Optimal Exchange Rate Policy

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Motivation

- What is the optimal exchange rate policy?
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  - exchange rate is not an *instrument* of the policy
    - what mix of monetary policy, FX interventions, capital controls?
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1. exchange rate is not an instrument of the policy
   — what mix of monetary policy, FX interventions, capital controls?

2. is exchange rate a target?
   — should it be stabilized/fixed? what is a float?
   — can inflation and exchange rate be simultaneous targets?

Build on a realistic GE model of exchange rates (Itskhoki-Mukhin 2021a,b)
— consistent with PPP, UIP, Backus-Smith, Meese-Rogoff, Mussa puzzles
— dual role of exchange rates
  a) expenditure switching in the goods market
  b) risk sharing in the financial market
show
Develop a rich framework for policy analysis
— intuitive linear-quadratic policy problem (cf. CGG'99, GM'05)
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  1. Exchange rate is not an *instrument* of the policy
     - what mix of monetary policy, FX interventions, capital controls?
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Main Results

1. **First best** requires:

   i. MP → output gap (inflation target.+ float)

2. Divine coincidence in an open economy: if the frictionless RER is stable, then MP alone can implement the first-best
   - fixed exchange rate ⇒ zero inflation
   - stabilizes output gap and eliminates risk sharing wedge

3. Without FX, optimal MP with commitment partially stabilizes ER
   - balances out output gap and UIP deviations

4. FX constraints can be relaxed via FX and ER forward guidance

5. Explore possibility of income and losses from FX interventions
Main Results

1. **First best** requires:
   
i. MP $\rightarrow$ output gap (inflation target. + float)

   ii. FX $\rightarrow$ UIP deviations (risk sharing wedges)
      
      — fixed exchange rate is not the goal
      — offset financial shocks, accommodate fundamental shocks

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4. FX constraints can be relaxed via **FX and ER forward guidance**

5. Explore possibility of income and losses from FX interventions
Relation to the Literature

- **Portfolio models:**

- **Optimal policy in open economy:**
SETUP
SOE with T and NT, segmented asset markets

**Households:**

\[
\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{Tt} + (1 - \gamma)(\log C_{Nt} - L_t) \right]
\]

s.t. \[
\frac{B_t}{R_t} + P_{Tt} C_{Tt} + P_{Nt} C_{Nt} = B_{t-1} + W_t L_t + \Pi_t + T_t
\]
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Firms:

1. **tradables**: exogenous endowment \( Y_{Tt} \), law of one price \( P_{Tt} = \mathcal{E}_t P^*_{Tt} = \mathcal{E}_t \)
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  \]

- **Firms:**
  1. **tradables:** exogenous endowment $Y_{Tt}$, law of one price $P_{Tt} = \mathcal{E}_t P^*_{Tt} = \mathcal{E}_t$
  2. **non-tradables:** technology $Y_{Nt} = A_t L_t$, fully sticky prices $P_{Nt} = 1$
Setup

- **SOE with T and NT, segmented asset markets**

- **Households:**

  \[
  \max \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_T + (1 - \gamma)(\log C_N - L) \right]
  \]

  s.t. \[ \frac{B_t}{R_t} + P_T C_T + P_N C_N = B_{t-1} + W_t L_t + \Pi_t + T_t \]

- **Firms:**

  1. **tradables**: exogenous endowment \( Y_T \), law of one price \( P_T = \mathcal{E}_t P_T^* = \mathcal{E}_t \)

  2. **non-tradables**: technology \( Y_N = A_t L_t \), fully sticky prices \( P_N = 1 \)

- **Financial sector:**

  ![Diagram of the financial sector](image.png)
Setup

- SOE with T and NT, segmented asset markets

- **Households:**
  \[
  \max \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{Tt} + (1 - \gamma)(\log C_{Nt} - L_t) \right]
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  s.t. \( \frac{B_t}{R_t} + P_{Tt} C_{Tt} + P_{Nt} C_{Nt} = B_{t-1} + W_t L_t + \Pi_t + T_t \)

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  2. **non-tradables:** technology \( Y_{Nt} = A_t L_t, \) fully **sticky prices** \( P_{Nt} = 1 \)

- **Financial sector:**

![Diagram of financial sector]
SOE with T and NT, segmented asset markets

**Households:**
\[
\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{Tt} + (1 - \gamma)(\log C_{Nt} - L_t) \right]
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s.t.
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\frac{B_t}{R_t} + P_{Tt} C_{Tt} + P_{Nt} C_{Nt} = B_{t-1} + W_t L_t + \Pi_t + T_t
\]

**Firms:**
1. **tradables:** exogenous endowment $Y_{Tt}$, law of one price $P_{Tt} = E_t P_{Tt}^* = E_t$
2. **non-tradables:** technology $Y_{Nt} = A_t L_t$, fully sticky prices $P_{Nt} = 1$

**Financial sector:**

Diagram showing the flow of goods and money between Home H/H, Foreign H/H, Arbitrageurs, Government, and Noise Traders.
Setup

- SOE with T and NT, segmented asset markets

- **Households:**
  \[
  \max \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_T + (1 - \gamma) (\log C_N - L) \right]
  \]

  s.t. \[
  \frac{B_t}{R_t} + P_T C_T + P_N C_N = B_{t-1} + W_t L_t + \Pi_t + T_t
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- **Firms:**
  1. **tradables:** exogenous endowment \( Y_T \), law of one price \( P_T = \mathbb{E}_t P_T^* = \mathbb{E}_t \)
  2. **non-tradables:** technology \( Y_N = A_t L_t \), fully sticky prices \( P_N = 1 \)

- **Financial sector:**
  - incomplete asset markets
  - segmented markets w/ risk-averse arbitrageurs

  \[
  \underbrace{B_t + N_t}_{h/h} + \underbrace{F_t}_{\text{noise traders}} + \underbrace{D_t}_{\text{FX interventions}} + \underbrace{A_t}_{\text{arbitrageurs}} = 0
  \]
Non-Tradable Sector

Equilibrium conditions:

- labor supply
  \[ C_{Nt} = \frac{W_t}{P_{Nt}} \]

