Precautionary Protectionism

Sharon Traiberman Martin Rotemberg*

March 23, 2022

Abstract

Should tariffs be used to protect industries that are crucial in times of national crises? In this paper we show that in a Ricardian setting the answer is basically no. We develop a dynamic extension of Dornbusch et al. (1977) with adjustment costs: during the crisis the home country faces relatively higher unit costs for goods it is not already producing. If during the crisis the home country would not produce a good in the absence of adjustment costs, then the optimal precautionary industrial policy does not encourage its production. Instead, we find that the optimal policy is to protect goods where comparative (dis)advantage is only weak, particularly for goods where protection induces production or exporting. Quantitatively, we find that sophisticated tariff policy can make up for around half of the welfare losses generated by adjustment costs.

1 Introduction

In March and April of 2020, there was a shortage of toilet paper on the shelves of supermarkets across the world. This was partially driven by a change in stockpiling behavior, but fundamentally it was not caused by total use of toilet paper, but by where it was used: at home instead of in the office. Offices tend to use larger toilet paper rolls, which do not fit in home bathroom holders. Factories scrambled to switch production, a costly and slow process. In 2020, factories had difficulty ramping up production of a variety of new products, including low-waste syringes, vaccines, and ventilators.[1] Concerns about future pandemics, environmental crises, and other shocks have raised the salience of having an industrial policy that mitigates the risks in turbulent times.[2] In this paper, we try to formally describe the implications that high short-run adjustment costs have for industrial policy.

To understand the economic forces at play, we add dynamics to the Ricardian model of Dornbusch et al. (1977), in order to study precautionary trade policy in a world with crises and adjustment costs.[3] Intuitively,

*New York University. We are grateful to Corina Boar, Arnaud Costinot, Gene Grossman, and Pete Klenow for valuable comments. Please contact st1012@nyu.edu and mr3019@nyu.edu with any questions or comments.

[1]See https://www.wsj.com/articles/kimberly-clark-to-reassign-makers-of-office-toilet-paper-to-meet-demand-for-softer-stuff-11587577564,
https://www.sciencemag.org/news/2021/03/syringe-size-and-supply-issues-continue-waste-covid-19-vaccine-doses-united-states,
https://www.washingtonpost.com/business/2021/03/31/vaccine-johnson-johnson-emergent/, and
https://www.ttnews.com/articles/teslas-ventilator-production-new-york-has-yet-begin.

[2]This is not just an academic concern: understanding the relationship between risk and policy is the subject of a recent Department of Homeland Security call for research proposals (https://cbts.tamu.edu/files/2020/11/Texas-AM-CBTS-Call-for-Proposals_Economics-of-SC-Risk.pdf).

[3]An alternative to protecting so-called critical industries could be national stockpiles, or other inventory management. An analysis integrating stockpiling and protectionism, characterizing when one policy or the other is useful, is likely important future work, but beyond the scope of our paper. We think tariff policy is likely to still be useful even allowing for other policy instruments. Some goods, such as those with short shelf lives or high depreciation rates, are not particularly amenable to stockpiling. Similarly, it may be difficult to predict in advance the exact goods that need to be stockpiled.

our model starts from the perspective that the a country's steady-state productivity for similar types of toilet paper, syringes, vaccines, or machines are correspondingly similar. However, in the short-run, adjustment costs means that the current productivity for a (narrowly defined) good is a function of whether it had been produced in the recent past. We focus on understanding tariff policy in a world with a risk of "foreign-biased" demand shocks: sharp increases in demand for goods which the home country has a comparative disadvantage.

While we do find a role for protectionism, it does not coincide with imposing tariffs for the specific products that are needed in times of crisis. Instead the guiding force for trade and trade policy is comparative advantage, reflecting results from the static case (Opp, 2010; Costinot et al., 2015). The logic is straightforward: if there are products which are particularly important in a crisis, it is valuable for policymakers in the home country to encourage their production in the places where they can be made at the lowest cost, which may be abroad. The role for protectionism is to improve the terms of trade, lowering the domestic price of the newly valuable goods. To do this, the optimal policy is to encourage the domestic production of products where the home country is only marginally less productive than the foreign country, which effectively frees up foreign labor in the crisis.

Formally, our approach adapts the set-up in Costinot et al. (2015) to consider production dynamics. We consider adjustment costs on the extensive margin: if in steady-state it takes a_i workers to produce a unit of good i, it takes $a_i\,(1+x)$ workers in the short run for goods that are not initially produced. We think this assumption reflects the state of the policy debate: should the United States promote domestic production of critical products, so that it can be scaled up quickly in a crisis (since the cost of scaling up discontinuously increases at the extensive margin).[4]

While perhaps the optimal solution would be to produce a small amount of everything, this isn't a policy lever available to the government. The only tool available is tariffs, and if a tariff is high enough to encourage domestic production, then firms will endogenously choose to satisfy domestic demand.[5]

The demand shock consists of a large increase in demand for a mass of goods for which the foreign country has a strong comparative advantage, and a secular decline in demand for the rest. To ground intuition, it is helpful to consider the baseline case with no adjustment costs, as in Costinot et al. (2015). This scenario implies no need for forward-looking policy making. Comparative advantage determines optimal trade policy: domestic-production follows a single cut-off rule in comparative advantage. A foreign-biased demand shock raises the cutoff in the second period (to a product where the home country is relatively less productive).

Our analysis starts with a simple extension to the baseline model, where we explicitly solve for optimal policy with adjustment costs. Our main conclusion is that if a product wouldn't be produced domestically in the absence of adjustment costs, there is no precautionary reason for industrial policy to reshore production. Similarly, if in the absence of adjustment costs a good would be protected pre-shock, then in a world with

[4] See, for instance, https://americanaffairsjournal.org/2020/05/reshoring-supply-chains-a-practical-policy-agenda/.

[5] While we do not model this explicitly, we implicitly assume that free entry and productivity spillovers mean that firms themselves do not internalize the dynamic productivity benefits of production (Hausmann and Rodrik, 2003; Mussa, 1982).

adjustment costs that product would to be produced domestically both pre and post-shock . We find more nuance between these cases: the goods which in the absence of adjustment costs would only be produced post-shock. For these intermediate goods, comparative advantage is no longer the only relevant force. The details of protectionism depend on the exact details of the environment, as described below.

Our benchmark model with adjustment-costs covers the deterministic two-period case, when the policy maker knows what sectors will be affected in the following period. Adjustment costs are infinite: if a product isn't produced in the first period, it cannot be produced in the second period. Optimal protection, and therefore production, leads to a single cut-off, which corresponds to an average of the initial and final steady-state cutoffs. We weaken each of these three assumptions in turn: adding uncertainty, finite adjustment costs, and more general consequences of a crisis.

In the benchmark case, we can split the region of goods between the pre and post crisis cutoffs into two sub-regions. The boundary between the sub-regions corresponds to the good where foreign consumers are indifferent between local or imported goods. In the first period, in both sub-regions, protected goods would only be consumed domestically. In one sub-region (the one corresponding to lower comparative advantage), in the second period, unprotected goods would be imported while protected goods would only be consumed domestically, since they are only cheaper at home because of the tariff. This sub-region corresponds to the endogenous non-traded region in Dornbusch et al. (1977). The other sub-region (to the left of the sub-region boundary), is new to our dynamic framework. In this sub-region, in the second period, any unprotected goods would be imported, while protected goods would be exported. Different shocks could lead to the same tariff policy, since it does not matter exactly what happens to goods above the second period cutoff.[6] As a result, one feature of the cutoff policy is that it is somewhat robust to the details of the crisis.

We then turn to finite adjustment costs: if a good isn't produced in the first period, its unit labor requirement in the second period increases by a factor of x. As x falls, the number of sub-regions increases (up to four). In order of increasing comparative advantage, the first two sub-regions are as described above. The next least relatively productive sub-region corresponds to goods where the home country will produce no matter what, but would only be productive enough to export if the good is protected in the first period. Finally, the most productive sub-region comprises goods which the home country will export no matter which technology is used. *Within* each sub-region there is a cutoff rule for protection. However, goods in the first period may be protected in a sub-region even when goods with *higher* comparative advantage are unprotected. The intuition for why the standard single-cutoff logic breaks down is because of the different effects of protectionism on second-period outcomes. For instance, in the most productive sub-region, the goods will be exported in the second period no matter what, so the gains from protectionism are small relative to the next-most productive sub-region, where protectionism induces exporting.

Next, we consider more flexible crises. First, we study flexible demand shocks, where the overall shock remains foreign biased but all goods potentially experience a shock to their demand. This also induces non-

[6] In a different setting, Itoh and Kiyono (1987) also argue that the appropriate products to target are the "marginal" ones.

monotonic cutoffs: in addition to the forces described in the benchmark case it is additionally valuable to onshore production of sectors the demand is particularly high in the crisis state. We also consider shocks that not only affect demand, but also lead to a secular decline in productivity. This can lead to more goods being protected with precautionary tariffs (depending on the intertemporal elasticity of substitution)–effectively "saving" through inefficiently high production before the crisis—but does not lead to production above the unconstrained post crises cutoff.

We then consider uncertainty over the probability of crisis, and exactly which goods will be affected by the crisis. The optimal policy is essentially an average of the optimal policies in each state of the world.

We apply our theoretical results to quantitatively understand the importance of dynamic considerations in policy making. We consider an economy with two periods and two countries: the United States and the Rest of the World, matching steady-state values as in Costinot et al. (2015). Motivated by the Covid-19 crisis, we consider a series of demand shocks in which (in the absence of policy) global consumption of foreign products increases by 1-10%. The optimal dynamic policy consistently closes around half of the welfare gap between myopic policies and the infeasible policy of removing adjustment costs. Any intervention is better than nothing at all: the optimal myopic policy achieves most of the gains of the infeasible one.

We build on the Dornbusch et al. (1977)-with-tariffs frameworks of Opp (2010) and Costinot et al. (2015) to add in dynamics on the demand side.[7] This complements a large literature studying dynamics on the production side, such as Hsieh et al. (2021), Beshkar and Shourideh (2020), Carroll and Hur (2020), Caselli et al. (2020), Naito (2019), Matsuyama (1992), and Krugman (1987). Our main contribution to this literature is considering short-run adjustment costs in productivity, although we do not specifically micro-found the adjustment costs as in Leamer (1980) and Furusawa and Lai (1999), nor do we consider other externalities such as pollution Weisbach et al. (2021). There is also a growing literature studying trade policy during a pandemic (Antràs et al., 2020; Argente et al., 2020; Bonadio et al., 2020; Fajgelbaum et al., 2020; Cunat and Zymek, 2020; Leibovici et al., 2021), although to our knowledge ours is the first to focus on comparative advantage and industrial policy.

In Section 2, we describe the environment, and define the no-tariff equilibrium for each period. In Section 3, we introduce demand shocks and frictions. In Section 4 we solve for optimal policy, and in Section 5 weaken the benchmark assumptions. Section 6 calibrates the welfare gains from policy.

2 Model

2.1 Environment

We consider the Ricardian environment of Dornbusch et al. (1977), augmented for trade policy as in Costinot et al. (2015). There are two countries, Home and Foreign, a continuum of goods indexed by $i \in [0, 1]$, and two periods, $t = 1, 2$. Foreign variables are denoted by an asterisk. Home and Foreign are each endowed with

[7] We assume homothetic preferences, see Matsuyama (2015) for a discussion of non-homothetic preferences.

an unchanged amount of inelastically supplied labor, L and L^*, respectively.

2.2 Preferences

There is a representative worker-consumer in each country. Statically, preferences are given by a Cobb-Douglas aggregator over goods:

$$u_t = \exp\left\{\int_0^1 \beta_{it}\log(c_{it})di\right\}, \tag{1}$$

where c_i is consumption of good i. For simplicity, we assume that $\beta_{it} = \beta^*_{it}$. Due to the Cobb-Douglas set-up, β_{it} is the expenditure share for good i in period t.

The consumer has log preferences over intertemporal consumption, discounting at rate δ. Hence, the utility for the household is given by,

$$U = \log(u_1) + \delta\log(u_2). \tag{2}$$

In Section 5 we weaken the assumptions on the elasticity of substitution, both within and across periods.

2.3 Technology and Adjustment Costs

The only input into production is labor. Unit input requirements for each good are given by a_{it}. Goods are ordered so that $A(i) \equiv a^*_{i1}/a_{i1}$ is decreasing in i. This ratio is the *comparative advantage schedule* (Dornbusch et al., 1977). We assume throughout that A is smooth and strictly decreasing.

m_{i1} indicates if Home imports good i in period t. Technology evolves according to the following assumption:

Assumption 2.1. External Economies of Scale:

$$a_{i2} = \begin{cases} a_{i1} & \textit{if } m_{i1} < 1 \\ a_{i1}(1+x) & \textit{if } m_{i1} = 1. \end{cases} \tag{3}$$

$$a^*_{i2} = a^*_{i1} \tag{4}$$

This equation states that at Home, if production does not occur in period 1, then producing the good in period 2 increases the unit labor requirement by $x\%$. Our benchmark case is $x = \infty$, so if Home does not produce in the first period, it cannot produce in the second. We turn to the case that x is finite in section 4.3.

There are two simplifications embedded in this setup. First, we assume that adjustment costs are only incurred at Home not in Foreign. The situation that interests us is one in which Home's terms of trade deteriorate in response to a shift in demand to Foreign. Second, we assume that x is constant across goods. Our main conclusions hold if we weaken either (or both), but doing so makes the expressions cumbersome without adding much insight.

2.4 Market Structure and Trade

Markets are perfectly competitive and workers are freely mobile between sectors. Let w and w^* be the wage paid to workers in Home and Foreign respectively. Thus, the unit costs of producing i in Home and Foreign are respectively wa_i and wa_i^*. We assume that firms optimize statically:

Assumption 2.2. Myopia: *Firms set* $\delta = 0$.

