Global demand for financial assets, falling real interest rates and macroeconomic instability

Enrique G. Mendoza
Univ. of Pennsylvania and NBER

Vincenzo Quadrini
Univ. of Southern California and CEPR

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Abstract

The sharp, secular decline in the world real interest rate of the past thirty years suggests that the observed surge in global demand for financial assets outpaced the growth in supply. We argue that this phenomenon was driven by (i) faster growth in emerging market economies, and (ii) changes in the financial structure of both emerging and advanced economies. We then show that the low-interest-rate environment made the world economy more vulnerable to financial crises. These findings are derived quantitatively using a two-region model in which financial assets provide direct services to production and private debt can be defaulted on.

1 Introduction

Four key facts illustrated in Figure 1 highlight major changes in the world economy during the last three decades. First, emerging economies grew significantly faster than advanced economies. As shown in the first panel, the GDP of emerging economies relative to that of advanced economies, measure in US dollars, rose from 28 to 68 percent between 1990 and 2020. Valuing GDP in PPP units, instead, yields an increase from 57 to 125 percent. Thus, the growth in the relative size of emerging economies is evident even setting aside real-exchange-rate movements.

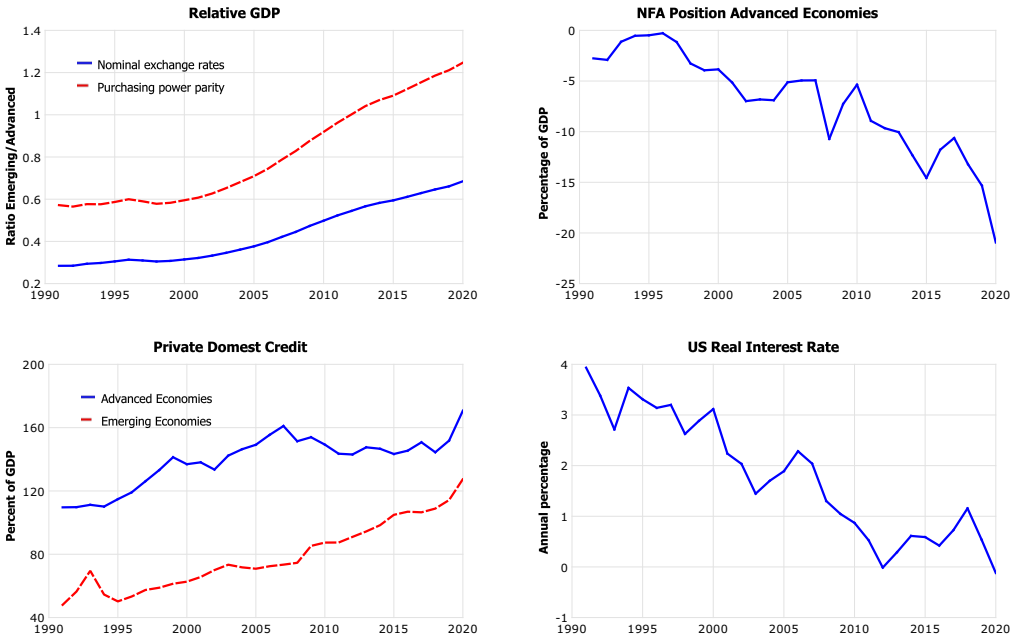

Figure 1: Real and Financial Trends in Advanced and Emerging Countries.

Note: **Emerging economies**: Argentina, Brazil, Bulgaria, Chile, China, Hong.Kong, Colombia, Estonia, Hungary, India, Indonesia, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Advanced economies**: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United.Kingdom, United.States. **Sources**: World Development Indicators (World Bank) and External Wealth of Nations database (Lane and Milesi-Ferretti (2018)).

The second fact, often labeled 'global imbalances', is the growth in net foreign liabilities of advanced economies. As the second panel of Figure 1 shows, the net foreign assets (NFA) of advanced economies, as a share of their collective GDP, fell from close to zero at the beginning of the 1990s to about -20 percent in 2020.

The third fact is that the financial structure of both emerging and advanced economies changed so as to produce significant growth in credit to the private sector. The third panel of Figure 1 shows that private domestic credit as a percentage of GDP roughly tripled in EMs in the last 30 years and grew about half as much (by a factor of 1.5) in advanced economies. Thus, although domestic credit as a share of GDP in EMs remains below that of advanced economies, the gap has narrowed markedly.

This significant growth in financial intermediation worldwide could be driven by both the growth in demand for financial assets and/or the growth in supply (i.e., issuance of liabilities). Whether the growth in demand outpaces the growth in supply or vice-versa is important for determining the direction of the response of the equilibrium interest rate, which brings us to the last key fact.

The fourth panel of Figure 1 plots the ex-post real interest rate on U.S. long-term public debt, a proxy for the risk-free world interest rate.[1] Starting from about 4 percent at the beginning of the 1990s, the real interest rate followed a declining trend reaching values close to zero at the end of 2020. Measures of expected real interest rates based on inflation expectations embedded in the pricing of inflation-indexed treasury bills also show significant declines. The market yield on 10-year U.S. TIPS at constant maturity fell from 2.29 percent in January 2003 to -1 percent at the end of 2020. The sharp drop in real interest rates suggests that the global demand for financial assets has increased at a faster pace than the supply.

The trends in the world economy documented in Figure 1 emerged during a period marked by financial globalization and a surge in the occurrence of financial crises. Well-established measures of de-jure and de-facto international capital mobility show the rapid progress of financial globalization as barriers to capital mobility were sharply reduced (see Chinn and Ito (2006)) and both gross external assets and liabilities grew in a large number of coun-

[1]The market yield is from FRED available at fred.stlouisfed.org/series/DFII10. The Cleveland Fed's expected real interest rate is available at www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations.aspx

tries (see Lane and Milesi-Ferretti (2007)).[2] The increase in the frequency of financial crises is documented in well-known empirical studies (e.g. Reinhart and Rogoff (2009)). They show that there were no financial crises in advanced economies between 1940 and 1973 and only a handful between 1973 and 1990. Since then, between 15 and 20 crises have occurred, depending on the study one considers. Crises in emerging economies were also rare between 1940 and the onset of the sovereign debt crises of the 1980s, and the number of crises rose sharply after 1990 (see the survey by Sufi and Taylor (2021)).

This paper has two main goals. The first is to identify and measure the factors that caused the rise in net demand for financial assets, relatively to the growth in supply, and the drop in the world real interest rate. The second is to assess the implications of these changes for global financial and macroeconomic volatility. We do that through the lenses of a two-region quantitative model, one representative of emerging economies and the other representative of advanced economies.

In each region there is a borrowing sector and a lending sector. Financial assets have features that make them akin to 'inside money' or to assets that embody a 'convenience yield' to the holders of the assets—the creditors. We formalize this parsimoniously by assuming that financial assets can be used as inputs in the production of goods. The issuers of financial assets—the debtors—cannot commit to repay and, as a result, private debt can be defaulted on.

A financial crisis occurs when the debt repayment is lower than the liquidation value of the debtors' real assets. This generates haircuts in credit recovery and, therefore, a financial crisis causes wealth redistribution from creditors to debtors. This redistribution of wealth is the central mechanism that causes real macroeconomic consequences. Importantly, the magnitude of the macroeconomic consequences depend on the changing structure of financial intermediation, which in the model is driven by exogenous structural changes as well as endogenous general equilibrium adjustments.

We consider three exogenous changes in each of the two regions: (i) productivity, (ii) structural parameter that affects the demand for financial assets, (iii) structural parameter that affects the supply of financial assets. We then use the model in conjunction with the data plotted in Figure 1 to iden-

[2]The latest update of the Chinn-Ito Index of financial openness is available at web.pdx.edu/ito/Chinn-Ito_website.htm and the latest update of the Lane-Milesi-Ferretti External Wealth of Nations database is available at www.brookings.edu/research/the-external-wealth-of-nations-database.

tify and measure these changes over the sample period. The final step is to assess their contribution to macroeconomic and financial volatility through counterfactual simulations.

We find that the measured exogenous changes in productivity and financial structure both contributed to increase macroeconomic and financial volatility over the 1991-2020 period. These exogenous changes raised the demand for financial assets relatively to the supply, causing the decline in the interest rate. The lower interest rate then encouraged higher borrowing and led to higher effective leverages. It is the higher leverage that increased financial and macroeconomic volatility.

The observed reduction in the interest rate and the dynamics of the NFA positions are key for the identification of the financial changes. As mentioned above, the expansion of the financial sector could be driven by both a higher relative demand for financial assets or a higher relative supply. The reduction in the interest rate, however, indicates that the worldwide growth in demand exceeded the growth in supply (at a fixed interest rate). NFA dynamics are important for determining in which country the demand for financial assets grew more than the supply. In particular, the fact that the net liabilities of advanced economies widened over the sample period indicates that the net demand for assets in these countries increased less than in emerging economies.

The organization of the paper is as follows. Section 2 describes the model and characterizes the equilibrium. Section 3 uses the model in conjunction with the data plotted in Figure 1 to construct empirical series for productivity and exogenous variables that affect the demand and supply of assets. We then show the quantitative implications of these changes. Section 4 concludes.

Related literature. Our work is related to three important strands of literature. On the side of international macroeconomics, there is the literature on global imbalances and the literature on financial crises or Sudden Stops. On the side of corporate finance, there is the literature on the growth of financial assets or cash held by nonfinancial corporations.

The global imbalances literature proposes several theories that provide explanations for the positive NFA positions of emerging economies. One explanation is based on the idea that emerging econmies have a lower ability to create viable saving instruments for inter-temporal smoothing (Caballero, Farhi, and Gourinchas (2008)). Another explanation is based on the idea

that emerging economies have a higher demand for assets due to lower insurance, or lower financial development related to weaker enforcement (Mendoza, Quadrini, and Ríos-Rull (2009)) or because of higher idiosyncratic uncertainty (Carroll and Jeanne (2009), Angeletos and Panousi (2011), Song, Storesletten, and Zilibotti (2011), Sandri (2014), Bacchetta and Benhima (2015), Fogli and Perri (2015)). The first theory highlights cross-country heterogeneity in the supply of assets while the second emphasizes heterogeneity in the demand. In both cases, emerging economies turn to advanced economies for the acquisition of saving instruments (financial assets).

Our model incorporates both types of heterogeneity between advanced and emerging economies. Importantly, the aim of our paper is not to examine why advanced economies have been borrowing from emerging economies, which is the focus of the above referenced studies. Our focus, instead, is on two issues that are relatively new to this literature: First, 'measuring' how the heterogeneity in both demand and supply has changed over time. Second, how the change has affected macroeconomic and financial 'stability'.

Various studies in the Sudden Stops literature examine the role of financial globalization, credit booms and high leverage as causing factors of financial crises (for example, Calvo and Mendoza (1996), Caballero and Krishnamurthy (2001), Gertler, Gilchrist, and Natalucci (2007), Edwards (2004), Mendoza and Quadrini (2010), Mendoza and Smith (2014), Fornaro (2018)).[3] Some of these studies emphasize mechanisms that cause financial crises because of equilibrium multiplicity due to self-fulfilling expectations (e.g., Aghion, Bacchetta, and Banerjee (2001), Perri and Quadrini (2018)). Crises in our model also follow from periods of fast credit and leverage growth, and they are also the result of self-fulfilling expectations. However, the mechanism that operates in our model differs in that it relies on the interaction between the inside-money-like role of financial assets for creditors with the debtors' lack of commitment to repay which could lead to debt renegotiation.

Several studies in the corporate finance literature document and provide explanations for the raising demand of financial assets. An example is the literature on the growing cash holdings of nonfinancial businesses (e.g., Busso, Fernández, and Tamayo (2016) and Bebczuk and Cavallo (2016)). Our model has a similar feature in that entrepreneurs hold positive positions in financial assets that expand as a result of faster growth of emerging economies and changes in financial structure in both emerging and advanced economies. Our

[3]See Bianchi and Mendoza (2020) for a survey of the literature.

focus, however, is on the macroeconomic implications. Through the lenses of the structural model we show that the increase in net demand for financial assets depresses the interest rate which in turn increases the incentives to leverage. While the higher leverage allows for sustained levels of financial intermediation and economic activity, it also makes the economies of emerging and advanced economies more vulnerable to crises (global macroeconomic instability).

