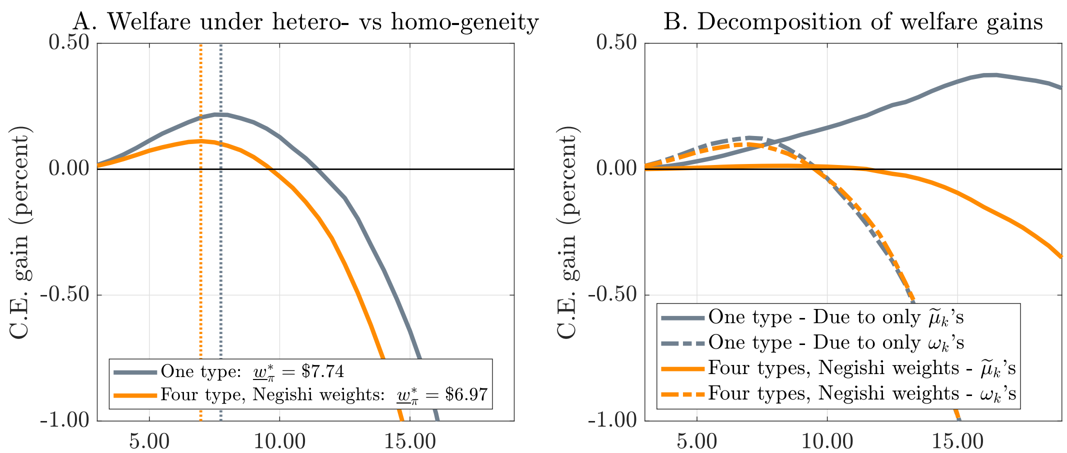


Figure 11: Minimum wages and welfare - Homogeneous vs. Heterogeneous households

redistribution (1- 0.17%/3.04%, in blue). Fifth, our method is robust to even extremely skewed social welfare weights, such as a 97 percent weight on high school workers.

Note that with transfers in Panel B, the optimal minimum wage and associated welfare gains coincide closely to the case of Negishi weights in Panel A. This should not be a surprise. The competitive equilibrium has zero lump-sum transfers, and the Negishi weights rationalize this as optimal for a planner with such weights. Therefore, moving weights, and adjusting transfers away from zero, delivers a similar welfare results. This can be verified in the third panel, where optimal transfers are roughly zero under Negishi weights.

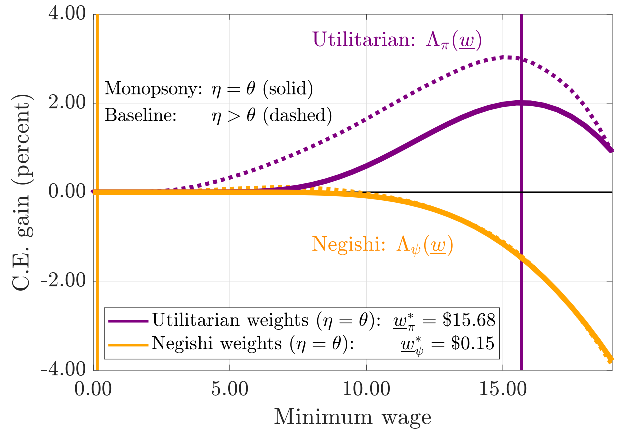


Figure 12: Minimum wages and welfare - Role of granular markets

### 7.4 Removing heterogeneity

The efficiency implications of minimum wages are largely independent of the distribution of households. To show this we recalibrate the model with no household heterogeneity and compute the optimal minimum wage.[40] With only one household, there is now no notion of social welfare weights.

In Figure 11 we compare welfare in the homogeneous household economy to the heterogeneous household model under Negishi weights, since, as discussed above, this coincides closely with the efficiency maximizing case. Panel A shows that absent household heterogeneity the optimal minimum wage is \$7.74, with a similar profile to welfare as the heterogeneous household economy under Negishi weights. Panel B shows that at the optimal minimum wage, the about two thirds of the welfare gains come from correcting misallocation, much less than in the heterogeneous household case. We conclude that heterogeneity can be enriched or removed, but the efficiency maximizing minimum wage is robust.

### 7.5 Removing granularity: $\eta = \theta$

The reallocation channel that is responsible for efficiency gains (see above, 'Decomposition' and Figure 10D), is due to direct reallocation to higher productivity firms within each labor market. To see this, we consider the optimal minimum wage in an economy in which $\eta = \theta$.[41] This is the monopsonistically competitive model that is frequently used in the literature. In our baseline, jobs lost at Region III firms due to a minimum wage are most likely reallocated to local firms with higher productivity. Due to the granularity of local labor markets, these can be much higher productivity firms. With $\eta = \theta$, workers are just as willing to substitute between the finite number of firms within a market as they are between the infinite number of firms outside of their market. Jobs lost at Region III firms minutely shift out labor supply of all firms, regardless of their productivity. Results from this exercise are shown in Figure 12.

Granularity with $\eta > \theta$ is crucial for capturing the efficiency benefits of the minimum wage. Assessed

[40] The parameters $\overline{Z}$, $\overline{\varphi}$ are still calibrated to match the same baseline targets: 29 percent of workers earn less than \$15 in the initial economy and average firm employment is 22.8 workers.

[41] We set $\eta = \theta = 3.02$, which gives the same aggregate labor share as the baseline economy. We also recalibrate all 'shifter' parameters, $\{\widetilde{\varphi}_k, \xi_k, \kappa_k, \}_{k=1}^{K}$, to match the same moments in Table 2.

under Utilitarian weights, we observe that the redistributive benefits of the minimum wage are largely unaffected. Under Negishi weights—which, as we have argued, are a shortcut to understanding the efficiency maximizing minimum wage at the end of Section 7.3—the optimal minimum wage almost goes to zero. With $\eta = \theta$ the within-market reallocation mechanism which generates efficiency gains in the baseline economy calibration is turned off.

**Summary.** We find that the efficiency gains from minimum wages are small, and—when recalibrated—robust to the amount of heterogeneity in the economy, and stem mostly from reallocation which operates within labor markets. In Section 8 we show that this is robust to (i) alternative preference parameters, (ii) region specific minimum wages, (iii) short- versus long-run effects. This is despite the fact that the model matches key empirical evidence disciplining the channels through which efficiency improvements under a minimum wage could occur: direct effects, spillovers, and reallocation.

### 7.6 Mechanisms

We conclude this section by providing some understanding into why the direct, spillover and misallocation channels have quantitatively small effects on efficiency.

**1. Decomposition.** Section 3 provided a characterization of the equilibrium of the economy that hinged on a set of shadow markdown and misallocation wedges for each type of worker. We therefore can understand the efficiency implications of minimum wages via these wedges, which we plot in Figure 13A and 13B. First, labor market power implies that as the minimum wage increases shadow markdowns narrow, generating welfare gains. However, these gains are limited and gradual on the way up, and then sharply decline. This non-linearity owes to the quick erosion of employment in Region III, and associated rapid tightening of the rationing constraint, which widens shadow markdowns. We use an example to expound on this below. Second, the gains from misallocation are even more swiftly undone, as firms in Region III contribute toward misallocation and employment reallocates from high to medium productivity firms.

To provide more detail, Panels C and D restrict attention to high school workers and add counterfactuals under which all markdowns are kept at their level under $\underline{w} = 0$, apart from firms in a particular region. If firms in Region I responded strongly to the binding minimum wages at their competitors in Region II, then the purple dashed line would steeply increase. However its increase is quantitatively small. Despite matching empirical evidence on spillovers across firms, an increase in the minimum wage has quantitatively negligible spillovers on the markdowns of unconstrained firms. There are two reasons. First, the firms in Region II are small, and so increases in their employment does not substantially shift the elasticity of unconstrained firms' residual supply curves. Second, firms in Region III shrink quickly, with declining shadow wages, which delivers more market power to firms in Region I, making their supply curves more inelastic which supports wider markdowns. The increase in the aggregate markdown is instead shaped by the narrowing markdowns of firms in Region II, up to about $15, and then by steeply widening shadow markdowns—capturing binding rationing constraints—in Region III.

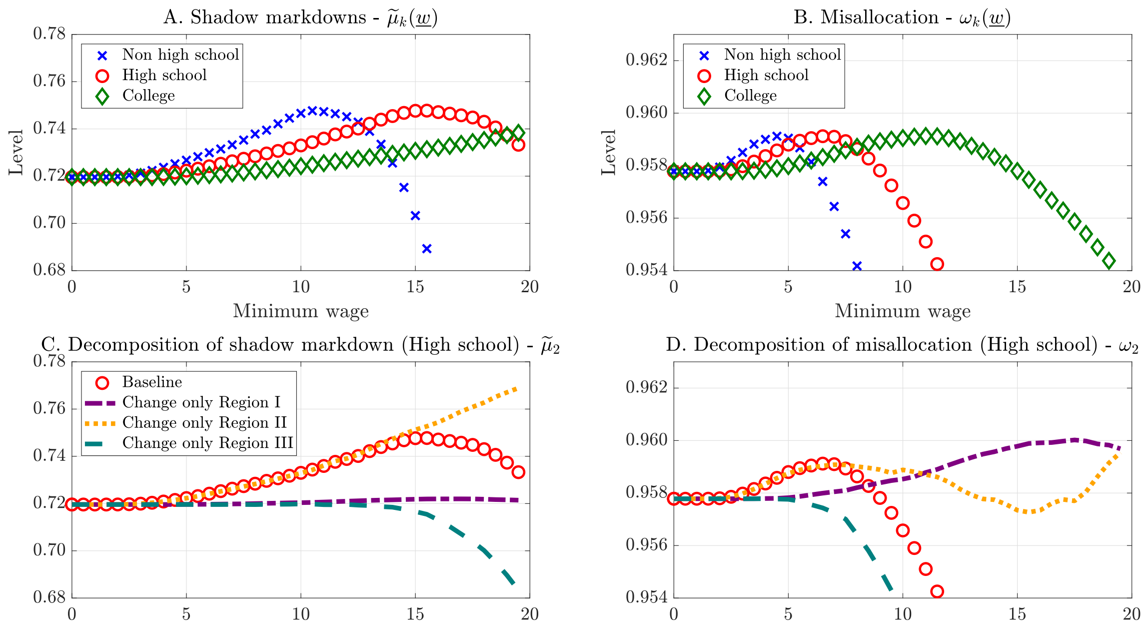


Figure 13: Decomposing efficiency wedges across the distribution of workers and firms

Notes: Panel A plots shadow markdowns faced by household types. Panel B plots misallocation by household types. Panel C decomposes the shadow markdown movements for high school grads into those due to each Region (I-III). Panel D repeats this exercise for misallocation.

**2. Example market.** To expand on the forces described above, Figure 14 provides an example of a market with 200 firms at a \$15 minimum wage. Each point is a different firm, with most being in Region III under $\underline{w} = \$15$ (panel A). We use this figure to make three points.

First, the range of productivity for which firms are in Region II is small (orange diamonds). This limits the *Direct* efficiency gains. Low productivity firms in Region II have relatively small market shares, and hence have a relatively elastic labor supply curve. Adapting our previous expression, in Region II:

$$\Delta \log \widetilde{\mu}_{ijk} = \left(1 + \varepsilon_{ijk}\left(\frac{1-\alpha}{1-(1-\gamma)\,\alpha}\right)\right)\Delta \log \underline{w}.$$

When $\widetilde{\mu}_{ijk} = 1$, the firm moves into Region III. Hence, with a high $\varepsilon_{ijk}$, the range over which the minimum wage situates them in Region II is relatively small. Small increases in the minimum wage quickly increase their employment toward the competitive level.

Second, once in Region III, the relatively elastic marginal revenue product of labor schedule, $\alpha = 0.94$, implies firms quickly shrink (Panel C). A high $\alpha$ increases the magnitude of this elasticity:

$$\Delta \log n_{ijk} = -\frac{1-(1-\gamma)\,\alpha}{1-\alpha}\Delta \log \underline{w}.$$

Panel C compares actual employment to the firm's partial equilibrium employment $n^*_{ij}$, if it were to behave competitively, absent a minimum wage. Holding market aggregates fixed, $n^*_{ij}$ equates firm labor supply and demand under a markdown of one:

$$w^*_{ij} = \widetilde{\alpha}\widetilde{z}_{ij} n^{*\,\widetilde{\alpha}-1}_{ij} \quad , \quad n^*_{ij} = \left(\frac{w^*_{ij}}{w_j}\right)^{\eta} n_j.$$

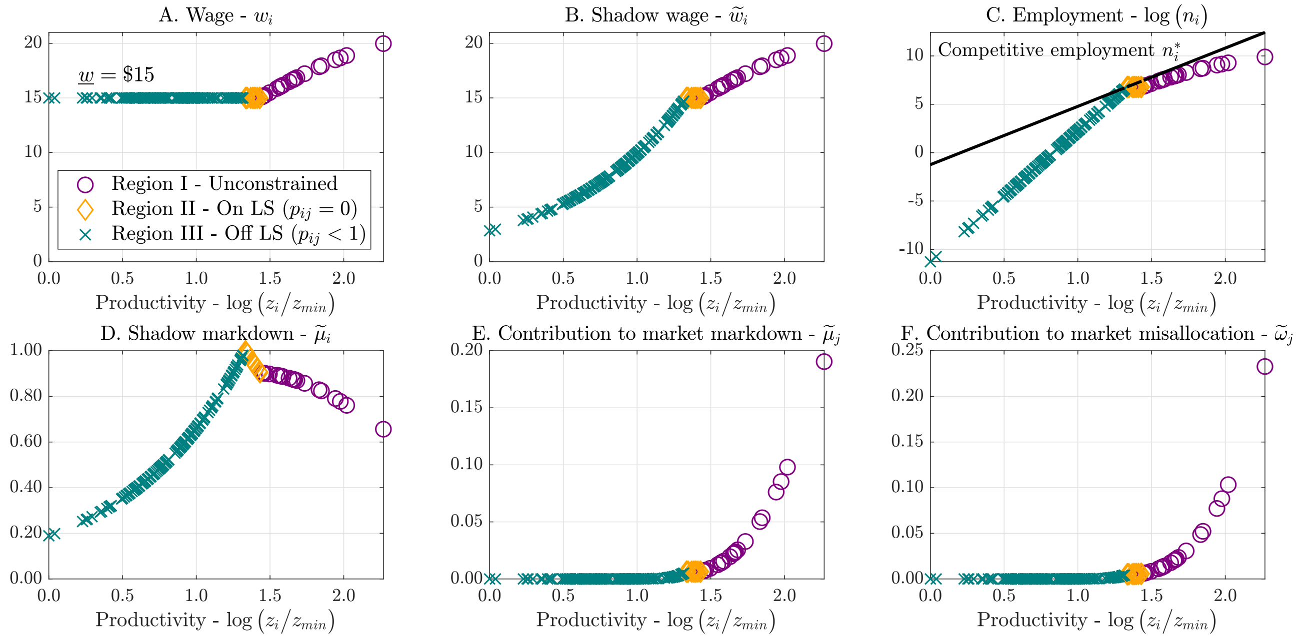


Figure 14: Example market - 200 firms, \$15 minimum wage and Non-high school workers

Notes: The pictured market has 200 firms and was drawn at random from markets with more than 150 firms.

In Region I, the most productive firms have wider markdowns, so their size is relatively more distorted away from the competitive level. In Region III, however firms are even further away from their competitive size, due to rationing. This asymmetry, leads to large efficiency losses from firms in Region III. Figures in Appendix A.1 plot the fraction of jobs and employment in each region for each worker type.

Third, Panels E and F plot the contribution of each firm to the market-level shadow markdown and misallocation, using the productivity weighted formulas from Section 3. The largest contributions are from firms that have unconstrained wages that are far away from the minimum wage. As discussed above, large firms respond little to the increase in wages of their low wage competitors, as their low wage competitors have small market shares. Panels E and F show that had these responses been large, then they would have large effects on the market. The empirical evidence in DNWT concerns competitors responding to wage changes at a large firm (Amazon), whereas the key question for the minimum wage is the large firm responses to small wage competitors, which we need a model to compute.

# 8 Robustness

First, we provide bounds on optimal minimum wages and welfare gains under different configurations of the aggregate elasticity of labor supply $\varphi$. Our key result of small efficiency gains holds across parameterizations. Second, we extend the model to multiple regions, calibrated to high, medium and low income US states. Quantitatively, we find little regional heterogeneity in region-specific $\underline{w}^*$. Interestingly, the efficiency maximizing $\underline{w}^*$ accords with intuition—lower in Mississippi, higher in high income states—while the Utilitarian optimal $\underline{w}^*$ varies unexpectedly depending on the exact distribution of income. Third, we consider short-run effects of minimum wages by keeping capital fixed, as opposed to

| Weights | Frisch | | Optimal minimum wage | | Welfare gain | | Frac. due to |
|---|---|---|---|---|---|---|---|
| $\psi_k$ | $\varphi$ | | No transfers | Transfers | No transfers | Transfers | redistribution |
| (1) | (2) | | (5) | (6) | (7) | (8) | (9) |
| **A. Utilitarian** | Baseline | 0.62 | $15.12 | $8.27 | 3.04% | 0.20% | 93.6% |
| | Low | 0.30 | $15.10 | $7.67 | 3.01% | 0.14% | 95.4% |
| | High | 0.86 | $15.17 | $8.87 | 3.05% | 0.18% | 94.0% |
| **B. Negishi** | Baseline | 0.62 | $6.97 | $7.76 | 0.11% | 0.17% | - |
| | Low | 0.30 | $6.87 | $7.43 | 0.10% | 0.14% | - |
| | High | 0.86 | $7.05 | $7.97 | 0.11% | 0.18% | - |

Table 4: Robustness exercise - Varying the elasticity of labor supply $\varphi$

Notes: In the case of Utilitarian social welfare weights, column (9) gives gains due to efficiency and reports one minus the welfare gains with transfers (column 8) divided by the welfare gains with no transfers (column 7).

the previous results which can be viewed as long-run effects. We characterize the theory of the short-run and quantitatively find that optimal minimum wages fall by only around one dollar.

### 8.1 Sensitivity to the elasticity of labor supply

We assess the role of the aggregate elasticity of labor supply in the efficiency implications of the minimum wage. Recall the aggregate labor supply curve for each type of worker: $n_k = \pi_k \widetilde{\varphi}_k \widetilde{w}_k^{\varphi} c_k^{-\sigma\varphi}$. A higher Frisch elasticity of labor supply $\varphi$, increases the positive employment effects of the minimum wage when shadow wages are increasing, and increases the negative disemployment effects of the minimum wage when shadow wages are falling.

**Approach.** We consider two values of $\varphi$ either side of the baseline value of 0.62. These values are informed by our exercise in Appendix D using data from Golosov, Graber, Mogstad, and Novgorodsky (2021). Their results imply larger $\varphi$ for high income households (lower MPC, higher MPE) than low income households (higher MPC, lower MPE). We consider the values for both groups: $\varphi \in \{0.30, 0.86\}$.[42] For each $\varphi$ we recalibrate all other shifters in Table 2 to match the same data as our baseline calibation.

**Results.** Table 4 shows that levels of $\varphi$ have essentially zero effect on our calculations. We conclude that our main results are robust to the Frisch elasticity of labor supply.

### 8.2 Sensitivity to heterogeneity in income across states

Lower average wages in a region, or a larger share of non-college workers will have implications for the optimal minimum wage. To understand the scope of these potential differences across regions, we ask how our answers for the optimal minimum wage and its decomposition into efficiency and redistributive elements might depend on the level and distribution of wages.

**Approach.** We split our economy into three separate regions, which we denote $r$ and consider a separate household type for each region. We make the simplifying assumption that labor is immobile across

[42] This range subsumes the range used by the Congressional Budget Office when modeling policy, which is around 0.30 to 0.53. See the following (link).

| Regions by income (1) | Average wage (2) | Fraction of workers < \$15 (3) | Non-HS (4) | HS (5) | Coll. (6) | Optimal minimum wage No transfers (7) | Transfers (8) | Welfare gain No transfers (9) | Transfers (10) | Frac. due to redistribution (11) |
|---|---|---|---|---|---|---|---|---|---|---|
| Baseline | \$19.69 | 29.3% | 12.4% | 52.8% | 34.8% | \$15.12 | \$8.27 | 3.04% | 0.17% | 93.6% |
| Mississippi | \$16.93 | 41.3% | 15.8% | 63.1% | 21.2% | \$14.89 | \$7.66 | 4.01% | 0.18% | 95.6% |
| Low | \$18.20 | 33.0% | 12.4% | 56.2% | 31.4% | \$14.91 | \$7.71 | 3.07% | 0.18% | 94.1% |
| Medium | \$19.46 | 29.8% | 12.0% | 53.5% | 31.4% | \$15.04 | \$8.25 | 3.01% | 0.17% | 94.4% |
| High | \$21.64 | 26.0% | 12.9% | 49.2% | 31.4% | \$14.90 | \$10.03 | 3.10% | 0.16% | 94.7% |

Table 5: Robustness exercise - Optimal minimum wages by US region - Utilitarian weights

Notes: In the case of Utilitarian social welfare weights, column (11) gives gains due to efficiency and reports one minus the welfare gains with transfers (column 10) divided by the welfare gains with no transfers (column 9). Baseline refers to the single region benchmark calibration from Tables 1 and 2. See footnote 44 for states included in Low, Medium and High income regions.

regions.[43] We calibrate each region to data from three sets of US states, grouped by median household income, such that each region contains approximately one third of the civilian labor force.[44] Across regions, we keep some preference and technology parameters the same, as well as the distribution of number of firms in a market: $\{\beta, \theta, \eta, \delta, \alpha, \gamma, G(M_j)\}$. We calibrate region-specific shifters in labor disutility and productivity $(\overline{\varphi}_r, \overline{Z}_r)$, corresponding type-parameters $\{\overline{\varphi}_{kr}, \xi_{kr}\}_{k=1,r=1}^{K,R}$ and measures $\{\pi_{kr}\}_{k=1,r=1}^{K,R}$, to match CPS data from each region: the fraction of workers earning less than \$15 an hour, distribution of worker types, their relative average earnings per hour, and their share of region total labor income. Since the SCF does not identify an individual's state, we impose two further restrictions across regions. First, we keep the target moments for the ratio of household capital to labor income constant. Nonetheless, since other parameters change, we recalibrate the share parameters in each region $\{\kappa_{kr}\}_{k=1,r=1}^{K,R}$ to match the benchmark targets. Second, we keep constant across states the total fraction of *all households* that are owners.[45] We also assume average firm size is constant across regions which determines $\overline{\varphi}_r$. Columns 2 to 6 of Table 5 describe some of these moments. Relative to High income states, in Low income states the average wage is 16 percent lower, 7 percent more of the workforce has a wage below \$15 per hour, and 6.6 percent fewer of the workers have a college degree, with relatively similar proportion of workers that do not complete high school

**Results.** Table 5 provides the results of this exercise. We consider only Utilitarian weights, with and without lump sum transfers. In the absence of transfers, the optimal minimum wage varies very little, but since the welfare gains depend so strongly on redistribution, $\underline{w}^*$ follows the distribution of workers. In particular, $\underline{w}^*$ is slightly lower in regions with more non-high-school workers, whose utility declines most at higher minimum wages (Figure 10).

[43] In this economy, capital and consumption goods are traded at the same rental rate and price across all regions.

