Precautionary Savings and the Stock-Bond Covariance

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Abstract

I show that the precautionary savings motive can account for the high-frequency variation in the stock-bond covariance. An increase in the price of risk lowers risky asset prices on account of an increase in risk premia; it lowers bond yields on account of the precautionary savings component. Consequently, times when the price of risk is volatile see a more negative stock-bond covariance. I demonstrate that a model of time-varying price of risk, calibrated to fit equity moments, matches well the evidence regarding both the nominal and real stock-bond covariance, even in the absence of inflation innovations. Empirically, I show that the stock-bond covariance co-moves with credit spreads and can predict excess returns on corporate bonds and on bond-like stocks. The calibrated model underlines the systematic nature of high-frequency changes in the stock-bond covariance and the first-order effect of risk compensation on safe rates.

1 Introduction

The stock market exposure of long safe assets displays both low- and high-frequency changes. On the longer time-frame, this key asset pricing moment has shown a steady decline from the 1980s through the present. On shorter time-frames, the stock-bond covariance has shown periodic large negative spikes: during and after the early 2000s recession, during the Global Financial Crisis, during the Eurozone crisis in 2011-12 and, most recently, during the Covid panic in early 2020. What is more, both nominal and real bonds have exhibited strikingly similar time variation in their exposure to stock market returns (I plot the recent behavior of the stock-bond covariance in Panel A of Figure 1).

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While it is straightforward to calibrate an asset pricing model to exhibit either a positive or negative stock-bond covariance, it is tougher to match such high frequency changes. In this paper I show that high-frequency variation in the stock-bond covariance arises naturally in a model with time varying volatility of the price of risk. Under reasonable preference parameter restrictions, the price of risk affects risky and safe asset prices in the opposite direction. Stock prices are decreasing in the price of risk on account of the risk premium term. Bond prices, in contrast, are increasing in the price of risk on account of the precautionary savings term—the dependence of present interest rates on the volatility of the pricing kernel. Any change in the price of risk, then, sees the stock and bond prices move in opposite direction.

This precautionary savings channel of stock-bond covariance is distinct from the majority of existing literature in that it does not rely on inflation dynamics to deliver the time-varying covariance. As shown in Campbell et al. (2017), the changing covariance between inflation and the real economy can account for the changing risk profile of long bonds. However, as Panel A of Figure 1 illustrates, the stock-bond covariance calculated with real bond prices (TIPS) is strikingly similar to that calculated with nominal bond prices, suggesting that inflation dynamics are not the sole driver of the stock-bond covariance, at least in the sample for which real bond yields are available. Indeed, I demonstrate in the model that the precautionary savings term can account for the high-frequency changes in the stock market exposure of both nominal and real bonds, regardless of the prevailing relationship between inflation and real growth.

In order to quantify the importance of the precautionary savings term on the stock-bond covariance, I write down a term structure model that features two key state variables: the price of risk and the aggregate dividend growth rate. I follow the approach in Lettau and Wachter (2007) and directly assume a price of risk process. By model assumption, the price of risk process displays time-varying volatility, while the variation in the forward-looking dividend growth rate has constant volatility. As a result, the relative share of volatility stemming from shocks to the two state variables changes over time. For reasons of tractability I specify that the price of risk follows a discrete time square root Cox, Ingersoll Jr. and Ross (1985) (CIR) process, and that dividend growth shocks are proportional to the price of risk. With these assumptions both the precautionary savings term of safe asset prices and the risk premium term of risky asset prices are affine in the state variables, allowing me to compute in closed form the term structure of stock as well as nominal and real bond prices.

With the model solution in hand, I pick parameters based on recent literature in order to ensure that the model matches the principal equity and bond market moments. I simulate the model at a daily frequency and calculate the implied stock-bond covariance. I show that under a price of risk process that matches the level and variation of the equity risk premium, the model is able to fit the main moments of both the real and nominal stock-bond covariance, as well as the stock market beta of the nominal and real bond, and the stock-bond correlation. For the main calibration I match empirical moments from 1999 to 2020, thus focusing on a period where inflation innovations have not been a major driver of the stock-bond covariance dynamics.