This assumption allows us to ignore how much firms internalize external economies of scale. The assumption is consistent with, for instance, free entry and no property rights on the productive technology, so there are no profits in the second period (Hausmann and Rodrik, 2003).[8]

The only trade barriers we consider are (weakly) positive tariffs t_{it} enacted by the Home government.[9] Consumers source from the lowest cost producer. The import status of good i at home in time t is given by

$$m_{it} = \arg\min \{wa_{it}, w^* a_{it}^* (1 + t_{it}.)\} \tag{5}$$

2.5 Equilibrium

An equilibrium in this model is a set of wages, $\{w_t, w_t^*\}$, prices, $\{p_{it}, p_{it}^*\}$, labor allocations, $\{l_{it}, l_{it}^*\}$, consumption decisions, $\{c_{it}, c_{it}^*\}$,tariffs, $\{t_{it}\}$, and import decisions, $\{m_{it}, m_{it}^*\}$ such that:

1. Households in home (and similarly in foreign) maximize (1) subject to the budget constraint:
$$\int_i p_{it} c_i \leq wL + \int_i p_{it}^* c_{it} m_{it} t_{it} di.$$

2. Prices are set competitively,
$$p_{it} = \min \{w_t a_{it}, w_t^* a_{it}^* (1 + t_{it})\}$$

3. Markets clear:
$$c_{it}^*(1 - m_{it}) + c_{it}^* m_{it}^* = l_{it}/a_{it}$$

4. Allocations are feasible:
$$\int_i a_{it} l_{it} di \leq L$$

5. Trade is balanced:
$$\int_i p_{it} c_{it}^* m_{it}^* di = \int_i p_{it}^* c_{it} m_{it} di$$

Given a set of tariffs, solving this equilibrium is straightforward and follows Dornbusch et al. (1977). In particular, given the ranking of i such that that A is monotonically decreasing, Foreign imports follow a

[8] Grossman and Rossi-Hansberg (2010) show that in the DFS model, the extent to which firms internalize externalities is determined by industry structure. However, in our context, firms internalizing the future lowers the need for tariff policy, and we want to set up the problem to make tariffs as potentially attractive as possible.

[9] In Section 6 we additionally consider iceberg trade costs

cutoff rule in $\omega \equiv w/w^*$, importing everything with $A(i) \leq \omega$. The equilibrium conditions are:

$$\omega_t = \ell \frac{\int_0^{A_t^{-1}(\omega_t)} \beta_{it} di}{\int_i \frac{\beta_{it} m_{it}}{1+t_{it}} di} \times \left(1 - \int_i \frac{\beta_{it} m_{it} t_{it}}{1+t_{it}} di\right) \tag{6}$$

$$m_{it} = \arg\min \{\omega_t, A_{it}^*(1+t_{it})\}, \tag{7}$$

where ℓ is the ratio of population on foreign to home, L^*/L. For the case where $\tau_{it} = 0$, this is exactly the solution of Dornbusch et al. (1977). Regardless of the τ_{it}s, there is a unique cutoff. The left hand side of equation (6) is (obviously) strictly increasing in ω. The right hand side is decreasing in ω (given Equation (7)). We now turn to the properties of a Foreign biased demand shock, and then discuss optimal policy when facing such a shock.

3 Foreign Biased Demand Shocks

To focus on the situation where Home wants to expand production after a shock, we assume that the demand shock only affects that Home would not normally want to produce. Specifically, we assume that for some $\underline{i}$ that is above the period 2 free trade equilibrium, $\int_{\underline{i}}^1 \beta_{i2} > \int_{\underline{i}}^1 \beta_{i1}$. We call this a *foreign biased* demand shock. To understand what happens with such a shock, consider the case of free trade with $t_{it} = 0$. In each period imports take a simple cutoff rule and equation (6) reduces to the standard system of equations from DFS:

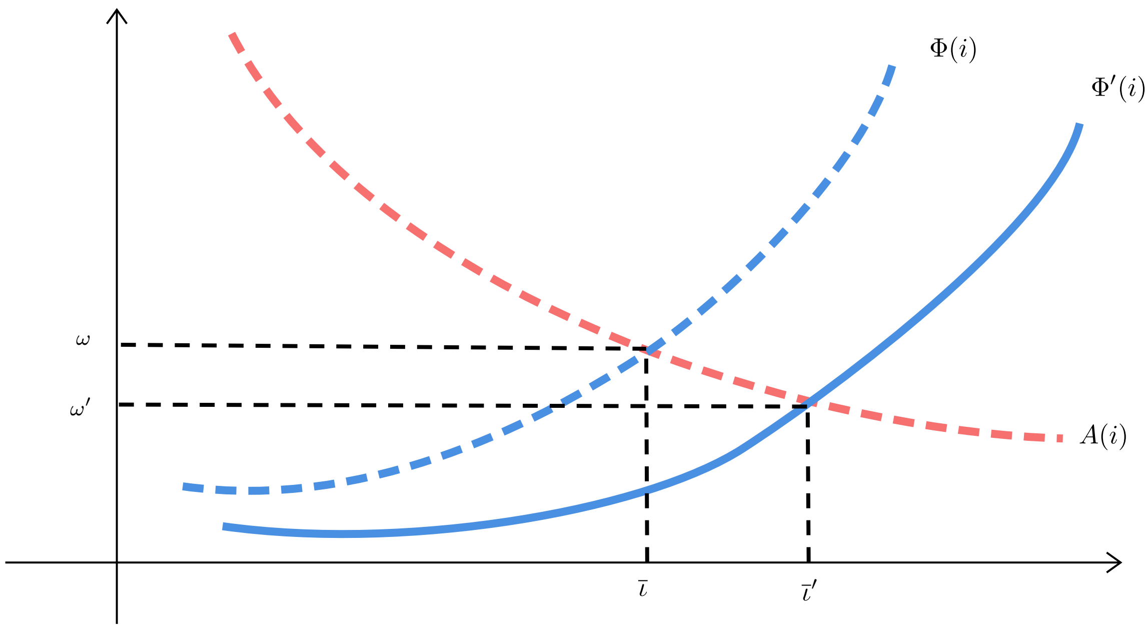
$$A_t(\bar{\iota}) = \bar{\omega}$$

$$\Phi(\bar{\iota}) \equiv \frac{\ell \int_0^{\bar{\iota}} \beta_{it}}{\int_{\bar{\iota}}^1 \beta_{it} dt} = \bar{\omega}.$$

Figure 1 plots the free trade equilibrium, holding A fixed, for two different schedules of β. The foreign biased demand shock shifts $\Phi(i)$ to the right. As a result, Home's demand for Foreign's good exceeds Foreign's demand for Home's; consequently, the initial equilibrium relative wage, $\bar{\omega}$, is no longer an equilibrium. In the new equilibrium, $\bar{\iota}$ is shifted to the right, as Home produces more goods. Intuitively, Foreign labor will be relatively more allocated in the now higher-demand high i goods. The home worker operating in the marginal industry is now less productive (as is the average home worker), while the opposite is true in foreign, which leads to a lower relative wage at home. In Figure 1, the variables denoted with a prime are after the foreign biased shock, and are drawn in solid lines.

The effect of a foreign biased demand shock changes when there are short run adjustment frictions. This situation with infinite adjustment costs is plotted in figure 2. As in Figure 1, the solid lines (and primed notation) represents the post-shock period. Because of the production shock, the second period comparative schedule is zero above the cutoff, and equal to its initial value before. The Φ schedules are unchanged relative to to Figure 1. However, because technology is fixed, the resulting equilibrium in period 2 is different to the

Figure 1: A Foreign Biased Demand Shock



This figure plots one comparative advantage schedule $A(i)$, and two different demand schedules Φ (which correspond to the share of global consumption on goods produced at home for any given relative wage). Φ' corresponds to a "foreign-biased" demand shock. The equilibrium relative wage in the first period is ω, and is ω') in the second period.

one without adjustment costs shown in Figure 1. In particular, the production constraint forces the cutoff to be the same as in the first period: $\bar{\imath}_2 = \bar{\imath}$ instead of $\bar{\imath}'$.

Adjustment costs lead workers in Home to be stuck making low demand goods. This misallocation lowers their relative wage even further after the demand shock (from ω' to ω''). While the relative foreign wage is higher, their absolute welfare is lower with adjustment frictions, since they also have misallocated workers (producing goods who would be made at Home without adjustment costs).

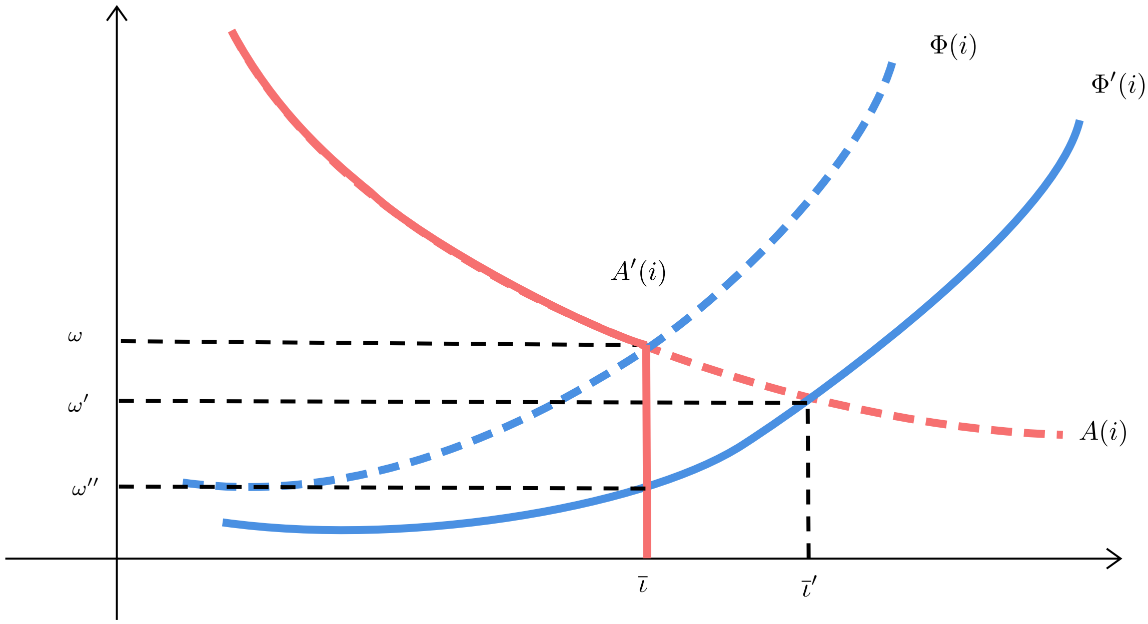
Even without adjustment costs, Home wants to introduce tariffs to improve its terms of trade (Opp, 2010; Costinot et al., 2015). In the next section, we describe a new dynamic motivation for protectionism: producing more goods in the first period results in less misallocation in the second period (at the cost of introducing misallocation in the first period).

4 Optimal Protectionism

4.1 Problem Setup

In this section we define the Home government's problem, and characterize optimal policy. As in Costinot et al. (2015) (CDVW), we focus on the primal problem of the planner. First, we solve for allocations while taking account of consumers' decision rules, and then we show how to decentralize the problem. The planner's

Figure 2: Foreign Biased Demand Shock with Frictions



This figure plots two different comparative advantage schedule $A(i)$, and two different demand schedules Φ (which correspond to the share of global consumption on goods produced at home for any given relative wage). The primes correspond to a "foreign-biased" demand shock. The equilibrium relative wage in the first period is ω, and is ω'') in the second period (and would be ω' without adjustment costs).

problem is to pick relative wages, consumption, tariffs, and sourcing decisions subject to consumer optimality, market clearing, and feasibility constraints (in Home and Foreign). We will treat the Foreign's wage, w_t^*, as the numeraire. However, it will often help with intuition to write down both domestic and foreign wages, not just their ratio ω_t. The planner's full problem (PP) is given by:

$$V \max_{w_t, w_t^*, c_{it}, m_{it}, \tau_{it}} L \int_i \beta_{i1} \log(c_{i1}/L) di + \delta L \int_i \beta_{i2} \log(c_{i2}/L) di \tag{PP}$$

$$\begin{aligned}
\text{subj. to} \quad & \int_0^1 a_i^* c_{it}^* (1 - m_{it}^*) di + \int_0^1 a_i^* c_{it} m_{it} di \le L^* \\
& \int_0^1 a_{it} c_{it}^* m_{it}^* di + \int_0^1 a_{it} c_{it} (1 - m_{it}) di \le L \\
& c_{it} = \beta_{it}(w_t L + T_t) / \min\{w_t a_{it}, w_t^* a_{it}^* (1 + \tau_{it})\} \\
& m_{it} = \arg\min\{w_t a_{it}, w_t^* a_{it}^* (1 + \tau_{it})\}, \\
& c_{it}^* = \beta_{it} w_t^* L^* / \min\{w_t^* a_{it}^*, w_t a_{it}\} \\
& m_{it}^* = \arg\min\{w_t^* a_{it}^*, w_t a_{it}\},
\end{aligned}$$

where a_{i2} is a function of m_{i1} and T_t is tariff revenue at Home.[10] Optimal consumption follows from the Cobb-Douglas preferences.

We will characterize optimal tariff and sourcing policy as a function of any choice of wages. This reduces the infinite dimensional problem above to a finite dimensional problem (that just depends on the choice of wages, as in CDVW). For a given set of wages, we can then check if it is feasible for the planner to achieve.