[44] States are allocated to regions as followed, ordered by 2019 median household income within each region: Low income states: MS, LA, NM, WV, AR, KY, AL, TN, GA, FL, OK, MT, MS, NC, SC, MI, SD. Medium income states: OH, WY, ID, IA, ME, IN, WI, TX, ND, RI, PA, AZ, NV, NY, CO, NE, KS, DE, VT. High income states: IL, OR, CA, AK, VA, MN, WA, UT, NH, CT, MA, NJ, HI, DC, MD.

[45] For example, if Group A has 37% of workers with a college degree, and Group B has 29%, then in both Group A and Group B we maintain that 6% of households are *college-owners* (Table 2) and set the share of households that are *college-workers* to 31% in Group A and 23% in Group B.

Turning on optimal lump sum transfers, we see more systematic variation in the optimal minimum wage which accords far more closely with our priors: lower $\underline{w}^*$ in low income states, and monotonically higher in high income states. With redistribution looked after, the region-government can push the minimum wage higher in high income states.[46] Despite this, the welfare gains are similar across states. Consistent with our baseline results, the efficiency gains—measured as welfare gains under optimal lump sum transfers—are similar and welfare gains under Utilitarian weights are mostly from redistribution.

An additional conclusion of this exercise (column 8) is that *absent additional redistributive policy*, and conditional on a set of welfare weights, the welfare losses from a national minimum wage are minimal relative to a set of region-specific minimum wages. Appendix Section A.5 describes welfare by worker type in each region $\{\lambda_{kr}(\underline{w})\}_{k=1,r=1}^{K,R}$, replicating Figure 10. Appendix Section A.6 describes region-level welfare $\{\Lambda_r(\underline{w})\}_{r=1}^{R}$ and national welfare $\Lambda(\underline{w})$, which is a weighted harmonic mean across regions. We compare aggregate welfare under $R$ optimal region minimum wages to aggregate welfare under one optimal national minimum wage. Under either Utilitarian or Negishi weights, welfare losses from a national minimum wage are small.

**Mississippi.** Mississippi (MS) has the lowest income per capita among the 50 U.S. states and a $15 minimum wage would bind for 41.3 percent of residents. There currently is no state-level minimum wage in Mississippi. *A priori*, one would expect Mississippi to be the most likely state to experience a welfare loss from a federal $15 minimum wage. However, we find that a federal minimum wage of $15 yields welfare gains in Mississippi, even after we recalibrate our model economy to match the fraction of workers earning less than $15 an hour (MS 41.3% vs. US 29.4%), distribution of worker types, their relative average earnings per hour, and each household types' share of total labor income.

Two off-setting forces lead to this result. Average wages are lower in MS, which would push toward a lower optimal minimum wage. However, despite the fact that only 21.2% of those in Mississippi have a college degree (versus 24.9% in the U.S.), there are significantly more individuals who have a high-school degree (63% in MS vs. 52.8% in U.S.). High-school graduates prefer a higher optimal minimum wage of roughly $18 (see Figure 10). These effects wash out and the optimal minimum wage is similar to that of the US as a whole.

These examples shed light on the relative stability of the optimal minimum wage. In states with fewer college educated, high-wage workers there are typically more high-school workers who actually prefer higher minimum wages. These offsetting compositional forces generate stable minimum wages across disparate regions.[47]

[46] We assume redistribution by lump sum taxes within, but not across, regions. This is equivalent to a balanced budget requirement at the region-level.

[47] As a proof of concept that the model can generate lower optimal minimum wages, Appendix Figure A10 shows that *counterfactually* large differences across regions in the moments used to calibrate the model can have significant effects on the optimal minimum wage. If we calibrate the model such that *65 percent of workers* have a wage less than $15, then the optimal minimum wage declines by about $3 under either Utilitarian or Negishi weights. This is counterfactual in that Mississippi has only 41.3 percent of workers at less than $15.

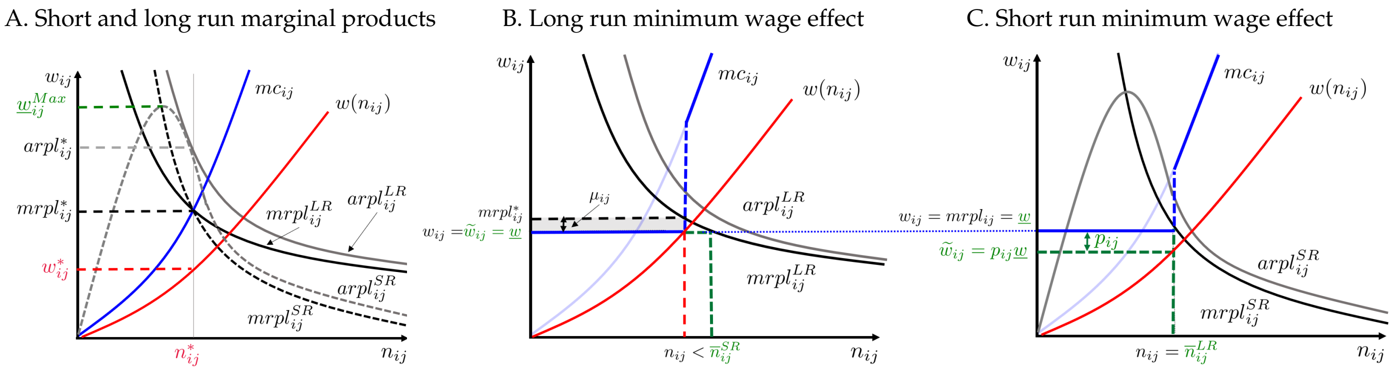


Figure 15: Partial equilibrium theory of minimum wage with capital fixed in the short-run

### 8.3 Sensitivity to short vs. long run

In comparing steady-states we are implicitly studying the long-run effects of the minimum wage. Our theory suggests a smaller optimal minimum wage in the short-run if the cost of labor increases but the level and distribution of capital across workers in each firm is slow to adjust. If we assume maximal stickiness in reallocation of capital, we find these effects are quantitatively small.

**Approach.** As we increase the minimum wage we keep capital at each firm for each type fixed at the allocation $\overline{k}_{ijk}$ under a zero minimum wage. We think of these as firm-worker specific installations of capital. Firm profits from each type are as follows, with three main implications:

$$\pi_{ijkt} = \overline{Z}\xi_k \left( z_{ij}\overline{k}_{ijk}^{(1-\gamma)\alpha} \right) n_{ijk}^{\gamma\alpha} - w_{ijk}n_{ijk} - R\overline{k}_{ijk}.$$

First, with fixed capital, the production function has sharper decreasing returns in labor: $\gamma\alpha < \widetilde{\alpha}$. Second, firms face overheard costs of pre-installed capital, $R\overline{k}_{ijk}$, which will cause firms to shut down non-profitable jobs at high minimum wages. This requires adding an endogenous margin of operation into the solution of the model.[48] Third, the aggregate equilibrium conditions are the same as in Lemma 3, minus the capital demand condition. Capital supply remains infinitely elastic at $R = 1/\beta + (1-\delta)$, but capital demand is pinned down at $\overline{K}(\underline{w}) = \sum_k \int \sum_i \chi_{ijk}(\underline{w})\overline{k}_{ijk}\, dj$, where $\chi_{ijk}(\underline{w}) \in \{0,1\}$ indicates whether the firm operates worker-type-$k$ capital in equilibrium under minimum wage $\underline{w}$.

**Theory.** Figure 15 characterizes the mechanism behind a lower optimal minimum wage in this environment. Panel A considers a firm in an economy without a minimum wage, where capital is fixed at the allocation consistent with long-run employment $n_{ij}^{*}$. Short-run marginal and average products coincide with long-run values at this point. Away from $n_{ij}^{*}$, short-run $mrpl_{ij}^{SR}$ is steeper due to sharper decreasing returns with fixed capital: if $n_{ij} > n_{ij}^{*}$, then $mrpl_{ij}^{SR} < mrpl_{ij}^{LR}$. With fixed overhead capital, the $arpl_{ij}^{LR}$

[48] Market-by-market we first assume that all firms enter, and then solve the Nash equilibrium of the market and general equilibrium of the economy. We then compute firm-type profits $\pi_{ijkt}$, which account for fixed capital costs. If any firm has profits $\pi_{ijkt} < 0$, we drop the lowest productivity firm in the market and then solve the market equilibrium again. With fewer firms, labor market power of the remaining firms increases, which increases profits, hence the need to remove only one firm at a time. We continue in this way until we reach a Cournot Nash equilibrium: no firm with shut-down jobs wishes to re-open them given competitor's operation and intensive margin labor decisions. This general equilibrium algorithm is similar to that in Loecker, Eeckhout, and Mongey (2021).

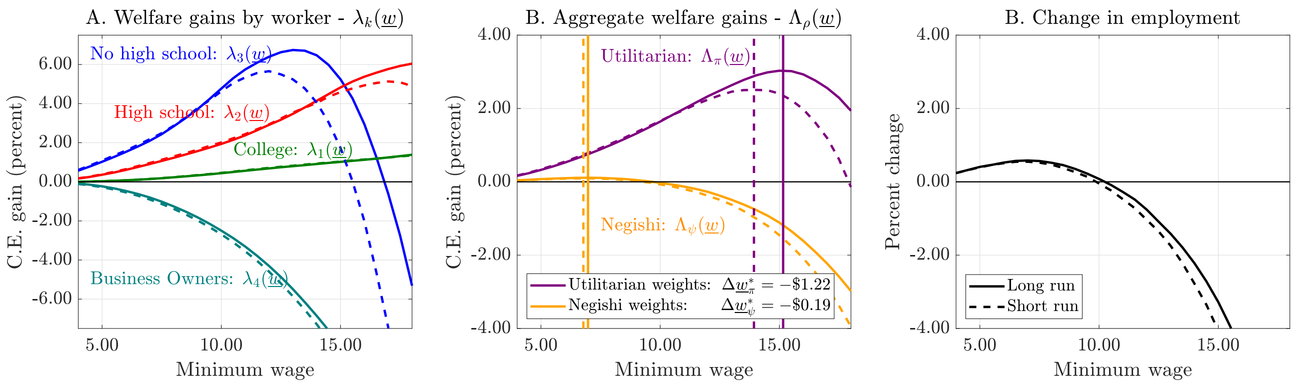


Figure 16: Short- and long-run effects of minimum wages

Notes: Long-run results, which are identical to Figure 10, in solid lines, short-run results in dashed lines. Legend in Panel B gives the decline in the optimal minimum wage when comparing short-run to long-run.

goes to zero as $n_{ij}$ goes to zero and overhead per worker explodes. The peak in $arpl^{SR}_{ij}$ intersects $mrpl^{SR}_{ij}$ and gives the maximum minimum wage the firm could afford and still operate type-$k$ capital: $\underline{w}^{Max}_{ij}$. At $\underline{w} > \underline{w}^{Max}_{ij}$, equating $\underline{w} = mrpl^{SR}_{ij}$ would imply $arpl^{SR}_{ij} < \underline{w}$ and shutdown is optimal.

Panels B and C show how these differences constrain the positive efficiency gains from narrowing $\widetilde{\mu}_k$. Take the firm in Panel A, in the long run, at the minimum wage pictured in Panel B, the firm is in Region II: employment is non-rationed $(n_{ij} < \overline{n}^{SR}_{ij})$, and wages are a narrower markdown on $mrpl^{LR}_{ij}$. A small increase in the minimum wage *increases employment* and *narrows shadow markdowns*. Panel C considers the short run, at the same minimum wage. The lower $mrpl^{SR}_{ij}$ places the firm in Region III, where employment is rationed. A small increase in the minimum wage *decreases employment* and *widens shadow markdowns*. In the short run, the range of $\underline{w}$ over which firms are in Region II is smaller. This constrains the efficiency gains from improvements in $\widetilde{\mu}_k$.

**Results.** Figure 16 plots the results. Panel B shows that the short-run optimal minimum wage under Utilitarian weights declines by about one dollar. Consistent with the theory, this is driven by a sharp decline in the welfare gains to non-college households, due to a smaller increase in $\widetilde{\mu}_k$. With capital unable to adjust, welfare losses to business owners are also slightly larger. Panel C shows that the positive employment effects of the minimum wage are slightly more limited. This exercise delivers the additional result that, quantitatively, short- and long-run elasticities in our model are similar which is reassuring for our earlier interpretations of empirical studies of short-run changes.

### 8.4 Incomplete markets

In our baseline model with limited household heterogeneity, the optimal federal minimum wage with transfers ranges from \$7 to \$10, very close to the current federal minimum wage of \$7.25. While our model only features market incompleteness *across* types, our framework can readily incorporate more types given the data necessary to discipline additional parameters. In previous iterations of this paper (available upon request), we treated high-school graduates and non-high school graduates as a single household type which could fully insure against employment losses *within* the household. We found a very similar efficiency maximizing minimum wage. As shown in the previous section, the efficiency maximizing minimum wage in an economy with heterogeneity is very similar to the efficiency maximizing minimum wage in an economy with no heterogeneity, where social welfare weights play no role. Adding more household heterogeneity alters redistributive motives but provides limited pressure on the efficiency maximizing minimum wage.

In on-going work, we relax the unitary households assumptions and incorporate oligopsony into a heterogeneous agent economy with search frictions Berger, Herkenhoff, Kostol, and Mongey (2022). These modifications pose significant computational challenges that require us to abstract from capital, wealth effects, and a number of key forces that shape the optimal minimum wage in the present setting.

# 9 Conclusion

This paper provides a theoretical framework for studying the effect of minimum wages on welfare and the allocation of employment across firms in the economy. The framework has three key features. First, each market features strategic interaction between firms, which we have shown to be important for (i) quantifying the reallocation effects of minimum wage policies, (ii) interpreting empirical evidence documenting such reallocation, and (iii) interpreting empirical evidence on employers' responses to competitors' minimum wages. Second, workers are of heterogeneous types, which allows us to decompose the heterogeneous impacts of minimum wages on employment, labor and capital income, and account for general equilibrium wealth effects. Third, we provide a parsimonious nesting of this market model into a general equilibrium economy and show how the economy aggregates, allowing for a succinct representation of the efficiency improvements and costs of minimum wages via *shadow markdown* $\widetilde{\mu}$, and misallocation $\omega$. When calibrated to US data, this model is consistent with a wide body of empirical research on the effects of minimum wage changes (recall additional exercises in Appendix B).

In such an economy we find that an optimal minimum wage exists. Quantitatively, we find that the efficiency maximizing minimum wage is around \$8 per hour, but that higher minimum wages can be justified through redistribution. Under Utilitarian social welfare weights, and ignoring alternative fiscal instruments for redistribution, we find an optimal minimum wage of around \$15 an hour. Under such a policy, 95 percent of welfare gains come from redistribution and only 5 percent from improved efficiency, with welfare gains equivalent to a 3 percent increase in TFP.

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# Appendix

This Appendix is organized as follows.

- Section A provides additional tables and figures referenced in the text.
- Section B provides additional validation exercises on (i) spillovers up the worker wage distribution, (ii) heterogeneity in employment responses by market concentration, (iii) disemployment effects of minimum wage increases from initially high minimum wages.
- Section C details the algorithm for solving the minimum wage economy.
- Section D contains derivations associated with our calibration of preference parameters.
- Section E contains proofs for the simplified monopsony and oligopsony economies from Section 2.
- Section F contains mathematical derivations for the full quantitative model from Section 3 and derivations associated with the solution of the government problem and its implementation via lump sum transfers.
- Section G studies a search model with frictional rationing that closely resembles our model.

# A Additional tables and figures

## A.1 Distribution of activity across regions

Figure A1 plots the fraction of production units (e.g. a firm-worker-type pair), and fraction of employment in each of the three Regions described in Section 3.

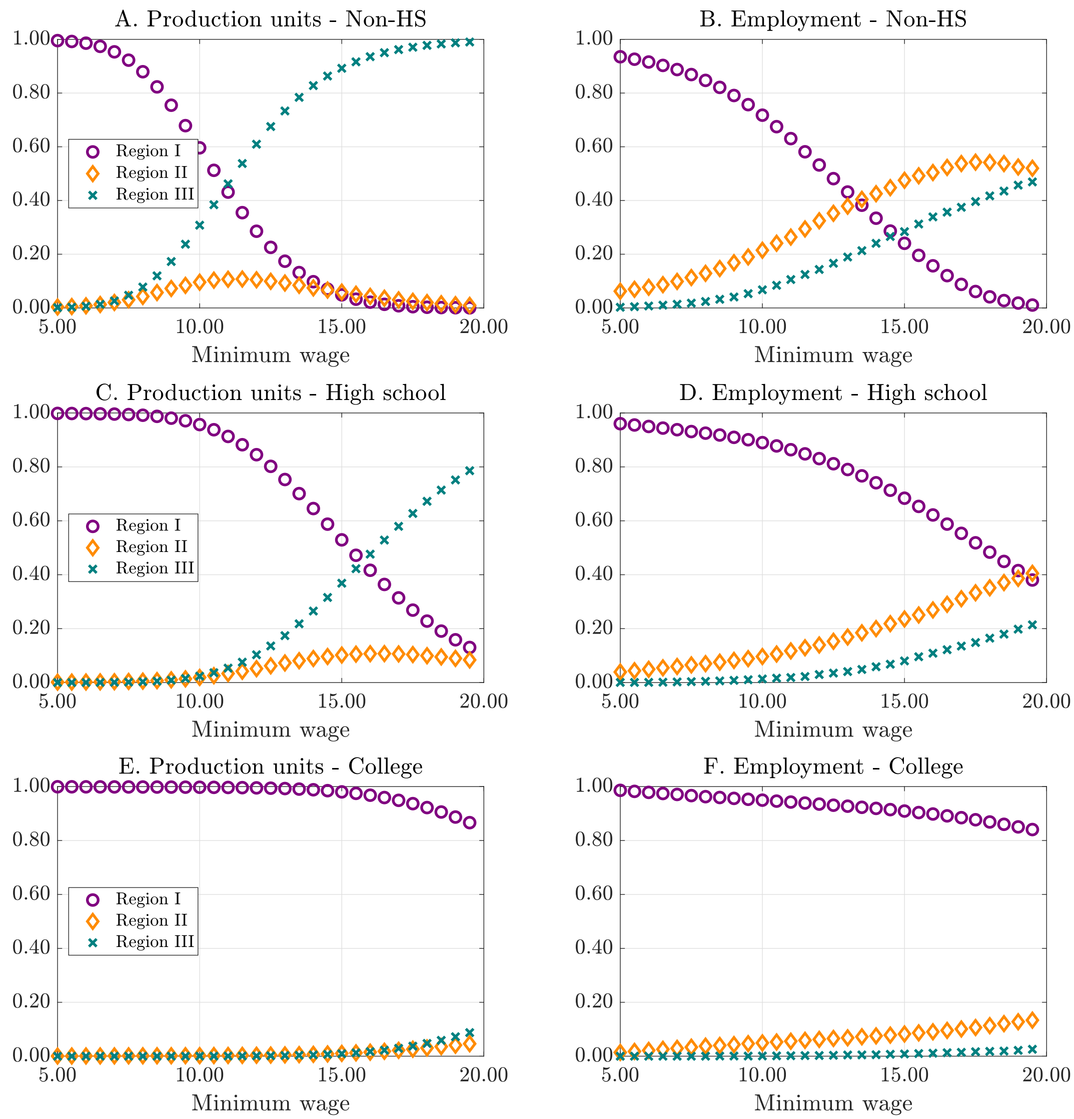


Figure A1: Distribution of positions and employment across regions - High school workers

Notes: A production unit is a firm-worker-type pair. For example, firm $i$ in market $j$ employs all four types of labor, and for each type of labor it may either be unconstrained by the minimum wage (Region I), constrained by the minimum wage but on its labor supply curve (Region II), constrained by the minimum wage but on its labor demand curve (Region III).

## A.2 Alternative calibration of worker-type productivity parameters $\{\xi_k\}_{k=1}^{K}$

Figures A2 and A3 give results for an alternative calibration of the model in which the the three values of $\xi_k$ are chosen to match the *average wage of each worker type*, and we normalize $\overline{Z} = 1$. Figures A2B compares the distribution of wages under this calibration to the baseline (thin lines). For all workers (black) and for non-high school workers (blue) the fraction of workers with a wage less than \$15 shifts from being higher than what is observed in the data (under the baseline), to less than what is observed in the data (under the alternative calibration). Nonetheless, Figure A3 shows similar magnitudes of welfare gains from the optimal minimum wage (compare to Figure 10). Moreover, the welfare gains associated with efficiency—which we have shown are consistent with welfare gains under the Negishi weights with no lump sum transfers—remain small and the optimal minimum wage reflecting efficiency is less than \$9.

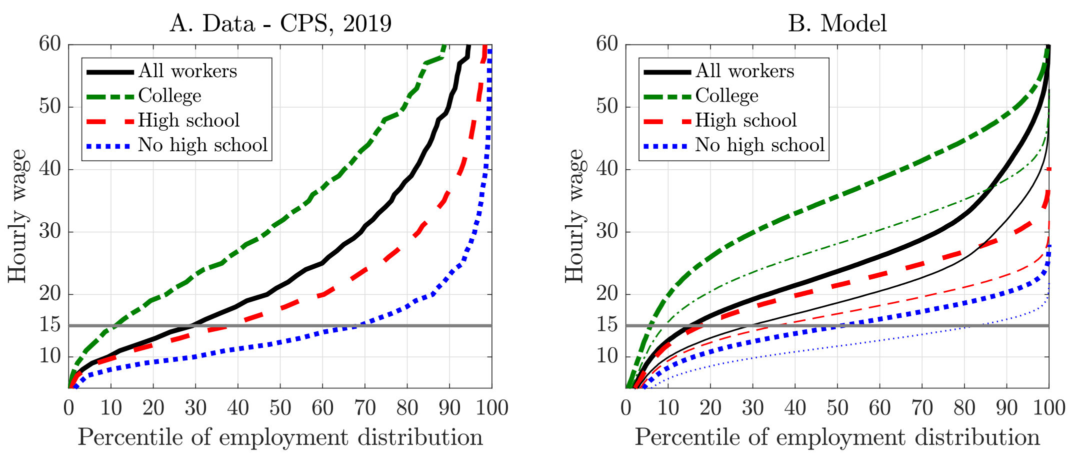


Figure A2: Distribution of wages by worker type and \$15 minimum wage - Alternative calibration

Notes: For Panel A see footnotes to Figure 5. Thick lines in Panel B refer to the alternative calibration of the model (see text). Thin lines in Panel B refer to the baseline calibration in the text, in which the two values of $\xi_k$ are chosen to match the ratio of average college-to-non-high-school and college-to-high-school worker wages and $\xi_k$ for college workers is normalized to one, and $\overline{Z}$ is chosen to match the fraction of workers for which a \$15 minimum wage would bind.

