

Bank Runs, Fragility, and Credit Easing

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Motivation

- Financial crises typically involve bank runs
- Short-term debt can make a bank vulnerable to a self-fulfilling run
- Empirically, runs more likely with weak aggregate fundamentals
 - General equilibrium feedbacks potentially important

★ Macroeconomic model essential to understand feedbacks

Q: What are the implications for government policy?

A Macroeconomic Model of Bank Runs

- Dynamic portfolio and equity decisions for banks
 - Depend on asset prices, determined in equilibrium
- Limited commitment and endogenous strategic default
 - Defaults triggered by fundamentals or runs
- Fragility linked to fundamentals, as in Gertler-Kiyotaki, but key differences:
 - Runs on individual banks
 - Maturity critical for fragility \Rightarrow role for lender of last resort

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 - Runs on individual banks
 - Maturity critical for fragility \Rightarrow role for lender of last resort
- Normative analysis

Preview of Main Normative Results

- Desirability of **credit easing** depends on source of the crisis
 - Welfare *reducing* if driven by fundamentals, but welfare *improving* if driven by runs
- **Key distinction:** Repaying banks are *net buyers* when crises are driven by fundamentals but *net sellers* when driven by runs
 - Increase in asset prices have opposite effects on the fraction of defaulting banks

Outline of the Talk

1. Environment without runs
2. Model with bank runs
3. Policy analysis

Environment

- Discrete time, infinite horizon, no aggregate risk
- Continuum of banks, preferences $\sum_{t=0}^{\infty} \beta^t \log(c_t)$.
- Creditors have linear utility, discount rate R
- Technology
 - Production of consumption good: $y = zk$
 - Capital in fixed supply \bar{K}
- Competitive market for assets and deposits
- No commitment to repay deposits

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Banks' Budget Constraints

All banks start at $t = 0$ with portfolio (b_0, \bar{K})

- If repay at time t :

$$c = (\bar{z} + p_t)k - Rb + q_t(b', k')b' - p_t k'.$$

- q_t price schedule of deposits
- p_t price of capital
- Deposits are one-period non-state contingent claims
 - Without loss for now, but will matter with runs
- Capital is liquid
 - Price determined in equilibrium

Banks' Budget Constraints

All banks start at $t = 0$ with portfolio (b_0, \bar{K})

- If default at time t :

$$c = (\underline{z} + p_t)k - p_t k'$$

- Permanent financial exclusion $b' = 0$
 - Restriction on saving w/o loss
- Productivity loss $y = \underline{z}k$
 - Evidence on losses of firms exposed to defaulting banks

Banks' Optimization: Values of Repayment and Default

$$V_t^R(b, k) = \max_{k', b', c} \log(c) + \beta V_{t+1}(b', k')$$

s.t. $c = (\bar{z} + p_t)k - Rb + q_t(b', k')b' - p_t k'$
No-Ponzi

$$V_t^D(k) = \max_{k', c} \log(c) + \beta V_{t+1}^D(k')$$

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- If $V_t^R(b, k) > V_t^D(k)$: repay

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s.t. $c = \underline{z}k + p_t(k - k')$

Repayment decision:

- If $V_t^R(b, k) = V_t^D(k)$: indifferent
 - Repay for $t > 0$
 - **Default with probability ϕ**

Equilibrium Consistent Borrowing Limit

- Equilibrium default only at $t = 0$
- Guess and verify that bank pays at $t + 1$ if

$$b_{t+1} \leq \gamma_t p_{t+1} k_{t+1}$$

where

$$\frac{\bar{z} + p_{t+1}(1 - \gamma_t R)}{\underline{z} + p_{t+1}} = \left(1 - \gamma_{t+1} \frac{p_{t+2}}{p_{t+1}}\right)^\beta$$

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- Potentially many solutions, but only one consistent with No-Ponzi

