

# Stubborn Beliefs in Search Equilibrium\*

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## Abstract

I study a search equilibrium model of the labor market in which workers have stubborn beliefs about their labor market prospects, i.e. beliefs about their probability of finding a job and the wage they will earn that do not respond to aggregate fluctuations in fundamentals. I show that, when workers have stubborn beliefs, the response of the wage bargained by a firm and a worker to aggregate shocks is dampened. As a result, the response of labor market tightness, job-finding probability, unemployment and vacancies to aggregate fluctuations is amplified. I show that stubborn beliefs generate cyclical inefficiencies in the labor market that can be corrected with countercyclical employment subsidies. I find that the response of the labor market to negative shocks is the same even if only a small fraction of workers has stubborn beliefs. In contrast, if the fraction of workers with stubborn beliefs is small, the response of the labor market to positive shocks is approximately the same as under rational expectations.

*JEL Codes:* J64.

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# 1 Introduction

In search-theoretic models of the labor market in the style of Pissarides (1985), the expectations that the workers hold about the probability of finding a job and about the wages they might earn once they find a job play a critical role in the determination of equilibrium outcomes. To see why this is the case, consider a worker and a firm who have just met and who are bargaining over the terms of trade. Whether the worker accepts or rejects the wage offered by the firm depends on the worker’s expectations about how quickly he could find another firm and what wage he could earn in case the current negotiation broke down. And since the worker’s expectations affect his bargaining strategy, they also affect the wages that come out of the bargaining game. Since the expectations held by workers affect actual wage outcomes, they also affect the firms’ actual incentives to create vacancies, the actual tightness of the labor market, and the workers’ actual job-finding probability.

In the search theory literature, it is standard to assume that workers have complete information and rational expectations (see, e.g., Pissarides 1985, Mortensen and Pissarides 1994 or Shimer 2005). That is, workers know the current realization and the stochastic process of all the time-varying fundamentals of the economy—such as, say, productivity, monetary or fiscal policy—as well as the time-invariant fundamentals—such as, say, the factor at which firms discount future profits, the cost that firms have to incur to maintain vacancies, and the bilateral matching process that brings unemployed workers and vacant jobs together. Using these pieces of information, workers can recover the mapping between current realizations of the fundamentals and equilibrium outcomes and, thus, form correct expectations about their labor market prospects. The assumption of rational expectations under complete information is very convenient for modelling. Since workers have complete information, the modeler does not need to specify neither the workers’ prior beliefs about the fundamentals nor the information that the workers use to update such beliefs. Since workers have rational expectations, the modeler does not need to exogenously specify the workers’ expectations.

The assumption that workers have rational expectations and complete information, however, need not be realistic. Mueller, Spinnewjin and Topa (2019) find that, among workers who just became unemployed, the average expected job-finding probability is quite close to the average realized job-finding probability. Yet, the workers who have the highest expected job-finding probability tend to be too optimistic, while the workers who have the lowest expected job-finding probability tend to be too pessimistic. Moreover, Mueller, Spinnewjin and Topa (2019) find that the expected job-finding probability of an individual worker does not change throughout a spell of unemployment. Similarly, they find that the expected job-finding probability does not respond to changes in macro-economic conditions. These findings suggest that workers have “stubborn beliefs” about their labor market prospects: beliefs that are correct on average, but that do not respond to either new individual information (the length of an unemployment spell) or to new

aggregate information (the state of the business cycle).

In this paper, I propose a simple search theoretic model of the labor market in which workers have stubborn beliefs. The fundamentals of the economy are constant over time, except for the aggregate component of productivity which follows a stochastic process. Yet, workers believe that all the fundamentals of the economy, including the aggregate component of productivity, are constant over time and equal to their unconditional mean. Based on their view of the economy, workers compute the equilibrium outcomes and, in turn, form expectations about the labor market tightness, the job-finding probability, the firms' bargaining strategy, and the outcome of the bargaining game. Firms, in contrast, have rational expectations and complete information about the fundamentals of the economy, including how workers form their expectations. When an individual worker meets an individual firm, I consider two alternative scenarios. In the first scenario, the worker observes the productivity of the firm with which he is bargaining and interprets any difference between the firm's productivity and the average of the aggregate component of productivity as a permanent, firm-specific component of productivity. In the second scenario, the worker does not observe the productivity of the firm with which he is bargaining and believes it to be given by the average of the aggregate component of productivity. In both scenarios, workers' expectations about their labor market prospects are correct on average, but they do not change in response to aggregate productivity shocks.

An equilibrium of the model with stubborn workers (SBE) is given by a system of equations for the actual value functions and equilibrium outcomes, and by a system of equations for the workers' perception of the value functions and the workers' expected equilibrium outcomes. The two systems of equations are linked by the outcome of the bargaining game. The bargaining game follows the same protocol as in Binmore, Rubinstein and Wolinsky (1986), in which the firm and the worker alternate in making wage offers and wage demands, and the negotiation may break down after a proposal is rejected. The wage outcome of the bargaining game is a weighted average between the worker's perceived productivity of the firm and the worker's perceived value of unemployment. In the first scenario, the worker's perceived productivity of the firm is the firm's actual productivity. Hence, in this scenario, the wage is sticky, in the sense that it is affected by aggregate productivity fluctuations only through their impact on the firm's actual productivity but not through their impact on the worker's actual value of unemployment. In the second scenario, the worker's perceived productivity of the firm is the unconditional mean of aggregate productivity. Hence, in this scenario, the wage is rigid, in the sense that it is completely unresponsive to aggregate productivity fluctuations. In both scenarios, the wage is entirely pinned down by the worker's beliefs. The intuition is simple. Since the firm knows that the worker's beliefs cannot be changed, it has no choice but to accommodate them.

The properties of equilibrium are different when workers have stubborn beliefs (SBE) rather than rational expectations (REE). If the aggregate component of productivity is at its unconditional mean, the labor market tightness, the job-finding probability, and

the unemployment and the vacancy rates are the same in an SBE as in an REE. This is because the wage is the same in an SBE and in an REE. The elasticities of the labor market tightness, the job-finding probability, the unemployment and the vacancy rate with respect to aggregate productivity are, however, higher in an SBE than in an REE. This is because the wage responds less to deviations of aggregate productivity from its unconditional mean in an SBE than in an REE. Moreover, since the wage in an SBE is sticky in the first scenario and rigid in the second scenario, the elasticity of labor market outcomes in an SBE is larger in the second scenario than in the first.

The welfare properties of equilibrium are also different when workers have stubborn beliefs rather than rational expectations. An REE is generically inefficient because, when a firm decides to create a vacancy, it does not internalize the negative effect of its decision on the probability that other vacancies are filled—the so-called congestion externality—and it does not internalize the positive effect of its decision on the worker who is eventually hired to fill the vacancy—the so-called thick market externality. An REE can be made efficient by means of an unemployment subsidy designed to make the firm internalize the congestion and the thick-market externalities. An SBE is also inefficient because there is a gap between the actual surplus of a firm-worker match and the surplus perceived by the worker—which is the one that determines the wage and the allocation of the gains from trade between the firm and the worker. An SBE can be made efficient by adding a countercyclical component to the unemployment subsidy that is optimal for an REE. For instance, when the Hosios’ condition holds, the REE is efficient and the optimal unemployment subsidy is zero. The SBE, however, is still inefficient, and the optimal unemployment subsidy involves making a transfers to firms in recessions and taxing firms in expansions.

I then consider an extension of the baseline model in which some workers have stubborn beliefs and some workers have rational expectations. When a firm and worker bargain over the wage, the firm knows the probability that the worker is stubborn and the probability that the worker is rational, but it does not know the type of the worker. Thus, the bargaining game between the firm and the worker is one of asymmetric information. The outcome of the bargaining game depends on whether a stubborn worker is more optimistic than a rational worker—which is the case when aggregate productivity is below its unconditional mean—or vice-versa—which is the case when aggregate productivity is above its unconditional mean. When a stubborn worker is more optimistic, the outcome of the bargaining game is such that both types of workers earn the wage that a stubborn worker would earn if the firm knew his type—that is, both types earn the same wage as in the baseline SBE. When a rational worker is more optimistic, the outcome of the bargaining game is such that each type of worker earns the same wage that he would earn if the firm knew his type—that is, a rational worker earns the same wage as in an REE and a stubborn worker earns the same wage as in the baseline SBE. Even though asymmetric information bargaining games are typically plagued by a multiplicity of equilibria, I show that this is the unique Perfect Sequential Equilibrium—an equilibrium concept proposed

by Grossman and Perry (1986a, 1986b).

The intuition behind the asymmetry of the bargaining outcomes is relatively simple. When a stubborn worker is more optimistic than a rational worker, the full-information wage is higher for a stubborn worker than for a rational one. For this reason, the rational worker finds it optimal to hide his type and make the same wage demands as a stubborn worker. The firm could screen the two types by making a wage offer that is acceptable only to the rational worker. The firm, however, does not find it optimal to do so, as the screening offer would have to be close to the full-information wage of a stubborn worker, which the rational worker can attain by rejecting the screening offer. When a rational worker is more optimistic than a stubborn worker, the full-information wage for a rational worker is higher than the full-information wage for a stubborn worker. For this reason, the rational worker would like to signal his type to the firm and can do so by making a wage offer that the firm could only credibly interpret as coming from him.

The asymmetric outcomes of the bargaining game cause the labor market to behave differently when the aggregate component of productivity is lower or higher than its unconditional mean. When aggregate productivity falls below its unconditional mean, the response of the wage paid by firms to both stubborn and rational workers and, hence, the average wage paid by firms is just as downward sticky or as rigid as in the baseline SBE. As a result, the response of the labor market tightness, the job-finding probability, and the unemployment and vacancy rates is just as large as in the baseline SBE, even though only a fraction of workers are stubborn. When aggregate productivity rises above its unconditional mean, the response of the wage paid by firms to stubborn workers is as sticky or rigid as in the baseline SBE, but the response of the wage paid by firms to rational workers is fully flexible as in an REE. As a result, the response of the average wage paid by firms is not as sticky as in the baseline SBE and not as flexible as in an REE. Consequently, the response of the labor market tightness, job-finding probability, unemployment and vacancy rates is higher than in an REE but lower than in the baseline SBE, and whether the response is closer to one or the other extreme depends on the fraction of stubborn workers in the population. If, for example, the fraction of stubborn workers is small, the labor market features the same large response to negative productivity shocks as in an SBE, and the same small response to positive productivity shocks as in an REE.

The paper makes two contributions. The first contribution of the paper is to build a search-theoretic equilibrium model of the labor market in which workers have stubborn beliefs. In order to build a model in which workers do not have rational expectations, I have to specify how workers form their expectations about equilibrium outcomes. I assume that workers form their expectations by solving the equilibrium of a model in which all of the fundamentals of the economy—including those that are stochastic—are constant and equal to their unconditional means. Given this specification of the workers' expectations, the equilibrium of the model can be represented by a system of equations describing the actual value functions and the actual equilibrium outcomes coupled with a system of equations describing the value functions perceived by the workers and the equilibrium

outcomes expected by the workers. The two system of equations come together in the solution of the bargaining game between a firm and a worker and, hence, in the equilibrium condition for the wage. The equilibrium is such that, on average, workers' expectations are correct, but they do not respond to fluctuations in fundamentals, as documented by Mueller, Spinewjin and Topa (2019).

As a departure from the assumption of rational expectations, the paper relates to the literature on behavioral macro (see Gabaix 2019 for an excellent survey and Gabaix 2020 for a modelling strategy that relates to mine). This literature, however, has been mainly applied to models in which trade is frictionless and, hence, it has not dealt with expectations about trading probability and bargaining outcomes. There are some search-theoretic models in which workers do not have rational expectations (e.g., Mueller, Spinewjin and Topa 2019 and Conlon et al. 2018). These models, in contrast to mine, are cast in partial equilibrium as the distribution of wages offered by firms is take to be exogenous.

The second contribution of the paper is to show that stubborn beliefs cause wages to be sticky. In a version of the model where all workers have stubborn beliefs, wages are sticky if workers observe the productivity of the firm with which they are bargaining, and rigid if they assume such productivity to be the unconditional mean of aggregate productivity. In a more general version of the model where only some workers have stubborn beliefs, wages are as sticky/rigid downward as they are when all workers have stubborn beliefs, while they are sticky upward only in proportion to the fraction of workers who have stubborn beliefs.

Therefore, the paper is related to theories of wage stickiness in frictional labor markets. Hall (2005) proposes a simple but radical theory of wage rigidity. When a firm and a worker meet, the wage is not determined by some bargaining game. Instead, the wage is given by some social norm that does not respond to cyclical fluctuations in fundamentals. Hall and Milgrom (2008) and Christiano, Eichenbaum and Trabandt (2016) propose a theory of wage stickiness based on an alternative bargaining protocol. The bargaining protocol behind the wage equation in standard search-theoretic models of the labor market is such that, if the negotiations between the firm and the worker break down, the two parties separate forever. Hall and Milgrom (2008) assume instead that, if the negotiations between the firm and the worker break down, the two parties do not trade in the current period but remain matched in the next period. This alternative protocol leads to an equilibrium wage that does not depend on the worker's value of unemployment and, hence, less responsive to aggregate fluctuations. Gertler and Trigari (2007) obtain aggregate wage stickiness by assuming that firms can adjust the wage paid to their employees—including the newly hired ones—only occasionally.

Menzio (2005) and Kennan (2010) propose theories of wage stickiness that are based on asymmetric information. In both theories, the premise is that, when a firm and a worker bargain over the wage, the worker knows the state of the economy, but does not know how the state of the economy affects the productivity of the firm. Menzio (2005)

shows that, if the outcome of the negotiation is observed by the other employees of the firm, the firm wants to mimic the bargaining strategy of a low-productivity firm so as to avoid revealing information to other employees and renegotiate their wage. As a result, the wage paid by the firm is rigid. Kennan (2010) shows that, if the gains from trade between the worker and the firm are large and the output gap between a high and a low-productivity firm is small, the worker finds it optimal to make a pooling wage demand. As a result, the wage paid by the firm is sticky, in the sense that it does not fully respond to changes in average productivity caused by aggregate shocks.

Menzio and Moen (2010) propose a theory of wage stickiness borrowing from the theory of implicit contracts (see, e.g., Azariadis 1975). When workers are risk averse and firms are risk neutral, firms want to insure workers against aggregate shock by offering them acyclical wages. If the firms cannot commit not to replace incumbent employees with new hires, though, the firms must commit to downward sticky wages for new hires in order to credibly provide insurance to incumbent employees. Fukui (2021) turns the theory of Menzio and Moen (2010) on its head. In a model where workers search on and off the job, the firm’s optimal wage offered to new hires depends on the distribution of wages earned by employed workers as well as by the reservation wage of unemployed workers. If the wages earned by employed workers are sticky because firms want to insure their risk-averse employees against productivity fluctuations, firms find it optimal to offer sticky wages to new hires.

Ljunqvist and Sargent (2017) argue that search-theoretic models that produce a high elasticity of unemployment to productivity shocks often rely on a small “fundamental surplus”—i.e. a small share of the flow of output produced by a firm-worker pair that “the invisible hand can allocate to vacancy creation.” Since the notion of fundamental surplus is somewhat fuzzy—as pointed out by Christiano, Eichenbaum and Trabandt (2021)—one might be able to interpret stubborn beliefs as making the fundamental surplus smaller than it would be under rational expectations. I believe, however, that this would not be a fruitful approach. The mechanisms at work here is conceptually different. Workers’ stubborn beliefs about labor market prospects affect their bargaining strategy and, in turn, make the bargained wage less responsive to productivity shocks. As a result, the profit margin enjoyed by firms becomes more responsive to productivity shocks. It is also worth pointing out that a small fundamental surplus and wage stickiness are not the only way to go from small productivity shocks to large unemployment fluctuations—see, e.g., Menzio and Shi (2011), Menzio and Kaplan (2016), Kehoe, Pastorino and Midrigan (2019), Golosov and Menzio (2020).

## 2 Environment and definition of equilibrium

In this section, I propose a search-theoretic model of the labor market in which the workers’ expectations about the probability of finding a job and the wage they would earn after finding a job are stubborn—in the sense that they do not change in response to fluctuations

in aggregate productivity. In terms of preferences, technology, and search frictions, I make the same assumptions as Pissarides (1985). In terms of expectations, I assume that workers believe that aggregate productivity is always equal to its “normal” value and they form expectations about the tightness of the labor market, the probability of finding a job, and the wage they will earn once they find a job by computing the equilibrium outcomes of a hypothetical labor market without aggregate productivity shocks. The workers’ expectations have an impact on the equilibrium outcomes of the actual labor market because they affect the worker’s bargaining strategy and, in turn, the wage and the tightness of the labor market.

## 2.1 Environment

The labor market is populated by a measure one of workers and by a positive measure of firms. A worker maximizes the present value of income discounted at some factor  $\beta$ , where  $\beta \in (0, 1)$ . A worker’s income is given by some wage  $w$  when he is employed, and by some value of leisure  $b$  when he is unemployed. A firm maximizes the present value of profits discounted at the factor  $\beta$ . A firm operates a constant return to scale technology that turns one unit of labor into  $y$  units of output. I will refer to  $y$  as aggregate productivity.

The labor market is subject to search frictions. Unemployed workers search the labor market to locate vacant jobs, and firms search the labor market to locate unemployed workers by opening and maintaining job vacancies at some unit cost  $k > 0$ . The outcome of the search process is a number  $M(u, v)$  of random bilateral meetings between unemployed workers and vacant jobs, where  $u$  denotes the measure of unemployed workers,  $v$  denotes the measure of vacant jobs, and  $M$  is a constant return to scale function of  $u$  and  $v$ . An unemployed worker meets a vacancy with probability  $p(\theta) \equiv M(1, \theta)$ , where  $\theta \equiv v/u$  denotes the tightness of the labor market, and  $p(\theta)$  is a strictly increasing and concave function with  $p(0) = 0$  and  $p(\infty) = 1$ . A vacancy meets an unemployed worker with probability  $q(\theta) = p(\theta)/\theta$ , where  $q(\theta)$  is a strictly decreasing function with  $q(0) = 1$  and  $q(\infty) = 0$ .

Upon meeting, a worker and a firm bargain over the wage. The bargaining game follows the alternating-offer protocol of Binmore, Rubinstein and Wolinsky (1986, henceforth BRW). Without loss in generality, assume that the game starts with the worker making a wage demand. If the firm accepts the worker’s demand, the game ends. If the firm rejects the worker’s demand, the negotiation breaks down with probability  $1 - \exp(-\lambda\Delta)$ , and it continues with probability  $\exp(-\lambda\Delta)$ , where  $\lambda > 0$  and  $\Delta > 0$ . If the negotiation breaks down, the game ends. If the negotiation continues, the firm makes a wage offer. If the worker accepts the wage offer, the game ends. If the worker rejects, the negotiation breaks down with probability  $1 - \exp(-\mu\Delta)$ , where  $\mu > 0$ . With probability  $\exp(-\mu\Delta)$ , the negotiation continues with the worker making another wage demand. The firm and the worker keep taking turns until either they reach an agreement or the negotiation breaks down. As standard in the bargaining literature, I will focus on the outcome of the



bargaining game in the limit for  $\Delta \rightarrow 0$ .

If the bargaining game ends with a break down, the worker moves back into unemployment and the firm's job remains vacant. If the bargaining game ends with an agreement at some wage  $w$ , the firm and the worker start producing output  $y$  and the firm pays the wage  $w$  to the worker. The employment relationship between the firm and the worker continues until it is dissolved with some probability  $\delta \in (0, 1)$ .

Aggregate productivity follows a simple stochastic process. If  $y = y^*$  in the current period, then next period's aggregate productivity  $y_+$  is equal to  $y^*$  with probability  $\phi_s$  and to some  $\tilde{y}$  with probability  $1 - \phi_s$ , where  $\phi_s \in [0, 1]$  and  $\tilde{y}$  is a drawn from a cumulative distribution function  $H(\tilde{y})$  with support  $[y_\ell, y_h]$  and mean  $y^*$ . If  $y \neq y^*$  in the current period, then next period's aggregate productivity  $y_+$  is equal to  $y$  with probability  $\phi_r$  and to  $y^*$  with probability  $1 - \phi_r$ , with  $\phi_r \in [0, 1]$ . The unconditional mean of the stochastic process for aggregate productivity is  $y^*$ . The parameter  $\phi_s$  controls the frequency at which productivity shocks happen, and the parameter  $\phi_r$  controls the duration of productivity shocks.

