Should Macroeconomists Use Seasonally Adjusted Time Series?

Structural Identification and Bayesian Estimation in Seasonal Vector Autoregressions

Ross Doppelt

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- Seasonality and identification
  - The X-11 filter and distortions to identification
  - Identification through seasonal heteroskedasticity
- Estimation: A Bayesian Approach
- Application: Supply and demand in U.S. labor markets

A Taxonomy of Seasonality

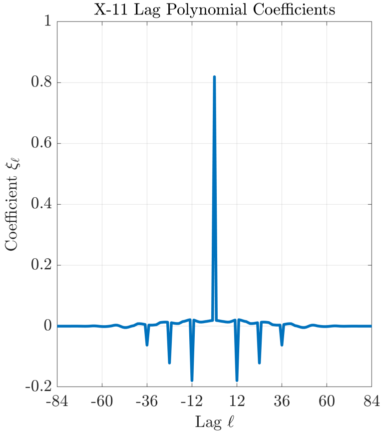
- Assume n_s seasons per year ($n_s = 12$ for monthly data)
- Let $\mathbf{y}_t$ be an $n \times 1$ vector time series

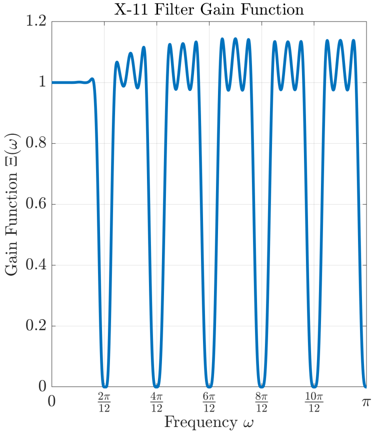
$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{s}_t + \tilde{\mathbf{y}}_t$$

  - $\mathbf{s}_t$ repeats annually: $\mathbf{s}_t = \mathbf{s}_{t-n_s}$
  - $\tilde{\mathbf{y}}_t$ is a purely non-deterministic stochastic process
- Deterministic seasonality: Captured by $\mathbf{s}_t$
- Stochastic seasonality: $\tilde{\mathbf{y}}_t$ can have seasonal spectral peaks
- X-11 seasonal adjustment has 2 main steps:
  - Estimate $\mathbf{s}_t$ and subtract it from $\mathbf{y}_t$
  - Apply a filter to $\tilde{\mathbf{y}}_t$ to suppress seasonal spectral peaks

An Overview of the X-11 Filter

- Suppose (for now) $\mathbf{y}_t$ has no deterministic terms ($\mathbf{y}_t = \tilde{\mathbf{y}}_t$)
- Time domain: $\mathbf{y}_t^{sa} \equiv \xi(L)\,\mathbf{y}_t$
- Freq. domain: $\mathbf{f}^{sa}(\omega) = \Xi(\omega)\,\mathbf{f}(\omega)$, $\Xi(\omega) \equiv |\xi(\exp\{-i\omega\})|^2$





Structural Identification Framework

- If $\mathbf{y}_t$ is stationary, it has a reduced-form representation:

$$\mathbf{y}_t = \hat{\mathbf{y}}_t + \mathbf{e}_t$$

  - $\hat{\mathbf{y}}_t$ is the projection of $\mathbf{y}_t$ on $\{\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots\}$
  - $\mathbf{e}_t$ is white noise with precision $\mathbf{Q} \equiv \mathbb{V}\left[\mathbf{e}_t\right]^{-1}$
- Assume that residuals are functions of structural shocks ϵ_t
  - $\mathbf{e}_t = \mathbf{\Psi}^{-1}\epsilon_t$ for some invertible $\mathbf{\Psi}$
  - $\mathbf{\Lambda} \equiv \mathbb{V}\left[\epsilon_t\right]^{-1}$ where $\mathbf{\Lambda}$ is diagonal
- An identification scheme is a mapping $\mathcal{I} : \mathbf{Q} \mapsto (\mathbf{\Psi}, \mathbf{\Lambda})$ such that $\mathbf{\Psi}'\mathbf{\Lambda}\mathbf{\Psi} = \mathbf{Q}$ whenever $(\mathbf{\Psi}, \mathbf{\Lambda}) = \mathcal{I}(\mathbf{Q})$
- Replace $\mathbf{y}_t$, $\hat{\mathbf{y}}_t$, $\mathbf{e}_t$, $\mathbf{Q}$ with $\mathbf{y}_t^{sa}, \hat{\mathbf{y}}_t^{sa}, \mathbf{e}_t^{sa}, \mathbf{Q}^{sa}$. How does $\mathcal{I}(\mathbf{Q}^{sa})$ compare to $\mathcal{I}(\mathbf{Q})$?