- market clearing
  \[ C_{Nt} = Y_{Nt} = A_t L_t \]
Non-Tradable Sector

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  - labor supply
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  - market clearing
  \[ C_{Nt} = Y_{Nt} = A_t L_t \]

- First-best = flexible prices:
  \[ C_{Nt} = \frac{W_t}{P_{Nt}} = \frac{W_t}{W_t/A_t} = A_t \]
Non-Tradable Sector

- Equilibrium conditions:
  - labor supply
    \[ C_{Nt} = \frac{W_t}{P_{Nt}} \]
  - market clearing
    \[ C_{Nt} = Y_{Nt} = A_t L_t \]
  - Euler equation
    \[ \beta R_t E_t \frac{C_{Nt}}{C_{Nt+1}} \frac{P_{Nt}}{P_{Nt+1}} = 1 \]

- Sticky prices:
  \[ P_{Nt} = 1 \quad \Rightarrow \quad \text{output gap} \quad x_t \equiv \log \frac{C_{Nt}}{C_{Nt}} = \log \frac{C_{Nt}}{A_t} \]
Non-Tradable Sector

- Equilibrium conditions:
  - labor supply
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    \[ \beta R_t E_t \frac{C_{Nt}}{C_{Nt+1}} \frac{P_{Nt}}{P_{Nt+1}} = 1 \]
  - optimal demand
    \[ \frac{\gamma}{1 - \gamma} \frac{C_{Nt}}{C_{Tt}} = \frac{E_t}{P_{Nt}} \]

- Sticky prices:
  \[ P_{Nt} = 1 \implies \text{output gap} \quad x_t \equiv \log \frac{C_{Nt}}{C_{Nt}} = \log \frac{C_{Nt}}{A_t} \]

- Exchange rate:
  \[ e_t = c_{Nt} - c_{Tt} \]
Non-Tradable Sector

Equilibrium conditions:

— labor supply

\[ C_{Nt} = \frac{W_t}{P_{Nt}} \]

— market clearing

\[ C_{Nt} = Y_{Nt} = A_t L_t \]

— Euler equation

\[ \beta R_t E_t \frac{C_{Nt}}{C_{Nt+1}} \frac{P_{Nt}}{P_{Nt+1}} = 1 \]

— optimal demand

\[ \frac{\gamma}{1 - \gamma} \frac{C_{Nt}}{C_{Tt}} = \frac{E_t}{P_{Nt}} \]

Sticky prices:

\[ P_{Nt} = 1 \implies \text{output gap} \quad x_t \equiv \log \frac{C_{Nt}}{C_{Nt}} = \log \frac{C_{Nt}}{A_t} \]

Exchange rate:

\[ e_t = \tilde{q}_t + x_t - z_t \]

— Consumption wedge for T: \[ z_t \equiv \log \frac{C_{Tt}}{C_{Tt}} \]

— Efficient RER: \[ \tilde{q}_t = a_t - \tilde{c}_{Tt} \]
Arbitrageurs choose portfolio \((D_t, D_t^*)\) w/

- zero net positions \(\frac{D_t}{R_t} + \frac{\mathcal{E}_t D_t^*}{R_t^*} = 0\),
- carry trade returns \(\tilde{R}_{t+1} \equiv R_t^* - R_t \frac{\mathcal{E}_t}{\bar{\mathcal{E}}_{t+1}}\)
- live for one period and transfer income to home h/h
Arbitrageurs choose portfolio \((D_t, D_t^*)\) w/

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Arbitrageurs’ problem:

\[
\max_{D_t^*} \quad \mathbb{E}_t(W_{t+1}) - \frac{\omega}{2} \text{var}_t(W_{t+1}), \quad W_{t+1} = \tilde{R}_{t+1} \frac{D_t}{R_t^*}
\]
Arbitrageurs choose portfolio \((D_t, D^*_t)\) \text{w/}

- zero net positions \(\frac{D_t}{R_t} + \frac{\varepsilon_t D^*_t}{R^*_t} = 0\),
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\]

International risk sharing:

\[
\frac{D_t}{R_t} = \frac{\mathbb{E}_t[\tilde{R}_{t+1}]}{\omega \sigma_t^2}, \quad \sigma_t^2 \equiv \text{var}_t(\tilde{R}_{t+1})
\]
Tradable Sector

- Arbitrageurs choose portfolio \((D_t, D_t^*)\) w/
  - zero net positions \(\frac{D_t}{R_t} + \frac{\varepsilon_t D_t^*}{R_t^*} = 0\),
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International risk sharing:

$$\beta R^*_t \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega \sigma^2_t \frac{B^*_t - N^*_t - F^*_t}{R^*_t}$$
Arbitrageurs choose portfolio \((D_t, D_t^*)\) w/

- zero net positions \(\frac{D_t}{R_t} + \frac{\varepsilon_t D_t^*}{R_t^*} = 0\),
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- live for one period and transfer income to home \(h/h\)

Arbitrageurs’ problem:

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\max_{D_t^*} \quad E_t(W_{t+1}) - \frac{\omega}{2} \text{var}_t(W_{t+1}), \quad W_{t+1} = \tilde{R}_{t+1} \left( \frac{D_t^*}{R_t^*} \right)
\]

International risk sharing:

\[
E_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*)
\]

\[
\sigma_t^2 = \text{var}_t(\Delta e_{t+1})
\]

- \(E_t \Delta z_{t+1} = i_t - i_t^* - E_t \Delta e_{t+1}\) (UIP deviations = RS wedge)
- first-order risk premium!
Arbitrageurs choose portfolio \( (D_t, D_t^*) \) w/

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- first-order risk premium!

Country’s budget constraint:

\[
\frac{B_t^*}{R_t^*} = B_{t-1}^* + Y_{Tt} - C_{Tt}
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Arbitrageurs choose portfolio \((D_t, D_t^*)\) w/
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- \(\mathbb{E}_t \Delta z_{t+1} = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}\) (UIP deviations = RS wedge)
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Country's budget constraint:
\[
\beta b_t^* = b_{t-1}^* - z_t
\]
**Lemma:** To the first-order approximation, the optimal policy solves

\[
\min_{\{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\}} \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
\]
**Lemma:** To the first-order approximation, the optimal policy solves

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\min_{\{z_t, x_t, b^*_t, f^*_t, \sigma^2_t\}} \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z^2_t + (1 - \gamma) x^2_t \right]
\]

s.t.

\[
\beta b^*_t = b^*_{t-1} - z_t
\]

\[
\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma^2_t (b^*_t - n^*_t - f^*_t)
\]

\[
\sigma^2_t = \text{var}_t(\tilde{q}_{t+1} - z_{t+1} + x_{t+1})
\]

- \(x_t\) – output gap
- \(z_t\) – consumption gap for T
- \(f^*_t\) – FX reserves

No Trilemma: it is possible to simultaneously have (i) independent MP, (ii) fixed ER, (iii) no capital controls.