2 Model

There are two countries/regions indexed by $j \in \{1, 2\}$. The first country is representative of advanced economies and the second is representative of emerging economies. In each country there are two sectors: (i) an entrepreneurial sector that produces final output, (ii) a consolidated household/business sector that holds capital and supplies labor. By having two sectors we can generate borrowing and lending. This allows us to have, in each country, a clear distinction between the 'demand' of financial assets (from the sector that has a positive financial position) and the 'supply' of financial assets (from the sector that has a negative financial position).[4]

Countries are heterogeneous in three dimensions: (i) economic size formalized by differences in aggregate productivity, $z_{j,t}$; and (ii) financial market structure that affects directly the 'demand' of financial assets, captured by the parameter $\phi_{j,t}$; (iii) financial market structure that affects directly the 'supply' of financial assets, captured by the parameter $\kappa_{j,t}$.

Although differences in economic size could be generated in the model by other factors, besides productivity (for example, population, real exchange rates, etc.), for the questions addressed in the paper, the other factors are isomorphic to productivity differences. This will become clear in the quantitative section. Productivity $z_{j,t}$ and financial parameters $\phi_{j,t}$ and $\kappa_{j,t}$ are time varying but not stochastic. Their changes over time are fully anticipated. The only source of uncertainty in the model derives from sunspot shocks as described below.

[4]We can interpret the business sector that is consolidated with the household sector as composed of firms that hold physical capital with high collateral value. In that sense, these firms are similar to households holding real estates. High collateral value allows both households and firms to borrow. We could have kept the household sector separate from the business sector but this would not make any important difference for the key properties of the model highlighted in this paper.

2.1 Entrepreneurial sector

In each country there is a unit mass of atomistic entrepreneurs that maximizes the expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_{j,t}),$$

where $c_{j,t}$ is consumption in country j at time t.

Entrepreneurs are business owners producing a single good with technology described below. Although the model is presented as if final production is carried out by privately owned businesses, we should think of the entrepreneurial sector broadly, that is, as also including public companies. Then, entrepreneurial consumption can be interpreted as dividend payments and the concavity of the utility function can be thought as reflecting the risk aversion of managers and/or major shareholders. The concavity could also reflect, in reduced form, the cost associated with financial distress: even if shareholders and managers are risk-neutral, a convex cost of financial distress would make the objective of the business concave. Since entrepreneurs are homogeneous, we can focus on the representative entrepreneur.

The production technology operated by entrepreneurs takes the form

$$y_{j,t} = z_{j,t}^{\gamma} m_{j,t}^{\alpha} l_{j,t}^{\gamma} k_{j,t}^{1-\alpha-\gamma}, \tag{1}$$

where $z_{j,t}$ is total factor productivity, $m_{j,t}$ is the financial wealth of the entrepreneur, $l_{j,t}$ is the input of labor, and $k_{j,t}$ is physical capital rented from households/firms.

Financial wealth provides working capital that is complementary to other production factors. The long-run growth rate of productivity $g - 1$ in both countries. In the short-run, however, there could be significant deviations from the long-run growth, especially in emerging economies.

Production also carries a cost $\phi_{j,t} m_{j,t}$. The cost captures expenses that increase with the production scale, as determined by the input $m_{j,t}$. One way to think about this cost is that it derives from the depreciation of capital: larger is the scale of production and larger are the expenses required to replenish the depreciated capital due to more intensive utilization. The time-varying parameter $\phi_{j,t}$ is exogenous and it plays an important role in determining the demand for financial assets. As we will see, lower is the value of $\phi_{j,t}$, and higher is the incentive for entrepreneurs to hold $m_{j,t}$.

Entrepreneurs have access to a market for bonds traded at price $q_{j,t}$. In equilibrium, the bonds held by entrepreneurs are liabilities issued by households. Even if there is capital mobility, the prices of bonds issued by the two countries differ because they are characterized by repayment risks that are specific to each of the two countries.

The representative entrepreneur in country j enters period t with bonds issued by country 1, $b_{1j,t}$, and bonds issued by country 2, $b_{2j,t}$. The first subscript denotes the country that issued the bond while the second subscript denotes the residency of the entrepreneur. In the event of a financial crisis, the entrepreneur incurs financial losses proportional to the owned bonds.

Denote by $\delta_{1,t}$ and $\delta_{2,t}$ the repayment fractions realized at the beginning of the period on bonds issued, respectively, by country 1 and country 2. The residual values of the two bonds are then $\delta_{1,t}b_{1j,t}$ and $\delta_{2,t}b^i_{2j,t}$. The repayment fractions $\delta_{1,t}$ and $\delta_{2,t}$ are endogenous stochastic variables determined in general equilibrium. Given the realization of these two variables, the after-default wealth of the entrepreneur is

$$m_{j,t} = \delta_{1,t}b_{1j,t} + \delta_{2,t}b_{2j,t}.$$

This is the financial wealth that enters the production function (1). In addition, the entrepreneur hires labor at the wage rate $w_{j,t}$ and rents physical capital from households at the rental rate $r_{j,t}$. The end-of-period wealth, after production, is

$$a_{j,t} = (1-\phi_{j,t})m_{j,t} + z^{\gamma}_{j,t}m^{\alpha}_{j,t}l^{\gamma}_{j,t}k^{1-\alpha-\gamma}_{j,t} - w_{j,t}l_{j,t} - r_{j,t}k_{j,t}.$$

Wealth is in part allocated to consumption, $c^i_{j,t}$, and in part to new bonds, $q_{1,t}b_{1j,t+1}$ and $q_{2,t}b_{2j,t+1}$. The budget constraint is

$$c_{j,t} + q_{1,t}b_{1j,t+1} + q_{2,t}b_{2j,t+1} = a_{j,t}. \tag{2}$$

While the input of labor $l_{j,t}$ depends on $m_{j,t}$, the portfolio decisions, $b_{1j,t+1}$ and $b_{2j,t+1}$, are functions of $a_{j,t}$. To make the timing of the model precise, we can think of a period as divided in three subperiods:

1. **Subperiod 1**: Entrepreneurs enter with financial assets $b_{1j,t}$ and $b_{2j,t}$, and observe the country-specific repayment fractions $\delta_{1,t}$ and $\delta_{2,t}$. The repayment fractions bring the residual wealth to $m_{j,t} = \delta_{1,t}b_{1j,t}+\delta_{2,t}b_{2j,t}$.

2. **Subperiod 2**: Given the residual wealth $m_{j,t}$, the entrepreneur chooses the inputs of labor $l_{j,t}$ and capital $k_{j,t}$. Market clearing determines the wage and rental rates $w_{j,t}$ and $r_{j,t}$.

3. **Subperiod 3**: The end-of-period wealth $a_{j,t}$ is in part consumed, $c_{j,t}$, and in part saved in bonds issued by country 1, $q_{1,t}b_{1j,t+1}$, and in bonds issued by country 2, $q_{2,t}b_{2j,t+1}$.

The following lemma characterizes the production decision (Subperiod 2) and the optimal portfolio decision (Subperiod 3).

Lemma 2.1 *The optimal entrepreneur's policies are*

$$\begin{aligned}
l_{j,t} &= z_{j,t}^{\frac{\gamma}{\alpha}} \left(\frac{\gamma}{w_{j,t}}\right)^{\frac{\alpha+\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{r_{j,t}}\right)^{\frac{1-\alpha-\gamma}{\alpha}} m_{j,t}, \\
k_{j,t} &= z_{j,t}^{\frac{\gamma}{\alpha}} \left(\frac{\gamma}{w_{j,t}}\right)^{\frac{\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{r_{j,t}}\right)^{\frac{1-\gamma}{\alpha}} m_{j,t}, \\
c_{j,t} &= (1-\beta)a_{j,t}^{i}, \\
q_{1,t}b_{1j,t+1} &= \beta\theta_t a_{j,t}, \\
q_{2,t}b_{2j,t+1} &= \beta(1-\theta_t)a_{j,t},
\end{aligned}$$

where θ_t solves the first order condition

$$\mathbb{E}_t \left\{ \frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_t \frac{\delta_{1,t+1}}{q_{1,t}} + (1-\theta_t)\frac{\delta_{2,t+1}}{q_{2,t}}} \right\} = 1.$$

Proof 2.1 *See Appendix A.*

The demand for labor is linear in financial wealth $m_{j,t}^{i}$. The proportional term depends positively on productivity, $z_{j,t}$, and negatively on the wage rate, $w_{j,t}$. Similarly, the demand for capital is linear in $m_{j,t}^{i}$, with the proportional term increasing in productivity $z_{j,t}$, and decreasing in the rental rate $r_{j,t}$.

Lemma 2.1 also indicates that entrepreneurs allocate their end-of-period wealth between consumption and saving according to the fixed factor β. This property derives from the log specification of the utility function. Finally, a fraction θ_t of savings is allocated to bonds issued by country 1 and the remaining fraction $1-\theta_t$ to bonds issued by country 2. The fraction θ_t

changes over time. However, it is the same for entrepreneurs of country 1 and country 2. This is indicated by the fact that θ_t does not have the j subscript. Therefore, entrepreneurs in both countries choose the same portfolio allocation between bonds issued by country 1 and bonds issued by country 2. Notice that, because θ_t is the same for the two countries, the last two conditions in the lemma are not equation identities.

2.2 Consolidated households/firms sector

In each country there is a consolidated sector with a unit mass of homogeneous households/firms. The reason we include some firms in the consolidated sector is to distinguish them from firms in the entrepreneurial sector (described in the previous subsection). We think of this second type of firms as large owners of collateralizable assets (capital). In this sense they are similar to households who are also own large collateralizable assets in the form of residential assets. Entrepreneurial firms, instead, are more representative of businesses that own few collateralizable assets (zero for simplicity in the model). As we will see, the consolidated sector will borrow in equilibrium while the entrepreneurial sector will lend.

Households/firms maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_{j,t} - z_{j,t} \frac{l_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right),$$

where $c_{j,t}$ is consumption/dividends, $l_{j,t}$ is the supply of labor and ν is the elasticity of labor supply.

The assumption that households/firms in the consolidated sector have linear utility in consumption/dividends simplifies the characterization of the equilibrium. It allows us to derive analytical results without affecting the key properties of the model. The dependence of the dis-utility from country-specific productivity $z_{j,t}$ guarantees balanced growth.

Consolidated households/firms hold $k_{j,t}$ units of capital. To keep the model tractable we assume that the aggregate supply of capital grows exogenously at the same rate as the average long-run growth of the economy, $g-1$. Therefore, capital in both countries evolves over time according to $K_{j,t} = \bar{K} g^t$. We interpret capital broadly including real estates and land. An important assumption is that capital is held by consolidated households/firms, not entrepreneurs. However, households/firms rent the capital

to domestic entrepreneurs at rate $r_{j,t}$. They can also trade the capital among households/firms at the market price $p_{j,t}$.

Debt and default. At the end of period $t-1$, households/firms can borrow $d_{j,t}/R_{j,t-1}$ where $R_{j,t-1}$ is the gross interest rate and $d_{j,t}$ is the 'promised' repayment at time t. At the beginning of time t, however, when the debt $d_{j,t}$ is due, households/firms could default. In the event of default creditors have the right to liquidate the $k_{j,t}$ units of capital owned by a defaulting household/firm. However, the liquidation value at the beginning of period t, when the repayment is due, could be smaller than the loan.

Denote by $\tilde{p}_{j,t}$ the liquidation price at the beginning of period t. The condition that the loan exceeds the liquidation value of capital is $\tilde{p}_{j,t}k_{j,t} \leq d_{j,t}$. This could happen because the economy ends up in a state in which the market for liquidated capital freezes and the liquidation price drops. The mechanism that generates a possible market freeze will be described in the next subsection. For the moment we should think of the liquidation price $\tilde{p}_t$ as a stochastic variable drawn from the probability distribution $f_{j,t}(\tilde{p}_{j,t})$. The probability distribution is endogenous in the model and will be determined in the general equilibrium as described in the next subsection.