On the empirical side, I document a series of regularities that are consistent with the model mechanism of risk dynamics being the principal high-frequency driver of the stock-bond covariance. The empirical exercises focus on the US market, though I additionally document that other advanced economies show strikingly similar dynamics in the stock-bond covariance calculated with local stock and bond data.

First, I show that the stock-bond covariance has a "level" effect on safe interest rates: large negative covariance episodes tend to see lower interest rates across maturities, corresponding to the level factor in interest rates. The stock-bond covariance also co-moves with the level of aggregate credit spreads. Times of negative covariance—which according to the model are also times when the level of risk premia is high—see high credit spreads of safe over risky fixed income assets. Correspondingly, the stock-bond covariance have some predictive power over excess returns on long maturity Treasuries, and substantial predictive power over excess returns of risky fixed income assets over equivalent maturity Treasury returns.

Secondly, I explore the cross section of stocks and currencies. I show that the stock-bond covariance is a good predictor of "bond-like" stock returns: stocks that have low volatility, are profitable, are mature, or pay stable dividends. I construct long-short portfolios of bond-like stocks following Baker and Wurgler (2012) and show that times of high negative stock-bond covariance are followed by low returns of such bond-like stocks, indicating that they are highly valued at times of negative stock-bond covariance. This predictability remains statistically significant controlling for other bond return predictors. Periods of low stock-bond covariance also predict high excess returns on the seven major currency pairs over the US Dollar.

Thirdly, employing options data on Treasury futures I measure implied volatility and jump probabilities of Treasury bond prices and show that times of high implied volatility of rates and high probability of a jump in Treasury prices coincide with negative covariance episodes.

Fourthly, I show that the stock-bond covariance captures variation in the investment grade share of total bond issuance. Additionally, the stock-bond covariance can also account for sectoral holdings of safe assets. During times of negative covariance broker-dealers and money market funds tend to increase the holdings of Treasuries.

Overall, these empirical regularities point towards large negative stock-bond covariance periods corresponding to times when the price of risk is high, consistent with the model mechanism.

To further explore the model mechanism, I study the stock-bond covariance, as well as the safe yield curve in the simulated data. Consistent with the above empirical results, I find that the rolling stock-bond covariance is a good proxy for the aggregate price of risk in the simulated data.

Additionally, the calibrated model allows me to explore the impact of the precautionary savings term on the term structure. I demonstrate that in the model a two-factor "level" and "slope" decomposition of safe interest rates emerges, reflecting the factor structure observed empirically. The simulated data reveals that the price of risk process has a strong impact on the level of interest rates. The time-varying dividend growth rate, in contrast, primarily determines the slope of the term structure. Just like in the data, the level factor corresponds to the majority of yield curve variation. Overall, the yield curve decomposition provides an distinct economic rationale of the first two yield curve principal factors and underlines the first-order impact of the price of risk process on the yield curve.

While the proposed model mechanism of changing stock-bond covariance is operative in a purely real model, I also explore the role of variation in the real-nominal relationship as highlighted by Campbell et al. (2017). I demonstrate that adjusting the real-nominal relationship has a large impact on the average level of the stock-bond covariance, all the while keeping intact the high-frequency dependence on risk prices. To do this, I simulate the model under an alternative calibrations that change the relationship between inflation innovations and realized dividend growth. To capture the inflation dynamics of the 1980s, I specify a calibration with a higher level of expected inflation, and crucially, a strong negative dependence of the real dividend growth on expected inflation rate, while keeping all other parameters unchanged. This parametrization allows me to match both real and nominal stock-bond covariance moments in the early part of the sample, spanning from 1973 to 1999. The model is therefore consistent with the empirical finding that the real and nominal stock-bond covariance move together at high frequencies, a fact evident both in the U.S. and U.K. data, and illustrated in Figures 1 and 2.

A number of papers have studied the stock-bond covariance. In recent work, Kozak (2015) specifies a production economy where two distinct production processes differ in riskiness and adjustment costs. One of the processes has relatively safe cash flows, the other relatively risky cash-flows. The model features time-varying risk aversion which affects the relative valuations of these two production processes and by the two-tree mechanism described in Cochrane et al. (2007), the aggregate market covariance with bond returns is rendered time-varying. In contrast to the model presented here, it is the change in the aggregate capital mix in the economy that drives changes in the covariance. In the model here all assets have constant loadings on the key sources of risk, but the relative share of total volatility stemming from the two state variables changes over time. In contrast to the rest of the literature, and like the model here, Kozak (2015) is able to generate a time-varying stock-bond covariance in a purely real model.