A Conceptual Issue

- Shocks extracted from the data
  - Using NSA series: $\boldsymbol{\epsilon}_t = \boldsymbol{\Psi}\mathbf{e}_t$, where $(\boldsymbol{\Psi}, \boldsymbol{\Lambda}) = \mathcal{I}(\mathbf{Q})$
  - Using SA series: $\boldsymbol{\epsilon}_t^{sa} = \boldsymbol{\Psi}^{sa}\mathbf{e}_t^{sa}$, where $(\boldsymbol{\Psi}^{sa}, \boldsymbol{\Lambda}^{sa}) = \mathcal{I}(\mathbf{Q}^{sa})$
- Recall that $\mathbf{y}_t^{sa} = \xi(L)\,\mathbf{y}_t$, and $\xi(L)$ is two-sided
  - $\mathbf{e}_t^{sa}$ ($\boldsymbol{\epsilon}_t^{sa}$) synthesized using past, present, and future $\mathbf{e}_t$ ($\boldsymbol{\epsilon}_t$)
  - By construction, $\boldsymbol{\epsilon}_t^{sa} \perp \left\{\mathbf{y}_{t-1}^{sa}, \mathbf{y}_{t-2}^{sa}, \ldots\right\}$, but $\boldsymbol{\epsilon}_t^{sa} \not\perp \{\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots\}$

- Kolmogorov's formula:

$$|\mathbf{Q}| = \exp\left\{-\frac{1}{2\pi}\int_{-\pi}^{\pi}\log\left(|2\pi\mathbf{f}(\omega)|\right)d\omega\right\}$$

with analogous relationship between $\mathbf{Q}^{sa}$ and $\mathbf{f}^{sa}(\cdot)$

- When the dimension of $\mathbf{y}_t$ is n:

$$|\mathbf{Q}^{sa}| = D^n|\mathbf{Q}|, \quad D \equiv \exp\left\{-\frac{1}{2\pi}\int_{-\pi}^{\pi}\log\left(\Xi(\omega)\right)d\omega\right\}$$

- For the X-11 filter shown earlier: $D \approx 2.83$
- Implication: $\mathcal{I}(\mathbf{Q}) \neq \mathcal{I}(\mathbf{Q}^{sa})$ or $\mathcal{I}(\mathbf{Q}) \cap \mathcal{I}(\mathbf{Q}^{sa}) = \emptyset$
- Example: With Cholesky identification, average log difference between $\frac{\partial \mathbf{y}_{k,t}}{\partial \epsilon_{k,t}}$ and $\frac{\partial \mathbf{y}^{sa}_{k,t}}{\partial \epsilon^{sa}_{k,t}}$ is about .52 ($\approx 68\%$)

Example: Labor Supply and Demand
Based on Baumeister and Hamilton (Econometrica, 2015)

$$\mathbf{y}_t = \begin{bmatrix} \Delta \log(\text{real wage}_t) \\ \Delta \log(\text{personhours}_t) \end{bmatrix}, \quad \boldsymbol{\epsilon}_t = \begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix}, \quad \boldsymbol{\Psi} = \begin{bmatrix} -\eta_d & 1 \\ -\eta_s & 1 \end{bmatrix}$$