It is important to point out that the liquidation price $\tilde{p}_t$ is determined at the beginning of the period and could differ from the market price at the end of the period, which we denote by p_t. This will become clear in the next subsection.

Once $\tilde{p}_{j,t}$ becomes known at the beginning of period t, consolidated households/firms could use the threat of default to renegotiate the outstanding debt $d_{j,t}$. Renegotiation will take place only if the debt is bigger than the liquidation value of the collateral, that is, $d_{j,t} > \tilde{p}_t k_{j,t}$. Under the assumption that borrowers have the whole bargaining power, the debt will be renegotiated to the liquidation value. Thus, the post-renegotiation debt is

$$\tilde{d}(d_{j,t}, \tilde{p}_{j,t}k_{j,t}) = \min \left\{ d_{j,t} \ , \ \tilde{p}_t k_{j,t} \right\} \tag{3}$$

Renegotiation carries a cost that is increasing and convex in the renegotiated value, that is, the difference between the nominal value of the debt and its repayment. Specifically, the cost takes the form,

$$\varphi \left(d_{j,t}, \tilde{p}_{j,t}k_{j,t} \right) = \eta \left[\frac{\max\{ 0 \ , \ d_{j,t} - \tilde{p}_t k_{j,t} \}}{d_{j,t}} \right]^2 d_{j,t}. \tag{4}$$

The cost is zero when the debt is fully repaid, that is, $d_{j,t} < \tilde{p}_{j,t}k_{j,t}$. It becomes positive if the borrower repays less than the nominal value, that is, $d_{j,t} > \tilde{p}_{j,t}k_{j,t}$. In this case the cost increases in the fraction that is not repaid according to a quadratic function.

After renegotiation, the market for capital returns to normal at the end of the period. The equilibrium price p_t at the end of the period could differ from the liquidation price $\tilde{p}_t$ formed at the beginning of the period. Subsection 2.3 provides the micro-foundation for the segmentation between beginning and end-of-period, which allows for the liquidation price $\tilde{p}_t$ to differ from the price at the end of the period p_t.

The assumption of an immediate fresh-start is a simplification that makes the model tractable. Under this assumption, the budget constraint for consolidated households/firms, after renegotiation, is

$$\tilde{d}(d_{j,t}, \tilde{p}_{j,t}k_{j,t}) + \varphi(d_{j,t}, \tilde{p}_{j,t}k_{j,t}) + p_{j,t}k_{j,t+1} + c_{j,t} = w_{j,t}l_{j,t} + r_{j,t}k_{j,t} + p_{j,t}k_{j,t}g + \frac{d_{j,t+1}}{R_{j,t}}.$$

The value of capital is multiplied by g because it grows at the same rate as the long-run growth of productivity. The additional asset is a new endowment added to the budget constraint without incurring any cost.

The gross interest rate $R_{j,t}$ paid by a household/firm depends on the borrowing decision. If the household/firm borrows more, relatively to the ownership of capital, the expected repayment rate could be lower in the next period. This will be reflected in a higher interest rate on the loan.

Denote by $\overline{R}_{j,t}$ the expected gross return from holding the debt issued in period t, and due at $t+1$, by all households/firms in country j. This represents the aggregate expected market return from holding a diversified portfolio of debt. Since households/firms are atomistic and financial markets are competitive, the expected return on the debt issued by an 'individual' household/firm must be equal to the aggregate expected return $\overline{R}_{j,t}$. Thus, the interest rate on the debt issued by an individual household satisfies

$$\frac{d_{j,t+1}}{R_{j,t}} = \frac{1}{\overline{R}_{j,t}} \mathbb{E}_t \tilde{d}(d_{j,t+1}, \tilde{p}_{j,t+1}k_{j,t+1}). \tag{5}$$

The left-hand-side is the amount borrowed in period t while the right-hand-side is the expected repayment in period $t+1$, discounted by the market return $\overline{R}_{j,t}$. Since the household/firm renegotiates the debt if $d_{j,t+1} >$

$\tilde{p}_{j,t+1}k_{j,t+1}$, the actual repayment $\tilde{d}(d_{j,t+1}, \tilde{p}_{j,t+1}k_{j,t+1})$ could differ from $d_{j,t+1}$. Competition in financial intermediation requires that the left-hand-side of (5) is equal to the right-hand-side.

Equation (5) determines the interest rate $R_{j,t}$ for an individual household/firm. In equilibrium all households/firms make the same decisions and they all borrow at the same rate. However, in order to characterize the optimal decision, we need to allow an individual household/firm to deviate from other households/firms, which implies a deviation of the individual borrowing rate from the aggregate rate as determined by (5).

First order conditions. As for entrepreneurs, households/firms' decisions are made sequentially. At the beginning of the period (Subperiod 1) households/firms decide whether to default and renegotiate the debt. After that (Subperiod 2), they chose the supply of labor. Finally, at the end of the period (Subperiod 3), they choose the new debt. Appendix B describes the optimization problem and derives the first order conditions

$$w_{j,t} = z_{j,t} l_{j,t}^{\frac{1}{\nu}}, \tag{6}$$

$$\frac{1}{R_{j,t}} = \beta \mathbb{E}_t \left[1 + \Phi \left(\frac{d_{j,t+1}}{\tilde{p}_{j,t+1}k_{j,t+1}} \right) \right], \tag{7}$$

$$p_{j,t} = \beta \mathbb{E}_t \left[r_{j,t+1} + g p_{j,t+1} + \Psi \left(\frac{d_{j,t+1}}{\tilde{p}_{j,t+1}k_{j,t+1}} \right) \right]. \tag{8}$$

The detailed functional forms for the functions $\mathbb{E}_t\Phi(.)$ and $\mathbb{E}_t\Psi(.)$, are derived in the appendix. The appendix shows that these two terms are increasing in the ratio $d_{j,t+1}/\tilde{p}_{j,t+1}k_{j,t+1}$, which is a measure of leverage.

Equation (7) posits a negative relation between the expected return on the debt (the interest rate) and leverage. Equation (8) establishes a positive relation between leverage and the price of capital. The two equations then imply that a decline in the interest rate increases leverage and generates an asset price boom.

2.3 Market for liquidated capital

So far we have used the liquidation price $\tilde{p}_{j,t}$ without explaining how it is formed in equilibrium. In this section we describe the market for liquidated capital and the determination of $\tilde{p}_t$. The market for liquidated capital takes place at the beginning of the period and it is based on two assumptions.

Assumption 1 *Capital can be sold to domestic households/firms or entrepreneurs. If sold to entrepreneurs, capital loses its functionality and will be converted to consumption goods at rate* $\kappa_{j,t}$.

This assumption formalizes the idea that capital may lose value when reallocated to non-specialized owners, provided that $\kappa_{j,t}$ is sufficiently low. In the model this is obtained with the simple assumption that entrepreneurs convert capital in consumption goods at rate $\kappa_{j,t}$, which we assume to be lower than $p_{j,t}$.

In order for capital to keep its functionality, it needs to be purchased by domestic households/firms, not foreign households/firms. With this assumption a crisis could be local, that is, it could take place in one country without spreading to the other country. However, even if a crisis takes place only in one country, it will have a real economic impact also in the other country due to portfolio cross-ownership.

Assumption 2 *Households/firms can purchase liquidated capital only if* $d_{j,t} < \tilde{p}_{j,t}k_{j,t}$.

If a household/firm starts with liabilities bigger than the liquidation value of the owned assets, that is, $d_{j,t} > \tilde{p}_{j,t}k_{j,t}$, it will be unable to raise additional funds to purchase the liquidated capital. Potential investors know that the new liabilities (as well as the outstanding liabilities) are not collateralized, and the debt will be renegotiated immediately by households/firms after taking the new debt. We refer to a household/firm with $d_{j,t} < \tilde{p}_{j,t}k_{j,t}$ as 'liquid' since it can raise extra funds at the beginning of the period. A household/firm with $d_{j,t} > \tilde{p}_{j,t}k_{j,t}$, instead, is 'illiquid'.

To better understand Assumptions 1 and 2, consider the condition for not renegotiating, $d_{j,t} \leq \tilde{p}_{j,t}k_{j,t}$. Furthermore, assume that $p_{j,t} > \kappa_{j,t}$, that is, the price at the end of the period is bigger than the liquidation price when the market freezes. If this condition is satisfied, households/firms have the ability to raise funds to purchase additional capital. This insures that the liquidation price is $\tilde{p}_{j,t} = p_{j,t}$. However, if $d_{j,t} > \kappa_{j,t}k_{j,t}$ for all households/firms, there will be no households/firms capable of buying the liquidated capital. Then, liquidated capital can only be purchased by entrepreneurs at price $\tilde{p}_{j,t} = \kappa_{j,t}$.

This shows that the market price for liquidated capital depends on the financial decision of households/firms, which in turn depends on the liquidation price. This interdependence could lead to self-fulfilling equilibria.

Proposition 2.1 *There exists multiple equilibria only if* $d_{j,t} > \kappa_{j,t} k_{j,t}$.

Proof 2.1 *See appendix C.*

When multiple equilibria are possible, that is, we have $d_{j,t} > \kappa_{j,t} k_{j,t}$, the equilibrium is selected by the random draw of sunspot shocks.

Let $\varepsilon_{j,t}$ be a variable that takes the value of 0 with probability λ and 1 with probability $1 - \lambda$. If the condition for multiplicity is satisfied, agents coordinate their expectations on the low liquidation price $\kappa_{j,t}$ when $\varepsilon_{j,t} = 0$. This implies that the probability distribution of the low liquidation price is

$$f_{j,t}\Big(\tilde{p}_{j,t} = \kappa_{j,t}\Big) = \begin{cases} 0, & \text{if} \quad d_{j,t} \leq \kappa_{j,t} k_{j,t} \\ \\ \lambda, & \text{if} \quad d_{j,t} > \kappa_{j,t} k_{j,t} \end{cases}$$

The ratio $d_{j,t}/\kappa_{j,t} k_{j,t}$ is the effective measure of leverage. If leverage is sufficiently small, households/firms remain liquid even if the (expected) liquidation price is $\kappa_{j,t}$. But then the liquidation price cannot be low and the realization of the sunspot shock is irrelevant for the equilibrium. Instead, when leverage is high, households/firms' liquidity depends on the liquidation price. The realization of the sunspot shock $\varepsilon_{j,t}$ then becomes important for selecting one of the two equilibria. When $\varepsilon_{j,t} = 0$—which happens with probability λ—the market expects that the liquidation price is $\kappa_{j,t}$, making the household's sector illiquid. On the other hand, when $\varepsilon_{j,t} = 1$—which happens with probability $1 - \lambda$—the market expects that households/firms are capable of participating in the liquidation market, validating the expectation of a higher liquidation price.

Notice that the argument is based on the assumption that $\kappa_{j,t}$ is sufficiently low (implying a low liquidation price if the capital market freezes). Also, the equilibrium value of capital without a freeze, $p_{j,t} k_{j,t}$, is always bigger than the debt $d_{j,t}$. Otherwise, households/firms would be illiquid with probability 1 and the equilibrium price is always $\kappa_{j,t}$. Condition (5) guarantees that this does not happen in equilibrium: if the probability of default is 1, the anticipation of the renegotiation cost will increase the interest rate, which deters households/firms from borrowing too much.

2.4 General equilibrium

Using capital letters to denote aggregate variables, the aggregate states are given by bonds held by entrepreneurs, $B_{11,t}$, $B_{21,t}$, $B_{12,t}$, $B_{22,t}$, and sunspot

shocks $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$. The aggregate debts issued by households/firms in the previous period are $D_{1,t} = B_{11,t} + B_{12,t}$ and $D_{2,t} = B_{21,t} + B_{22,t}$. In addition, we also have the whole sequence of productivity $z_{1,t}$ and $z_{2,t}$ and financial variables $\phi_{1,t}, \phi_{2,t}, \kappa_{1,t}, \kappa_{2,t}$. Since the changes in these variables are deterministic and perfectly anticipated, the whole sequence is part of the state space. We denote the sequence starting at time t with the time superscript. For example, z_j^t represents the sequence of productivity in country j from time t to infinity. To use a compact notation we denote the state vector by

$$\mathbf{s}_t \equiv (z_1^t, z_2^t, \phi_1^t, \phi_2^t, \kappa_1^t, \kappa_2^t, B_{11,t}, B_{21,t}, B_{12,t}, B_{22,t}, \varepsilon_{1,t}, \varepsilon_{2,t}).$$

The equilibrium is determined sequentially in the three subperiods as shown in Figure 2.

