Optimal Exchange Rate Policy

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    b) risk sharing in the financial market ▸ show

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- Develop a rich framework for policy analysis
  — intuitive linear-quadratic policy problem (cf. CGG'99, GM'05)

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4. FX constraints can be relaxed via FX and ER forward guidance
5. Explore possibility of income and losses from FX interventions

Relation to the Literature

- Portfolio models:
  - Segmented markets: Kouri (1976), Blanchard, Giavazzi & Sa (2005), Alvarez, Atkeson & Kehoe (2002, 2009), Pavlova & Rigobon (2008), Vutz (2020), Jeanne & Rose (2002), Gabaix & Maggiori (2015), Gourinchas, Ray & Vayanos (2021), Itskhoki & Mukhin (2021)
  - Currency crisis: Krugman (1979), Morris & Shin (1998), Fornaro (2021)
- Optimal policy in open economy:
  - Monetary policy: Obstfeld & Rogoff (1995), Clarida, Gali & Gertler (1999, 2001, 2002), Devereux & Engel (2003), Benigno & Benigno (2003), Gali & Monacelli (2005), Engel (2011), Goldberg & Tille (2009), Corsetti, Dedola & Leduc (2010, 2018), Fanelli (2018), Egorov & Mukhin (2021)
  - Capital controls: Jeanne & Korinek (2010), Bianchi (2011), Farhi & Werning (2012, 2013, 2016, 2017), Costinot, Lorenzoni & Werning (2014), Schmitt-Grohe & Uribe (2016), Basu, Boz, Gopinath, Roch & Unsal (2020)
  - FX interventions: Jeanne (2013), Cavallino (2019), Amador, Bianchi, Bocola & Perri (2016, 2020), Fanelli & Straub (2021)

SETUP

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- SOE with T and NT, segmented asset markets
- **Households**:

$$\max \ \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big[\gamma \log C_{Tt} + (1-\gamma)(\log C_{Nt} - L_t) \Big]$$

$$\text{s.t.} \quad \frac{B_t}{R_t} + P_{Tt} C_{Tt} + P_{Nt} C_{Nt} = B_{t-1} + W_t L_t + \Pi_t + T_t$$

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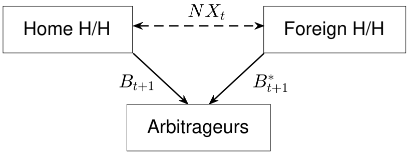

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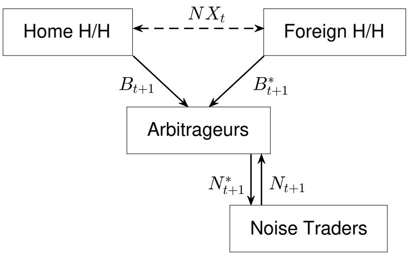

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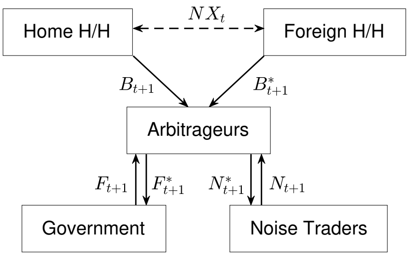

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- **Financial sector**:

— incomplete asset markets

— segmented markets w/ risk-averse arbitrageurs

$$\underbrace{B_t}_{\text{h/h}} + \underbrace{N_t}_{\text{noise traders}} + \underbrace{F_t}_{\text{FX interventions}} + \underbrace{D_t}_{\text{arbitrageurs}} = 0$$

Non-Tradable Sector

- Equilibrium conditions:
  - labor supply

$$C_{Nt} = \frac{W_t}{P_{Nt}}$$

  - market clearing

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- First-best = flexible prices:

$$C_{Nt} = \frac{W_t}{P_{Nt}} = \frac{W_t}{W_t / A_t} = A_t$$

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$$P_{Nt} = 1 \qquad \Rightarrow \qquad \text{output gap} \quad x_t \equiv \log \frac{C_{Nt}}{\tilde{C}_{Nt}} = \log \frac{C_{Nt}}{A_t}$$

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- Exchange rate:
  $$\boxed{e_t = \tilde{q}_t + x_t - z_t}$$
  - Consumption wedge for T: $z_t \equiv \log \frac{C_{Tt}}{\tilde{C}_{Tt}}$
  - Efficient RER: $\tilde{q}_t = a_t - \tilde{c}_{Tt}$