In other recent work, Campbell et al. (2017) study a term structure model with a constant price of risk and attribute the changing sign of the stock-bond covariance to the change in covariance between the nominal interest rate and the real economy. Campbell et al. (2020) focus on the interaction of preferences and changing inflation exposure of the real economy to demonstrate the importance of risk premia in accounting for the time-series of stock-bond return covariance. Pflueger et al. (2020) construct a measure of aggregate risk appetite by comparing the relative valuations of high and low volatility stock portfolios. They document that this measure has strong explanatory power over the short interest rate and predicts investment. I show that their measure of relative valuations follows the high-frequency stock-bond covariance very closely in the recent data. I therefore interpret the relative valuation measure in Pflueger et al. (2020) as being driven by the same underlying phenomenon than the stock-bond covariance.

As I summarize further in the Literature section, there is a long history of work seeking to jointly understand stock and bond returns. Papers that study stock and bond returns in affine term structure models include Bekaert et al. (2010), Bansal and Shaliastovich (2013) though they do not specifically address the time-varying nature of stock-bond covariance.

The literature has focused less on the precautionary savings term. Many standard asset pricing models assume away the precautionary savings term for analytical convenience. One well-known example is Campbell and Cochrane (1999) where the functional form of the habit formation process ensures that the precautionary savings term of the risk-free rate exactly cancels out the intertemporal substitution term (this assumption is subsequently relaxed in Wachter (2006) who studies the term structure under a general process of the surplus consumption ratio). Similarly, in Lettau and Wachter (2007) the risk-free rate is assumed to be constant at all maturities, despite the time variation in the price of risk. I show that the inclusion of the precautionary savings term materially alters the stock-bond covariance in the model—though in many settings the inclusion of this term comes at the cost of analytical tractability.

I organize the paper in three main sections. In the first section document a number empirical regularities regarding the stock-bond covariance in recent data. In the second section I write down a term structure model that can account for the high-frequency variation the stock-bond covariance. I show that the model can fit the observed level and variation in the covariance, correlation, and beta of the long bond. In the third section I further explore model-generated data. I show that in the model a two-factor structure of safe yields emerges and the price of risk process is a major determinant of the "level" factor of interest rates. The model presented here therefore highlights the first order importance of precautionary savings in determining safe interest rates. By emphasizing the quantitatively large effect of time-varying risk aversion on intertemporal substitution, the calibration underlines the role of precautionary savings in escaping the "macro-finance separation" of Tallarini (2000). A spike in risk aversion—everything else equal—changes the price of intertemporal substitution.

2 Empirics

In this section I describe the behavior of the stock-bond covariance and show how it relates to the level of safe and risky interest rates, the returns on safe and risky assets, risk-neutral moments of the Treasury return distribution, and holdings and issuance of fixed income assets.

For the majority of empirical facts I employ monthly data from 1973 to 2020. I choose the start date to reflect the availability of credit spreads, particularly the Gilchrist and Zakrajšek (2012) credit spread. A number of the regression estimates are presented separately for the sample from 1997 to 2020. The reason is twofold. Many of the data items, such as real bond prices in the US, holdings data, and issuance data have only become available recently. More importantly, this restricts to a time period where inflation has been under control.

2.1 Stock-Bond Covariance Over Time and Across Countries

Panel A of Figure 1 plots the stock-bond covariance from 1973 to 2020. I calculate the covariance between the return on the CRSP value-weighted portfolio, and the return on a 10-year zero coupon constant maturity real or nominal Treasury bond in a rolling 30 trading day window. I then average the daily calculated covariances to a monthly frequency. The covariances are reported on an annualized basis in percent units[1]. Both real and nominal zero-coupon bond prices are from the daily fitted yield curve constructed in Gürkaynak et al. (2007) and Gürkaynak et al. (2010).