We initially solve a relaxed version of the planner's problem, ignoring the constraints on c_{it} and m_{it}. We show that the solution to the relaxed problem is feasible even in the full planner's problem, and therefore is a solution to the full planner's problem

The relaxed problem (RP) is the following Lagrangian as a function of relative wages:

$$\begin{aligned}\mathcal{L}(\omega_1, \omega_2) = \int_0^1 \Big[& \beta_{i1} \log c_{i1}/L + \delta\beta_{i2} \log c_{i2}/L + \\ & \nu_1^*(L^* - a_{i1}^* c_{i1}^*(1 - m_{i1}^*) - a_{i1}^* c_{i1} m_{i1}) + \nu_1(L - a_{i1} c_{i1}^* m_{i1}^* - a_{i1} c_{i1}(1 - m_{i1})) + \\ & \delta\nu_2^*(L^* - a_{i2}^* c_{i2}^*(1 - m_{i2}^*) - a_{i2}^* c_{i2} m_{i2}) + \delta\nu_2(L - a_{i2} c_{i2}^* m_{i2}^* - a_{i2} c_{i2}(1 - m_{i2})) \Big],\end{aligned} \tag{RP}$$

where m_{it}^* are the (known) import decision rules for Foreign. We start by solving the relaxed problem in the benchmark case where $x = \infty$, and then turn to more general assumptions.

4.2 Benchmark: Infinite Adjustment Frictions

In our benchmark case, we will assume that $x = \infty$, and that $\beta_{i2}/\beta_{i1} = \lambda \in (0, 1)$ for $i < \iota^*$ and is equal to $\frac{1-\lambda\iota^*}{1-\iota^*}$ otherwise. Furthermore, $\beta_{i1} = 1$ for all i (Eaton and Kortum, 2002). The first assumption implies that if the planner does not produce the good in the first period, the good *must* be imported in the second. Similarly, Foreign will produce the good domestically. The second and third assumptions are simplifications that will make characterizing the solution tractable, and be relaxed at the end of this section. As in the previous section, we continue to assume throughout that ι^* is large, to ensure that Home would only manufacture ι^* with policy.

In order to solve (RP) we first characterize optimal consumption and import tariffs conditional on importing. We then solve for optimal import policy (which uses prohibitive tariffs). Since RP is linear in i and concave in each i conditional on m and m^*, it can be solved using first order conditions:

$$\frac{\partial \mathcal{L}}{\partial c_{it}} : c_{it} = \frac{\beta_{it} L}{\nu_t^* a_{it}^* m_{it} + \nu_t a_{it}(1 - m_{it})}. \tag{8}$$

The ratio of Equation 8 for any two goods that are imported is

$$\frac{c_{it}}{c_{i't}} = \frac{\beta_{it}}{\beta_{i't}} \times \frac{a_{i't}^*}{a_{it}}.$$

[10] To economize on space, we have omitted writing the market clearing condition. The market clearing constraints are redundant, as they are implied by the feasibility constraints and optimal sourcing and consumption rules.

This exactly matches the consumer's decision rule if and only if $1 + \tau_{it} = 1 + \tau_{i't}$, implying that optimal policy is one single tariff for all imported goods (Opp, 2010; Costinot et al., 2015). However, not all goods are imported, so prohibitive tariffs may be optimal for some goods.

To identify which goods should have infinite tariffs, we work backwards. In the second period, the value function is differentiable in m_{i2}, treated as a variable on $[0, 1]$. The associated first order condition is

$$\frac{\partial \mathcal{L}}{\partial m_{i2}} : m_{i2} = 1 \Leftrightarrow -\nu_2^* a_i^* + \nu_2 a_{i2} > 0.$$

As a result, the planner will import if $\frac{a_i^*}{a_{i2}} < \frac{\nu_2}{\nu_2^*}$. This solution to (RP) will be a solution for (PP) only if $\tau_2 = \frac{\omega_2 \nu_2^*}{\nu_2} - 1$. Since $a_{i2} = \infty$ if $m_{i1} = 1$, the optimal importing strategy in period 2 will be given by:

$$m_{i2} = \begin{cases} 1 & \text{if } m_{i1} = 1 \\ 0 & \text{if } m_{i1=0} \text{ and } A_i \leq \nu_2/\nu_2^*. \end{cases} \tag{9}$$

Foreign's import strategy will similarly be given by,

$$m_{i2}^* = \begin{cases} 0 & \text{if } m_{i1} = 1 \\ 1 & \text{if } m_{i1=0} \text{ and } A_i > \omega_2. \end{cases} \tag{10}$$

Thus, given ν_2/ν_2^*, period 2 imports for good i only depend on period 1 import behavior for good i— implying that period 2 decisions are separable across goods.

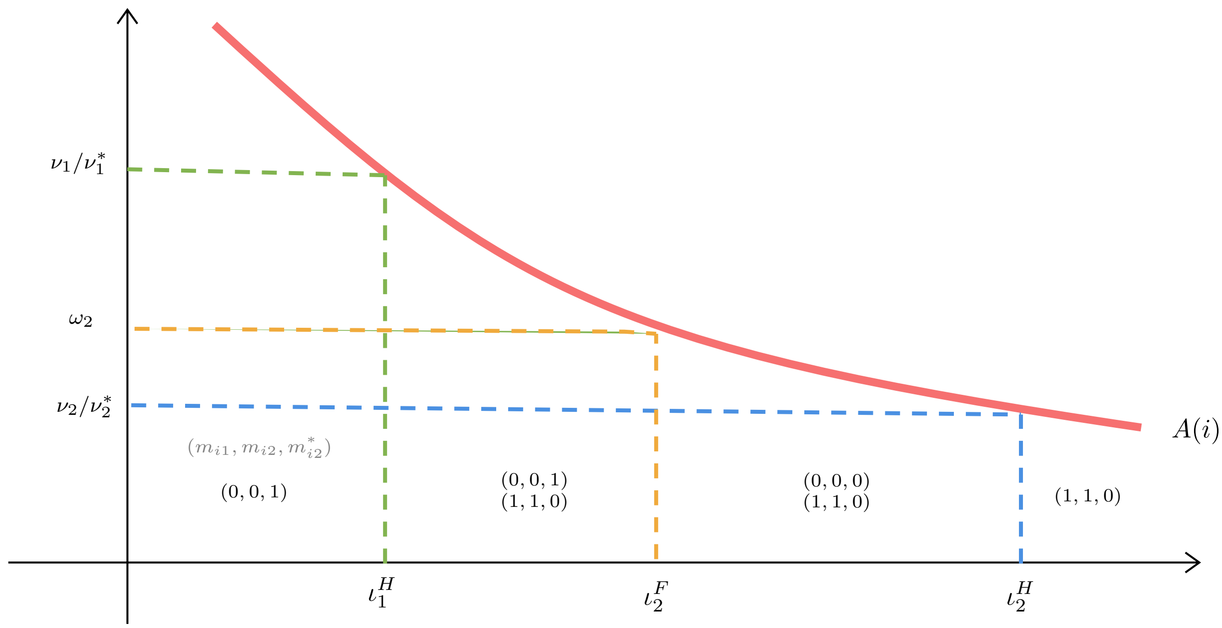
Turning to the first period, the Lagrangian is no longer differentiable. Hence, while for *any* choice of m_{i1} we can solve for consumption and period 2 policies, we cannot use first order methods to find the optimal policy in period 1. However, the linearity of $\mathcal{L}$ in i, and the fact that period 2 decisions are separable across goods, means that we can proceed good-by-good, taking account of the optimal choice of c_{i1}, m_{i2}, and m_{i2} conditional on m_{i1}.

In particular, define $\iota_t^H = A^{-1}(\nu_t/\nu_t^*)$ and $\iota_t^F = A^{-1}(\omega_t)$. These are the optimal production cutoffs in the absence of adjustment costs. Every good will fall into a partition of the unit interval given by $[0, \iota_1^H]$, $[\iota_1^H, \iota_2^F], [\iota_2^F, \iota_2^H]$, and $[\iota_2^H, 1]$. Within each grouping (given wages), each first period importing decision m_{i1} corresponds to exactly one combination of Home and Foreign importing decisions.

We depict this situation in figure 3. The ordered triples in each region are the options the planner considers. In the first region, the goods to the left of ι_1^H, Home will produce in the first period regardless of the tariff (and correspondingly will also produce in the second period). In the fourth region, to the right of ι_2^H, Home will import all goods in the second period regardless of production status, so there is no dynamic motivation for protectionism in the first period.[11] For the second region, goods j in $[\iota_1^H, \iota_2^F]$, Home will be

[11]Formally, one must check that $A_i > \nu_1/\nu_1^* > \nu_2/\nu_2^*$ in the first region and the opposite in the fourth, ensuring that the planner does indeed choose to produce in the former and import in the latter.

Figure 3: Cutoffs With Infinite Adjustment Costs



This figure plots the different possibilities for importing behavior given a comparative advantage schedule $A(i)$. ν_t/ν_t^*, is the equilibrium relative wage times the optimal tariff in period t, and ω_2 is the equilibrium relative wage in period 2. The triplets correspond to the possible import decisions that can be chosen through tariff policy.

the only producer of j in the second period if and only if it produces j in the first period. In the third region, the partition $[\iota_2^F, \iota_2^H]$, Foreign will always produce all of the goods. Home will produce all (and only) of the goods in the second period that it produced in the first period. Optimal policy consists of a cutoff rule within each of the regions.

Proposition 4.1. *For a given choice of admissible wages, the optimal policy is given by:*

$$m_{i1} = 1 \Leftrightarrow A_i \leq \begin{cases} \frac{\nu_1}{\nu_1^*} & \text{if } i \in [0, \iota_1^H] \\ \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{1}{1+\delta\lambda}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\lambda}{1+\delta\lambda}} \times \exp\left(-\frac{\delta\lambda}{1+\delta\lambda}\frac{L^*}{L}\frac{\Phi_H}{\Phi_F}\left(1 - \frac{\nu_2/\nu_2^*}{\omega_2}\right)\right) & \text{if } i \in [\iota_1^H, \iota_2^F] \\ \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{1}{1+\delta\lambda}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\lambda}{1+\delta\lambda}} & \text{if } i \in [\iota_2^F, \iota_2^H] \\ \frac{\nu_2}{\nu_2^*} & \text{if } i \in [\iota_2^H, 1], \end{cases} \tag{11}$$

where $\Phi_{H,t}$ and $\Phi_{F,t}$ are Home and Foreign import expenditure shares. Moreover, there exists a monotonically decreasing map $B(\nu_1/\nu_1^*)$ with a fixed point at the optimum. This map can be used to check if the optimal policy is feasible at a given choice of wages.

We leave the full derivation to the appendix, but present a sketch of the proof. Let $\mathcal{L}_i^{m_{i1},m_{i2}}$ be the contribution to the Lagrangian of good i given choices of m_{i1}, m_{i2}, and m_{i2}^*. The optimal consumption decision is

$$\begin{aligned}\mathcal{L}_i^{m_{i1},m_{i2},m_{i2}^*} =&\kappa_i - \beta_{i1}\log(a_{i1}\nu_1(1-m_{i1}) + a_i^*\nu_1^* m_{i1}) - \delta\beta_{i2}\log(a_{i2}\nu_2(1-m_{i2}) + a_i^*\nu_2^* m_{i2}) \\ &- \delta\beta_{i2}[\nu_2^*(1-m_{i2}^*) + \nu_2 w_2^*/w_2 m_{i2}^*],\end{aligned} \tag{12}$$

where κ_i is a constant independent of any choices. Since m_{i2} and m_{i2}^* are known conditional on m_{i1}, optimal consumption is only a function of m_{i1}. The cutoff rules follow from manipulating Equation 12. Within each bin, the cutoffs have economic interpretations. The ratios of the multipliers are the statically optimal cutoffs for each period—they denote the bounds on the comparative advantage schedule where Home is either too productive (the lowest cutoff) or too unproductive (the highest cutoff) for dynamic considerations to matter. The middle regions each contain the geometric mean of the two cutoffs weighted by the demand shock—reflecting that the planner combines the two cutoffs, using the demand shock to determine importance. The cutoff for the second region contains one additional term, which reflects that policy will not only induce production in the second period, but also exports. As a result the cutoff for the second region is smaller than the cutoff for the third region. Since A is decreasing in i, this implies that only one cutoff will be binding, so overall production will follow a single cutoff rule.

Equation (11) is defined implicitly by the import shares in period 2, which are a function of the import shares in period 1. The problem has a fixed point for any choice of the ratio of multipliers (which can be reached by iterating on the import shares). This establishes that for *any* set of wages and multipliers, there exists an import policy that solves the planner's problem. The final step is to derive the optimal multipliers that are consistent with wages.