- Combine the above with reduced-form projection:

$$\begin{aligned} \Delta \log(\text{personhours}_t) &= \eta_d \times \Delta \log(\text{real wage}_t) + \phi^d(L)' \mathbf{y}_t + \epsilon_t^d \\ \Delta \log(\text{personhours}_t) &= \eta_s \times \Delta \log(\text{real wage}_t) + \phi^s(L)' \mathbf{y}_t + \epsilon_t^s \end{aligned}$$

- Identified Set:

$$\mathcal{I}(\mathbf{Q}) = \left\{ (\boldsymbol{\Psi}, \boldsymbol{\Lambda}) \middle| \boldsymbol{\Psi} = \begin{bmatrix} -\eta_d & 1 \\ -\eta_s & 1 \end{bmatrix}, \boldsymbol{\Lambda} = \begin{bmatrix} \lambda_d & 0 \\ 0 & \lambda_s \end{bmatrix}, \begin{array}{c} \boldsymbol{\Psi}' \boldsymbol{\Lambda} \boldsymbol{\Psi} = \mathbf{Q} \\ \eta_s, \lambda_d, \lambda_s > 0 > \eta_d \end{array} \right\}$$

- Maybe $(\lambda_d, \lambda_s) = (\lambda_d^{sa}, \lambda_s^{sa})$, but then $\frac{|\eta_d^{sa}| + |\eta_s^{sa}|}{|\eta_d| + |\eta_s|} = D \approx 2.83$

- Maybe $(\eta_d, \eta_s) = (\eta_d^{sa}, \eta_s^{sa})$, but then $\left(\frac{\lambda_d^{sa}}{\lambda_d} \frac{\lambda_s^{sa}}{\lambda_s} \right)^{\frac{1}{2}} = D \approx 2.83$

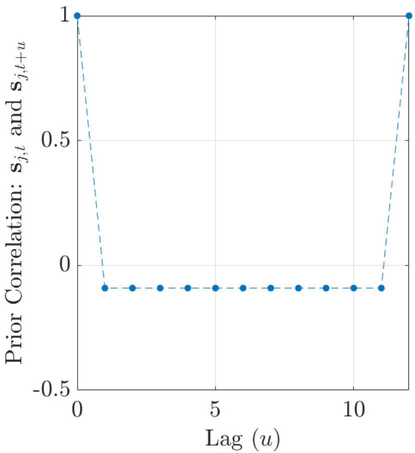
Identification via Seasonal Heteroskedasticity

- Allow $\mathbb{V}\left[\epsilon_t\right]^{-1} = \boldsymbol{\Lambda}_t$, where $\boldsymbol{\Lambda}_t = \boldsymbol{\Lambda}_{t'}$ if $t \overset{\text{mod } n_s}{=} t'$
- Precision of reduced-form residuals: $\mathbf{Q}_t \equiv \mathbb{V}\left[\mathbf{e}_t\right]^{-1} = \boldsymbol{\Psi}'\boldsymbol{\Lambda}_t\boldsymbol{\Psi}$
- Standard ID through heteroskedasticity argument
  - Notice $\mathbf{Q}_t\mathbf{Q}_{t'}^{-1} = \boldsymbol{\Psi}'\boldsymbol{\Lambda}_t\boldsymbol{\Lambda}_{t'}^{-1}\boldsymbol{\Psi}'^{-1}$
  - If $\boldsymbol{\Lambda}_t\boldsymbol{\Lambda}_{t'}^{-1}$ has distinct diagonal elements, rows of $\boldsymbol{\Psi}$ are (proportional to) eigenvectors of $\mathbf{Q}_t\mathbf{Q}_{t'}^{-1}$
- Rigobon (2003): "Probabilistic instruments"

- $\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{s}_t + \tilde{\mathbf{y}}_t$
- Sample-size issue
  - 50 years of monthly data: $T = 600$
  - Need to estimate January-specific mean with only 50 Januarys
- Alternative: Fit a model to $\mathbf{y}_t - \mathbf{y}_{t-12}$
  - Need to check for up to 12 unit roots
  - Frequentist tests can pose practical challenges
- Want: A prior for seasonal processes
  - Favor smoothness in $\mathbf{s}_t$
  - Favor seasonal unit roots, or spectral peaks, in $\tilde{\mathbf{y}}_t$