To put the recent data into historical context, I'm also including a longer time-series in Panel B of Figure 1. Here I'm using monthly return data and the stock-bond covariance is calculated using a rolling window of 120 observations from 1880 to 2020. Panel B shows that while the large negative covariance is mostly a recent phenomenon, the 1960s and assorted periods before then have also seen negative stock-bond covariance. The very high values of stock-bond covariance in the 1980s are an outlier with respect to historical experience.

As already documented in prior work, the covariance between real bonds and the stock market is close to identical to the nominal bond covariance. In the US, Treasury Inflation Protected Securities (TIPS) have traded since the late 1990s and outside the most severe negative spikes the covariance with the stock market of the real bonds has been tracking closely that of nominal Treasuries, as shown on Figure 1 Panel A. Indeed, the correlation between the two monthly series is .84 in the sample starting in 1999 (TIPS started trading in 1997 but the Gürkaynak et al. (2010) fitted yield curve data begins in 1999).

I summarize both the real and nominal stock-bond covariance moments in Table 1. Panel A

[1]For instance, if the daily stock return volatility is 2% and daily bond return volatility is .5% and the correlation between the two return series is -.6, the covariance would be reported as $.02 \times .005 \times 252 \times -.6 = -1.512\%$.

characterizes the real and nominal stock-bond covariance for both the United States and the United Kingdom in the 1999-2020 sample. Just like in the U.S. data, both the real and nominal covariance in the U.K. data have negative means in this sample. The standard deviation of the U.S. measure is almost double that of the U.K.'s but that is mostly on account of a handful of extreme negative realizations. The U.K. data further resembles the U.S. in that the nominal and real measures are highly correlated: .90 in the sample since 1999.

The similarity between real and nominal measures is surprising in light of the inflation-based accounts of the stock-bond covariance put forth in the literature. A simple, though unlikely, explanation is that the Bureau of Labor Studies measured inflation that determines the inflation-protected bond variable coupon rate is a poor measure of true inflation, and in effect the TIPS payout is that of a nominal bond plus noise. A more plausible explanation is that in recent times inflation has has been under control and surprises in the inflation rate have been small, and the the observed variation in both covariances is mostly not driven by inflation dynamics. (The impact of inflation shocks on bond returns can be mild if realized inflation close to expectations, or if inflation shocks command a small price of risk.)

This explanation—inflation innovations being relatively tame in the recent sample—is also in line with the longer time series of real bond yields available in the U.K. The correlation between the real and nominal measures is weaker (.74) in the 1985-1998 sample in which inflation was more variable. All the while, the nominal and real measures still exhibit substantial co-movement, consistent with the model proposed in this paper. The 1985-1998 sample also saw a larger gap in the level of the real and nominal stock-bond covariance, a fact that is consistent with a higher importance of inflation shocks, as I show in the model. The stock-bond covariance in the 1985-1998 sample for the U.S. and U.K. are summarized in Panel C of 1.

Panel B of Table 1 documents the stock-bond covariance in a selection of advanced economies. I calculate local stock-bond covariances using daily data on equity market returns, and 10-year nominal bond returns, both downloaded from DataStream. All of the countries show a substantially negative stock-bond covariance in this sample. Moreover, the standard deviations of the respective measures all hover around .5, consistent with the U.K. data, though somewhat lower than the standard deviation in the U.S. In part the higher S.D. in the U.S. reflects extreme downward spikes during the Global Financial Crisis, and during the Coronavirus panic.

I plot some of these international stock-bond covariances in Figure 2. In Panel A I show the stock-bond covariance in the U.K., using both nominal and real bond prices. As already noted, there is substantial commonality in the U.S.- and U.K.-based measures. Note too that the real and nominal covariances in the U.K. tend to substantially co-move throughout the sample, consistent with the precautionary savings channel proposed here.

In Panel B of Figure 2 I plot the data from the U.S., Germany, Japan, and Spain. I highlight these three countries for the following reasons. The German data shows striking similarity to that calculated with U.S. data, even though the stock and bond markets taken separately are not that highly correlated. Japan was the first to witness negative covariance in the early 1990s when local interest rates first hit zero and inflation crossed into negative territory. Spain saw the local stock-bond covariance wander into large positive territory precisely when its bonds became risky during the Eurozone crisis, emphasizing the importance of low default risk in delivering the negative covariance.