To complete the proof we solve for the multipliers and show that they are a fixed point of the mapping,

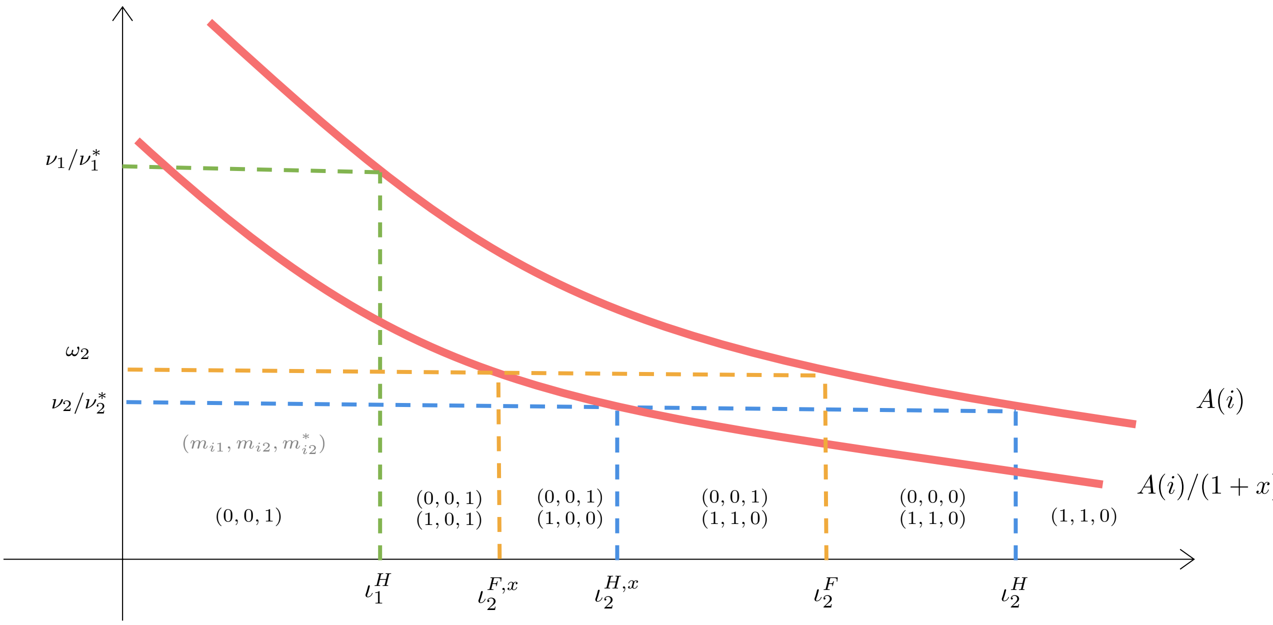
$$B_t(\nu_t/\nu_t^*) = \omega_t \times \frac{1-\Phi_{H,t}}{\Phi_{H,t}} \times \frac{\Phi_{F,t}}{\frac{wL}{w^*L^*} - \Phi_{F,t}},$$

where the import shares are a function of the multipliers through (11). This mapping makes clear that there may be implicit restrictions on the feasible set of wages in period 2 as it must be that $\nu_t/\nu_t^* \in [0, \omega_t]$. In the appendix we prove that B is continuous and monotonically decreasing on each partition of the unit interval. We show that B_1, taking into account the fixed point of B_2, is monotonically decreasing on $[0, 1]$, so that a fixed point exists, ensuring a solution.

4.3 Finite Adjustment Frictions

The case with finite adjustment costs is similar to the one with infinite adjustment costs, however the potential number of regions that the planner must consider in the first period increases. To understand the new regions, note that with finite adjustment costs some goods may be optimally produced in Home in the second period even if they *were not* locally produced in the first period (which would be technologically impossible with infinite adjustment costs). This will occur if $A_i/(1+x) \geq \nu_2/\nu_2^*$. The good will be exported

Figure 4: Optimal Cutoff Rules with finite adjustment costs



if $A_i/(1+x) \geq \omega_2$. This implies two new regions for goods that would be produced in the second period regardless of what happens in the first.[12] We can still use the equivalent of equation (12) to solve this problem, except now $a_{i2} = a_i(1+x)$ if $m_{i1} = 1$.

Figure 4 plots the new set of regions the planner must consider. The lower comparative advantage schedule is the one for goods assuming that nothing was produced in the first period. The values of ν_1/ν_1^*, ω_2, and ν_2/ν_2^*, partition the space into the six intervals. As in Figure 3 within each interval the second period import choices are the same given first period decisions. Similarly, optimal policy follows a cutoff within each interval.

Proposition 4.2. *For a given choice of wages, if the optimal policy is feasible it is given by:*

$$m_{i1} = 1 \Leftrightarrow A_i \leq \begin{cases} \frac{\nu_1}{\nu_1^*} & \text{if } i \in [0, \iota_1^H] \\ \frac{\nu_1}{\nu_1^*} \times (1+x)^{-\delta\lambda} & \text{if } i \in [\iota_1^H, \iota_2^{F,x}] \\ \frac{\nu_1}{\nu_1^*} \times (1+x)^{-\delta\lambda} \times \exp\left(-\frac{\delta\lambda}{1+\delta\lambda}\frac{L^*}{L}\frac{\Phi_H}{\Phi_F}\left(1 - \frac{\nu_2/\nu_2^*}{\omega_2}\right)\right) & \text{if } i \in [\iota_2^{F,x}, \iota_2^{H,x}] \\ \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{1}{1+\delta\lambda}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\lambda}{1+\delta\lambda}} \times \exp\left(-\frac{\delta\lambda}{1+\delta\lambda}\frac{L^*}{L}\frac{\Phi_H}{\Phi_F}\left(1 - \frac{\nu_2/\nu_2^*}{\omega_2}\right)\right) & \text{if } i \in [\iota_1^{H,x}, \iota_2^F] \\ \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{1}{1+\delta\lambda}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\lambda}{1+\delta\lambda}} & \text{if } i \in [\iota_2^F, \iota_2^H] \\ \frac{\nu_2}{\nu_2^*} & \text{if } i \in [\iota_2^H, 1]. \end{cases} \tag{13}$$

Moreover, for a given choice of the ratio of multipliers, import shares exist and are continuous. Finally, there are bounds on x for which the operator from import shares onto themselves is monotonic and smooth,

[12] The new regions may be empty if ι_1^H is large enough (for instance, in the limit as x approaches ∞).

allowing for computation of the optimal policy (as well as checking if feasible given wages).

The proof of this proposition proceeds in the same fashion as in the proof with infinite adjustment costs. Relative to the regions from Proposition 4.1, the second and third regions are the new ones, which correspond to goods where Home might produce in the second period even if Home does not produce them in the first period. The second region corresponds to goods that home will export in the second period regardless. The third region corresponds to goods where Home will always produce in the second period, but only export if it produces in the first period. The cutoff within each region is monotonically increasing in Φ_H, but the cutoff in the third region might be below the cutoff in the second region. This means that optimal policy might lead to non-monotonic production patterns: for instance in period 1 producing some of the goods in the third region, but not all of the goods in the second region. As in the results from Proposition 4.1, the planner does not optimally protect goods they would import if there were no adjustment constraints.

We now turn to several extensions of our model—removing restraints on β, adding in stochastic demand shocks, and introducing aggregate productivity shocks (while also relaxing the functional form on utility).

5 Extensions of the Main Model

5.1 Unrestricted Demand Shifts

In the previous section, we focused on the case that $\beta_{i1} = 1$ and that β_{i2} was a uniform decrease to λ for some goods, and a jump up for the remainder. These assumptions ensures that one can check if ω_t is feasible. However, the general method introduced above can be extended to allow for unrestricted β_i, as in both DFS and CDVW. For the optimal solution, all that changes is that λ is replaced everywhere by β_{i2}/β_{i1}, so the solution becomes:

Corollary 5.1. *With flexible demand parameters, if the optimal policy is feasible it is given by:*

$$m_{i1} = 1 \Leftrightarrow A_i \leq \begin{cases} \frac{\nu_1}{\nu_1^*} & \text{if } i \in [0, \iota_1^H] \\ \frac{\nu_1}{\nu_1^*} \times (1+x)^{-\delta \frac{\beta_{i2}}{\beta_{i1}}} & \text{if } i \in [\iota_1^H, \iota_2^{F,x}] \\ \frac{\nu_1}{\nu_1^*} \times (1+x)^{-\delta \frac{\beta_{i2}}{\beta_{i1}}} \times \exp\left(-\frac{\delta\beta_{i2}}{\beta_{i1}+\delta\beta_{i2}} \frac{L^*}{L} \frac{\Phi_H}{\Phi_F}\left(1 - \frac{\nu_2/\nu_2^*}{\omega_2}\right)\right) & \text{if } i \in [\iota_2^{F,x}, \iota_2^{H,x}] \\ \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{\beta_{i1}}{\beta_{i1}+\delta\beta_{i2}}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\beta_{i2}}{\beta_{i1}+\delta\beta_{i2}}} \times \exp\left(-\frac{\delta\beta_{i2}}{\beta_{i1}+\delta\beta_{i2}} \frac{L^*}{L} \frac{\Phi_H}{\Phi_F}\left(1 - \frac{\nu_2/\nu_2^*}{\omega_2}\right)\right) & \text{if } i \in [\iota_1^{H,x}, \iota_2^F] \\ \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{\beta_{i1}}{\beta_{i1}+\delta\beta_{i2}}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\beta_{i2}}{\beta_{i1}+\delta\beta_{i2}}} & \text{if } i \in [\iota_2^F, \iota_2^H] \\ \frac{\nu_2}{\nu_2^*} & \text{if } i \in [\iota_2^H, 1]. \end{cases} \tag{14}$$

The feasibility of the solution to Corollary 5.2 is difficult to check because integrating across β can lead to discontinuities.[13]

[13] In practice, if the shocks are "on average" small enough, the optimum ω will be such that $\iota_H^1 > \iota_2^F$, so that the integral terms do not appear in any cutoffs. In this case, the B map behaves exactly as in the previous section. and feasibility is straightforward to verify.

The unrestricted case introduces a new force behind optimal policy (when there is a feasible solution): the change in a good's demand affects the value of its protection. There is no longer a simple cutoff rule in productivity (within each partition). For a given β_{i2}, goods that have lower β_{i1} require less first period reallocation to protect a good. As a result, those goods are relatively more valuable to protect.[14].

Nevertheless, the solution to Corollary 5.2 has many of the same features as the cases with a secular change in demand. The level of adjustment costs only matters where comparative advantage is already relatively strong, and Home only potentially protects goods that it would produce in the absence of adjustment costs.

5.2 Stochastic Demand Shocks

In our next extension, we turn to the case of stochastic demand shocks, as in Eaton and Grossman (1985). For the sake of exposition, we return to the case that x is infinite and that β's are restricted. To this end, suppose there is uncertainty over λ. Let $s \in \mathcal{S}$ index the set of possible states, λ_s the expenditure coefficients in each state, and $G(s)$ the cumulative distribution function on $\mathcal{S}$. We will assume that all relevant integrals exist and that *every* realization of s is a foreign biased demand shock.[15]

Given the realization of the state s in the second period, the formation of optimal policy (given m_{i1}) follow CDVW. Letting $\bar{i} \leq 1$ solve $\sup_s \{A(i) = \nu_2(s)/\nu_2^*(s)\}$. Then for $i > \bar{i}$ the planner will clearly always import. For i such that $A(i) \geq \nu_1/\nu_1^*$, the planner will always produce at home.

For $i \in [\iota_1^H, \bar{i}]$ there is the possibility of the planner choosing to produce domestically or at home, depending on demand conditions. Now there will be three regions to consider. The first region will be the case that $i \in [\iota_1, \iota_2^F(s)]$, the second will be that $i \in [\iota_2^F(s), \iota_2^H(s)]$, and finally there will be the region $i \in [\iota_2^H(s), \bar{i}]$. This last region is new, and is the region where if the planner produces in the first period, in the second period they may still import. This would happen if the planner *ex post* made a mistake and protected too many goods, which is impossible in the deterministic case. Denote the regions by $R_I(s)$, $R_{II}(s)$, and $R_{III}(s)$ respectively, and let $S_j(i) = \{s \in \mathcal{S} : i \in R_j(s)\}$, be the set of realizations for which good i is in set j. These are mutually exclusive regions so let $p_{j,i}$ be the probability of being in that region, so that $\sum p_{j,i} = 1$, and let $E_{ji}(\cdot)$ be the conditional expectation operator over $S_j(i)$.

We can transform equation (12) into an expectation in the first period and rearrange to arrive at the following cutoff rule for goods:

Corollary 5.2. *With uncertainty over the demand shock, if the optimal policy is feasible it is given by:*

$$m_{i1} = 1 \Leftrightarrow \log A_i \leq \frac{\log\left(\frac{\nu_1}{\nu_1^*}\right) + p_{I,i}\delta E_{I,i}\left(\lambda \log\left(\frac{\nu_2}{\nu_2^*}\right) + \lambda \frac{\Phi_{H,2}}{\Phi_{F,2}}\frac{L^*}{L}\left(1 - \frac{\nu_2 \nu_2^*}{\omega_2}\right)\right) + p_{II,i}\delta E_{II,i}\left(\lambda \log\left(\frac{\nu_2}{\nu_2^*}\right)\right)}{p_{I,i}\delta E_{I,i}(\lambda) + p_{II,i}\delta E_{II,i}(\lambda)} \tag{15}$$

This equation is the weighted average of the various cutoffs from (11). The probability of a good being in

[14] Similarly, for a given β_{i1}, goods with a higher β_{i2} are more valuable to protect)

[15] This is actually irrelevant, but obviously some probability that the demand shock favors Home will militate against any precautionary action.

a particular region in the second period depends on which goods are protected in period 1. Since the optimal first period cutoff depends on second period probabilities, as in the deterministic case the above cutoff is implicit and the solution to a fixed point.

The presence of the expectation prevents exponentiation from simplifying the expression as in the non-stochastic case. However, the right hand side of the inequality is the probability weighted average of the expected cutoff for region I, and the expected cutoff for region II. Important for thinking about trade policy is that these weights do *not* sum to 1. This reflects the fact that with probability $p_{III} = 1 - p_I - p_{II}$, the planner can make ex-post mistakes by protecting a good in the first period that is nevertheless imported in the second period. Wanting to avoid overprotection tilts the planner towards placing *more* weight on the first period. That said, without more information on G, it is difficult to know if uncertainty makes the planner more or less likely to use industrial policy than if the shock is $E(\lambda)$ with probability 1. This is because there are ultimately two forces at play: the planner is more cautious on account of risk, but also wants to avoid making mistakes.