- limits to arbitrage (cf. ABBP'20, Fanelli-Straub'21)
- distortionary \(n^*_t\) shocks
- two channels of monetary policy
**Lemma:** To the first-order approximation, the optimal policy solves

\[
\min_{\{z_t,x_t,b_t^*,f_t^*,\sigma_t^2\}} \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
\]

s.t. \( \beta b_t^* = b_{t-1}^* - z_t \)

\[
\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*)
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\sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1})
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s.t. \[\beta b_t^* = b_{t-1}^* - z_t\]

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- limits to arbitrage (cf. ABBP’20, Fanelli-Straub’21)
- distortionary \(n_t^*\) shocks
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TWO POLICY INSTRUMENTS
Optimal Policy

Planner’s problem:

$$\min_{\{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]$$

s.t. \( \beta b_t^* = b_{t-1}^* - z_t \)

\( \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*) \)

\( \sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1}) \)

Proposition (First best)

The optimal policy implements the first best: i) MP close the output gap \( x_t = 0 \), ii) FX interventions eliminate the risk-sharing wedge \( f_t^* = b_t^* - n_t^* \).
Optimal Policy

- Planner’s problem:
  \[
  \min_{\{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
  \]
  s.t. \[\beta b_t^* = b_{t-1}^* - z_t\]
  \[\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*)\]
  \[\sigma_t^2 = \text{var}_t(\tilde{q}_{t+1} - z_{t+1} + x_{t+1})\]

Proposition (First best)

The optimal policy implements the first best: i) MP close the output gap \(x_t = 0\), ii) FX interventions eliminate the risk-sharing wedge \(f_t^* = b_t^* - n_t^*\).

Optimal targets:
**Optimal Policy**

Planner’s problem:

\[
\min \{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\} \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
\]

s.t.

\[
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\]

\[
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**Optimal targets:** MP → inflation/output, FX policy → UIP deviations

— targeting ER is suboptimal, but equilibrium ER volatility is lower
Optimal Policy

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Responses to shocks:
Optimal Policy

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Proposition (First best)

The optimal policy implements the first best: i) MP close the output gap \( x_t = 0 \), ii) FX interventions eliminate the risk-sharing wedge \( f_t^* = b_t^* - n_t^* \).

Optimal targets: MP → inflation/output, FX policy → UIP deviations

— targeting ER is suboptimal, but equilibrium ER volatility is lower

Responses to shocks: FX policy offsets \( n_t^* \) and accommodates \( \tilde{q}_t \)

— \( \tilde{q}_t \) depends on \( a_t, yT_t, r_t^* \)

— unobservable \( \tilde{q}_t \) and \( n_t^* \) (cf. potential output, NAIRU, natural rate)
MONETARY POLICY
Monetary policy problem:

\[
\min_{\{z_t, x_t, b^*_t, \sigma^2_t\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
\]

s.t. \( \beta b^*_t = b^*_{t-1} - z_t, \)

\[
\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma^2_t (b^*_t - n^*_t),
\]

\[
\sigma^2_t = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1}), \quad \tilde{q}_t \equiv a_t - \tilde{c}_t
\]

"Divine coincidence": if \( \tilde{q}_t = 0 \), the optimal MP implements \( x_t = z_t = 0 \)

— peg ≻ inflation targeting due to multiple equilibria

— optimal currency areas (Mundell’61)
Monetary policy problem:

$$\min_{\{z_t, x_t, b^*_t, \sigma^2_t\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]$$

s.t. $$\beta b_t^* = b_{t-1}^* - z_t,$$

$$\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma^2_t (b_t^* - n_t^*),$$

$$\sigma^2_t = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1}), \quad \tilde{q}_t \equiv a_t - \tilde{c}_t$$

Can monetary policy alone close the two gaps?

— no in general case: conditional on $$\sigma^2_t$$, $$z_t \perp x_t$$

— important exception: $$\sigma^2_t = 0 \Rightarrow z_t = 0 \Rightarrow x_t = -\tilde{q}_t$$
Divine Coincidence

- **Monetary policy problem:**

\[
\min_{\{z_t, x_t, b^*_t, \sigma^2_t\}} \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
\]

s.t. \( \beta b^*_t = b^*_{t-1} - z_t, \)

\[
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\]

\[
\sigma^2_t = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1}), \quad \tilde{q}_t \equiv a_t - \tilde{\zeta}_t
\]

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- "Divine coincidence": if \( \tilde{q}_t = 0 \), the optimal MP implements \( x_t = z_t = 0 \)
  - \( \tilde{q}_t = 0 \) requires that i) \( a_t = y_T \), ii) both follow RW, iii) \( r^*_t = 0 \)
Monetary policy problem:

$$\min_{\{z_t, x_t, b_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]$$

s.t. \( \beta b_t^* = b_{t-1}^* - z_t \),

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**“Divine coincidence”**: if \( \tilde{q}_t = 0 \), the optimal MP implements \( x_t = z_t = 0 \)

--- \( \tilde{q}_t = 0 \) requires that i) \( a_t = y_{T_t} \), ii) both follow RW, iii) \( r_t^* = 0 \)
--- peg \( \succ \) inflation targeting due to multiple equilibria
Divine Coincidence

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s.t. \[\beta b_t^* = b_{t-1}^* - z_t,\]