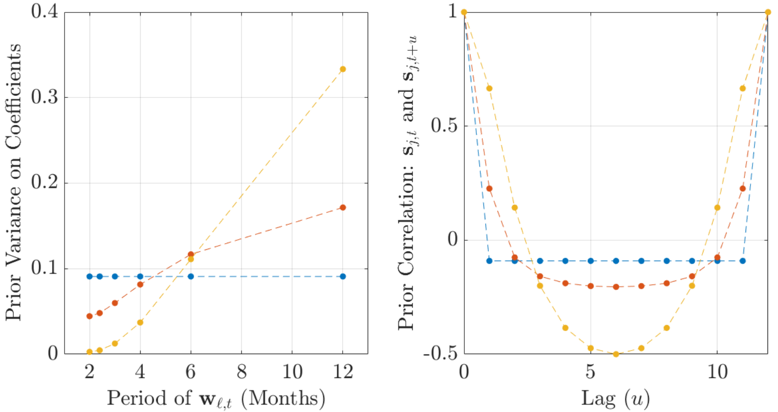
Why Not Seasonal Dummies?

- Consider $\mathbf{B}_d \mathbf{d}_t = \boldsymbol{\mu} + \mathbf{s}_t$ for seasonal dummies $\mathbf{d}_t$
- Consider the prior $\text{vec}\left(\mathbf{B}_d\right) \sim \text{N}\left(\mathbf{0}, \sigma_d^2 \mathbf{I}\right)$
- Then: $\mathbb{E}_{prior}\left[\frac{1}{T}\sum_t \mathbf{s}_{j,t}^2\right] = \frac{n_s - 1}{n_s}\sigma_d^2$



A Prior for Deterministic Seasonality

- $\mathbf{s}_t = \mathbf{B}\mathbf{w}_t$, where $\mathbf{w}_t$ contains $n_s - 1$ seasonal sinusoids (periods of 1 year, $\frac{1}{2}$ year, $\frac{1}{3}$ year, etc.)



A Prior for Stochastic Seasonality

Beliefs About The Spectrum

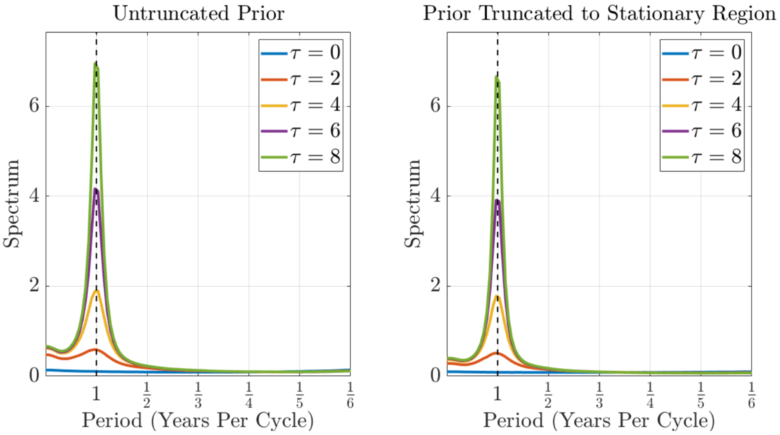
- $\mathbf{A}(L)\,\tilde{\mathbf{y}}_t = \boldsymbol{\epsilon}_t$, with $\mathbf{A}(L) \equiv \boldsymbol{\Psi} - \sum_{\ell=1}^{m} \boldsymbol{\Phi}_\ell L^\ell$

  Spectrum of $\tilde{\mathbf{y}}_t$: $\quad \mathbf{f}(\omega) = \frac{1}{2\pi}\left[\mathbf{A}(\exp\{i\omega\})' \boldsymbol{\Lambda}\mathbf{A}(\exp\{-i\omega\})\right]^{-1}$

- Seasonal unit root: $|\mathbf{A}(\exp\{i\omega^*\})| = 0$
  - Implies $|\mathbf{f}(\exp\{i\omega^*\})| \to \infty$ as $\omega \to \omega^*$
  - Oscillations at frequency ω^* important for variation in $\tilde{\mathbf{y}}_t$