Tradable Sector

- Arbitrageurs choose portfolio (D_t, D_t^*) w/
  - zero net positions $\frac{D_t}{R_t} + \frac{\mathcal{E}_t D_t^*}{R_t^*} = 0$,
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- **Lemma**: To the first-order approximation, the optimal policy solves ▸ proof

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   - limits to arbitrage (cf. ABBP'20, Fanelli-Straub'21)
   - distortionary n_t^* shocks
   - two channels of monetary policy

TWO POLICY INSTRUMENTS

Optimal Policy

- Planner's problem:

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Proposition (First best)

The optimal policy implements the first best: i) MP close the output gap $x_t = 0$, ii) FX interventions eliminate the risk-sharing wedge $f_t^* = b_t^* - n_t^*$.

Optimal Policy

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Proposition (First best)

The optimal policy implements the first best: i) MP close the output gap $x_t = 0$, ii) FX interventions eliminate the risk-sharing wedge $f_t^* = b_t^* - n_t^*$.

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3. **Responses to shocks**: FX policy offsets n_t^* and accommodates $\tilde{q}_t$
   - $\tilde{q}_t$ depends on a_t, y_{Tt}, r_t^*
   - unobservable $\tilde{q}_t$ and n_t^* (cf. potential output, NAIRU, natural rate)

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   — optimal currency areas (Mundell'61)

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- More generally, the optimal monetary rule is

$$(1-\gamma)\underbrace{x_t}_{\text{output gap}} = -\gamma\bar{\omega}\underbrace{\lambda_{t-1}(b^*_{t-1} - n^*_{t-1})}_{\geq 0}\left(\underbrace{e_t - \mathbb{E}_{t-1}e_t}_{\text{ER volatility}}\right),$$

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6. **Time consistency**: optimal *discretionary* policy closes output gap $x_t = 0$

▸ Markov problem

FX POLICY

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  — cf. macroprudential policy under AD externality (Farhi-Werning'16)

7. **FX trade-off**: FX interventions are unlikely to mitigate output gap
8. **Time consistency**: FX policy does not require commitment ▸ show

INTERNATIONAL SPILLOVERS

International Spillovers

- Global equilibrium:
  - continuum of SOEs trading dollar bonds
  - unchanged risk sharing condition

$$\mathbb{E}_t \Delta z_{it+1} = -\bar{\omega}\sigma_{it}^2 \left(b_{it}^* - n_{it}^* - f_{it}^*\right)$$

  - endogenous p_{Tt} and r_t^*

$$r_t^* \equiv i_t^* - \mathbb{E}_t \Delta p_{Tt+1}, \qquad \int c_{Tit} \mathrm{d}i = \int y_{Tit} \mathrm{d}i$$

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9. **Gains from cooperation**:
   i) first-best policies $\Rightarrow$ NE is efficient
   ii) second-best policies $\Rightarrow$ negative spillovers

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10. **Anchor currency**: countries import U.S. MP under second-best policy

$$e_{it} = \tilde{q}_{it} - p_{Tt} + x_{it} - z_{it}$$

  — currency of debt $\Rightarrow$ anchor/reserve currency ▸ IRR'2019
  — cf. *gold standard* with $i_t^* = 0$ and p_{Tt} determined from $c_{Tt} = y_{Tt}$

EXTENSIONS

Extensions

Extensions relax assumptions of the baseline model:

1. Home traders → int'l transfers ▸ show
2. T and NT goods → ToT effects ▸ show
3. Fully sticky prices → NKPC ▸ show
4. Noise traders → risk-premium shocks ▸ show
5. Log-linear preferences → complementarities

Transfers

- No redistributional effects in the baseline model
- Assume **foreign** arbitrageurs and noise traders
- Country's budget constraint:

$$\frac{B_t^*}{R_t^*} = B_{t-1}^* + Y_{Tt} - C_{Tt} - \mathcal{T}_t \left(N_{t-1}^* + \frac{\mathbb{E}_{t-1}\mathcal{T}_t}{\omega \sigma_t^2} \right), \quad \mathcal{T}_t \equiv \frac{R_{t-1}}{R_t^*} \frac{\mathcal{E}_{t-1}}{\mathcal{E}_t} - 1$$