2.2 Term Structure of Safe Rates

Let me now turn to documenting relationships between the stock-bond covariance and term structure of safe rates. Stock-bond covariance corresponds to a "level" effect on the term structure of interest rates. In Table 3 I regress monthly changes in the 1-, and 10-year zero coupon Treasury yields on the changes in the contemporaneous stock-bond covariance. These rates (as well as other maturities) tend to be lower when stock-bond covariance goes down, roughly corresponding to the level factor of interest rates. This conclusion holds when I estimate the regression controlling for changes in stock return volatility measured over the same look-back window.

In order to summarize the impact on the entire yield curve I also estimate regressions with forward curve factors as left-hand-side variables. I start with the Gürkaynak et al. (2007) fitted yield curve data and decompose the daily 1- to 15-year instantaneous forward rates by principal component analysis. Consistent with the literature (for instance, Cochrane and Piazzesi (2005)) I find that the first three components—traditionally labeled level, slope, and curvature—account for 99.96% of the variation in the forward rate curve. I again estimate monthly regressions in first differences, controlling for changes in stock volatility calculated over the same look-back window.

The estimates in Table 3 underline that the stock-bond covariance has a level effect on the forward curve. When the backwards-looking stock-bond covariance is large and negative, all yields tend to be depressed as a function of their loading on the level factor. Low covariance times tend to see a higher slope factor, though that effect is not statistically significant.

Because of the level effect on the term structure of interest rates, the stock-bond covariances calculated with other maturity zero-coupon bond result in strikingly similar, net of the duration effect. In other words, the stock bond covariance calculated with a 10-year bond rate is roughly double of the corresponding covariance calculated with a 5-year rate.

2.3 Credit Spreads

Corporate bond credit spreads show a strong correlation with the stock-bond covariance: during times when the covariance is negative, credit spreads tend to be high. In Table 4 I estimate regressions of ratings-based corporate bond spreads over Treasuries on the stock-bond covariance. I also include the GZ Spread as well as the Excess Bond Premium (EBP) constructed in Gilchrist and Zakrajšek (2012) and updated by the Federal Reserve Board[2]. The stock-bond covariance explains over 13% of the variation in the GZ spread and over 30% of the variation in the High Yield-Treasury spread. Again the regressions are estimated in first differences, and the conclusion holds true controlling for changes in the market volatility over the same look-back window. In the model I interpret times of negative stock-bond covariance as times when the price of risk is high—which in turn implies wide credit spreads.

2.4 Aggregate and Relative Return Predictability

Cochrane and Piazzesi (2005) find that risk premia across the term structure of Treasury rates is captured by a single variable that affects the term structure via the level factor. Any variable, then, that can explain movements in the level factor can potentially serve as a predictor of bond returns.

As shown in Table 5 Panel A, the stock-bond covariance has some predictive power over longer maturity Treasury returns, particularly in the sample since 1997 when inflation has been tame. Negative covariance episodes predict poor Treasury returns going forward, consistent with the view that safe assets are particularly valued at those times.

Panel B of the same Table find some evidence of aggregate stock return predictability, again concentrated in the sample since 1997.

The stock-bond covariance does considerably better in predicting relative returns. Specifically, I show that it can predict returns of credit-rating sorted bond portfolios, of "bond-like" stocks and, of foreign currency excess returns over the U.S. Dollar. The estimates are in Tables 6, 7, and 8.

In Table 6 I show that the stock-bond covariance predicts with a negative sign the 12-month returns on bond portfolios that are long corporate bonds and short Treasuries of equivalent maturity. The regressions reveal a substantial predictability in the relative returns, with R^2 over 12% for Baa-rated bonds in the recent sample. The predictive power of stock-bond covariance is not subsumed by other predictors such as the Cochrane and Piazzesi (2005) factor (constructed like in Hodrick et al. (2021)) or the sentiment factor of Baker and Wurgler (2006).

