Bank Runs, Fragility, and Credit Easing

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Motivation

- Financial crises typically involve bank runs
- Short-term debt can make a bank vulnerable to a self-fulfilling run
- Empirically, runs more likely with weak aggregate fundamentals
  - General equilibrium feedbacks potentially important

★ Macroeconomic model essential to understand feedbacks

**Q:** What are the implications for government policy?
A Macroeconomic Model of Bank Runs

• Dynamic portfolio and equity decisions for banks
  • Depend on asset prices, determined in equilibrium

• Limited commitment and endogenous strategic default
  ○ Defaults triggered by fundamentals or runs

• Fragility linked to fundamentals, as in Gertler-Kiyotaki, but key differences:
  • Runs on individual banks
  • Maturity critical for fragility ⇒ role for lender of last resort
A Macroeconomic Model of Bank Runs

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  - Runs on individual banks
  - Maturity critical for fragility $\Rightarrow$ role for lender of last resort

- Normative analysis
Preview of Main Normative Results

- Desirability of **credit easing** depends on source of the crisis

  - Welfare *reducing* if driven by fundamentals, but welfare *improving* if driven by runs

- **Key distinction**: Repaying banks are *net buyers* when crises are driven by fundamentals but *net sellers* when driven by runs

  - Increase in asset prices have opposite effects on the fraction of defaulting banks
Outline of the Talk

1. Environment without runs

2. Model with bank runs

3. Policy analysis
Environment

• Discrete time, infinite horizon, no aggregate risk

• Continuum of banks, preferences $\sum_{t=0}^{\infty} \beta^t \log(c_t)$.

• Creditors have linear utility, discount rate $R$

• Technology
  • Production of consumption good: $y = zk$
  • Capital in fixed supply $\bar{K}$

• Competitive market for assets and deposits

• No commitment to repay deposits
Environment

- Discrete time, infinite horizon, no aggregate risk
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- Technology
  - Production of consumption good: $y = zk$
  - Capital in fixed supply $\bar{K}$
- Competitive market for assets and deposits
- No commitment to repay deposits
All banks start at $t = 0$ with portfolio $(b_0, \bar{K})$

- If repay at time $t$:
  \[ c = (\bar{z} + p_t)k - Rb + q_t(b', k')b' - p_t k'. \]

  - $q_t$ price schedule of deposits
  - $p_t$ price of capital

- Deposits are one-period non-state contingent claims
  - Without loss for now, but will matter with runs

- Capital is liquid
  - Price determined in equilibrium
Banks’ Budget Constraints

All banks start at $t = 0$ with portfolio $(b_0, \overline{K})$

- If default at time $t$:

  \[ c = (z + p_t)k - p_t k' \]

- Permanent financial exclusion $b' = 0$
  - Restriction on saving w/o loss

- Productivity loss $y = zk$
  - Evidence on losses of firms exposed to defaulting banks
Banks’ Optimization: Values of Repayment and Default

\[ V_t^R(b, k) = \max_{k', b', c} \log(c) + \beta V_{t+1}(b', k') \]

s.t. \[ c = (\bar{z} + p_t)k - Rb + q_t(b', k')b' - p_t k' \]

No-Ponzi

\[ V_t^D(k) = \max_{k', c} \log(c) + \beta V_{t+1}^D(k') \]

s.t. \[ c = zk + p_t(k - k') \]
Banks’ Optimization: Values of Repayment and Default

\[ V^R_t(b, k) = \max_{k', b', c} \log(c) + \beta V^R_{t+1}(b', k') \]

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No-Ponzi

\[ V^D_t(k) = \max_{k', c} \log(c) + \beta V^D_{t+1}(k') \]

s.t. \[ c = \underline{z}k + p_t(k - k') \]

Repayment decision:

- If \( V^R_t(b, k) > V^D_t(k) \): repay
Banks’ Optimization: Values of Repayment and Default

\[ V_t^R(b, k) = \max_{k', b', c} \log(c) + \beta V_{t+1}(b', k') \]
\[ \text{s.t. } c = (\bar{z} + p_t)k - Rb + q_t(b', k')b' - p_t k' \]

No-Ponzi

\[ V_t^D(k) = \max_{k', c} \log(c) + \beta V_{t+1}^D(k') \]
\[ \text{s.t. } c = zk + p_t(k - k') \]

Repayment decision:

- If \( V_t^R(b, k) < V_t^D(k) \): default
Banks’ Optimization: Values of Repayment and Default

\[
V_t^R(b, k) = \max_{k', b', c} \log(c) + \beta V_{t+1}(b', k') \\
\text{s.t. } c = (\bar{z} + p_t)k - Rb + q_t(b', k')b' - p_t k'
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Repayment decision:

- If \( V_t^R(b, k) = V_t^D(k) \): indifferent
Banks’ Optimization: Values of Repayment and Default

\[ V_t^R(b, k) = \max_{k', b', c} \log(c) + \beta V_{t+1}(b', k') \]

s.t. \( c = (\bar{z} + p_t)k - Rb + q_t(b', k')b' - p_t k' \)

No-Ponzi

\[ V_t^D(k) = \max_{k', c} \log(c) + \beta V_{t+1}^D(k') \]

s.t. \( c = zk + p_t(k - k') \)

Repayment decision:

- If \( V_t^R(b, k) = V_t^D(k) \): indifferent
  - Repay for \( t > 0 \)
  - Default with probability \( \phi \)
Equilibrium Consistent Borrowing Limit

- Equilibrium default only at $t = 0$

- Guess and verify that bank pays at $t + 1$ if

$$b_{t+1} \leq \gamma_t p_{t+1} k_{t+1}$$

where

$$\bar{z} + p_{t+1} (1 - \gamma_t R) \frac{1}{\bar{z} + p_{t+1}} = \left( 1 - \gamma_{t+1} \frac{p_{t+2}}{p_{t+1}} \right)^\beta$$
Equilibrium Consistent Borrowing Limit

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- Potentially many solutions, but only one consistent with No-Ponzi
Solving for $\gamma_t$ for Constant Price

$$\gamma_{t+1} = 1 - \left( \frac{R^k(p)/R - \gamma_t}{R^D(p)/R} \right)^{\frac{1}{\beta}} \equiv H(\gamma_t)$$
Solving for $\gamma_t$ for Constant Price

$$\gamma_{t+1} = 1 - \left( \frac{R^k(p)/R - \gamma_t}{R^D(p)/R} \right)^\frac{1}{\beta} \equiv H(\gamma_t)$$

• Partial eqm. does not exist if return on capital is too high
  ○ No borrowing limit

(b) No fixed point
Solving for $\gamma_t$ for Constant Price

\[
\gamma_{t+1} = 1 - \left( \frac{R^k(p)/R - \gamma_t}{R^D(p)/R} \right)^{\frac{1}{\beta}} \equiv H(\gamma_t)
\]

- If eqm $\exists$, two fixed points but only smallest satisfies No-Ponzi
  - First fixed point unstable $\Rightarrow \gamma_t = \gamma^*$
  - $\gamma^*$ is increasing in $(\beta, \bar{z})$ and decreasing in $(R, \underline{z}, p)$

(a) Two fixed points

(b) No fixed point
Outline of the Talk

1. Environment without runs
   • Bank problem in partial equilibrium
   • General equilibrium

2. Model with bank runs

3. Policy analysis
• Market clearing for capital

\[ \phi K_t^D + (1 - \phi)K_t^R = \bar{K} \]

where \( \phi \in [0, 1] \) are the banks that default at \( t = 0 \)
General Equilibrium

- Market clearing for capital

\[ \phi K_t^D + (1 - \phi)K_t^R = \bar{K} \]

where \( \phi \in [0, 1] \) are the banks that default at \( t = 0 \)

- Recall a bank is indifferent at \( t = 0 \) if \( b_0 = \gamma_{-1}p_0k_0 \)

where

\[ \frac{\bar{z} + p_0(1 - \gamma_{-1}R)}{\bar{z} + p_0} = \left(1 - \gamma_0 \frac{p_1}{p_0}\right)^\beta \]

- Therefore,

\[ \phi = \begin{cases} 
1 & \text{if } B_0 > \gamma_{-1}p_0\bar{K}, \\
0 & \text{if } B_0 < \gamma_{-1}p_0\bar{K}, \\
\in [0, 1] & \text{if } B_0 = \gamma_{-1}p_0\bar{K}
\end{cases} \]
Type of equilibrium depends on $B_0$

- **Repayment eqm.**
  \[ \gamma^R p^R K \]

- **Mixed eqm.**
  \[ \gamma^D p^D K \]

- **Default eqm**

In the paper:
- Analytical characterization of thresholds
- Unique stationary eqm. and unique transition results
- Repaying banks are net buyers of $k$ in the mixed eqm.
Type of equilibrium depends on $B_0$

- Repayment eqm.
- Mixed eqm.
- Default eqm

$\gamma^R p^R K$  $\gamma^D p^D K$

• Generalize Kehoe-Levine, by allowing initial defaults
• Analytical characterization of thresholds
• Unique stationary eqm. and unique transition results
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Type of equilibrium depends on $B_0$

Repayment eqm.  
$\gamma^R p^R K$

Mixed eqm.  