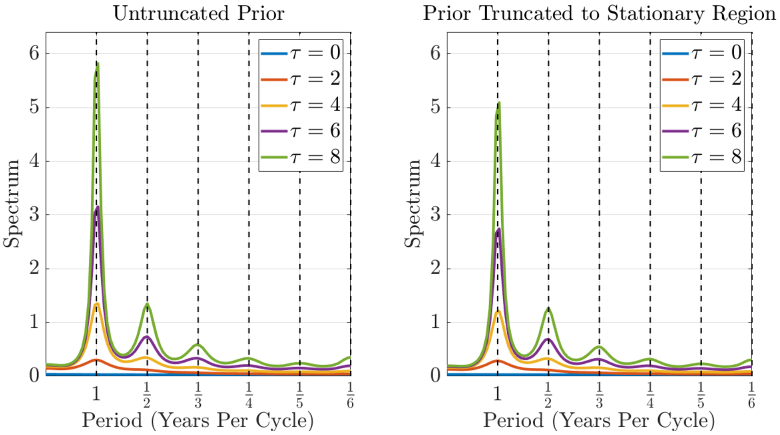
- The prior will favor, but not impose, $\mathbf{A}(\exp\{i\omega^*\}) = \mathbf{0}$ for seasonal ω^*

- Stochastic linear restrictions: $\mathbf{A}(\exp\{i\omega^*\}) \sim$ Complex normal with zero mean ▸ Details


Untruncated Prior
Prior Truncated to Stationary Region
$\tau = 0$
$\tau = 2$
$\tau = 4$
$\tau = 6$
$\tau = 8$
Spectrum
0
2
4
6
1
$\frac{1}{2}$
$\frac{1}{3}$
$\frac{1}{4}$
$\frac{1}{5}$
$\frac{1}{6}$
Period (Years Per Cycle)

A Prior for Stochastic Seasonality

Implied Prior Over the Spectrum



Application: Labor Supply and Demand

Based on Baumeister and Hamilton (Econometrica, 2015)

- Structural VAR

$$\mathbf{\Psi}\left(\mathbf{y}_{t-\ell}-\boldsymbol{\mu}-\mathbf{s}_t\right)=\sum_{\ell=1}^{m}\mathbf{\Phi}_\ell\left(\mathbf{y}_{t-\ell}-\boldsymbol{\mu}-\mathbf{s}_{t-\ell}\right)+\boldsymbol{\epsilon}_t,\ \boldsymbol{\epsilon}_t\overset{\text{i.i.d.}}{\sim}\mathrm{N}\left(\mathbf{0},\mathbf{\Lambda}_t^{-1}\right)$$

with:

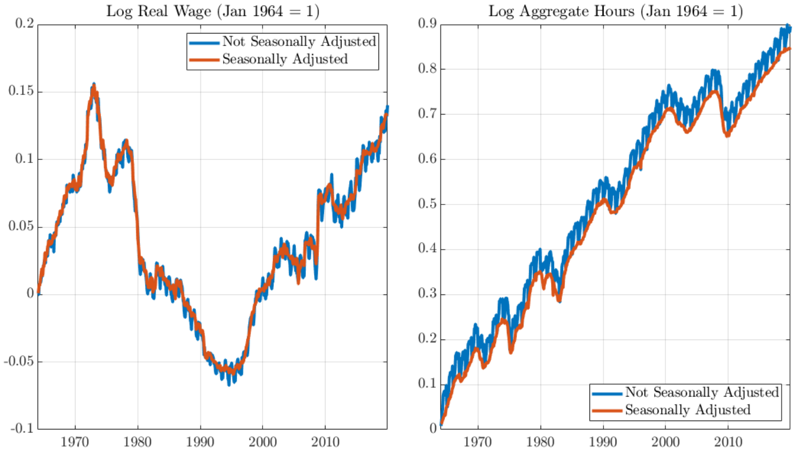
$$\mathbf{y}_t=\begin{bmatrix}\Delta\log\left(\text{real wage}_t\right)\\ \Delta\log\left(\text{personhours}_t\right)\end{bmatrix},\quad \boldsymbol{\epsilon}_t=\begin{bmatrix}\epsilon_t^d\\ \epsilon_t^s\end{bmatrix},\quad \mathbf{\Psi}=\begin{bmatrix}-\eta_d & 1\\ -\eta_s & 1\end{bmatrix}$$