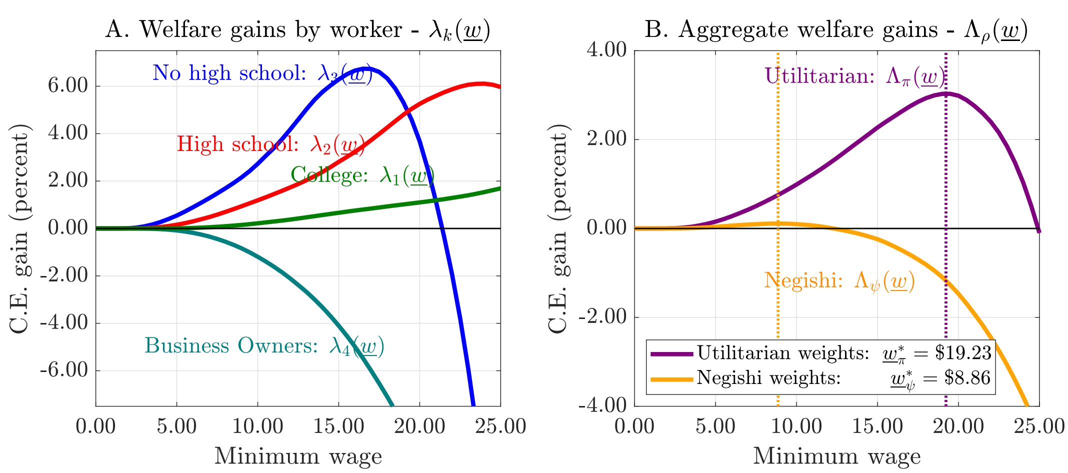


Figure A3: Minimum wages and welfare - Alternative calibration

Notes: For details see footnote to Figure 10.

### A.3 Effective social welfare weights for alternative minimum wage benchmarks

In the main text we compute the welfare gains relative to a zero minimum wage economy. This implies that the social welfare weights $\psi_k$ are benchmarked to a zero minimum wage economy. For example, when using *Negishi weights*, then the welfare weights can be shown to be equal to consumption shares in the benchmark (i.e. zero minimum wage economy): $\psi_k = \psi_k^*(0)$. Figure A4 shows that our findings are robust to alternative benchmarks since, quantitatively, the effective weights move very little with the minimum wage. Panel A plots the implied Negishi weights $\psi_k^*(\underline{w})$, and shows that they vary little as the minimum wage changes, while in Panel B, Utilitarian weights are constant. Panels C and D show how these map into the *effective weights*, described below, which are also relatively flat.

Recall the main formula from the text, but now written to reflect that the welfare gains in an economy with minimum wage $\underline{w}_1$ are being computed with respect to a benchmark economy indexed by a minimum wage $\underline{w}_0$:

$$1 + \Lambda(\underline{w}_1) = \left[\sum_k \widetilde{\psi}_k(\underline{w}_0)\,(1+\lambda_k(\underline{w}_1))^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \quad , \quad \widetilde{\psi}_k(\underline{w}_0) := \frac{\psi_k(\underline{w}_0)\left(\frac{c_k(\underline{w}_0)}{\pi_k}\right)^{1-\sigma}}{\sum_k \psi_k(\underline{w}_0)\left(\frac{c_k(\underline{w}_0)}{\pi_k}\right)^{1-\sigma}}. \tag{A1}$$

For different $\underline{w}_0$, Figure A4 plots in panels A and B the underlying weights $\psi_k(\underline{w}_0)$ for A. Negishi weights, and B. Utilitarian weights. Panels C and D plot the *effective weights* $\widetilde{\psi}_k(\underline{w}_0)$ for C. Negishi weights, and D. Utilitarian weights. The main conclusion is that $\widetilde{\psi}_k(0)$, which are the effective weights used as the benchmark in the text, are very similar to $\widetilde{\psi}_k(\underline{w}_0)$ for alternative benchmark minimum wages $\underline{w}_0 \in [0, 20]$.

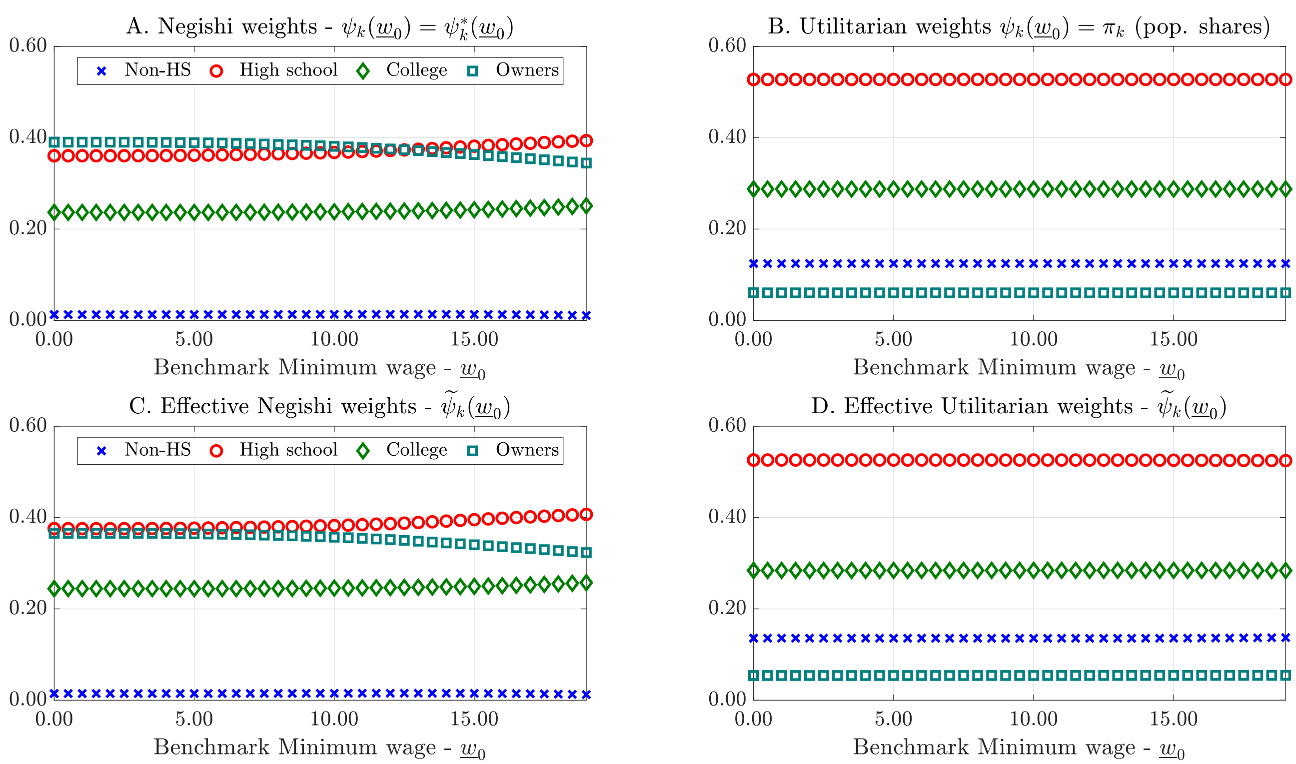


Figure A4: Negishi and Utilitarian weights as the minimum wage varies

## A.4 Alternative calibration of heterogeneity - Homogeneous worker calibration

Our baseline model has four types of workers, here we show results for an alternative specification with only one type of worker. The parameters $\overline{Z}$, $\overline{\varphi}$ are still calibrated to match the same baseline targets as the benchmark model: 29 percent of workers earn less than \$15 in the initial economy, and average firm employment is 22.8 workers. We compare aggregate welfare and the decomposition of welfare for this homogeneous worker economy to the benchmark economy under the implied Negishi weights.

Figure A5 plots the results. The orange lines are those for the benchmark economy with heterogeneous workers, and are identical to the orange lines in Figure 10B and 10D. The grey lines are those of the homogeneous worker economy. The main result is that the welfare gains from the minimum wage are similar in shape and size in the homogeneous worker economy and the heterogeneous economy under the implied Negishi weights. They also imply optimal minimum wages that are within one dollar of each other. This further reflects the extent to which minimum wages that are more than \$10 are welfare improving precisely due to their interaction with the heterogeneity in the economy via redistribution, rather than through efficiency. We view this as a further check on our efficiency results.

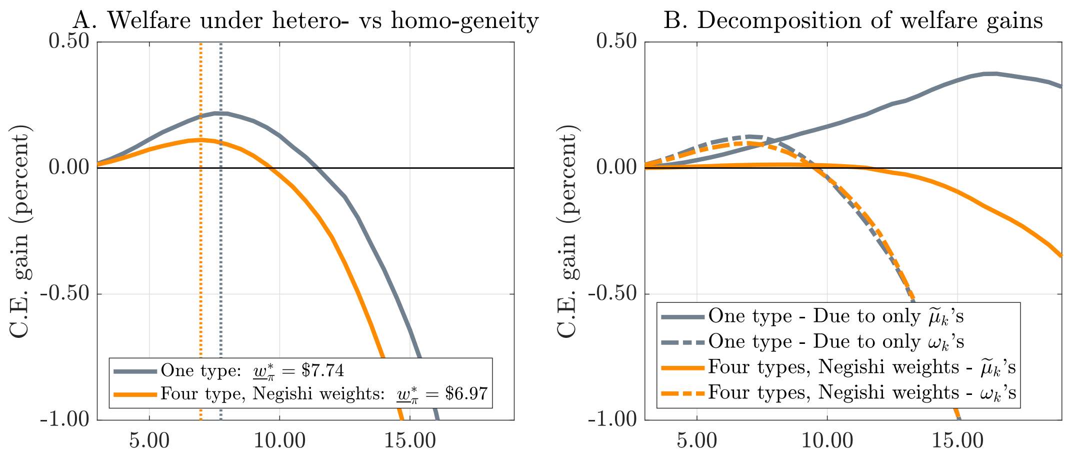


Figure A5: Minimum wages and welfare - Homogeneous vs. Heterogeneous households

Notes: For details see text.

## A.5 Within region welfare results for heterogeneous state calibration

Figure A6 to A8 plot within-region welfare comparative statics and region-specific optimal minimum wages.

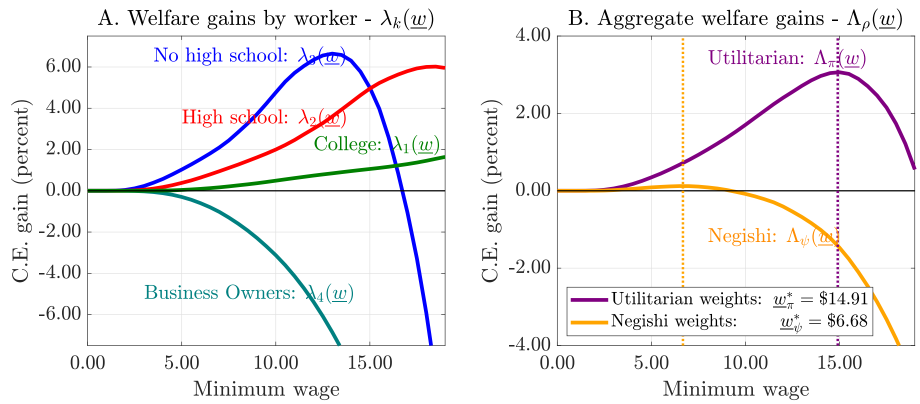


Figure A6: Low income states

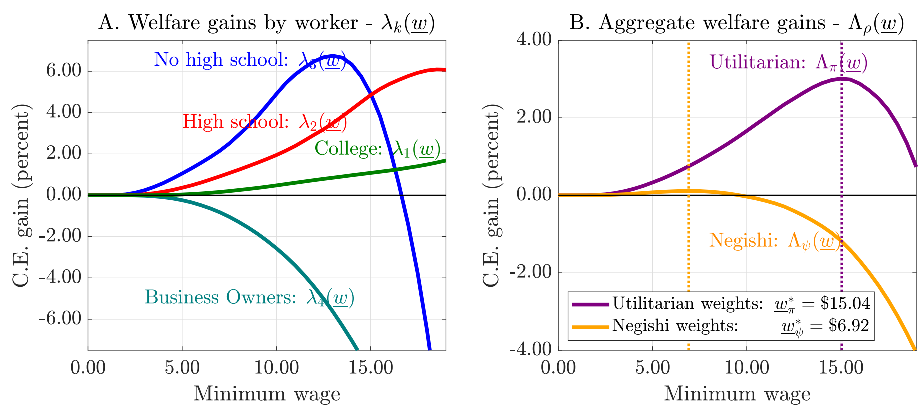


Figure A7: Medium income states

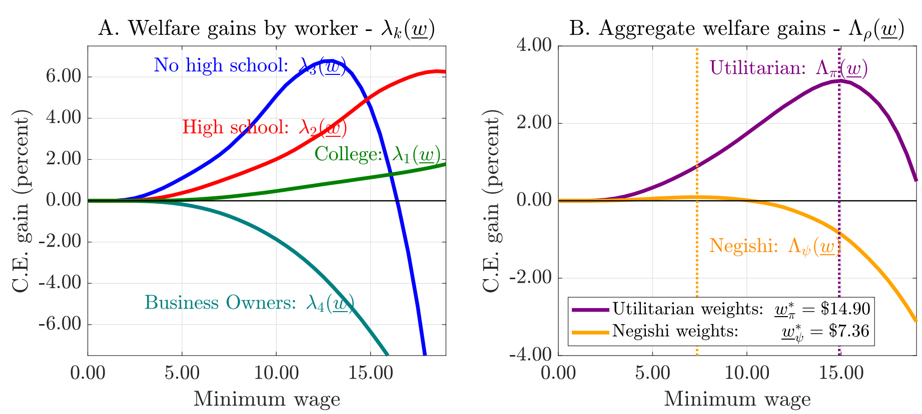


Figure A8: High income states

## A.6 Welfare gains under region-specific and national minimum wage

In Section 8 we computed optimal minimum wages in a set of three regions. Figure A9 plots the welfare gains in each region, along with aggregate welfare gains. Aggregate welfare gains are simply the harmonic mean of region welfare gains, since each region accounts for one third of the total population. The red crosses in each panel denote the *average of region-specific optimal minimum wages*, these are the minimum wages that attain the maximum of welfare in each state, and the aggregate welfare associated with the region-optimal minimum wages. The black line gives aggregate welfare under a *national minimum wage*, and the black circle gives the associated welfare maximizing minimum wage and welfare. In both cases the aggregate welfare gains associated with region specific optimal minimum wages are two orders of magnitude less than the level. Under utilitarian weights, the welfare gains from a national minimum wage are 3.059 percent, and 3.060 percent under region-specific minimum wages. Under Negishi weights, the welfare gains from a national minimum wage are 0.108 percent, and 0.109 percent under region-specific minimum wages.

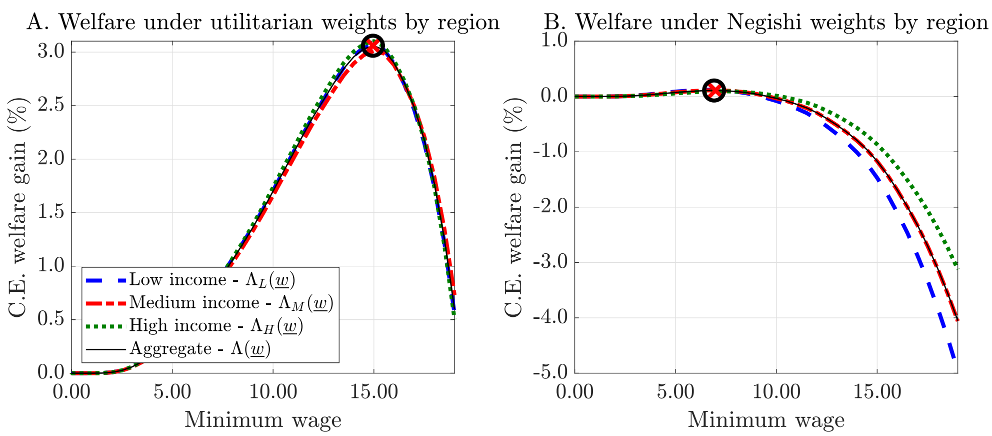


Figure A9: Minimum wages and welfare in a multiregion calibration

Notes: For details see text.

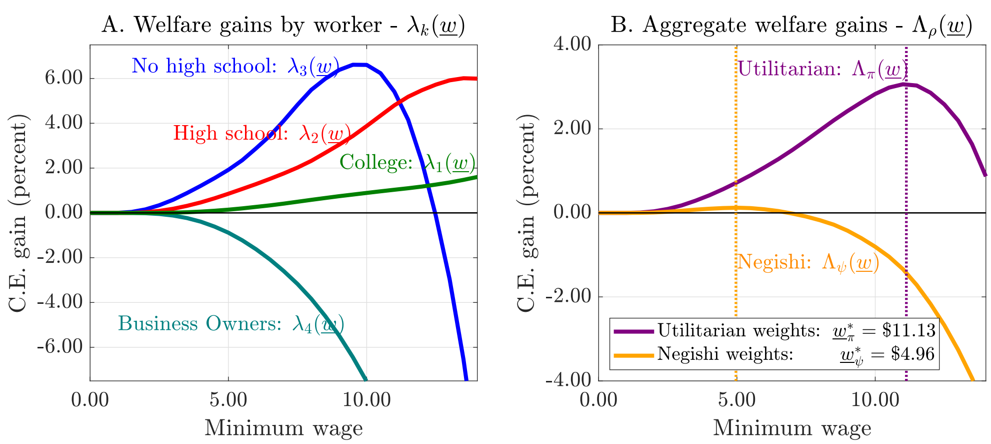


Figure A10: Counterfactual calibration such that 65 percent of workers initially earn less than \$15 per hour

Notes: For details see text.

# B Additional validation exercises

In the main text we document that spillovers of wages across firms are consistent with the data. In this appendix we first document that our model generates empirically reasonable measures of spillovers from the perspective of workers. Second, since our paper contributes a model of minimum wages in concentrated markets, we also show that the model generates heterogeneity in employment responses to the minimum wage by market concentration consistent with recent evidence. Third, we replicate the Seattle minimum wage increase studied in Jardim et. al. (2022).

## B.1 Spillovers

**Empirical setting.** Rather than consider evidence from short-run employment responses to small minimum wage changes in the US (Autor, Manning, and Smith, 2016), where spillovers are observed up to around the 20th percentile, we consider recent leading empirical studies of the large minimum wage increase over two decades in Brazil: Engbom and Moser (2021), Haanwickel (2020).[49] Both papers follow the procedure of Autor, Manning, and Smith (2016). We focus on Engbom and Moser (2021) since the paper contains additional summary statistics that aid our replication. They compute that in 1996 the minimum wage was 34 percent of the median wage, and then increased by 119 percent between 1996 and 2012 (Engbom and Moser, 2021, page 11). To replicate this experience we solve our economy under a minimum wage of \$6.34, which is 34.9 percent of the median wage, and then increase it to \$14.23 which is a 119 percent increase. We denote these period zero and period one.

**Statistic.** Let $\overline{p}$ be a reference percentile of the wage distribution, and let $w_{p,t}$ be the percentile $p$ wage in period $t$. We compute spillovers at $p$ by

$$Spillover_p = \frac{\log(w_{p,1}/w_{\overline{p},1}) - \log(w_{p,0}/w_{\overline{p},0})}{\log(\underline{w}_1/w_{\overline{p},1}) - \log(\underline{w}_0/w_{\overline{p},0})} \tag{A1}$$

By construction $Spillover_{\overline{p}} = 0$. If wages below $\overline{p}$ compress upward, then $Spillover_p > 0$. If wages above $\overline{p}$ compress upward, then $Spillover_p < 0$. Engbom and Moser (2021) use a regression framework to obtain estimates of $Spillover_p$, whereas we simply compute $Spillover_p$ non-parametерically via (A1).[50] We compute results for non-High school workers. As shown by Engbom and Moser (2021, Figure A2), as far up as at the 70th percentile of the earnings distribution more than 80 percent of workers have not completed high school in Brazil.[51]

**Results.** Figure B1 plots $Spillover_p$ for $p \in [10, 12, \ldots, 90]$ and compares estimates to those from Engbom and Moser (2021, Figure 4) for the case where reference percentiles are $\overline{p} = 50$ (panel A) and $\overline{p} = 90$ (panel B).[52] We find very similar patterns of spillovers, with compression far up into the wage distribution. Again, we find that the model is consistent with key empirical facts that arise in the discussion of minimum wages.

[49] See the former, Figure 4, and the latter Figure 8.

[50] Note that if one wanted to estimate $Spillover_p$ at percentile $p$ via regression, its a regression of $\Delta Gap_{p,t}$, where $Gap_{p,t} = \log w_{p,t} - \log w_{\overline{p},t}$, on the commonly named 'Kaitz' index $\Delta Kaitz_{p,t}$, where $Kaitz_{p,t} = \log \underline{w}_t - \log w_{\overline{p},t}$, which measures the 'bite' of the minimum wage. Estimating this $p$-by-$p$ would be excessively demanding of the data, so in practice this is implemented parametерically, with instruments for $Kaitz_{p,t}$. For details see Engbom and Moser (2021, Section 3.2).

[51] To clarify: take all workers in the 70th percentile of the earnings distribution. Of these workers, more than 80 percent had not completed high school. Another statistic that reflect this is as follows: at the 90th percentile of the earnings distribution, around 90 percent of workers do not have a college degree.

[52] We compare our results to their IV specification that controls for state-level trends and state fixed effects. This delivers similar results to their specification with state-level fixed effects only.

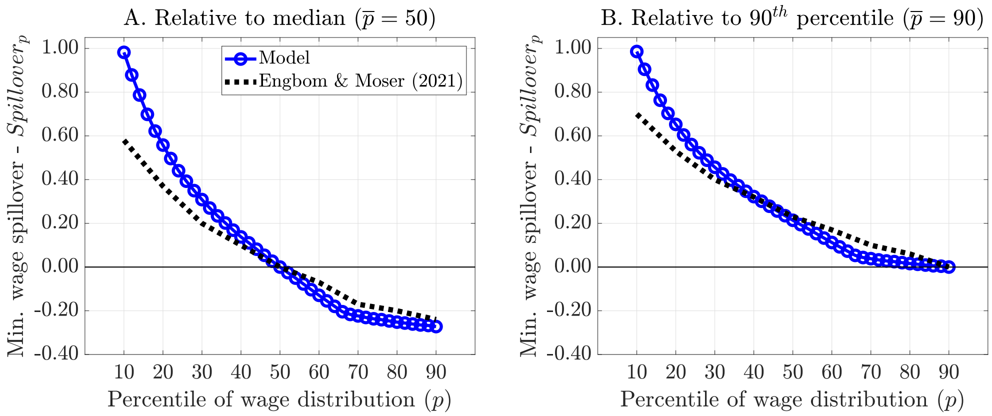


Figure B1: Common measure of spillovers from a large increase in the minimum wage

Notes: Consistent with the minimum wage in Brazil in 1996, the initial minimum wage is 34 percent of the median wage. Consistent with the minimum wage increase in Brazil from 1996 to 2012, the minimum wage increases by 119 percent. These statistics are reported in Engbom and Moser (2021, page 12). We compare model results to those of Engbom and Moser (2021), Figure 4, under the State Fixed Effects plus IV specification.