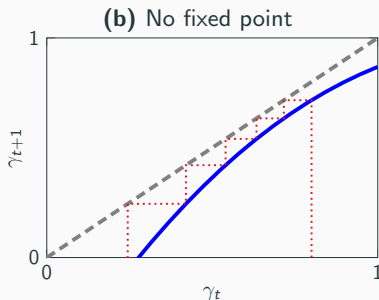
Solving for γ_t for Constant Price

$$\gamma_{t+1} = 1 - \left(\frac{R^k(p)/R - \gamma_t}{R^D(p)/R} \right)^{\frac{1}{\beta}} \equiv H(\gamma_t)$$

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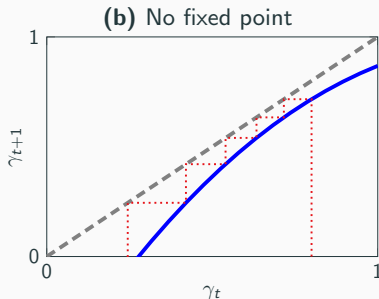
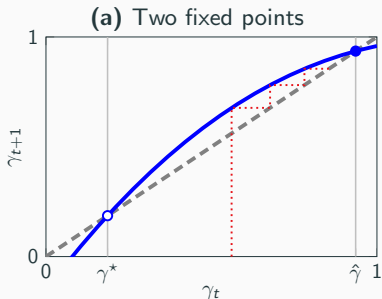
- Partial eqm. does not exist if return on capital is too high
 - No borrowing limit



Solving for γ_t for Constant Price

$$\gamma_{t+1} = 1 - \left(\frac{R^k(p)/R - \gamma_t}{R^D(p)/R} \right)^{\frac{1}{\beta}} \equiv H(\gamma_t)$$

- If eqm \exists , two fixed points but only smallest satisfies No-Ponzi
 - First fixed point unstable $\Rightarrow \gamma_t = \gamma^*$
 - γ^* is increasing in (β, \bar{z}) and decreasing in (R, \underline{z}, p)



Outline of the Talk

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- Bank problem in partial equilibrium
- General equilibrium

2. Model with bank runs

3. Policy analysis

General Equilibrium

- Market clearing for capital

$$\phi K_t^D + (1 - \phi) K_t^R = \bar{K}$$

where $\phi \in [0, 1]$ are the banks that default at $t = 0$

General Equilibrium

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$$\phi K_t^D + (1 - \phi) K_t^R = \bar{K}$$

where $\phi \in [0, 1]$ are the banks that default at $t = 0$

- Recall a bank is indifferent at $t = 0$ if $b_0 = \gamma_{-1} p_0 k_0$

where

$$\frac{\bar{z} + p_0(1 - \gamma_{-1}R)}{\underline{z} + p_0} = \left(1 - \gamma_0 \frac{p_1}{p_0}\right)^\beta$$

- Therefore,

$$\phi = \begin{cases} 1 & \text{if } B_0 > \gamma_{-1} p_0 \bar{K}, \\ 0 & \text{if } B_0 < \gamma_{-1} p_0 \bar{K}, \\ \in [0, 1] & \text{if } B_0 = \gamma_{-1} p_0 \bar{K} \end{cases}$$

General Equilibrium

Type of equilibrium depends on B_0



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Mixing within thresholds: Fraction ϕ defaults and $1 - \phi$ repays.