When a worker is negotiating with a firm and deciding whether to accept or reject a wage offer, he makes an intertemporal calculation—a calculation which involves comparing the value of being employed at the wage offered by the firm and the value of remaining unemployed and having to search for another firm. For this reason, the worker's expectations about future labor market outcomes affect his current bargaining strategy and, in turn, the outcome of the current bargaining game. In the search-theoretic literature, it is standard to assume that the worker's expectations about future labor market outcomes are correct—i.e. the worker knows the law of motion and the current realization of aggregate productivity, and he correctly computes the mapping between aggregate productivity and labor market tightness, job-finding probability, firms' bargaining strategies, and wage outcomes.

Motivated by the empirical evidence in Mueller, Spinnewjin and Topa (2019), I assume that a worker's expectations about labor market outcomes are incorrect. Specifically, a worker incorrectly believes that aggregate productivity is always equal to  $y^*$ . Based on this belief, the worker forms expectations about the tightness of the labor market, the job-finding probability, the firm's bargaining strategy, and the wage by computing the equilibrium outcomes of a hypothetical labor market in which aggregate productivity is always equal to  $y^*$ . When a worker finds himself negotiating with a particular firm, I consider two alternative scenarios—which, as I will show, lead to different outcomes of the bargaining game. In the first scenario, the worker observes the actual productivity  $y$  of the firm and, if different from  $y^*$ , he rationalizes  $y - y^*$  as a permanent firm-specific component of productivity. In the second scenario, the worker does not observe the actual productivity  $y$  of the firm and, instead, believes it to be  $y^*$ . In contrast to workers, firms have correct expectations about aggregate productivity, labor market tightness, worker's bargaining strategy, and wages. In particular, firms know that the worker's beliefs are

incorrect.

## 2.2 Equilibrium conditions and bargaining outcomes

In order to define an equilibrium, I need some notation. Let  $V_0(y)$  denote the worker's actual value of unemployment, and  $\hat{V}_0$  the worker's perceived value of unemployment—that is, the value of unemployment calculated based on the worker's beliefs. Let  $V_1(w, y)$  denote the worker's actual value of employment at the wage  $w$ , and with  $\hat{V}_1(w)$  the worker's perceived value of employment at the wage  $w$ . Let  $J(w, y)$  denote the firm's value from employing a worker at the wage  $w$ , and with  $\hat{J}(w, y)$  the worker's perception of that value. I denote as  $\theta(y)$  the actual tightness of the labor market, and as  $\hat{\theta}$  the tightness expected by the worker. Lastly, I denote as  $w(y)$  the actual wage outcome of the bargaining game between a worker and a firm, and as  $\hat{w}$  the wage outcome expected by the worker.

The worker's actual and perceived values of unemployment are, respectively, given by

$$V_0(y) = b + \beta \mathbb{E}_{y_+} [p(\theta(y_+))V_1(w(y_+), y_+) + (1 - p(\theta(y_+)))V_0(y_+)], \quad (2.1)$$

$$\hat{V}_0 = b + \beta [p(\hat{\theta})\hat{V}_1(\hat{w}) + (1 - p(\hat{\theta}))\hat{V}_0]. \quad (2.2)$$

Consider (2.1). In the current period, the worker's income is  $b$ . In the next period, the worker meets a firm with probability  $p(\theta(y_+))$ . In this case, the worker and the firm agree to the wage  $w(y_+)$  and the worker's continuation value is  $V_1(w(y_+), y_+)$ . With probability  $1 - p(\theta(y_+))$ , the worker does not meet a firm. In this case, the worker remains unemployed and his continuation value is  $V_0(y_+)$ . Now, consider (2.2). In the current period, the worker's income is  $b$ . In the next period, the worker expects to meet a firm with probability  $p(\hat{\theta})$ . Conditional on meeting a firm, the worker expects to agree to the wage  $\hat{w}$  and to enjoy the continuation value  $\hat{V}_1(\hat{w})$ . The worker expects to not meet a firm with probability  $1 - p(\hat{\theta})$ . Conditional on not meeting a firm, the worker expects a continuation value of  $\hat{V}_0$ .

The worker's actual and perceived values of employment at the wage  $w$  are given by

$$V_1(w, y) = w + \beta \mathbb{E}_{y_+} [(1 - \delta)V_1(w, y_+) + \delta V_0(y_+)], \quad (2.3)$$

$$\hat{V}_1(w) = w + \beta [(1 - \delta)\hat{V}_1(w) + \delta \hat{V}_0]. \quad (2.4)$$

Consider the Bellman equation for  $V_1(w, y)$ . In the current period, the worker's income is  $w$ . In the next period, the worker becomes unemployed with probability  $\delta$ . In this case, the worker's continuation value is  $V_0(y_+)$ . The worker remains employed with probability  $1 - \delta$ . In this case, the worker's continuation value is  $V_1(w, y_+)$ . The Bellman equation for  $\hat{V}_1(w)$  is the same as for  $V_1(w, y)$ , except that the worker expects continuation values  $\hat{V}_0$  and  $\hat{V}_1(w)$  rather than  $V_0(y_+)$  and  $V_1(w, y_+)$ .

The firm's actual value from having an employee and the worker's perception of such

value are given by

$$J(w, y) = y - w + \beta \mathbb{E}_{y_+} [(1 - \delta)J(w, y_+)], \quad (2.5)$$

$$\hat{J}(w, \hat{y}) = \hat{y} - w + \beta(1 - \delta)\hat{J}(w, \hat{y}). \quad (2.6)$$

Consider (2.5). In the current period, the firm earns a profit of  $y - w$  from the employee. In the next period, the firm loses the employee with probability  $\delta$ , in which case its continuation value is zero. The firm retains the employee with probability  $1 - \delta$ , in which case its continuation value is  $J(w, y_+)$ . Now, consider (2.6). Let  $\hat{y}$  denote the worker's perception of the firm's productivity. In the current period, the worker perceives the firm's profit to be  $\hat{y} - w$ . In the next period, the worker expects the firm to retain the employee with probability  $1 - \delta$ , in which case he expects the firm's continuation value to be  $\hat{J}(w, \hat{y})$ .

The actual tightness of the labor market is given by

$$k = q(\theta(y))J(w(y), y). \quad (2.7)$$

That is, the actual tightness  $\theta(y)$  is such that the firm's cost from opening a vacancy,  $k$ , is equal to the benefit,  $q(\theta(y))J(w(y), y)$ —which is the product between the firm's probability of filling the vacancy and the firm's value of having an extra worker employed at the wage  $w(y)$ .

In contrast, the tightness of the labor market expected by the worker is given by

$$k = q(\hat{\theta})\hat{J}(\hat{w}, y^*). \quad (2.8)$$

That is, the tightness  $\hat{\theta}$  is such that, from the worker's perspective, the firm's cost from opening a vacancy,  $k$ , is equal to the benefit,  $q(\hat{\theta})\hat{J}(\hat{w}, y^*)$ —which is the product between the firm's probability of filling a vacancy and the firm's value of having an extra worker employed at the wage  $\hat{w}$ , as perceived by the worker.

Up to this point, the equilibrium condition that describe the actual agents' values and the actual aggregate outcomes of the economy do not interact with the equilibrium conditions that describe the workers' perception of the agents' values and the workers' expectations about aggregate outcomes. The two blocks of conditions interact in the outcome of the bargaining game between a worker and a firm, which I am going to characterize next.

I start with the characterization of the outcome of the bargaining game for the scenario in which the worker observes the productivity of the firm  $y$  and rationalizes the difference  $y - y^*$  as a permanent, firm-specific component of productivity. In this scenario, the worker believes that the value of reaching an agreement at the wage  $w$  is  $\hat{V}_1(w)$  for himself and  $\hat{J}(w, y)$  for the firm, and that the value of not reaching an agreement is  $\hat{V}_0$  for himself and 0 for the firm.

Given the worker's beliefs about the values from trading and not trading, his optimal

bargaining strategy is the same as in BRW. Specifically, when the worker makes a proposal, he demands the wage  $w_d^*$  and, when the worker receives a proposal, he accepts any wage offer  $w_o$  greater than  $\hat{w}_o$ , where  $w_d^*$  and  $\hat{w}_o$  are such that

$$\hat{J}(w_d^*, y) = e^{-\lambda\Delta} \hat{J}(\hat{w}_o, y), \quad (2.9)$$

$$\hat{V}_1(\hat{w}_o, y) = (1 - e^{-\mu\Delta})\hat{V}_0 + e^{-\mu\Delta}\hat{V}_1(w_d^*, y). \quad (2.10)$$

Equation (2.9) states that  $w_d^*$  is such that, from the perspective of the worker, the firm is indifferent between accepting  $w_d^*$  (left-hand side) and rejecting  $w_d^*$  (right-hand side). From the perspective of the worker, the firm's value of rejecting  $w_d^*$  is equal to the probability that the negotiation continues times the firm's value of reaching an agreement at the wage  $\hat{w}_o$ . Equation (2.10) states that  $\hat{w}_o$  is such that the worker is indifferent—according to his perception—between accepting  $\hat{w}_o$  (left-hand side) and rejecting  $\hat{w}_o$  (right-hand side). From the perspective of the worker, the value of rejecting  $\hat{w}_o$  is equal to the probability that the negotiation breaks down times the value of unemployment plus the probability that the negotiation continues times the value of reaching an agreement at the wage  $w_d^*$ .

The firm's optimal strategy is the best response to the worker's. Assuming that  $J(\hat{w}_o, y) \geq 0$ , the firm finds it optimal to offer the wage  $w_o^* = \hat{w}_o$ . It is easy to see why this is the case. If the firm offers a wage  $w_o = \hat{w}_o$ , the worker accepts  $\hat{w}_o$  and the firm's payoff is  $J(\hat{w}_o, y)$ . If the firm offers any wage  $w_o > \hat{w}_o$ , the worker accepts  $w_o$  and the firm's payoff is  $J(w_o, y)$ , which is strictly smaller than  $J(\hat{w}_o, y)$ . If the firm offers any wage  $w_o < \hat{w}_o$ , the worker rejects  $w_o$  and demands the wage  $w_d^*$ . If the firm accepts  $w_d^*$ , its payoff is  $\exp(-\mu\Delta)J(w_d^*, y)$ , which is strictly smaller than  $J(\hat{w}_o, y)$  since (2.9) implies that  $w_d^* > \hat{w}_o$ . If the firm rejects  $w_d^*$ , the firm finds itself again in the position of making a wage offer. Since the worker's strategy is stationary, the firm has nothing to gain from having delayed the trade.

The firm finds it optimal to accept any wage demand  $w_d$  such that

$$J(w_d, y) \geq e^{-\lambda\Delta} J(w_o^*, y). \quad (2.11)$$

Since  $J(w, y) \neq \hat{J}(w, y)$ , the firm may find it optimal to reject the worker's equilibrium wage demand  $w_d^*$ . In particular, if  $y < y^*$  and  $\phi_r < 1$ ,  $J(w, y) > \hat{J}(w, y)$  since the worker believes  $y$  to be a permanent component of productivity while  $y$  is an aggregate component of productivity that reverts back to the mean. Therefore, if  $y < y^*$ , the firm finds it optimal to accept  $w_d^*$ . Conversely, if  $y > y^*$ , the firm finds it optimal to reject  $w_d^*$ .

The equilibrium wage demand  $w_d^*$  and the equilibrium wage offer  $w_o^*$  are given by (2.8) and (2.9). Solving (2.8) and (2.9) with respect to  $w_d^*$  and  $w_o^*$  and taking the limit for  $\Delta \rightarrow 0$ , I find  $w_d^* = w_o^* = w(y)$ , where

$$w(y) = \frac{\lambda}{\lambda + \mu} y + \frac{\mu}{\lambda + \mu} (1 - \beta) \hat{V}_0. \quad (2.12)$$

In words, the wage demanded by the worker and the wage offered by the firm are iden-

tical, and they are equal to a weighted average between the worker's annuitized value of unemployment and the firm's current productivity, as perceived by the worker. The weight on the worker's value of unemployment is  $\mu/(\lambda + \mu)$  and the weight on the firm's productivity is  $\lambda/(\lambda + \mu)$ . I will denote  $\lambda/(\lambda + \mu)$  as  $\gamma$  and refer to it as the worker's bargaining power. I will denote  $\mu/(\lambda + \mu)$  as  $1 - \gamma$  and refer to it as the firm's bargaining power.

I am now in the position to characterize the solution of the bargaining game in the scenario where the worker observes the firm's productivity  $y$ . In this scenario, the firm and the worker reach an agreement with probability 1 at the wage  $w(y)$ . Similarly, in the scenario where the worker does not observe the firm's productivity  $y$  and, instead, believes it to be equal to  $y^*$ , the firm and the worker reach an agreement with probability 1 at the wage  $w(y) = w(y^*)$ .

I summarize the characterization of the bargaining game in the following proposition.

**Proposition 1.** (*Bargaining outcomes*). *The equilibrium outcome of the bargaining game is as follows:*

1. *If the worker observes the firm's productivity  $y$ , the worker and the firm reach an agreement with probability 1 at the wage  $w(y)$ , where  $w(y)$  is given by (2.12);*
2. *If the worker believes that the firm's productivity is  $y^*$ , the worker and the firm reach an agreement with probability 1 at the wage  $w(y^*)$ ;*
3. *When searching the market, the worker expects to earn the wage  $\hat{w} = w(y^*)$  upon meeting a firm.*

A few comments about Proposition 1 are in order. The equilibrium wage is determined entirely by the worker's beliefs. The worker's optimal bargaining strategy is the best response to the firm's bargaining strategy expected by the worker—i.e., the bargaining strategy that a firm would follow if the worker's and firm's agreement payoffs were  $\hat{V}_1(w)$  and  $\hat{J}(w, \hat{y})$  and the disagreement payoffs were  $\hat{V}_0$  and 0. The worker's best response to the firm's bargaining strategy expected by the worker is to demand the wage  $w_d^*$  and to accept any wage offer above  $\hat{w}_o$ , with  $w_d^* = \hat{w}_o = w(\hat{y})$  and  $\hat{y} = y$  in the first scenario, and  $\hat{y} = y^*$  in the second scenario. The firm understands that the worker's beliefs are incorrect and, hence, anticipates the worker's strategy. And given the worker's strategy, the firm has no choice but to trade at the wage  $w(\hat{y})$ . In some sense, its awareness of the worker's beliefs forces the firm to passively respond to the worker's bargaining strategy.

The equilibrium wage is an average between the worker's annuitized value of unemployment and the firm's productivity, as perceived by the worker. In the first scenario, the equilibrium wage does not respond to changes in the worker's actual value of unemployment caused by aggregate productivity fluctuations. Since the worker's actual value of unemployment moves in the same direction as aggregate productivity, the equilibrium wage tends to be “sticky.” This is the same type of stickiness obtained in Hall and Milgrom (2008), albeit through an entirely different channel. Hall and Milgrom (2008) assume

that, when the negotiation between the worker and the firm breaks down, the two parties cannot trade in the current period but they enter the next period matched. For this reason, the worker's and firm's bargaining strategies and, in turn, the wage outcome are independent from the worker's value of unemployment and, hence, from its changes. Here, the equilibrium wage does not respond to changes in the worker's value of unemployment because the worker believes that the value of unemployment to be constant.

In the second scenario, the equilibrium wage does not respond to changes in the firm's productivity nor to changes in the worker's value of unemployment caused by fluctuations in aggregate productivity. Hence, in the second scenario, the equilibrium wage is "rigid." Hall (2005) obtains the same result, but through a different mechanism. Hall (2005) assumes that the wage is not given by the outcome of a bargaining game between the worker and the firm, but rather it is determined by a social norm. Since the social norm is assumed to be invariant to aggregate productivity fluctuations, the wage is rigid. Here, the wage is rigid because the worker—no matter what aggregate productivity might be—is convinced that the aggregate state of productivity and the productivity of the firm with which he is bargaining are always equal to  $y^*$ .

Finally, note that the equilibrium wage equation (2.12) links the conditions that describe the workers' perception of the agents' values and the worker's expectations of aggregate outcomes with the conditions that describe the actual agents' values and aggregate outcomes. In fact, the equilibrium wage—which is entirely determined by the workers' beliefs—affects the firms' benefit from opening vacancies and, in turn, the actual market tightness.

I am now in the position to define a Stubborn Beliefs Equilibrium (SBE).

**Definition 2.** (SBE) An SBE is given by actual and perceived values  $\{V_0, V_1, J, \hat{V}_0, \hat{V}_1, \hat{J}\}$ , actual and expected market tightness  $\{\theta, \hat{\theta}\}$ , and actual and expected wages  $\{w, \hat{w}\}$  such that:

1. The values  $\{V_0, V_1, J, \hat{V}_0, \hat{V}_1, \hat{J}\}$  satisfy conditions (2.1)-(2.6);
2. The tightnesses  $\theta$  and  $\hat{\theta}$  satisfy conditions (2.7) and (2.8);
3. The wage  $w$  satisfies condition (2.12) and  $\hat{w} = w(y^*)$ .

It will be useful to compare the SBE with the standard Rational Expectations Equilibrium (REE), in which the workers' expectations are correct. An REE is defined as follows.

**Definition 3.** (REE) A REE is given by values  $\{V_0, V_1, J\}$ , tightness  $\theta$ , and wage  $w$  such that:

1. The values  $\{V_0, V_1, J\}$  satisfy conditions (2.1), (2.3) and (2.5);
2. The tightness  $\theta$  satisfies condition (2.7);
3. The wage  $w$  is such that the gains from trade accruing to the firm,  $J(w(y), y)$ , are equal to a fraction  $1 - \gamma$  of  $V_1(w(y), y) + J(w(y), y) - V_0(y)$ .

Before moving to the characterization of an SBE and its comparison with an REE, let me point out a couple of properties of equilibrium. First, the worker's beliefs about market tightness, job-finding probability, and wages are all correct when aggregate productivity  $y$  is equal to  $y^*$ . To see why this is the case, note that the worker's expected wage  $\hat{w}$  is equal to  $w(y^*)$ . Moreover, the worker's perception of the firm's value  $\hat{J}(w, y^*)$  is equal to the firm's actual value  $J(w, y^*)$  which, through equilibrium conditions (2.7) and (2.8), implies that the worker's expected market tightness  $\hat{\theta}$  is equal to  $\theta(y^*)$  and, hence, the worker's expected job-finding probability  $p(\hat{\theta})$  is equal to  $p(\theta(y^*))$ . Since the unconditional mean of aggregate productivity is  $y^*$ , the fact that the worker's expectations are correct when  $y = y^*$  implies that, up to a first-order approximation, the worker's expectations are correct on average.

Thus, there are two alternative interpretations for an SBE: (i) the worker believes aggregate productivity  $y$  is always equal to  $y^*$ , and he forms expectations about tightness, job-finding probability, and wages by solving for the equilibrium outcomes of a hypothetical labor market in which  $y$  never moves away from  $y^*$ ; (ii) the worker believes that aggregate productivity is always equal to  $y^*$ , and he forms expectations about tightness, job-finding probability, and wages based on their actual long-run averages. While I defined an SBE based on the first interpretation, it turns out to be consistent with the second interpretation as well.

Second, when  $y = y^*$ , the market tightness, job-finding probability, and wages in an SBE coincide with the market tightness, job-finding probability, and wages in an REE. To see why this is the case, it is sufficient to notice that, when  $y = y^*$ , the equilibrium conditions for an SBE coincide—up to a first-order approximation—with the equilibrium conditions for an REE. This property of an SBE implies that—when the economy is in its “normal” state  $y^*$ —the equilibrium outcomes in an SBE are the same as in a version of the model where workers have rational expectations.