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In the paper:

- Analytical characterization of thresholds
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General Equilibrium

Type of equilibrium depends on $B_0$

- Repayment eqm. $\gamma^R p^{R\overline{K}}$
- Mixed eqm. $? p^{D\overline{K}}$
- Default eqm $\gamma^D p^{D\overline{K}}$

Mixing within thresholds: Fraction $\phi$ defaults and $1 - \phi$ repays.

- Generalize Kehoe-Levine, by allowing initial defaults
Type of equilibrium depends on $B_0$

Repayment eqm.  Mixed equilibrium  Default eqm

$\gamma^R p^R K$  $\gamma^D p^D K$

Mixing within thresholds: Fraction $\phi$ defaults and $1 - \phi$ repays.

- Generalize Kehoe-Levine, by allowing initial defaults

In the paper:
- Analytical characterization of thresholds
- Unique stationary eqm. and unique transition results
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Mixed Equilibrium Simulations

Price of Capital $p_t$

Leverage Threshold $\gamma_t$

Capital Holdings

- Repaying banks
- Defaulting banks
Outline of the Talk

1. Environment without runs

2. Model with bank runs

3. Policy analysis
Coordination problem between creditors a la Cole-Kehoe

- Creditors may refuse to rollover $\Rightarrow$ repayment more costly
- If optimal to default during a run, a bank is “vulnerable”
Bank facing a run needs to de-lever:

\[ \hat{V}_{t}^{\text{Run}}(n) = \max_{k' \geq 0, c} \log(c) + V_{t+1} \left( (\bar{z} + p_{t+1})k' \right) \]

\[ s.t \quad c = n + b' - p_{t}k' \]
Multiplicity of Equilibria

Bank facing a run needs to de-lever:

\[ \hat{V}_{t}^{Run}(n) = \max_{k' \geq 0, c} \log(c) + V_{t+1}((\bar{z} + p_{t+1})k') \]

\[ s.t \quad c = n + b' - p_t k' \]

Safe bank faces tighter constraint:

\[ \hat{V}_{t}^{Safe}(n) = \max_{b', k' \geq 0, c} \log(c) + \beta \hat{V}_{t+1}((\bar{z} + p_{t+1})k' - R b') \]

\[ s.t \quad c = n + b' - p_t k' \]

\[ \hat{V}_{t+1}^{Run}(n') \geq V_{t+1}^{D}(k') \]
Bank facing a run needs to de-lever:

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subject to

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subject to

$$c = n + b' - p_t k'$$

$$\hat{V}_{t+1}^{Run}(n') \geq V_{t+1}^D(k')$$

- Multiplicity: $$\hat{V}_t^{Safe}(n) > \hat{V}_t^D(k) > \hat{V}_t^{Run}(n)$$
Multiplicity of Equilibria

Bank facing a run needs to de-lever:

$$\hat{V}_t^{\text{Run}}(n) = \max_{k' \geq 0, c} \log(c) + V_{t+1} \left( (\bar{z} + p_{t+1})k' \right)$$

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Safe bank faces tighter constraint:

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$$s.t \quad c = n + b' - p_t k'$$

$$\hat{V}_{t+1}^{\text{Run}}(n') \geq V_{t+1}^{D}(k')$$

- Multiplicity: $$\hat{V}_t^{\text{Safe}}(n) > \hat{V}_t^{D}(k) > \hat{V}_t^{\text{Run}}(n)$$

  - Assume that if a bank is vulnerable for $$t > 0$$, a run happens
The Effects of Bank Runs

- Financial fragility, default region expands $\downarrow \gamma^D$
  - Repayment region contracts $\gamma^R \downarrow$ if and only if $\beta R < 1$

<table>
<thead>
<tr>
<th>Repayment eqm.</th>
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The Effects of Bank Runs

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Repayment eqm. \quad Mixed eqm. \quad Default eqm

$\gamma^R p^R K$ \quad $\gamma^D p^D K$
Financial fragility, default region expands $\downarrow \gamma^D$

- Repayment region contracts $\gamma^R \downarrow$ if and only if $\beta R < 1$

\[
\begin{align*}
\text{Repayment eqm.} & \quad \gamma^R p^R K \\
\text{Mixed eqm.} & \quad \gamma^D p^D K \\
\text{Default eqm} & \quad \gamma^D p^D K
\end{align*}
\]

- Lower price of capital
  - Lower $\gamma$, implies lower demand by repaying banks
  - More defaulting banks, which have lower demand for capital
Outline of the Talk

1. Basic environment without bank runs
   - Bank problem in partial equilibrium
   - General equilibrium

2. Introduce bank runs

3. Policy analysis
Introduce government purchases of assets $K^g$ at $t = 0$

Assume that government makes losses:

- Productivity $z^g < z$ and return $(z^g + p_1)/p_0 < R$

$\Rightarrow$ Investors they do not purchase $k$ if same productivity as govt.