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  - no in general case: conditional on \(\sigma_t^2, z_t \perp x_t\)
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- "Divine coincidence": if \(\tilde{q}_t = 0\), the optimal MP implements \(x_t = z_t = 0\)
  - \(\tilde{q}_t = 0\) requires that i) \(a_t = y_{T_t}\), ii) both follow RW, iii) \(r_t^* = 0\)
  - peg > inflation targeting due to multiple equilibria
  - optimal currency areas (Mundell’61)
More generally, the optimal monetary rule is

$$(1 - \gamma) \underbrace{x_t}_{\text{output gap}} = -\gamma \overline{\omega} \lambda_{t-1} (b^*_{t-1} - n^*_{t-1}) \underbrace{(e_t - \mathbb{E}_{t-1} e_t)}_{\geq 0},$$

ER volatility
More generally, the optimal monetary rule is

$$(1 - \gamma) x_t = -\gamma \bar{\omega} \lambda_{t-1} (b_{t-1}^* - n_{t-1}^*) (e_t - \mathbb{E}_{t-1} e_t),$$

where $x_t$ represents the output gap, $\gamma$ is the weight on the output gap, $\bar{\omega}$ is the risk aversion parameter, $\lambda_{t-1}$ is the adjustment speed, $b_{t-1}^*$ and $n_{t-1}^*$ are the long-run equilibrium values of the variables, and $e_t$ and $\mathbb{E}_{t-1} e_t$ are the exchange rate and its expected value.

**Proposition**

The optimal monetary policy closes the average output gap $\mathbb{E} x_t = 0$, but deviates from targeting $x_t$ state-by-state to lower exchange rate volatility $\sigma_t^2$. 

MP trade-off: given one policy instrument, the optimal policy leans against the wind and implements a crawling peg that is tighter when
- the economy is more open ($\gamma$)
- arbitrageurs are more risk averse ($\bar{\omega}$)
- volatility of $n_{t-1}^*$ and $\tilde{q}_t$ is higher

Time consistency: optimal discretionary policy closes output gap $x_t = 0$.
More generally, the optimal monetary rule is

\[
(1 - \gamma) x_t = -\gamma \bar{\omega} \lambda_{t-1} (b_{t-1}^* - n_{t-1}^*) (e_t - E_{t-1}e_t),
\]

output gap \geq 0 ER volatility

**Proposition**

*The optimal monetary policy closes the average output gap \( \mathbb{E} x_t = 0 \), but deviates from targeting \( x_t \) state-by-state to lower exchange rate volatility \( \sigma_t^2 \).*

**MP trade-off:**
Monetary Peg

More generally, the optimal monetary rule is

\[(1 - \gamma) x_t = -\gamma \bar{\omega} \lambda_{t-1} (b_{t-1}^* - n_{t-1}^*) (e_t - \mathbb{E}_{t-1} e_t) \geq 0\]

output gap

\[\geq 0\]

ER volatility

Proposition

The optimal monetary policy closes the average output gap $\mathbb{E}x_t = 0$, but deviates from targeting $x_t$ state-by-state to lower exchange rate volatility $\sigma_t^2$.

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**Time consistency**:
More generally, the optimal monetary rule is

\[
(1 - \gamma) \times_t = -\gamma \bar{\omega} \lambda_{t-1} (b^*_{t-1} - n^*_{t-1}) (e_t - \mathbb{E}_{t-1} e_t) \geq 0
\]

output gap  \hspace{1cm} ER volatility

**Proposition**

The optimal monetary policy closes the average output gap \( \mathbb{E}x_t = 0 \), but deviates from targeting \( x_t \) state-by-state to lower exchange rate volatility \( \sigma_t^2 \).

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- economy is more open \( \gamma \)
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**Time consistency:** optimal discretionary policy closes output gap \( x_t = 0 \)
FX POLICY
FX Policy

FX policy problem:

\[
\min_{\{x_t, z_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma)x_t^2 \right]
\]

s.t.

\[
\beta b_t^* = b_{t-1}^* - z_t
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\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*)
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\[
\sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1})
\]

\[x_t \in \Gamma(z_t, s_t)\]

Restrictions \(\Gamma(\cdot)\) on MP:

a) \(\text{peg}\)

b) \(\text{ZLB}\)
FX Policy

FX policy problem:

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\min_{\{x_t, z_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
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s.t. \( \beta b_t^* = b_{t-1}^* - z_t \)

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\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*)
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\[
\sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1})
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\( x_t \in \Gamma(z_t, s_t) \)

Restrictions \( \Gamma(\cdot) \) on MP:

a) peg \( \Rightarrow \) \( z_t \) is exogenous

b) ZLB \( \Rightarrow \) \( x_t \) is exogenous

\[\Rightarrow \text{no FX divine coincidence} \]
FX Policy

FX policy problem:

\[
\min_{\{x_t, z_t, b_t^*, f_t^*, \sigma_t^2\}} \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
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\[
\sigma_t^2 = \text{var}_t (q_{t+1} - z_{t+1} + x_{t+1})
\]

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\]

Restrictions \( \Gamma(\cdot) \) on MP:

a) peg \( \Rightarrow \) \( z_t \) is exogenous

b) ZLB \( \Rightarrow \) \( x_t \) is exogenous

\[ \left\{ \begin{array}{c}
\text{no FX divine coincidence}
\end{array} \right. \]

cf. macroprudential policy under AD externality (Farhi-Werning’16)

FX trade-off: FX interventions are unlikely to mitigate output gap

Time consistency: FX policy does not require commitment
INTERNATIONAL SPILLOVERS
Global equilibrium:

— continuum of SOEs trading dollar bonds
— unchanged risk sharing condition

\[ \mathbb{E}_t \Delta z_{it+1} = -\tilde{\omega} \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*) \]

— endogenous \( p_{Tt} \) and \( r_t^* \)

\[ r_t^* \equiv i_t^* - \mathbb{E}_t \Delta p_{Tt+1}, \quad \int c_{Tit} di = \int y_{Tit} di \]
Global equilibrium:

— continuum of SOEs trading dollar bonds
— unchanged risk sharing condition
\[ \mathbb{E}_t \Delta z_{it+1} = -\bar{\omega} \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*) \]
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\[ r_t^* \equiv i_t^* - \mathbb{E}_t \Delta p_{Tt+1}, \quad \int c_{Tt} \, di = \int y_{Tt} \, di \]