### 3 Properties of equilibrium

In this section, I characterize the properties of an SBE and compare them with the properties of an REE. I am particularly interested in comparing the elasticity of the market tightness, the job-finding probability, the unemployment and vacancy rates with respect to aggregate productivity when workers have stubborn or rational beliefs about their labor market prospects. I find that the elasticity of the market tightness, job-finding probability, unemployment and vacancy rates are all higher in an SBE than in an REE.

#### 3.1 Properties of an REE

Let me start with the characterization of an REE. Let me denote as  $S(y)$  the surplus of a match between a worker and a firm, which is defined as the difference between the sum of the values to a worker and a firm if they do trade,  $V_1(w, y) + J(w, y)$ , and the sum of

their values if they do not trade,  $V_0(y)$ . Using (2.1), (2.3), (2.5) and the outcome of the bargaining game, I can write  $S(y)$  as

$$S(y) = y - b - \beta \mathbb{E}_{y_+} [p(\theta(y_+))\gamma S(y_+)] + \beta \mathbb{E}_{y_+} [(1 - \delta)S(y_+)]. \quad (3.1)$$

Equation (3.1) is a Bellman equation for the surplus of a match. In the current period, the flow of surplus is given by the difference between the joint income of a worker and a firm if they are matched and their joint income if they are not matched,  $y - b$ , net of the worker's option value of searching,  $\beta \mathbb{E}_{y_+} [p(\theta(y_+))\gamma S(y_+)]$ . In the next period, the match breaks with probability  $\delta$  and survives with probability  $1 - \delta$ . In the first case, the continuation surplus is zero. In the second case, the continuation surplus is  $S(y_+)$ .

Using the definition of surplus and the outcome of the bargaining game, I can write (2.7) as

$$k = q(\theta(y))(1 - \gamma)S(y). \quad (3.2)$$

Equation (3.2) states that the market tightness  $\theta(y)$  is such that the firm's cost of opening a vacancy is equal to the firm's probability of filling the vacancy times the firm's value of filling the vacancy—which, given the outcome of the bargaining game, is equal to a fraction  $1 - \gamma$  of the surplus of the match between the firm and a worker.

Taken together, equations (3.1) and (3.2) characterize an REE. In order to understand the cyclical properties of an REE, I differentiate (3.1) and (3.2) with respect to the aggregate productivity  $y$  and derive expressions for the elasticities of the surplus of a match and the tightness of the labor market with respect to  $y$  in a neighborhood of  $y^*$ . Differentiating (3.1) with respect to  $y$  yields

$$\begin{aligned} \frac{S'(y^*)y^*}{S(y^*)} &= \frac{1 - \beta [1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta \phi_r [1 - \delta - p(\theta(y^*))\gamma]} \cdot \frac{y^*}{y^* - b} \\ &\quad - \frac{\beta \phi_r p(\theta(y^*))\gamma \epsilon}{1 - \beta [1 - \delta - p(\theta(y^*))\gamma]} \cdot \frac{\theta'(y^*)y^*}{\theta(y^*)}, \end{aligned} \quad (3.3)$$

where  $\epsilon$  denotes the elasticity of the job-finding probability  $p$  with respect to  $\theta$ . Equation (3.3) states that the elasticity of the surplus with respect to  $y$  is given by the difference of two terms. The first term is proportional to the elasticity of  $y - b$  with respect to  $y$ , and captures the effect of an increase in  $y$  on the difference between the joint income generated by a firm and a worker when they are matched rather than unmatched. The second term is proportional to the elasticity of  $\theta$  with respect to  $y$ , and captures the effect of an increase in  $y$  on a worker's option value of searching.

Differentiating (3.2) with respect to  $y$  yields

$$\frac{\theta'(y^*)y^*}{\theta(y^*)} = \frac{1}{1 - \epsilon} \cdot \frac{S'(y^*)y^*}{S(y^*)}, \quad (3.4)$$

where  $1 - \epsilon$  is the elasticity of the job-filling probability  $q$  with respect to  $\theta$ . Equation (3.4) states that the elasticity of the labor market tightness with respect to  $y$  is proportional



to the elasticity of the surplus with respect to  $y$ . The constant of proportionality is the inverse of the elasticity of  $q$  with respect to  $\theta$  and captures the extent to which the tightness needs to move for the job-filling probability to absorb a given change in the surplus of a match.

Using (3.3) and (3.4), I can solve for the elasticity of the surplus and the elasticity of the market tightness. Specifically, these elasticities are

$$\frac{S'(y^*)y^*}{S(y^*)} = \frac{1 - \beta [1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta\phi_r [1 - \delta - p(\theta(y^*))\gamma/(1 - \epsilon)]} \cdot \frac{y^*}{y^* - b}, \quad (3.5)$$

and

$$\frac{\theta'(y^*)y^*}{\theta(y^*)} = \frac{1}{1 - \epsilon} \cdot \frac{1 - \beta [1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta\phi_r [1 - \delta - p(\theta(y^*))\gamma/(1 - \epsilon)]} \cdot \frac{y^*}{y^* - b}. \quad (3.6)$$

The elasticity of the job-finding probability,  $p(\theta(y))$ , the elasticity of the stationary unemployment rate,  $u(y) = \delta/(\delta + p(\theta(y)))$ , and the elasticity of the stationary vacancy rate,  $v(y) = u(y)\theta(y)$ , are all proportional to the elasticity of the market tightness. Specifically, these elasticities are

$$\frac{p'(\theta(y^*))\theta'(y^*)y^*}{p(\theta(y^*))} = \epsilon \cdot \frac{\theta'(y^*)y^*}{\theta(y^*)}, \quad (3.7)$$

$$\frac{u'(y^*)y^*}{u(y^*)} = -\epsilon \cdot \frac{p(\theta(y^*))}{\delta + p(\theta(y^*))} \cdot \frac{\theta'(y^*)y^*}{\theta(y^*)}, \quad (3.8)$$

$$\frac{v'(y^*)y^*}{v(y^*)} = \frac{\theta'(y^*)y^*}{\theta(y^*)} + \frac{u'(y^*)y^*}{u(y^*)}. \quad (3.9)$$

### 3.2 Properties of an SBE

I now characterize an SBE in the first scenario—i.e. when workers observe the productivity  $y$  of the firm with which they are bargaining and they rationalize any difference between  $y$  and  $y^*$  as a permanent and firm-specific component of productivity.

Let me denote as  $\hat{S}(y)$  the worker's perceived surplus of a match with a firm. I define  $\hat{S}(y)$  as the worker's perceived difference between the sum of his and the firm's value if they do trade,  $\hat{V}_1(w) + \hat{J}(w, y)$ , and the sum of their values if they do not trade,  $\hat{V}_0$ . Using (2.4) and (2.6), it follows that  $\hat{S}(y)$  is given by

$$\hat{S}(y) = \frac{y - (1 - \beta)\hat{V}_0}{1 - \beta(1 - \delta)}. \quad (3.10)$$

Using (3.10) and (2.12), it follows that the outcome of the bargaining game is such that, from the worker's perspective, he captures a fraction  $\gamma$  of the surplus and the firm captures a fraction  $1 - \gamma$  of the surplus. That is,

$$V_1(w(y)) - V_0 = \gamma\hat{S}(y), \text{ and } \hat{J}(w(y), y) = (1 - \gamma)\hat{S}(y). \quad (3.11)$$

Using (2.2) and (3.11) and the fact that  $\hat{\theta} = \theta(y^*)$ , it follows that the worker's perceived value of unemployment  $\hat{V}_0$  is such that

$$(1 - \beta)\hat{V}_0 = b + \beta p(\theta(y^*))\gamma\hat{S}(y). \quad (3.12)$$

Combining (3.10) and (3.12), I can write the worker's perceived surplus as

$$\hat{S}(y) = y - b - \beta p(\theta(y^*))\gamma\hat{S}(y^*) + \beta(1 - \delta)\hat{S}(y). \quad (3.13)$$

Equation (3.13) is a Bellman equation for the perceived surplus of a match. In the current period, the perceived flow of surplus is given by the difference between the income produced by the worker and the firm when they are matched rather than unmatched,  $y - b$ , net of the worker's perceived option value of searching,  $\beta p(\theta(y^*))\gamma\hat{S}(y^*)$ . In the next period, the match breaks up with probability  $\delta$  and continues with probability  $1 - \delta$ . In the first case, the perceived continuation surplus is zero. In the second case, the perceived continuation surplus is  $\hat{S}(y)$ .

Combining (2.7) and (3.11), I can write (2.7) as

$$k = q(\theta(y)) \left[ (1 - \gamma)\hat{S}(y) - \frac{\beta(1 - \delta)(1 - \phi_r)}{1 - \beta\phi_r(1 - \delta)} \cdot \frac{y - y^*}{1 - \beta(1 - \delta)} \right]. \quad (3.14)$$

Equation (3.14) states that the tightness of the labor market is such that the firm's cost of opening vacancy is equal to the firm's probability of filling a vacancy times the firm's value of filling a vacancy. In turn, the firm's value of filling a vacancy is equal to a fraction  $1 - \gamma$  of the perceived surplus  $\hat{S}(y)$  plus the difference between the firm's actual value of filling a vacancy and the worker's perception of it—a difference that exists because the worker interprets  $y - y^*$  to be a permanent firm-specific component of productivity while  $y - y^*$  is a transitory aggregate component of productivity.

Taken together, equations (3.13) and (3.14) characterize an SBE. In order to understand the cyclical behavior of an SBE, I differentiate (3.13) and (3.14) with respect to  $y$  and derive expressions for the elasticities of the perceived surplus of a match and the actual tightness of the market with respect to aggregate productivity  $y$  in a neighborhood of  $y^*$ . Differentiating (3.13) with respect to  $y$ , yields

$$\frac{\hat{S}'(y^*)y^*}{\hat{S}(y^*)} = \frac{1 - \beta[1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta(1 - \delta)} \cdot \frac{y^*}{y^* - b}. \quad (3.15)$$

Differentiating (3.14) with respect to  $y$ , yields

$$\frac{\theta'(y^*)y^*}{\theta(y^*)} = \frac{1}{1 - \epsilon} \cdot \frac{1 - \beta(1 - \delta)(1 + (1 - \phi_r)\gamma/(1 - \gamma))}{1 - \beta\phi_r(1 - \delta)} \cdot \frac{\hat{S}'(y^*)y^*}{\hat{S}(y^*)}. \quad (3.16)$$

Using (3.15) to substitute out  $\hat{S}'(y^*)y^*/\hat{S}(y^*)$  in (3.16), I can write the elasticity of

the labor market tightness as

$$\begin{aligned} \frac{\theta'(y^*)y^*}{\theta(y^*)} &= \frac{1}{1-\epsilon} \cdot \frac{1-\beta(1-\delta)(1+(1-\phi_r)\gamma/(1-\gamma))}{1-\beta\phi_r(1-\delta)} \\ &\quad \cdot \frac{1-\beta[1-\delta-p(\theta(y^*))\gamma]}{1-\beta(1-\delta)} \cdot \frac{y^*}{y^*-b}. \end{aligned} \quad (3.17)$$

Equation (3.17) states that the elasticity of the labor market tightness is proportional to the elasticity of the difference  $y - b$  between the income generated by the worker and the firm when they are matched and the income generated by the worker and the firm when they are not matched. The constant of proportionality is the product of three terms. The first term captures the extent to which the market tightness needs to change for the firm's probability of filling a vacancy to absorb changes in the firm's value of filling a vacancy. The second term captures the relationship between changes in the firm's value of filling a vacancy and changes in the perceived surplus of a match. The last term captures the relationship between changes in the perceived surplus of a match and changes in  $y - b$ .

The elasticity of the labor market tightness in an SBE is different than in an REE because of two workers' misperceptions. First, in an SBE, the worker does not recognize the existence of fluctuations in aggregate productivity and, hence, believes that his option value of searching is constant. For this reason, the worker's bargaining strategy is unaffected by the changes in the option value of searching caused by aggregate productivity fluctuations. This misperception tends to make the wage less sensitive and the market tightness more sensitive to aggregate productivity shocks. Second, in an SBE, the worker believes a firm has productivity  $y \neq y^*$  because  $y - y^*$  is a permanent firm-specific component of productivity rather than a transitory shock to the aggregate component of productivity. For this reason, the worker's bargaining strategy responds too much to the changes in the current productivity of the firm caused by aggregate productivity fluctuations. This misperception tends to make the wage more sensitive and the market tightness less sensitive to aggregate productivity shocks. Overall, the elasticity of the market tightness in an SBE may be higher or lower than in an REE. Yet, if aggregate productivity shocks are persistent enough, the second misperception is small and the elasticity of the market tightness is higher than in an SBE.

Next, I characterize the properties of an SBE in the second scenario—i.e. when workers do not observe the productivity  $y$  of the firm with which they are bargaining and, instead, they believe the productivity to be equal to  $y^*$ . In this scenario, the worker's perceived surplus from a match is given by

$$\hat{S} = y^* - b - \beta p(\theta(y^*))\gamma \hat{S} + \beta(1-\delta)\hat{S}. \quad (3.18)$$

Equation (3.18) is a Bellman equation for the perceived surplus. In the current period, the perceived flow of surplus is given by the difference between the perceived income generated by the worker and the firm when they are matched rather than unmatched,  $y^* - b$ , net

of the worker's perceived option value of searching,  $\beta p(\theta(y^*))\gamma\hat{S}$ . In the next period, the match breaks up with probability  $\delta$ , in which case the perceived continuation surplus is zero, and survives with probability  $1 - \delta$ , in which case the perceived continuation surplus is  $\hat{S}$ .

The tightness of the labor market is given by

$$k = q(\theta(y)) \left[ (1 - \gamma)\hat{S} + \frac{1 - \beta(1 - \delta)(1 - \phi_r)}{1 - \beta\phi_r(1 - \delta)} \cdot \frac{y - y^*}{1 - \beta(1 - \delta)} \right]. \quad (3.19)$$

Equation (3.19) states that the market tightness is such that the firm's cost of opening a vacancy is equal to the probability of filling the vacancy times the value of filling the vacancy. In turn, the firm's value of filling a vacancy is equal to a fraction  $1 - \gamma$  of the perceived surplus  $\hat{S}$  plus the difference between the firm's actual value of filling a vacancy and the worker's perception of such value—a difference that exists because the worker believes the firm's productivity to be  $y^*$  now and in the future, while the firm's productivity is  $y$  now and possibly in the future.

Differentiating (3.19) with respect to  $y$ , yields the following expression for the elasticity of the market tightness with respect to  $y$  in a neighborhood of  $y^*$

$$\frac{\theta'(y^*)y^*}{\theta(y^*)} = \frac{1}{1 - \epsilon} \cdot \frac{1}{1 - \gamma} \cdot \frac{1 - \beta[1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta\phi_r(1 - \delta)} \cdot \frac{y^*}{y^* - b}. \quad (3.20)$$

The elasticity of the market tightness is again proportional to the elasticity of  $y - b$  with respect to  $y$ . The constant of proportionality is the product of three terms. The first term captures the extent to which the market tightness needs to change for the firm's probability of filling a vacancy to absorb changes in the firm's value of filling a vacancy. The second and third terms captures the relationship between changes in  $y - b$  and changes in the firm's value of filling a vacancy.

Also in the second scenario, the elasticity of the market tightness is different than in an REE because of two workers' misperceptions. First, the worker does not recognize the existence of fluctuations in aggregate productivity and, hence, believes that his option value of searching is constant. For this reason, the worker's bargaining strategy is unaffected by changes in the option value of searching caused by aggregate productivity fluctuations. Second, the worker believes that the firm's productivity  $y^*$  rather than  $y$ . For this reason, the worker's bargaining strategy is also unaffected by the changes in the productivity of the firm caused by aggregate productivity fluctuations. Overall, in the second scenario, the worker's bargaining strategy and the bargained wage do not respond at all to changes in aggregate productivity and, hence, the elasticity of the market tightness is higher than in an REE. The elasticity of the market tightness in the second scenario is also higher than in the first scenario, as workers make the same mistake in calculating the option value of searching but underestimate, rather than overestimate, the present value of the changes in the productivity of the firm with which they are bargaining.

Since the elasticities of the job-finding probability, the unemployment rate, and the

vacancy rate depend on the elasticity of the market tightness in exactly the same way in an SBE and in an REE, the proof of the following proposition is complete.

**Proposition 4.** (*Labor market fluctuations*). *There exists a  $\phi_r^* \in (0, 1)$  such that for all  $\phi_r > \phi_r^*$ :*

1. *The elasticity of  $\theta(y)$ ,  $p(\theta(y))$ ,  $u(y)$  and  $v(y)$  with respect to  $y$  is greater in an SBE than in an REE.*
2. *In an SBE, the elasticity of  $\theta(y)$ ,  $p(\theta(y))$ ,  $u(y)$  and  $v(y)$  with respect to  $y$  is higher if workers do not observe the productivity  $y$  of the firm with which they are bargaining, but rather believe such productivity to be  $y^*$ .*

A simple back-of-the-envelope calibration reveals the importance of different assumptions about workers' beliefs on the elasticity of the labor market variables with respect to aggregate productivity shocks. Assume that the average UE rate  $p(\theta(y^*))$  is 30% per month, the average EU rate  $\delta$  is 2% per month, the elasticity  $\epsilon$  of the job-finding probability with respect to tightness is 0.5, that the worker's bargaining power  $\gamma$  is 0.5, and that the unemployment income  $b$  is half of the unconditional mean of aggregate productivity  $y^*$ . While these values are somewhat arbitrary, they are similar to the values typically used to calibrate search-theoretic models of the labor market (see, e.g., Shimer 2005, Menzio and Shi 2011, Martellini, Menzio and Visschers 2021). For the sake of simplicity, let me assume that the discount factor  $\beta$  is close to 1. Moreover, let me assume that productivity shocks are rare and permanent, in the sense that  $\phi_s$  and  $\phi_r$  are close to 1.

For  $\beta$ ,  $\phi_s$ ,  $\phi_r \rightarrow 1$ , the elasticity of the market tightness with respect to aggregate productivity shocks in an REE simplifies to

$$\frac{\theta'(y^*)y^*}{\theta(y^*)} = \underbrace{\frac{1}{1-\epsilon}}_2 \cdot \underbrace{\frac{\delta + p(\theta(y^*))\gamma}{\delta + p(\theta(y^*))\gamma/(1-\epsilon)}}_{.53} \cdot \underbrace{\frac{y^*}{y^* - b}}_2 = 2.12 \quad (3.21)$$

The elasticity is equal to 2.12. That is, a 1% decline in the aggregate productivity of labor leads to a 2.12% decline in the tightness of the labor market. In turn, the elasticity of the UE rate is about 1%, and so are the elasticities of the unemployment and vacancy rates. As a point of comparison, note that the ratio of the standard deviation of the cyclical component of the labor market tightness to the standard deviation of the cyclical component of productivity is about 20 in the post-war US. The relative elasticity of the cyclical component of the UE rate is about 6, and the relative elasticities of the unemployment and vacancy rates are both close to 10.

For  $\beta$ ,  $\phi_s$ ,  $\phi_r \rightarrow 1$ , the elasticity of the market tightness in an SBE where workers observe the productivity of their employer simplifies to

$$\frac{\theta'(y^*)y^*}{\theta(y^*)} = \frac{1}{1-\epsilon} \cdot \underbrace{\frac{\delta + p(\theta(y^*))\gamma}{\delta}}_{8.5} \cdot \frac{y^*}{y^* - b} = 34 \quad (3.22)$$

The elasticity is equal to 34. That is, a 1% decline in the aggregate productivity of labor leads to a 34% decline in the tightness of the labor market. The elasticity of the UE rate is about 17, and the elasticities of the unemployment and vacancy rates are both close to 16. All these elasticities are 16 times higher than in an REE.

For  $\beta, \phi_s, \phi_r \rightarrow 1$ , the elasticity of the market tightness in an SBE where workers do not observe the productivity of their employer simplifies to

$$\frac{\theta'(y^*)y^*}{\theta(y^*)} = \frac{1}{1-\epsilon} \cdot \underbrace{\frac{1}{1-\gamma}}_2 \cdot \underbrace{\frac{\delta + p(\theta(y^*))\gamma}{\delta}}_{8.5} \cdot \frac{y^*}{y^* - b} = 68 \quad (3.23)$$

The elasticity is equal to 68. In turn, the elasticity of the UE rate is about 34, and so are the elasticities of the unemployment and vacancy rates. These elasticities are 30 times higher than in an REE.