- Implies a demand curve and a supply curve:

$$\begin{aligned}\Delta\log\left(\text{personhours}_t\right) &= c_d+\eta_d\times\Delta\log\left(\text{real wage}_t\right)+\boldsymbol{\delta}_d'\mathbf{w}_t+\phi^d(L)'\,\mathbf{y}_t+\epsilon_t^d\\ \Delta\log\left(\text{personhours}_t\right) &= c_s+\eta_s\times\Delta\log\left(\text{real wage}_t\right)+\boldsymbol{\delta}_s'\mathbf{w}_t+\phi^s(L)'\,\mathbf{y}_t+\epsilon_t^s\end{aligned}$$

- "Seasonally adjusted model": Fit to SA time series
- "Seasonal model": Fit to NSA time series
- Heteroskedasticity by season & heteroskedasticity à la Brunnermeier et al. (2021) ▸ Details

A Look at the Data



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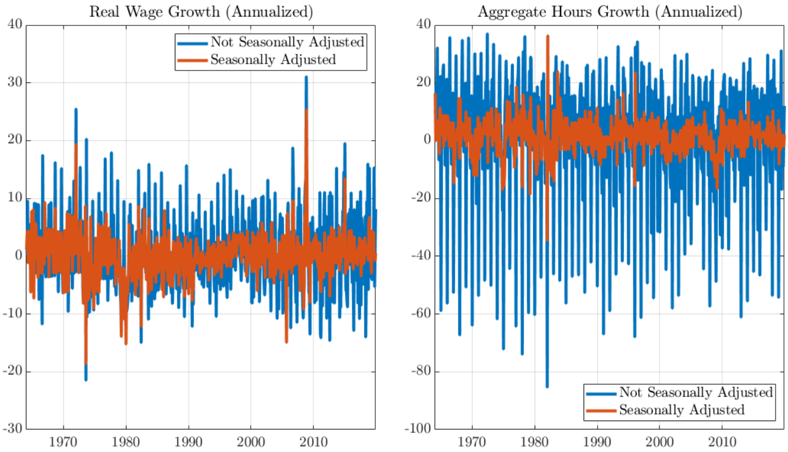
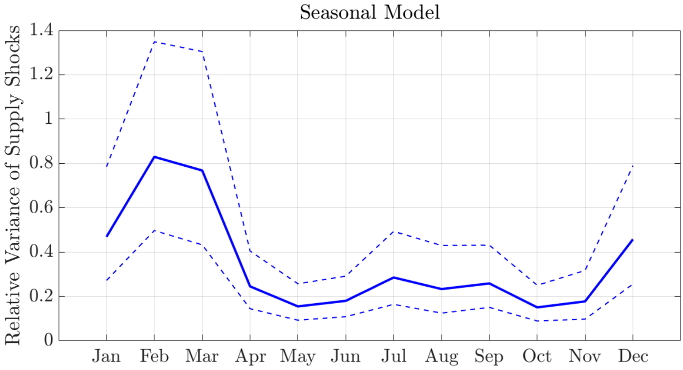


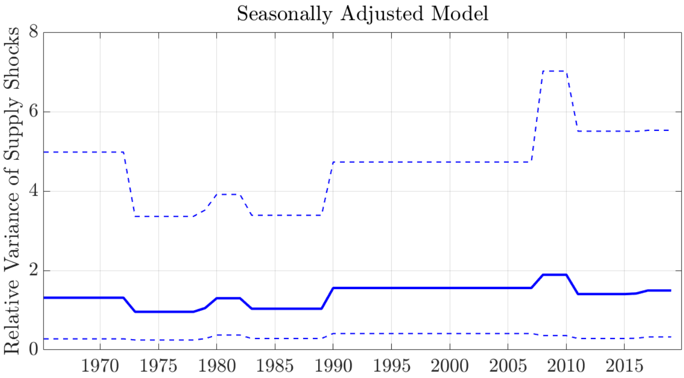
Evidence of Seasonal Heteroskedasticity

- Identification requires $\mathbb{V}\left[\epsilon_t^s\right] / \mathbb{V}\left[\epsilon_t^d\right]$ to vary over time