Gains from cooperation:

i) first-best policies \( \Rightarrow \) NE is efficient
ii) second-best policies \( \Rightarrow \) negative spillovers
\[ \mathbb{E}_t \Delta \tilde{c}_{T_{it+1}} = r_t^*, \quad r_t^* = \mathbb{E}_t \Delta y_{T_{t+1}} + \bar{\omega} \int \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*) \, di \]
Global equilibrium:

- continuum of SOEs trading dollar bonds
- unchanged risk sharing condition

\[
\mathbb{E}_t \Delta z_{it+1} = -\bar{\omega} \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*)
\]

- endogenous \( p_{Tt} \) and \( r_t^* \)

\[
r_t^* \equiv i_t^* - \mathbb{E}_t \Delta p_{Tt+1}, \quad \int c_{Tit} di = \int y_{Tit} di
\]

9 Gains from cooperation:

i) first-best policies \( \Rightarrow \) NE is efficient

ii) second-best policies \( \Rightarrow \) negative spillovers

\[
\mathbb{E}_t \Delta \tilde{c}_{Tit+1} = r_t^*, \quad r_t^* = \mathbb{E}_t \Delta y_{Tt+1} + \bar{\omega} \int \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*) di
\]

10 Anchor currency: countries import U.S. MP under second-best policy

\[
e_{it} = \tilde{q}_{it} - p_{Tt} + x_{it} - z_{it}
\]

- currency of debt \( \Rightarrow \) anchor/reserve currency

- cf. gold standard with \( i_t^* = 0 \) and \( p_{Tt} \) determined from \( c_{Tt} = y_{Tt} \)
EXTENSIONS
Extensions relax assumptions of the baseline model:

1. **Home traders** \(\rightarrow\) **int’l transfers**

2. **T and NT goods** \(\rightarrow\) **ToT effects**

3. **Fully sticky prices** \(\rightarrow\) **NKPC**

4. **Noise traders** \(\rightarrow\) **risk-premium shocks**

5. **Log-linear preferences** \(\rightarrow\) **complementarities**
Transfers

- No redistributational effects in the baseline model
- Assume **foreign** arbitrageurs and noise traders
- Country’s budget constraint:

\[
\frac{B_t^*}{R_t^*} = B_{t-1}^* + Y_{Tt} - C_{Tt} - \mathcal{T}_t \left( N^*_{t-1} + \frac{E_{t-1} \mathcal{T}_t}{\omega \sigma^2_t} \right), \quad \mathcal{T}_t \equiv \frac{R_{t-1}}{R_t^*} \frac{E_{t-1}}{E_t} - 1
\]
Transfers

- No redistribuional effects in the baseline model

- Assume foreign arbitrageurs and noise traders

- Country’s budget constraint:
  \[
  \frac{B_t^*}{R_t^*} = B_{t-1} + Y_{Tt} - C_{Tt} - T_t \left( N_{t-1}^* + \frac{E_{t-1}T_t}{\omega \sigma_t^2} \right), \quad T_t \equiv \frac{R_{t-1} E_{t-1}}{R_t^* E_t} - 1
  \]

- Loss function depends on UIP deviations \( \tau_{t-1} \equiv r_t - r_t^* - E_{t-1} \Delta e_t \):
  \[
  \mathcal{L} = \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 + 2 \gamma \tau_{t-1} \left( n_{t-1}^* + \frac{\tau_{t-1}}{\omega \sigma_t^2} \right) \right]
  \]
  - extends Fanelli-Straub’21 to stochastic shocks
  - to the SOA, welfare depends on ex-ante UIP deviations
  - if local noise traders, \( n_t^* = 0 \) in \( \mathcal{L} \) and any \( \tau_t \neq 0 \) lower the welfare
  - if \( n_t^* \neq 0 \), the planner can extract rents
Transfers

Planer’s problem:

\[
\min \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
\]

s.t. \( \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*) \)

\( \beta b_t^* = b_{t-1}^* - z_t \)

\( \sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} + x_{t+1} - z_{t+1}) \)
Transfers

- Planner’s problem:
  \[
  \min \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma)x_t^2 + 2\gamma \tau_{t-1} \left(n_{t-1} - \frac{\tau_{t-1}}{\bar{\omega}\sigma_{t-1}^2}\right) \right]
  \]
  s.t. \[
  \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}\sigma_t^2 \left(b_t^* - n_t^* - f_t^*\right) = \tau_t
  \]
  \[
  \beta b_t^* = b_{t-1}^* - z_t
  \]
  \[
  \sigma_t^2 = \text{var}_t \left(\tilde{q}_{t+1} - z_{t+1} + x_{t+1}\right)
  \]

- **First-best**: implementation of the first best generically requires three instruments – i) monetary policy, ii) FX interventions, iii) capital controls
Transfers

- **Planner’s problem:**

\[
\min \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 + 2\gamma \tau_{t-1} \left( n_{t-1}^* + \frac{\tau_{t-1}}{\bar{\omega} \sigma_{t-1}^2} \right) \right]
\]

s.t. \( \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^*) = \tau_t \)

\( \beta b_t^* = b_{t-1}^* - z_t \)

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- **First-best:** implementation of the first best generically requires three instruments – i) monetary policy, ii) FX interventions, iii) capital controls

- **Divine coincidence:** if \( \bar{q}_t = 0 \) and either i) local noise traders or ii) \( b_t^*/n_t^* \approx 0 \), then MP alone can achieve the first-best
Transfers

Planner’s problem:

$$\min \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma)x_t^2 + 2\gamma \tau_{t-1} \left( n_{t-1}^* + \frac{\tau_{t-1}}{\bar{\omega}\sigma_{t-1}^2} \right) \right]$$

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\( \beta b_t^* = b_{t-1}^* - z_t \)

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**FX policy**: given \( \sigma^2 \), the optimal FX policy trades off rents for efficient risk sharing and smooths the optimal UIP deviations \( \bar{\tau}_t \equiv \frac{2}{\bar{\omega}\sigma^2} \tau_t + n_t^* \) in time:

$$\bar{\tau}_t = \lambda \bar{\tau}_{t-1} + \frac{\bar{\omega}\sigma^2\lambda}{2} \sum_{j=0}^{\infty} (\beta\lambda)^j \mathbb{E}_t n_{t+j}^*, \quad \lambda \in (0, 1)$$
Transfers