5.3 Wealth Effects and the Elasticity of Intertemporal Substitution

Finally, we consider a generalization of our previous setup, allowing for general CES preferences over goods within a period, and CRRA preferences over consumption bundles across periods. We maintain the simplifying assumptions that $x = \infty$, $\beta_{i1} = 1$, and that $\beta_{i2} = \lambda$ for $i < i^*$ and $\beta_{i2} = \frac{1-\lambda i^*}{1-i^*}$ otherwise. The objective for the planner is to now maximize,

$$\tilde{V}\left(\{\sigma_i\}\right) = \left(\int_0^1 c_{i1}^{\frac{\epsilon-1}{\epsilon}}\right)^{(1-\rho)\frac{\epsilon}{\epsilon-1}} + \delta\left(\int_0^1 \beta_{i2} c_{i2}^{\frac{\epsilon-1}{\epsilon}}\right)^{(1-\rho)\frac{\epsilon}{\epsilon-1}}, \tag{16}$$

where ρ is the elasticity of intertemporal substitution (EIS) and ϵ is the elasticity of substitution across goods (EOS). The constraints of the planner are the same as before, with foreign and home's demand functions being adapted to the CES case. Our interest in this section is studying *symmetric* productivity shocks that arise at the same time as the demand shock. By symmetric, we mean that a_i and a_i shift by the same (proportional) amount. Such a shock leaves A_i unchanged, while lowering the global PPF. Let σ_i be the shock, so that second period unit labor requirements are given by $\sigma_i a_i$ and $\sigma_i a_i^*$. The first thing to notice in this setting is that *regardless* of first period policy, the second period will still adopt the setting of CDVW, and so can be characterized by a cutoff rule in A_i, and a uniform tariff. The tariff and the cutoff rule do not depend on the level of a_i or a_i^*, and so the solution is invariant to σ_i.

What then is the role of the supply shock? Given the CES structure of preferences, welfare in the equilibrium economy will contain a shifter term given by, $\Sigma^{-1} = \left(\int_0^1 \sigma_i^{1-\epsilon}\right)^{\frac{1}{\epsilon-1}}$. For σ_i constant, this will just be $1/\sigma$, so that total output is shifted down by σ. In this case, we can rewrite the planner's objective as

$$V(\Sigma) = \left(\int_0^1 c_{i1}^{\frac{\epsilon-1}{\epsilon}}\right)^{(1-\rho)\frac{\epsilon}{\epsilon-1}} + \delta\Sigma^{\rho-1}\left(\int_0^1 \beta_{i2} c_{i2}^{\frac{\epsilon-1}{\epsilon}}\right)^{(1-\rho)\frac{\epsilon}{\epsilon-1}}, \tag{17}$$

Equation 17 shows that the role of supply shocks is to make the planner *more* patient, by effectively increasing the discount rate. If, as in the previous sections, $\rho = 1$ (so intertemporal preferences take the log form) then the σ shock does not affect policy.

As in the main case discussed in Proposition 4.1, the infinite adjustment costs ensure a simple cutoff rule in protectionism. Similarly, the planner will want uniform tariffs on imported goods in both periods to avoid distorting relative marginal utilities. We show in the appendix that in the second period, the optimal tariff policy conditional on a cutoff rule chosen in the first period, ι, is the inverse of the export supply elasticity. Given ι, the planner in the second period follows CDVW, so the first period planner's problem is

$$V(\Sigma) = \max_{\tau,\iota} \left(\int_0^1 c_{i1}^{\frac{\epsilon-1}{\epsilon}} \right)^{(1-\rho)\frac{\epsilon}{\epsilon-1}} + \delta \Sigma^{\rho-1} V_2(\iota). \tag{18}$$

If the second period problem is globally concave in ι (which depends on the shape of A) then the full problem will be concave in ι, and so by the Theorem of the Maximum, ι will be increasing in Σ.[16] This means that so long as $\rho > 1$ there is strictly *more* protectionism when there is also an aggregate productivity shock. This suggests that in a situation where global shocks shift both demand and lower productivity, there is more of a precautionary incentive to protect.

6 An Illustration with Covid-19 Pandemic

In this section, we quantify the theoretical results, focusing on crisis of the order of magnitude of the Covid-19 pandemic (where demand for goods such as masks and ventilators spiked). We are both interested in quantifying the costs of adjustment frictions, as well as the gains from policy. To map to our model, we combine all countries into a Rest of World composite. We use the version of the model from 5.3, adding symmetric trade costs, d. For calibrating the static equilibrium, we model unit labor requirements as,

$$a(i) = \left(\frac{i}{T}\right)^{1/\theta}$$
$$a^*(i) = \left(\frac{1-i}{T^*}\right)^{1/\theta}.$$

We use the same parameters as CDVW, setting $\theta = 5$ and $T/T^* = 5194.8$. Furthermore we set $\sigma = 2.5$, $L^*/L = 19.2$ and $d = 1.44$.[17]

Matching the model to the actual crisis is difficult, since there were many changes to the global economy in 2020. In addition to the direct effects of the pandemic, there were also tightening of trade and migration restrictions, as well as large and changes in the location of work. Nevertheless, there is evidence of a foreign-

[16] Concavity of the second period problem in ι is also a requirement for the problem to have a solution at the first order conditions.

[17] The parameters CDVW use target a US GDP in world output of 26.2% and a US import share of 24.7%. The model is somewhat sensitive to rounding. At these parameter values we have a GDP share of 24.7% and an import share of 23.6%. If we were to target the initial moments ourselves, we find $d = 1.42$ and $T/T^* = 7961.8$, with otherwise very similar numbers. We use their numbers for comparability.

biased demand shock: the decline in imports was about half of the decline in exports (imports over GDP declined in the US by 12% from 2020 Q1 to 2020 Q2, while exports declined 20%).

There is additional evidence that the relative increase in imports was related to the pandemic. For instance, imports of COVID-19 related goods increased by around $15 a person at the peak of the pandemic,[18] where those goods had previously made up around four percent of the consumption basket. We can rationalize this change with a λ of around .99, with further discussion in Appendix C. In order to show the effects of policy under even larger demand shocks, we additionally show counterfactuals with $\lambda = .95$ and .9.

Turning to the remaining dynamic parameters, we calibrate σ to match the decline in global GDP over the period. We target the World Bank's global GDP decline of 3.5% against an average growth rate of 2.9% per year, adjusted to the quarterly level. This leads to $\sigma = 1.08$.[19] We set $\rho = 2$. Finally, we consider both a myopic government ($\delta = 0$) and a forward looking government ($\delta = .95$).

Table 1: Gains From Trade Policy

	Change in Utility Relative to Laissez-Faire		
	1% shock	5% shock	10% shock
	(1)	(2)	(3)
Panel A. NPV Utility Increase			
Infeasible	1.100%	1.548%	1.966%
Myopic	1.086%	1.370%	1.569%
Sophisticated	1.093%	1.445%	1.720%
Panel B. Utility Change, Period 1			
Infeasible	0.974%	0.974%	0.974%
Myopic	0.974%	0.974%	0.974%
Sophisticated	0.967%	0.889%	0.792%
Panel C. Utility Change, Period 2			
Infeasible	1.224%	2.128%	2.975%
Myopic	1.197%	1.769%	2.170%
Sophisticated	1.216%	2.005%	2.663%

Table 2: Set of Imported Goods

	Change Relative to Laissez-Faire		
	1% shock	5% shock	10% shock
	(1)	(2)	(3)
Infeasible	-31.76%	-31.76%	-31.76%
Myopic	-31.76%	-31.76%	-31.76%
Sophisticated	-35.74%	-45.38%	-51.23%

To analyze the importance of dynamically optimal tariffs, in Table 1 we show the welfare gains of three different policies to the benchmark of laissez-faire. First, to show the best-case scenario, we show the gains from the infeasible optimal policy of removing adjustment costs and applying tariffs (which turns out to be

[18] https://unctad.org/system/files/official-document/ditcinf2020d4_en.pdf

[19] This is close to the OECD's estimate for an average Q2 decline in GDP of around 10.2%—they estimate a rebound in in latter half of the year, while we assumed zero growth. Thus, we think our approximation is reasonable.

applying CDVW each period). Not surprisingly, the gains are increasing in the size of the shock (Panel A), from a 1.1% welfare increase in the 1.6% shock scenario to a 2.0% increase in the 10% shock scenario. This masks heterogeneity across periods, shown in Panels B (period 1) and C (period 2). The gains in the first period are unaffected by the size of the period 2 shock, just under one percent.[20] The gains in the second period increase from 1.2% under the smaller shock to 3.0% under the bigger one.

Second, we turn to a myopic planner who faces adjustment costs. For our estimated shock, which is small, the decline in the gains relative to the infeasible case are small. However, the larger shocks demonstrate the potentially large losses from adjustment costs—with the gains from protectionism declining by 20% $\approx \frac{1.569}{1.966}$ for the 10% shock to λ. This is entirely driven by the adjustment costs, as the first period first best is still achieved.

Finally, we use the dynamically optimal policy derived in our model. For our 1% shock, the sophisticated policy closes half of the gap between the myopic planner and the infeasible optimum. This pattern is the same for larger shocks. Table 2 shows the size of the set of imported goods relative to Laissez-Faire across different regimes. The sophisticated planner uses prohibitive tariffs to increase the set of goods produced domestically by about 4%, which lowers the import share about 2.25% in the first period. For a larger shock, the amount of industrial policy required to achieve the optimal policy is much larger. We conclude from this exercise that, in levels, the value of industrial policy is limited—as the model generally predicts low gains to policy—however, that industrial policy can close about half the gap between myopic policy and a world of no adjustment costs.

7 Conclusion

When elite athletes return from a break (after injury or the off-season), they are often "rusty." It's not that they have lost their skill, but that it takes time and practice to return to full speed. This paper takes a similar view of production: there are many products for which countries would be productive if they regularly made them, but getting to that level of productivity takes time. Our analysis is based on the Dornbusch et al. (1977) set-up, and includes some simplifying assumptions that are important to highlight. First, the technological constraints we have assumed are entirely on the extensive margin. It is likely that intensive margin adjustments are also costly, due to short-run external diseconomies of scale (Bartelme et al. 2019 discuss more broadly trade policy with economies of scale). Similarly, we have also ignored supply chains (Shih, 2020; Grossman and Helpman, 2021), focusing on trade in final goods. That said, we suspect that even in these situations the qualitative intuition from our set-up will still hold: trade policy should encourage production of goods in the location of their comparative advantage. A additional caveat is that we have ignored any kind of strategic interaction or bargaining between countries (Ossa, 2014; Beshkar and Lashkaripour, 2020). Provisions for emergencies will likely be an important part of international agreements,

[20] CDVW find approximately the same number using uniform tariffs.

and ex-post countries discussed export-bans for crisis-related products. This is, for instance, an important motivation for the protection of war-related goods (Johnson, 1960).

In this paper, we have considered the problem of a planner in a two country, two period world, facing a demand shock in the second. Due to adjustment costs and myopia on the part of producers, the planner benefits from using trade policy to promote production in the first period, in order to facilitate better terms of trade in the second. We think of this as a reasonable approximation to how a planner may think of a pandemic or other large crisis: there is a small probability of large demand shocks occurring, and (potential) producers are unlikely to internalize this global risk *ex ante*.

While the future crisis does motivate industrial policy, the existence of adjustment costs does not motivate the production of "new" products: if the planner would not want to optimally produce a product without adjustment costs, it also would not want to produce it with adjustment costs. In both periods, optimal policy leads to domestic production of produces goods for which Home has a strong comparative advantage, and imports goods in which Home's comparative disadvantage is large. In between this range, we find more nuance: the planner bases the decision (of whether to produce domestically, using prohibitive tariffs, or to buy from abroad) on a weighted average of the rules that the planner would statically use in period 1 and period 2. The implications for this characterization carry several lessons for policy makers considering designing policy for crises, in particular the value of identifying the products that are on the margin of being produced domestically or exported. We provide a graphical representation of where those products lie in comparative advantage space, since there may be many non-contiguous regions of products worth protecting.

We show that the intuition underlying optimal policy under uncertainty is similar. The optimal domestic policy does not depend on the specific identity of which foreign-produced goods face demand shocks, only on the effect of demand for good that would be produced at home in the absence of adjust costs. If the magnitude of the shock is unknown, then the optimal policy is between the policies of the possible states of the world. If there are idiosyncratic demand shocks for goods produced at home, then the optimal policy both reflects comparative advantage forces, as well as favoring products whose demand is increasing the most in the second period. Our results solidify the intuition that the role of home production is predominantly about improving the terms of trade with the least misallocation of workers.

References

Antràs, P., Redding, S. J., and Rossi-Hansberg, E. (2020). Globalization and pandemics.

Argente, D. O., Hsieh, C.-T., and Lee, M. (2020). The cost of privacy: welfare effects of the disclosure of covid-19 cases. *Working Paper*.

Bartelme, D. G., Costinot, A., Donaldson, D., and Rodriguez-Clare, A. (2019). The textbook case for industrial policy: Theory meets data. Technical report, National Bureau of Economic Research.

Beshkar, M. and Lashkaripour, A. (2020). The cost of dissolving the wto: The role of global value chains. *Working Paper*.

Beshkar, M. and Shourideh, A. (2020). Optimal trade policy with trade imbalances. *Journal of Monetary Economics*, 109:65–82.

Bonadio, B., Huo, Z., Levchenko, A. A., and Pandalai-Nayar, N. (2020). Global supply chains in the pandemic. *Working Paper*.

Carroll, D. R. and Hur, S. (2020). On the heterogeneous welfare gains and losses from trade. *Journal of Monetary Economics*, 109:1–16.

Caselli, F., Koren, M., Lisicky, M., and Tenreyro, S. (2020). Diversification through trade. *The Quarterly Journal of Economics*, 135(1):449–502.

Costinot, A., Donaldson, D., Vogel, J., and Werning, I. (2015). Comparative Advantage and Optimal Trade Policy. *The Quarterly Journal of Economics*, 130(2):659–702.

Cunat, A. and Zymek, R. (2020). The (structural) gravity of epidemics. *CESifo Working Paper*.

Dornbusch, R., Fischer, S., and Samuelson, P. A. (1977). Comparative advantage, trade, and payments in a ricardian model with a continuum of goods. *The American Economic Review*, 67(5):823–839.