## B.2 Heterogeneity of employment effects by market concentration

**Empirical setting.** Azar, Huet-Vaughn, Marinescu, Taska, and von Wachter (2019) compute the response of employment in low wage occupations to changes in state minimum wages, but stratify responses by the concentration of the labor market for each occupation. They estimate statistically significant positive effects in markets in the upper tercile of concentration, and statistically significant negative effects in markets in the lower tercile of concentration. We show that the same results hold in our economy.[53]

**Statistic.** Holding aggregates fixed, we increase the minimum wage by fifty cents, compute the increase in employment in each market $j$, and regress the change in market employment $\Delta \log n_j$ on the change in the minimum wage $\Delta \log \underline{w}$, interacted with dummies for the tercile of market concentration $\overline{hhi}_j^n$:[54]

$$\Delta \log n_j = \psi_1 \Delta \log \underline{w} + \sum_{k=2}^{3} \psi_k \mathbf{1}[\overline{hhi}_j^n \in \text{Tercile}_k] \times \Delta \log \underline{w}.$$

This is equivalent to the main specification with market fixed effects estimated in the paper, but where we have computed concentration in employment as opposed to in job postings, which is observed in their study. In their sample, the average pre- and post-policy minimum wages are \$7.43 and \$7.83.[55] Given the focus of Azar, Huet-Vaughn, Marinescu, Taska, and von Wachter (2019) on low wage jobs (Stock Clerks, Retail Sales, and Cashiers), we compute market $hhi_j^n$ and $n_j$ using non-Highschool and Highschool workers. To understand potential heterogeneity by the level of the initial minimum wage, we repeat this exercise for initial minimum wages $\underline{w}_0$ between \$2 and \$10 per hour.

**Results.** Figure B2 plots the estimated coefficients for low ($\widehat{\psi}_1$) and high ($\widehat{\psi}_1 + \widehat{\psi}_3$) concentration markets, holding the increase in the minimum wage constant (50c), but varying the initial minimum wage $\underline{w}_0$ on the horizontal

[53] We are unable to replicate their study directly and conduct a quantitative comparison due to concentration being computed using Burning Glass data on job openings.

[54] As in their paper, the measure of concentration we use to determine terciles is the average of concentration pre- and post-policy change

[55] We thank the authors for sharing these two moments with us.

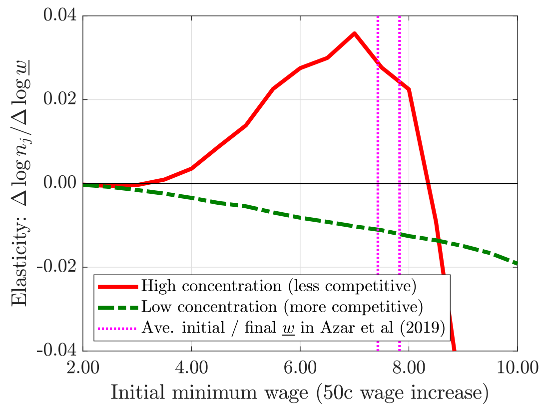


Figure B2: Effect of a minimum wage increase on employment, by concentration of labor market

Notes: Horizontal axis gives the initial minimum wage $\underline{w}_0$. The minimum wage is then increased by 50 cents. Red solid line plots estimated elasticity in high concentration markets ($\widehat{\psi}_1 + \widehat{\psi}_3$). Green dashed line plots estimated elasticity in low concentration markets ($\widehat{\psi}_1$).

axis. For initial minimum wages consistent with the settings the paper studies, i.e. less than \$8.00 per hour, the model is consistent with its key empirical findings. High concentration markets see large, positive, employment effects, and low concentration markets see small negative employment effects. Low productivity firms in more concentrated markets have more market power, wider markdowns, and hence have larger positive employment gains available in Region II before shrinking in Region III. In less concentrated markets these firms have initially narrow markdowns and move quickly into Region III, incurring employment losses.

### B.3 Disemployment effects of a high minimum wage on low wage jobs.

**Empirical setting.** Jardim et. al. (2022) study the minimum wage increases in Seattle in 2015 and 2016. These are useful benchmarks as (i) they are minimum wage increases from initially high minimum wages, (ii) the authors have access to hours data which most closely maps into our model concept of $n_{ijk}$ from an efficiency perspective since it is the object that enters production. The authors study two minimum wage increases: *"The minimum wage rose from the state's minimum of \$9.47 to as high as \$11 on April 1, 2015, and again to as high as \$13 on January 1, 2016"* (page 266, and Table 1). The authors compare firms in Seattle to those in Washington state, and compute the elasticity of employment in jobs that pay less than \$19 per hour, which account for 63 percent of the workforce (page 269, and Table 2). In Tables 6A and 6B the authors present estimates of causal effects in percent changes on wages and hours. Their results vary across specifications. We summarize them as ranges via their text:

1. **Wages** - We associate the first minimum wage increase with wage effects of 1.1 to 2.2 percent, averaging 1.7 percent, the second increase is associated with a larger 3.0 to 3.9 percent, averaging 3.4 percent wage effect. (page 290)
2. **Hours** - Point estimates for the \$11 period range between +0.8 and -2.7 percent, averaging -1.0 percent, the subsequent increase to \$13 is associated with larger reductions between 4.6 and 9.9 percent, averaging -7.0 percent. (page 292-3)

**Replication.** Our economy is calibrated to 2019, so we first deflate all wages to 2015 levels at 1.55 percent inflation using the 2015-2019 CPI. We take an economy with a minimum wage of \$9.47 to match the pre-2015 baseline. We then consider a \$1.53 minimum wage increase, corresponding to the first raise, and \$2 minimum wage increase corresponding to the second raise. We keep all jobs of all worker types that had a pre-policy wage

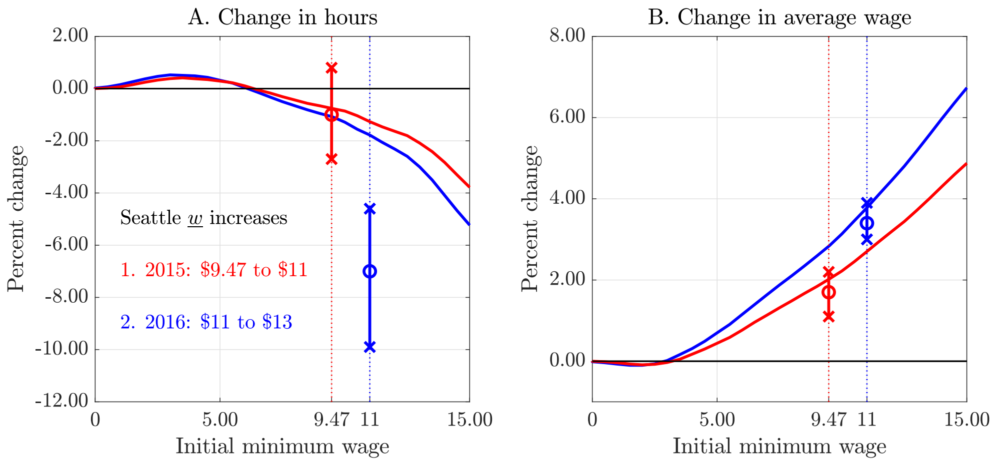


Figure B3: Disemployment effects on low wage employment from high initial minimum wages - Seattle
Notes: Vertical lines denote the range of point estimates described by the authors in Jardim et. al. (2022), see main text of this section.

less than $19.85, to match the 63 percent of employment in the study, which had a very similar cut-off of $19. We then compute the percent change in total employment—which corresponds to hours in their study—and the average wage. The benefit of the model is that we can conduct this for multiple initial minimum wages. We do not recalibrate any other parameters to Seattle data.

**Results.** Figure B3 presents our results. The vertical lines denote the forementioned ranges of point estimates and average estimate from the authors. The horizontal axis plots the initial minimum wage. The red line plots percent changes in hours and average wage following a $1.53 increase in $\underline{w}$, and the blue line following a $2.00 increase.

First, consistent with the authors we obtain negative effects on hours and positive effects on wages. Second, the model has similar non-linear employment effects as found in the data. Effects on hours are small following the first increase, and large following the second increase. The model understates the large negative effects on the second increase found in Seattle, but would obtain similar estimates from a $2 increase from $13 to $15 per hour. Third, the increase in average wages is also larger on the second increase, with wage changes very close to the authors' empirical estimates.

These results give us confidence that the non-linearities in the model observed in our welfare exercises are consistent with the data, and kick in at the empirically relevant range of minimum wages, around $10 to $13 per hour.

# C Algorithm for the minimum wage economy

The aim of this section is to clearly lay out the algorithm for solving the minimum wage equilibrium, and to present a full solultion of a simplified model, which may be pedagogically useful relative to the extensive derivations in Appendix D. The algorithm for the minimum wage equilibrium is nested in the broader solution to the equilibrium of the model described in Appendix F.

For ease of exposition, we lay out the minimum wage problem (i) ignoring capital, (ii) consider an economy with a single type of household, (iii) to simplify exposition we also consider GHH preferences, which are not used in the main text, (iv) as well as a static environment, (v) set the coefficient on labor in utility $\overline{\varphi} = 1$. We derive conditions for this simplified economy and then present the algorithm.

## C.1 Model

- Consider the household problem with the rationing constraint $n_{ij} \leq \overline{n}_{ij}$. For ease of interpretation we attach multiplier $\zeta_{ij} = \lambda w_{ij} \left(1 - p_{ij}\right)$ to the rationing constraint, normalized by the household budget multiplier $\lambda$:

$$U_0 = \max_{\left\{n_{ij}, c_{ij}\right\}} u \left( C - \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right)$$

$$\begin{aligned}
C &= \int \sum_{i \in j} w_{ij} n_{ij} \, dj + \Pi \qquad [\lambda] \\
n_{ij} &\leq \overline{n}_{ij} \qquad [\lambda w_{ij} \left(1 - p_{ij}\right)] \\
C &= \int \sum_{i \in j} c_{ij} dj \\
N &= \left[ \int n_j^{\frac{\theta+1}{\theta}} dj \right]^{\frac{\theta}{\theta+1}} \\
n_j &= \left[ \sum_{i \in j} n_j^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}
\end{aligned}$$

- The first order condition for $n_{ij}$ yields

$$\begin{aligned}
\lambda w_{ij} - \lambda w_{ij} \left(1 - p_{ij}\right) &= u'(\cdot) \left( \frac{\partial n_j}{\partial n_{ij}} \right) \left( \frac{\partial N}{\partial n_j} \right) N^{\frac{1}{\varphi}} \\
\lambda w_{ij} p_{ij} &= u'(\cdot) \left( \frac{\partial n_j}{\partial n_{ij}} \right) \left( \frac{\partial N}{\partial n_j} \right) N^{\frac{1}{\varphi}}
\end{aligned}$$

- The first order condition for consumption yields $u'(\cdot) = \lambda$.

- Define the *shadow wage* $\widetilde{w}_{ij} = p_{ij} w_{ij}$, use the first order condition for consumption $u'(\cdot) = \lambda$, and use the derivatives of $N$ and $n_j$:

$$\widetilde{w}_{ij} = \left( \frac{n_{ij}}{n_j} \right)^{\frac{1}{\eta}} \left( \frac{n_j}{N} \right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \qquad (*)$$

- Now define the *shadow wage indexes*

$$\widetilde{w}_j = \left[\sum_{i\in j} \widetilde{w}_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{W} = \left[\int \widetilde{w}_j^{1+\theta} dj\right]^{\frac{1}{1+\theta}} .$$

- Using these definitions in $(*)$ along with the definition of $n_j$:

$$\sum_{i\in j} \widetilde{w}_{ij}^{1+\eta} = \left[\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right]^{1+\eta} \sum_{i\in j} \left(\frac{n_{ij}}{n_j}\right)^{\frac{1+\eta}{\eta}}$$
$$\widetilde{w}_j = \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$

- Using this along with the definition of $N$:

$$\int \widetilde{w}_j^{1+\theta} dj = \left[N^{\frac{1}{\varphi}}\right]^{1+\theta} \int \left(\frac{n_j}{N}\right)^{\frac{1+\theta}{\theta}} dj$$
$$\widetilde{W} = N^{\frac{1}{\varphi}}$$

- Note that $\widetilde{W}N \neq \int \sum_{i\in j} w_{ij} n_{ij} dj$, however the aggregate labor supply $N = \widetilde{W}^{\varphi}$ is *as if*, the household had maximized

$$U_0 = \max_{C,N} u \left(C - \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) \quad \text{subject to} \quad C = \widetilde{W}N + \Pi.$$

  This makes clear the extent to which the shadow wage index $\widetilde{W}$ captures the full distribution of binding minimum wages.

- Note that shadow wages aggregate:

$$\widetilde{w}_{ij} n_{ij} = n_{ij}^{\frac{1+\eta}{\eta}} \left(\frac{1}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$
$$\sum_{i\in j} \widetilde{w}_{ij} n_{ij} = \left[\sum_{i\in j} n_{ij}^{\frac{1+\eta}{\eta}}\right] \left(\frac{1}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$
$$\sum_{i\in j} \widetilde{w}_{ij} n_{ij} = n_j \widetilde{w}_j$$

- **Shadow shares** - We can define the shadow share $\widetilde{s}_{ij}$ as

$$\widetilde{s}_{ij} := \frac{\widetilde{w}_{ij} n_{ij}}{\sum_{i\in j} \widetilde{w}_{ij} n_{ij}}.$$

Substituting in the labor supply system $(*)$ for $\widetilde{w}_{ij}$

$$\widetilde{s}_{ij} := \frac{n_{ij}^{\frac{1+\eta}{\eta}}}{\sum_{i\in j} n_{ij}^{\frac{1+\eta}{\eta}}} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1+\eta}{\eta}} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{1+\eta}$$

- The firm's problem is

$$\pi_{ij} = \max_{n_{ij}} z_{ij} n_{ij}^{\alpha} - w_{ij} n_{ij}$$

subject to

$$n_{ij} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N$$
$$w_{ij} \geq \underline{w}$$

- Let $r_{ij} \in \{1, 2, 3\}$ denote the region that the firm is in.
- **Region I -** If the firm is in Region I, then its wage is the optimal markdown on the marginal revenue product of labor

$$w_{ij} = \mu_{ij} \alpha z_{ij} n_{ij}^{\alpha-1}$$
$$p_{ij} = 1$$
$$\widetilde{w}_{ij} = w_{ij}$$
$$n_{ij} = \left(\frac{w_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$

where the markdown depends on its shadow share of the labor market. That is, $\mu_{ij} = \mu\left(\widetilde{s}_{ij}\right)$, where $\mu\left(\widetilde{s}_{ij}\right) = \frac{\varepsilon\left(\widetilde{s}_{ij}\right)}{\varepsilon\left(\widetilde{s}_{ij}\right)+1}$. We have shown that

$$\widetilde{s}_{ij} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{1+\eta} \implies \widetilde{w}_j = \widetilde{w}_{ij} \widetilde{s}_{ij}^{-\frac{1}{1+\eta}}$$

Using these, we can write:

$$w_{ij} = \left[\mu\left(\widetilde{s}_{ij}\right) \alpha z_{ij} \widetilde{s}_{ij}^{-\frac{(1-\alpha)(\eta-\theta)}{1+\eta}} \widetilde{W}^{(1-\alpha)(\theta-\varphi)}\right]^{\frac{1}{1+\theta(1-\alpha)}}$$

- **Region II -** In Region II, then

$$\begin{aligned} w_{ij} &= \underline{w} \\ p_{ij} &= 1 \\ \widetilde{w}_{ij} &= \underline{w} \\ n_{ij} &= \left(\frac{\underline{w}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N \end{aligned}$$

- **Region III -** In Region III, then

$$\begin{aligned} w_{ij} &= \alpha z_{ij} n_{ij}^{\alpha-1} \\ p_{ij} &< 1 \\ \widetilde{w}_{ij} &= p_{ij}\underline{w} \\ n_{ij} &= \left(\frac{p_{ij}\underline{w}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N \end{aligned}$$

## C.2 Minimum wage solution algorithm

We implement the following solution algorithm. We denote the *Region* that a firm is in by $r_{ijt} \in \{I, II, III\}$. Initialize the algorithm by (i) guessing a value for $\widetilde{W}^{(0)}$, (ii) assuming all firms are in *Region I*, $r_{ij}^{(0)} = I$, which implies guessing $p_{ij}^{(0)} = 1$. These will all be updated in the algorithm.

1. **Solve all market equilibria in shadow shares**

   (a) Guess shadow shares $\widetilde{s}_{ij}^{(0)}$.

   (b) *Region I* - Using the above optimality condition

   $$w_{ij} = \left[\mu\left(\widetilde{s}_{ij}\right) \alpha z_{ij} \widetilde{s}_{ij}^{(0)-\frac{(1-\alpha)(\eta-\theta)}{1+\eta}} \widetilde{W}^{(0)(1-\alpha)(\theta-\varphi)}\right]^{\frac{1}{1+\theta(1-\alpha)}}$$

   (c) *Regions II, III* - Here the minimum wage is binding so set $w_{ij} = \underline{w}$.

   (d) Given the guess $p_{ij}^{(k)}$ and $w_{ij}$, compute the shadow wage: $\widetilde{w}_{ij} = p_{ij} w_{ij}$.

   (e) With all shadow wages in hand, update shadow shares using $\widetilde{w}_{ijt}$:

   $$\widetilde{s}_{ij}^{(l+1)} = \frac{\widetilde{w}_{ij}^{1+\eta}}{\sum_{i\in j} \widetilde{w}_{ij}^{1+\eta}}.$$

   (f) Iterate over (b)-(e) until shadow shares converge: $\widetilde{s}_{ij}^{(l+1)} = \widetilde{s}_{ij}^{(l)}$.

2. **Recover employment** - Here we use the wages from the previous step plus the current guess of each firms' region. First aggregate $\widetilde{w}_{ij}$ to compute $\widetilde{w}_j$ and $\widetilde{W}$. Then by region $r_{ijt}^{(k)}$:

(a) *Region I* - Firm is unconstrained:

$$n_{ij} = \left(\frac{w_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$

(b) *Region II* - Firm is constrained and $n_{ij}$ determined by household labor supply curve at $\underline{w}$:

$$n_{ij} = \left(\frac{\underline{w}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$

(c) *Region III* - Firm is constrained and $n_{ijt}$ determined by firm labor demand curve at $\underline{w}$:

$$\underline{w} = \alpha z_{ij} n_{ij}^{\alpha-1} \quad \Longrightarrow \quad n_{ij} = \left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}.$$

3. Update the multipliers: $p_{ij}^{(k)}$

(a) Aggregate $n_{ij}$ to compute $n_j$ and $N$.

(b) Update $p_{ij}$ from the *household's* first order conditions: $\widetilde{w}_{ij} = p_{ij} w_{ij}$

$$p_{ij}^{(k+1)} = \frac{\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}}{w_{ij}}$$

4. Update $\widetilde{W}^{(k)}$:

(a) Compute $\widetilde{w}_{ij} = p_{ij}^{(k+1)} w_{ij}$

(b) Use $\widetilde{w}_{ij}$ to update the aggregate shadow wage index to $\widetilde{W}^{(k+1)}$.

5. Update firm regions. For each region:

(a) Compute the marginal product of labor of all firms $mrpl_{ij} = \alpha z_{ij} n_{ij}^{\alpha-1}$.

(b) If in market $j$ there exists a firm in *Region I* with $w_{ij} < \underline{w}$, then move the firm with the lowest wage into *Region II*.

(c) If in market $j$ there exists a firm that was initially in *Region II* and has a marginal product of labor that is less than marginal cost $(\underline{w})$, move that firm into *Region III*.

6. Iterate over (1) to (5) until $p_{ij}^{(k+1)} = p_{ij}^{(k)}$ and $\widetilde{W}^{(k+1)} = \widetilde{W}^{(k)}$ and $r_{ij}^{(k+1)} = r_{ij}^{(k)}$.

# D Disciplining preference parameters

This Section details how we use recent evidence from Golosov, Graber, Mogstad, and Novgorodsky (2021) to discipline preference parameters $\sigma$ and $\varphi$.

**Background.** Consider a budget constraint, where $b_i$ is unearned income and $\mathcal{T}$ gives taxes and transfers which depend on pre-tax labor income $y_i$:

$$c_i = y_i - \mathcal{T}(y_i) + b_i$$

Totally differentiating with respect to $b_i$:

$$\frac{dc_i}{db_i} = \frac{dy_i}{db_i} - \frac{d\mathcal{T}_i}{db_i} + 1$$
$$MPC_i = MPE_i - MPT_i + 1$$

Table 4.1 of Golosov, Graber, Mogstad, and Novgorodsky (2021, henceforth GGMN) gives estimates of the marginal propensity to consume $(MPC)$ and marginal propensity to earn $(MPE)$ for different income groups, where lottery winnings are used as an instrument for the endogenous variable $b_i$. For example, results are of the type: *An extra dollar in unearned income leads to a $MPE = -0.52$ cent reduction in labor earnings.* We show how their results can be used to discipline preference parameters $(\varphi, \sigma)$ in a simple labor supply setting that is consistent with our model.

**Derivation.** Consider the following individual problem, where preferences are as in the main text, and $y = wn$, where $w$ is taken as given:

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{1}{\overline{\varphi}^{1/\varphi}} \frac{n^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \tag{D1}$$
$$c = wn + \mathcal{T}(wn) + b$$

Optimality conditions for $c$ and $n$ give labor supply, which can be expressed in terms of earnings:

$$y = \overline{\varphi} c^{-\varphi\sigma} w^{\varphi+1} \left(1 - \mathcal{T}'(y)\right)^{\varphi}$$

Totally differeniating with respect to $b$

$$\frac{dy}{db} = -\varphi\sigma \frac{dc}{db}\left(\frac{y}{c}\right) - \varphi\left(\frac{\mathcal{T}''(y)\, y}{1-\mathcal{T}'(y)}\right)\frac{dy}{db}.$$

Now suppose that post-tax labor earnings were of the form used in Heathcote, Storesletten, and Violante (2020, henceforth HSV): $y - \mathcal{T}(y) = \lambda y^{1-\tau}$. In this case, the elasticity term is simply the progressivity of taxes, $\tau$.

$$\frac{dy}{db} = -\varphi\sigma \frac{dc}{db}\left(\frac{y}{c}\right) - \varphi\tau \frac{dy}{db}.$$

Using the definitons of $MPC$, $MPE$, the average propensity to consume $APC = c/y$, and after rearranging, we have a closed-form relationship between $\sigma$ and $\varphi$, given data on $\{MPC, MPE, APC, \tau\}$:

$$\varphi = -\frac{1}{\sigma \frac{MPC}{MPE}\frac{1}{APC} - \tau}. \tag{D2}$$

If we let $\sigma = 1$ and $\tau = 0$, it is straightforward to observe that a lower $MPC$ and higher $MPE$ in absolute terms (as will be the case for richer households), requires a higher $\varphi$.

$$\varphi = \frac{|MPE|}{MPC} APC.$$

**Data.** We use BLS data to compute $APC$ for non-high-school, high-school, and college completion households. We map these into the four quartiles of income groups in GGMN Table 4.1 as given in the following table. We take a value of $\tau = 0.086$ from HSV (JEEA, 2020).[56] This uses *pre-government-transfer* income for $y$, that is $y$ only considers labor income earnings.

| | **All** | **Group** | | |
|---|---|---|---|---|
| BLS category<br>GGMN category | | Non-High School<br>Q1 | High school<br>Q2-Q3 | Completed college<br>Q4 |
| $APC$ (BLS) | 0.69 | 0.73 | 0.71 | 0.67 |
| $MPE$ (GGMN) | -0.5227 | -0.3080 | -0.5549 | -0.6735 |
| $MPC$ (GGMN) | 0.5836 | 0.7315 | 0.5429 | 0.4990 |

Table D1: Data used in calibrating preference parameters

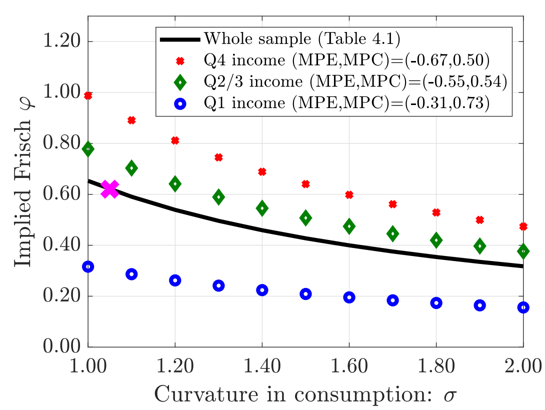


Figure D1: Implied parameters

Notes: Given a value for the coefficient of relative risk aversion $\sigma$, this figure plots the Frisch elasticity of labor supply $\varphi$ required for the optimality conditions of the simple labor supply model D1 to be consistent with (i) empirical measures of the marginal propensity to earn and marginal propensity to consume following changes in unearned income from Golosov, Graber, Mogstad, and Novgorodsky (2021), (ii) estimates of the average propensity to consume from the BLS, (iii) estimates of the progressivity of post-tax labor income to pre-tax-and-transfer income from Heathcote, Storesletten, and Violante (2020).