- Planner’s problem:

  \[
  \min \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma)x_t^2 + 2\gamma \tau_{t-1} \left( n_{t-1}^* + \frac{\tau_{t-1}}{\bar{\omega}\sigma_{t-1}^2} \right) \right] \\
  \text{s.t.} \quad \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}\sigma_t^2 (b_t^* - n_t^* - f_t^*) = \tau_t \\
  \beta b_t^* = b_{t-1}^* - z_t \\
  \sigma_t^2 = \text{var}_t (\bar{q}_{t+1} - z_{t+1} + x_{t+1})
  \]

- **First-best**: implementation of the first best generically requires three instruments – i) monetary policy, ii) FX interventions, iii) capital controls

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  \[
  \bar{\tau}_t = \lambda \bar{\tau}_{t-1} + \frac{\bar{\omega}\sigma^2 \lambda}{2} \sum_{j=0}^{\infty} (\beta \lambda)^j \mathbb{E}_t n_{t+j}^*, \quad \lambda \in (0, 1)
  \]

  — commitment is important, but differently from the NK models
Baseline model assumes T and NT:

- might be a good approximation for commodity exporters
- contrasts with OR’95, DE’03, GM’05, etc.
Terms of Trade

- Baseline model assumes T and NT:
  - might be a good approximation for commodity exporters
  - contrasts with OR’95, DE’03, GM’05, etc.

- Allow for home and foreign goods:
  \[ C_t = C_{Ht}^{1-\gamma} C_{Ft}^\gamma, \quad C_{Ht}^* = P_{Ht}^{-\varepsilon} C_t^* \]
  - log-linear preferences for simplicity
  - optimal steady-state production subsidies
  - three shocks: \( n_t^*, a_t, c_t^* \)
Baseline model assumes T and NT:

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- contrasts with OR’95, DE’03, GM’05, etc.

Allow for home and foreign goods:

\[ C_t = C_{Ht}^{1-\gamma} C_{Ft}^{\gamma}, \quad C_{Ht}^* = P_{Ht}^{-\varepsilon} C_t^* \]

- log-linear preferences for simplicity
- optimal steady-state production subsidies
- three shocks: \( n_t^* \), \( a_t \), \( c_t^* \)

Currency of invoicing:

1. producer (PCP) = sticky wages
2. dominant (DCP)
Optimal Policy: PCP

- Planner’s problem under PCP:

$$\min_{\{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \kappa \left( z_t^2 \right) + x_t^2 \right]$$

$$c_{Ft} - \bar{c}_{Ft}$$

$$y_t - \bar{y}_t$$
Optimal Policy: PCP

- Planner’s problem under **PCP**: 

\[
\min_{\{z_t, x_t, b^*_t, f^*_t, \sigma^2_t\}} \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \kappa \left( z^2_t \right) + x^2_t \right] \\
\text{s.t.} \quad \beta b^*_t = b^*_{t-1} - z_t + x_t, \\
\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma^2_t \left( b^*_t - n^*_t - f^*_t \right), \\
\sigma^2_t = \text{var}_t \left( \tilde{q}_{t+1} - (1 - \tilde{\gamma}) z_{t+1} + x_{t+1} \right), \quad \tilde{q}_t \equiv a_t - \tilde{c}_F t
\]
Optimal Policy: PCP

- Planner’s problem under **PCP**:

\[
\min_{\{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \kappa \left( z_t^2 - c_{Ft} - \tilde{c}_{Ft} \right) + \chi_t^2 \left( y_t - \tilde{y}_t \right) \right]
\]

s.t. \( \beta b_t^* = b_{t-1}^* - z_t + x_t, \)

\( \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*), \)

\( \sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - (1 - \bar{\gamma})z_{t+1} + x_{t+1}), \quad \tilde{q}_t \equiv a_t - \tilde{c}_{Ft} \)

- **FX policy**: same motives as in the baseline model
Optimal Policy: PCP

- Planner’s problem under **PCP**:

  \[
  \min_{\{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\}} \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \sum_{u=0}^{\infty} \beta^u \left( \kappa z_u^2 \right) + \beta^u x_u^2 \right]
  \]

  s.t. \( \beta b_t^* = b_{t-1}^* - z_t + x_t, \)

  \( \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*), \)

  \( \sigma_t^2 = \text{var}_t \left( \tilde{q}_{t+1} - (1 - \bar{\gamma})z_{t+1} + x_{t+1} \right), \quad \tilde{q}_t \equiv a_t - \tilde{c}_{Ft} \)

- **FX policy**: same motives as in the baseline model

- **Divine coincidence**: if \( a_t = c_t^* \) and follow a random walk, then \( \tilde{q}_t = 0 \) and the MP alone can implement the first-best allocation \( x_t = z_t = 0 \)
Optimal Policy: PCP

- Planner’s problem under **PCP**:  
  \[
  \min_{\{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \kappa \left( z_t^2 \right)_{c_F - \tilde{c}_F} + x_t^2 \right]_{y_t - \tilde{y}_t} 
  \]
  \[
  \text{s.t. } \beta b_t^* = b_{t-1}^* - z_t + x_t, \\
  \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*), \\
  \sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - (1 - \bar{\gamma}) z_{t+1} + x_{t+1}), \quad \tilde{q}_t \equiv a_t - \tilde{c}_{Ft}
  \]

- **FX policy**: same motives as in the baseline model

- **Divine coincidence**: if \( a_t = c_t^* \) and follow a random walk, then \( \tilde{q}_t = 0 \) and the MP alone can implement the first-best allocation \( x_t = z_t = 0 \)

- **One instrument**: neither \( f_t^* \) nor \( \sigma_t^2 = 0 \) are sufficient to implement \( z_t = 0 \) because of suboptimal exports
Optimal Policy: DCP