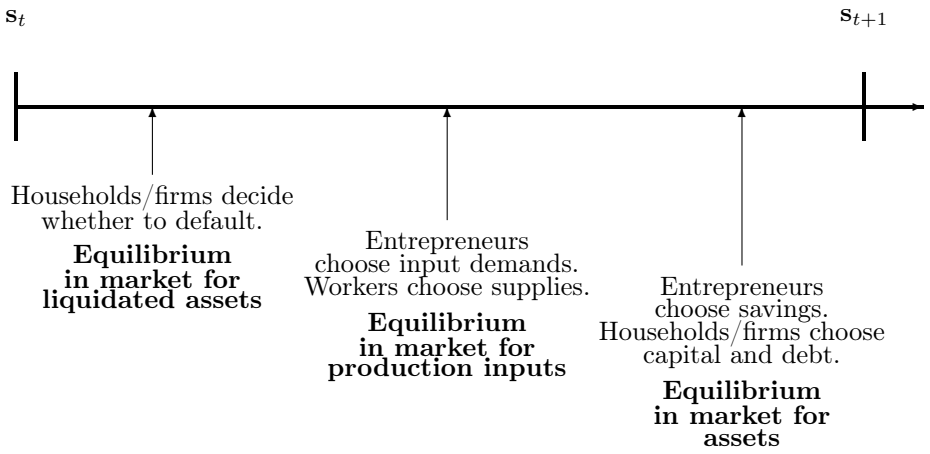

Figure 2: Timing within period t.

1. **Subperiod 1**: Given the sunspot shock $\varepsilon_{j,t}$ in country j, agents form (self-fulfilling) expectations about the liquidation price $\tilde{p}_{j,t}$. Households/firms then choose whether to default. The renegotiated debt is

$$\tilde{D}_{j,t} = \begin{cases} \kappa_{j,t} K_{j,t}, & \text{if} \quad D_{j,t} \geq \kappa_{j,t} K_{j,t} \ \text{ and } \ \varepsilon_{j,t} = 0 \\ D_{j,t}, & \text{otherwise} \end{cases}.$$

The post-default wealth held by entrepreneurs in each country is proportional to their holdings prior to default, that is,

$$M_{j,t} = \left(\frac{\tilde{D}_{1,t}}{D_{1,t}}\right) B_{1j,t} + \left(\frac{\tilde{D}_{2,t}}{D_{2,t}}\right) B_{2j,t}$$

2. **Subperiod 2**: Given the post-default wealth $M_{j,t}$, entrepreneurs in country j choose the inputs of labor and capital, and households/firms choose the supplies. The aggregate input demands in country j are obtained from the individual demands derived in Lemma 2.1,

$$\begin{aligned} L_{j,t} &= z_{j,t}^{\frac{\gamma}{\alpha}} \left(\frac{\gamma}{w_{j,t}}\right)^{\frac{\alpha+\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{r_{j,t}}\right)^{\frac{1-\alpha-\gamma}{\alpha}} M_{j,t}, \\ K_{j,t} &= z_{j,t}^{\frac{\gamma}{\alpha}} \left(\frac{\gamma}{w_{j,t}}\right)^{\frac{\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{r_{j,t}}\right)^{\frac{1-\gamma}{\alpha}} M_{j,t}. \end{aligned}$$

The aggregate supply of labor is derived from the household's first order condition (6). Imposing $l_{j,t} = L_{j,t}$ and inverting we obtain

$$L_{j,t} = \left(\frac{w_{j,t}}{z_{j,t}}\right)^{\nu}.$$

The supply of capital is exogenous and equal to $K_{j,t} = \bar{K} g^t$. Market clearing will then determine the wage rate $w_{j,t}$, the rental rate $r_{j,t}$, and employment $L_{j,t}$ in each country.

3. **Subperiod 3**: The end-of-period wealth of entrepreneurs is

$$A_{j,t} = (1-\phi_j) M_{j,t} + z_{j,t}^{\gamma} M_{j,t}^{\alpha} L_{j,t}^{\gamma} K_{j,t}^{1-\alpha-\gamma} - w_{j,t} L_{j,t} - r_{j,t} K_{j,t}.$$

A fraction $1-\beta$ is consumed while the remaining fraction β will be saved in new bonds, $q_{1,j} B_{1j,t+1}$ and $q_{2,j} B_{2j,t+1}$. Households/firms choose new debt $D_{j,t+1}$ and new holding of capital $K_{j,t+1}$.

Market clearing in financial assets requires

$$B_{11,t+1} + B_{12,t+1} = D_{1,t+1}, \tag{9}$$

$$B_{21,t+1} + B_{22,t+1} = D_{2,t+1}. \tag{10}$$

Because of capital mobility and cross-country heterogeneity, the net foreign asset positions of the two countries could be different from zero, that is, $B_{1j,t+1} + B_{2j,t+1} \neq D_{j,t+1}$. Competition also implies that the price paid by entrepreneurs to purchase households/firms' debt is consistent with the interest rate, that is,

$$q_{j,t} = \frac{1}{R_{j,t}}.$$

Since $\overline{R}_{j,t} = R_{j,t}\mathbb{E}_{t+1}\delta_{j,t+1}$, the above condition relates the price of bonds $q_{j,t}$ to their expected return.

Using the optimal savings of entrepreneurs derived in Lemma 2.1 and aggregating, we obtain the demand for bonds in country j,

$$q_{1,j}B_{1j,t+1} + q_{2,j}B_{2j,t+1} = \beta A_{j,t}. \tag{11}$$

The supply of bonds is derived from the borrowing decisions of households/firms. From the first order condition (7) we have

$$\frac{1}{\overline{R}_{j,t}} = \beta \left[1 + \Phi\left(\frac{D_{j,t+1}}{\kappa_{j,t+1}K_{j,t+1}}\right)\right].$$

Because in equilibrium $\overline{R}_{j,t} = R_{j,t}\mathbb{E}\delta_{j,t+1}$ and $q_{j,t} = 1/R_{j,t}$, the first order condition can be rewritten as

$$q_{j,t} = \beta \left[1 + \Phi\left(\frac{D_{j,t+1}}{\kappa_{j,t+1}K_{j,t+1}}\right)\right]\mathbb{E}\delta_{j,t+1}. \tag{12}$$

The market for capital must also clear, that is, the demand $K_{j,t+1}$ must be equal to the exogenous supply $\bar{K}g^t$. The first order condition (7) will then determine the (end-of-period) price p_t. Since this equation depends on future prices, we cannot derive a close-form expression but we will compute it numerically in the quantitative analysis.

The equilibrium is characterized by entrepreneurs holding the debt issued by households/firms even if the price is higher than β (or equivalently, the interest rate is lower than the inter-temporal discount rate). This is because bonds represent financial wealth that contribute to production. Therefore, bonds generate a profit in addition to interest payments.

Because $z_{j,t}, \phi_{t,t}, \kappa_{j,t}$ are time-varying and households/firms can default, the economy does not reach a steady state but displays stochastic dynamics driven by sunspot shocks. In particular, sunspot shocks can lead to financial crises with default where bonds are only partially repaid. This redistributes wealth from lenders (entrepreneurs) to borrowers (households/firms). When entrepreneurs hold less wealth $M_{j,t}$, they demand less labor and in equilibrium there will be lower employment and production. This is the main mechanism through which financial crises have real macroeconomic effects. A lower value of $M_{j,t}$ also decreases the demand for capital which lowers the rental rate $r_{j,t}$. The lower return on capital then reduces its price p_t. Therefore, financial crises have also a negative impact on asset prices.

2.5 Discussion and remarks

Before proceeding, it would be helpful to clarify the importance of some modeling assumptions and associated properties.

In equilibrium entrepreneurs are net savers and households/firms are net borrowers. Although having producers with positive net financial wealth might appear counterfactual at first, it is not inconsistent with the recent changes in the financial structure of US corporations. It is well known that during the last two and half decades, the corporate sector has increased the holdings of financial assets. This suggests that the proportion of financially dependent firms has declined significantly over time, which is consistent with the empirical findings of Shourideh and Zetlin-Jones (2012) and Eisfeldt and Muir (2016).

The large accumulation of financial assets by producers (often referred to 'cash') is related to the significance of business savings. Busso et al. (2016) document the share of savings done by firms both in advanced and emerging economies and present evidence that in Latin America this share is even larger than in advanced economies. The importance of business savings is also documented in Bebczuk and Cavallo (2016). Using data for 47 countries over 1995–2013 they show that the contribution of businesses to national savings is on average more than 50%. Our entrepreneurial sector captures the growing importance of firms that are not very dependent on external financing.

The second remark is that the equilibrium property for which producers are net lenders does not rely on the assumption that households/firms are risk neutral. What is crucial is that the overall return of bonds for en-

trepreneurs is greater than the interest rate. For borrowing households/firms, instead, bonds are valuable only because they pay interests. With risk averse households/firms, bonds could also provide an insurance benefit. However, as long as the extra return that entrepreneurs receive from holding bonds is sufficiently large, they would continue to be lenders while households/firms would continue to be borrowers.

3 Quantitative analysis

We now use the model to assess quantitatively how the unprecedented growth of emerging economies and the changes in financial structure experienced by both emerging and advanced economies affected financial and macroeconomic volatility. The quantitative application uses data for advanced economies (country 1 in the model) and emerging economies (country 2 in the model) over the period 1991-2020. Starting in 1991, we simulate the model until 2020. The countries included in the groups of emerging and advanced economies are listed in Figure 1.

3.1 Calibration

The model is calibrated annually and the discount factor is set to $\beta = 0.96$, implying an annual intertemporal discount rate of about 4%. We set the elasticity of labor supply to $\nu = 1$, a number often used in macroeconomics.

The probability of a negative sunspot shock ($\varepsilon = 0$) is set to $\lambda = 0.04$. Provided that leverage is sufficiently high, crises are low probability events: on average, every twenty-five years. This is within the range of crisis probabilities used in the literature (see, for example, Bianchi and Mendoza (2018)). Notice that, since sunspot shocks are country-specific, that is, they are independent across countries, a global financial crisis that arises contemporaneously in both countries is a rare event that happens with probability $0.04 \times 0.04 = 0.016$.

We calibrate next the share parameters in the production function. We set the labor share to $\gamma = 0.6$, which is a standard value. We interpret the cost $\phi_{j,t} M_{j,t}$ as depreciation of capital.[5] To pin down α, then, we use the depreciation-output ratio as a calibration target. More specifically, we

[5]Higher input of financial assets increases production which leads to more intensive utilization of capital. This increases capital depreciation.

assume that the worldwide average of $\phi_{j,t}M_{j,t}$ as a fraction of worldwide output over the sample period 1991-2020 is 0.2.[6] To determine the precise value of α, however, we need to use an iterative procedure: After fixing α, we calibrate all other parameters including the sequences of $z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$, and check whether the average depreciation in the model is 20% the value of output. We then update α until we reach the calibration target of 0.2.

Differences in size and financial structure between the two regions are generated by the deterministic sequences of $z_{j,t}$, $\phi_{j,t}$, and $\kappa_{j,t}$, with $j \in \{1, 2\}$. We construct these sequences to replicate the empirical time series showed in Figure 1 over the period 1991-2020. The required sequences, however, depend on the realizations of the sunspot shocks over the sample period. Therefore, the first step is to choose a sequence of sunspot shocks for the period 1991-2000.