## 4 Optimal policy

In this section, I analyze the efficiency properties of an SBE and derive a formula for the optimal employment subsidy, i.e. the subsidy that makes an SBE efficient. I start the analysis by solving the problem of a utilitarian social planner, and derive the efficient tightness of the labor market as a function of aggregate productivity. I then solve for the SBE in the presence of an arbitrary employment subsidy that is allowed to depend on aggregate productivity. Lastly, I derive a formula for the employment subsidy that makes an SBE efficient. I find that that, even at the Hosios' condition, the SBE is inefficient and the optimal employment subsidy is countercyclical.

### 4.1 Social planner problem

A utilitarian social planner controls the tightness of the labor market to maximize the present value of aggregate income discounted at the factor  $\beta$ . Given a measure  $e$  of employed workers, a measure  $u$  of unemployed workers, and an aggregate productivity of  $y$ , the value  $W(e, u, y)$  of the social plan is such that

$$\begin{aligned} W(e, u, y) &= ey + ub + \beta \mathbb{E}_{y_+} \left[ \max_{\theta \geq 0} -k\theta u + W(e_+, u_+, y_+) \right], \\ \text{s.t. } e_+ &= e(1 - \delta) + up(\theta), \text{ and } u_+ = u(1 - p(\theta)) + e\delta. \end{aligned} \quad (4.1)$$

In the current period, aggregate income is the sum of the income produced by the workers who are employed,  $ey$ , and the income produced by the workers who are unemployed,  $ub$ . In the next period,  $k\theta u$  units of income are spent to create vacancies and the continuation value of the social plan is  $W(e_+, u_+, y_+)$ , where  $e_+$  and  $u_+$  denote next period's measures

of employed and unemployed workers. As shown in Menzio and Shi (2011),  $W(e, u, y)$  is linear in  $u$  and  $e$ .

Let  $S_P(y)$  denote the difference between the value to the planner of an additional employed worker and the value to the planner of an additional unemployed worker. It is easy to verify that  $S_P(y)$  is such that

$$S_P(y) = y - b - \beta \mathbb{E}_{y_+} [p(\theta_P(y_+))S_P(y_+) - k\theta_P(y_+)] + \beta \mathbb{E}_{y_+} [(1 - \delta)S_P(y_+)]. \quad (4.2)$$

In the current period, the difference between the social value of an employed worker and the social value of an unemployed worker is the difference in the income that they produce,  $y - b$ , net of the social value generated by the search of the unemployed worker,  $p(\theta_P(y_+))S_P(y_+) - k\theta_P(y_+)$ . In the next period, the employed worker becomes unemployed with probability  $\delta$ , in which case the continuation difference in the social value of the two workers is zero, and he remains employed with probability  $1 - \delta$ , in which case the continuation difference in the social value of the two workers is  $S_P(y_+)$ .

Let  $\theta_P(y)$  denote the optimal market tightness. It is easy to verify that  $\theta_P(y)$  is such that

$$\begin{aligned} k &= p'(\theta_P(y))S_P(y) \\ &= q(\theta_P(y))\epsilon(\theta_P(y))S_P(y) \end{aligned} \quad (4.3)$$

The optimal market tightness is such that the marginal cost of increasing the tightness,  $k$ , is equal to the marginal benefit—which is given by the increase in the probability that an unemployed worker becomes employed,  $p'(\theta_P(y))$ , times the difference between the social value of an employed worker and the social value of an unemployed worker,  $S_P(y)$ . The second line makes use of the fact that  $q(\theta) = p(\theta)/\theta$  and of the definition of  $\epsilon(\theta)$ .

It is useful to take a linear approximation of (4.2) around  $y = y^*$ , which yields

$$S_P(y^*) = y^* - b - \beta p(\theta_P(y^*))(1 - \epsilon(\theta_P(y^*)))S_P(y^*) + \beta(1 - \delta)S_P(y^*), \quad (4.4)$$

$$S'_P(y^*) = 1 - \beta \phi_r p(\theta_P(y^*))S'_P(y^*) + \beta \phi_r (1 - \delta)S'_P(y^*). \quad (4.5)$$

The expression in (4.4) is derived from a linear approximation of (4.2) making use of the fact that (4.3) implies that  $p(\theta_P(y^*))S_P(y^*) - k\theta_P(y^*)$  is equal to  $p(\theta_P(y^*))(1 - \epsilon(\theta_P(y^*)))S_P(y^*)$ . The expression in (4.5) is derived from a linear approximation of (4.2) making use of the fact that (4.3) implies that  $k$  is equal to  $p'(\theta_P(y))S_P(y)$ .

## 4.2 Optimal employment subsidy

I now want to derive a formula for the optimal employment subsidy  $t(y)$ . The subsidy is a flow transfer from the government to every firm that is currently employing a worker. The subsidy is financed through a lump-sum tax levied on all workers, irrespective of whether they are currently employed or unemployed. The subsidy is optimal when it is such that

the market tightness in the equilibrium,  $\theta(y)$ , coincides with the market tightness in the solution of the social planner's problem,  $\theta_P(y)$ . In order to simplify the characterization of the optimal employment subsidy, I will assume that the elasticity  $\epsilon(\theta)$  of the job-finding probability with respect to the market tightness is a constant  $\epsilon$  in a neighborhood of  $\theta_P(y^*)$ .

Let me start by characterizing the optimal employment subsidy  $t(y)$  in an REE. Given the subsidy  $t(y)$ , the surplus  $S(y)$  and the tightness  $\theta(y)$  are such that

$$S(y) = y + t(y) - b - \beta \mathbb{E}[p(\theta(y_+))\gamma S(y_+)] + \beta \mathbb{E}[(1 - \delta)S(y_+)], \quad (4.6)$$

$$k = q(\theta(y))(1 - \gamma)S(y). \quad (4.7)$$

A linear approximation of (4.6) around  $y^*$  yields

$$S(y^*) = y^* + t(y^*) - b - \beta p(\theta(y^*))\gamma S(y^*) + \beta(1 - \delta)S(y^*), \quad (4.8)$$

$$\begin{aligned} S'(y^*) &= 1 + t'(y^*) - \beta \phi_r p'(\theta(y^*))\theta'(y^*)\gamma S(y^*) \\ &\quad - \beta p(\theta(y^*))\gamma S'(y^*) + \beta \phi_r(1 - \delta)S'(y^*). \end{aligned} \quad (4.9)$$

A comparison between (4.3) and (4.7) reveals that the employment subsidy  $t(y)$  is optimal if and only if the firm's benefit from opening a vacancy,  $q(\theta(y))(1 - \gamma)S(y)$ , is equal to the planner's benefit from opening a vacancy,  $q(\theta_P(y))\epsilon S_P(y)$ , for  $\theta(y) = \theta_P(y)$ . Using the fact that  $S(y^*)$  is approximately equal to (4.8) and  $S_P(y^*)$  is approximately equal to the expression in (4.4), the optimal  $t(y)$  for  $y = y^*$  can be written as

$$t(y^*) = \left\{ \frac{\epsilon}{1 - \gamma} \cdot \frac{1 - \beta[1 - \delta - p(\theta_P(y^*))\gamma]}{1 - \beta[1 - \delta - p(\theta_P(y^*))\gamma/(1 - \epsilon)]} - 1 \right\} (y^* - b). \quad (4.10)$$

Using the fact that  $S'(y^*)$  is approximately equal to (4.9) and that  $S'_P(y^*)$  is approximately equal to (4.5), the optimality condition for  $t(y)$  in a neighborhood of  $y^*$  can be written as

$$1 + t'(y^*) = \frac{\epsilon}{1 - \gamma} \cdot \frac{1 - \beta \phi_r [1 - \delta - p(\theta_P(y^*))\gamma/(1 - \epsilon)]}{1 - \beta \phi_r [1 - \delta - p(\theta_P(y^*))\gamma]}. \quad (4.11)$$

The formula in (4.10) implies that the optimal employment subsidy  $t(y^*)$  is positive if  $\epsilon > 1 - \gamma$ , negative if  $\epsilon < 1 - \gamma$ , and zero if  $\epsilon = 1 - \gamma$ . The properties of  $t(y^*)$  are an immediate consequence of the well-known efficiency properties of search-theoretic models (e.g., Mortensen 1982, Hosios 1990). When a firm opens a vacancy, it creates a negative congestion externality on the other firms searching the labor market. The congestion externality is equal to the difference between the probability that the vacancy is filled,  $M(u, v)/v$ , and the number of additional matches created by the vacancy,  $M_v(u, v)$ , times the firm's share of the surplus,  $(1 - \gamma)S(y)$ . When a firm opens a vacancy, it also creates a positive thick-market externality on the workers searching the labor market. The thick-market externality is equal to the number of additional matches created by the vacancy,  $M_v(u, v)$ , times the worker's share of the surplus,  $\gamma S(y)$ . Therefore, if the elasticity of



the matching function with respect to vacancies,  $\epsilon$ , exceeds the firm's bargaining power,  $1 - \gamma$ , the thick-market externality dominates and the equilibrium market tightness is inefficiently low. In this case, the optimal employment subsidy is positive. If  $\epsilon > 1 - \gamma$ , the congestion externality dominates, the equilibrium market tightness is inefficiently high, and the optimal employment subsidy is negative. At the Hosios' condition, where  $\epsilon = 1 - \gamma$ , the equilibrium is efficient and the optimal employment subsidy is zero.

The formula in (4.11) implies that the derivative  $t'(y^*)$  of the optimal employment subsidy is positive if  $\epsilon > 1 - \gamma$ , negative if  $\epsilon < 1 - \gamma$ , and zero if  $\epsilon = 1 - \gamma$ . The properties of  $t'(y^*)$  are also easy to understand. An increase in aggregate productivity  $y$  leads to an increase in the surplus. If  $\epsilon > 1 - \gamma$ , the increase in the surplus magnifies the difference between the thick-market and the congestion externalities, and the optimal employment subsidy increases. If  $\epsilon < 1 - \gamma$ , the increase in the surplus magnifies the difference between the congestion and the thick-market externalities, and the optimal employment subsidy decreases. If  $\epsilon = 1 - \gamma$ , the equilibrium remains efficient and the optimal employment subsidy remains equal to zero.

Now, let me characterize the optimal employment subsidy in an SBE. I begin by considering the scenario in which the worker observes the productivity  $y$  as well as the employment subsidy  $t(y)$  of the firm with which he is bargaining. In this scenario, the worker's perceived surplus  $\hat{S}(y)$  and the market's actual tightness  $\theta(y)$  are such that

$$\hat{S}(y) = y + t(y) - b - \beta p(\theta(y^*))\gamma \hat{S}(y^*) + \beta(1 - \delta)\hat{S}(y), \quad (4.12)$$

$$k = q(\theta(y)) \left[ (1 - \gamma)\hat{S}(y) - \frac{\beta(1 - \delta)(1 - \phi_r)}{1 - \beta\phi_r(1 - \delta)} \cdot \frac{1 + t'(y^*)}{1 - \beta(1 - \delta)} \cdot (y - y^*) \right], \quad (4.13)$$

where the second term on the right-hand side of (4.13) is the difference between the firm's actual value of filling a vacancy and the worker's perception of such value, computed using a linear approximation of  $t(y)$  around  $y^*$ .

The optimal employment subsidy is such that the firm's value of filling a vacancy, i.e. the right-hand side of (4.13), is equal to the planner's value of filling a vacancy, i.e. the right-hand side of (4.3). For  $y = y^*$ , the optimality condition for the employment subsidy yields (4.10). For  $y = y^*$ , the equilibrium conditions for an SBE coincide with the equilibrium condition for an REE and, hence, the optimal employment subsidy is the same in an SBE as in an REE. For  $y$  in a neighborhood of  $y^*$ , the optimality condition for the employment subsidy yields

$$1 + t'(y^*) = \frac{\epsilon}{1 - \gamma} \cdot \frac{1 - \beta\phi_r[1 - \delta - p(\theta_P(y^*))\gamma/(1 - \epsilon)]}{1 - \beta[1 - \delta - p(\theta_P(y^*))]} \cdot \frac{1 - \beta\phi_r(1 - \delta)}{1 - \beta\phi_r[1 - \delta - p(\theta_P(y^*))\gamma/(1 - \epsilon)]} \cdot \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta)[1 + (1 - \phi_r)\gamma/(1 - \gamma)]}. \quad (4.14)$$

The first term on the right-hand side of (4.14) is equal to  $1 + t'(y^*)$  in an REE. The second term on the right-hand side of (4.14) is equal to the elasticity of the market tightness with

respect to  $y$  in an REE relative to the elasticity of the market tightness with respect to  $y$  in an SBE. If the market tightness is less elastic in an SBE than in an REE, the second term is larger than 1. This is the case if the persistence  $\phi_r$  of aggregate productivity shock is smaller than  $\phi_r^*$ . If the market tightness is more elastic in an SBE than in an REE, the second term is smaller than 1. This is the case if  $\phi_r$  is greater than  $\phi_r^*$ .

The formula in (4.14) implies that, as long as  $\phi_r > \phi_r^*$ , the optimal employment subsidy in an SBE is the given by the product between the optimal employment subsidy in an REE and a countercyclical term. For  $\epsilon > 1 - \gamma$ , the derivative of the optimal employment subsidy with respect to aggregate productivity  $y$  is positive in an REE, and it is either positive but smaller or altogether negative in an SBE. For  $\epsilon < 1 - \gamma$ , the derivative of the optimal employment subsidy with respect to  $y$  is negative in an REE, and it is more negative in an SBE. For  $\epsilon = 1 - \gamma$ , the case in which the optimal employment subsidy in an REE is equal to zero for all  $y$ , the derivative of the optimal employment subsidy with respect to  $y$  is negative. These properties are easy to understand. In an SBE, workers incorrectly perceive that the value of searching the labor market is acyclical and, hence, they bargain wages that are less procyclical than in an REE. As a result of the lower procyclicality of wages, the labor market tightness is more procyclical than in an REE. In order to correct for the higher procyclicality of the labor market tightness, the employment subsidy must incorporate an additional countercyclical term.

Lastly, I consider the scenario in which the worker does not observe neither the productivity  $y$  nor the employment subsidy  $t(y)$  of the firm with which he is bargaining, but rather believes that the firm's productivity is  $y^*$  and the employment subsidy is  $t(y^*)$ . In this scenario, the worker's perceived surplus  $\hat{S}$  and the market's actual tightness  $\theta(y)$  are such that

$$\hat{S} = y^* + t(y^*) - b - \beta p(\theta(y^*))\gamma \hat{S} + \beta(1 - \delta)\hat{S}, \quad (4.15)$$

$$k = q(\theta(y)) \left[ (1 - \gamma)\hat{S} + \frac{1 + t'(y^*)}{1 - \beta(1 - \delta)} \cdot (y - y^*) \right], \quad (4.16)$$

where the second term on the right-hand side of (4.16) is the difference between the firm's actual value of filling a vacancy and the worker's perception of such value, computed using a linear approximation of  $t(y)$  around  $y^*$ .

The optimal employment subsidy is such that the firm's value of filling a vacancy, i.e. the right-hand side of (4.16), is equal to the planner's value of filling a vacancy, i.e. the right-hand side of (4.3). For  $y = y^*$ , the optimality condition for the employment subsidy yields (4.10). For  $y$  in a neighborhood of  $y^*$ , the optimality condition for the employment subsidy yields

$$\begin{aligned} 1 + t'(y^*) &= \frac{\epsilon}{1 - \gamma} \cdot \frac{1 - \beta\phi_r[1 - \delta - p(\theta_P(y^*))\gamma/(1 - \epsilon)]}{1 - \beta[1 - \delta - p(\theta_P(y^*))]} \\ &\quad \cdot (1 - \gamma) \frac{1 - \beta\phi_r(1 - \delta)}{1 - \beta\phi_r[1 - \delta - p(\theta_P(y^*))\gamma/(1 - \epsilon)]}. \end{aligned} \quad (4.17)$$

The first term on the right-hand side of (4.17) is equal to  $1 + t'(y^*)$  in an REE. The second term on the right-hand side of (4.17) is equal to the elasticity of the market tightness with respect to  $y$  in an REE relative to the elasticity of the market tightness with respect to  $y$  in an SBE. The second term is smaller than 1. Hence, the optimal employment subsidy is more countercyclical in an SBE than in an REE. The second term is also smaller than its analogue in (4.14). Hence, the optimal employment subsidy is even more countercyclical in an SBE if workers do not observe the productivity of the firm with which they are bargaining.

The following proposition summarizes the characterization of the optimal employment subsidy.

**Proposition 5.** (*Optimal policy*). *For all  $\phi_r > \phi_r^*$ :*

1. *In an REE, the optimal employment subsidy is positive and procyclical if  $\epsilon > 1 - \gamma$ , negative and countercyclical if  $\epsilon < 1 - \gamma$ , and always equal to zero if  $\epsilon = 1 - \gamma$ .*
2. *In an SBE, the optimal employment subsidy is more countercyclical than in an REE. The optimal employment subsidy is more countercyclical if workers do not observe the productivity  $y$  of the firm with which they are bargaining, but rather believe such productivity to be  $y^*$ .*

In order to appreciate the impact of different assumptions about workers' expectations on the design of the optimal employment subsidy, it is useful to return to our back-of-the-envelope calibration. Recall that we calibrate the model to an average UE rate  $p(\theta(y^*))$  of 30% per month, an average EU rate  $\delta$  of 2% per month, an elasticity  $\epsilon$  of the job-finding probability with respect to tightness of 0.5, a worker's bargaining power  $\gamma$  of 0.5, and an unemployment income  $b$  equal to half of  $y^*$ .

Since the Hosios condition  $\epsilon = 1 - \gamma$  holds, the optimal employment subsidy at  $y^*$  is equal to zero both in an REE and in an SBE. For  $\beta, \phi_s, \phi_r \rightarrow 1$ , the derivative of the optimal employment subsidy in an REE simplifies to

$$t'(y^*) = \underbrace{\frac{\epsilon}{1-\gamma} \frac{\delta + \frac{\gamma}{1-\epsilon} p(\theta_P(y^*))}{\delta + p(\theta_P(y^*))}}_1 - 1 = 0. \quad (4.18)$$

The derivative of the optimal employment subsidy in an SBE where workers observe the productivity of their employer simplifies to

$$t'(y^*) = \underbrace{\frac{\epsilon}{1-\gamma} \frac{\delta + \frac{\gamma}{1-\epsilon} p(\theta_P(y^*))}{\delta + p(\theta_P(y^*))}}_1 \cdot \underbrace{\frac{\delta}{\delta + \frac{\gamma}{1-\epsilon} p(\theta_P(y^*))}}_{0.06} - 1 = -0.94. \quad (4.19)$$

The derivative of the optimal employment subsidy in an SBE where workers do not observe

the productivity of their employer simplifies to

$$t'(y^*) = \underbrace{\frac{\epsilon}{1-\gamma} \frac{\delta + \frac{\gamma}{1-\epsilon} p(\theta_P(y^*))}{\delta + p(\theta_P(y^*))}}_1 \cdot \underbrace{\frac{\delta(1-\gamma)}{\delta + \frac{\gamma}{1-\epsilon} p(\theta_P(y^*))}}_{0.03} - 1 = -0.97. \quad (4.20)$$

In an REE, the optimal employment subsidy remains equal to zero in response to changes in aggregate labor productivity. In an SBE where workers observe the productivity of their employer, the optimal employment subsidy decreases by 94 cents for any 1 dollar increase in the aggregate component of productivity. In other words, the optimal employment subsidy is such that the post-subsidy output of a firm-worker match increases by only 6 cents for every 1 dollar of increase in pre-subsidy output. In an SBE where workers do not observe the productivity of their employer, the optimal employment subsidy decreases by 97 cents for any 1 dollar of increase in the aggregate component of productivity. In other words, the optimal employment subsidy is such that the post-subsidy output of a firm-worker match increases by only 6 cents for every 1 dollar of increase in pre-subsidy output. These findings show that almost all of the volatility of the labor market induced by productivity shocks in an SBE is inefficient, and the optimal employment subsidy is tasked with undoing almost all of productivity fluctuations.