- Planner’s problem under DCP:

\[
\min_{\{z_t, x_t, b^*_t, f^*_t, \sigma^2_t\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \frac{z_t^2}{c_{Ft} - \tilde{c}_{Ft}} + (1 - \gamma) \frac{x_t^2}{\gamma_{Ht} - \tilde{\gamma}_{Ht}} \right]
\]
Planner’s problem under DCP:

\[
\min \left\{ z_t, x_t, b^*_t, f^*_t, \sigma^2_t \right\} \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \left( z^2_t \right) + (1 - \gamma) \left( x^2_t \right) \right] \\
\text{s.t.} \quad \beta b^*_t = b^*_{t-1} - z_t + \kappa \tilde{q}_t, \\
\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma^2_t \left( b^*_t - n^*_t - f^*_t \right), \\
\sigma^2_t = \text{var}_t \left( \tilde{q}_{t+1} - z_{t+1} + x_{t+1} \right), \quad \tilde{q}_t = a_t - \tilde{c}_{Ft}
\]
Optimal Policy: DCP

- Planner’s problem under DCP:

\[
\min_{\{z_t, x_t, b^*_t, f^*_t, \sigma^2_t\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \frac{z^2_t}{c_{Ft} - \tilde{c}_{Ft}} + (1 - \gamma) \frac{x^2_t}{y_{Ht} - \tilde{y}_{Ht}} \right]
\]

s.t. \( \beta b^*_t = b^*_{t-1} - z_t + \kappa \tilde{q}_t, \)

\( \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma^2_t (b^*_t - n^*_t - f^*_t), \)

\( \sigma^2_t = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1}), \quad \tilde{q}_t = a_t - \tilde{c}_{Ft} \)

- FX policy: can no longer implement \( z_t = 0 \), but still focuses on the wedge in the risk-sharing condition
Optimal Policy: DCP

- Planner’s problem under DCP:

\[
\min_{\{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma \overbrace{z_t^2}^{c_{Ft} - \tilde{c}_{Ft}} + (1 - \gamma) \overbrace{x_t^2}^{y_{Ht} - \tilde{y}_{Ht}} \right]
\]

s.t.

\[
\beta b_t^* = b_{t-1}^* - z_t + \kappa \tilde{q}_t,
\]

\[
\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*),
\]

\[
\sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1}), \quad \tilde{q}_t = a_t - \tilde{c}_{Ft}
\]

- **FX policy**: can no longer implement \( z_t = 0 \), but still focuses on the wedge in the risk-sharing condition

- **Divine coincidence**: if \( a_t = c_t^* \) and follow a random walk, then \( \tilde{q}_t = 0 \) and the MP alone can close the two gaps \( x_t = z_t = 0 \)

- **Monetary policy**: same motives as in the baseline model
Adjusting Prices

- Replace fully sticky prices with Calvo friction
Adjusting Prices

- Replace fully sticky prices with Calvo friction

- Planner’s problem:

\[
\min \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma)(x_t^2 + \alpha \pi_{Nt}^2) \right]
\]

s.t. \( \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}\sigma_t^2 (b_t^* - n_t^* - f_t^*) \)

\( \beta b_t^* = b_{t-1}^* - z_t \)

\( \pi_{Nt} = \kappa x_t + \beta \mathbb{E}_t \pi_{Nt+1} + \nu_t \)

\( \sigma_t^2 = \text{var}_t (\bar{q}_{t+1} - z_{t+1} + x_{t+1} + \pi_{Nt+1}) \)

Divine Coincidence: if \( \nu_t = 0 \), then isomorphic to the baseline model

Markup shocks: the optimal policy does not result in long-term price targeting

\( p_{Nt} \not\rightarrow 0 \) back
Adjusting Prices

- Replace fully sticky prices with Calvo friction
- Planner’s problem:

\[
\min \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma)(x_t^2 + \alpha \pi_{Nt}^2) \right]
\]

s.t. \[ \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b^*_t - n^*_t - f^*_t) \]

\[ \beta b^*_t = b^*_{t-1} - z_t \]

\[ \pi_{Nt} = \kappa x_t + \beta \mathbb{E}_t \pi_{Nt+1} + \nu_t \]

\[ \sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1} + \pi_{Nt+1}) \]

Divine Coincidence: if \( \nu_t = 0 \), then isomorphic to the baseline model

Markup shocks: the optimal policy does not result in long-term price targeting \( p_{Nt} \not\to 0 \)
Adjusting Prices

- Replace fully sticky prices with Calvo friction

- Planner’s problem:

\[
\begin{align*}
\min & \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ x_t^2 + \pi_{Nt}^2 \right] \\
\text{s.t.} & \quad \pi_{Nt} = \kappa x_t + \beta \mathbb{E}_t \pi_{Nt+1} + \nu_t \\
& \quad x_0 + \pi_{N0} = \hat{x}_0
\end{align*}
\]
- Replace fully sticky prices with Calvo friction

- Planner’s problem:

$$\min \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) \delta (\hat{x}_t - \eta \epsilon_t)^2 \right]$$

s.t. $$\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*)$$

$$\beta b_t^* = b_{t-1}^* - z_t$$

$$\sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + \hat{x}_{t+1})$$
Replace fully sticky prices with Calvo friction

Planner’s problem:

$$\min \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) \delta (\hat{x}_t - \eta \epsilon_t)^2 \right]$$

s.t. $\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*)$

$\beta b_t^* = b_{t-1}^* - z_t$

$\sigma_t^2 = \text{var}_t(\bar{q}_{t+1} - z_{t+1} + \hat{x}_{t+1})$

**Divine Coincidence**: if $\nu_t = 0$, then isomorphic to the baseline model
Adjusting Prices

- Replace fully sticky prices with Calvo friction

- Planner’s problem:

\[
\begin{align*}
\min \quad & \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) \delta (\hat{x}_t - \eta \varepsilon_t)^2 \right] \\
\text{s.t.} \quad & \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b^*_t - n^*_t - f^*_t) \\
& \beta b^*_t = b^*_{t-1} - z_t \\
& \sigma_t^2 = \text{var}_t (\bar{q}_{t+1} - z_{t+1} + \hat{x}_{t+1})
\end{align*}
\]

- **Divine Coincidence**: if \( \nu_t = 0 \), then isomorphic to the baseline model

- **Markup shocks**: the optimal policy does not result in long-term price targeting \( p_{Nt} \to 0 \)
Risk-Premium Shocks

- Baseline model focuses on noise-trader shocks
Baseline model focuses on noise-trader shocks

Arbitrageurs as drivers of UIP deviations:

1. Risk-aversion shocks (Gabaix-Maggiori’15):

\[ E_t \Delta z_{t+1} = -\bar{\omega}_t \sigma_t^2 (b^*_t - n^*_t - f^*_t) \]
Risk-Premium Shocks