Non-Seasonal Heteroskedasticity

- Identification requires $\mathbb{V}[\epsilon_t^s]/\mathbb{V}\left[\epsilon_t^d\right]$ to vary over time



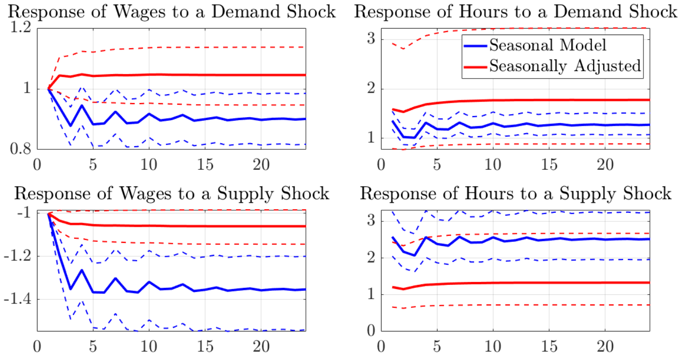
Structural Parameters

	η_d	$\frac{1}{T}\sum_t \mathbb{V}\left[\epsilon_{d,t}\right]$	η_s	$\frac{1}{T}\sum_t \mathbb{V}\left[\epsilon_{s,t}\right]$
Seasonal Model	-2.58 $[-3.27, -2.06]$	2.00 [1.49, 2.80]	1.36 [1.19, 1.55]	0.62 [0.55, 0.71]
Seasonally Adjusted Model	-1.21 $[-2.44, -0.67]$	0.40 [0.28, 0.94]	1.59 [0.79, 2.92]	0.53 [0.30, 1.24]

Posterior Median Estimates. 10^{th} & 90^{th} Posterior Quantiles in Brackets.

Homoskedastic Estimates

Impulse Responses



Variance Decompositions

	Wage Growth		Hours Growth	
	Seasonal	Seasonally Adjusted	Seasonal	Seasonally Adjusted
Unconditional Variance	25 [19, 32]	57 [24, 81]	52 [43, 60]	43 [19, 75]
Low Frequencies	41 [30, 53]	57 [24, 83]	55 [45, 64]	42 [18, 74]
Business-Cycle Frequencies	40 [29, 50]	57 [24, 82]	55 [46, 64]	42 [18, 75]
Irregular Frequencies	23 [17, 30]	57 [24, 81]	51 [43, 59]	43 [19, 75]

Percent Attributable to Supply Shocks.

Posterior Median Estimates. 10^{th} and 90^{th} Posterior Quantiles in Brackets.

Business-Cycle Frequencies: Periodicities between 1.5 and 8 years.
Low frequencies (irregular frequencies): All periodicities longer (shorter) than business cycles.

Thanks!

- Questions/Comments/Suggestions?

Selected Literature ▸ Return

- Bayesian priors for seasonality: Canova (1992, 1993), Raynaud and Simonato (1993), Gersovitz and McKinnon (1978)
- Seasonality and causality in distributed-lag models: Sims (1974), Wallis (1974), Granger (1978)
- Seasonality and identification in equilibrium models: Sargent (1978), Ghysels (1988), Hansen and Sargent (1993), Sims (1993), Christiano and Todd (2002), Saijo (2013)
- Filtering and interpreting economic models: Nelson and Kang (1981), King and Rebelo (1993), Cogley and Nason (1995), Hamilton (2018), Ashley and Verbrugge (2022)