We assume that ε_{jt} is equal to 1 (no crisis) in all years with only few exceptions. For emerging economies it takes the value of zero (possibility of crisis) in 1997 and 2009. These two years correspond, respectively, to the 1997 crisis in Asia and to the financial crisis that started in 2008 and extended to 2009. Both crises had an impact on emerging economies. For advanced economies, instead, it takes the value of zero only in 2009 reflecting, again, the 2008-2009 financial crisis. It is important to point out that, even though we calibrate the model assuming a specific sequence of sunspot shocks, agents do not anticipate them and, therefore, they make decisions based on the random distribution of these shocks. We start describing how we construct the productivity series.

Productivity. The productivity series $z_{1,t}$ and $z_{2,t}$ are constructed as Solow residuals from the production function. To do so we need measurements for production inputs and outputs. For output we use GDP measured at nominal exchange rates, not PPP. Since movements in nominal exchange rates affect the purchasing power of a country in the acquisition of foreign assets, our productivity measure should also reflect movements in exchange rates. Another factor that contributes to differences in aggregate GDP is population growth. However, because population is not explicitly formalized in the model, $z_{1,t}$ and $z_{2,t}$ will also reflect changes in population.

Denote by $P_{j,t}$ the nominal price index for country j expressed in US

[6] If the average depreciation rate is 0.08 and the capital-output ratio is 2.5, then the depreciation-output ratio is 0.2.

dollar. The price is calculated by multiplying the price in local currency by the dollar exchange rate. We can then define the 'nominal' aggregate output of country j as

$$P_{j,t}Y_{j,t} = P_{j,t}\hat{z}_{j,t}^{\gamma}M_{j,t}^{\alpha}L_{j,t}^{\gamma}K_{j,t}^{1-\alpha-\gamma}N_{j,t},$$

where $\hat{z}_{j,t}$ is actual productivity, $M_{j,t}$ is per-capita financial assets, $L_{j,t}$ is per-capita employment, $K_{j,t}$ is per-capita capital, $N_{j,t}$ is population. Notice that the above definition of output assumes that physical capital increases with population.

If we deflate the nominal GDP of both countries by the price index in country 1, we obtain

$$\begin{aligned} Y_{1,t} &= \hat{z}_{1,t}^{\gamma}M_{1,t}^{\alpha}L_{1,t}^{\gamma}K_{1,t}^{1-\alpha-\gamma}N_{1,t}, \\ \frac{P_{2,t}Y_{2,t}}{P_{1,t}} &= \left(\frac{P_{2,t}\hat{z}_{2,t}^{\gamma}}{P_{1,t}}\right)M_{2,t}^{\alpha}L_{2,t}^{\gamma}K_{2,t}^{1-\alpha-\gamma}N_{2,t}, \end{aligned}$$

Therefore, aggregate productivity in the model corresponds to

$$\begin{aligned} z_{1,t} &= \hat{z}_{1,t}N_{1,t}^{\frac{1}{\gamma}}, \\ z_{2,t} &= \hat{z}_{2,t}\left(\frac{P_{2,t}N_{2,t}}{P_{1,t}}\right)^{\frac{1}{\gamma}}. \end{aligned}$$

This shows that $z_{1,t}$ and $z_{2,t}$ also reflect cross-country differences in prices and population. The productivity series in the model can be calculated from the data as

$$z_{1,t} = \left(\frac{Y_{1,t}}{M_{1,t}^{\alpha}L_{1,t}^{\gamma}K_{1,t}^{1-\alpha-\gamma}}\right)^{\frac{1}{\gamma}}, \tag{13}$$

$$z_{2,t} = \left(\frac{P_{2,t}Y_{2,t}/P_{1,t}}{M_{2,t}^{\alpha}L_{2,t}^{\gamma}K_{2,t}^{1-\alpha-\gamma}}\right)^{\frac{1}{\gamma}}. \tag{14}$$

The numerator is total real GDP, deflated by the nominal price in advanced economies. If the price in emerging economies grows more than the price in advanced economies, this will be reflected in higher relative productivity in emerging economies. Although this does not increase actual

productivity, it raises the ability of these countries to purchase assets in advanced economies, which is important for general equilibrium effects. Also notice that the change in relative price could simply be the result of movements in nominal exchange rates. Still, when the currencies of emerging economies appreciate, assets created in advanced economies become cheaper for emerging economies.

In order to use equations (13) and (14) to construct the productivity sequences, we need empirical counterparts for $Y_{1,t}$, $P_{2,t}Y_{2,t}/P_{1,t}$, $M_{1,t}$, $M_{2,t}$ $L_{1,t}$, $L_{2,t}$, $K_{1,t}$, and $K_{2,t}$.

The output variables $Y_{1,t}$ and $P_{2,t}Y_{2,t}/P_{1,t}$ are obtained by aggregating the GDP of advanced and emerging economies, both expressed at constant US dollars. To construct $M_{j,t}$ we use domestic private credit together with the net foreign asset positions. Both variables are expressed in constant US dollars, divided by population over 15 years of age. More specifically, denoting by $D_{j,t}$ domestic credit and $NFA_{j,t}$ the net foreign asset position of country j, financial assets used in production are $M_{j,t} = D_{j,t} + NFA_{j,t}$.

For the labor input $L_{j,t}$ we use employment-to-population ratio (population over 15 years of age). The variable $K_{j,t}$ grows in the model at the constant rate $g-1$. Therefore, we can express the stock of capital as $K_{j,t} = \bar{K}g^t$. Notice that the constant growth rate of capital is the same in the two regions. We set this rate to the average growth rate of aggregate GDP in advanced economies which, over the period 1991-2020, is 1.89%. We take this number as the long-run growth rate for both advanced and emerging economies (after convergence). Data is from the World Development Indicators (WDI). The resulting productivity series are plotted in the top panel of Figure 3.

As expected, productivity has increased faster in emerging economies. The fact that the productivity of emerging economies is now higher than in advanced economies does not mean that emerging economies have a more efficient technology. Remember that $z_{j,t}$ reflects also the size of the population, which is much larger in emerging economies.

Financial structure. The next step is to construct sequences for $\phi_{j,t}$ and $\kappa_{j,t}$. The first variable, $\phi_{j,t}$, is important for the 'demand' of financial assets (in the spirit of Mendoza et al. (2009)): Lower values of $\phi_{j,t}$ increase the demand for financial assets since their use in production is less costly. The parameter $\kappa_{j,t}$, instead, affect more directly the 'supply' of financial assets (in the spirit of Caballero et al. (2008)): Higher values of $\kappa_{j,t}$ increase the

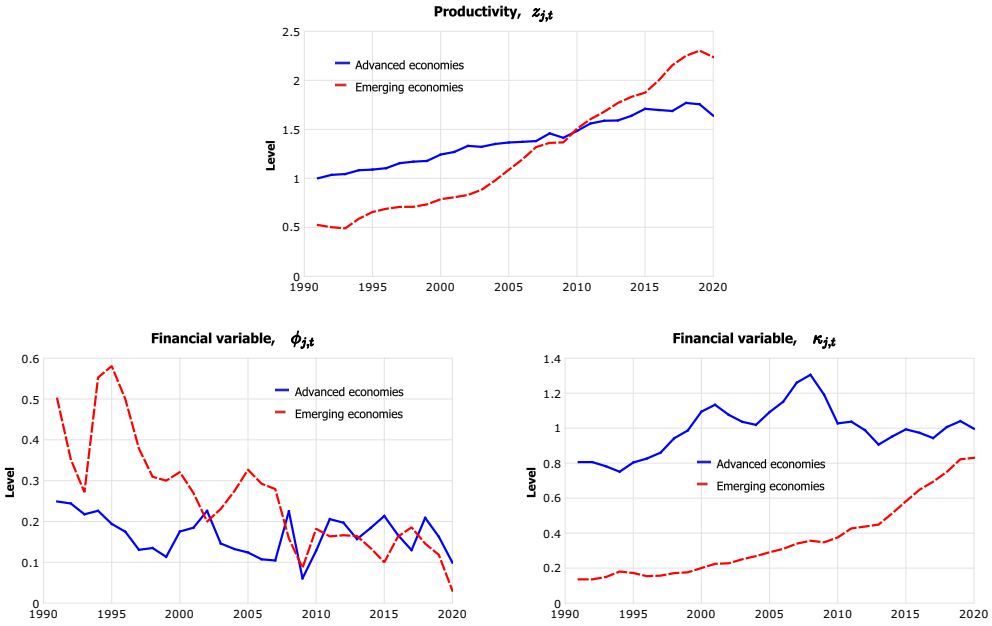

Figure 3: Computed productivity and financial variables series for advanced and emerging economies, 1990-2020.

incentive of households/firms to borrow.

We construct the sequences of $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$ so that the model replicates four empirical series over the period 1991-2020: (i) domestic credit-to-GDP ratio in advanced economies, (ii) domestic credit-to-GDP ratio in emerging economies, (iii) net foreign asset position of advanced economies, (iv) US risk-free real interest rate. These are the empirical series shown in the last three panels of Figure 1). The mapping of these four empirical targets to the corresponding variables in the model is as follows:

$$\text{Credit-to-GDP AEs} = \frac{q_{1,t}D_{1,t+1}}{Y_{1,t}}, \tag{15}$$

$$\text{Credit-to-GDP EEs} = \frac{q_{2,t}D_{2,t+1}}{Y_{2,t}}, \tag{16}$$

$$\text{NFA-to-GDP AEs} = \frac{q_{1,t}B_{11,t+1} + q_{2,t}B_{21,t+1} - q_{1,t}D_{1,t+1}}{Y_{1,t}}, \tag{17}$$

$$\text{US real interest rate} = \frac{\mathbb{E}_t\delta_{j,t+1}}{q_{1,t}} - 1. \tag{18}$$

The terms on the right-hand-side are equilibrium objects that we can compute after fixing $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$. Given the structure of the model, we can solve for the equilibrium in period t, independently of future equilibria, as if the model were static. More specifically, given the states $\mathbf{s}_t$ and given the values of $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$, we can find the equilibrium variables at time t by solving the system of nonlinear equations described in Appendix D.[7] The procedure to find $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$ will then apply two nested nonlinear solvers: the inner solver finds the equilibrium variables given the values of $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$ (as described in Appendix D) and the outer solver finds the values of $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$ using conditions (15)-(18).

The computed series for $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$ and $\kappa_{2,t}$ are plotted in the bottom panels of Figure 3. The first panel shows that ϕ does not display any significant trend for advanced economies while it trends downward for emerging economies. Recall that a reduction in the value of ϕ leads to an increase in the demand for financial assets. Therefore, the higher growth of emerging economies has been accompanied to a structural change that increased the demand for financial assets more than in advanced economies. The second panel shows that the variable κ has increased for both advanced and emerging economies. Since a higher value of κ raises the supply of assets, the computed series indicate that financial constraints have been relaxed in both advanced and emerging economies.

The full set of parameter values are listed in Table 1. The parameter α in the production function is set to 0.294. This is the value for which the model generates an average depreciation of capital of 20% the value of output (calibration target).

Table 1: Parameter values.

Description	*Parameter*	*Value*
Discount factor	β	0.960
Share of financial wealth in production	α	0.294
Share of labor in production	γ	0.600
Elasticity of labor supply	ν	1.000
Probability of low sunspot shock	λ	0.040
Renegotiation cost	η	2.000

[7] We can solve for all equilibrium variables sequentially at any time t, except for the price of capital $p_{j,t}$. However, the price of capital does not affect the equilibrium variables that are mapped to the four empirical targets listed in (15)-(18).

3.2 Productivity growth vs. structural financial changes

In this section we explore how the changes in productivity and financial structure—the variables $z_{j,t}$, $\phi_{j,t}$, $\kappa_{j,t}$ shown in Figure 3—contributed to real and financial dynamics. We do so by conducting counterfactual simulations over the sample period 1991-2020, where we allow only one factor to change, while keeping the other factors fixed.

We start with productivity. We impose that $\phi_{j,t}$ and $\kappa_{j,t}$ remain constant at their 1991 values for the whole simulation period, while $z_{j,t}$ takes the values constructed in the calibration section. The series generated by the counterfactual simulation are plotted in Figure 4.