**Results.** Using equation (D2), we can then determine $\varphi$ given $\sigma$. Figure D1 plots $\varphi(\sigma)$ for $\sigma \in [1, 2]$. As a benchmark, with log preferences, and when calibrated to the whole sample values, $\varphi(1) = 0.65$. For low income (Q1) households $\varphi(1) = 0.32$, for high income households $\varphi(1) = 0.987$. High income (Q4) households have higher MPE's, and their MPC is lower, reducing $|MPC/MPE|$, and requiring a higher $\varphi$. The pink cross corresponds to $(\sigma, \varphi) = (1.05, 0.62)$, which are the values used in the baseline calibration of our model (see Table 1).

[56] See: http://violante.mycpanel.princeton.edu/Workingpapers/JEEA_final.pdf, Table 1, Row 5

# E Proofs for simple monopsony and simple oligopsony economies

## E.1 Proofs: Simple Monopsony

In this appendix, we provide derivations of (1) the *perceived inverse labor supply schedule* and (2) weakly optimal rationing constraint. Recall the firm problem:

$$\begin{aligned} \max_{\overline{n}_i, n_i, w_i} \; & z_i n_i^{\alpha} - w_i n_i \\ \text{subject to} \quad & w_i \geq \max\left\{\underline{w}, w\left(n_i, \overline{n}_i, N\right)\right\} \\ & n_i \leq \overline{n}_i \end{aligned}$$

where the inverse labor supply schedule is a correspondence given by:

$$w\left(n_i, \overline{n}_i, N\right) = \begin{cases} \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} & , n_i \in \left[0, \overline{n}_i\right) \\ \in \left[\left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}, \infty\right) & , n_i = \overline{n}_i. \end{cases}.$$

We defined a *legal wage* to be any wage $w_i \geq \underline{w}$.

**(1) Perceived labor supply curve.** **Lemma 1** - *Consider some level of employment $n_i \leq \overline{n}_i$. A firm would never pay a wage that is greater than the lowest legal wage necessary to deliver $n_i$.*

- *Proof*: Conditional on $n_i$, profits are strictly decreasing in $w_i$. ■

**Lemma 2** - *Consider some level of employment $n_i \leq \overline{n}_i$. The lowest legal wage that delivers $n_i$, is given by*:

$$\max\left\{\underline{w}, \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\} \quad , \quad \forall n_i \in \left[0, \overline{n}_i\right]$$

- *Proof*: If $n_i = \overline{n}_i$, then $w\left(n_i, \overline{n}_i, N\right)$ requires $w_i \geq \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$. Given **Lemma 1**, the firm will pay $w_i = \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$. If this is not legal, then the firm pays $w_i = \max\left\{\underline{w}, \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\}$. If $n_i < \overline{n}_i$, then then firm will pay $w_i = \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$. If this is not legal, then the firm pays $w_i = \max\left\{\underline{w}, \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\}$. ■

Lemma 2 maps employment to legal wages. We call this mapping the firm's *perceived inverse labor supply curve* $w^p\left(n_i, \overline{n}_i, N\right)$, which is the inverse labor supply curve that the firm faces *conditional on* a choice $\overline{n}_i$, and also taking account of the minimum wage:

$$w^p\left(n_i, \overline{n}_i, N\right) = \begin{cases} \max\left\{\underline{w}, \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\} & , \quad n_i \in \left[0, \overline{n}_i\right) \\ \max\left\{\underline{w}, \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\} & , \quad n_i = \overline{n}_i \end{cases}$$

We can further simplify $w^P(\cdot)$. Fix $\overline{n}_i$. Marginal disutility $\left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$ is strictly increasing in $n_i$. Therefore, if $\underline{w} \geq \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$, then $\underline{w} \geq \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$ for all $n_i \in \left(0, \overline{n}_i\right]$. Using this we obtain the expression in the main text:

$$w^p\left(n_i, \overline{n}_i, N\right) = \begin{cases} \underline{w} & \text{if } \underline{w} > \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \\ \max\left\{\underline{w}, \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \end{cases}$$

We have therefore simplified the firm problem, as presented in the text, which can be expressed as (i) choosing $\overline{n}_i$ and then, (ii) choosing $n_i$ subject to the perceived labor supply schedule implied by $\overline{n}_i$:

$$\begin{aligned} & \max_{\overline{n}_i} \ \pi\left(\overline{n}_i\right) \\ \pi\left(\overline{n}_i\right) = & \max_{n_i} \ z_i n_i^{\alpha} - w^p\left(n_i, \overline{n}_i, N\right) n_i \\ \text{subject to} \quad & n_i \leq \overline{n}_i \\ w^p\left(n_i, \overline{n}_i, N\right) = & \begin{cases} \underline{w} & \text{if } \underline{w} > \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \\ \max\left\{\underline{w}, \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \end{cases} \end{aligned}$$

**(2) Optimal rationing constraint.** We can solve the problem $\max_{n_i}$ for any value of $\overline{n}_i$. First, consider the case of $\overline{n}_i = \infty$:

$$\pi(\infty) = \max_{n_i} \ z_i n_i^{\alpha} - \max\left\{\underline{w}, \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\} n_i.$$

We can partition $n_i$ into two sets. Let $\widetilde{n}_i$ be such that $\underline{w} = \left(\frac{\widetilde{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$. Again using the monotonicity of the disutility of supplying $n_i$ to firm $i$:

$$\max\left\{\underline{w}, \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\} = \begin{cases} \underline{w} & \text{if } n_i < \widetilde{n}_i \\ \underbrace{\left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}}_{:=\widehat{w}(n_i)} & \text{if } n_i \geq \widetilde{n}_i. \end{cases}$$

There are three possible first-order conditions, depending on the firm's optimal choice of $n_i$ relative to $\widetilde{n}_i$,

$$\underbrace{\alpha z_i n_i^{\alpha-1}}_{mrpl(n_i)} = \begin{cases} \underline{w} & n_i < \widetilde{n}_i \\ \in \left[\underline{w}, \widehat{w}'\left(n_i\right) n_i + \widehat{w}\left(n_i\right)\right] & n_i = \widetilde{n}_i \\ \widehat{w}'\left(n_i\right) n_i + \widehat{w}\left(n_i\right) & n_i > \widetilde{n}_i. \end{cases} \tag{E1}$$

Lemma 3 characterizes the firm's optimal choice of $n_i$.

**Lemma 3** - *If $\overline{n}_i = \infty$, then at the firm's optimal choice of $n_i$, $mrpl\left(n_i\right) \geq \underline{w}$ .*

- *Proof:* Consider each case:
  - If $n_i < \widetilde{n}_i$ then $mrpl\left(n_i\right) = \underline{w}$.
  - If $n_i = \widetilde{n}_i$ then $mrpl\left(n_i\right) \geq \underline{w}$, since $\widehat{w}'\left(\widetilde{n}_i\right) \widetilde{n}_i + \widehat{w}\left(\widetilde{n}_i\right) = \widehat{w}'\left(\widetilde{n}_i\right) \widetilde{n}_i + \underline{w} > \underline{w}$.

– If $n_i > \widetilde{n}_i$, then $mrpl\left(n_i\right) > \underline{w}$ if $\widehat{w}'\left(n_i\right)n_i + \widehat{w}\left(n_i\right) > \widehat{w}'\left(\widetilde{n}_i\right)\widetilde{n}_i + \widehat{w}\left(\widetilde{n}_i\right)$, since $\widehat{w}'\left(\widetilde{n}_i\right)\widetilde{n}_i + \widehat{w}\left(\widetilde{n}_i\right) > \underline{w}$. We can write the marginal cost the firm as follows:

$$\begin{aligned} mc\left(n_i\right) &= w'\left(n_i\right)n_i + w\left(n_i\right) = \left(\varepsilon^{Inv}\left(n_i\right) + 1\right)w\left(n_i\right) \\ \varepsilon_I\left(n_i\right) &= \frac{w'\left(n_i\right)n_i}{w\left(n_i\right)} > 0 \end{aligned}$$

Therefore a sufficient condition for $mc'\left(n_i\right) > 0$ is that the inverse labor supply elasticity $\varepsilon_I\left(n_i\right)$ is weakly increasing in $n_i$. In the monopsony case, $\varepsilon_I\left(n_i\right) = 1/\eta$. In the oligopsony case, this is also true as

$$\varepsilon_I\left(n_i\right) = \frac{1}{\theta}s_i + \left(1 - s_i\right)\frac{1}{\eta}$$

which—holding $n_{-ij}$ fixed—is increasing in $n_i$. Higher $n_i$ also increases $w_i$, since $w'\left(n_i\right)$ constant, which increases $s_i$ and pushes $\varepsilon_I\left(n_i\right)$ toward $1/\theta > 1/\eta$ . ■

The three first order conditions in Lemma 3 define value maximizing choices of $n_i$ under $\overline{n}_i = \infty$. Consider an $\overline{n}_i < \infty$. If the three first order conditions in (E1) remain unchanged, then so does the firm's value: $\pi(\overline{n}_i) = \pi(\infty)$. Lemma 3 says that if $mrpl(\overline{n}_i) = \underline{w}$ then this is the case, since $n_i \leq \overline{n}_i$ implies $mrpl(n_i) \geq mrpl(\overline{n}_i) = \underline{w}$. That is, such a choice of $\overline{n}_i$ does not distort first order conditions relative to the $\overline{n}_i = \infty$ case. Hence we can always set $\overline{n}_i$ by $mrpl\left(\overline{n}_i\right) = \underline{w}$ and this rationing constraint is non-binding away from the minimum wage, and weakly binding at the optimal value of employment when the firm is constrained by the minimum wage. Lemma 4 formally proves this result.

**Lemma 4 -** *It is (weakly) optimal for the firm to impose a rationing constraint $\overline{n}_i$ such that $mrpl\left(\overline{n}_i\right) = \underline{w}$.*

- *Proof:* As above, define $\overline{n}_i$ by $mrpl\left(\overline{n}_i\right) = \underline{w}$, with $mrpl\left(n\right)$ decreasing in $n$. Given $\left(n_i, \overline{n}_i\right)$, the inverse labor supply curve can be split into three regions:

$$w^p\left(n_i, \overline{n}_i, N\right) = \begin{cases} \underbrace{\underline{w}}_{\text{Region III}} & \text{if } \underline{w} > \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \\ \max\left\{\underbrace{\underline{w}}_{\text{Region II}}, \underbrace{\left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}}_{\text{Region I}}\right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \end{cases}$$

  – We solve for the optimal $n_i$ in each Region and show that $\overline{n}_i$ is weakly binding, and thus weakly optimal.

  – **Region I -** Suppose $\left(n_i, \overline{n}_i\right)$ are such that the firm is in Region I. Then it solves the problem

$$\max_{n_i \leq \overline{n}_i} z_i n_i^{\alpha} - \widehat{w}\left(n_i\right)n_i \quad , \quad \widehat{w}\left(n_i\right) = \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$$

and hence has first order condition

$$\begin{aligned}
mrpl\left(n_i^*\right) &= \widehat{w}'\left(n_i^*\right) n_i^* + \widehat{w}\left(n_i^*\right) &&, \quad \text{since } \widehat{w}'\left(n_i\right) > 0 \\
&> \widehat{w}\left(n_i^*\right) &&, \quad \text{since in Region I, then } \widehat{w}\left(n_i^*\right) > \underline{w} \\
&> \underline{w} &&, \quad \text{by the conjectured } \overline{n}_i \\
mrpl\left(n_i^*\right) &> mrpl\left(\overline{n}_i\right) &&, \quad \text{by declining } mrpl(n_i) \\
n_i^* &< \overline{n}_i .
\end{aligned}$$

Therefore the constraint is slack. Note also that $n_i^*$ and firm profits are independent of $\overline{n}_i$.

- **Region II -** Suppose $(n_i, \overline{n}_i)$ are such that the firm is in Region II. Define $\widetilde{n}_i$ by the maximum $n_i$ such that the firm is still in Region II: $\underline{w} = \left(\frac{\widetilde{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$. For $n_i > \widetilde{n}_i$, $\left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} > \underline{w}$, and the firm is in Region I. Since the firm is in Region II, then it is also the case that $\underline{w} \leq \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$. Hence $\left(\frac{\widetilde{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} = \underline{w} \leq \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$, and so $n_i \leq \widetilde{n}_i \leq \overline{n}_i$. Therefore the constraint is slack. Again, firm profits and $n_i^*$ are independent of $\overline{n}_i$.
- **Region III -** Suppose $(n_i, \overline{n}_i)$ are such that the firm is in Region III. Then $w_i = \underline{w}$ for all $n_i \leq \overline{n}_i$. Therefore the firm solves:

$$\max_{n_i} z_i n_i^{\alpha} - \underline{w} n_i \quad \text{with first order condition} \quad mrpl\left(n_i^*\right) = \underline{w} = mrpl\left(\overline{n}_i\right)$$

therefore the constraint is weakly binding.

We have therefore shown that setting $\overline{n}_i$ such that $mrpl\left(\overline{n}_i\right) = \underline{w}$ is weakly optimal in Regions I and II, and optimal in Region III.■

Applying Lemma 4, we can write the firm problem with the constraint $\overline{n}_i = \left(\frac{\alpha z_i}{\underline{w}}\right)^{\frac{1}{1-\alpha}}$ imposed:

$$\max_{n_i} z_i n_i^{\alpha} - w^p\left(n_i, \overline{n}_i, N\right) n_i$$

$$\text{subject to} \qquad n_i \leq \overline{n}_i \quad , \quad \overline{n}_i = \left(\frac{\alpha z_i}{\underline{w}}\right)^{\frac{1}{1-\alpha}}$$

$$w^p\left(n_i, \overline{n}_i, N\right) = \begin{cases} \underline{w} & \text{if } \underline{w} > \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \\ \max\left\{\underline{w}, \left(\frac{n_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}\right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_i}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}} \end{cases}$$

## E.2 Proofs: Simple Oligopsony

In this appendix, we provide derivations of (1) the perceived inverse labor supply curve and (2) optimal rationing constraint.

**(1) Perceived labor supply curve.** **Lemma 0:** *Given competitor employment $\{n_{-ij}\}$, competitor rationing constraints $\{\overline{n}_{-ij}\}$ are payoff irrelevant for firm ij.*

*Proof:* $\{\overline{n}_{-ij}\}$ do not enter the Cournot oligopsony firm problem. ■

**Lemma 1** - *Consider some level of employment $n_{ij} \leq \overline{n}_{ij}$. Given competitor employment $\left\{n_{-ij}\right\}$, a firm would never pay a wage that is greater than the lowest legal wage necessary to deliver $n_{ij}$.*

*Proof:* Conditional on $n_{ij}$ and $\left\{n_{-ij}\right\}$, profits are strictly decreasing in $w_{ij}$. ■

**Lemma 2** - *Consider some level of employment $n_{ij} \leq \overline{n}_{ij}$. Given competitor employment $\left\{n_{-ij}\right\}$,the lowest legal wage that delivers $n_{ij}$, is given by*

$$\max\left\{\underline{w}, \min\left\{w_L\left(n_{ij}, n_j, N\right), w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)\right\}\right\}$$

*where* $w_L\left(n_{ij}, n_j, N\right) = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$.

*Proof:* Given competitor employment $\left\{n_{-ij}\right\}$, the mapping from wages to employment is one-to-one except when $\overline{n}_i = n_i$. In that case $\min\left\{w_L\left(n_{ij}, n_j, N\right), w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)\right\}$ is the lowest wage that delivers $n_{ij} = \overline{n}_{ij}$ employees. This wage may not be legal. The lowest legal wage that delivers $n_{ij} = \overline{n}_{ij}$ employees is therefore $\max\left\{\underline{w}, \min\left\{w_L\left(n_{ij}, n_j, N\right), w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)\right\}\right\}$. ■

Lemma 2 maps employment to legal wages. We call this mapping the firm's *perceived inverse labor supply curve*, which is the inverse labor supply curve that the firm faces *conditional on* a choice $\overline{n}_{ij}$, and also taking account of the minimum wage. Given competitor employment $\left\{n_{-ij}\right\}$ and substituting the firms choice of $w_{ij}$ conditional on choices of $\left(n_{ij}, \overline{n}_{ij}, n_j\right)$, we can write the firm's problem as

$$\max_{\overline{n}_{ij}, n_{ij}} z_{ij} n_{ij}^{\alpha} - w_{ij} n_{ij}$$

subject to

$$w_{ij} = w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) = \begin{cases} \max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} & , \quad n_{ij} \in \left(0, \overline{n}_{ij}\right] \\ \max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} & , \quad n_{ij} = \overline{n}_{ij} \end{cases}$$

Using the monotonicity of $\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$ in $n_{ij}$,[57] we know that the *highest wage possible* for $n_{ij} \in \left[0, \overline{n}_{ij}\right]$ is at $\overline{n}_{ij}$. If $\left(\frac{\overline{n}_{ij}}{N}\right)^{\frac{1}{\eta}} N^{\frac{1}{\varphi}}$ is less than $\underline{w}$, then it must be the case that $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} < \underline{w}$ for all $n_{ij} \in \left[0, \overline{n}_{ij}\right]$. Using this, and given $\overline{n}_{ij}$, we can write the perceived labor supply curve on $n_{ij} \in \left[0, \overline{n}_{ij}\right]$ as follows

$$w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) = \begin{cases} \underline{w} & \text{if } \underline{w} > \left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \\ \max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}. \end{cases}$$

Note that the perceived labor supply curve is not a function of $w_{ij}$. Given competitor employment $\left\{n_{-ij}\right\}$, the Cournot firm problem becomes,

$$\max_{\overline{n}_{ij}, n_{ij}} z_{ij} n_{ij}^{\alpha} - w_{ij} n_{ij}$$

[57] Note $w\left(n_{ij}, \left\{n_{-ij}\right\}\right) = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} = n_{ij}^{\frac{1}{\eta}} n_j^{\frac{1}{\theta} - \frac{1}{\eta}} N^{\frac{1}{\varphi} - \frac{1}{\theta}}$ and that $\frac{\partial n_j}{\partial n_{ij}} > 0$ and $\eta > \theta$. Therefore, given competitor employment $\left\{n_{-ij}\right\}$, $\frac{\partial w\left(n_{ij}, \left\{n_{-ij}\right\}\right)}{\partial n_{ij}} > 0$.

subject to

$$n_{ij} \leq \overline{n}_{ij}$$

and the *perceived inverse labor supply curve*

$$w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) = \begin{cases} \underline{w} & \text{if } \underline{w} > \left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \\ \max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}. \end{cases}$$

**(2) Optimal rationing constraint.** Consider the case of $\overline{n}_{ij} = \infty$. Given competitor employment $\{n_{-ij}\}$, the firm solves the following problem:

$$\max_{n_{ij}} z_{ij} n_{ij}^{\alpha} - \max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} n_{ij}.$$

We can partition $n_{ij}$ into two sets. Let $\widetilde{n}_{ij}$ be such that

$$\underline{w} = \left(\frac{\widetilde{n}_{ij}}{n_j\left(\widetilde{n}_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(\widetilde{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$

where

$$\max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} = \begin{cases} \underline{w} & \text{if } n_{ij} < \widetilde{n}_{ij} \\ \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} & \text{if } n_{ij} \geq \widetilde{n}_{ij}. \end{cases}$$

Note that the marginal revenue product is well defined and differentiable for all $n_i$. Total labor costs are differentiable everywhere except at $n_i = \widetilde{n}_i$. However, the unconstrained labor supply curve is differentiable everywhere (note that $n_j$ depends on $n_{ij}$ and we suppress dependence on aggregates and competitor employment, both of which are taken as given),

$$\widehat{w}\left(n_{ij}\right) = \left(\frac{n_{ij}}{n_j\left(n_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(n_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}.$$

. There are three possible first-order conditions, depending on the firm's optimal choice of $n_{ij}$ relative to $\widetilde{n}_{ij}$,

$$mrpl\left(n_{ij}\right) = \begin{cases} \underline{w} & n_{ij} < \widetilde{n}_{ij} \\ \in \left[\underline{w}, \widehat{w}'\left(n_{ij}\right) n_{ij} + \widehat{w}\left(n_{ij}\right)\right] & n_{ij} = \widetilde{n}_{ij} \\ \widehat{w}'\left(n_{ij}\right) n_{ij} + \widehat{w}\left(n_{ij}\right) & n_{ij} > \widetilde{n}_{ij}. \end{cases}$$

Lemma 3 characterizes the firm's optimal choice of $n_{ij}$.

**Lemma 3** - *Given competitor employment $\{n_{-ij}\}$, the firm's optimal choice of $n_{ij}$ satisfies $mrpl\left(n_{ij}\right) \geq \underline{w}$ .*

*Proof:* If $n_{ij} < \widetilde{n}_{ij}$ then $mrpl\left(n_{ij}\right) = \underline{w}$. If $n_{ij} = \widetilde{n}_{ij}$ then $mrpl\left(n_{ij}\right) \geq \underline{w}$ where we have used $\widehat{w}'\left(\widetilde{n}_{ij}\right)\widetilde{n}_{ij} + \widehat{w}\left(\widetilde{n}_{ij}\right) = \widehat{w}'\left(\widetilde{n}_{ij}\right)\widetilde{n}_{ij} + \underline{w} > \underline{w}$. If $n_{ij} > \widetilde{n}_{ij}$, we need to show that show that $\widehat{w}'\left(n_{ij}\right) n_{ij} + \widehat{w}\left(n_{ij}\right)$ is also increasing in $n_{ij}$, therefore $mrpl\left(n_{ij}\right) = \widehat{w}'\left(n_{ij}\right) n_{ij} + \widehat{w}\left(n_{ij}\right) > \widehat{w}'\left(\widetilde{n}_{ij}\right)\widetilde{n}_{ij} + \widehat{w}\left(\widetilde{n}_{ij}\right) > \underline{w}$. We can rewrite the marginal cost the firm as follows:

$$\begin{aligned} mc\left(n_{ij}\right) &= w'\left(n_{ij}\right)n_{ij} + w\left(n_{ij}\right) \\ &= \left[\frac{w'\left(n_{ij}\right)n_{ij}}{w\left(n_{ij}\right)} + 1\right] w\left(n_{ij}\right) \\ &= \left[\varepsilon^{Inv}\left(n_{ij}\right) + 1\right] w\left(n_{ij}\right). \end{aligned}$$

We can then show that $mc'(n_{ij}) > 0$ so long as $\varepsilon^{Inv\prime}(n_{ij}) > 0$:

$$mc'\left(n_{ij}\right) = \underbrace{\varepsilon^{Inv\prime}\left(n_{ij}\right)}_{\text{RHS positive if this is positive}} w\left(n_{ij}\right) + \left[\varepsilon^{Inv}\left(n_{ij}\right) + 1\right] w'\left(n_{ij}\right)$$

Following the derivations in BHM this is true in the Cournot oligopsony problem of the firm since we have

$$\varepsilon^{Inv}\left(n_{ij}\right) = \frac{1}{\theta}s_{ij} + \left(1 - s_{ij}\right)\frac{1}{\eta}$$

which is increasing (holding $n_{-ij}$ fixed), as higher $n_{ij}$ increases also $w_{ij}$, which increases $s_{ij}$, which pushes toward the larger $1/\theta$ term which is $> 1/\eta$.■

Define $\overline{n}_{ij}$ by $mrpl\left(\overline{n}_{ij}\right) = \underline{w}$. Then by Lemma 3 we know that

$$\begin{aligned} mrpl\left(n_{ij}\right) \geq \underline{w} = mrpl\left(\overline{n}_{ij}\right) \\ n_{ij} \leq \overline{n}_{ij} \end{aligned}$$

Therefore we can always set $\overline{n}_{ij}$ by $mrpl\left(\overline{n}_{ij}\right) = \underline{w}$ and this rationing constraint is non-binding away from the minimum wage, and weakly binding at the optimal value of employment when the firm is constrained by the minimum wage. Lemma 4 formally proves this result.