- Generalize Kehoe-Levine, by allowing initial defaults

General Equilibrium

Type of equilibrium depends on B_0



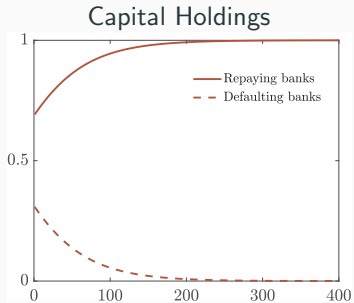
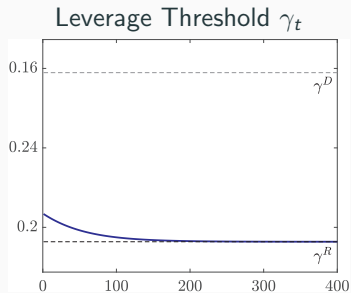
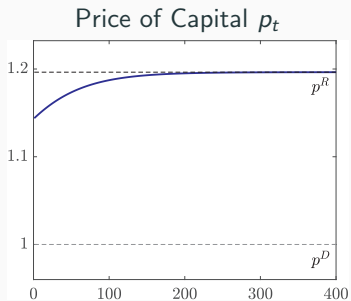
Mixing within thresholds: Fraction ϕ defaults and $1 - \phi$ repays.

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In the paper:

- Analytical characterization of thresholds
- Unique stationary eqm. and unique transition results
- Repaying banks are net buyers of k in the mixed eqm.

Mixed Equilibrium Simulations



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Self-Fulfilling Bank Runs

Coordination problem between creditors a la Cole-Kehoe


- Creditors may refuse to rollover \Rightarrow repayment more costly
- If optimal to default during a run, a bank is “vulnerable”

Multiplicity of Equilibria

Bank facing a run needs to de-lever:

$$\hat{V}_t^{Run}(n) = \max_{k' \geq 0, c} \log(c) + V_{t+1}((\bar{z} + p_{t+1})k')$$

s.t. $c = n + \cancel{b} - p_t k'$



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Safe bank faces tighter constraint:

$$\hat{V}_t^{Safe}(n) = \max_{b', k' \geq 0, c} \log(c) + \beta \hat{V}_{t+1}^{Safe}((\bar{z} + p_{t+1})k' - Rb')$$

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$$\hat{V}_{t+1}^{Run}(n') \geq V_{t+1}^D(k')$$

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$$\hat{V}_{t+1}^{Run}(n') \geq V_{t+1}^D(k')$$

- Multiplicity: $\hat{V}_t^{Safe}(n) > \hat{V}_t^D(k) > \hat{V}_t^{Run}(n)$

Multiplicity of Equilibria

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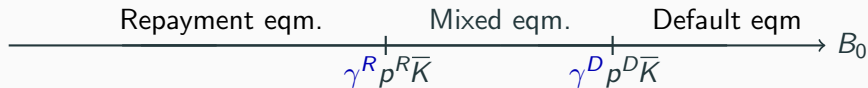
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- Multiplicity: $\hat{V}_t^{Safe}(n) > \hat{V}_t^D(k) > \hat{V}_t^{Run}(n)$

- Assume that if a bank is vulnerable for $t > 0$, a run happens

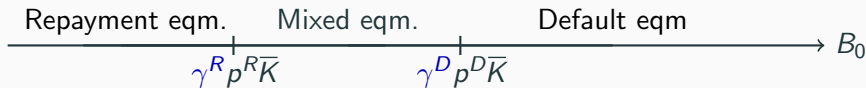
The Effects of Bank Runs

- Financial fragility, default region expands $\downarrow \gamma^D$
 - Repayment region contracts $\gamma^R \downarrow$ if and only if $\beta R < 1$



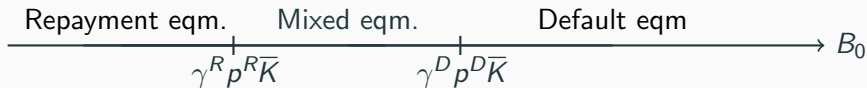
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 - Repayment region contracts $\gamma^R \downarrow$ if and only if $\beta R < 1$



- Lower price of capital
 - Lower γ , implies lower demand by repaying banks
 - More defaulting banks, which have lower demand for capital

Outline of the Talk

1. Basic environment without bank runs
 - Bank problem in partial equilibrium
 - General equilibrium
2. Introduce bank runs
3. Policy analysis

Credit Easing

Introduce government purchases of assets K^g at $t = 0$

Assume that **government makes losses**:

- Productivity $z^g < \underline{z}$ and return $(z^g + p_1)/p_0 < R$

⇒ Investors they do not purchase k if same productivity as govt.