The first four panels plot domestic credit in advanced and emerging economies, net foreign asset position in advanced economies, and the risk-free real interest rate (which is common in the two regions). The continuous line is the original data shown in Figure 1. This is also the series generated by the baseline model with productivity and financial variables taking the values plotted in Figure 3. The dashed line is the model generated data when only productivity changes. The dotted line also plots the simulated series when only productivity changes but with an additional assumption: We impose that productivity in emerging economies grows at the same rate as in advanced economies. The comparison of the dashed and dotted lines illustrates the importance of faster growth experienced by emerging economies.

The higher growth of emerging economies accounts for most of the imbalance (the dynamics of the net foreign asset position) and a significant portion of the decline in the interest rate. The spike in the interest rate in 2009 is caused by the financial crisis. The growing size of emerging economies also generates an increase in the domestic credit of advanced economies (as a percentage of GDP), while it declines in emerging economies. However, the decline in emerging economies is due to the fact that GDP (the denominator) grows faster than domestic credit (the numerator). In absolute terms, domestic credit increases also in emerging economies when only productivity changes.

The last two panels plots the 'effective leverage'. This is the ratio of the debt, $D_{j,t+1}$, and its recovery value in a financial crisis, $\kappa_{j,t+1}K_{j,t+1}$. Besides the temporary drop after the financial crisis, the model predicts an increasing trend in effective leverage in response to the change in productivity (dashed line). This is directly related to the change in the interest rate: a lower interest rate is always associated with a higher effective leverage. Remember

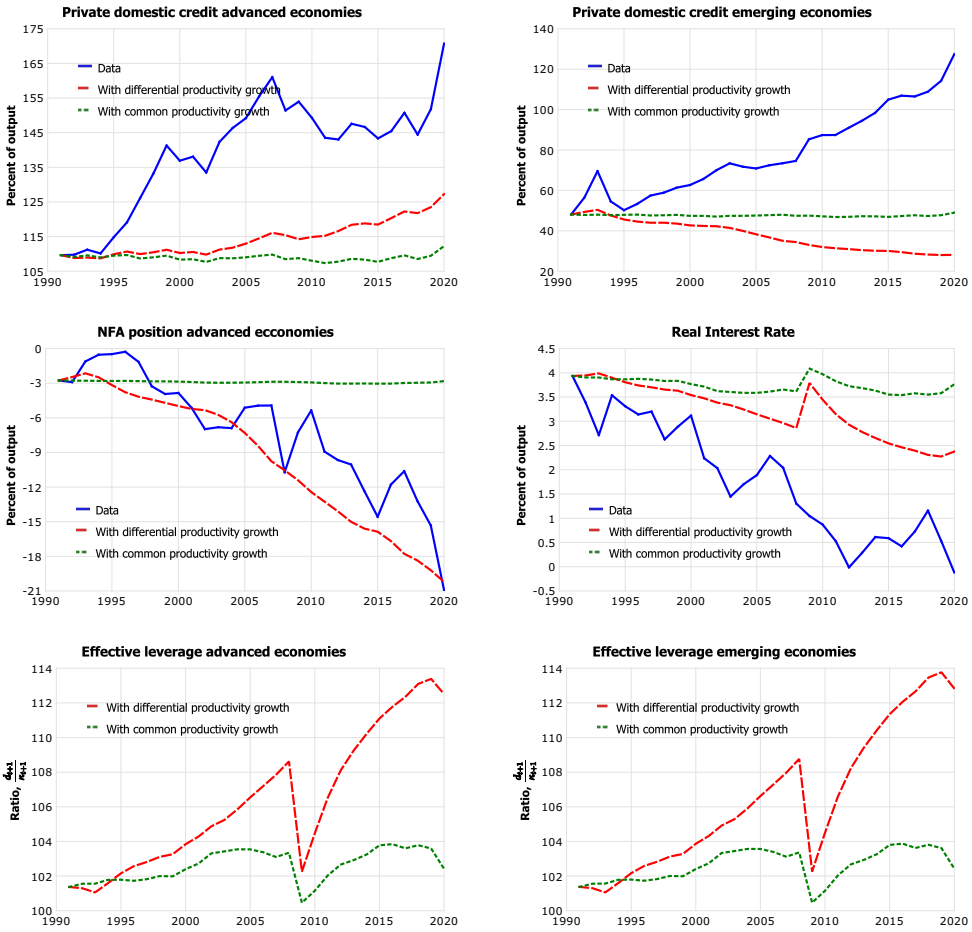

Figure 4: Counterfactual simulation with constant financial structure, 1990-2020.

that the interest rate satisfies condition (7).

As we will see, the increase in effective leverage plays an important role for aggregate volatility. However, the upward trend would not arise if emerging economies had grown at the same (lower) rate experienced by advanced economies (see dotted line).

The main take away from the counterfactual exercise shown in Figure 4 is that the faster growth of emerging economies has been an important force for global imbalance and declining worldwide interest rates. Faster growth generates profits that increase entrepreneurial wealth and, therefore, the demand for financial assets. But when $\kappa_{j,t}$ does not change, the supply remains

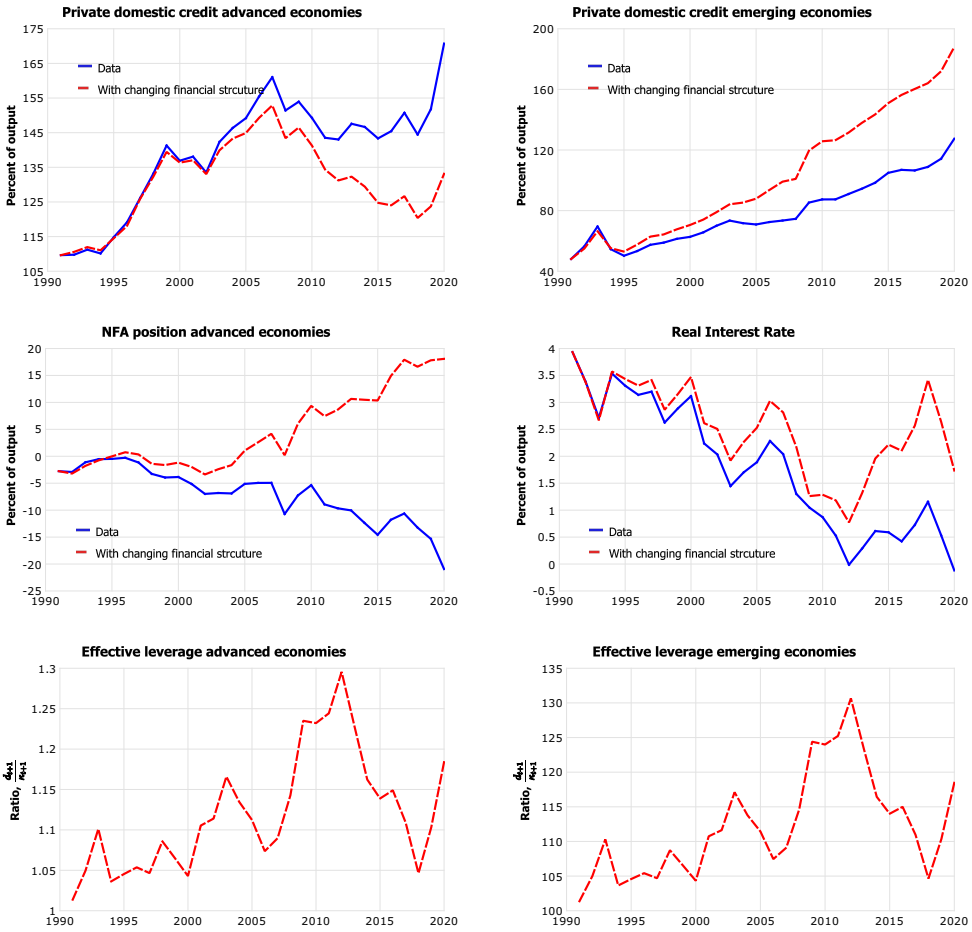

Figure 5: Counterfactual simulation with constant and common productivity growth, 1990-2020.

the same. To clear the market, then, the interest rate has to drop. The faster growth of entrepreneurial wealth in emerging economies also implies that part of that wealth is invested abroad, which explains the imbalance.

We now conduct another counterfactual exercise to explore the importance of the changes in financial structure. We fix the productivity of the two countries and explore the implications of the changes in financial parameters $\phi_{j,t}$ and $\kappa_{j,t}$. More specifically, we assume that, starting from the values in 1991, $z_{1,t}$ and $z_{2,t}$ both grow at the long-run rate $g - 1 = 0.0189$. This is the average GDP growth of advanced economies over the sample period

1991-2020. The financial parameters $\phi_{j,t}$ and $\kappa_{j,t}$, however, take the values shown in Figure 3. The simulated variables are plotted in Figure 5.

The changes in financial structure are important for capturing the growing size of financial intermediation (higher credit-to-GDP ratios). They are also important for generating a large decline in the real interest rate. However, they do not explain the observed imbalance. In fact, in absence of differential productivity growth, advanced economies would accumulate positive net foreign asset positions.

The changes in financial structure also lead to an increase in effective leverage. As observed earlier, a lower interest rate is always associated to a higher effective leverage (see condition (7)). This is important for understanding the impact of the structural changes on aggregate volatility.

3.3 Macroeconomic volatility

We now explore the main question addressed in this paper, that is, how the faster growth of emerging economies and the changes in financial structure impacted macroeconomic and financial volatility.

To compute measures of volatility, we simulate the model for 130 years in response to random draws of sunspot shocks: $\varepsilon_{j,t} = 0$ with probability $\lambda = 0.04$ and $\varepsilon_{j,t} = 1$ with probability $1 - \lambda = 0.96$. During the first 100 years, the variables $\phi_{j,t}$ and $\kappa_{j,t}$ remain constant at their 1991 values, and productivity in both countries grows at the same long-run rate $g - 1$, that is, the average growth rate of GDP for advanced economies over the period 1991-2020. The first 100 simulated years are used to derive the invariant distribution. The remaining 30 years of simulation correspond to the sample period 1991-2020 where $z_{j,t}$, $\phi_{j,t}$ and $\kappa_{j,t}$ take the values plotted in Figure 3. The simulation is then repeated 10,000 times, each time with a new sequence of random draws of the sunspot shocks.

The first two columns of Figure 6 plot the mean and volatility of aggregate output computed over the 10,000 repeated simulations. By repeating the simulation, we obtain 10,000 data points for every year. The mean in year t is the average of the 10,000 data points in year t resulting from the repeated simulations. Denoting by $Y_{i,t}$ the value of output generated by the model in year t with simulation i, the mean is computed as

$$\overline{Y}_t = \frac{1}{10,000} \sum_{i=1}^{10,000} Y_{i,t}.$$

Volatility is computed as the difference between the 5th and 95th percentile of the 10,000 points generated by the repeated simulation in year t, in percentage of the mean of the variable in the same year. More specifically, denoting by $P_t(5)$ the threshold for the 5th percentile of the 10,000 points in year t, and $P_t(95)$ the threshold for the 95th percentile, output volatility at time t is computed as

$$VOL_t = \left(\frac{P_t(95) - P_t(5)}{\overline{Y}_t} \right) \times 100.$$

The top panels of Figure 6 are for the baseline model where both productivity and financial structure change over time (as shown in Figure 3). Both regions experience an increase in volatility. The last panel plots the average value of the effective leverage. It shows that the increase in volatility is directly related to the increase in average (effective) leverage. A financial crises leads to debt restructuring which causes a redistribution of wealth from borrowers (households/firms) to lenders (entrepreneurs). The reduction in entrepreneurial wealth, then, reduces employment and production. Since the magnitude of the redistribution increases with leverage, the model generates an increase in volatility as a consequence of the increase in leverage.

The next panels illustrate the factors that contributed to the growth in volatility. The graphs in the middle rows show the importance of faster productivity growth in emerging economies. The growth in productivity, keeping $\phi_{j,t}$ and $\kappa_{j,t}$ unchanged, contributed about 40 percent to the increase in volatility. It is important to emphasize that the increase in volatility would not arise if emerging economies experienced the same productivity growth as advanced economies. This is shown in the third row of the figure, which is constructed under the counterfactual assumption that emerging economies experienced the same productivity growth as advanced economies. In this case volatility does not change significantly over the simulated period. This is again related to the fact that effective leverage remains almost unchanged (see the panel in the third column). This shows that the faster growth of emerging economies has been important for generating higher macroeconomic and financial volatility.