**Lemma 4 -** *It is (weakly) optimal for the firm to impose a rationing constraint of* $\overline{n}_{ij} = \left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}$.

*Proof:* As above, define $\overline{n}_{ij}$ by $mrpl\left(\overline{n}_{ij}\right) = \underline{w}$. Note that $mrpl\left(n_{ij}\right) = \alpha z_{ij} n_{ij}^{\alpha-1}$, thus $\underline{w} = \alpha z_{ij}\overline{n}_{ij}^{\alpha-1}$, and $mrpl\left(\overline{n}_{ij}\right) = \underline{w}$. Also note that $mrpl\left(n\right)$ is *decreasing* in $n$. Conditional on competitor employment, we define three regions (I,II, and III) on the perceived inverse labor supply curve:

$$w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) = \begin{cases} \underbrace{\underline{w}}_{\text{Region III}} & \text{if } \underline{w} > \left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \\ \max\left\{ \underbrace{\underline{w}}_{\text{Region II}}, \underbrace{\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}}_{\text{Region I}} \right\} & \text{if } \underline{w} \leq \left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}. \end{cases}$$

Let **Region I** be the case that $\left(w_{ij}, n_{ij}, \overline{n}_{ij}, n_j\right)$ are such that the firm is on the *second* part of the second branch of $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$. Let **Region II** be the case that $\left(w_{ij}, n_{ij}, \overline{n}_{ij}, n_j\right)$ are such that the firm is on the *first* part of the second branch of $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$. Let **Region III** be the case that $\left(w_{ij}, n_{ij}, \overline{n}_{ij}, n_j\right)$ are such that the firm is on the first branch of $w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right)$ (note that this *does not* require that $n_{ij} = \overline{n}_{ij}$, although this will be the case under firm optimality). We proceed by solving for the optimal $n_{ij}$ in each Region and show that $\overline{n}_{ij}$ is weakly binding, and thus weakly optimal.

**Region I -** Suppose the firm is in Region I, then it is solving the problem (taking competitor employment as given),

$$\max_{n_{ij}\leq\overline{n}_{ij}} z_{ij}n_{ij}^{\alpha}-\widehat{w}\left(n_{ij}\right)n_{ij}\quad,\quad\widehat{w}\left(n_{ij}\right)=\left(\frac{n_{ij}}{n_{j}\left(n_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_{j}\left(n_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}$$

and hence has first order condition

$$\begin{aligned}
mrpl\left(n_{ij}^{*}\right) &= \widehat{w}'\left(n_{ij}^{*}\right)n_{ij}^{*}+\widehat{w}\left(n_{ij}^{*}\right)\quad,\quad\text{since }\widehat{w}'\left(n_{ij}\right)>0\\
&> \widehat{w}\left(n_{ij}^{*}\right)\quad,\quad\text{since in Region I, then }\widehat{w}\left(n_{ij}^{*}\right)>\underline{w}\\
&> \underline{w}\quad,\quad\text{by the conjectured }\overline{n}_{ij}\\
mrpl\left(n_{ij}^{*}\right) &> mrpl\left(\overline{n}_{ij}\right)\\
n_{ij}^{*} &< \overline{n}_{ij}
\end{aligned}$$

therefore the constraint is slack. Note also that the value is independent of $\overline{n}_{i}$.

**Region II -** Suppose the firm is in Region II, then $w_{ij}=\underline{w}$ and $\underline{w}\leq\left(\frac{\overline{n}_{ij}}{n_{j}\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_{j}\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}$. Define $\widetilde{n}_{ij}$ such that

$$\underline{w}=\left(\frac{\widetilde{n}_{ij}}{n_{j}\left(\widetilde{n}_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_{j}\left(\widetilde{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}.$$

Note that for $n_{ij}\leq\widetilde{n}_{ij}$, then $\max\left\{\underline{w},\left(\frac{n_{ij}}{n_{j}\left(n_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_{j}\left(n_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}\right\}=\underline{w}$, and hence the firm is in Region II, while the firm is not in Region II for $n_{ij}>\widetilde{n}_{ij}$. Since the firm is in Region II, then also have $\underline{w}\leq\left(\frac{\overline{n}_{ij}}{n_{j}\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}}\left(\frac{n_{j}\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}}N^{\frac{1}{\varphi}}$, which by monotonicity of the labor supply curve implies that $\widetilde{n}_{ij}\leq\overline{n}_{ij}$. Therefore the $n_{ij}$ for which the firm is in Region II are all weakly less than $\overline{n}_{ij}$. Note that this *does not* require knowing anything about the $mrpl_{ij}$, its simply by definition of Region II. Note also that the value is independent of $\overline{n}_{ij}$.

**Region III -** Suppose the firm is in Region III, then $w_{ij}=\underline{w}$ for all $n_{ij}\leq\overline{n}_{ij}$. Therefore the firm is solving:

$$\max_{n_{ij}} z_{ij}n_{ij}^{\alpha}-\underline{w}n_{ij}$$

and hence has the first order condition

$$mrpl\left(n_{ij}^{*}\right)=\underline{w}=mrpl\left(\overline{n}_{ij}\right)$$

therefore the constraint is weakly binding.■

Applying Lemma 4, we can write the firm problem with the constraint $\overline{n}_{ij}=\left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}$ imposed:

$$\max_{n_{ij}} z_{ij}n_{ij}^{\alpha}-w\left(n_{ij},\overline{n}_{ij},n_{j},N\right)n_{ij}$$

subject to

$$
\begin{aligned}
n_{ij} &\leq \overline{n}_{ij} \\
\overline{n}_{ij} &= \left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}
\end{aligned}
$$

and the *perceived labor supply curve*

$$
w\left(n_{ij}, \overline{n}_{ij}, n_j, N\right) = \begin{cases} \underline{w} & \textit{if } \underline{w} > \left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \\ \max\left\{\underline{w}, \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right\} & \textit{if } \underline{w} \leq \left(\frac{\overline{n}_{ij}}{n_j\left(\overline{n}_{ij}\right)}\right)^{\frac{1}{\eta}} \left(\frac{n_j\left(\overline{n}_{ij}\right)}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \end{cases}
$$

# F Mathematical details - Full quantitative model

- We first derive results for the competitive equilibrium, then the government's allocation problem. We then use results from the competitive equilibrium to prove that the solution to the government's allocation problem can be decentralized in a competitive equilibrium with revenue neutral lump sum taxes

## F.1 Competitive equilibrium

### F.1.1 Household problem - Labor supply system, shadow wages

- In the competitive equilibrium, household $k$ solves the following problem:

$$\max_{c_{kt},h_{kt}} \sum_{t=0}^{\infty} \beta^t \left[ \frac{(c_{kt}/\pi_k)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

where $\widetilde{\varphi}_k = \overline{\varphi}_k \pi_k^{1+\varphi}$ is adjusted for the measure of workers of the household,

$$n_{kt} = \left[ \int n_{jkt}^{\frac{\theta+1}{\theta}} dj \right]^{\frac{\theta}{\theta+1}} \quad , \quad n_{jkt} = \left[ \sum_{i\in j} n_{ijkt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

subject to the budget constraint

$$c_{kt} + k_{kt+1} = \int \sum_{i\in j} w_{ijkt} n_{ijkt} dj + R_t k_{kt} + (1-\delta) k_{kt} + \kappa_k \Pi_t.$$

with the initial condition $k_{k0} = \kappa_k K_0$.

- Since we focus on steady-state we normalize the price of consumption to one.
- In the text we refer to these preferences as $u^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)$:

$$u^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right) = \frac{(c_{kt}/\pi_k)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

- The household is also subject to the firm by firm rationing constraints: $n_{ijkt} \leq \overline{n}_{ijkt}$.
- Let $\beta^t \nu_{kt}$ be the multiplier on the household's budget constraint and write the multiplier on the ratioining constraint as $\zeta_{ijkt} = \beta^t \nu_{kt} w_{ijkt} \left(1 - p_{ijkt}\right)$.
- The the household's Lagrangean features the following terms in $n_{ijkt}$

$$\mathcal{L} = \cdots + \beta^t u^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right) + \cdots + \beta^t \nu_{kt} w_{ijkt} n_{ijkt} + \beta^t \nu_{kt} w_{ijkt} \left(1 - p_{ijkt}\right) \left[\overline{n}_{ijkt} - n_{ijkt}\right] + \ldots$$

$$\mathcal{L} = \cdots + u^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right) + \cdots + \beta^t \nu_{kt} \left\{w_{ijkt} p_{ijkt}\right\} n_{ijkt} + \beta^t \nu_{kt} w_{ijkt} \left(1 - p_{ijkt}\right) \overline{n}_{ijkt} + \ldots$$

- The first order condition for consumption is

$$u_c^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right) = \nu_{kt}$$

- The first order condition for labor supply is

$$\nu_{kt} w_{ijkt} p_{ijkt} = -u_n^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right) \frac{\partial n_{kt}}{\partial n_{jkt}} \frac{\partial n_{jkt}}{\partial n_{ijkt}}$$

$$w_{ijkt} p_{ijkt} = -\frac{u_n^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}$$

- Define the *shadow wage* by $\widetilde{w}_{ijkt} := w_{ijkt} p_{ijkt}$.
- Then

$$\widetilde{w}_{ijkt} = -\frac{u_n^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}.$$

- Now define the following *shadow wage indexes*:

$$\widetilde{w}_{jkt} = \left[\sum_{i\in j} \widetilde{w}_{ijkt}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{w}_{kt} = \left[\int \widetilde{w}_{jkt}^{1+\theta} dj\right]^{\frac{1}{1+\eta}}$$

- Using this

$$\sum_{i\in j} \widetilde{w}_{ijkt}^{1+\eta} = \left[-\frac{u_n^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}\right]^{1+\eta} \sum_{i\in j} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1+\eta}{\eta}}$$

$$\widetilde{w}_{jkt} = -\frac{u_n^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}$$

$$\widetilde{w}_{jkt}^{1+\theta} = \left[-\frac{u_n^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}\right] \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1+\theta}{\theta}}$$

$$\int \widetilde{w}_{jkt}^{1+\theta} dj = \left[-\frac{u_n^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}\right] \int \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1+\theta}{\theta}} dj$$

$$\widetilde{w}_{kt} = -\frac{u_n^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}{u_c^k\left(\frac{c_{kt}}{\pi_k}, n_{kt}\right)}$$

- Using our form of preferences, this gives the household $k$ labor supply curve:

$$n_{kt} = \overline{\varphi}_k \pi_k \widetilde{w}_{kt}^{\varphi} \left( \frac{c_{kt}}{\pi_k} \right)^{-\varphi\sigma}$$

- Using this we can show that shadow wages aggregate, as claimed in the text,
- First across markets:

$$\begin{aligned}
\widetilde{w}_{ijkt} &= \widetilde{w}_{kt} \left( \frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \left( \frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1}{\eta}}. \\
\widetilde{w}_{ijkt}^{1+\eta} &= \left[ \widetilde{w}_{kt} \left( \frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \right]^{1+\eta} \left( \frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1+\eta}{\eta}} \\
\left[ \sum_{i \in j} \widetilde{w}_{ijkt}^{1+\eta} \right]^{\frac{1}{1+\eta}} &= \widetilde{w}_{kt} \left( \frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \\
\widetilde{w}_{jkt} &= \widetilde{w}_{kt} \left( \frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \\
\widetilde{w}_{jkt} n_{jkt} &= \widetilde{w}_{kt} n_{jkt} \times \left( \frac{n_{jkt}}{n_{kt}} \right)^{\frac{1+\theta}{\theta}} \\
\int \widetilde{w}_{jkt} n_{jkt} dj &= \widetilde{w}_{kt} n_{jkt}
\end{aligned}$$

- Then using these results, across firms within a market:

$$\begin{aligned}
\widetilde{w}_{ijkt} &= \widetilde{w}_{kt} \left( \frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \left( \frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1}{\eta}} \\
\widetilde{w}_{ijkt} &= \widetilde{w}_{jkt} \left( \frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1}{\eta}} \\
\widetilde{w}_{ijkt} n_{ijkt} &= \widetilde{w}_{jkt} n_{jkt} \times \left( \frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1+\eta}{\eta}} \\
\sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt} &= \widetilde{w}_{jkt} n_{jkt}
\end{aligned}$$

- Summarizing results so far, we have:

$$\widetilde{w}_{ijkt} = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \widetilde{w}_{jkt}$$
$$\widetilde{w}_{jkt} = \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \widetilde{w}_{kt}$$
$$\widetilde{w}_{kt} n_{jkt} = \int \widetilde{w}_{jkt} n_{jkt} dj$$
$$\widetilde{w}_{jkt} n_{jkt} = \sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt}$$

- Note that these can be combined to give the entire labor supply system of household $k$ in shadow wages:

$$n_{ijkt} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{\eta} \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}$$
$$n_{kt} = \overline{\varphi}_k \pi_k^{1+\varphi\sigma} \widetilde{w}_{kt}^{\varphi} c_{kt}^{-\varphi\sigma}$$

- A key result, used below, is that if the household received lump sum transfers $T_k$, then the same labor supply system would be obtained.
- Now consider our results regarding shadow shares. We define the *shadow share* as

$$\widetilde{s}_{ijkt} := \frac{\widetilde{w}_{ijkt} n_{ijkt}}{\sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt}}.$$

- Using the above aggregation results, labor supply system, and definition of the aggregator $n_{jkt}$:

$$\widetilde{s}_{ijkt} = \frac{\widetilde{w}_{ijkt} n_{ijkt}}{\widetilde{w}_{jkt} n_{jkt}} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{1+\eta} = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1+\eta}{\eta}} = \frac{\partial \log n_{ijkt}}{\partial \log n_{jkt}}$$

which we use below in the firm optimality conditions.

### F.1.2 Firm optimality

- **Simplifying the firm problem -** First we simplify the firm problem by separating it out across types and optimizing out capital for each type of worker:
- Consider the maximization problem of the firm in the text:

$$\pi_{ij} = \max_{\left\{n_{ijk}, k_{ijk}\right\}_{k=1}^{K}} \overline{Z} z_{ij} \sum_{k=1}^{K} \left( \left[\xi_k n_{ijk}\right]^{\gamma} k_{ijk}^{1-\gamma} \right)^{\alpha} - R \sum_{k=1}^{K} k_{ijk} - \sum_{k=1}^{K} w_{ijk} n_{ijk}$$

subject to the labor supply system and minimum wage constraints.

- First observe that this can separated out by type of worker $k$.

- The problem for type $k$ labor at the firm is

$$\pi_{ijk} = \max_{n_{ijk},k_{ijk}} \overline{Z} z_{ij} \left( \left[ \xi_k n_{ijk} \right]^{\gamma} k_{ijk}^{1-\gamma} \right)^{\alpha} - R k_{ijk} - w_{ijk} n_{ijk}$$

- We first optimize out capital. This yields the objective function

$$\pi_{ijk} = \max_{n_{ijk}} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}} - w_{ijk} n_{ijk}$$

where

$$\begin{aligned}
\widetilde{Z} &= \overline{Z}^{\frac{1}{1-(1-\gamma)\alpha}} \\
\widetilde{z}_{ij} &= \left[1 - (1-\gamma)\alpha\right] \left( \frac{(1-\gamma)\alpha}{R} \right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} z_{ij}^{\frac{1}{1-(1-\gamma)\alpha}} \\
\widetilde{\xi}_k &= \xi_k^{\widetilde{\alpha}} \\
\widetilde{\alpha} &= \frac{\gamma\alpha}{1-(1-\gamma)\alpha}
\end{aligned}$$

- We denote output net of capital expenses as $\widetilde{y}_{ijk} := \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}}$.
- We can also define a market-level aggregate $\widetilde{y}_{jk} = \sum_{i \in j} \widetilde{y}_{ijk}$, and a type-level aggregate $\widetilde{y}_k = \int \widetilde{y}_{jk} dj$.
- Note that

$$y_{ijk} = \frac{\widetilde{y}_{ijk}}{1-(1-\gamma)\alpha} \quad , \quad y_{jk} = \frac{\widetilde{y}_{jk}}{1-(1-\gamma)\alpha} \quad , \quad y_k = \frac{\widetilde{y}_k}{1-(1-\gamma)\alpha}.$$

- Using the simplified problem we now consider optimality of the firm in each of the three regions described in the text.
- **Region I - Unconstrained**
  - Consider an unconstrained firm. Its problem is

$$\pi_{ijk} = \max_{n_{ijk}} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}} - w_{ijk} n_{ijk}$$

    subject to its wage being given by the above labor supply system:

$$w\left(n_{ijkt}\right) = \left( \frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1}{\eta}} \left( \frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} X_{kt}.$$

    where $X_{kt}$ are aggregates the firm takes as given.

– The first order condition is

$$
\begin{aligned}
w_{ijk} + w'\left(n_{ijk}\right) n_{ijk} &= \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1} \\
w_{ijk}\left(1 + \frac{w'\left(n_{ijk}\right) n_{ijk}}{w_{ijk}}\right) &= \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1} \\
w_{ijk}\left(1 + \frac{1}{\varepsilon_{ijk}}\right) &= \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1} \\
w_{ijk} &= \frac{\varepsilon_{ijk}}{1+\varepsilon_{ijk}}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}
\end{aligned}
$$

where using the inverse labor supply curve gives

$$
\begin{aligned}
\frac{1}{\varepsilon_{ijk}} := \frac{w'\left(n_{ijk}\right) n_{ijk}}{w_{ijk}} = \frac{\partial \log w_{ijk}}{\partial \log n_{ijk}} &= \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\frac{\partial \log n_{jk}}{\partial \log n_{ijk}} = \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\widetilde{s}_{ijk} \\
\varepsilon_{ijk} &= \left[\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\widetilde{s}_{ijk}\right]^{-1}.
\end{aligned}
$$

– Therefore

$$
w_{ijk} = \mu_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}
$$

where the markdown depends on the firms' elasticity of labor supply.

– Note that since $p_{ijk} = 1$ since the firm is unconstrained, then $\widetilde{w}_{ijk} = p_{ijk}w_{ijk} = w_{ijk}$, so

$$
\widetilde{w}_{ijk} = \mu_{ijk} \times \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}
$$

- **Region III - Constrained, on labor demand curve**

  – Now consider a constrained firm in Region III, this firm's problem is

$$
\pi_{ijk} = \max_{n_{ijk}} \widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}} - \underline{w}n_{ijk}
$$

  – The solution to this problem is to choose employment to equate the marginal revenue product of labor to the minimum wage:

$$
\underline{w} = \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}
$$

  – For convenience when aggregating, we can express this in terms of shadow wages by multiplying through by the equilibrium multiplier on the rationing constraint

$$
\begin{aligned}
\underline{w}p_{ijk} &= p_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1} \\
\widetilde{w}_{ijk} &= p_{ijk} \times \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}
\end{aligned}
$$

- **Region II - Constrained, on labor supply curve**

  – Now consider a constrained firm in Region II, this firm simply has labor determined by the labor supply curve, but since the rationing constraint is slack, $\widetilde{w}_{ijk} = p_{ijk}w_{ijk} = \underline{w}$. Using our characterization results we can write this in terms of shadow wages:

$$n_{ijk} = \left(\frac{\underline{w}}{\widetilde{w}_{jk}}\right)^{\eta} \left(\frac{\widetilde{w}_{jk}}{\widetilde{W}_k}\right)^{\theta} n_k.$$

– Nonetheless, we can express the shadow wage of the firm as

$$\begin{aligned}
\widetilde{w}_{ijk} &= \widetilde{\mu}_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1} \\
\widetilde{\mu}_{ijk} &= \frac{\underline{w}}{\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}} \quad , \quad n_{ijk} = \left(\frac{\underline{w}}{\widetilde{w}_{jk}}\right)^{\eta} \left(\frac{\widetilde{w}_{jk}}{\widetilde{W}_k}\right)^{\theta} n_k.
\end{aligned}$$

– Therefore, in all three regions, we can express the *shadow wage* as a *shadow markdown* on the marginal revenue product of labor:

$$\widetilde{w}_{ijk} = \widetilde{\mu}_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}.$$

### F.1.3 Aggregation of output and labor demand conditions

- Using the above results for firm optimality and the household's labor supply system, where the first order conditions have been characterized in terms of shadow wages, we can aggregate the optimality conditions of agents. This is a key step in solving the government problem and optimal transfers, which we describe below.
- **Aggregation - Firm-Type to Market-Type**
  – From the above we have the following set of five conditions at the firm and market level:
  – **Firm level**:

$$\begin{aligned}
\widetilde{y}_{ijk} &= \widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}} \\
\widetilde{w}_{ijk} &= \widetilde{\mu}_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}. \\
n_{ijk} &= \left(\frac{\widetilde{w}_{ijk}}{\widetilde{w}_{jk}}\right)^{\eta} n_{jk}
\end{aligned}$$

  – **Aggregates**:

$$\begin{aligned}
\widetilde{y}_{jk} &= \sum_{i\in j} \widetilde{y}_{ijk} \\
\widetilde{w}_{jk} &= \left[\sum_{i\in j} \widetilde{w}_{ijk}^{1+\eta}\right]^{\frac{1}{1+\eta}}
\end{aligned}$$

– Following steps from Berger, Herkenhoff, Mongey (2022), these can be combined to yield:

$$\begin{aligned}
\widetilde{y}_{jk} &= \omega_{jk}\widetilde{Z}\widetilde{\xi}_k\widetilde{z}_j n_{jk}^{\widetilde{\alpha}} \\
\widetilde{w}_{jk} &= \widetilde{\mu}_{jk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_j\widetilde{\xi}_k n_{jk}^{\widetilde{\alpha}-1} \\
n_{jk} &= \left(\frac{\widetilde{w}_{jk}}{\widetilde{w}_k}\right)^{\theta} n_k
\end{aligned}$$

where the three wedges $\left\{\widetilde{z}_j, \widetilde{\mu}_{jk}, \omega_{jk}\right\}$ are given by

$$\begin{aligned}
\widetilde{z}_j &= \left[\sum_{i\in j} \widetilde{z}_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}} \\
\widetilde{\mu}_{jk} &= \left[\sum_{i\in j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \widetilde{\mu}_{ijk}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}} \\
\omega_{jk} &= \sum_{i\in j} \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}} \left(\frac{\widetilde{\mu}_{ijk}}{\widetilde{\mu}_{jk}}\right)^{\frac{\eta\widetilde{\alpha}}{1+\eta(1-\widetilde{\alpha})}}
\end{aligned}$$

– Note that this implies that if $\left\{\widetilde{z}_j, \widetilde{\mu}_{jk}, \widetilde{\omega}_{jk}\right\}$ are known, then $\left\{n_{jk}, \widetilde{w}_{jk}, \widetilde{y}_{jk}\right\}$ can be determined.