**Q:** How does credit easing affect $\phi$ and welfare?
Welfare ↓ if defaults due to fundamentals

\[
\frac{dW}{dK_g} = \left[ \phi \frac{dV^D}{dK_g} - (1 - \phi) \frac{dV^R}{dK_g} \right] - \left( V^R - V^D \right) \frac{d\phi}{dK_g}
\]
Welfare ↓ if defaults due to fundamentals

\[
\frac{dW}{dK_g} = \left[ \phi \frac{dV^D}{dK_g} - (1 - \phi) \frac{dV^R}{dK_g} \right] - \left( V^R - V^D \right) \frac{d\phi}{dK_g}
\]
Welfare $\downarrow$ if defaults due to **fundamentals**

$$\frac{dW}{dK_g} = \left[ \phi \frac{dV^D}{dK_g} - (1 - \phi) \frac{dV^R}{dK_g} \right] - \left( V^R - V^D \right) \frac{d\phi}{dK_g}$$
Welfare ↓ if defaults due to fundamentals

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Without runs:
- \( V^R = V^D \) \( \Rightarrow \) \( d\phi \) irrelevant
Welfare \( \uparrow \) if defaults due to runs

\[
\frac{dW}{dK_g} = \left[ \phi \frac{dV^D}{dK_g} - (1 - \phi) \frac{dV^R}{dK_g} \right] - \left( V^R - V^D \right) \frac{d\phi}{dK_g} \rightarrow 0
\]

Without runs:

- \( V^R = V^D \Rightarrow d\phi \) irrelevant

- Given \( \{p_1, p_2, \ldots\} \), \( dV^R = dV^D = dW = \left. \frac{dT_0}{dK_g} \right|_{K_g=0} < 0 \)
Welfare \uparrow if defaults due to runs

\[
\frac{dW}{dK_g} = \left[ \phi \frac{dV^D}{dK_g} - (1 - \phi) \frac{dV^R}{dK_g} \right] - \left( V^R - V^D \right) \frac{d\phi}{dK_g} < 0
\]

Without runs:

- \( V^R = V^D \) \( \Rightarrow \) \( d\phi \) irrelevant

- Given \( \{p_1, p_2, \ldots\} \), \( dV^R = dV^D = dW = \left. \frac{dT_0}{dK_g} \right|_{K_g=0} < 0 \)

With runs:

- \( V^R = V^{Safe} \geq V^{Run} = V^D \)

\( \Rightarrow \) If \( d\phi < 0 \), possibility that \( \uparrow W \)

A repaying banks facing a run is a net seller of assets

\( \Rightarrow \) benefits from intervention that \( \uparrow p_0 \Rightarrow d\phi < 0 \)
Welfare ↑ if defaults due to runs

\[
\frac{dW}{dK_g} = \left[ \phi \frac{dV^D}{dK_g} - (1 - \phi) \frac{dV^R}{dK_g} \right] - \left( V^R - V^D \right) \frac{d\phi}{dK_g}
\]

Without runs:

- \( V^R = V^D \) ⇒ \( d\phi \) irrelevant
- Given \( \{p_1, p_2...\} \), \( dV^R = dV^D = dW = \frac{dT_0}{dK^g} \bigg|_{K^g=0} < 0 \)

With runs:

- \( V^R = V^{Safe} > V^{Run} = V^D \)

⇒ If \( d\phi < 0 \), possibility that \( W \) ↑

A repaying banks facing a run is a net seller of assets

⇒ benefits from intervention that \( p_0 \) ↑ ⇒ \( d\phi < 0 \)
Credit Easing: Self-Fulfilling vs. Fundamentals

**Self-Fulfilling Runs**

\[ p_0 \]

\[ \phi \]

**Fundamentals**

\[ p_0 \]

\[ \phi \]
Credit Easing: Self-Fulfilling vs. Fundamentals

**Self-Fulfilling Runs**

\[ p_0 \]

\[ \phi \]

**Fundamentals**

\[ p_0 \]

\[ \phi \]
Other Policies

- Controlling default decisions: Details
  - Higher $\phi$ w/o runs and lower $\phi$ w/runs

- Tax on purchases of capital at $t = 0$ rebated lump sum
  - Irrelevant: after-tax price remains constant and has no effects