Eaton, J. and Grossman, G. M. (1985). Tariffs as insurance: Optimal commercial policy when domestic markets are incomplete. *Canadian Journal of Economics*, pages 258–272.

Eaton, J. and Kortum, S. (2002). Technology, geography, and trade. *Econometrica*, 70(5):1741–1779.

Fajgelbaum, P. D., Khandelwal, A., Kim, W., Mantovani, C., and Schaal, E. (2020). Optimal lockdown in a commuting network. *CESifo Working Paper*.

Furusawa, T. and Lai, E. L.-C. (1999). Adjustment costs and gradual trade liberalization. *Journal of International Economics*, 49(2):333–361.

Grossman, G. and Helpman, E. (2021). When tariffs disrupt global supply chains. *Working Paper*.

Grossman, G. and Rossi-Hansberg, E. (2010). External economies and international trade redux. *The Quarterly Journal of Economics*, 125(2):829–858.

Hausmann, R. and Rodrik, D. (2003). Economic development as self-discovery. *Journal of development Economics*, 72(2):603–633.

Hsieh, C.-T., Klenow, P. J., and Nath, I. B. (2021). A global view of creative destruction. *Working Paper*.

Itoh, M. and Kiyono, K. (1987). Welfare-enhancing export subsidies. *Journal of Political Economy*, 95(1):115–137.

Johnson, H. G. (1960). The cost of protection and the scientific tariff. *Journal of Political Economy*, 68(4):327–345.

Krugman, P. (1987). The narrow moving band, the dutch disease, and the competitive consequences of mrs. thatcher: Notes on trade in the presence of dynamic scale economies. *Journal of Development Economics*, 27(1-2):41–55.

Leamer, E. E. (1980). Welfare computations and the optimal staging of tariff reductions in models with adjustment costs. *Journal of International Economics*, 10(1):21–36.

Leibovici, F., Santacreu, A. M., et al. (2021). *International trade of essential goods during a pandemic*. Federal Reserve Bank of St. Louis, Research Division.

Matsuyama, K. (1992). Agricultural productivity, comparative advantage, and economic growth. *Journal of economic theory*, 58(2):317–334.

Matsuyama, K. (2015). The home market effect and patterns of trade between rich and poor countries.

Mussa, M. (1982). Government policy and the adjustment process. In *Import competition and response*, pages 73–122. University of Chicago Press.

Naito, T. (2019). A larger country sets a lower optimal tariff. *Review of International Economics*, 27(2):643–665.

Opp, M. M. (2010). Tariff wars in the ricardian model with a continuum of goods. *Journal of International Economics*, 80(2):212 – 225.

Ossa, R. (2014). Trade wars and trade talks with data. *American Economic Review*, 104(12):4104–46.

Shih, W. C. (2020). Global supply chains in a post-pandemic world. *Harvard Business Review*, 98(5):82–89.

Weisbach, D., Kortum, S., and Wang, M. (2021). Optimal unilateral carbon policy. *Working Paper*.

A Tables Appendix

Table 3: Optimal Trade Policy

	Tariff on Imported Goods		
	1% shock	5% shock	10% shock
	(1)	(2)	(3)
Panel A. Tariff, Period 1			
Infeasible	21.04%	21.04%	21.04%
Myopic	21.04%	21.04%	21.04%
Sophisticated	21.01%	20.93%	20.88%
Panel B. Tariff, Period 2			
Infeasible	21.18%	21.82%	22.81%
Myopic	21.24%	22.04%	23.16%
Sophisticated	21.21%	21.95%	23.02%

B Technical Details

B.1 Setting Up The Relaxed Problem

The full problem of the planner is given by,

$$
\begin{aligned}
V = \max_{w_t, w_t^*, c_{it}, m_{it}, \tau_{it}} & \int_i u_{i1}(c_{i1})di + \delta \int_i u_{i2}(c_{i2})di \\
\text{subj. to} \quad & \int_0^1 a_i^* c_{i1}^* (1 - m_{i1}^*)di + \int_0^1 a_i^* c_{i1} m_{i1} di \leq L^* \\
& \int_0^1 a_i^* c_{i2}^* (1 - m_{i2}^*)di + \int_0^1 a_i^* c_{i2}^* m_{i1} di \leq L^* \\
& \int_0^1 c_{i1}^* w_1 a_i^* (1 - m_{i1}^*)di + \int_0^1 c_{i1}^* w_1^* a_i m_{i1}^* di \leq w_1^* L^* \\
& \int_0^1 c_{i2}^* w_1 a_i^* (1 - m_{i12}^*)di + \int_0^1 c_{i1}^* w_1^* a_{i2}^* m_{i1}^* di \leq w_2^* L^* \\
& \int_0^1 a_i c_{i1}^* m_{i2}^* di + \int_0^1 a_i c_{i1}(1 - m_{i1})di \leq L \\
& \int_0^1 a_i c_{i2}^* m_{i1}^* + \int_0^1 a_{i2} c_{i2}(1 - m_{i2})di \leq L \\
& c_{it} = \beta_{it}[w_t L + T_t] / \min\{w_t a_{it}, w_t^* a_{it}^*(1 + \tau_{it})\} \\
& m_{it} = \arg\min\{w_t a_{it}, w_t^* a_{it}^*(1 + \tau_{it})\} \\
& c_{it}^* = \beta_{it} w_t^* L / \min\{w_t a_{it}, w_t^* a_{it}^*)\} \\
& m_{it}^* = \arg\min\{w_t^* a_{it}^*, w_t a_{it})\},
\end{aligned}
$$

and the home budget constraint will hold by Walras's law. First we get to the planner's problem of the text. To do so, we first consider breaking up the optimization problem into an outer and an inner problem.

The inner problem will fix wages, and solve for allocations and tariffs conditional on this choice. Then we can optimize over wages given the solution to the problem. Two things to notice before proceeding. First, if foreign optimality holds then the foreign budget constraint holds automatically. This is because the Cobb-Douglas assumption ensures that *whatever* the optimal sourcing strategy of Foreign, they will spend a constant expenditure share β_{it} on good i at t. Hence, the budget constraint can be dropped as long as we keep the optimality restrictions on Foreign's consumption and sourcing rules. Second, given wages in the first period, Foreign's decision rule in the first period is pegged down independently of Home's actions. In particular, they will use a cutoff rule to determine their import behavior, solving $A(\iota_1^F) = \frac{w_t}{w_t^*}$, where $A(i) = a_i^*/a_i$. Hence, we can drop this constraint as well. In the second period, Foreign's sourcing decision is a function of Home's sourcing decisions because the comparative advantage schedule becomes endogenous. With these fixes in mind, we can write the inner problem as:

$$
\begin{aligned}
V(w_t, w_t^*) = \max_{c_{it}, m_{it}, \tau_{it}} & \int_i u_{i1}(c_{i1})di + \delta \int_i u_{i2}(c_{i2})di \\
\text{subj. to} \quad & \int_0^1 a_i^* c_{i1}^*(1 - m_{i1}^*)di + \int_0^1 a_i^* c_{i1} m_{i1} di \leq L^* \\
& \int_0^1 a_i^* c_{i2}^*(1 - m_{i2}^*)di + \int_0^1 a_i^* c_{i2}^* m_{i1} di \leq L^* \\
& \int_0^1 a_i c_{i1}^* m_{i2}^* di + \int_0^1 a_i c_{i1}(1 - m_{i1})di \leq L \\
& \int_0^1 a_i c_{i2}^* m_{i1}^* + \int_0^1 a_{i2} c_{i2}(1 - m_{i2})di \leq L \\
& c_{it} = \beta_{it}[w_t L + T_t] / \min\{w_t a_{it}, w_t^* a_{it}^*(1 + \tau_{it})\} \\
& m_{it} = \arg\min\{w_t a_{it}, w_t^* a_{it}^*(1 + \tau_{it})\},
\end{aligned}
$$

where we have not written out the constraints for c_{i2}^* and m_{i2}^*, but understand them to be functions of m_{i1}. Our goal is to solve the inner problem and then maximize over wages, subject to market clearing.

To make progress on the inner problem we follow CDVW and focus on the relaxed planner's problem, dropping the constraints on c_{it} and m_{it}. We will then construct a solution to this relaxed problem and show that it is feasible in the full inner problem. The relaxed problem Lagrangian is thus given by:

$$
\begin{aligned}
\mathcal{L}(w_t, w_t^*) = \int_0^1 \Big[& \beta_{i1} \log c_{i1} + \delta \beta_{i2} \log c_{i2} + \\
& \nu_1^*(L^* - a_{i1}^* c_{i1}^*(1 - m_{i1}^*) - a_{i1}^* c_{i1} m_{i1}) + \nu_1(L - a_{i1} c_{i1}^* m_{i1}^* - a_{i1} c_{i1}(1 - m_{i1})) + \\
& \delta \nu_2^*(L^* - a_{i2}^* c_{i2}^*(1 - m_{i2}^*) - a_{i2}^* c_{i2} m_{i2}) + \delta \nu_2(L - a_{i2} c_{i2}^* m_{i2}^* - a_{i2} c_{i2}(1 - m_{i2})) \Big],
\end{aligned}
$$

where $\delta^{t-1}\nu_t$ and $\delta^{t-1}\nu_t^*$ are the multipliers on the feasibility constraints. We refer to each term in the integrand as the good-specific Lagrangian and denote it $\mathcal{L}_i$.

B.2 Proof of Propositions 1 and 2

Since Proposition 1 is a special case of Proposition 2, we solve the more general case here. Turning to consumption. The objective is concave in c conditional on m, and the first order condition implies,

$$\frac{\beta_{it}}{c_{it}} = \nu_t a_{it}(1 - m_{it}) + \nu_t^* a_{it}^* m_{it}.$$

Explicitly deriving the value of the multipliers will be important in characterizing the solution for m. To recover these, plug c_{it} and $c_{it}^* = \beta_{it} w^* L^*/(a_{it}^* w_t^*) = \beta_{it} L^*/a_{it}^*$ into the Foreign feasibility constraint:

$$\int_0^1 a_{it}^* \frac{\beta_{it} L^*}{a_{it}^*}(1 - m_{it}^*) + \int_0^1 a_{it}^* \frac{\beta_{it}}{a_{it}^* \nu_t^*} m_{it} du = L^*.$$

Rearranging, and using the fact that $\int \beta_{it} di = 1$, yields,

$$\nu_t^* = \frac{1}{L^*} \times \frac{\int_0^1 \beta_{it} m_{it} di}{\int_0^1 \beta_{it} m_{it}^* di}. \tag{19}$$

I.e., the Foreign multiplier is the ratio of Home's import share to Foreign's import share. By a similar calculation,

$$\nu_t = \frac{1 - \int_0^1 \beta_{it} m_{it} di}{L - L^*/\omega_t \times \int_0^1 \beta_{it} m_{it}^* di}. \tag{20}$$

From the first order condition on consumption, for any two imported goods,

$$\frac{c_{it}}{c_{i't}} = \frac{a_{it}^*}{a_{i't}^*}.$$

From consumer optimality, this is equal to,

$$\frac{c_{it}}{c_{i't}} = \frac{a_{it}^*(1 + \tau_{it})}{a_{i't}^*(1 + \tau_{i't})}.$$

Hence, if the solution to the relaxed problem solves the inner problem, it must do so with a uniform tariff on imported goods.

Turning to the import decision, we work backwards. As in CDVW, for the second period we could further relax the problem and take the FOC with m_{i2}, treating it *as if* it were a continuous choice. Doing so condition on m_{i1} yields the CDVW optimal decision rule:

$$m_{i2} = 1 \Leftrightarrow \frac{a_i^*}{a_{i2}} \leq \frac{\nu_t}{\nu_t^*}.$$

Given m_{i1}, this corresponds to two simple cutoff rules. For i such that $m_{i1} = 1$, the cutoff solves $A(\iota_2^{H,x})/(1+x) = \frac{\nu_t}{\nu_t^*}$ (goods below the cutoff are produced at home, above are imported). For i such that $m_{i1} = 0$, the cutoff solves $A(\iota_2^H) = \frac{\nu_t}{\nu_t^*}$.

This cutoff rule will correspond to consumer optimality if we set the uniform tariff to solve,

$$\frac{\nu_2}{\nu_2^*} = \frac{w_2}{w_2^*(1+\tau_2)}.$$

I.e.,

$$\tau_2 = \frac{w_2 \nu_2^*}{w_2^* \nu_2} - 1.$$

Using this result, one can verify that given market clearing, this result is consistent with the multipliers that we solved for above. Summarizing the second period policy:

$$\begin{aligned}
\tau_2 &= \frac{w_2 \nu_2^*}{w_2^* \nu_2} - 1 \\
\iota_2^{H,x} &= A^{-1}\left(\frac{\nu_2}{\nu_2^*}(1+x)\right) \\
\iota_2^{H} &= A^{-1}\left(\frac{\nu_2}{\nu_2^*}\right).
\end{aligned}$$

This gives us the problem of the planner for period 2 given the optimal policy in period 1. The problem has been reduced now to solving solely for the period 1 problem. As the Lagrangian is not differentiable in m_{i1}, we directly compare the choice of m_{i1} good by good, using the period 2 problem. The linearity of $\mathcal{L}$ in i allows us to optimize good-by-good to solve this problem, taking account of the optimal choice of c_{i1}, m_{i2}, and m_{i2} conditional on m_{i1}.