- **Aggregation - Market-Type to Type**

  – The same approach can be followed to aggregate to the household level, which delivers:

$$\begin{aligned}
\widetilde{y}_k &= \omega_k\widetilde{Z}\widetilde{\xi}_k\widetilde{z}_k n_k^{\widetilde{\alpha}} \\
\widetilde{w}_k &= \widetilde{\mu}_k\widetilde{\alpha}\widetilde{Z}\widetilde{z}_k\widetilde{\xi}_k n_k^{\widetilde{\alpha}-1}
\end{aligned}$$

  where

$$\begin{aligned}
\widetilde{z}_k &= \left[\int \widetilde{z}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}} \\
\widetilde{\mu}_k &= \left[\int \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \widetilde{\mu}_{jk}^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}} \\
\omega_k &= \int \left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \left(\frac{\widetilde{\mu}_{jk}}{\widetilde{\mu}_k}\right)^{\frac{\theta\widetilde{\alpha}}{1+\theta(1-\widetilde{\alpha})}} \omega_{jk}
\end{aligned}$$

- The conditions derived thus far all hold in a competitive equilibrium with lump sum transfers.

## F.2 General equilibrium

Combining the above, we can state the general equilibrium conditions of the economy, where the wedges $\{\widetilde{\mu}_k, \omega_k\}_{k=1}^{K}$ are determined by the market-level Nash equilibria described above, aggregated up to the household $k$ level.

1. **Macro to micro** - Suppose the following are determined by market equilibria for all types of workers $k$ and in all markets $j$, where firms take aggregate quantities $\{C_k, N_k, Y_k, K_k\}_{k=1}^K$ and prices $\{\widetilde{W}_k\}_{k=1}^K$, $R$ as given

$$\underbrace{\widetilde{z}_k := \left[\int \widetilde{z}_{jk}^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}}_{\text{1. Type productivity}}, \underbrace{\widetilde{\mu}_k := \left[\int \left(\frac{\widetilde{z}_{jk}}{\widetilde{z}_k}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \widetilde{\mu}_{jk}^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}}_{\text{2. Type shadow markdown}}, \underbrace{\omega_k := \int \left(\frac{\widetilde{z}_{jk}}{\widetilde{z}_k}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \left(\frac{\widetilde{\mu}_{jk}}{\widetilde{\mu}_k}\right)^{\frac{\theta\alpha}{1+\theta(1-\widetilde{\alpha})}} \omega_{jk}}_{\text{3. Type misallocation}}.$$

2. **Micro to macro** - For each $k$, under $\{\widetilde{z}_k, \widetilde{\mu}_k, \omega_k\}_{k=1}^K$, aggregate quantities $\{C_k, N_k, Y_k, K_k\}_{k=1}^K$ and prices $\{\widetilde{W}_k\}_{k=1}^K$, $R$ satisfy:

$$\textit{Output}: \quad \widetilde{Y}_k = \omega_k \widetilde{Z}\widetilde{\xi}_k \widetilde{z}_k N_k^{\widetilde{\alpha}} \quad , \quad Y_k = \frac{1}{1-(1-\gamma)\alpha}\widetilde{Y}_k$$

$$\textit{Capital supply and demand}: \quad 1 = \beta(R + (1-\delta)) \quad , \quad R = \alpha(1-\gamma)\frac{Y_k}{K_k} \quad , \quad K_k = \kappa_k K$$

$$\textit{Labor supply and demand}: \quad N_k = \pi_k \widetilde{\varphi}_k \left(\frac{\widetilde{W}_k}{P}\right)^{\varphi} C_k^{-\sigma\varphi} \quad , \quad \widetilde{W}_k = \widetilde{\mu}_k \widetilde{\alpha} \widetilde{Z}\widetilde{\xi}_k \widetilde{z}_k N_k^{\widetilde{\alpha}-1}$$

$$\textit{Budget constraint}: \quad C_k + \delta K_k = \int \sum_{i\in k} w_{ijk} n_{ijk}\, dj + RK_k + \kappa_k \Pi$$

where aggregate profits are consistent:

$$\Pi = \sum_k Y_k - \int \sum_{i\in k} w_{ijk} n_{ijk}\, dj - R\sum_k K_k$$

These conditions yield three results. First, they show that the market-level lesson of focusing on the shadow markdown and misallocation carries over to the aggregate economy, when these wedges are appropriately aggregated. Second, they provide an algorithm to solve the competitive equilibrium, given $\{z_k, \widetilde{\mu}_k, \omega_k\}_{k=1}^K$, which later will allow us understand the role of different wedges in aggregate welfare. Third, they show how the shadow wages that we have constructed are allocative for quantities. Household labor supply $N_k$ is pinned down by the shadow wage $\widetilde{W}_k$ that the household faces.

## F.3 Government problem - Summary and brief results

*This section should be read as a summary, with the full derivations in the following subsection.*

To separate out the redistribution and efficiency effects of a minimum wage, we consider the problem of a government with social welfare weights $\{\psi_k\}_{k=1}^k$. The government faces prices determined by the imperfectly competitive labor market where firms are subject to the minimum wage. The government is given access to lump-sum taxes $\{T_k\}_{k=1}^K$, with the restriction that total lump sum taxes add to zero. We take the standard approach of solving for the optimal allocation, then the transfers that implement it.

**Problem.** The government chooses allocations of consumption and labor to maximize

$$\mathcal{U} = \sum_k \psi_k \sum_{t=0}^{\infty} u^k\left(\frac{C_{kt}}{\pi_k}, N_{kt}\right) = \sum_k \psi_k \sum_{t=0}^{\infty} \left[\frac{C_{kt}^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{N_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right].$$

We can define the following aggregate consumption and labor indices, and use these to write social welfare as follows:

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[ \frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\overline{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right] \quad , \quad \mathcal{C}_t := \left[ \sum_k \psi_k \left( \frac{C_{kt}}{\pi_k} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad , \quad \mathcal{N}_t := \left[ \sum_k \frac{\psi_k}{\widetilde{\varphi}_k^{1/\varphi}} N_{kt}^{\frac{1+\varphi}{\varphi}} \right]^{\frac{\varphi}{1+\varphi}}$$

This problem can be solved subject to an aggregate budget constraint, and then implemented using lump sum taxes. The government also takes labor rationing constraints into account. Under this approach, the government is endowed with $K_0$ units of capital, and maximizes social welfare subject to:

$$\sum_k C_{kt} + K_{t+1} = \sum_k \int \sum_{i \in j} w_{ijkt} n_{ijkt} \, dj + R_t K_t + (1-\delta) K_t + \Pi_t \quad , \quad n_{ijkt} \leq \overline{n}_{ijkt} \tag{F1}$$

Optimization delivers an identical set of labor supply conditions to firms as the competitive equilibrium given in the main text. These involve $\{\widetilde{W}_k, N_k, C_k\}$, which in the decentralized economy were determined by each household's labor supply curve and budget constraint. In the government's allocation problem these are instead determined by the government's optimality conditions:

$$C_{kt} = \pi_k \left( \frac{\psi_k}{\pi_k} \right)^{\frac{1}{\sigma}} \left( \frac{1}{\mathcal{P}_t} \right)^{-\frac{1}{\sigma}} \mathcal{C}_t \quad , \quad N_{kt} = \pi_k \widetilde{\varphi}_k \left( \frac{\psi_k}{\pi_k} \right)^{-\varphi} \left( \frac{\widetilde{W}_{kt}}{\widetilde{\mathcal{W}}_t} \right)^{\varphi} \mathcal{N}_t \quad , \quad \mathcal{N}_t = \overline{\varphi} \left( \frac{\widetilde{\mathcal{W}}_t}{\mathcal{P}_t} \right)^{\varphi} \mathcal{C}_t^{-\sigma\varphi}. \tag{F2}$$

Higher social welfare weights relative to population shares entail a higher share of consumption and less labor supply, where the latter is offset if relative wages of the type are higher, or disutility of work is lower (higher $\widetilde{\varphi}_k$). The aggregate shadow price $\mathcal{P}_t$—which is such that $\mathcal{P}_t \mathcal{C}_t = \sum_k C_{kt}$—and aggregate shadow wage $\widetilde{\mathcal{W}}_t$ indexes are given by

$$\mathcal{P}_t = \left[ \sum_k \psi_k^{\frac{1}{\sigma}} \pi_k^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad , \quad \widetilde{\mathcal{W}}_t = \left[ \sum_k \widetilde{\varphi}_k \pi_k \left( \frac{\psi_k}{\pi_k} \right)^{-\varphi} \widetilde{W}_{kt}^{1+\varphi} \right]^{\frac{1}{1+\varphi}}.$$

The derivation of all of these results are given in the following subsection.

**Implementation.** The planner can implement this allocation in a competitive economy in which households are endowed with shares of capital and profits, by choosing lump sum transfers $T_k$. These can be read off of each household's budget constraint under equilibrium prices and the government's desired allocation:

$$T_k = \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj + (R+\delta) \kappa_k K + \kappa_k \Pi - C_k. \tag{F3}$$

To see that this implements the government's solution, observe that combining conditions in (F2) yields the decentralized household labor supply curves, and that the government's steady-state Euler equation coincides with each household's in the competitive equilibrium. Since taxes are lump-sum, their presence does not distort these conditions.[58] Finally, summing budget constraints (F3) obtains the planner's budget constraint (F1), and hence transfers sum to zero. The following subsection steps through these.

[58] As is standard, comparing households' and the planner's first order conditions for consumption reveal that the social welfare weights map into multipliers on households budget constraints, which are constant in steady-state. Denote these multipliers $\nu_k$. Normalize $\psi_1 = 1$, then $\psi_k = \nu_1/\nu_k$. Hence, starting with some social welfare weights, the implied allocation can be decentralized by budget-neutral lump-sum taxes. Lump sum transfers tighten and loosen budget constraints, so can be chosen to align multipliers with the planner's social welfare weights.

**Aggregates.** To solve the government's problem still requires the determination of aggregates $\mathcal{C}$, $\widetilde{\mathcal{W}}$, and $\mathcal{N}$. Under a given set of social welfare weights, market equilibria can be aggregated to obtain shadow markdowns for all types: $\{\widetilde{\mu}_k, \omega_k\}_{k=1}^{K}$. These can be further aggregated, where $\phi_k = \pi_k^{1+\varphi}\widetilde{\varphi}_k\psi_k^{-\varphi}$:

$$\widetilde{z} = \left[\sum_k (\xi_k \widetilde{z}_k)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}}\right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}}, \widetilde{\mu} = \left[\sum_k \left(\frac{\xi_k\widetilde{z}_k}{\widetilde{z}}\right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \widetilde{\mu}_k^{\frac{1+\varphi}{1+\varphi(1-\alpha)}}\right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}}, \omega = \sum_k \left(\frac{\xi_k\widetilde{z}_k}{\widetilde{z}}\right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \left(\frac{\widetilde{\mu}_k}{\widetilde{\mu}}\right)^{\frac{\varphi\alpha}{1+\varphi(1-\alpha)}} \omega_k$$

Weights now account for productivity shifters $\xi_k$, household measures $\pi_k$, labor disutility $\widetilde{\varphi}_k$ and social welfare weights $\psi_k$. Given $\{\mathcal{P}, \widetilde{z}, \widetilde{\mu}, \omega\}$, the following can be solved for $\mathcal{C}, \widetilde{\mathcal{W}}$, and $\mathcal{N}$ in closed form:

$$\underbrace{Y = \frac{\omega \widetilde{Z}\widetilde{z}\mathcal{N}^{\widetilde{\alpha}}}{1-(1-\gamma)\alpha}}_{\text{Output}} \quad , \quad \underbrace{\mathcal{P}\mathcal{C} = Y - \delta K}_{\text{Resource constraint}} \quad , \quad \underbrace{\widetilde{\mathcal{W}} = \widetilde{\mu}\widetilde{\alpha}\widetilde{Z}\widetilde{z}\mathcal{N}^{\widetilde{\alpha}-1}}_{\text{Labor demand}} \quad , \quad \underbrace{\mathcal{N} = \overline{\varphi}\left(\frac{\widetilde{\mathcal{W}}}{\mathcal{P}}\right)^{\varphi} \mathcal{C}^{-\varphi\sigma}}_{\text{Labor supply}} \tag{F4}$$

To allocate these aggregates across households, we use the government's first order conditions (F2).

**Negishi weights.** Our baseline calibration of the model is a competitive equilibrium with zero lump sum taxes. This yields an allocation of labor, consumption and capital. Note that there exists a vector of social welfare weights $\{\psi_k^*\}_{k=1}^{K}$ such that a government with these weights would choose the same allocation, also with zero lump sum taxes. As is standard, we refer to this vector of social welfare weights as the *Negishi weights*. Computing the Negishi weights associated with the benchmark competitive equilibrium is a key step in our welfare exercise. Optimal policy under this benchmark can be compared to optimal policy under alternative weights, such as Utilitarian weights. Incidentally, we also exploit the associated *Negishi algorithm* to make feasible the computation of the competitive equilibrium with $K$ types.[59]

## F.4 Government problem - Details

- We consider the government primal problem where it chooses an allocation of (i) labor from each household to all firms (ii) consumption of all households, (iii) investment. We then show that the government can decentralize this allocation in a competitive equilibrium by choosing appropriate lump sum transfers.
- This has the flavor of a 'partial' planning problem. 'Partial' in the sense that the government takes as given the prices of firms in the economy, and firms' rationing constraints, where these are due to the market power of firms. The government therefore faces a *budget constraint* rather than a resource constraint.

[59] In particular, we can guess a set of Negishi weights, normalizing $\psi_1^* = 1$. First, we solve market equilibria, to obtain $\{\widetilde{\mu}_k, \omega_k\}_{k=1}^{K}$. Using the guessed Negishi weights we can compute $\widetilde{z}, \widetilde{\mu}, \omega, \mathcal{P}$ from the above expressions, and then use these to solve for $Y, \mathcal{W}, \mathcal{C}, \mathcal{N}$ using equations (F4). Using the planner's first order conditions (F2), we can allocate $\mathcal{C}$ among households, and hence compute implied household consumption $C_k$. We can also compute firm wages and employment. Following the tradition of the Negishi algorithm, we then compute the implied residual in the household's budget constraint—under $T_k = 0$—and update our guess of $\{\psi_k^*\}_{k\neq 1}^{K}$, until this residual is zero. We lower $\psi_k^*$ for households with a deficit, and increase $\psi_k^*$ for households with a surplus.

### F.4.1 Allocation problem

- The government is endowed with $K_0$, takes prices $\left\{w_{ijkt}\right\}$, profits $\{\Pi_t\}$, rationing constraints $\left\{\overline{n}_{ijkt}\right\}$ as given and chooses directly $\left\{c_{kt}, n_{ijkt}, K_{t+1}\right\}$ to maximize

$$U_0 = \sum_{t=0}^{\infty} \beta^t \sum_k \psi_k \left[ \frac{(c_{kt}/\pi_k)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

where

$$n_{kt} = \left[ \int n_{jkt}^{\frac{\theta+1}{\theta}} dj \right]^{\frac{\theta}{\theta+1}}$$

$$n_{jkt} = \left[ \sum_{i \in j} n_{ijkt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

subject to its budget constraint

$$\sum_k c_{kt} + K_{t+1} = \sum_k \int \sum_{i \in j} w_{ijkt} n_{ijkt} dj + R_t K_t + (1-\delta) K_t + \Pi_t$$

rationing constraints

$$n_{ijk} \leq \overline{n}_{ijk}$$

- Here $\widetilde{\varphi}_k = \overline{\varphi}_k \pi_k^{\varphi+1}$ is adjusted for the measure of workers of the household.
- We can rewrite the objective function as

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left[ \frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\widehat{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

where

$$\mathcal{C}_t = \left[ \sum_k \psi_k \left( \frac{c_{kt}}{\pi_k} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

$$\mathcal{N}_t = \left[ \sum_k \left( \frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}} \right) n_{kt}^{\frac{\varphi+1}{\varphi}} \right]^{\frac{\varphi}{\varphi+1}}$$

and $\widehat{\varphi}_k = \widetilde{\varphi}_k / \widehat{\varphi}$.

- Let $\beta^t \Lambda_t$ be the multiplier on the government's budget constraint.

### F.4.2 Allocation problem - Consumption

- The first order condition for $c_{kt}$ gives the following:

$$\psi_k \left(\frac{c_{kt}}{\pi_k}\right)^{1-\sigma} = \Lambda_t p_{kt} c_{kt}$$
$$c_{kt} = \pi_k \left(\frac{\Lambda_t p_{kt} \pi_k}{\psi_k}\right)^{-\frac{1}{\sigma}}$$

- Suppose there exists some $\mathcal{P}_t$ such that aggregate consumption $C_t = \sum_k c_{kt} = \mathcal{P}_t \mathcal{C}_t$.
- Using the first order condition we can obtain:

$$\Lambda_t = \frac{\mathcal{C}_t^{-\sigma}}{\mathcal{P}_t}$$
$$c_{kt} = \pi_k \left(\frac{\psi_k}{\pi_k}\right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}_t}\right)^{-\frac{1}{\sigma}} \mathcal{C}_t$$
$$\mathcal{P}_t = \left[\sum_k \psi_k^{\frac{1}{\sigma}} \pi_k^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}}$$

- We can substitute $\sum_k c_{kt} = \mathcal{P}_t \mathcal{C}_t$ into the planner's problem to obtain the following problem, where the distribution of $\mathcal{C}_t$ among households is determined by

$$c_{kt} = \pi_k \left(\frac{\psi_k}{\pi_k}\right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}_t}\right)^{-\frac{1}{\sigma}} \mathcal{C}_t$$

- The government's reduced problem is therefore to choose $\left\{\mathcal{C}_t, n_{ijkt}\right\}$ to maximize

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left[\frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\widehat{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right]$$
$$\mathcal{N}_t = \left[\sum_k \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) n_{kt}^{\frac{\varphi+1}{\varphi}}\right]^{\frac{\varphi}{\varphi+1}}$$
$$n_{kt} = \left[\int n_{jkt}^{\frac{\theta+1}{\theta}} dj\right]^{\frac{\theta}{\theta+1}}$$
$$n_{jkt} = \left[\sum_{i\in j} n_{ijkt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$$

subject to

$$\mathcal{P}_t \mathcal{C}_t + K_{t+1} = \sum_k \int \sum_{i\in j} w_{ijkt} n_{ijkt} dj + R_t K_t + (1-\delta) K_t + \Pi_t$$

and rationing constraints

$$n_{ijk} \leq \overline{n}_{ijk}$$

- The planner's first order condition for $\mathcal{C}_t$ is then

$$U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right) = \Lambda_t \mathcal{P}_t$$

where

$$U\left(\mathcal{C}_t, \mathcal{N}_t\right) = \frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\widehat{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}.$$

### F.4.3 Allocation problem - Labor

- Consider the terms in the government's Lagrangean that feature $n_{ijkt}$
- Write the multiplier on the rationing constraint as $\zeta_{ijkt} = \beta^t \Lambda_t w_{ijkt} \left(1 - p_{ijkt}\right)$
- These terms are

$$\mathcal{L} = \cdots + \beta^t U\left(\mathcal{C}_t, \mathcal{N}_t\right) + ... + \beta^t \Lambda_t w_{ijkt} n_{ijkt} + \beta^t \Lambda_t w_{ijkt} \left(1 - p_{ijkt}\right) \left[\overline{n}_{ijkt} - n_{ijkt}\right] + ...$$
$$\mathcal{L} = \cdots + \beta^t U\left(\mathcal{C}_t, \mathcal{N}_t\right) + ... + \beta^t \Lambda_t w_{ijkt} p_{ijkt} n_{ijkt} + \beta^t \Lambda_t w_{ijkt} \left(1 - p_{ijkt}\right) \overline{n}_{ijkt} + ...$$

- The first order condition for consumption is as above:

$$\Lambda_t = U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right) / \mathcal{P}_t$$

- The first order condition for $n_{ijkt}$ is

$$w_{ijkt} p_{ijkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right) / \mathcal{P}_t} \left(\frac{\partial \mathcal{N}_t}{\partial n_{kt}}\right) \left(\frac{\partial n_{kt}}{\partial n_{jkt}}\right) \left(\frac{\partial n_{jkt}}{\partial n_{ijkt}}\right)$$

- Using the definitions of aggregators $\mathcal{N}_t$, $n_{kt}$, $n_{jkt}$:

$$w_{ijkt} p_{ijkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right) / \mathcal{P}_t} \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) \left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}$$

- Define the *shadow wage* $\widetilde{w}_{ijkt} := w_{ijkt} p_{ijkt}$.
- Using this definition:

$$\widetilde{w}_{ijkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right) / \mathcal{P}_t} \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) \left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \qquad (*)$$

- We now define the following *shadow wage indexes* at the market, type and aggregate level:

$$\widetilde{w}_{jkt} := \left[\sum_{i\in j} \widetilde{w}_{ijkt}^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

$$\widetilde{w}_{kt} := \left[\int \widetilde{w}_{jkt}^{1+\theta} dj\right]^{\frac{1}{1+\theta}}$$

$$\widetilde{\mathcal{W}}_{t} := \left[\sum_{k} \left(\frac{\widehat{\varphi}_k}{\psi_k^{\varphi}}\right) \widetilde{w}_{kt}^{1+\varphi}\right]^{\frac{1}{1+\varphi}}$$

- Using the definition of $\widetilde{w}_{jkt}$ and $n_{jkt} = \left[\sum_{i\in j} n_{ijkt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$ in $(*)$ we have

$$\widetilde{w}_{jkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right)/\mathcal{P}_t} \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) \left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}$$

- Then using the definition of $\widetilde{w}_{kt}$ and $n_{kt} = \left[\int n_{jkt}^{\frac{\theta+1}{\theta}} dj\right]^{\frac{\theta}{\theta+1}}$:

$$\widetilde{w}_{kt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right)/\mathcal{P}_t} \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) \left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}}$$

- Then using the definition of $\widetilde{\mathcal{W}}_t$ and $\mathcal{N}_t = \left[\sum_k \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) n_{kt}^{\frac{\varphi+1}{\varphi}}\right]^{\frac{\varphi}{\varphi+1}}$:

$$\widetilde{\mathcal{W}}_t = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right)/\mathcal{P}_t}$$

- Using $U\left(\mathcal{C}_t, \mathcal{N}_t\right)$ we can obtain what we will refer to as the *aggregate labor supply curve*

$$\widetilde{\mathcal{W}}_t = \widehat{\varphi}^{-\frac{1}{\varphi}} \mathcal{P}_t \mathcal{C}_t^{\sigma} \mathcal{N}_t^{\frac{1}{\varphi}}$$

$$\mathcal{N}_t = \widehat{\varphi} \left(\frac{\widetilde{\mathcal{W}}_t}{\mathcal{P}_t}\right)^{\varphi} \mathcal{C}_t^{-\varphi\sigma}$$

- A key result is that this is the labor supply curve that would obtain from a government that maximizes $U\left(\mathcal{C}_t, \mathcal{N}_t\right)$ subject to a budget constraint

$$\mathcal{P}_t \mathcal{C}_t + K_{t+1} = \widetilde{\mathcal{W}}_t \mathcal{N}_t + R_t K_t + \left(1-\delta\right) K_t + \Pi_t$$

  and faced no rationing constraints. However such a budget constraint is incorrect, in that $\widetilde{\mathcal{W}}_t \mathcal{N}_t \neq \sum_k \int \sum_{i\in j} w_{ijkt} n_{ijkt} dj$. Nonetheless, the interpretation of the aggregate labor supply curves holds, and shows exactly the extent to which the economy supplies labor *as if* it faced a wage $\widetilde{\mathcal{W}}_t$.