- Deposit insurance: deters runs, but requires borrowing limits

- Lender of last resort: must cover all banks to be effective
Conclusions

- A dynamic macroeconomic model of self-fulfilling bank runs
- General equilibrium effects crucial to assess govt. policies
- Desirability of credit easing depends on whether a crisis is driven by fundamentals or self-fulfilling runs
- Agenda:
  - Anticipation effects of credit easing
  - Use framework for other policies, such as macroprudential
Government picks $\phi$ at $t = 0$

Banks’ welfare

$$W = (1 - \phi) V^R + \phi V^D$$
Government picks $\phi$ at $t = 0$

Banks’ welfare

$$W = (1 - \phi) V^R + \phi V^D$$

- Assume only $p_0$ changes in response to policy:

$$\left. \frac{dW}{d\phi} \right|_{\phi = \phi^E} = (V^D(p_0^E) - V^R(p_0^E)) + \left[ (1 - \phi) \left. \frac{dV^R(p_0)}{dp_0} \right|_{p_0 = p_0^E} + \phi \left. \frac{dV^D(p_0)}{dp_0} \right|_{p_0 = p_0^E} \right] \frac{dp_0}{d\phi}$$
Government picks $\phi$ at $t = 0$

Banks’ welfare

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$\phi$ reduces $p_0$ and helps repaying banks that have high $u'$. 
Government picks $\phi$ at $t = 0$

Banks’ welfare

$$W = (1 - \phi)V^R + \phi V^D$$

- Assume only $p_0$ changes in response to policy:

$$\frac{dW}{d\phi} \bigg|_{\phi = \phi^E} = (V^D(p_0^E) - V^R(p_0^E)) +$$

$$\left[ (1 - \phi) \left. \frac{dV^R(p_0)}{dp_0} \right|_{p_0 = p_0^E} + \phi \left. \frac{dV^D(p_0)}{dp_0} \right|_{p_0 = p_0^E} \right] \frac{dp_0}{d\phi}$$

$$\left. \frac{dV^R(p_0)}{dp_0} \right|_{\phi = \phi^E} = u'(c^R)(\bar{K} - k^R(p_0^E)), \quad \left. \frac{dV^D(p_0)}{dp_0} \right|_{\phi = \phi^E} = u'(c^D)(\bar{K} - k^D(p_0^E)).$$
Government picks $\phi$ at $t = 0$

Banks’ welfare

$$W = (1 - \phi)V^R + \phi V^D$$

- Assume only $p_0$ changes in response to policy:

$$\left. \frac{dW}{d\phi} \right|_{\phi = \phi^E} = \left[ V^D(p_0^E) - V^R(p_0^E) \right]$$

$$- (1 - \phi) \left[ u'(c^R(p_0^E)) - u'(c^D(p_0^E)) \right] \left( k^R(p_0^E) - \bar{K} \right)$$

↑ $\phi$ reduces $p_0$ and helps repaying banks that have high $u'$
Government picks $\phi$ at $t = 0$

Banks’ welfare

$$W = (1 - \phi)V^R + \phi V^D$$

- Assume only $p_0$ changes in response to policy:

$$\frac{dW}{d\phi} \bigg|_{\phi = \phi^E} = [V^D(p_0^E) - V^R(p_0^E)] > 0$$

$$- (1 - \phi)[u'(c^R(p_0^E)) - u'(c^D(p_0^E))] (k^R(p_0^E) - \bar{K}) > 0$$

$$\frac{dp_0}{d\phi} < 0$$

$\uparrow \phi$ reduces $p_0$ and helps repaying banks that have high $u'$

- Without runs: optimal to have more banks defaulting
Government picks $\phi$ at $t = 0$

Banks’ welfare

$$W = (1 - \phi)V^R + \phi V^D$$

• Assume only $p_0$ changes in response to policy:

$$\frac{dW}{d\phi} \bigg|_{\phi=\phi^E} = [V^D(p_0^E) - V^R(p_0^E)]$$

$$- (1 - \phi)[u'(c^R(p_0^E)) - u'(c^D(p_0^E))] (k^R(p_0^E) - \bar{K})$$

$\uparrow \phi$ reduces $p_0$ and helps repaying banks that have high $u'$

• Without runs: optimal to have more banks defaulting

• With runs: may be optimal to reduce defaults
Fundamentals

(a) Welfare

(b) \( p_0 \)

(c) \( \gamma_0 p_1 \)

Self-Fulfilling Runs

(d) Welfare

(e) \( p_0 \)

(f) \( \gamma_0 p_1 \)