- Baseline model focuses on noise-trader shocks
- Arbitrageurs as drivers of UIP deviations:

1. **Risk-aversion shocks** (Gabaix-Maggiori’15):

\[ \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}_t \sigma_t^2 (b_t^* - n_t^* - f_t^*) \]

⇒ optimal policy remains largely unchanged
Baseline model focuses on noise-trader shocks

Arbitrageurs as drivers of UIP deviations:

1. **Risk-aversion shocks** (Gabaix-Maggiori’15):

   \[ \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}_t \sigma_t^2 (b^*_t - n^*_t - f^*_t) \]

   ⇒ optimal policy remains largely unchanged

2. **Expectation shocks**

   \[ \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}_t \sigma_t^2 (b^*_t - n^*_t - f^*_t) + \psi_t \]
Risk-Premium Shocks

- Baseline model focuses on noise-trader shocks

- Arbitrageurs as drivers of UIP deviations:
  
  1. **Risk-aversion shocks** (Gabaix-Maggiori’15):

     \[ E_t \Delta z_{t+1} = -\bar{\omega}_t \sigma_t^2 (b_t^* - n_t^* - f_t^*) \]

     \[ \Rightarrow \text{optimal policy remains largely unchanged} \]

  2. **Expectation shocks**

     \[ E_t \Delta z_{t+1} = -\bar{\omega}_t \sigma_t^2 (b_t^* - n_t^* - f_t^*) + \psi_t \]

     \[ \Rightarrow \text{no divine coincidence} \]

     \[ \Rightarrow \text{same optimal policy} \]
Conclusion

- Shall exchange rate be fixed or freely float?
  - with MP and FX available, eliminate output gap and UIP deviation, but not exchange rate volatility
  - nonetheless, do eliminate non-fundamental exchange rate volatility from noise traders
    - possibly the dominant portion of exchange rate volatility and UIP deviations under laissez faire
  - explicit partial peg when FX is unavailable

- Divine coincidence:
  - fix exchange rate with MP

- Without divine coincidence:
  - neither fully fixed nor freely floating is optimal
APPENDIX
Mussa Puzzle Redux

\[ \Delta q_t: \]

Peg \hspace{2cm} Float

⇒ \hspace{1cm} IRBC
(flex prices)

\[ i_t - i^*_t - E_t \Delta e_{t+1} = \chi(\sigma^2_{e_t}) \cdot \psi_t \]
Mussa Puzzle Redux

$\Delta q_t$: Peg

$\Delta c_t$: Float

$\Rightarrow \times \text{ IRBC (flex prices)}$

$\Rightarrow \times \text{ NKOE (sticky prices)}$
Mussa Puzzle Redux

\[ \Delta q_t: \]

\[ \Delta c_t: \]

⇒ \( \times \) IRBC (flex prices)

⇒ \( \times \) NKOE (sticky prices)

\( i_t - i_t^* - E_t \Delta e_{t+1} = \chi(\sigma_e^2) \cdot \psi_t \)

\( \checkmark \) ER Disconnect
\[
i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \chi(\sigma_e^2) \cdot \psi_t
\]
**Lemma**: Let $\tilde{x}$ solve $\max_x F(x)$ s.t. $g(x) = 0$. Then the second-order approximation to the problem is given by

$$\mathcal{L}(dx) \propto \frac{1}{2} dx' \left[ \nabla^2 F(\tilde{x}) + \tilde{\lambda} \nabla^2 g(\tilde{x}) \right] dx,$$

where $\tilde{\lambda}$ is the steady-state values of the Lagrange multipliers.

- **Non-tradable sector** (NK block):

  $$\mathcal{L}_N = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \log C_{Nt} + \lambda_t (A_t L_t - C_{Nt}) \right] \propto -\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left( \frac{c_{Nt} - \tilde{c}_{Nt}}{x_t} \right)^2$$

- ** Tradable sector** (portfolio choice):

  $$\mathcal{L}_T = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \log C_{Tt} + \lambda_t \left( B_{t-1}^* + Y_t - C_{Tt} - \frac{B_t^*}{R^*} \right) \right] \propto -\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left( \frac{c_{Tt} - \tilde{c}_{Tt}}{z_t} \right)^2$$

- **Total welfare**:

  $$\mathcal{L} = \gamma \mathcal{L}_T + (1 - \gamma) \mathcal{L}_N \propto -\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]$$
Discretionary Policy

- Markov problem:

\[ V(b^*, s) = \min_{z, x, b^*'} \gamma z^2 + (1 - \gamma) x^2 + \beta \mathbb{E} V(b'^*, s') \]

s.t. \[ \mathbb{E} z(b'^*, s') = z - \omega \sigma^2 (b'^* - n^*) , \]

\[ \beta b'^* = b^* - z , \]

\[ \sigma^2 = \text{var} \left( \tilde{q}' + x(b'^*, s') - z(b'^*, s') \right) , \]

\[ \Rightarrow \text{path of } \{ z_t, b_t^* \} \text{ is independent of } x_t \]

\[ \Rightarrow \text{optimal policy focuses on closing the output gap} \]
Optimal FX Policy

FX policy problem:

\[
\min_{\{z_t, b_t^*\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t z_t^2 \\
\text{s.t.} \quad \beta b_t^* = b_{t-1}^* - z_t
\]
Optimal FX Policy

- FX policy problem:
  \[
  \min_{\{z_t, b^*_t\}} \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t z_t^2
  \]
  \[s.t. \quad \beta b^*_t = b^*_{t-1} - z_t\]

- Has standard recursive formulation:
  \[
  V(b^*) = \min_{b^*'} \frac{1}{2} (b^* - \beta b^*')^2 + \beta V(b^*')
  \]

Proposition

Optimal FX policy is time consistent and implements efficient risk sharing \(z_t = 0\).
Exchange Rate Regime

Source: Ilzetzki, Reinhart, and Rogoff (2019)
Anchor Currencies

US GDP as a share of world GDP (percent, right scale)

Share of countries where the US dollar is the principal anchor currency (percent, left scale)

Source: Ilzetzki, Reinhart, and Rogoff (2019)