### F.4.4 Implied labor supply system to firms

- Using the above results we can refine the labor supply system.
- Using the aggregate labor supply curve in the type-level expression above, we have

$$\widetilde{w}_{kt} = \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right)\left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}} \widetilde{\mathcal{W}}_t$$

$$n_{kt} = \left(\frac{\widehat{\varphi}_k}{\psi_k^{\varphi}}\right)\left(\frac{\widetilde{w}_{kt}}{\widetilde{\mathcal{W}}_t}\right)^{\varphi} \mathcal{N}_t$$

- Using this in the market-type-level expression above:

$$\widetilde{w}_{jkt} = \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \widetilde{w}_{kt}$$

$$\implies n_{jkt} = \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}$$

- Using this in the firm-market-type-level expression $(*)$ above, we then recover the same labor supply system as the competitive equilibrium:

$$\widetilde{w}_{ijkt} = \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}\left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \widetilde{w}_{kt}$$

  which can then be written:

$$n_{ijkt} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{\eta}\left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}$$

$$\widetilde{w}_{jkt} = \left[\sum_{i\in j} \widetilde{w}_{ijkt}^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

$$\widetilde{w}_{kt} = \left[\int \widetilde{w}_{jkt}^{1+\theta} dj\right]^{\frac{1}{1+\theta}}$$

- **Result A** - *This corresponds to the labor supply system from type-k household optimality in a competitive equilibrium with lump sum transfers $T_k$.*
  - This follows immediately from our derivation in the competitive equilibrium, and the fact that the presence of lump sum transfers in household $k$ 's budget constraint do not affect any such derivations.

### F.4.5 Implied household labor supply curves

- Combining the planner's allocation of consumption

$$c_{kt} = \pi_k \left(\frac{\psi_k}{\pi_k}\right)^{\frac{1}{\sigma}}\left(\frac{1}{\mathcal{P}_t}\right)^{-\frac{1}{\sigma}} \mathcal{C}_t$$

the aggregate labor supply curve

$$\mathcal{N}_t = \widehat{\varphi}\left(\frac{\widetilde{\mathcal{W}}_t}{\mathcal{P}_t}\right)^{\varphi} \mathcal{C}_t^{-\varphi\sigma}$$

and the planner's allocation of labor

$$n_{kt} = \left(\frac{\widehat{\varphi}_k}{\psi_k^{\varphi}}\right)\left(\frac{\widetilde{w}_{kt}}{\widetilde{\mathcal{W}}_t}\right)^{\varphi} \mathcal{N}_t$$

by substituting out the planner's social welfare weight $\psi_k$, obtains

$$n_{kt} = \pi_k^{\varphi(\sigma-1)} \widehat{\varphi}\widehat{\varphi}_k \widetilde{w}_{kt}^{\varphi} c_{kt}^{-\varphi\sigma}$$

- Using definition of $\widehat{\varphi}_k = \widetilde{\varphi}_k / \widehat{\varphi}$

$$n_{kt} = \pi_k^{\varphi(\sigma-1)} \widetilde{\varphi}_k \widetilde{w}_{kt}^{\varphi} c_{kt}^{-\varphi\sigma}$$

- Using the definition of $\widetilde{\varphi}_k = \overline{\varphi}_k \pi_k^{\varphi+1}$:

$$n_{kt} = \pi_k \overline{\varphi}_k \widetilde{w}_{kt}^{\varphi} \left(\frac{c_{kt}}{\pi_k}\right)^{-\varphi\sigma}$$

- **Result B** - *This corresponds to the household labor supply curve from type-k household optimality in a competitive equilibrium with lump sum transfers $T_k$.*
  - This follows immediately from our derivation in the competitive equilibrium, and the fact that the presence of lump sum transfers in household $k$ 's budget constraint do not affect any such derivations.

### F.4.6 Further conditions

- From the above we have obtained the aggregate supply curve. We also have the aggregate resource constraint, which in steady-state is:

$$Y_t = \sum_k c_{kt} + \delta K_t.$$

- Using the consumption results from above, this can be written

$$Y_t = \mathcal{P}_t \mathcal{C}_t + \delta K_t.$$

- Recall also, that we have the aggregation of output

$$Y_t = \frac{1}{1-\gamma(1-\alpha)} \widetilde{Y}_t$$
$$\widetilde{Y}_t = \sum_k \widetilde{y}_{kt}$$

- The steady-state Euler equation of the government is

$$1 = \beta\left[R + (1-\delta)\right]$$

- **Result C** - *This corresponds to the household Euler equation from type-k household optimality in a competitive equilibrium with lump sum transfers $T_k$.*
  - This follows immediately from our derivation in the competitive equilibrium, and the fact that the presence of lump sum transfers in household $k$ 's budget constraint do not affect any such derivations.

### F.4.7 Aggregating labor demand and output

- From **Result A** above, the labor supply system for type $k$ labor from the solution to the government's primal (allocation) problem corresponds to the labor supply system in the competitive equilibrium.
- Firm optimality conditions will therefore be the same as in the competitive equilibrium, and the aggregation results derived earlier hold up to the type-$k$ level.
- Recall that these results yielded the following. For type-$k$, output, the shadow wage index, labor supply are as follows, where the third line is the new solution to the government's supply of type-$k$ labor

$$\widetilde{y}_k = \omega_k \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_k n_k^{\widetilde{\alpha}}$$
$$\widetilde{w}_k = \widetilde{\mu}_k \widetilde{\alpha} \widetilde{Z} \widetilde{z}_k \widetilde{\xi}_k n_k^{\widetilde{\alpha}-1}$$
$$n_k = \phi_k \left( \frac{\widetilde{w}_k}{\widetilde{\mathcal{W}}} \right)^{\varphi} \mathcal{N}$$

where $\phi_k = \pi_k^{1+\varphi} \left( \frac{\overline{\varphi}_k}{\widehat{\varphi}} \right) \psi_k^{-\varphi}$, where $\{\omega_k, \widetilde{\mu}_k, \widetilde{z}_k\}$ are as in the competitive equilibrium, derived above.

- We also have two aggregation conditions, where we have substituted in

$$\widetilde{Y} = \sum_k \widetilde{y}_k$$
$$\widetilde{\mathcal{W}} = \left[ \sum_k \phi_k \widetilde{w}_{kt}^{1+\varphi} \right]^{\frac{1}{1+\varphi}}$$

- In the same way as we used the sets of 5 conditions to aggregate output and labor demand in the competitive equilibrium we can also use the same approach on this set of conditions.
- The result is that aggregate output and the aggregate shadow wage can be expressed using aggregate shadow markdown, productivity and misallocation wedges:

$$\widetilde{Y} = \omega \widetilde{Z} \widetilde{z} \mathcal{N}^{\alpha}$$
$$\widetilde{\mathcal{W}} = \widetilde{\mu} \alpha \widetilde{Z} \widetilde{z} \mathcal{N}^{\alpha-1}$$

where

$$\widetilde{z} = \left[ \sum_k \left( \xi_k \widetilde{z}_k \right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}}$$

$$\widetilde{\mu} = \left[ \sum_k \left( \frac{\xi_k \widetilde{z}_k}{\widetilde{z}} \right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \widetilde{\mu}_k^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}}$$

$$\omega = \sum_k \left( \frac{\xi_k \widetilde{z}_k}{\widetilde{z}} \right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \left( \frac{\widetilde{\mu}_k}{\widetilde{\mu}} \right)^{\frac{\varphi\alpha}{1+\varphi(1-\alpha)}} \omega_k$$

- Capital demand is as in the competitive equilibrium:

$$Rk_{ijk} = \alpha \left(1 - \gamma\right) y_{ijk}$$

which when aggregated yeilds

$$RK = \alpha \left(1 - \gamma\right) Y$$

### F.4.8 Full set of conditions for the solution of government allocation problem and competitive equilibrium

- Equilibrium under the government allocation problem, can therefore be summarized in the following conditions, given the wedges $\{z, \widetilde{\mu}, \omega\}$:

$$\begin{aligned}
\widetilde{Y} &= \omega \widetilde{Z} \widetilde{z} \mathcal{N}^{\alpha} \\
\mathcal{W} &= \widetilde{\mu} \alpha \widetilde{Z} \widetilde{z} \mathcal{N}^{\alpha-1} \\
\mathcal{N} &= \widehat{\varphi} \left( \frac{\widetilde{\mathcal{W}}}{\mathcal{P}} \right)^{\varphi} \mathcal{C}^{-\varphi\sigma} \\
Y &= \mathcal{P}\mathcal{C} + \delta K \\
Y &= \frac{1}{1 - \gamma\left(1 - \alpha\right)} \widetilde{Y} \\
\mathcal{P} &= \left[ \sum_k \psi_k^{\frac{1}{\sigma}} \pi_k^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}} \\
1 &= \beta \left[ R + (1 - \delta) \right] \\
RK &= \alpha \left(1 - \gamma\right) Y
\end{aligned}$$

- This system of 8 equations in 8 unknowns $\left\{ \widetilde{Y}, Y, \mathcal{N}, \mathcal{C}, \mathcal{P}, R, K, \mathcal{W} \right\}$ can be solved in closed form.

- Once solved, and given $\{\widetilde{w}_k\}_{k=1}^{K}$, household type variables can be determined from the government's first order conditions:

$$n_k = \phi_k \left(\frac{\widetilde{w}_k}{\widetilde{\mathcal{W}}}\right)^{\varphi} \mathcal{N}$$

$$c_k = \pi_k \left(\frac{\psi_k}{\pi_k}\right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}}\right)^{-\frac{1}{\sigma}} \mathcal{C}$$

- The allocation of labor $n_k$ to firms is then determined by the labor supply system:

$$n_{ijk} = \left(\frac{\widetilde{w}_{ijk}}{\widetilde{w}_{jk}}\right)^{\eta} \left(\frac{\widetilde{w}_{jk}}{\widetilde{w}_k}\right)^{\theta} n_k$$

### F.4.9 Implementation with lump-sum taxes

- **Results A, B, C** above imply that the government's optimality conditions of its allocation problem coincide with those of households in a competitive equilibrium with lump sum transfers $T_k$.
- Therefore the government can choose arbitrary lump sum transfers and yield the same set of optimality conditions.
- The only thing that is left is to determine the lump sum transfers themselves.
- These can simply be read off of the household's budget constraints, which in steady state are:

$$c_k = \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj + \kappa_k \left[(R - \delta) K + \Pi\right] + T_k$$

$$T_k = c_k - \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj - \kappa_k \left[(R - \delta) K + \Pi\right]$$

- Transfers clearly sum to zero since summing the household budget constraint yeilds the government budget constraint if and only if $\sum_k T_k = 0$.

## F.5 Leveraging the government solution to solve the competitive equilibrium

- In practice we leverage the government problem described above to solve the competitive equilibrium of the economy.
- We do this in the trandition of the *Negishi algorithm.*
- The above section described how we can first *fix social welfare weights*, then solve the government problem, then determine the required lump-sum transfers.
- The competitive equilibrium can be solved under *guessing of social welfare weights*, then solve the government problem, then determine the required lump-sum transfers, and then *iterating on the guess of social welfare weights*, until the implied lump sum transfers are all equal to zero.
- Under the social welfare weights that deliver zero lump sum transfers, the competitive equilibrium budget constraints of all households hold by construction:

$$c_k = \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj + \kappa_k \left[(R - \delta) K + \Pi\right]$$

and all remaining competitive equilibrium conditions also hold (i.e. each household's Euler equation, labor supply system to firms, household labor supply curve, resource constraint, capital and labor demand).

- The solution of the government problem, which can be achieved largely in closed form, is therefore a key part of our computational strategy.

# G Search model

In this appendix, we show how the simple monopsony model can be recast in terms of search effort. In this environment, firms and workers take equilibrium meeting rates as given.

**Household problem.** The household takes matching rates $\{p_i\}_{i\in[0,1]}$, wages $\{w_i\}_{i\in[0,1]}$ and profits $\Pi$ as given. The household chooses search effort $\{s_i\}_{i\in[0,1]}$ sent to each firm. The household's problem is:

$$\max_{\{s_i\}_{i\in[0,1]}} u(C,S) \quad , \quad S = \left[\int s_i^{\frac{1+\eta}{\eta}} di\right]^{\frac{\eta}{1+\eta}}$$

subject to its budget constraint, which captures that only $s_i$ units of search effort yield a match at firm $i$:

$$C = \int w_i p_i s_i \, di + \Pi$$

The first order conditions for $s_i$ and $C$ yields the optimality condition for household search effort at firm, which we refer to as the *inverse labor supply schedule:*

$$w_i = \frac{1}{p_i}\left(\frac{s_i}{S}\right)^{\frac{1}{\eta}}\left(-\frac{U_S}{U_C}\right)$$

**Firm problem.** Firm $i$ takes as given the aggregate search index $S$ and the matching rate $p_i$, and chooses its (i) wage $w_i$, and (ii) employment $n_i = p_i s_i$ to maximize profits. The firm is constrained by (a) the minimum wage $w_i \geq \underline{w}$, and (b) the inverse search supply curve of households. Therefore the firm problem is given by:

$$\max_{s_i} z_i (p_i s_i)^\alpha - w(s_i, p_i, S)\, p_i s_i$$

$$\text{subject to} \quad w(s_i, p_i, S) \geq \underline{w}$$

$$w(s_i, p_i, S) = \frac{1}{p_i}\left(\frac{s_i}{S}\right)^{\frac{1}{\eta}}\left(-\frac{U_S}{U_C}\right)$$

Under our assumption of Cournot competition, the firm understands $\frac{\partial w(s_i,p_i,S)}{\partial s_i} \neq 0$, yielding monopsony power.

**Matching technology.** We assume a constant returns to scale matching function given by the *short-side matching function*. This gives $p_i = \min\{s_i, n_i\}/s_i$ from the household's perspective. If $s_i < n_i$, then $p_i = 1$ and all units of search are converted into a job. If $s_i > n_i$, then $p_i = n_i/s_i < 1$ and only some of the units of search effort lead to a job.

**Comparison.** A comparison of the two economies is then as follows

- In the monopsony model, aggregation can be written in terms of

$$Y = \omega Z N^\alpha \quad , \quad \widetilde{W} = \widetilde{\mu}\alpha Z N^{\alpha-1} \quad , \quad N = \overline{\varphi}\widetilde{W}^{\varphi} C^{-\varphi\sigma} \quad , \quad C = Y$$

- In the search model, aggregation can be written in terms of

$$Y = \omega Z S^\alpha \quad , \quad \widetilde{W} = \widetilde{\mu}\alpha Z S^{\alpha-1} \quad , \quad S = \overline{\varphi}\widetilde{W}^{\varphi} C^{-\varphi\sigma} \quad , \quad C = Y$$

Table G1: Comparison of monopsony and search models

| | **Monopsony** | **Search** |
|---|---|---|
| Preferences | $U(C,N)$ | $U(C,S)$ |
| Disutility | *... of labor* | *... of search effort* |
| | $N=\left[\int n_i^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$ | $S=\left[\int s_i^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$ |
| Budget constraint | $C=\int w_i n_i di+\Pi$ | $C=\int p_i w_i s_i+\Pi$ <br> $=\widetilde{W}S+\Pi$ |
| Optimality condition | $p_i w_i=\left(\frac{n_i}{N}\right)^{\frac{1}{\eta}}N^{\frac{1}{\varphi}}$ | $p_i w_i=\left(\frac{s_i}{S}\right)^{\frac{1}{\eta}}S^{\frac{1}{\varphi}}$ |
| Wedge | *Rationing multiplier* | *Job finding rate* |
| | $p_i\in(0,1]$ if $n_i=\overline{n}_i$ | $p_i<1$ if $s_i>n_i$ |
| Wage determinative of $n_i$ | *Shadow wage* | *Expected wage* |
| | $\widetilde{w}_i=p_i w_i$ | $\widetilde{w}_i=p_i w_i$ |
| Aggregation | $\widetilde{W}=\left[\int\widetilde{w}_i^{1+\eta}\right]^{\frac{1}{1+\eta}}$ | $\widetilde{W}=\left[\int\widetilde{w}_i^{1+\eta}\right]^{\frac{1}{1+\eta}}$ |
| Supply | $\widetilde{W}=-U_S/U_C$ | $\widetilde{W}=-U_N/U_C$ |
| Production | $y_i=z_i n_i^{\alpha}$ | $y_i=z_i(p_i s_i)^{\alpha}$ |
| Optimality | $\widetilde{w}_i=\widetilde{\mu}_i\alpha z_i n_i^{\alpha-1}$ | $\widetilde{w}_i=\widetilde{\mu}_i\alpha z_i s_i^{\alpha-1}$ |
| Shadow-markdown | $\widetilde{\mu}_i=\begin{cases}\mu_i & \text{Region I}\\ p_i & \text{Region III}\end{cases}$ | $\widetilde{\mu}_i=\begin{cases}\mu_i & \text{Region I}\\ p_i^{\alpha} & \text{Region III}\end{cases}$ |

- If $\{Z,\omega,\widetilde{\mu}\}$ are identical across these two economies, then it must be that $\left\{Y,\widetilde{W},C\right\}$ are the same and $N=S$, and hence welfare $U(C,N)=U(C,S)$ is the same. The appropriate measure of aggregate productivity is, however, lower in the search economy. In the search economy $y_i=p_i^{\alpha}\times z_i s_i^{\alpha}$,and hence

$$Z^S=\left[\int\left(p_i^{\alpha}z_i\right)^{\frac{1}{1+\eta(1-\alpha)}}di\right]^{\frac{1}{1+\eta(1-\alpha)}}<\left[\int z_i^{\frac{1}{1+\eta(1-\alpha)}}di\right]^{\frac{1}{1+\eta(1-\alpha)}}=Z^M.$$

The search economy has lower productivity due to frictional non-employment $p_i<0$ at firms in Region III.

**Discussion.** The main difference between our benchmark economy and the search economy is that, in the presence of a minimum wage, there may be *frictional* rationing of workers in the search economy which leads to search effort expended at some firms where $n_i<s_i$ matches are generated. For such firms in Region III, $s_i$ is positive. Absent firm heterogeneity this frictional rationing is aptly thought of as *unemployment*, but with firm heterogeneity the search framework imposes additional welfare costs by having large volumes of costly search effort at firms that do not seek to employ it. We show below that the meeting rate $p_i$ is declining in productivity in Region III. For example, a corner-store in Region III is characterized by $(n_i,s_i,p_i)=(1,5,0.20)$, while a super-market is characterized by $(n_i,s_i,p_i)=(9,12,0.75)$. The four units of excess search effort incur utility costs, while providing no additional output.

From a normative perspective, we view this as problematic for welfare computations. Our approach of rationing constraints implies that there is no costly search effort being wasted in equilibrium. The corner-store has a sign for $\overline{n}_i=1$ worker, and 1 worker applies. Firms stop recruiting when full and households do not send excess labor in equilibrium. Hence we remove a welfare cost of minimum wages that would appear in the search model. This places the focus of inefficiencies on minimum wages forcing firms in Region III to shrink below their efficient size, rather than the frictional non-employment this creates.

From an empirical perspective, the primary challenge to estimating the search model is calibrating the labor disutility parameter(s) $\eta$ (and $\theta$ with nested-CES preferences over search effort). Search effort is not observed, nor are plant level matching rates. A richer model would also introduce parameters via the matching function if we deviate from short-sided matching. Hence we believe it would be difficult to make progress on this model. The rationing constraint model introduces no new parameters.

With the above caveats, the above mapping leads us to expect very similar results in such a model for values of $\eta$ and $\theta$ close to our benchmark economy. Since search effort contributes additional welfare costs without any additional benefits we view the welfare gains from our rationing constraint model as an upper bound on welfare gains in the corresponding search model.

**Comparative statics.** We can compute comparative statics of $\{p_i, \widetilde{w}_i, s_i\}$ in each of the three regions as $\{z_i, \underline{w}\}$ vary, *ceterus parabus.*

1. **Region I** - We have

$$\begin{aligned}\Delta \log p_i &= 0 \\ \Delta \log \widetilde{w}_i &= \frac{1}{1+\eta(1-\alpha)}\Delta \log z_i \\ \Delta \log s_i &= \frac{\eta}{1+\eta(1-\alpha)}\Delta \log z_i\end{aligned}$$

2. **Region II** - We have

$$\begin{aligned}\Delta \log p_i &= 0 \\ \Delta \log \widetilde{w}_i &= \Delta \log \underline{w} \\ \Delta \log s_i &= \eta \Delta \log \underline{w}\end{aligned}$$

3. **Region III -** Supply and demand are:

$$\begin{aligned}\widetilde{w}_i = p_i \underline{w} &= \left(\frac{s_i}{S}\right)^{\frac{1}{\eta}} \widetilde{W}^{\frac{1}{\varphi}} \\ s_i &= \frac{1}{p_i}\left(\frac{\alpha z_i}{\underline{w}}\right)^{\frac{1}{1-\alpha}}\end{aligned}$$

Then holding $\left\{S, \widetilde{W}\right\}$ fixed

$$\begin{aligned}\Delta \log p_i &= \frac{1}{(1-\alpha)(1+\eta)}\Delta \log z_i - \underbrace{\frac{1+(1-\alpha)\eta}{(1-\alpha)(1+\eta)}}_{>1}\Delta \log \underline{w} \\ \Delta \log \widetilde{w}_i &= \frac{1}{(1-\alpha)(1+\eta)}\Delta \log z_i - \frac{\alpha}{(1-\alpha)(1+\eta)}\Delta \log \underline{w} \\ \Delta \log s_i &= \frac{\eta}{[1+\eta(1-\alpha)]-\alpha}\Delta \log z_i - \frac{\eta\alpha}{(1-\alpha)(1+\eta)}\Delta \log \underline{w}\end{aligned}$$

First, at firm $i$, as $\underline{w}$ increases, the matching rate $p_i$ declines monotonically from 1 to 0 and declines sufficiently quickly that the expected wage $\widetilde{w}_i = p_i \underline{w}$ declines. With the expected wage falling, the household allocates lower search effort.

Second, holding $\underline{w}$ fixed and comparing firms $i$ and $j$ in Region III with $z_i < z_j$, then the matching rate $p_i < p_j$, hence the expected wage is lower at firm $i$, and search effort is also lower.

Combined, search effort is increasing quickly in $z_i$ in Region III, then flat in Region II, and increasing at a slower rate in Region III. The matching rate is constant and 1 in Region I and II and then as productivity falls, $p_i$ falls in Region III. Hence *lower productivity firms* are characterized by (i) shorter queues, (ii) lower meeting rates, (iii) lower employment.