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- Loss function depends on UIP deviations $\tau_{t-1} \equiv r_t - r_t^* - \mathbb{E}_{t-1}\Delta e_t$:

$$\mathcal{L} = \frac{1}{2}\mathbb{E}\sum_{t=0}^{\infty}\beta^t \left[\gamma z_t^2 + (1-\gamma)x_t^2 + 2\gamma\tau_{t-1}\left(n_{t-1}^* + \frac{\tau_{t-1}}{\bar{\omega}\sigma_t^2} \right) \right]$$

  - extends Fanelli-Straub'21 to stochastic shocks
  - to the SOA, welfare depends on **ex-ante** UIP deviations
  - if local noise traders, $n_t^* = 0$ in $\mathcal{L}$ and any $\tau_t \neq 0$ lower the welfare
  - if $n_t^* \neq 0$, the planner can extract rents

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  — commitment is important, but differently from the NK models

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- Currency of invoicing:
  1. producer (PCP) = sticky wages
  2. dominant (DCP)

Optimal Policy: PCP

- Planner's problem under **PCP**:

$$\min_{\{z_t, x_t, b_t^*, f_t^*, \sigma_t^2\}} \quad \frac{1}{2}\, \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big[\kappa \underbrace{z_t^2}_{c_{Ft} - \tilde{c}_{Ft}} + \underbrace{x_t^2}_{y_t - \tilde{y}_t} \Big]$$

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- **One instrument**: neither f_t^* nor $\sigma_t^2 = 0$ are sufficient to implement $z_t = 0$ because of suboptimal exports

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Adjusting Prices

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$$x_0+\pi_{N0}=\hat{x}_0$$

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- **Divine Coincidence**: if $\nu_t = 0$, then isomorphic to the baseline model
- **Markup shocks**: the optimal policy does not result in long-term price targeting $p_{Nt} \nrightarrow 0$ back

Risk-Premium Shocks

- Baseline model focuses on noise-trader shocks

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- Arbitrageurs as drivers of UIP deviations:

  1. Risk-aversion shocks (Gabaix-Maggiori'15):

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  2. Expectation shocks

  $$\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*) + \psi_t$$

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$$\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (b_t^* - n_t^* - f_t^*) + \psi_t$$

     $\Rightarrow$ no divine coincidence

     $\Rightarrow$ same optimal policy

back

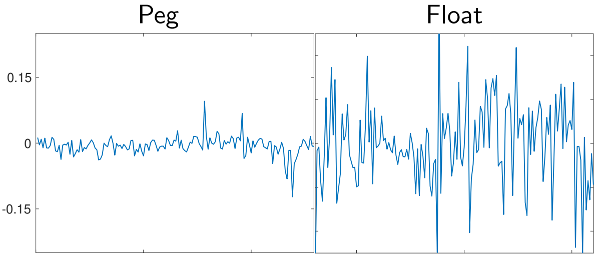
Conclusion

- Shall exchange rate be fixed or freely float?
  - with MP and FX available, eliminate output gap and UIP deviation, but not exchange rate volatility
  - nonetheless, do eliminate non-fundamental exchange rate volatility from noise traders
    - possibly the dominant portion of exchange rate volatility and UIP deviations under laissez faire
  - explicit partial peg when FX is unavailable
- Divine coincidence:
  - fix exchange rate with MP
- Without divine coincidence:
  - neither fully fixed nor freely floating is optimal

APPENDIX

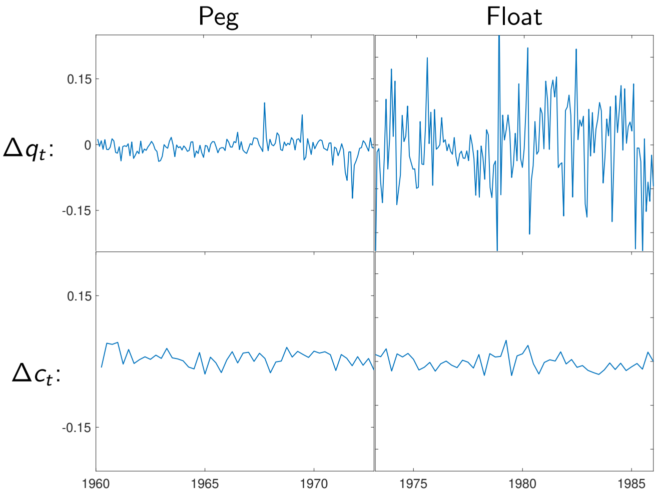
Mussa Puzzle Redux

Δq_t:



$\Rightarrow$ ✗ IRBC (flex prices)

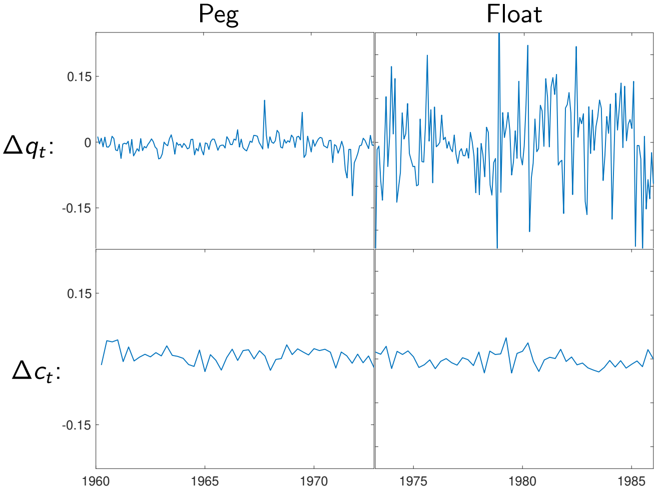
Mussa Puzzle Redux



⇒ ✗ IRBC (flex prices)

⇒ ✗ NKOE (sticky prices)

Mussa Puzzle Redux



$\Rightarrow$ ✗ IRBC (flex prices)

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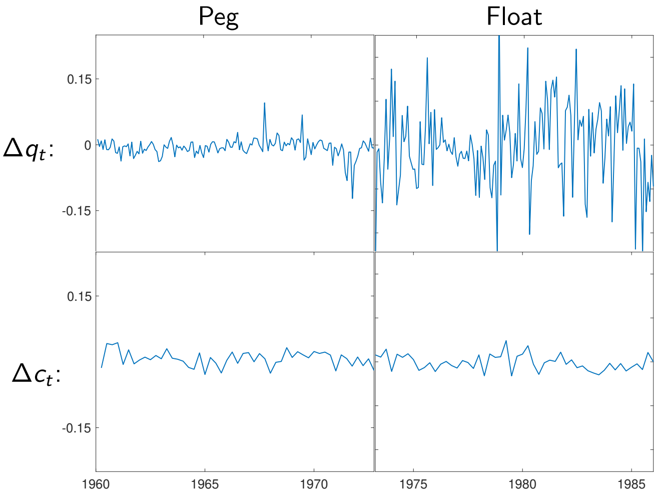
$\Downarrow$

✓ ER Disconnect

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \chi(\sigma_e^2) \cdot \psi_t$$

Mussa Puzzle Redux



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back

Quadratic Loss Function

- **Lemma**: Let $\tilde{x}$ solve $\max_x \ F(x)$ s.t. $g(x) = 0$. Then the second-order approximation to the problem is given by

$$\mathcal{L}(dx) \propto \frac{1}{2} dx' \left[\nabla^2 F(\tilde{x}) + \bar{\lambda} \nabla^2 g(\tilde{x})\right] dx,$$

where $\bar{\lambda}$ is the steady-state values of the Lagrange multipliers.

- Non-tradable sector (NK block):

$$\mathcal{L}_N = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\log C_{Nt} + \lambda_t \left(A_t L_t - C_{Nt} \right) \right] \propto -\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big(\underbrace{c_{Nt} - \tilde{c}_{Nt}}_{x_t} \Big)^2$$

- Tradable sector (portfolio choice):

$$\mathcal{L}_T = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\log C_{Tt} + \lambda_t \left(B^*_{t-1} + Y_t - C_{Tt} - \frac{B^*_t}{R^*} \right) \right] \propto -\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big(\underbrace{c_{Tt} - \tilde{c}_{Tt}}_{z_t} \Big)^2$$