- $\mathbf{A}(\exp\{i\omega^*\}) = \mathbf{0}$ requires zero real and imaginary parts:

$$\mathbf{A}(\exp\{i\omega^*\}) = \underbrace{\boldsymbol{\Psi} - \sum_{\ell=1}^{m} \boldsymbol{\Phi}_\ell \cos(\omega^*\ell)}_{\Re(\mathbf{A}(\exp\{i\omega^*\}))} + i \underbrace{\sum_{\ell=1}^{m} \boldsymbol{\Phi}_\ell \sin(\omega^*\ell)}_{\Im(\mathbf{A}(\exp\{i\omega^*\}))}$$

- Prior treats each column of $\Re(\mathbf{A}(\exp\{i\omega^*\}))$ and $\Im(\mathbf{A}(\exp\{i\omega^*\}))$ as $\mathrm{N}\left(\mathbf{0}, (\tau_{\omega^*}^2 \boldsymbol{\Lambda})^{-1}\right)$, so $\mathbf{A}(\exp\{i\omega^*\})$ is mean-zero complex normal

- Dummy observations implementation:

$$\begin{aligned}
\bar{\mathbf{Y}}_{\omega^*}\boldsymbol{\Psi}' &= \bar{\mathbf{X}}_{\omega^*}\boldsymbol{\Phi}' + \bar{\boldsymbol{\mathcal{E}}}_{\omega^*}, \quad \left(\bar{\boldsymbol{\mathcal{E}}}_{\omega^*}\right)_{j,k} \overset{\text{i.i.d.}}{\sim} \mathrm{N}(0, \lambda_k) \\
\bar{\mathbf{Y}}_{\omega^*} &\equiv \tau_{\omega^*} \begin{bmatrix} \mathbf{I}_n \\ \mathbf{0}_{n\times n} \end{bmatrix} \\
\bar{\mathbf{X}}_{\omega^*} &\equiv \tau_{\omega^*} \begin{bmatrix} \cos(\omega^*1) & \cos(\omega^*2) & \cdots & \cos(\omega^*m) \\ \sin(\omega^*1) & \sin(\omega^*2) & \cdots & \sin(\omega^*m) \end{bmatrix} \otimes \mathbf{I}_n
\end{aligned}$$

Non-Seasonal Heteroskedasticity

Following Brunnermeier, Palia, Sastry, and Sims (AER, 2021) Return

		Start	End
1	Pre-Stagflation	Jan. 1967	Dec. 1972
2	Stagflation	Jan. 1973	Sep. 1979
3	Volcker Disinflation	Oct. 1979	Dec. 1982
4	S&L Crisis	Jan. 1983	Dec. 1989
5	Great Moderation	Jan. 1990	Dec. 2007
6	Financial Crisis	Jan. 2008	Dec. 2010
7	ZLB & Recovery	Jan. 2011	Nov. 2016
8	Interest-Rate Takeoff	Dec. 2016	Dec. 2019

- My sample: Jan. 1967 – Dec. 2019. Brunnermeier et al.'s sample: Jan. 1973 – Jun. 2015.

Structural Parameters

The Role of Heteroskedasticity

		η_d	$\frac{1}{T}\sum_t \mathbb{V}\left[\epsilon_{d,t}\right]$	η_s	$\frac{1}{T}\sum_t \mathbb{V}\left[\epsilon_{s,t}\right]$
Seasonal Model	Hetero-skedastic	-2.58 $[-3.27, -2.06]$	2.00 [1.49, 2.80]	1.36 [1.19, 1.55]	0.62 [0.55, 0.71]
	Homo-skedastic	-2.19 $[-3.29, -1.52]$	1.64 [1.10, 2.85]	1.46 [1.15, 1.83]	0.66 [0.55, 0.83]
Seasonally Adj. Model	Hetero-skedastic	-1.21 $[-2.44, -0.67]$	0.40 [0.28, 0.94]	1.59 [0.79, 2.92]	0.53 [0.30, 1.24]
	Homo-skedastic	-1.37 $[-2.28, -0.84]$	0.49 [0.34, 0.93]	1.37 [0.84, 2.20]	0.48 [0.33, 0.86]

Posterior Median Estimates. 10^{th} & 90^{th} Posterior Quantiles in Brackets.

Return