## 5 Rational and stubborn workers

I now study a version of the model in which workers with stubborn beliefs coexist with workers with rational expectations. The extension is natural, since presumably some workers are aware of aggregate shocks and adjust accordingly their expectations about their probability of finding a job and the wage they would earn when hired. More importantly, the extension provides new and surprising insights. It would be natural to conjecture that a model in which some workers have stubborn beliefs and some workers have rational expectations behaves like a mixture of an REE and an SBE. The conjecture, however natural, turns out to be wrong. A model in which some workers have stubborn beliefs and some workers have rational expectations behaves exactly like an SBE in recessions, and it behaves like a mixture of an REE and an SBE only in expansions. The intuition behind this result is relatively simple. In recessions, a firm cannot successfully wage discriminate between rational and stubborn workers, since a rational worker can mimic the strategy of a stubborn worker and earn the same wage. As a result, in response to a negative productivity shock, the average wage is as downward sticky or rigid as in an SBE and the market tightness, job-finding probability, unemployment and vacancy rates are as elastic as in an SBE. In expansions, a rational worker will signal his type to the firm and, hence, earn a different wage than a stubborn worker. As a result, in response to a positive productivity shock, the average wage is upward sticky or rigid proportionally to the fraction of stubborn workers in the economy. Hence, the market tightness, job-finding probability, unemployment and vacancy rates are not as elastic as in an SBE.

## 5.1 Equilibrium conditions

I consider a version of the model in which there are two types of workers: stubborn and rational. Stubborn workers ( $S$ ) believe that the aggregate component of productivity  $y$  is always equal to  $y^*$  and, based on such belief, they compute the equilibrium of a hypothetical labor market and use it to form expectations about the tightness of the labor market, the job-finding probability, the firm's bargaining strategy, and the wage. Rational workers ( $R$ ) know the actual law of motion for aggregate productivity, they know the current realization of aggregate productivity, and they know the economic environment—including the measure and beliefs of workers of type  $S$ . Therefore, they know the actual equilibrium mapping between aggregate productivity, market tightness, job-finding probability, firm's bargaining strategy and wages. The measure of workers of type  $S$  is  $\sigma$  and the measure of workers of type  $R$  is  $1 - \sigma$ , with  $\sigma \in (0, 1)$ . Firms, like workers of type  $R$ , know the law of motion and the current realization of aggregate productivity and the economic environment—including the measure and beliefs of workers of type  $S$  and  $R$ . However, when they meet a worker, firms do not know his type.

To define an equilibrium for this version of the model, I need some extra notation. Let  $V_{i,0}(y)$  and  $V_{i,1}(w, y)$  denote the actual values of unemployment and employment for a worker of type  $i$ , and let  $\hat{V}_{S,0}$  and  $\hat{V}_{S,1}(w)$  denote the values of unemployment and employment perceived by a worker of type  $S$ . Let  $J(w, y)$  denote the actual value of a worker to a firm, and let  $\hat{J}_S(w, y)$  denote the firm's value as perceived by a worker of type  $S$ . Let  $\theta(y)$  and  $\hat{\theta}$  denote the actual tightness of the labor market, and the tightness of the labor market expected by a worker of type  $S$ . Lastly, let  $w_i(y)$  the actual wage for a worker of type  $i$ , and with  $\hat{w}_S$  the wage expected by a worker of type  $S$ .

The worker's actual and perceived values of unemployment,  $V_{i,0}(y)$  and  $\hat{V}_{S,0}$ , are

$$V_{i,0}(y) = b + \beta \mathbb{E}_{y_+} [p(\theta(y_+))V_{i,1}(w_i(y_+), y_+) + (1 - p(\theta(y_+)))V_{i,0}(y_+)], \quad (5.1)$$

$$\hat{V}_{S,0} = b + \beta \left[ p(\hat{\theta})\hat{V}_{S,1}(\hat{w}_S) + (1 - p(\hat{\theta}))\hat{V}_{S,0} \right]. \quad (5.2)$$

The worker's actual and perceived values of employment,  $V_{i,1}(w, y)$  and  $\hat{V}_{S,1}(w)$ , are

$$V_{i,1}(w, y) = w + \beta \mathbb{E}_{y_+} [(1 - \delta)V_{i,1}(w, y_+) + \delta V_{i,0}(y_+)], \quad (5.3)$$

$$\hat{V}_{S,1}(w) = w + \beta \left[ (1 - \delta)\hat{V}_{S,1}(w) + \delta \hat{V}_{S,0} \right]. \quad (5.4)$$

The firm's actual and perceived values of employing a worker,  $J(w, y)$  and  $\hat{J}_S(w, \hat{y})$ , are

$$J(w, y) = y - w + \beta \mathbb{E}_{y_+} [(1 - \delta)J(w, y_+)], \quad (5.5)$$

$$\hat{J}_S(w, \hat{y}) = \hat{y} - w + \beta(1 - \delta)\hat{J}_S(w, \hat{y}). \quad (5.6)$$

The values for workers of type  $S$  and the values for the firm are the same as in the definition of an SBE in Section 2, and need no further comment. The values for workers of type  $R$  are the same as in the definition of an REE, since these workers have rational

expectations.

The actual market tightness,  $\theta(y)$ , and the market tightness expected by workers of type  $S$ ,  $\hat{\theta}_S$ , are such that

$$k = q(\theta(y)) [\sigma J(w_S(y), y) + (1 - \sigma) J(w_R(y), y)], \quad (5.7)$$

$$k = q(\hat{\theta}_S) \hat{J}_S(\hat{w}_S, y^*). \quad (5.8)$$

Consider (5.7). The firm pays the cost  $k$  to open a vacancy. The firm fills the vacancy with probability  $q(\theta(y))$ . With probability  $\sigma$ , the firm fills the vacancy with a worker of type  $S$ , to whom it pays a wage  $w_S(y)$ . With probability  $1 - \sigma$ , the firm fills the vacancy with a worker of type  $R$ , whom it pays a wage  $w_R(y)$ . The actual market tightness is such that the firm's cost and benefit from opening a vacancy are equal. Now consider (5.8). From the  $S$ -worker's perspective, the firm's cost of opening a vacancy is  $k$ . The firm's benefit of opening a vacancy is the probability of filling the vacancy,  $q(\hat{\theta}_S)$ , times the value of filling the vacancy, which is  $\hat{J}_S(\hat{w}_S, y^*)$  since the worker expects the firm to have productivity  $y^*$  and to pay the wage  $\hat{w}_S$ . The market tightness expected by a worker of type  $S$  is such that, from his perspective, the firm's cost and benefit from opening a vacancy are equal.

## 5.2 Bargaining outcomes

I now turn to the analysis of the bargaining game between a firm and a worker. The protocol of the game is the same as in Section 2. Now, though, the game is one of asymmetric information, since the firm knows that there is a probability  $\sigma$  that the worker with whom it is bargaining is of type  $S$ , a probability  $1 - \sigma$  that the worker with whom it is bargaining is of type  $R$ , but it does not know the worker's actual type. In order to simplify the analysis of the game, I restrict attention to the case in which productivity shocks are nearly permanent, i.e.  $\phi_r \rightarrow 1$  and  $\phi_s \rightarrow 1$ .

As an equilibrium concept, I adopt the Perfect Sequential Equilibrium (PSE) proposed by Grossman and Perry (1986 a and b). A PSE is a meta-strategy for the firm and the worker contingent on the beliefs of the firm, and an updating rule for the firm's belief. The meta-strategy of the firm and the worker is optimal conditional on the belief of the firm, for any belief of the firm.<sup>1</sup> The updating rule for the firm's belief is given by Bayes' rule if the firm observes an action from the worker that occurs with positive probability given the firm's belief and the worker's strategy. When the firm observes an action from the worker that occurs with probability zero given the firm's belief and the worker's strategy, the posterior belief is credible—i.e. the firm seeks for a subset of the worker types in the support of its prior belief that would be better off taking the off-equilibrium action rather than playing the equilibrium action if the firm believes that the action comes from

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<sup>1</sup>I only consider PSE in pure strategies. For this reason, I restrict attention to firm's beliefs  $\hat{\sigma} = 0, \sigma$  or 1.

that subset of types. If there is no such subset of worker types, the only restriction on the posterior belief is that its support should be contained in the support of the prior belief.<sup>2</sup> The above definition of a PSE needs to be amended to account for workers of type  $S$ . In particular, I do not require the meta-strategy of an  $S$ -worker to be optimal. Rather, the meta-strategy of an  $S$ -worker is the best response to the strategy of the firm that he expects, i.e. the strategy that the firm would follow in a world where aggregate productivity was always equal to  $y^*$ .

I start by characterizing a PSE in the scenario where the  $S$ -worker observes the productivity  $y$  of the firm and interprets the difference  $y - y^*$  as a permanent firm-specific component of productivity. As a preliminary step, it is useful to define some wages. Let  $w_{S,d}$  and  $w_{S,o}$  denote the wage demand and the wage offer in the equilibrium of the bargaining game between a firm and a worker of type  $S$  under full information, i.e. the game between a firm and an  $S$ -worker in which the firm believes that the worker is of type  $S$  with probability 1. This is the game analyzed in Section 2 and, given  $\phi_r \rightarrow 1$  and  $\phi_s \rightarrow 1$ , the equilibrium wages are

$$w_{S,d} = \frac{1 - e^{-\lambda\Delta}}{1 - e^{-(\lambda+\mu)\Delta}}y + \frac{1 - e^{-\mu\Delta}}{1 - e^{-(\lambda+\mu)\Delta}}e^{-\lambda\Delta}(1 - \beta)\hat{V}_{S,0}, \quad (5.9)$$

$$w_{S,o} = \frac{1 - e^{-\lambda\Delta}}{1 - e^{-(\lambda+\mu)\Delta}}e^{-\mu\Delta}y + \frac{1 - e^{-\mu\Delta}}{1 - e^{-(\lambda+\mu)\Delta}}(1 - \beta)\hat{V}_{S,0}. \quad (5.10)$$

Similarly, let  $w_{R,d}$  and  $w_{R,o}$  denote the wage demand and the wage offer in the equilibrium of the bargaining game between the firm and a worker of type  $R$  under full information, i.e. the game between a firm and an  $R$ -worker in which the firm believes that the worker is of type  $R$  with probability 1. This is the same game analyzed by Binmore, Rubinstein and Wolinsky (1986) and the equilibrium wages are

$$w_{R,d} = \frac{1 - e^{-\lambda\Delta}}{1 - e^{-(\lambda+\mu)\Delta}}y + \frac{1 - e^{-\mu\Delta}}{1 - e^{-(\lambda+\mu)\Delta}}e^{-\lambda\Delta}(1 - \beta)V_{R,0}(y), \quad (5.11)$$

$$w_{R,o} = \frac{1 - e^{-\lambda\Delta}}{1 - e^{-(\lambda+\mu)\Delta}}e^{-\mu\Delta}y + \frac{1 - e^{-\mu\Delta}}{1 - e^{-(\lambda+\mu)\Delta}}(1 - \beta)V_{R,0}(y). \quad (5.12)$$

Notice that  $w_{S,o} < w_{S,d}$  and  $w_{R,o} < w_{R,d}$ . Furthermore, under the conjecture that  $V_{R,0}(y)$  is continuous and increasing in  $y$  and such that  $V_{R,0}(y) < \hat{V}_{S,0}$  for  $y < y^*$  and  $V_{R,0}(y) > \hat{V}_{S,0}$  for  $y > y^*$ ,  $w_{R,d} < w_{S,o}$  for  $y < y^*$  and  $w_{S,d} < w_{R,o}$  for  $y > y^*$  for all  $\Delta$  small enough.

Consider a continuation game in which the firm's belief is  $\hat{\sigma} = 0$ . Since in a PSE the support of the posterior belief is always contained in the support of the prior, the firm's updated belief is at  $\hat{\sigma}_+ = 0$ . In this continuation game, the strategy of a worker of type  $S$  is, as it always is, to accept a wage offer  $w_o$  if and only if  $w_o \geq w_{S,o}$ , and to make the wage demand  $w_{S,d}$ . The strategy of a worker of type  $R$  is to accept a wage offer  $w_o$  if and only if  $w_o \geq w_{R,o}$ , and to make the wage demand  $w_{R,d}$ . The strategy of the firm is to

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<sup>2</sup>The credibility requirement for the updating of beliefs is designed to avoid situations in which the firm can insist on a particular wage offer by threatening to revise its beliefs optimistically whenever a different wage offer is made.

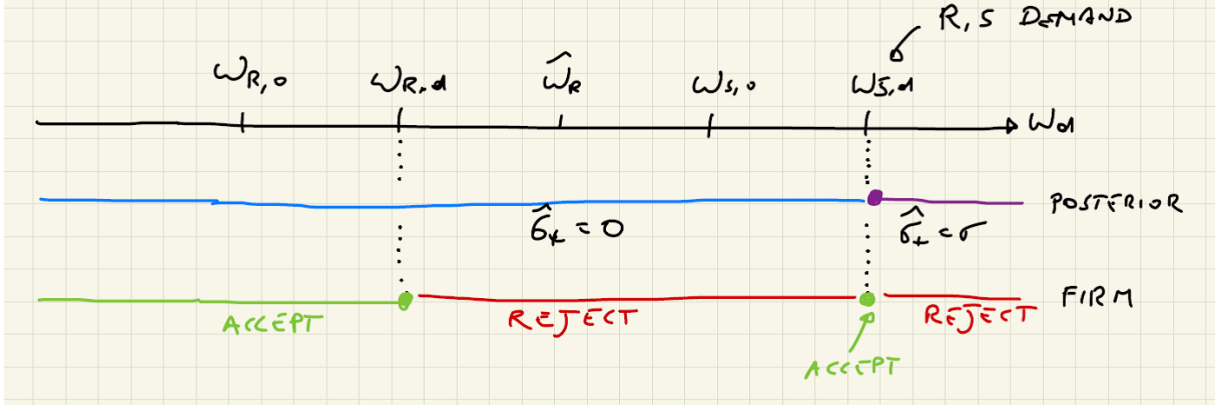


Figure 1: Strategies and belief updates when  $\hat{\sigma} = \sigma$  and  $y < y^*$ .

accept a wage demand  $w_d$  if and only if  $w_d \leq w_{R,d}$ , and to make the wage offer  $\leq w_{R,o}$ . It is immediate to verify that the above strategies and belief updates are the unique PSE for the continuation game with  $\hat{\sigma} = 0$ .

Consider a continuation game in which the firm's belief is  $\hat{\sigma} = 1$ . Since in a PSE the support of the posterior belief is always contained by the support of the prior belief, the firm's updated belief is  $\hat{\sigma}_+ = 1$ . In this continuation game, the strategy of a worker of type  $S$  is, as always, to accept a wage offer  $w_o$  if and only if  $w_o \geq w_{S,o}$ , and to make the wage demand  $w_{S,d}$ . The strategy of the firm is to accept a wage demand  $w_d$  if and only if  $w_d \leq w_{S,d}$ , and to make the wage offer  $w_{S,o}$ . The strategy of a worker of type  $R$  is to accept a wage offer  $w_o$  if and only if  $w_o \geq \tilde{w}_R$ , and to make the wage demand  $w_{S,d}$ , with  $\tilde{w}_R$  such that

$$V_{R,1}(\tilde{w}_R, y) = (1 - e^{-\mu\Delta})V_{R,0}(y) + e^{-\mu\Delta}V_{R,1}(w_{S,d}, y). \quad (5.13)$$

It is immediate to verify that the above strategies and belief updates are the unique PSE for the continuation game with  $\hat{\sigma} = 1$ .

Consider the game in which the firm's belief is  $\hat{\sigma} = \sigma$ . The outcome of the game depends on whether  $y$  is smaller or greater than  $y^*$ . Suppose that  $y < y^*$ , so that  $w_{R,o} < w_{R,d} < w_{S,o} < w_{S,d}$  and  $\tilde{w}_R \in (w_{R,d}, w_{S,o})$  for all  $\Delta$  small enough. The belief updates are as follows. The firm's updated belief is  $\hat{\sigma}_+ = 0$  if the worker makes a wage demand  $w_d < w_{S,d}$ , and it is  $\hat{\sigma}_+ = \sigma$  if the worker makes a wage demand  $w_d \geq w_{S,d}$ . The firm's updated belief is  $\hat{\sigma}_+ = \sigma$  if the worker rejects a wage offer  $w_o < \tilde{w}_R$ , and it is  $\hat{\sigma}_+ = 1$  if the worker rejects a wage offer  $w_o > \tilde{w}_R$ . The strategy of a worker of type  $S$  is to accept a wage offer  $w_o$  if and only if  $w_o \geq w_{S,o}$ , and to make the wage demand  $w_{S,d}$ . The strategy of a worker of type  $R$  is to accept a wage offer  $w_o$  if and only if  $w_o \geq \tilde{w}_R$ , and to make the wage demand  $w_{S,d}$ . The strategy of the firm is to accept a wage demand  $w_d$  if and only if  $w_d \leq w_{R,d}$  or  $w_d = w_{S,d}$ , and to make the wage offer  $w_{S,o}$ .

I now need to check that the strategy of the firm and of a worker of type  $R$  are optimal. Let me consider the acceptance strategy of the firm. If the worker makes a wage demand  $w_d < w_{S,d}$ , the firm updates its belief to  $\hat{\sigma}_+ = 0$ . If the firm accepts the demand, it gets the



payoff  $J(w_d, y)$ . If the firm rejects the demand, it expects the payoff  $\exp(-\lambda\Delta)J(w_{R,o}, y)$ , where  $J(w_{R,o}, y)$  is the firm's equilibrium payoff in the continuation game in which the firm's belief is  $\hat{\sigma} = 0$  and the firm makes an offer. Since  $J(w_d, y) = \exp(-\lambda\Delta)J(w_{R,o}, y)$  for  $w_d = w_{R,d}$  and is decreasing in  $w_d$ , the firm finds it optimal to accept  $w_d$  if  $w_d \leq w_{R,d}$  and to reject  $w_d$  if  $w_d \in (w_{R,d}, w_{S,d})$ . If the worker makes a wage demand  $w_d \geq w_{S,d}$ , the firm updates its belief to  $\hat{\sigma}_+ = \sigma$ . If the firm accepts the demand, its payoff is  $J(w_d, y)$ . If the firm rejects the demand, it expects the payoff  $\exp(-\lambda\Delta)J(w_{S,o}, y)$ , where  $J(w_{S,o}, y)$  is the firm's equilibrium payoff in the continuation game in which the firm's belief is  $\hat{\sigma} = \sigma$  and the firm makes an offer. Since  $J(w_d, y) = \exp(-\lambda\Delta)J(w_{S,o}, y)$  for  $w_d = w_{S,d}$ , the firm finds it optimal to accept  $w_d$  if  $w_d = w_{S,d}$  and to reject  $w_d$  if  $w_d > w_{S,d}$ .