The last row of Figure 6 is based on the counterfactual simulation in which productivity in both regions grows at the same long-run rate of $g-1 = 0.0189$. What change are the variables $\phi_{j,t}$ and $\kappa_{j,t}$ which we interpret as reflecting structural changes in the financial sector. These changes contributed about 60 percent to the increase in volatility. Also in this case the increase in

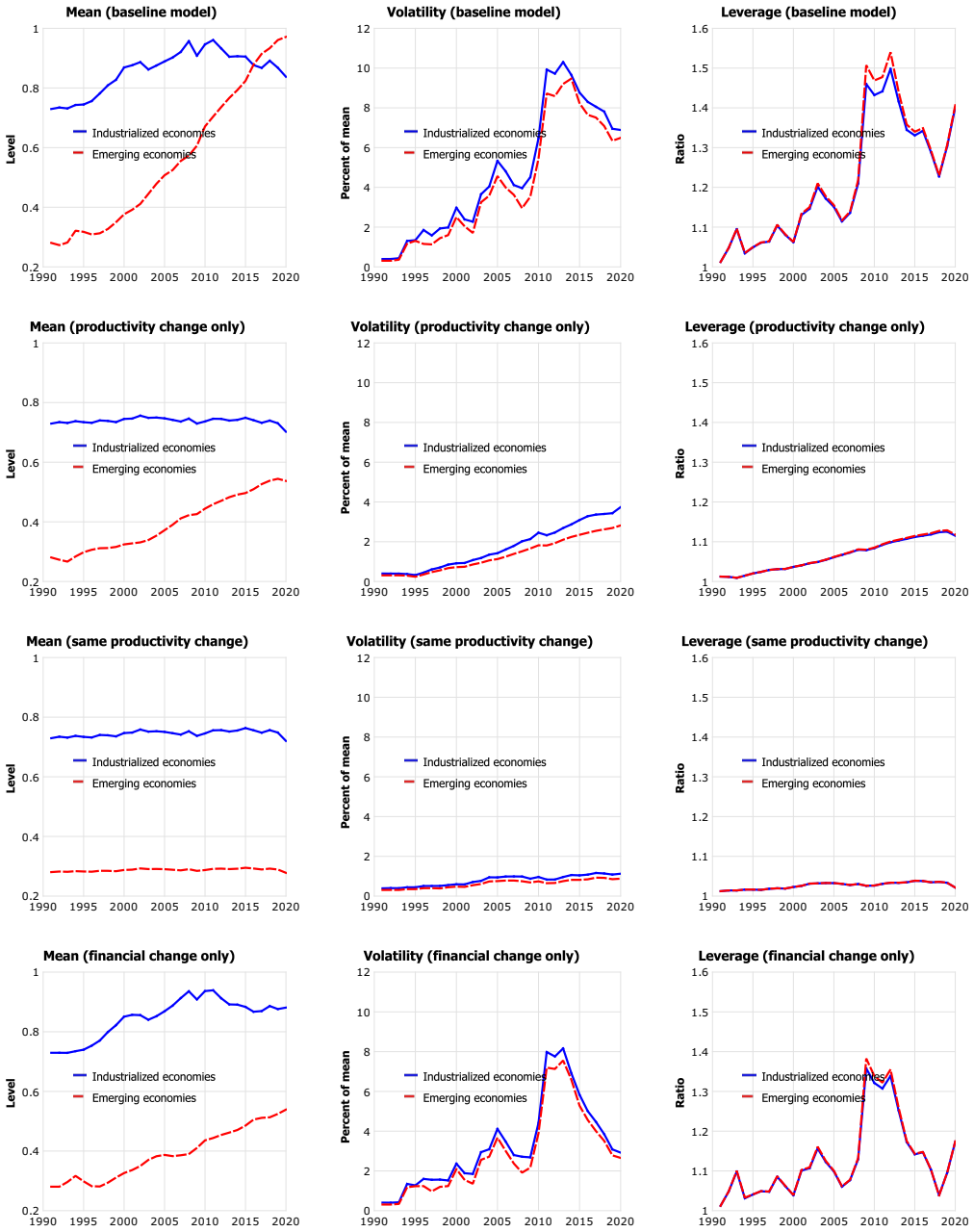

Figure 6: Simulated mean and volatility of output over the period 1991-2020. The mean is the average in every year over the 10,000 repeated simulation. The volatility is the difference between the 5th and 95th percentile as a percentage of the mean.

volatility is related to the increase in effective leverage (see last panel). The changes in financial structure led to a worldwide increase in net demand for financial assets. This caused a decline in the interest rate, which in turn increased effective leverage.

Notice that the changes in financial structure affected not only volatility but also the level of output. As can be seen from the panels in the first column of the figure, the mean for advanced economies does not change significantly when the financial structure does not change and the growth of emerging economies is (counter-factually) smaller. This is because the expansion of the financial sector allowed by the changes in $\phi_{j,t}$ and $\kappa_{j,t}$ created more financial assets. Since financial assets enter the production function, the supply change caused an increase in output.

4 Discussion and conclusion

An implication of the increased size of emerging economies is that they are more influential in the world economy. The view that countries in emerging markets are a collection of small open economies with negligible impact on advanced economies is no longer a valid approximation.

There are many channels through which countries in emerging markets could affect the rest of the world. In this paper we emphasized one of these channels: the increased demand for financial assets traded in globalized capital markets. In particular, we have shown that the worldwide increase in the demand for financial assets raises the incentives to leverage. On the one hand, this allows for the expansion of the financial sector with positive effects on real macroeconomic activities. On the other, it increases the fragility of the financial system, raising the probability and/or the consequences of a crisis. From a policy perspective there is a trade-off: the benefit of an expanded financial system versus the potential cost of more severe crises. A similar mechanism also arises in models with asset price bubbles and borrowing constraints as in Miao and Wang (2011).

Appendix

A Proof of Lemma 2.1

The optimization problem of an entrepreneur in country j is

$$\max_{\{l_{j,t},k_{j,t},c_{j,t},b_{1j,t+1},b_{2j,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_{j,t})$$

subject to

$$\begin{aligned} m_{j,t} &= \delta_{1,t} b_{1j,t} + \delta_{2,t} b_{2j,t}, \\ a_{j,t} &= (1-\phi_{j,t}) m_{j,t} + z_{j,t}^{\gamma} m_{j,t}^{\alpha} l_{j,t}^{\gamma} k_{j,t}^{1-\alpha-\gamma} - w_{j,t} l_{j,t} - r_{j,t} k_{j,t}, \\ c_{j,t} &= a_{j,t} - q_{1,t} b_{1j,t+1} - q_{j,t} b_{2j,t+1}. \end{aligned}$$

The first order conditions for $l_{j,t}$ and $k_{j,t}$ are

$$\begin{aligned} \gamma z_{j,t}^{\gamma} m_{j,t}^{\alpha} l_{j,t}^{\gamma-1} k_{j,t}^{1-\alpha-\gamma} &= w_{j,t} \\ (1-\alpha-\gamma) z_{j,t}^{\gamma} m_{j,t}^{\alpha} l_{j,t}^{\gamma} k_{j,t}^{-\alpha-\gamma} &= r_{j,t} \end{aligned}$$

These two conditions give us the first two equations in Lemma 2.1. Since the inputs of labor and capital are linear functions of $m_{j,t}$, the end of period wealth is also linear in $m_{j,t}$, that is, $a_{j,t} = \pi_{j,t} m_{j,t}$. Here the term $\pi_{j,t}$ is a function of parameters and aggregate prices that are taken as given by an individual entrepreneur. Since $m_{j,t} = \delta_{1,t} b_{1j,t} + \delta_{2,t} b_{2j,t}$, we can write the end-of-period wealth at time t and at $t+1$ as

$$\begin{aligned} a_{j,t} &= \pi_{j,t}(\delta_{1,t} b_{1j,t} + \delta_{2,t} b_{2j,t}), \\ a_{j,t+1} &= \pi_{j,t+1}(\delta_{1,t+1} b_{1j,t+1} + \delta_{2,t+1} b_{2j,t+1}). \end{aligned}$$

We can now derive the first order conditions with respect to $b_{1j,t+1}$ and $b_{2j,t+1}$,

$$\frac{q_{1,t}}{c_{j,t}} = \beta \mathbb{E}_t \left(\frac{\pi_{j,t+1} \delta_{1,t+1}}{c_{j,t+1}} \right), \tag{19}$$

$$\frac{q_{2,t}}{c_{j,t}} = \beta \mathbb{E}_t \left(\frac{\pi_{j,t+1} \delta_{2,t+1}}{c_{j,t+1}} \right). \tag{20}$$

In the next step we guess that optimal consumption is a fraction $1-\beta$ of wealth,

$$c_{j,t} = (1-\beta) a_{j,t}.$$

The saved wealth is allocated bonds issued by country 1 and to bonds issued by country 2. Denoting by $\theta_{j,t}$ the share allocated to country 1, we have

$$q_{1,t}b_{1j,t+1} = \theta_{j,t}\beta a_{j,t} \qquad \text{and} \qquad q_{2,t}b_{2j,t+1} = (1-\theta_{j,t})\beta a_{j,t}. \tag{21}$$

Multiplying equation (19) by $b_{1j,t+1}$ and equation (20) by $b_{2j,t+1}$, adding the resulting expressions, and using the equations that define consumption and next period wealth, we obtain

$$q_{1,t}b_{1j,t+1} + q_{2,t}b_{2j,t+1} = \beta a_{j,t}.$$

This is obviously satisfied given (21). Thus, the Euler equation is satisfied if consumption is a fraction $1-\beta$ of wealth, which verifies our guess.

We now replace the guess for $c_{j,t}$ into equation (20), to obtain

$$\mathbb{E}_t \left\{ \frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{j,t}\frac{\delta_{1,t+1}}{q_{1,t}} + (1-\theta_{j,t})\frac{\delta_{2,t+1}}{q_{2,t}}} \right\} = 1. \tag{22}$$

This condition determines the share of savings invested in the bonds of the two countries. Since the condition is the same for entrepreneurs in country 1 and in country 2, it must be that $\theta_{1,t} = \theta_{2,t} = \theta_t$. ■

B First order conditions for households/firms

The optimization problem of households/firms can be written recursively as

$$V(d,k) = \max_{l,c,d'} \left\{ c - z\frac{l^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \beta \mathbb{E} V(d',k') \right\},$$
$$\text{subject to}$$

$$\tilde{d}(d,\tilde{p}k) + \varphi(d,\tilde{p}k) + pk' + c \;=\; wl + rk + pkg + \frac{1}{R}\mathbb{E}\tilde{d}(d',\tilde{p}'k'),$$

where the function $\tilde{d}(d,\tilde{p}k)$ is defined in (3) and the function $\varphi(d,\tilde{p}k)$ in (4).