- Total welfare:

$$\mathcal{L} = \gamma \mathcal{L}_T + (1-\gamma) \mathcal{L}_N \propto -\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\gamma z_t^2 + (1-\gamma) x_t^2 \right]$$

back

Discretionary Policy

- Markov problem:

$$V(b^*, s) = \min_{z,x,b^{*\prime}} \quad \gamma z^2 + (1-\gamma)x^2 + \beta \mathbb{E} V(b^{*\prime}, s')$$

$$\text{s.t.} \quad \mathbb{E}z(b^{*\prime}, s') = z - \omega\sigma^2\left(b^{*\prime} - n^*\right),$$

$$\beta b^{*\prime} = b^* - z,$$

$$\sigma^2 = \mathrm{var}\Big(\tilde{q}' + x(b^{*\prime}, s') - z(b^{*\prime}, s')\Big),$$

$\Rightarrow$ path of $\{z_t, b_t^*\}$ is independent of x_t

$\Rightarrow$ optimal policy focuses on closing the output gap

back

Optimal FX Policy

- FX policy problem:

$$\min_{\{z_t, b_t^*\}} \quad \frac{1}{2}\,\mathbb{E}\sum_{t=0}^{\infty}\beta^t z_t^2$$

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- Has standard recursive formulation:

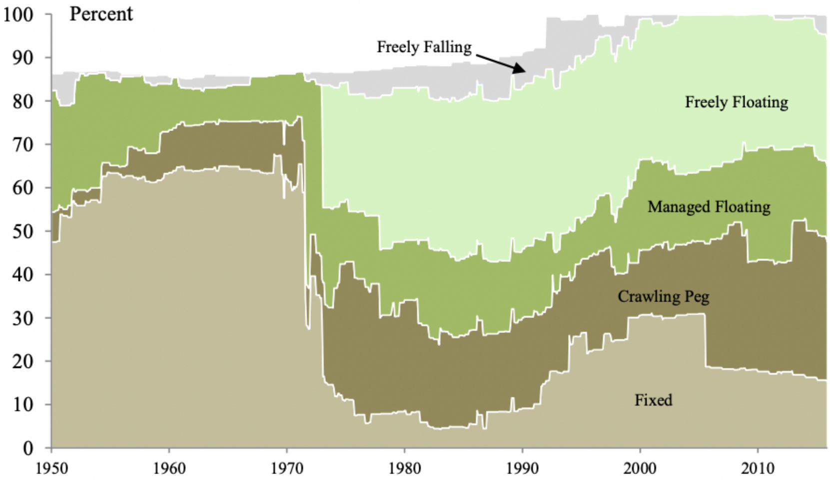
$$V(b^*) = \min_{b^{*\prime}} \, \frac{1}{2} \left(b^* - \beta b^{*\prime} \right)^2 + \beta V(b^{*\prime})$$

Proposition

Optimal FX policy is time consistent and implements efficient risk sharing $z_t = 0$.

back

Exchange Rate Regime



Source: Ilzetzki, Reinhart, and Rogoff (2019)

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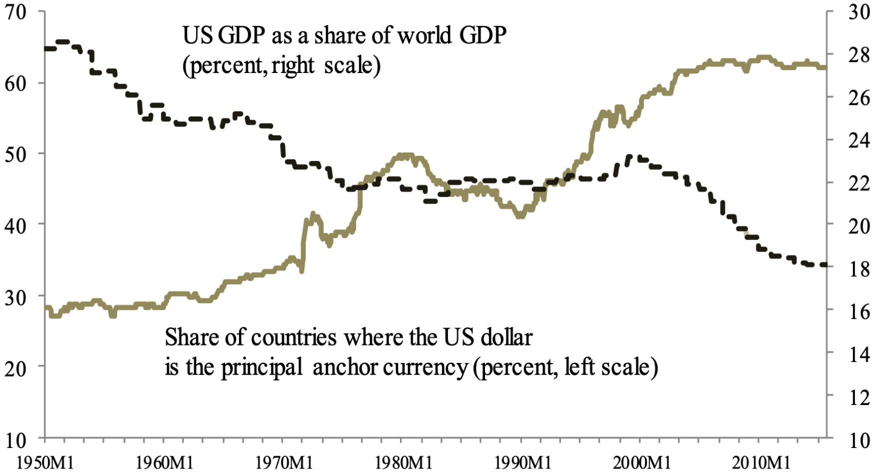
Anchor Currencies



Source: Ilzetzki, Reinhart, and Rogoff (2019) back