Q: How does credit easing affect ϕ and welfare?

Welfare ↓ if defaults due to **fundamentals**

$$\frac{dW}{dK_g} = \left[\phi \frac{dV^D}{dK_g} - (1 - \phi) \frac{dV^R}{dK_g} \right] - (V^R - V^D) \frac{d\phi}{dK_g}$$

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Without runs:

- $V^R = V^D \Rightarrow d\phi$ irrelevant

Welfare \uparrow if defaults due to runs

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With runs:

- $V^R = V^{Safe} > V^{Run} = V^D$
 \Rightarrow If $d\phi < 0$, possibility that $\uparrow W$

A repaying banks facing a run is a net seller of assets

\Rightarrow benefits from intervention that $\uparrow p_0 \Rightarrow d\phi < 0$

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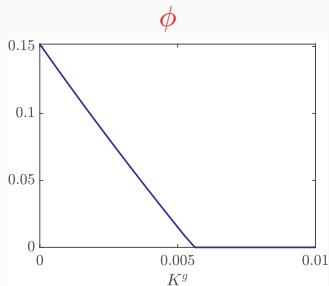
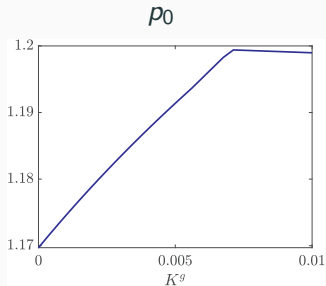
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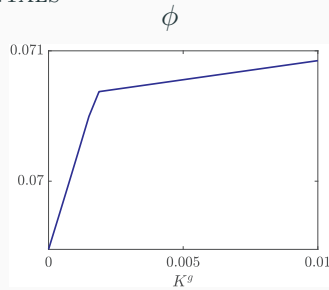
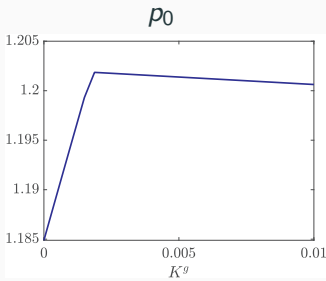
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Credit Easing: Self-Fulfilling vs. Fundamentals

SELF-FULFILLING RUNS

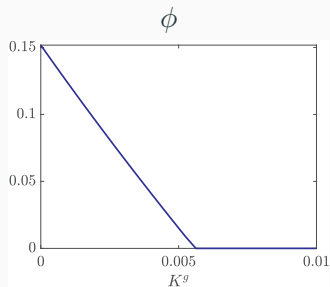
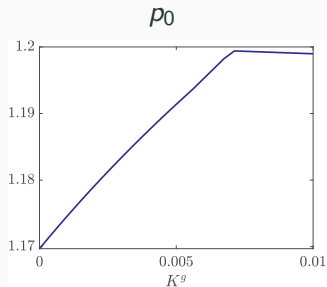


FUNDAMENTALS

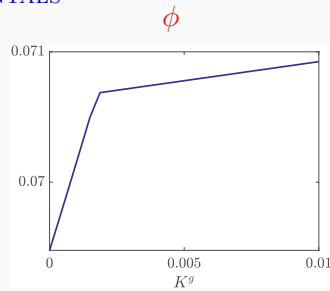
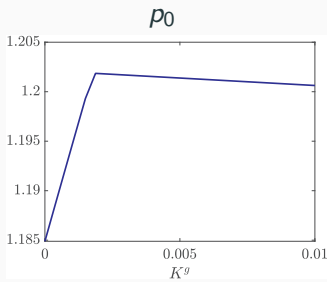