Step 1: Solving for Cutoff Rules

Let $\mathcal{L}_i^{m_{i1},m_{i2},m_{i2}^*}$ the contribution to the Lagrangian of good i given choices m_{i1}, m_{i2}, and m_{i2}^*. Plugging in the optimal consumption decision and rearranging yields,

$$\begin{aligned}
\mathcal{L}_i^{m_{i1},m_{i2},m_{i2}^*} =& \kappa_i - \beta_{i1}\log(a_{i1}\nu_1(1-m_{i1}) + a_{i1}^*\nu_1^* m_{i1}) - \delta\beta_{i2}\log(a_{i2}\nu_2(1-m_{i2}) + a_{i2}^*\nu_1^* m_{i2}) \\
&- \delta\beta_{i2}[\nu_2^*(1-m_{i2}^*) + \nu_2 w_2^*/w_2 m_{i2}^*]
\end{aligned}, \tag{21}$$

where κ_i is a constant independent of any choices. This formulation of the problem in the first period naturally lends itself to a solution strategy: every good i will be in some interval of the space partitioned by all of the cutoffs, $\iota_1^H, \iota_2^{F,x}, \iota_2^{H,x}, \iota_2^F, \iota_2^H$. These are formed by partitioning the unit interval according to the inverse image of A_i and $A_i/(1+x)$ at ν_1/ν_1^*, ω_2, and ν_2/ν_2^*. Depending on which region good i is in, the planner makes a binary choice between producing (by using a prohibitive tariff) or importing (by setting the uniform tariff).

By way of example, consider the space carved out between $\iota_2^{F,x}$ and $\iota_2^{H,x}$. Here, if the planner uses a prohibitive tariff to produce in the first period, then in the second period, i will be to the left of both relevant cutoffs, ι_2^F and ι_2^H, so Home will again produce and Foreign will import—leading to the ordered triple $(0, 0, 1)$. On the other hand, if the planner imports, in the next period it will *still* be optimal to produce since i is to the left of $\iota_2^{H,x}$, even though the planner will operate the inferior technology; however, Foreign will not import since i would be to the right of $\iota_2^{F,x}$—hence the ordered triplet, $(1,0,0)$. We will assume that the cutoffs are ranked in the order above. However, by shifting ν_1/ν_1^* down until ι_1^H falls to the right of $\iota_2^{F,x}$, it is clear that this need not be the case. However, this is the maximal number of partitions and for any smaller partition, the cutoffs would be the same (but some will never bind—we will demonstrate an example below). With the partition of the space above, one can solve for the optimal first period strategy using this setup. We proceed case-by-case.

$[\mathbf{0}, \iota_\mathbf{1}^\mathbf{H}]$

There is no internal disagreement with the planner in either period, nor will Foreign's actions change, and so in this region the planner will always produce. Formally, the planner will use the same cutoff rule as in the next subsection, but it will always hold trivially for goods where $A_i > \nu_1/\nu_1^*$.

$[\iota_\mathbf{1}^\mathbf{H}, \iota_\mathbf{2}^\mathbf{F,x}]$

In this region, the comparative advantage of Home is so high that, in the second period, even with the adjustment cost technology, Foreign will choose to import and Home will choose to produce. However, in the first period, this is suboptimal for Home (this is because the first period cutoff, ignoring the second period terms, would be at ι_1^H). Comparing the planner's options:

$$\mathcal{L}^{1,0,1} - \mathcal{L}^{0,0,1} = \beta_{i1} \log \frac{a_i \nu_1}{a_i^* \nu_1^*} + \delta\beta_{i2} \log \frac{a_i \nu_2}{a_i(1+x)\nu_2}$$

Simplifying the above expression yields the following cutoff:

$$m_{i1} = 1 \Leftrightarrow A_i \leq \left(\frac{\nu_1}{\nu_1^*}\right)(1+x)^{-\frac{\delta\beta_{i2}}{\beta_{i1}}}$$

This says that the static cutoff is scaled by the second term.[21]

$[\iota_\mathbf{2}^\mathbf{F,x}, \iota_\mathbf{2}^\mathbf{H,x}]$

In this region, Foreign will change their sourcing decision depending on Home's actions. One can show by a similar calculation as before that,

$$\mathcal{L}^{1,0,0} - \mathcal{L}^{0,0,1} = \beta_{i1} \log \frac{a_i \nu_1}{a_i^* \nu_1^*} + \delta\beta_{i2} \log \frac{a_i \nu_2}{a_i(1+x)\nu_2} + \delta\beta_{i2} L^* [\nu_2 w_2^*/w_2 - \nu_2^*].$$

[21] We pause for a sanity check: if x is larger, the cutoff shrinks, because this raises the cost of not producing in the second period. If $\beta_{i1} \to 0$, then this goes to 0 (so one always produces, because very little labor is needed to cover production). If $\delta\beta_{i2} \to 0$, one gets back the static cutoff.

Rearranging, simplifying and using the definitions of the tariff yields the cutoff,

$$m_{i1} = 1 \Leftrightarrow A_i \leq \left(\frac{\nu_1}{\nu_1^*}\right)(1+x)^{-\frac{\delta\beta_{i2}}{\beta_{i1}}} \exp\left(-\frac{\delta\beta_{i2}}{\beta_{i1}} \times \frac{\tau_2}{1+\tau_2} \times L^*\nu_2^*\right)$$

Notice that since period 2 actions depends on period 1 actions, and the first period cutoff depends on second period outcomes, the above cutoff is implicit and the solution to a fixed point.

$[\iota_{\mathbf{2}}^{\mathbf{H,x}}, \iota_{\mathbf{2}}^{\mathbf{F}}]$

In this region, both home and foreign will change their sourcing decision depending on what happens in the first period. In this interval, home will either produce the entirety of the good in period 2, or none of the good in period 2, depending on their action. Plugging in,

$$\mathcal{L}^{1,1,0} - \mathcal{L}^{0,0,1} = \beta_{i1} \log \frac{a_i\nu_1}{a_i^*\nu_1^*} + \delta\beta_{i2} \log \frac{a_i\nu_2}{a_i^*\nu_2^*} + \delta\beta_{i2} L^*[\nu_2 w_2^*/w_2 - \nu_2^*].$$

Rearranging and simplifying yields the cutoff,

$$m_{i1} = 1 \Leftrightarrow A_i \leq \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{\beta_{i1}}{\beta_{i1}+\delta\beta_{i2}}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\beta_{i2}}{\beta_{i1}+\delta\beta_{i2}}} \times \exp\left(-\frac{\delta\beta_{i2}}{\beta_{i1}+\delta\beta_{i2}} \times \frac{\tau_2}{1+\tau_2} \times L^*\nu_2^*\right).$$

This expression is similar to before, but now it does not depend on x at all—this is because the inferior technology is never operated (but x determines the size of this region). Instead, there is a weighted geometric mean between the two cutoffs.

As before, we have a fixed point in the import share.

$[\iota_{\mathbf{2}}^{\mathbf{F}}, \iota_{\mathbf{2}}^{\mathbf{H}}]$

In this region, Foreign will always produce domestically, so there is no margin to induce Foreign to change their action. However, Home's action in period 2 will depend on their action in period 1. Comparing the two situations,

$$\mathcal{L}^{1,1,0} - \mathcal{L}^{0,0,1} = \beta_{i1} \log \frac{a_i\nu_1}{a_i^*\nu_1^*} + \delta\beta_{i2} \log \frac{a_i\nu_2}{a_i^*\nu_2^*}.$$

This leads to the policy,

$$m_{i1} = 1 \Leftrightarrow A_i \leq \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{\beta_{i1}}{\beta_{i1}+\delta\beta_{i2}}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\beta_{i2}}{\beta_{i1}+\delta\beta_{i2}}}.$$

This is a simple geometric weighted average of the two cutoff rules.

$[\iota_{\mathbf{2}}^{\mathbf{H}}, \mathbf{1}]$

In this interval, the planner will always import regardless. Comparative advantage dominates all other concerns.

Step 2: Solving for the Implicit Cutoffs

Above we have described 6 regions and 6 cutoff rules. However, in 2 regions, these cutoffs are implicitly defined by the cutoff rules across all goods. That is to say, there is interdependence across the goods in this case. It is not obvious that a solution to this exists, such that the resulting multipliers (which are functions of the import shares) are consistent with the cutoff rules. Nevertheless, this is key to the solution, as it is necessary for us to be able to decentralize the problem. Here we offer a constructive proof that conditional on w_t, w_t^*, and ν_t/ν_t^*, a solution exists and is unique. First, note that by plugging in the definition of ν_2^*, the relevant term can be written as $\exp\left(a - b\frac{\Phi_H}{\Phi_F}\right)$, where Φ_H is the home import share, Φ_F is the foreign import share, and a and b are constants (that will depend on the multipliers and wages). Since m_{i2} is increasing in m_{i1} and m_{i2}^* is decreasing in m_{i1}, ν_2^* itself is monotonically increasing in the first period's import share, dictated by m_{i1}. Consider the operator $B(\nu^*)$ which constructs the ratio of second period import shares using the cutoff rules with ν^*. This is monotonically decreasing in ν^* by the argument above, since a higher ν^* tightens the cutoffs for importing. Since B is monotonically decreasing, if it has a fixed point, will be a unique fixed point. It remains to prove this fixed point exists. To solve for this, notice that ν^* is bounded above by $1/\int_0^{\iota_2^{F,x})\beta_{i2}}$. That is to say, since wages are fixed, the *least* that Foreign will import is the import share if Home operated *only* the bad technology in period 1; similarly, the largest import share that Home can have is 1. Let ν_0^* be this upper bound. Consider $B(\nu_0^*)$. Since we have set the *tightest* possible bounds on importing activity, $B(\nu_0^*)$ yields the *smallest* possible import share. Suppose this weren't the case and the optimal import share were smaller. Let i be a good that is imported in the case of $T(\nu_0^*)$ but exported in the other case. Then A_i is above the tightest possible cutoff for importation, and so its importing could not possibly be optimal. By the same logic, $T(T(\nu_0^*))$ yields the largest possible import shares. However, since we have already constructed the largest possible import share, it must be that $T(T(\nu_0^*)) \leq \nu_0^*$. One can continue this pattern iteratively: since success applications of T either tighten or loosen the constraints less than previous applications, $T^{2N}(\nu_0^*)$ will be a decreasing sequence. This sequence is bounded below by 0 (autarky), and so must converge to a limit. Hence, $\nu^* = T^{\infty}\left(\frac{1}{\int_0^{\iota_2^{F,x}} \beta_{i2} di}\right)$ is the unique fixed point that solves this system.

Step 3: Solving for the Second Period Cutoff

At this point we have established that the cutoff rules in Propositions 1 and 2 exist conditional on the ratio of the multipliers. Now we show how to recover the values of the second period multipliers conditional on the first period. Consider the map,

$$T(\nu_2/\nu_2^*) = \omega_2 \times \frac{1-\Phi_{H,2}}{\Phi_{H,2}} \times \frac{\Phi_{F,2}}{\frac{w_2 L}{w_2^* L^*} - \Phi_{F,2}},$$

where one uses the import rule to generate the import shares. We need to find a fixed point of T. We proceed by arguing that T is monotone in each partition of $[0,1]$ in the $x=\infty$ case, and then deriving some sufficient conditions for the $x<\infty$ case. In the $x=\infty$ case the cutoff rules are given by:

$$m_{i1} = 1 \Leftrightarrow A_i \leq \begin{cases} \frac{\nu_1}{\nu_1^*} & \text{if } i \in [0, \iota_1^H] \\ \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{1}{1+\delta\lambda}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\lambda}{1+\delta\lambda}} \times \exp\left(-\frac{\delta\lambda}{1+\delta\lambda}\frac{L^*}{L}\frac{\Phi_H}{\Phi_F}\left(1-\frac{\nu_2/\nu_2^*}{\omega_2}\right)\right) & \text{if } i \in [\iota_1^H, \iota_2^F] \\ \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{1}{1+\delta\lambda}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\lambda}{1+\delta\lambda}} & \text{if } i \in [\iota_2^F, \iota_2^H] \\ \frac{\nu_2}{\nu_2^*} & \text{if } i \in [\iota_2^H, 1], \end{cases}$$

First, as A is decreasing and the cutoff in region 2 is smaller than the cutoff in region 1, only one cutoff will bind. For a value of ν_2/ν_2^* that turns on the cutoff in region 3, T is monotone: raising ν_2/ν_2^* will create more imports in period 1, which necessarily generates more importing activity in period 2 and less exporting activity. This pushes up $\Phi_{H,2}$ and down $\Phi_{F,2}$ and so in this region T is decreasing. It is also continuous, except at maybe finite points, because the integral over β_{i2} is assumed to be continuous, and this determines the change in the import share. Suppose that instead the cutoff is turned on in region 2. We will argue that T must be decreasing by a contradiction argument. Two observations to this result are important. First, rewrite the operator as:

$$T(\nu_2/\nu_2^*) \rightarrow \frac{\frac{1/\Phi_{F,2}}{\Phi_{H,2}/\Phi_{F,2}} - 1}{\frac{w/w^* \times L/L^*}{\Phi_{F,2}} - 1}$$

This function is decreasing in the ratio of Home import share to Foreign import share as long $\nu_2/\nu_2^* > 0$, which is a restriction that must be placed on ω_2. It is increasing in Φ_F, holding the ratio fixed, if $wL\Phi_H > w^* L^* \Phi_F$, which will be the case whenever $\nu_2/\nu_2^* < \omega_2$, another restriction we have also already placed on admissible parameters (i.e., the tariff must be positive). The second observation is that in region 2, there are no non-traded goods. And in fact,

$$\Phi_{F,2} = \int_0^{C_{II}} \beta_{i2} di = 1 - \Phi_{H,2} = 1 - \int_{C_{II}}^1 \beta_{i2} di,$$

where C_{II} is the cutoff. From the first observation, if T were increasing, then raising ν_2/ν_2^* from some initial point must lead to a decline in $\Phi_{H,2}/\Phi_{F,2}$, or a compensating rise in $\Phi_{F,2}$. First, suppose that the ratio decreases and ν_2/ν_2^* increases. Then the cutoff rule would be strictly looser (for importing) than at the initial point and so imports would have to rise, or exports decline, or both, relative to the initial point, a contradiction of Φ_H/Φ_F decreasing. Hence, the ratio must increase. There must be a large increase in $\Phi_{F,2}$ to compensate. But this would imply a decline in $\Phi_{H,2}$ and the ratio could not increase. Another contradiction.