Consider the optimality of the firm's wage offer strategy. If the firm makes the offer  $w_o \geq w_{S,o}$ , every worker accepts and the firm gets the payoff  $J(w_o, y)$ . If the firm makes an offer  $w_o < \tilde{w}_R$ , every worker rejects, and the firm gets the payoff  $\exp(-\mu\Delta)J(w_{S,d}, y)$ , where  $J(w_{S,d}, y)$  is the firm's equilibrium payoff in the continuation game in which the firm's belief is  $\hat{\sigma} = \sigma$  and the worker makes a demand. If the firm makes an offer  $w_o \in [\tilde{w}_R, w_{S,o})$ , a worker of type  $R$  accepts, a worker of type  $S$  rejects, and the firm gets the payoff  $(1 - \sigma)J(w_o, y) + \sigma \exp(-\mu\Delta)J(w_{S,d}, y)$ , where  $J(w_{S,d}, y)$  is the firm's equilibrium payoff in the continuation game in which the firm's belief is  $\hat{\sigma} = 1$  and the worker makes a demand. Since  $J(w_o, y)$  is decreasing in  $w_o$  and  $J(w_{S,o}, y) > J(w_{S,d}, y)$ , the firm prefers making the offer  $w_{S,o}$  than any offer  $w_o > w_{S,o}$  or any offer  $w_o < \tilde{w}_R$ . Since  $J(w_o, y)$  is decreasing in  $w_o$ , the firm prefers making the offer  $\tilde{w}_R$  than any offer  $w_o \in (\tilde{w}_R, w_{S,o})$ . The firm prefers making the offer  $w_{S,o}$  rather than the offer  $\tilde{w}_R$  if  $J(w_{S,o})$  is greater than  $(1 - \sigma)J(\tilde{w}_R, y) + \sigma \exp(-\mu\Delta)J(w_{S,d}, y)$ . This is the case as long as

$$\sigma(y - (1 - \beta)\hat{V}_{S,0}) \geq (1 - \sigma)(1 - \beta)(\hat{V}_{S,0} - V_{R,0}(y)). \quad (5.14)$$

Notice that, under the conjecture that  $V_{R,0}(y)$  is continuous and increasing in  $y$  and  $V_{R,0}(y) = \hat{V}_{S,0}$  for  $y = y^*$ , condition (5.14) holds for all  $y$  that are smaller than  $y^*$  and sufficiently close to  $y^*$ . In particular, for any given fraction  $\sigma > 0$  of  $S$ -workers, there exists a left neighborhood of  $y^*$  such that condition (5.14) holds.

I now turn to the optimality of the acceptance strategy for a worker of type  $R$ . Suppose that the firm makes a wage offer  $w_o$ . If the worker accepts the offer, he gets the payoff  $V_{R,1}(w_o, y)$ . If the worker rejects the offer, the firm's posterior belief is  $\hat{\sigma}_+ = \sigma$  if  $w_o < \tilde{w}_R$  or  $\hat{\sigma}_+ = 1$  if  $w_o \geq \tilde{w}_R$ . In either case, if the worker rejects the offer  $w_o$ , he makes the wage demand  $w_{S,d}$  and the firm accepts it. Therefore, if the worker rejects the offer  $w_o$ , he gets the payoff  $(1 - \exp(-\mu\Delta))V_{R,0}(y) + \exp(-\mu\Delta)V_{R,1}(w_{S,d}, y)$ . Since the acceptance and rejection payoffs are equal for  $w_o = \tilde{w}_R$ , the worker finds it optimal to accept  $w_o$  if and only if  $w_o \geq \tilde{w}_R$ .

Consider the proposal strategy for a worker of type  $R$ . If the worker makes a wage demand  $w_d \leq w_{R,d}$ , the firm accepts, and the worker gets the payoff  $V_{R,1}(w_d, y)$ . If the worker makes a wage demand  $w_d \in (w_{R,d}, w_{S,d})$ , the firm rejects, and the worker gets the

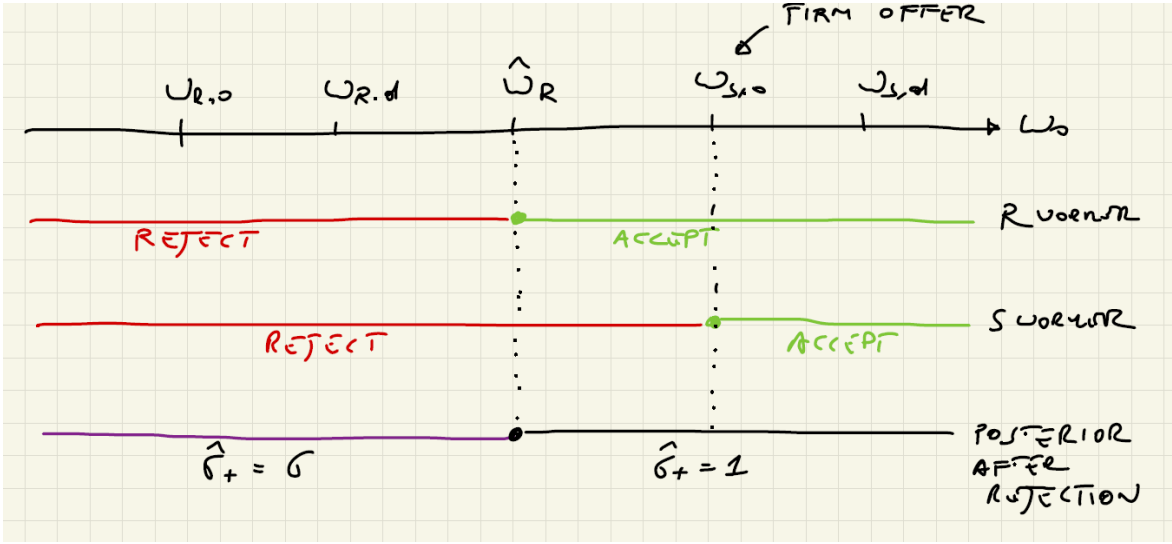


Figure 2: Strategies and belief updates when  $\hat{\sigma} = \sigma$  and  $y < y^*$ .

payoff is  $(1 - \exp(-\lambda\Delta))V_{R,0}(y) + \exp(-\lambda\Delta)V_{R,1}(w_{R,o}, y)$ , which is the worker's equilibrium payoff in the continuation game where the firm's belief is  $\hat{\sigma} = 0$  and the firm makes an offer. If the worker makes a wage demand  $w_d = w_{S,d}$ , the firm accepts, and the worker gets the payoff  $V_{R,1}(w_{S,d}, y)$ . If the worker makes a wage demand  $w_d > w_{S,d}$ , the firm rejects, and the worker gets the payoff  $(1 - \exp(-\lambda\Delta))V_{R,0}(y) + \exp(-\lambda\Delta)V_{R,1}(w_{S,o}, y)$ , where  $V_{R,1}(w_{S,o}, y)$  is the worker's equilibrium payoff in the continuation game in which the firm's belief is  $\hat{\sigma} = \sigma$  and the firm makes an offer. Clearly, the worker finds it optimal to demand the wage  $w_{S,d}$ .

Finally, I need to check that the firm updates its belief according to Bayes' rule if possible, and credibly otherwise. Consider the way in which the firm updates its belief after a wage offer  $w_o$  is rejected. For  $w_o < \tilde{w}_R$ , the firm updates its belief from  $\hat{\sigma} = \sigma$  to  $\hat{\sigma}_+ = \sigma$ , which is consistent with Bayes' rule. For  $w_o \in [\tilde{w}_R, w_{S,o})$ , the firm updates its belief from  $\hat{\sigma} = \sigma$  to  $\hat{\sigma}_+ = 1$ , which is also consistent with Bayes' rule. For  $w_o > w_{S,o}$ , no worker is expected to reject the offer and Bayes' rule is not applicable. In this case, the firm should seek a subset of types that would be better off rejecting the offer than accepting it, if the firm believed that the rejection came from this subset of types. If the firm were to believe the rejection comes from an  $S$ -worker, the  $S$ -worker would be better off accepting than rejecting  $w_o$  and getting  $w_{S,d}$ . If the firm were to believe that the rejection comes from an  $R$ -worker, the  $R$ -worker would be better off accepting than rejecting  $w_o$  and getting  $w_{R,d}$ . If the firm were to believe that the rejection comes from both  $S$  and  $R$ -workers, both types would be better off accepting it. Overall, the credibility condition does not impose any further constraint on the firm's off-equilibrium beliefs.

Now, consider the way in which the firm updates its belief after a wage demand  $w_d$ . For  $w_d = w_{S,d}$ , the firm updates its belief from  $\hat{\sigma} = \sigma$  to  $\hat{\sigma}_+ = \sigma$  in accordance with Bayes' rule. For  $w_d \neq w_{S,d}$ , Bayes' rule is not applicable. In this case, the firm should seek a

subset of types that would be better off making the wage demand  $w_d$  rather than  $w_{S,d}$ , if the firm believed that the demand  $w_d$  came from this subset of types. Notice that, no matter how the firm updates its belief, a worker of type  $R$  and a worker of type  $S$  are better off making the wage demand  $w_{S,d}$  than any wage demand  $w_d < w_{S,d}$ , which might be either accepted or rejected by the firm, and then countered with an offer non-greater than  $w_{S,o}$ . Hence, the credibility condition does not impose any constraint on the firm's off-equilibrium beliefs for  $w_d < w_{S,d}$ . Similarly, irrespective of how the firm updates its belief, the firm rejects a wage demand  $w_d > w_{S,d}$ . Hence, a worker of type  $R$  and a worker of type  $S$  are better off demanding  $w_{S,d}$  than demanding  $w_d > w_{S,d}$  and then settling for at most  $w_{S,o}$ . Also for  $w_d > w_{S,d}$ , the credibility condition does not impose any further constraint on the firm's off-equilibrium beliefs.

I have thus shown that the strategies and belief updates above constitute a PSE for  $y < y^*$  and  $y$  close enough to  $y^*$ . It is straightforward to compute the outcome of the PSE. In particular, in the limit for  $\Delta \rightarrow 0$ , the outcome of the PSE is such that the firm and a worker of type  $i$  reach an agreement with probability 1 at the wage  $w_{S,d} = w_{S,o} = w_i(y)$ , with

$$w_i(y) = \gamma y + (1 - \gamma)(1 - \beta)V_{S,0}, \quad (5.15)$$

where  $\gamma \equiv \lambda/(\lambda + \mu)$  is the worker's bargaining power and  $1 - \gamma = \mu/(\lambda + \mu)$  is the firm's bargaining power.

Now, I analyze the game in which the firm's belief is  $\hat{\sigma} = \sigma$ , for the case  $y > y^*$ . In this case, for all  $\Delta$  small enough,  $w_{S,o} < w_{S,d} < w_{R,o} < w_{R,d}$  and  $\tilde{w}_R \in (w_{S,d}, w_{R,o})$ . The belief updates are as follows. The firm's updated belief is  $\hat{\sigma}_+ = 1$  if the worker makes a wage demand  $w_d \leq w_{S,d}$ , and  $\hat{\sigma}_+ = 0$  if the worker makes a wage demand  $w_d > w_{S,d}$ . The firm's updated belief is  $\hat{\sigma}_+ = \sigma$  if the worker rejects a wage offer  $w_o < w_{S,o}$ , and  $\hat{\sigma}_+ = 0$  if the worker rejects a wage offer  $w_o \geq w_{S,o}$ . The strategy of a worker of type  $S$  is to accept a wage offer  $w_o$  if and only if  $w_o \geq w_{S,o}$ , and to make the wage demand  $w_{S,d}$ . The strategy of a worker of type  $R$  is to accept a wage offer  $w_o$  if and only if  $w_o \geq w_{R,o}$ , and to make the wage demand  $w_{R,d}$ . The strategy of the firm is to accept a wage demand  $w_d$  if and only if  $w_d \leq w_{R,d}$ , and to make the wage offer  $w_{S,o}$ .

I now check that the acceptance strategy of the firm is optimal. If the worker makes a wage demand  $w_d \leq w_{S,d}$ , the firm updates its belief to  $\hat{\sigma}_+ = 1$ . If the firm accepts the demand, its payoff is  $J(w_d, y)$ . If the firm rejects the demand, its payoff is  $\exp(-\lambda\Delta)J(w_{S,o}, y)$ , where  $J(w_{S,o}, y)$  is the firm's equilibrium payoff in the continuation game in which the firm's belief is  $\hat{\sigma} = 1$  and the firm makes an offer. Since  $J(w_d, y) \geq \exp(-\lambda\Delta)J(w_{S,o}, y)$  for all  $w_d \leq w_{S,d}$ , the firm finds it optimal to accept any wage demand  $w_d \leq w_{S,d}$ . If the worker makes a wage demand  $w_d > w_{S,d}$ , the firm updates its belief to  $\hat{\sigma}_+ = 0$ . If the firm accepts the demand, its payoff is  $J(w_d, y)$ . If the firm rejects the demand, it expects the payoff  $\exp(-\lambda\Delta)J(w_{R,o}, y)$ , where  $J(w_{R,o}, y)$  is the firm's equilibrium payoff in the continuation game in which the firm's belief is  $\hat{\sigma} = 0$  and the firm makes an offer. Since  $J(w_d, y) = \exp(-\lambda\Delta)J(w_{R,o}, y)$  for  $w_d = w_{R,d}$ , the firm

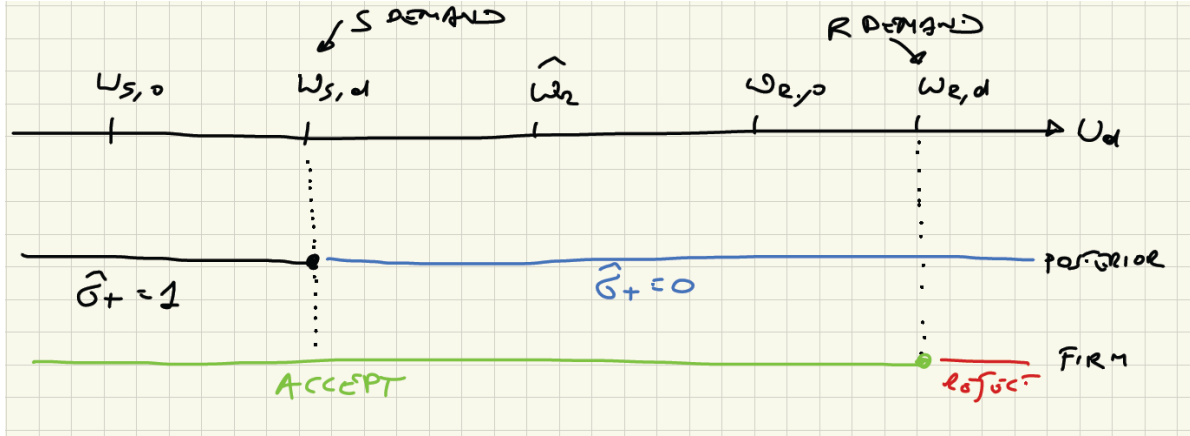


Figure 3: Strategies and belief updates when  $\hat{\sigma} = \sigma$  and  $y > y^*$ .

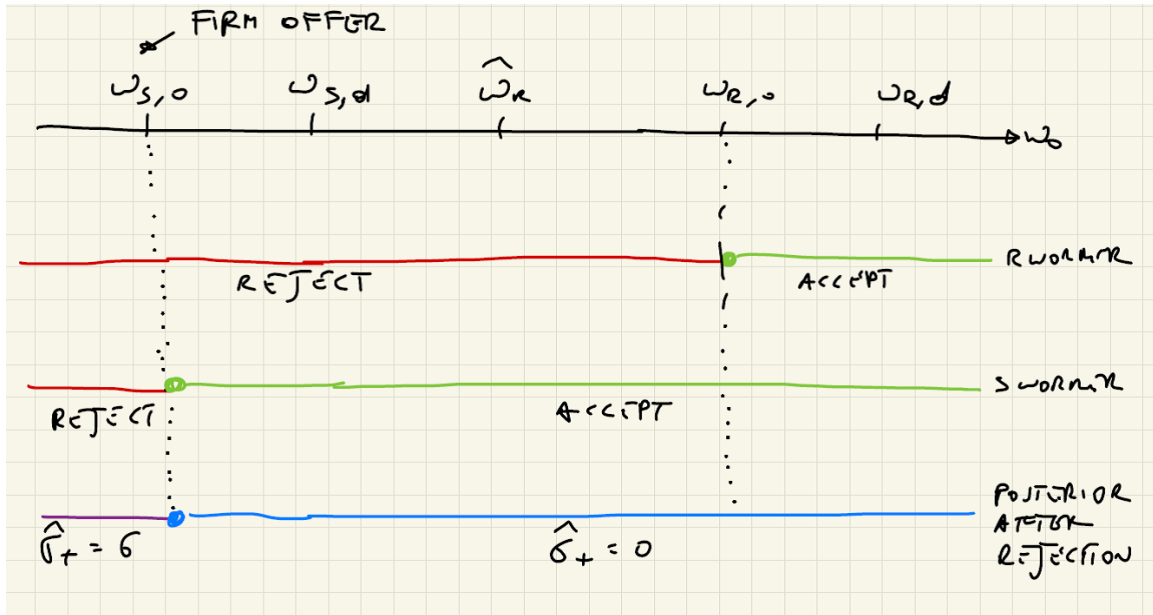


Figure 4: Strategies and belief updates when  $\hat{\sigma} = \sigma$  and  $y > y^*$ .

finds it optimal to accept  $w_d$  if  $w_d \in (w_{S,d}, w_{R,d}]$  and to reject if  $w_d > w_{R,d}$ .

Consider the optimality of the firm's wage offer strategy. If the firm makes an offer  $w_o < w_{S,o}$ , every worker rejects, and the firm's payoff is  $\exp(-\mu\Delta)\{\sigma J(w_{S,d}, y) + (1 - \sigma)J(w_{R,d}, y)\}$ . If the firm makes an offer  $w_o \in [w_{S,o}, w_{R,o})$ , a worker of type  $R$  rejects, a worker of type  $S$  accepts, and the firm's payoff is  $\sigma J(w_o, y) + (1 - \sigma)\exp(-\mu\Delta)J(w_{R,d}, y)$ . If the firm makes an offer  $w_o \geq w_{R,o}$ , every worker accepts, and the firm's payoff is  $J(w_o, y)$ . Since  $J(w_o, y)$  is decreasing in  $w_o$  and  $w_{S,o} < w_{S,d}$ , the firm prefers making the offer  $w_{S,o}$  than any other offer  $w_o < w_{R,o}$ . Since  $J(w_o, y)$  is decreasing, the firm prefers making the offer  $w_{R,o}$  than any offer  $w_o > w_{R,o}$ . To figure out whether the firm prefers making the offer  $w_{S,o}$  or the offer  $w_{R,o}$ , notice that the distance between  $w_{R,d}$  and  $w_{R,o}$  vanishes for  $\Delta \rightarrow 0$ , but the distance between  $w_{R,o}$  and  $w_{S,o}$  is bounded away from zero for  $\Delta \rightarrow 0$ . Hence, for all  $\Delta$  small enough, the firm prefers making the offer  $w_{S,o}$  rather than the offer  $w_{R,o}$ .

Consider the optimality of the acceptance strategy for a worker of type  $R$ . Suppose that the firm makes a wage offer  $w_o$ . If the worker accepts the offer, he attains the payoff  $V_{R,1}(w_o, y)$ . If the worker rejects the offer, the firm's posterior belief is  $\hat{\sigma}_+ = \sigma$  if  $w_o < w_{S,o}$  or  $\hat{\sigma}_+ = 0$  if  $w_o \geq w_{S,o}$ . In either case, after rejecting the wage offer  $w_o$ , the worker makes the wage demand  $w_{R,d}$  and the firm accepts it. Hence, if the worker rejects  $w_o$ , he gets the payoff  $(1 - \exp(-\mu\Delta))V_{R,0}(y) + \exp(-\mu\Delta)V_{R,1}(w_{R,d}, y)$ . Since the acceptance and rejection payoffs are equal for  $w_o = w_{R,o}$ , the worker finds it optimal to accept  $w_o$  if and only if  $w_o \geq w_{R,o}$ .

Consider the optimality of the proposal strategy for a worker of type  $R$ . If the worker makes a wage demand  $w_d \leq w_{R,d}$ , the firm accepts and the worker's payoff is  $V_{R,1}(w_d, y)$ . If the worker makes a wage demand  $w_d > w_{R,d}$ , the firm rejects and the worker's payoff is  $(1 - \exp(-\lambda\Delta))V_{R,0}(y) + \exp(-\lambda\Delta)V_{R,1}(w_{R,o}, y)$ . Since  $V_{R,1}(w_d, y)$  is increasing in  $w_d$  and equals  $(1 - \exp(-\lambda\Delta))V_{R,0}(y) + \exp(-\lambda\Delta)V_{R,1}(w_{R,o}, y)$  for  $w_d = w_{R,d}$ , the worker finds it optimal to demand the wage  $w_{R,d}$ .