[2] `https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/updating-the-recession-risk-and-the-excess-bond-premium-20161006.html`

Table 7 documents predictability in long-short portfolios that seek to capture "bond-like" stocks. The portfolios are constructed following Baker and Wurgler (2012). The seven portfolios are: *age*—long old firms, short young firms; *dbe*—long high dividend to book equity firms; *bm*—long high book to market firms; *ebe*–long high earnings to book equity firms; *efa*—long high external finance firms; *gs*—long high sales growth firms; *rvol*—long high volatility firms.[3]

I find statistically significant return predictability in the expected direction for many of these portfolios at the 60-month horizon. Older firms, high dividend firms, high earnings firms, and low volatility firms tend to underperform after episodes of large negative stock-bond covariance, consistent with the view that these assets are highly valued during such periods. Like with the bond predictability returns the conclusions are robust to including known predictors such as the Cochrane and Piazzesi (2005) factor or Baker and Wurgler (2006) sentiment factor. I also construct an "aggregate" bond-like portfolio by combining the seven bond-like stock portfolios with equal weights. Such an aggregate bond-like stock portfolio is strongly predictable by the stock-bond covariance. Similar aggregate predictability holds on the 12-month horizon.

In Table 8 I demonstrate FX return predictability with the U.S. stock-bond covariance. I construct 12-month excess returns of the 7 major currency pairs over the USD. Times of negative stock-bond covariance in the U.S. see positive excess returns on foreign currencies going forward. This evidence is consistent with the view that negative stock-bond covariance periods see increases in demand for USD, a flight to the safe-haven currency. This predictability is particularly strong in the sample starting in 1997 when inflation innovations have been tame. Again I construct and "aggregate" currency pair portfolio by combining the seven individual currencies. Just like with bond-like stocks, the aggregate portfolio return is strongly predictable by the stock-bond covariance.

2.5 Risk Neutral Distribution of Safe Returns

With the availability of traded options on Treasury futures it is possible to construct forward-looking measures of the risk-neutral Treasury return distribution. In this section I show that the stock-bond covariance is more negative when implied volatility is high, and when the probability of a jump in Treasury prices (a flight-to-quality episode) is high.

Using Treasury options data from the CME Group, I follow Choi et al. (2017) and calculate the 10-year Treasury implied volatility as:

$$IV_t = \sqrt{\frac{2e^{rT}}{T}\left(\sum_{K_i=0}^{F_{t,T}} \frac{P_{t,T}(K_i)}{K_i^2}\Delta K + \sum_{K_i=F_{t,T}}^{\infty} \frac{C_{t,T}(K_i)}{K_i^2}\Delta K\right)} \tag{1}$$

[3]Note that Baker and Wurgler (2012) also include book-to-market (BM) sorted portfolios. However, in the updated sample BM portfolio is not predictable by the sentiment factor, nor is it predictable by the covariance measure.

where $C_{t,T}(K)$ and $P_{t,T}(K)$ are the time t prices of calls and puts with strike K maturing at time T, r is the continuously compounded risk-free rate, and F_t is the time t price of the underlying futures contract.

I follow Bollerslev and Todorov (2011) to estimate the probability of large jumps in the underlying future from out-of-the-money option prices. Specifically, I calculate:

$$LT_t(k) = \frac{e^{rT} C_{t,T}(K)}{(T-t)F_t} \quad \text{and} \quad RT_t(k) = \frac{e^{rT} P_{t,T}(K)}{(T-t)F_t} \tag{2}$$

where $k = \frac{F_t}{K}$ is the moneyness of the underlying option and the remaining notation is as before. $LT_t(k)$ measures the time t risk-neutral probability of an upward jump to price $k \times F_t$ or higher of the underlying futures contract. Similarly, $RT_t(k)$ measures the probability of a downward jump to price $k \times F_t$ or lower of the underlying futures contract, corresponding to an increase in the interest rate.

As shown in Table 9, implied volatility of the 10-year bond price coincides with negative covariance episodes. Intriguingly, a high risk-neutral probability of a jump in bond prices (meaning a flight-to-quality episode) is associated with larger negative stock-bond covariance. In contrast, a high probability of a drop in bond prices is associated with a positive covariance.