By the same assumption on the integral of β_{it} as before, T is continuous in ν_2/ν_2^*, except at maybe finite points. Hence on each region, this function is monotonically decreasing and discontinuity can be checked for crossings. This does not guarantee existence, because not every possible value of ω_2 can be supported given a choice of technology. For example, T could be above the diagonal on the range where the second region cutoff binds and below the diagonal on the range where the third region cutoff binds. In this case, one would need to shift ω_2 up or down until T can extend past the diagonal. There could also be two crossings. However, this is easily dealt with: check the objective at the crossing for each region.

For the case of $x < \infty$, the cutoff rule is summarized by:

$$m_{i1} = 1 \Leftrightarrow A_i \leq \begin{cases} \frac{\nu_1}{\nu_1^*} & \text{if } i \in [0, \iota_1^H] \\ \frac{\nu_1}{\nu_1^*} \times (1+x)^{-\delta\lambda} & \text{if } i \in [\iota_1^H, \iota_2^{F,x}] \\ \frac{\nu_1}{\nu_1^*} \times (1+x)^{-\delta\lambda} \times \exp\left(-\frac{\delta\lambda}{1+\delta\lambda}\frac{L^*}{L}\frac{\Phi_H}{\Phi_F}\left(1-\frac{\nu_2/\nu_2^*}{\omega_2}\right)\right) & \text{if } i \in [\iota_2^{F,x}, \iota_2^{H,x}] \\ \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{1}{1+\delta\lambda}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\lambda}{1+\delta\lambda}} \times \exp\left(-\frac{\delta\lambda}{1+\delta\lambda}\frac{L^*}{L}\frac{\Phi_H}{\Phi_F}\left(1-\frac{\nu_2/\nu_2^*}{\omega_2}\right)\right) & \text{if } i \in [\iota_1^{H,x}, \iota_2^F] \\ \left(\frac{\nu_1}{\nu_1^*}\right)^{\frac{1}{1+\delta\lambda}} \times \left(\frac{\nu_2}{\nu_2^*}\right)^{\frac{\delta\lambda}{1+\delta\lambda}} & \text{if } i \in [\iota_2^F, \iota_2^H] \\ \frac{\nu_2}{\nu_2^*} & \text{if } i \in [\iota_2^H, 1]. \end{cases}$$

Now, there is no clear ranking of the cutoffs and multiple cutoffs could be operative at the same time. However, if the cutoff in region 3 is inoperative, then this is similar to the case above and T will be decreasing. To see this, foreign will *always* import in regions 1 and 2, and can be induced to import in regions 3 and 4. If the constraint in 3 is never binding, then foreign will import all the way into the third region no matter what, and regardless of Home's choice in region 2, the import shares in period 2 will be governed by a single cutoff. The difference is that in region 2, in this economy, importing behavior 1 does not change behavior in period 2, it *only* changes productivity. However, determining the cutoffs only depends on import activity. The role of region 2 instead will determine optimal ω_2. Hence, *if* the constraint in region 3 never binds, then T is monotonic. For this to be the case we would need that for all ν_2/ν_2^* under consideration,

$$A(\nu_2/\nu_2^*)/(1+x) \geq \frac{\nu_1}{\nu_1^*} \times (1+x)^{-\delta\lambda} \times \exp\left(-\frac{\delta\lambda}{1+\delta\lambda}\frac{L^*}{L}\frac{\Phi_H}{\Phi_F}\left(1-\frac{\nu_2/\nu_2^*}{\omega_2}\right)\right)$$

We now characterize T in the case that for a given ν_2/ν_2^* the constraint binds for some i. We modify our characterization of the import shares as follows:

$$\Phi_{F,2} = \int_0^{C_{III}} \beta_{i2} di + \int_{\iota_2^{H,x}}^{C_{IV}} \beta_{i2} di$$
$$\Phi_{H,2} = \int_{C_{IV}}^1 \beta_{i2} di$$

There is now a "hole" in the import share of Foreign in between C_{III} and $\iota_2^{H,x}$ where goods are non-traded. As before, if T were increasing at some initial point, then for a small increase in ν_2/ν_2^* either Φ_H/Φ_F would have to decrease or both import shares would rise with Φ_F rising by less. For the same reason as before, it cannot be that Φ_H/Φ_F decreases. If this happened, then constraints would become strictly looser on importing in all regions. Now suppose that $\Phi_{H,2}/\Phi_{F,2}$ increases. For T to increase, it must be that $\Phi_{F,2}$ increases as well. If the ratio increases, then what happens to the term in the exponential is ambiguous. If this term increases relative to the initial point, then there will be more importing done in regions 4 and in region 3, but then $\Phi_{F,2}$ could not be increasing, a contradiction. However, suppose this term becomes smaller. Then the constraint in region 3 is stricter, and Home will produce more in period 1, inducing Foreign to buy, increasing $\Phi_{F,2}$ by filling in the "hole." If the same term does *not* become so much smaller that increasing ν_2/ν_2^* still increases the threshold in region 4, then $\Phi_{H,2}$ will also rise and *if* the increase in Foreign buying in region 3 offsets the decrease in Foreign buying in region 4,then the net effect could be an increase in Foreign purchasing that nevertheless allows the ratio to rise. Hence, there need not be monotonicity in this region. The issue is that Home can induce Foreign to import without changing its own importing behavior in one region. A sufficient condition for this problem to not occur is that x either be so large that $\iota_2^{H,x}$ is pushed to the left of ι_1^H, or that x be so small region 3 be irrelevant. In this intermediate range, there are problems. This does not mean that there is no solution to the planner's problem that achieves the desired terms of trade, but there is no guarantee that T is monotonic.

With stronger assumption on β_{i1} one can better characterize the conditions on x. For example, if $\beta_{i1} = 1$, then

$$\Phi_{H,2} = 1 - \lambda A^{-1}(C_{IV})$$

Similarly,

$$\Phi_{F,2} = \lambda \left[A^{-1}(C_{IV}) + A^{-1}(C_{III}) - A^{-1} \left((1+x)\frac{\nu_2}{\nu_2^*} \right) \right].$$

Using the implicit function theorem, one can take the derivative of Φ_H and Φ_F w/r/t ν_2/ν_2^* and then verify that T is monotonic.

Step 4: Solving for the First Period Cutoff

Assuming one is in the parameter space where ν_2/ν_2^* can be solved for, one can solve for ν_1/ν_1^*. Now $\Phi_{F,1}$ is fixed by $\int_0^{A^{-1}(\omega_1)}$, so that $T(\nu_1/\nu_1^*)$ is only a function of the resulting import share, and is decreasing in the import share. As before we'll argue that T, taking the solution for ν_2/ν_2^* into account, is monotonically decreasing. Suppose that T were increasing at a point, $\tilde{\nu}^o$. Let $\tilde{\nu}_2^o$ be the ratio of multipliers in the second period at this point. For T to be increasing, the change in $\tilde{\nu}^o$ must *lower* imports. Since all cutoffs are increasing in $\tilde{\nu}^o$, this could *only* happen through a very large decline in $\tilde{\nu}_2^o$. However, this large decline would similarly tighten constraints for period 2 behavior, lowering Φ_H and raising Φ_F which could not be consistent with $\tilde{\nu}_2^o$ decreasing.

Step 5: Decentralizing the Problem

We have established that there exists a unique solution for m_{it}, c_{it}, ν_t^*, ν_t, and t_{it} that solves the relaxed problem, for admissible ω_1, ω_2. Moreover, we have shown that the solution fulfills all restrictions of the constrained problem. Hence, this must also be a solution of the constrained problem. One can also verify that at the solution, markets clear. The Planner uses prohibitive tariffs inside the region bounded by $A^{-1}(\nu_2/\nu_2^*)$ and $A^{-1}(\nu_1/\nu_1^*)$ and charges uniform tariffs on imported goods with the tariff equal to,

$$1 + \tau_t = \frac{\omega_2}{\nu_2/\nu_2^*}.$$

Through iteration one can solve for the implied tariff and cutoffs. Then the planner can solve for optimal wages by searching first over w_2 given first period wages, then over first period wages.

B.3 Optimal Tariff Given ι^H

To solve for the optimal tariff we define several important functions. First, log welfare is given by:

$$U = \log \omega - \log \left(1 - \frac{\tau}{1+\tau}\Phi_H\right) - \log P, \tag{22}$$

where Φ_H is home import share, P is the price index, ω is relative wage, and τ are tariffs. The price index and home's import share *given* ι are given by:

$$P = \left(\omega^{1-\epsilon} \underbrace{\int_0^\iota \beta_i^\epsilon a_i^{1-\epsilon} di}_{\mathcal{A}} + (1+\tau)^{1-\epsilon} \underbrace{d^{1-\epsilon} \int_\iota^1 \beta_i^\epsilon (a_i^*)^{1-\epsilon} di}_{\mathcal{A}^*}\right)^{\frac{1}{1-\epsilon}} \tag{23}$$

$$\Phi_H = \frac{(1+\tau)^{1-\epsilon}\mathcal{A}^*}{\omega^{1-\epsilon}\mathcal{A} + (1+\tau)^{1-\epsilon}\mathcal{A}^*}, \tag{24}$$

where terms that do not depend on price and tariff care collected into the constants, $\mathcal{A}$ and $\mathcal{A}^*$. Finally, the relative wage is an implicit function of tariffs given through market clearing:

$$\omega = \ell \frac{\Phi_F}{\Phi_H} \times (1 + \tau - \tau\Phi_H), \tag{25}$$

where ℓ is the population ratio and Φ_F is Foreign's import share. To solve for the optimal tariff we first differentiate (22):

$$\frac{d \log U}{d\tau} = \frac{d\omega}{d\tau}\frac{1}{\omega} + \frac{1}{1 - \frac{\tau}{1+\tau}\Phi_H} \times \frac{\tau(1+\tau)\frac{d\Phi_H}{d\tau} + \Phi_H}{(1+\tau)^2} - \frac{dP}{d\tau}\frac{1}{P}.$$

Next, differentiating (23) yields,

$$\frac{dP}{d\tau} = P \times \left(\frac{d\omega}{d\tau}\frac{1}{\omega}(1 - \Phi_H) + \frac{1}{1+\tau}\Phi_H\right)$$

Here is where we use the fact that ι is fixed. However, notice that if A^{-1} were differentiable around ι, then by the envelope theorem, the expression would be the same. Combining these two equations, rearranging, and simplifying yields:

$$\frac{d\omega}{d\tau}\frac{1}{\omega} = \frac{\tau}{1+\tau} \times \frac{(1-\Phi_H) - (1+\tau)\frac{d\Phi_H}{d\tau}\frac{1}{\Phi_H}}{1+\tau-\tau\Phi_H}. \tag{26}$$

To make progress, now differentiate the log of the market clearing equation:

$$\frac{d\omega}{d\tau}\frac{1}{\omega} = \frac{d\Phi_F}{d\omega}\frac{1}{\Phi_F}\frac{d\omega}{d\tau} - \frac{1}{\Phi_H}\frac{d\Phi_H}{d\tau} + \frac{(1-\Phi_H) - \tau\frac{d\Phi_H}{d\tau}}{1+\tau-\tau\Phi_H}.$$

Define the import demand elasticity to be $\eta_F = \frac{d\Phi_F}{d\omega} \times \frac{\omega}{\Phi_F}$. Then, the above equation can be rearranged, simplified, and rewritten as:

$$\frac{d\omega}{d\tau}\frac{1}{\omega} \times (1-\eta_F) = \frac{(1-\Phi_H) - (1+\tau)\frac{d\Phi_H}{d\tau}\frac{1}{\Phi_H}}{1+\tau-\tau\Phi_H}. \tag{27}$$

Combining equations (26) and (27) yields,

$$\frac{d\omega}{d\tau}\frac{1}{\omega} = \frac{\tau}{1+\tau}\frac{d\omega}{d\tau}\frac{1}{\omega} \times (1-\eta_F)$$

Rearranging yields, the main result:

$$\tau^o = -\frac{1}{\eta_F}. \tag{28}$$

When $\beta = 1$ and productivity is Frechet with shape θ, then Foreign's imports only depend on θ and $\eta_F = -\frac{1}{\theta \times (1-\Phi_F)}$. When $\beta_i \neq 1 \forall i$ the expression is more complicated.

C Calibrating the COVID-19 demand shock.

In this section, we describe how we measured the size of the demand shock for COVID-19 related products. The United Nations Conference on Trade and Development estimated around a \$15 per person increase in pandemic related goods.[22]. We estimate using the same WCO/WHO classification of COVID-19 medical supplies[23] that in 2019 they had made up around 20 percent of 2019 US imports in COMTRADE, or around 5% of consumption given the import penetration values from Costinot et al. (2015). We measure steady-state per capita merchandise consumption in the Consumer Expenditure Survey[24], assuming a median household size of 2.54 (from the CPS).[25]. We estimate that consumption fell by 14 % in the National Income and Product Accounts.[26], from \$522 per person per month to \$452.

[22] https://unctad.org/system/files/official-document/ditcinf2020d4_en.pdf
[23] http://www.wcoomd.org/en/media/newsroom/2020/june/new-edition-of-the-wco-who-hs-classification-list-for-covid-19-medical-supplies-now-available.aspx
[24] https://www.bls.gov/cex/tables/top-line-means.htm
[25] https://www2.census.gov/programs-surveys/demo/tables/families/2020/cps-2020/tabavg1.xls
[26] https://fred.stlouisfed.org/series/PCE