The first order conditions with respect to l, d', k' are, respectively,

$$\begin{gathered} zl^{\frac{1}{\nu}} = w, \\ \tfrac{1}{R}\mathbb{E}\left\{\tfrac{\partial \tilde{d}(d',\tilde{p}'k')}{\partial d'}\right\} + \beta\mathbb{E}\left\{\tfrac{\partial V(d',k')}{\partial d'}\right\} = 0, \\ \tfrac{1}{R}\mathbb{E}\left\{\tfrac{\partial \tilde{d}(d',\tilde{p}'k')}{\partial k'}\right\} + \beta\mathbb{E}\left\{\tfrac{\partial V(d',k')}{\partial k'}\right\} = p_{j,t}. \end{gathered}$$

The envelope conditions are

$$\begin{aligned}\frac{\partial V(d,k)}{\partial d} &= -\frac{\partial \tilde{d}(d,\tilde{p}k)}{\partial d} - \frac{\partial \varphi(d,\tilde{p}k)}{\partial d},\\ \frac{\partial V(d,k)}{\partial k} &= r + pg - \frac{\partial \tilde{d}(d,\tilde{p}k)}{\partial k} - \frac{\partial \varphi(d,\tilde{p}k)}{\partial k}.\end{aligned}$$

Updating by one period and substituting in the first order conditions for debt and capital we obtain

$$\frac{1}{R} = \beta\left[1 + \frac{\mathbb{E}\left\{\frac{\partial \varphi(d',\tilde{p}'k')}{\partial d'}\right\}}{\mathbb{E}\left\{\frac{\partial \tilde{d}(d',\tilde{p}'k')}{\partial d'}\right\}}\right], \tag{23}$$

$$p_{j,t} = \frac{1}{R}\mathbb{E}\left\{\frac{\partial \tilde{d}(d',\tilde{p}'k')}{\partial k'}\right\} + \beta\mathbb{E}\left\{r' + gp' - \frac{\partial \tilde{d}(d',\tilde{p}'k')}{\partial k'} - \frac{\partial \varphi(d',\tilde{p}'k')}{\partial k'}\right\}. \tag{24}$$

Using the functional forms for the functions $\tilde{d}(d,\tilde{p}k)$ and $\varphi(d,\tilde{p}k)$ defined, respectively, in (3) and (4), we derive the analytical expressions for the for derivatives:

$$\frac{\partial \tilde{d}(d,\tilde{p}k)}{\partial d} = \begin{cases} 0, & \text{if} \quad d \geq \tilde{p}k \ \ \& \ \ \varepsilon = 0 \\ 1, & \text{otherwise} \end{cases}$$

$$\frac{\partial \tilde{d}(d,\tilde{p}k)}{\partial k} = \begin{cases} \tilde{p}, & \text{if} \quad d \geq \tilde{p}k \ \ \& \ \ \varepsilon = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\frac{\partial \varphi(d,\tilde{p}k)}{\partial d} = \begin{cases} 2\eta\left(1 - \frac{\tilde{p}k}{d}\right)\frac{\tilde{p}k}{d} + \eta\left(1 - \frac{\tilde{p}k}{d}\right)^2, & \text{if} \quad d \geq \tilde{p}k \ \ \& \ \ \varepsilon = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\frac{\partial \varphi(d,\tilde{p}k)}{\partial k} = \begin{cases} -2\eta\left(1 - \frac{\tilde{p}k}{d}\right)^2\tilde{p}, & \text{if} \quad d \geq \tilde{p}k \ \ \& \ \ \varepsilon = 0 \\ 0, & \text{otherwise} \end{cases}$$

We now assume that the equilibrium is always characterized by $d \geq \tilde{p}k$. This will be the case in the parameterized model. Under this assumption we always have default when $\varepsilon = 0$, which arises with probability λ. The expected values of

the above derivatives can then be written as

$$\begin{aligned}
\mathbb{E}\left\{\frac{\partial \tilde{d}(d,\tilde{p}k)}{\partial d}\right\} &= 1-\lambda \\
\mathbb{E}\left\{\frac{\partial \tilde{d}(d,\tilde{p}k)}{\partial k}\right\} &= \lambda\kappa \\
\mathbb{E}\left\{\frac{\partial \varphi(d,\tilde{p}k)}{\partial d}\right\} &= 2\lambda\eta\left(1-\frac{\kappa k}{d}\right)\frac{\kappa k}{d}+\lambda\eta\left(1-\frac{\kappa k}{d}\right)^2 \\
\mathbb{E}\left\{\frac{\partial \varphi(d,\tilde{p}k)}{\partial k}\right\} &= -2\lambda\eta\left(1-\frac{\kappa k}{d}\right)^2\kappa
\end{aligned}$$

We can now use these expressions in the first order conditions (23) and (24). After rearranging, conditions (23) and (24) can be written as

$$\frac{1}{\overline{R}} = \beta\mathbb{E}\left[1+\Phi\left(\frac{d'}{\tilde{p}'k'}\right)\right], \tag{25}$$

$$p = \beta\mathbb{E}\left[r'+gp'+\Psi\left(\frac{d'}{\tilde{p}'k'}\right)\right], \tag{26}$$

where

$$\begin{aligned}
\mathbb{E}\Phi\left(\frac{d'}{\tilde{p}'k'}\right) &= \left(\frac{\lambda}{1-\lambda}\right)\left[2\eta\left(1-\frac{\kappa'k'}{d'}\right)\frac{\kappa'k'}{d'}+\eta\left(1-\frac{\kappa'k'}{d'}\right)^2\right], \\
\mathbb{E}\Psi\left(\frac{d'}{\kappa'k'}\right) &= \lambda\left[\Phi\left(\frac{d'}{\kappa'k'}\right)+2\eta\left(1-\frac{\kappa'k'}{d'}\right)\right]\kappa'.
\end{aligned}$$

By taking derivatives we can verify that they are increasing in d' and decreasing in both k' and $\tilde{p}'$. ■

C Proof of Proposition 2.1

At the beginning of the period households/firms choose whether to renegotiate the debt. Given the initial states d_t and k_t, the renegotiation decision boils down to a take-it or leave-it offer made to creditors for the repayment of the debt.

Denote by $\tilde{d}_t = \psi(d_t, k_t, \tilde{p}_t)$ the offered repayment. This depends on the individual liabilities, d_t, individual capital, k_t, and the price for liquidated capital, $\tilde{p}_t$. The price of the liquidated capital is the price at which the lender could sell

the capital after rejecting the offer from the borrower. The best offer made by the household/firm is

$$\psi(d_t, k_t, \tilde{p}_t) = \begin{cases} d_t, & \text{if} \quad d_t \leq \tilde{p}_t k_t \\ \\ \tilde{p}_t^e k_t, & \text{if} \quad d_t > \tilde{p}_t k_t \end{cases}, \tag{27}$$

which is accepted by creditors if they cannot sell at a price higher than $\tilde{p}_t$.

For the moment we assume that the equilibrium is symmetric, that is, all households/firms start with the same ratio d_t/k_t. At this stage this is only an assumption. However, we will show below that households/firms do not have an incentive to deviate from the ratio chosen by other households/firms.

Given the assumption that the equilibrium is symmetric (all households/firms choose the same ratio d_t/k_t), multiple equilibria arise if $d_t/k_t \in [\kappa_t, p_t)$. If the market expects that the liquidation price is $\tilde{p}_t = \kappa_t$, all households/firms are illiquid and they choose to renege their liabilities (given the renegotiation policy (27)). As a result, there will be no households/firms that can purchase the liquidated capital of other households/firms. The only possible liquidation price that is consistent with the expected price is $\tilde{p}_t = \kappa_t$. On the other hand, if the market expects $\tilde{p}_t = p_t$, households/firms are liquid and, if one household/firm reneges, creditors can sell the liquidated assets to other households/firms at the liquidation price $\tilde{p}_t = p_t$. Therefore, it is optimal for households/firms not to renegotiate.

We now address the issue of whether individual households/firms have an incentive to deviate from the symmetric equilibrium and choose a different ratio d_t/k_t in the previous period $t-1$. In particular, we need to show that, in the anticipation that the liquidation price could be $\tilde{p}_t = \kappa_t$, a household/firm does not find convenient to borrow less at time $t-1$ so that it could purchase the liquidated capital if the price drops to κ_t.

The first point to consider is that, in equilibrium, capital is never liquidated. The low liquidation price κ_t simply represents the threat value for creditors. However, in equilibrium all creditors accept the renegotiation offer and no capital is ever liquidated.

What would happen if there is a household/firm that is liquid and, therefore, has the ability to purchase the liquidated capital at a higher price than κ_t? This would arise if a household/firm deviates from the symmetric equilibrium. In this case debtors know that their creditors could liquidate the capital and sell it at a higher price than κ_t. Knowing this, debtors will offer a higher repayment and, as a result, capital is not liquidated. Potentially, this could drive the liquidation price to p_t. This shows that a household/firm cannot make any profit by remaining liquid. Therefore, there is no incentive to deviate from the symmetric equilibrium. ■

D Equilibrium system of equations at time t

Given the values of $\phi_{1,t}$, $\phi_{2,t}$, $\kappa_{1,t}$, $\kappa_{2,t}$, $\kappa_{1,t+1}$, $\kappa_{2,t+1}$, and the stochastic states $\mathbf{s}_t$, we can find the values of $\delta_{j,t}$, $M_{j,t}$, $L_{j,t}$, $K_{j,t}$, $w_{j,t}$, $r_{j,t}$, $q_{j,t}$, $A_{j,t}$, $B_{j1,t+1}$, $B_{j2,t+1}$, $D_{j,t+1}$ and θ_t, by solving the following system of equations:

$$\delta_{j,t} = \begin{cases} \min\left\{1\ ,\ \frac{\kappa_{j,t}K_{j,t}}{D_{j,t}}\right\}, & \text{if} \quad \varepsilon_{j,t}=0 \\ 1, & \text{if} \quad \varepsilon_{j,t}=1 \end{cases} \tag{28}$$

$$M_{j,t} = \delta_{1,t}B_{1,t} + \delta_{2,t}B_{2,t} \tag{29}$$

$$L_{j,t} = z_{j,t}^{\frac{\gamma}{\alpha}} \left(\frac{\gamma}{w_{j,t}}\right)^{\frac{\alpha+\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{r_{j,t}}\right)^{\frac{1-\alpha-\gamma}{\alpha}} M_{j,t}, \tag{30}$$

$$K_{j,t} = z_{j,t}^{\frac{\gamma}{\alpha}} \left(\frac{\gamma}{w_{j,t}}\right)^{\frac{\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{r_{j,t}}\right)^{\frac{1-\gamma}{\alpha}} M_{j,t}, \tag{31}$$

$$L_{j,t} = \left(\frac{w_{j,t}}{z_{j,t}}\right)^{\nu}, \tag{32}$$

$$K_{j,t} = \bar{K}g^t, \tag{33}$$

$$A_{j,t} = (1-\phi_j)M_{j,t} + z_{j,t}^{\gamma} M_{j,t}^{\alpha} L_{j,t}^{\gamma} K_{j,t}^{1-\alpha-\gamma} - w_{j,t}L_{j,t} - r_{j,t}K_{j,t}, \tag{34}$$

$$B_{1j,t+1} = \frac{\theta_t \beta A_{j,t}}{q_{1,t}}, \tag{35}$$

$$B_{2j,t+1} = \frac{(1-\theta_t)\beta A_{j,t}}{q_{2,t}}, \tag{36}$$

$$1 = \mathbb{E}_t \left\{ \frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_t \frac{\delta_{1,t+1}}{q_{1,t}} + (1-\theta_t)\frac{\delta_{2,t+1}}{q_{2,t}}} \right\}, \tag{37}$$

$$D_{j,t+1} = B_{j1,t+1} + B_{j2,t+1}, \tag{38}$$

$$q_{j,t} = \beta \left[1 + \Phi\left(\frac{D_{j,t+1}}{\kappa_{j,t+1}K_{j,t+1}}\right)\right] \mathbb{E}_t \delta_{j,t+1}. \tag{39}$$

Equation (28) defines the optimal renegotiation strategy (the fraction of the debt repaid). Equation (29) defines entrepreneurial wealth after default. Equations (30) and (31) are the demand for labor and capital from entrepreneurs, given the prices $w_{j,t}$ and $r_{j,t}$, and their wealth $M_{j,t}$. Equations (32) and (33) are the supplies of labor and capital from households/workers. Equation (34) defines the end-of-period wealth of entrepreneurs after production. This is allocated to bonds issued by the two countries as indicated in equations (35) and (36). Equation (37) is the condition that determines the investment share θ_t. This is the Euler

equation derived from the optimization problem of entrepreneurs. Equation (38) is equilibrium in the bond market. The final equation (39) is the Euler equation for the households/firms determining the price of bonds.

The above system determines all equilibrium variables except the price of capital $p_{j,t}$. To solve for the price of capital we need to use condition (26) where the current price $p_{j,t}$ depends on the future price $p_{j,t+1}$. This implies that we cannot solve for the equilibrium price in the current period without solving for the equilibrium in the future. Therefore, we need to use an iterative procedure. However, since the current price $p_{j,t}$ does not affect other variables in the current period, we can use the above system to solve for the equilibrium in period t ignoring the price. Notice that this would not be the case if the liquidation value of capital was a function of $p_{j,t}$.

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