Finally, I need to check that the firm updates its belief according to Bayes' rule if possible, and credibly otherwise. Consider the way in which the firm updates its belief after a wage offer  $w_o$  is rejected. For  $w_o < w_{S,o}$ , the firm updates its belief from  $\hat{\sigma} = \sigma$  to  $\hat{\sigma}_+ = \sigma$ , which is consistent with Bayes' rule. For  $w_o \in [w_{S,o}, w_{R,o})$ , the firm updates its belief from  $\hat{\sigma} = \sigma$  to  $\hat{\sigma}_+ = 0$ , which is also consistent with Bayes' rule. For  $w_o \geq w_{R,o}$ , no worker is expected to reject the offer and Bayes' rule is not applicable. Suppose that the firm were to believe that the rejection of  $w_o$  comes from an  $S$ -worker. In this case, the  $S$ -worker would be better off accepting  $w_o$  rather than rejecting it and trading at  $w_{S,d}$ . Suppose that the firm were to believe that the rejection of  $w_o$  comes from an  $R$ -worker. In this case, the  $R$ -worker would be better off accepting  $w_o$  rather than rejecting it and trading at  $w_{R,d}$ . If the firm were to believe that the rejection of  $w_o$  comes from both  $S$  and  $R$ -workers, then both  $S$  and  $R$ -worker would be better off accepting  $w_o$  rather than rejecting it. Overall, the credibility condition does not impose any further constraint on

the firm's off-equilibrium beliefs.

Consider the way in which the firm updates its belief after a wage demand  $w_d$ . For  $w_d = w_{S,d}$ , the firm updates its belief from  $\hat{\sigma} = \sigma$  to  $\hat{\sigma}_+ = 1$ , which is consistent with Bayes' rule. For  $w_d = w_{R,d}$ , the firm updates its belief from  $\hat{\sigma} = \sigma$  to  $\hat{\sigma}_+ = 0$ , which is also consistent with Bayes' rule. For  $w_d \neq w_{S,d}$  and  $w_{R,d}$ , Bayes' rule is not applicable. Notice that a worker of type  $S$  is better off making the equilibrium wage demand  $w_{S,d}$  rather than any off-equilibrium wage demand  $w_d$ , irrespective of how the firm updates its beliefs. If the worker demands  $w_d < w_{S,d}$ , the worker expects the firm to accept  $w_d$ . Hence, the worker is worse off demanding  $w_d < w_{S,d}$  than  $w_{S,d}$ . If the worker demands  $w_d > w_{S,d}$ , the worker expects the firm to reject  $w_d$  and counter with  $w_{S,o}$ . Hence, the worker is worse off demanding  $w_d > w_{S,d}$  rather than  $w_{S,d}$ . Similarly, a worker of type  $R$  is better off making the equilibrium wage demand  $w_{R,d}$  rather than any off-equilibrium wage demand  $w_d$ , irrespective of how the firm updates its beliefs. If the worker demands  $w_d < w_{R,d}$ , the firm may accept  $w_d$  or reject it and counter with an offer non-greater than  $w_{R,o}$ . Hence, the worker is worse off demanding  $w_d < w_{R,d}$  rather than  $w_{R,d}$ . If the worker demands  $w_d > w_{R,d}$ , the firm rejects  $w_d$  and counters with an offer non-greater than  $w_{R,o}$ . Hence, the worker is worse off demanding  $w_d > w_{R,d}$  rather than  $w_{R,d}$ . Since there is no type of worker that is better off making an off-equilibrium demand, irrespective of how the firm updates its belief, the credibility condition does not impose any further constraint on the firm's off-equilibrium beliefs.

I have thus shown that the strategies and belief updates above constitute a PSE for  $y > y^*$ . It is straightforward to compute the outcome of the PSE. In particular, in the limit for  $\Delta \rightarrow 0$ , the outcome of the PSE is such that the firm and a worker of type  $S$  reach an agreement with probability 1 at the wage  $w_{S,d} = w_{S,o} = w_S(y)$ , with

$$w_S(y) = \gamma y + (1 - \gamma)(1 - \beta)\hat{V}_{S,0}. \quad (5.16)$$

In contrast, in the limit for  $\Delta \rightarrow 0$ , the outcome of the PSE is such that the firm and a worker of type  $R$  reach an agreement with probability 1 at the wage  $w_{R,d} = w_{R,o} = w_R(y)$ , with

$$w_R(y) = \gamma y + (1 - \gamma)(1 - \beta)V_{R,0}(y). \quad (5.17)$$

I summarize the characterization of the bargaining game in the following proposition.

**Proposition 6.** (*Asymmetric information bargaining I*) *Consider the scenario in which a worker of type  $S$  observes the productivity  $y$  of the firm with which he is bargaining. For any  $\sigma > 0$ , there exists a unique PSE of the bargaining game. The PSE is such that:*

1. *For any  $y < y^*$  with  $y$  sufficiently close to  $y^*$ , the firm and the worker reach an agreement with probability 1. The worker is paid  $w_S(y)$  if his type is  $S$ , and  $w_R(y)$  if his type is  $R$ , where  $w_S(y)$  and  $w_R(y)$  are such that  $w_S(y) = w_R(y)$  and given by (5.15);*
2. *For any  $y > y^*$ , the firm and the worker reach an agreement with probability 1. The*

worker is paid  $w_S(y)$  if his type is  $S$ , and  $w_R(y)$  if his type is  $R$ , where  $w_S(y)$  and  $w_R(y)$  are such that  $w_S(y) < w_R(y)$  and given by (5.16) and (5.17);

3. When searching the market, the worker expects to earn the wage  $\hat{w}_S = w_S(y^*)$  upon meeting a firm.

Some comments about Proposition 7 are in order. For  $y < y^*$ , an  $R$ -worker and an  $S$ -worker earn the same wage. This common wage is the same as in the full information game between a firm and an  $S$ -worker. For  $y > y^*$ , an  $R$ -worker and an  $S$ -worker earn different wages. An  $R$ -worker earns the same wage as in the full information game between a firm and an  $R$ -worker. An  $S$ -worker earns the same wage as in the full information game between a firm and an  $S$ -worker. Note that the full information wage of an  $R$ -worker is fully flexible, in the sense that it responds to both changes in the productivity of the firm and changes in the value of unemployment that are caused by aggregate productivity shocks. In contrast, the full information wage of an  $S$ -worker is sticky, in the sense that it does not respond to changes in the value of unemployment caused by aggregate productivity shocks. Therefore, under asymmetric information, the responsiveness of the average wage paid by the firm to a worker is different depending on whether shocks are negative or positive. In response to a negative shock, the average wage features the same degree of stickiness as in a version of the model where every worker is of type  $S$ . In response to a positive shock, the degree of stickiness of the average wage is proportional to the fraction of  $S$ -workers in the population. Overall, the average wage is stickier downward than upward. When the fraction of  $S$ -workers in the population is small, the average wage is only downward sticky.

Let me now provide some intuition for the properties of the equilibrium wages. When  $y < y^*$ , the full information wage of an  $R$ -worker is lower than the full information wage of an  $S$ -worker. For this reason, an  $R$ -worker does not want to reveal his type to the firm, and it can do so by making the same wage demand  $w_{S,d}$  as an  $S$ -worker. The firm can then try to screen the two types of workers by making a wage offer  $\tilde{w}_R$  that is acceptable to an  $R$ -worker but not to an  $S$ -worker, or it can pool the two types of workers by making a wage offer  $w_{S,o}$  that is acceptable to both types. Since an  $R$ -worker has the option to reject the screening offer  $\tilde{w}_R$  and, thus, convince the firm that he is an  $S$ -worker,  $\tilde{w}_R$  must be close to the full information wage of an  $S$ -worker. Since  $\tilde{w}_R$  is close to the full information wage of an  $S$ -worker, the return to the firm from screening the two types is small and, under condition (5.14), the firm prefers making the pooling offer  $w_{S,o}$ .

When  $y > y^*$ , the full information wage of an  $R$ -worker is higher than the full information wage of an  $S$ -worker. For this reason, an  $R$ -worker wants to reveal his type to the firm, and it can do so by making a wage offer  $w_{R,d}$ . Indeed, after observing the wage offer  $w_{R,d}$ , the firm must believe that the offer is coming from an  $R$ -worker, since it realizes that making such an offer would be in the interest of an  $R$ -worker if the firm interpreted as coming from an  $R$ -worker, while making such an offer would never be in the interest of an  $S$ -worker—since the  $S$ -worker expects the offer to be rejected. Once the

$R$ -worker has revealed his type by offering  $w_{R,d}$ , the firm finds it optimal to accept. Once the  $S$ -worker has revealed his type by offering  $w_{S,d}$ , the firm finds it optimal to accept. The asymmetry between the nature of the equilibrium in the case of  $y < y^*$ —where the  $R$ -worker mimics the  $S$ -worker—and in the case of  $y > y^*$ —where the  $S$ -worker does not mimic the  $R$ -worker—is due to the fact that  $S$ -workers do not understand that aggregate productivity and, in turn, the value of unemployment is different in the two cases.

It is easy to show that the PSE is unique—up to the specification of some off-equilibrium belief updates that do not affect the equilibrium payoffs. Consider the case in which  $y < y^*$ . In any PSE, an  $R$ -worker and an  $S$ -worker make the same wage demand,  $w_{S,d}$ . To see why this is the case, suppose there was a PSE in which the  $R$ -worker made a different wage demand than an  $S$ -worker. In such a PSE, the firm would accept the wage demand  $w_{S,d}$  of an  $S$ -worker, and it would either accept or reject the wage demand of an  $R$ -worker. In either case, the  $R$ -worker would not earn more than  $w_{R,d}$ . Since  $w_{R,d} < w_{S,d}$ , an  $R$ -worker would be better off deviating from the equilibrium by making the wage demand  $w_{S,d}$ . In any PSE, the firm makes the pooling wage offer,  $w_{S,o}$ . To see why this is the case, suppose there was a PSE in which the firm makes a screening wage offer that is accepted by an  $R$ -worker and rejected by an  $S$ -worker. If an  $R$ -worker rejects the offer, the firm believes that the worker is of type  $S$  and accepts  $w_{S,d}$ . Therefore, the screening offer cannot be lower than  $\tilde{w}_R$ . And, under condition (5.14), the firm prefers making the pooling offer  $w_{S,o}$  than any screening offer greater or equal to  $\tilde{w}_R$ .

In the case of  $y > y^*$ , uniqueness follows immediately from the credibility restriction on belief updating. In any PSE, an  $R$ -worker and an  $S$ -worker make different wage demands. To see why this is the case, suppose there was a PSE in which an  $R$ -worker and an  $S$ -worker make the same wage demand  $w_{S,d}$ . If the firm accepted  $w_{S,d}$ , an  $R$ -worker would be better off deviating from the equilibrium and making the wage offer  $w_{R,d}$ , which—because of the credibility restriction on the belief updates—the firm must interpret as coming from an  $R$ -worker and, hence, accept. For the same reason, if the firm rejected  $w_{S,d}$ , an  $R$ -worker would be better off making the wage offer  $w_{R,d}$ . Since, in any PSE, an  $R$ -worker reveals his type by making a wage offer different from  $w_{S,d}$ , his payoff is the same as in a full information game. In turn, the payoff of an  $S$ -worker is the same as in a full information game.

Finally, let me comment on the existence of a PSE. For  $y > y^*$ , a PSE always exists and is unique. For  $y < y^*$ , a PSE exists and is unique as long as condition (5.14) holds. If the condition does not hold, the firm prefers making the screening offer  $\tilde{w}_R$  than the pooling offer  $w_{S,o}$ . Since the firm prefers offering  $\tilde{w}_R$  than  $w_{S,o}$ , it does reject the wage demand  $w_{S,d}$  when its belief is  $\hat{\sigma} = \sigma$ . Now, consider a wage offer  $w_o$  that is slightly lower than  $\tilde{w}_R$ . Suppose that the firm believes that  $w_o$  is rejected by both type of workers. In this case, an  $R$ -worker who rejects the offer will make the demand  $w_{S,d}$ , which the firm will reject, and he will end up accepting  $\tilde{w}_R$ . Therefore, if the firm believes that  $w_o$  is rejected by both type of workers, an  $R$ -worker should accept it and the firm's belief would violate Bayes' rule. Suppose that the firm believes that  $w_o$  is rejected only by  $S$ -workers.



In this case, an  $R$ -worker who rejects the offer will make the wage demand  $w_{S,d}$ , which the firm will accept. Therefore, if the firm believes that  $w_o$  is rejected by  $S$ -workers, an  $R$ -worker would reject it too and the firm's belief would violate Bayes' rule. This argument suggests that, when condition (5.14) does not hold, a pure-strategy PSE may not exist. A mixed-strategy PSE could be quite complicated and involve multiple rounds of screening. Yet, in light of the insights on bargaining under asymmetric information of Gul and Sonnenschein (1988), one can conjecture that, even in a mixed-strategy PSE, an  $R$ -worker would end up with a wage that is arbitrarily close to the full information wage of an  $S$ -worker.

Without going into details, let me briefly discuss the outcome of the bargaining game in the scenario where a worker of type  $S$  believes that the productivity of the firm with which he is bargaining is  $y^*$ . In this alternative scenario, the unique PSE of the bargaining game is similar to the one described above, both in terms of meta-strategies and belief updating rules. The two PSE differ because, in the alternative scenario, the strategy of an  $S$ -worker is to accept any wage offer  $w_o \geq w_{S,o}$ , where  $w_{S,o}$  is given by (5.10) with  $y^*$  replacing  $y$ , and to make the wage demand  $w_{S,d}$ , where  $w_{S,d}$  is given by (5.9) with  $y^*$  replacing  $y$ . Hence, in the alternative scenario, the full information wage for an  $S$ -worker is different. For  $y > y^*$ , this leads to a different wage paid to  $S$ -workers. For  $y < y^*$ , this leads to a different wage paid to both  $R$  and  $S$ -workers. Moreover, the condition under which the firm prefers making the pooling offer  $w_{S,o}$  than the screening offer  $\tilde{w}_R$  becomes

$$(\sigma - \gamma)(y - (1 - \beta)\hat{V}_{S,0}) \geq \sigma(y^* - y) + (1 - \sigma)(1 - \beta)(\hat{V}_{S,0} - V_{R,0}(y)). \quad (5.18)$$

The following proposition contains a characterization of the bargaining outcomes.

**Proposition 7.** (*Asymmetric information bargaining II*) *Consider the scenario in which a worker of type  $S$  does not observe the productivity  $y$  of the firm with which he is bargaining and believes such productivity to be  $y^*$ . For any  $\sigma > \gamma$ , there exists a unique PSE of the bargaining game. The PSE is such that:*

1. *For any  $y < y^*$  with  $y$  sufficiently close to  $y^*$ , the firm and the worker reach an agreement with probability 1. The worker is paid  $w_S(y)$  if his type is  $S$ , and  $w_R(y)$  if his type is  $R$ , where  $w_S(y)$  and  $w_R(y)$  are such that  $w_S(y) = w_R(y)$  and are given by*

$$w_i(y) = \gamma y^* + (1 - \gamma)(1 - \beta)\hat{V}_{S,0}; \quad (5.19)$$

2. *For any  $y > y^*$ , the firm and the worker reach an agreement with probability 1. The worker is paid  $w_S(y)$  if his type is  $S$ , and  $w_R(y)$  if his type is  $R$ , where  $w_S(y)$  and  $w_R(y)$  are such that  $w_S(y) < w_R(y)$  and are given by*

$$w_R(y) = \gamma y + (1 - \gamma)(1 - \beta)V_{R,0}(y). \quad (5.20)$$

$$w_S(y) = \gamma y^* + (1 - \gamma)(1 - \beta)\hat{V}_{S,0}; \quad (5.21)$$

3. *When searching the market, the worker expects to earn the wage  $\hat{w}_S = w_S(y^*)$  upon*

*meeting a firm.*

Let me comment on Proposition 8. For  $y < y^*$ , an  $R$ -worker and an  $S$ -worker both earn the full information wage of an  $S$ -worker. For  $y > y^*$ , an  $R$ -worker earns the full information wage of an  $R$ -worker, and an  $S$ -worker earns the full information wage of an  $S$ -worker. The full information wage of an  $R$ -worker is fully-flexible, in the sense that it responds to both the changes in the productivity of the firm and the changes in the value of unemployment that are caused by aggregate shocks. In contrast, the full information wage of an  $S$ -worker is rigid, as it responds to neither the changes in the productivity of the firm nor the changes in the value of unemployment that are caused by aggregate shocks. Therefore, under asymmetric information, the average wage paid by the firm to a worker does not respond at all to negative shocks to aggregate productivity. The average wage is rigid downwards. In contrast, the average wage paid by the firm responds to positive shocks in proportion to the fraction of  $R$ -workers in the population. If the fraction is large, the average wage is essentially fully flexible upwards.

I am now in the position to formally define an equilibrium for the model in which some workers have rational expectations and some workers have stubborn beliefs. I refer to this equilibrium as a Partially Rational Expectations Equilibrium (PREE).

**Definition 8.** (PREE) A PREE is given by actual and perceived values  $\{V_{i,0}, V_{i,1}, J, \hat{V}_{S,0}, \hat{V}_{S,1}, \hat{J}_S\}$ , actual and expected market tightness  $\{\theta, \hat{\theta}_S\}$ , and actual and expected wages  $\{w_i, \hat{w}_S\}$  such that:

1. The values  $\{V_{i,0}, V_{i,1}, J, \hat{V}_{S,0}, \hat{V}_{S,1}, \hat{J}_S\}$  satisfy conditions (5.1)-(5.6);
2. The tightnesses  $\theta$  and  $\hat{\theta}_S$  satisfy conditions (5.7) and (5.8);
3. The wage  $w_i$  is given in Proposition 4 or 5, and  $\hat{w}_S = w_S(y^*)$ .

### 5.3 Properties of equilibrium

I now want to characterize the properties of a PREE, and compare them with the properties of an SBE and with the properties of an REE. Let me start with the analysis of the scenario in which an  $S$ -worker observes the productivity  $y$  of the firm with which he is bargaining and interprets the difference  $y - y^*$  as a permanent and firm-specific component of productivity.

Let  $S_R(y)$  denote the surplus of a match between a firm and an  $R$ -worker. That is,  $S_R(y)$  denotes  $V_{R,1}(w, y) + J(w, y) - V_{R,0}(y)$ . Similarly, let  $\hat{S}_S(y)$  denote the surplus of a match between a firm and an  $S$ -worker, as perceived by the worker. That is,  $\hat{S}_S(y)$  denotes  $\hat{V}_{S,1}(w) + \hat{J}_S(w, y) - \hat{V}_{S,0}$ . Inserting the equilibrium wages  $w_R(y)$  and  $w_S(y)$  for  $y < y^*$  into  $V_{R,1}(w, y)$ ,  $\hat{V}_{S,1}(w)$  and  $J(w, y)$  and using the definitions of  $S_R(y)$  and  $\hat{S}_S(y)$

yields

$$\hat{V}_{S,1}(w_S(y)) - \hat{V}_{S,0} = \gamma \hat{S}_S(y), \quad (5.22)$$

$$V_{R,1}(w_R(y), y) - V_{R,0}(y) = \gamma \hat{S}_S(y) + S_R(y) - \hat{S}_S(y), \quad (5.23)$$

$$J(w_S(y), y), J(w_R(y), y) = (1 - \gamma) \hat{S}_S(y). \quad (5.24)$$

For  $y < y^*$ , the outcome of the bargaining game between a firm and an  $S$ -worker is such that the firm captures a fraction  $1 - \gamma$  of the surplus perceived by the worker, and the worker captures a fraction  $\gamma$  of it. The outcome of the bargaining game between a firm and an  $R$ -worker is such that the firm captures a fraction  $1 - \gamma$  of the surplus perceived by an  $S$ -worker, and the worker captures a fraction  $\gamma$  of the surplus perceived by an  $S$ -worker plus the difference between the actual and perceived surpluses.