2.6 Issuance and Holdings

As documented above, negative stock-bond covariance episodes see a wider spread between prices of safe and risky fixed income assets. It is therefore reasonable to expect that potential issuers of safe corporate bonds would find it an opportune time to increase issuance. This is what I find. As shown in the first three columns of Table 10 Panels A and B, negative stock-bond covariance episodes see a larger share of safe asset issuance over total corporate issuance. Similarly, the share of financial sector investment grade issuance over GDP is higher when the covariance is negative.

Combined with the previously discussed result on bond return predictability, these facts offer a way to interpret the results in Greenwood and Hanson (2013). They find that a high share of non investment grade issuance in the corporate bond market predicts poor corporate bond returns going forward. As shown above, negative stock-bond covariance times see an increase in the wedge between safe and risky bond prices—inducing more issuance from high grade issuers. Going forward, on average prices tend to converge as the price of risk returns to normal levels. As a consequence, the non investment grade issuance share negatively predicts bond returns.

Stock-bond covariance can also account for time-series variation in the sectoral holdings of safe assets. I construct quarterly holdings of Treasury assets based on data from *Financial Accounts of the United States* (formerly the *Flow of Funds.*) As shown in Table 10 columns 4 to 6, times of negative covariance broker-dealers and money market funds tend to increase the holdings of Treasuries. I interpret these findings as evidence in high demand during bad times for safe collateral by the shadow banking system, as modeled in Moreira and Savov (2017).

2.7 Pandemic Evidence

Market behavior during the recent global pandemic illustrates the striking time variation in stock-bond covariance. In Figure 8 I plot the stock-bond covariance since the beginning of 2020. The calculation is slightly different than the main variable used in the rest of the paper. I use the prices of the SPDR SP500 ETF (ticker: SPY) and the iShares 7-10 year Treasury ETF (ticker:IEF) to calculate returns in 15-minute intervals, and calculate the stock-bond covariance over a 2-day lookback window.

As the Figure shows, the initial drop in market values in late February did not see a spike in the stock-bond covariance, despite increased volatility as measured by VIX. The first three weeks of March, in contrast, saw unprecedented levels of stock-bond covariance with the two-day calculation using intraday returns reaching values of -36, more than twice the highest levels of the same quantity during the 2008 financial crisis. This underlines the importance of discount rate volatility at the height of the market turmoil in spring 2020. In line with the return predictability documented in this section, the subsequent stock market returns from April onwards were substantial.

3 Model

The goal of the theory is to construct a tractable model that delivers a high-frequency variation in the stock-bond covariance. To this end, I follow the approach in Lettau and Wachter (2007) and Lettau and Wachter (2011) and work with an exogenously specified process for the price of risk. The calibrated model employs parameters that can match the aggregate equity risk premium in order to demonstrate that such variation in the compensation for risk, studied extensively in the literature, can also match the observed variation in the stock-bond covariance.

The main departure from these papers lies in that I employ a price of risk state variable x_t that follows a discrete time Cox, Ingersoll Jr. and Ross (1985) (CIR) process. The CIR process features time-varying volatility: in this formulation the price of risk is volatile when it is high. It therefore provides a convenient way to impose time-varying volatility of the aggregate risk premium.

Let me set up the model. The log stochastic discount factor (SDF) is given by

$$m_{t,t+1} = a - \rho(g + z_t) - \sqrt{x_t}\sigma_c\epsilon_{t+1}. \tag{3}$$

while the aggregate dividend growth rate is given by

$$\ln\left(\frac{D_{t+1}}{D_t}\right) = \Delta d_{t+1} = g + z_t + \theta y_t + \sqrt{x_t}\sigma_d \epsilon_{t+1} - .5\sigma_d^2 x_t. \tag{4}$$

These two equations contain all three state variables of the economy: x_t, the square of the price of risk; z_t the time-varying component of the growth rate of aggregate dividends; y_t the time-varying component of the inflation rate.