Credit Easing: Self-Fulfilling vs. Fundamentals

SELF-FULFILLING RUNS



FUNDAMENTALS



Other Policies

- Controlling default decisions: [▶ Details](#)
 - Higher ϕ w/o runs and lower ϕ w/runs
- Tax on purchases of capital at $t = 0$ rebated lump sum
 - Irrelevant: after-tax price remains constant and has no effects
- Deposit insurance: deters runs, but requires borrowing limits
- Lender of last resort: must cover *all* banks to be effective

Conclusions

- A dynamic macroeconomic model of self-fulfilling bank runs
- General equilibrium effects crucial to assess govt. policies
- Desirability of credit easing depends on whether a crisis is driven by fundamentals or self-fulfilling runs
- Agenda:
 - Anticipation effects of credit easing
 - Use framework for other policies, such as macroprudential

Government picks ϕ at $t = 0$

Banks' welfare

$$W = (1 - \phi)V^R + \phi V^D$$

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- Assume only p_0 changes in response to policy:

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$$\left. \frac{dV^R(p_0)}{dp_0} \right|_{\phi=\phi^E} = u'(c^R)(\bar{K} - k^R(p_0^E)), \quad \left. \frac{dV^D(p_0)}{dp_0} \right|_{\phi=\phi^E} = u'(c^D)(\bar{K} - k^D(p_0^E)).$$

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$\uparrow \phi$ reduces p_0 and helps repaying banks that have high u'

Government picks ϕ at $t = 0$

Banks' welfare

$$W = (1 - \phi)V^R + \phi V^D$$

- Assume only p_0 changes in response to policy:

$$\frac{dW}{d\phi} \Big|_{\phi=\phi^E} = [V^D(p_0^E) - V^R(p_0^E)] - (1 - \phi) \underbrace{[u'(c^R(p_0^E)) - u'(c^D(p_0^E))]}_{>0} \underbrace{[k^R(p_0^E) - \bar{K}]}_{>0} \underbrace{\frac{dp_0}{d\phi}}_{<0}$$

$\uparrow \phi$ reduces p_0 and helps repaying banks that have high u'

- Without runs: optimal to have more banks defaulting

Government picks ϕ at $t = 0$

Banks' welfare

$$W = (1 - \phi)V^R + \phi V^D$$

- Assume only p_0 changes in response to policy:

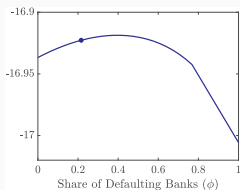
$$\frac{dW}{d\phi} \Big|_{\phi=\phi^E} = [V^D(p_0^E) - V^R(p_0^E)] - (1 - \phi) \underbrace{[u'(c^R(p_0^E)) - u'(c^D(p_0^E))]}_{>0} \underbrace{[k^R(p_0^E) - \bar{K}]}_{>0} \underbrace{\frac{dp_0}{d\phi}}_{<0}$$

$\uparrow \phi$ reduces p_0 and helps repaying banks that have high u'

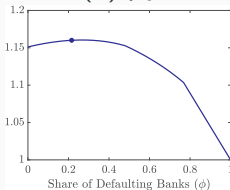
- Without runs: optimal to have more banks defaulting
- With runs: may be optimal to reduce defaults

FUNDAMENTALS

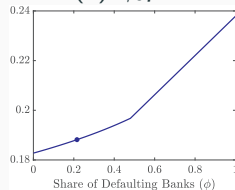
(a) Welfare



(b) p_0

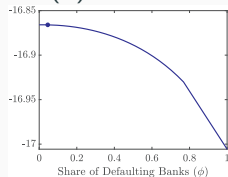


(c) $\gamma_0 p_1$

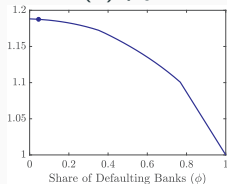


SELF-FULFILLING RUNS

(d) Welfare



(e) p_0



(f) $\gamma_0 p_1$