Let  $y > y^*$ . Inserting the equilibrium wages  $w_R(y)$  and  $w_S(y)$  into  $V_{R,1}(w, y)$ ,  $\hat{V}_{S,1}(w)$  and  $J(w, y)$  and using the definitions of  $S_R(y)$  and  $\hat{S}_S(y)$  yields

$$\hat{V}_{S,1}(w_S(y)) - \hat{V}_{S,0} = \gamma \hat{S}_S(y), \quad (5.25)$$

$$V_{R,1}(w_R(y), y) - V_{R,0}(y) = \gamma S_R(y), \quad (5.26)$$

$$J(w_S(y), y) = (1 - \gamma) \hat{S}_S(y), \quad (5.27)$$

$$J(w_R(y), y) = (1 - \gamma) S_R(y). \quad (5.28)$$

For  $y > y^*$ , the outcome of the bargaining game between a firm and an  $S$ -worker is such that the firm captures a fraction  $1 - \gamma$  of the surplus perceived by the worker, and the worker captures a fraction  $\gamma$  of it. The outcome of the bargaining game between a firm and an  $R$ -worker is such that the firm captures a fraction  $1 - \gamma$  of the actual surplus, and the worker captures a fraction  $\gamma$  of the actual surplus.

Using the definition of the surplus and the outcome of the bargaining game, I can characterize the equilibrium values for  $S_R(y)$ ,  $\hat{S}_S(y)$  and  $\theta(y)$ . For  $y < y^*$ , they are

$$S_R(y) = y - b - \beta p(\theta(y)) \left[ \gamma \hat{S}_S(y) + S_R(y) - \hat{S}_S(y) \right] + \beta(1 - \delta) S_R(y), \quad (5.29)$$

$$\hat{S}_S(y) = y - b - \beta p(\theta(y^*)) \gamma \hat{S}_S(y^*) + \beta(1 - \delta) \hat{S}_S(y), \quad (5.30)$$

$$k = q(\theta(y))(1 - \gamma) \hat{S}_S(y). \quad (5.31)$$

For  $y > y^*$ ,  $S_R(y)$ ,  $\hat{S}_S(y)$  and  $\theta(y)$  are given by

$$S_R(y) = y - b - \beta p(\theta(y)) \gamma S_R(y) + \beta(1 - \delta) S_R(y), \quad (5.32)$$

$$\hat{S}_S(y) = y - b - \beta p(\theta(y^*)) \gamma \hat{S}_S(y^*) + \beta(1 - \delta) \hat{S}_S(y), \quad (5.33)$$

$$k = q(\theta(y))(1 - \gamma) \left[ \sigma \hat{S}_S(y) + (1 - \sigma) S_R(y) \right]. \quad (5.34)$$

The expressions (5.30) and (5.33) for the surplus perceived by an  $S$ -worker make use of the fact that  $\hat{\theta}_S = \theta(y^*)$  and that  $\hat{w}_S = w_S(y^*)$ . It is immediate to verify that the system of equations (5.29)-(5.31) and the system of equations (5.32)-(5.34) are identical at  $y = y^*$ ,

and they are both equal to the system of equations that describes a Rational Expectation Equilibrium.

In order to characterize the cyclical properties of a PREE, I differentiate  $S_R(y)$ ,  $\hat{S}_S(y)$  and  $\theta(y)$  with respect to  $y$  around  $y^*$  and derive expressions for their elasticities with respect to  $y$ . Since  $S_R(y)$ ,  $\hat{S}_S(y)$  and  $\theta(y)$  satisfy different conditions for  $y < y^*$  and for  $y > y^*$ , I need to distinguish between left and right derivatives. Let me start with the left derivatives. Differentiating (5.30) and (5.31) with respect to  $y$ , yields

$$\frac{\hat{S}'_S(y^*-)y^*}{\hat{S}_S(y^*)} = \frac{1 - \beta [1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta(1 - \delta)} \cdot \frac{y^*}{y^* - b}, \quad (5.35)$$

$$\frac{\theta'(y^*-)y^*}{\theta(y^*)} = \frac{1}{1 - \epsilon} \cdot \frac{1 - \beta [1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta(1 - \delta)} \cdot \frac{y^*}{y^* - b}. \quad (5.36)$$

Now consider the right derivatives. Differentiating (5.32), (5.33) and (5.34) yields

$$\frac{\theta'(y^*)y^*}{\theta(y^*)} = \frac{1}{1 - \epsilon} \cdot \left[ \sigma \frac{\hat{S}'_S(y^+)y^*}{\hat{S}_S(y^*)} + (1 - \sigma) \frac{S'_R(y^+)y^*}{S_R(y^*)} \right], \quad (5.37)$$

$$\frac{\hat{S}'_S(y^+)y^*}{\hat{S}_S(y^*)} = \frac{1 - \beta [1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta(1 - \delta)} \cdot \frac{y^*}{y^* - b}, \quad (5.38)$$

$$\frac{S'_R(y^+)y^*}{S_R(y^*)} = \frac{y^*}{y^* - b} - \frac{\beta p(\theta(y^*))\gamma \epsilon}{1 - \beta [1 - \delta - p(\theta(y^*))\gamma]} \cdot \frac{\theta'(y^+)y^*}{\theta(y^*)}, \quad (5.39)$$

where the expression in (5.37) makes use of the fact that the surplus perceived by an  $S$ -worker,  $\hat{S}_S(y)$ , is equal to the actual surplus of an  $R$ -worker,  $S_R(y)$ , at  $y = y^*$ . Substituting (5.38) and (5.39) into (5.37) gives

$$\begin{aligned} \frac{\theta'(y^*)y^*}{\theta(y^*)} &= \frac{1}{1 - \epsilon} \cdot \frac{1 - \beta [1 - \delta - p(\theta(y^*))\gamma\sigma]}{1 - \beta(1 - \delta)} \\ &\quad \cdot \frac{1 - \beta [1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta [1 - \delta - p(\theta(y^*))\gamma \frac{1 - \epsilon\sigma}{1 - \epsilon}]} \cdot \frac{y^*}{y^* - b}. \end{aligned} \quad (5.40)$$

The elasticity (5.36) of the market tightness with respect to a negative shock to aggregate productivity is the same as the elasticity (3.17) in an SBE, irrespective of what the fraction  $\sigma$  of  $S$ -workers in the population might be. This property of equilibrium is easy to understand. When the economy is hit by a negative productivity shock, an  $S$ -worker overestimates the value of unemployment and, hence, insists on making wage demands and accepting wage offers that are too high. An  $R$ -worker knows that the strategy of the  $S$ -worker is suboptimal, but he is better off mimicking the strategy of an  $S$ -worker than revealing his own type. In response to the strategy of the workers, the firm ends up agreeing to the wage demand of both types of worker. Since the wage demanded by both types of workers is the wage demanded by an  $S$ -worker and does not depend on the fraction of  $S$ -workers in the population, the wage is just as downward sticky as in an SBE. Hence, the elasticity of the market tightness is just the same as in an SBE, and so are the

elasticities of the job-finding probability, the unemployment, and the vacancy rates.

The elasticity (5.40) of the market tightness with respect to a positive shock to aggregate productivity lies between the elasticity (3.6) in an REE and the elasticity (3.17) in an SBE, and its exact value depends on the fraction  $\sigma$  of  $S$ -workers in the population. In particular, the elasticity (5.40) converges to the elasticity (3.6) in an REE for  $\sigma \rightarrow 0$ , converges to the elasticity (3.17) for  $\sigma \rightarrow 1$ , and is increasing in  $\sigma$ . These properties of equilibrium are also intuitive. When the economy is hit by a positive productivity shock, an  $S$ -worker underestimates the value of unemployment and, hence, insists on making wage demands and accepting wage offers that are too low. An  $R$ -worker knows this and signals its type to the firm by making higher wage demands. The firm ends up paying each type of worker their full-information wage. Since the full-information wage of an  $S$ -worker is sticky, but the full information wage of an  $R$ -worker is flexible, it follows that the average wage paid by the firm has an intermediate degree of stickiness. And, hence, the market tightness, the job-finding probability, the unemployment and vacancy rates have an elasticity that is in between an REE and an SBE. If  $\sigma$  is low, the average wage is upward flexible, and the elasticity of the labor market outcomes with respect to a positive shock to productivity is close to an REE.

The asymmetric response of wages and, in turn, of market tightness, job-finding probability, unemployment, and vacancy rates calls for an asymmetric employment subsidy. The optimal subsidy when  $y = y^*$  is the same as in REE or in an SBE. The optimal subsidy when  $y < y^*$  is the same as in an SBE, which is more countercyclical (i.e. higher) than the optimal subsidy in an REE. The optimal subsidy when  $y > y^*$  is between the optimal subsidy in an REE and the optimal subsidy in an SBE, which is more countercyclical (i.e. lower) than the optimal subsidy  $t(y)$  in an REE. For instance, if the Hosios' condition holds and  $\sigma$  is small, the optimal subsidy is zero for  $y = y^*$ , i.e.  $t(y^*) = 0$ , it is positive for  $y < y^*$ , i.e.  $t'(y^*-) < 0$ , and approximately equal to zero for  $y > y^*$ , i.e.  $t'(y^*+) = 0$ .

For the sake of completeness, let me now characterize the properties of a PREE in the second scenario—the one where an  $S$ -worker does not observe the productivity  $y$  of the firm with which he is bargaining and believes such productivity to be equal to  $y^*$ . In this second scenario, the equilibrium values for  $\hat{S}_S$  is such that

$$\hat{S}_S = y - b - \beta p(\theta(y^*))\gamma \hat{S}_S + \beta(1 - \delta)\hat{S}_S. \quad (5.41)$$

For  $y < y^*$ ,  $S_R(y)$  and  $\theta(y)$  are such that

$$S_R(y) = y - b - \beta p(\theta(y)) \left[ \gamma \hat{S}_S + S_R(y) - \hat{S}_S \right] + \beta(1 - \delta)S_R(y), \quad (5.42)$$

$$k = q(\theta(y)) \left[ (1 - \gamma)\hat{S}_S(y) + \frac{y - y^*}{1 - \beta(1 - \delta)} \right]. \quad (5.43)$$

For  $y > y^*$ ,  $S_R(y)$  and  $\theta(y)$  are such that

$$S_R(y) = y - b - \beta p(\theta(y))\gamma S_R(y) + \beta(1 - \delta)S_R(y), \quad (5.44)$$

$$k = q(\theta(y)) \left\{ \sigma \left[ (1 - \gamma)\hat{S}_S(y) + \frac{y - y^*}{1 - \beta(1 - \delta)} \right] + (1 - \sigma)(1 - \gamma)S_R(y) \right\} \quad (5.45)$$

Differentiating (5.43) with respect to  $y$ , I find that the elasticity of the market tightness with respect to a negative productivity shock is

$$\frac{\theta'(y^*-)y^*}{\theta(y^*)} = \frac{1}{1 - \epsilon} \cdot \frac{1}{1 - \gamma} \cdot \frac{1 - \beta[1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta(1 - \delta)} \cdot \frac{y^*}{y^* - b}. \quad (5.46)$$

Differentiating (5.44) and (5.45) with respect to  $y$ , I find that the elasticity of the market tightness with respect to a positive productivity shock is

$$\begin{aligned} \frac{\theta'(y^+)y^*}{\theta(y^*)} &= \frac{1}{1 - \epsilon} \cdot \frac{1 - \beta[1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta[1 - \delta - p(\theta(y^*))\gamma^{\frac{1-\sigma\epsilon}{1-\epsilon}}]} \\ &\cdot \left\{ \frac{\sigma}{1 - \gamma} \frac{1 - \beta[1 - \delta - p(\theta(y^*))\gamma]}{1 - \beta(1 - \delta)} + 1 - \sigma \right\} \cdot \frac{y^*}{y^* - b}. \end{aligned} \quad (5.47)$$

Also in this scenario, the elasticity of the market tightness with respect to a negative shock to  $y$  is the same in a PREE as in an SBE, irrespective of the fraction  $\sigma$  of  $S$ -workers in the population. Since the elasticity of market tightness in an SBE is higher in this scenario than in the previous one, so is the elasticity of the market tightness with respect to a negative shock to  $y$  in a PREE. The elasticity of the market tightness with respect to a positive shock to  $y$  in a PREE is between the elasticity in an REE and the elasticity in an SBE, and the elasticity increases with the fraction  $\sigma$  of  $S$ -workers in the population. It is easy to check that the elasticity of the market tightness in response to a positive shock to  $y$  is higher in the second scenario than in the first one.

The following proposition summarizes the characterization of a PREE. The following proposition summarizes the characterization of an REE.

**Proposition 9.** (*Asymmetric labor market fluctuations*). Let  $\phi_r \rightarrow 1$  and  $\phi_s \rightarrow 1$ .

1. The elasticity of  $\theta(y)$ ,  $p(\theta(y))$ ,  $u(y)$  and  $v(y)$  with respect to a negative  $y$ -shock is the same in a PREE as in an SBE, it is greater than in an REE, and does not depend on  $\sigma$ .
2. The elasticity of  $\theta(y)$ ,  $p(\theta(y))$ ,  $u(y)$  and  $v(y)$  with respect to a positive  $y$ -shock is greater in a PREE than in an REE, it is smaller than in an SBE, and it goes from one extreme to the other as  $\sigma$  increases.
3. The elasticity of  $\theta(y)$ ,  $p(\theta(y))$ ,  $u(y)$  and  $v(y)$  with respect to  $y$  in a PREE is higher if  $S$ -workers do not observe the productivity  $y$  of the firm with which they are bargaining.

To assess the extent of the asymmetry of labor market fluctuations in response to

negative and positive shocks to aggregate productivity, let me return to our back-of-the-envelope calibration. As a reminder, the calibration targets an average UE rate of 30%, an average EU rate of 2%, an elasticity of the job-finding probability with respect to tightness of 0.5, a worker's bargaining power of 0.5, and an unemployment income that is half of the unconditional mean of labor productivity.

In an REE, the elasticity of the labor market tightness is the same with respect to positive and negative shocks to productivity, and it is equal to 2.1 for  $\beta, \phi_s, \phi_r \rightarrow 1$ . Now consider a PREE in which stubborn workers observe the productivity of their employer. For  $\beta, \phi_s, \phi_r \rightarrow 1$  and for a fraction  $\sigma$  of stubborn workers equal to 10% of the population, the elasticity of the market tightness with respect to a negative shock to aggregate productivity is

$$\frac{\theta'(y^*-)y^*}{\theta(y^*)} = \frac{1}{1-\epsilon} \cdot \underbrace{\frac{\delta + p(\theta(y^*))\gamma}{\delta}}_{8.5} \cdot \frac{y^*}{y^* - b} = 34, \quad (5.48)$$

and the elasticity of the market tightness with respect to a positive shock is

$$\frac{\theta'(y^*+)y^*}{\theta(y^*)} = \frac{1}{1-\epsilon} \cdot \underbrace{\frac{\delta + p(\theta(y^*))\gamma}{\delta} \frac{\delta + p(\theta(y^*))\gamma\sigma}{\delta + p(\theta(y^*))\gamma\frac{1-\epsilon\sigma}{1-\epsilon}}}_{.94} \cdot \frac{y^*}{y^* - b} = 3.7. \quad (5.49)$$

The elasticity of the labor market tightness is about 10 times larger in response to a negative shock than in response to a positive shock, and about 15 times larger than in an REE. Similarly, the elasticity of the UE rate, the unemployment rate and the vacancy rate are also 15 times larger in response to a negative shock than in response to a positive shock, and 10 times larger than in an REE.

Next, consider a PREE in which stubborn workers do not observe the productivity of their employer. For  $\sigma = 0.1$ , the no screening condition does not hold. This does not necessarily mean that the bargaining outcome would be different, only that the characterization of the equilibrium would be different and involve mixed strategies. To be on the safe side, however, let me choose  $\sigma = 0.55$ , a value for which the no screening condition holds. For  $\beta, \phi_s, \phi_r \rightarrow 1$ , the elasticity of the market tightness with respect to a negative shock to aggregate productivity is

$$\frac{\theta'(y^*-)y^*}{\theta(y^*)} = \frac{1}{1-\epsilon} \cdot \underbrace{\frac{1}{1-\gamma}}_2 \cdot \underbrace{\frac{\delta + p(\theta(y^*))\gamma}{\delta}}_{8.5} \cdot \frac{y^*}{y^* - b} = 68, \quad (5.50)$$

and the elasticity of the market tightness with respect to a positive shock is

$$\frac{\theta'(y^*+)y^*}{\theta(y^*)} = \frac{1}{1-\epsilon} \cdot \underbrace{\frac{\delta + p(\theta(y^*))\gamma}{\delta + p(\theta(y^*))\gamma\frac{1-\sigma\epsilon}{1-\epsilon}}}_{7} (\dots) \frac{y^*}{y^* - b} = 28. \quad (5.51)$$

The elasticity of the labor market tightness is about 3 times larger in response to a negative shock than in response to a positive shock, and about 30 times larger than in an REE, and so are the elasticities of the UE, unemployment and vacancy rates.

## 6 Conclusions

In this paper, I developed a search-theoretic model of the labor market in which workers have incorrect expectations about their probability of finding a job and the wage they will earn once they are hired. I modelled workers with incorrect expectations as agents who believe that aggregate productivity economy is always at its normal level, and who construct expectations about the tightness of the labor market, the job-finding probability, and the firms' bargaining strategy and the wage according to such belief. The worker's expectations are correct on average, but they are irrationally optimistic when aggregate productivity is below its normal level, and irrationally pessimistic when aggregate productivity is above its normal level. The worker's expectations affect bargaining outcomes, specifically leading to wages that are either too sticky or outright rigid. On the positive side, the behavior of wages leads to excess cyclical volatility of the tightness of the labor market, job-finding probability, unemployment and vacancies. On the normative side, the behavior of wages calls for a countercyclical employment subsidy, even when firms happen to internalize the congestion and thick market externalities associated with vacancy creation.

The model is amenable to several natural extensions. In the paper, I considered a version of the model in which some workers have rational expectations, some workers have stubborn beliefs, and firms cannot observe the type of worker with which they are bargaining. In this version of the model, the outcome of the bargaining game is qualitatively different depending on whether aggregate productivity is below or above its normal level. When aggregate productivity is below its normal level, the outcome of the bargaining game is such that both types of workers earn the same wage that a worker with stubborn beliefs would earn if the firm knew its type. When aggregate productivity is above its normal level, the outcome of the bargaining game is such that each type of worker earns the same wage that he would earn if the firm knew its type. As a result, when aggregate productivity falls below its normal level, the average wage is as sticky or as rigid as it would be in a model where all workers have stubborn beliefs. When aggregate productivity rises above its normal level, the average wage is sticky or rigid only in proportion to the fraction of workers with stubborn beliefs. The asymmetry in the response of wages leads to a response of market tightness, job-finding probability, unemployment and vacancies that is more pronounced in response of negative than in response to positive shocks.

Other extensions of the model seem worth exploring. Let me mention two of them. For example, it would be interesting to consider a version of the model in which the output produced by a firm-worker pair depends on a component of productivity that is aggregate



and one that is specific to the firm-worker pair (as in, say, Mortensen and Pissarides 1994). In this version of the model, the stubbornness of the workers' beliefs would not only lead to inefficiencies in job creation but also in job destruction. In particular, when aggregate productivity is below its normal level, a stubborn worker would have expectations about his job-finding probability that are too optimistic, and he would quit matches that have positive surplus. When aggregate productivity is above its normal level, a stubborn worker would have expectations that are overly pessimistic and he would stay in matches that have a negative surplus. As a result, job destruction would be amplified in response to negative shocks and it would be muted in response to positive shocks.

Another interesting extension would be to consider workers who all believe that the job-finding probability and the wage are constant, but who differ with respect to the level of job-finding probability and wage that they expect. This version of the model would be especially interesting when job-destruction is endogenous. The most optimistic workers would be the most likely to quit a job and the least likely to find a job. The least optimistic workers would be the least likely to keep a job and the most likely to find a job. Therefore, heterogeneity in workers' beliefs could provide an explanation for the systematic heterogeneity in the workers' pattern of transitions across employment states observed in the data (see, e.g., Hall and Kudlyak 2019 and Gregory, Menzio and Wiczer 2021).

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