Equation (3) specifies that the only priced risk is ϵ_{t+1}, the innovation to the aggregate dividend growth. This modeling choice follows Lettau and Wachter (2007) and restricts the degrees of freedom in the model. The ϵ_{t+1} innovation is multiplied by $\sqrt{x_t}\sigma_c$, the time-varying price of risk. For comparison, under CRRA utility the corresponding term would be $\gamma\sigma_c\epsilon_{t+1}$ and so $\sqrt{x_t}$ can be thought of as a risk aversion coefficient. The model does not specify the underlying economic mechanism that gives rise to the time variation in the price of risk—it could stem from time-varying disaster risk, habits, intermediation frictions, behavioral effects, and so on. The parameter ρ in the SDF multiplies the growth rate $(g + z_t)$ of aggregate dividend process and is akin to one over the elasticity of intertemporal substitution. Comparing with CRRA again, the corresponding term of the SDF would be $\gamma(g + z_t)$. The term a can be interpreted as a time discount term. The SDF is specified in real terms: it discounts future payoffs described in real terms into real present values.

Equation (4) specifies the aggregate dividend process. The dividends are specified in real terms. The expected growth rate consists of a constant term, g, and a time-varying part, z_t. Innovations to the growth rate are represented by the term $\sqrt{x_t}\sigma_d\epsilon_{t+1}$. The inclusion of the $\sqrt{x_t}$ tern ensures that the equity risk premium is linear in x_t, allowing me to derive prices of risky assets in closed form. In addition, the growth rate is exposed to the level of inflation via the term θy_t. This term captures whether high inflation tends to coincide with low growth (as in the 1980s), or with high growth (as in the 2010s). As I explore below, this parameter allows me to capture the gradual change in the real-nominal relationship that has taken place over the past 50 years. Finally, the term $.5\sigma_d^2 x_t$ ensures that the level of expected dividend growth rate does not depend on the level of the price of risk x_t.

Let me specify the functional form of the other three state variables. The price of risk process x_t evolves according to a CIR process by which its volatility is proportional to the square root of the current value

$$x_{t+1} = (1 - \phi_x)\bar{x} + \phi_x x_t + \sqrt{x_t}\sigma_\eta \eta_{t+1}. \tag{5}$$

The time-varying dividend growth rate follows

$$z_{t+1} = \phi_z z_t + \sigma_\xi \xi_{t+1} - .5\sigma_\xi^2. \tag{6}$$

Note that I have included the term $.5\sigma_\xi^2$ to ensure that the expected growth rate does not depend on the volatility. Finally, the price level Π_t evolves according to

$$\ln\left(\frac{\Pi_{t+1}}{\Pi_t}\right) = \Delta\pi_{t+1} = i + y_t \tag{7}$$

where i is the unconditional expected inflation rate and the the time-varying component y_t follows

$$y_{t+1} = \phi_y y_t + \sigma_\chi \chi_{t+1} - .5\sigma_\chi^2. \tag{8}$$

Note that the only shock that is directly priced is the dividend growth shock ϵ, following Lettau and Wachter (2007) (for an empirical investigation of the price of risk of innovations in x_t see Kozak and Santosh (2020)).

With this setup I can find closed-form expressions for asset prices using recursive substitution, as shown in the subsequent sections.

3.1 Real Bond Prices

Let me start by deriving the term structure of real interest rates. At time t the price of a zero-coupon bond with maturity n periods from now—denoted $P_{n,t}^R$—has to satisfy:

$$P_{n,t}^R = \mathrm{E}_t\left[\exp\{m_{t,t+1}\}P_{n-1,t+1}^R\right] \tag{9}$$

Let me guess and verify that bond prices are affine in the state variables (except for y_t as this is a real bond). The pricing equation then reads

$$\begin{aligned}
P_{n,t}^R &= \exp\left\{A_{(n)}^R + B_{(n)}^R x_t + C_{(n)}^R z_t\right\} = \\
&= \mathrm{E}\left[\exp\{a - \rho(g + z_t) - \sqrt{x_t}\sigma_c\epsilon_{t+1}\}\exp\left\{A_{(n-1)}^R + B_{(n-1)}^R x_{t+1} + C_{(n-1)}^R z_{t+1}\right\}\right] \\
&= \mathrm{E}\Big[\exp\{a - \rho(g + z_t) - \sqrt{x_t}\sigma_c\epsilon_{t+1}\} \\
&\exp\left\{A_{(n-1)}^R + B_{(n-1)}^R\left((1-\phi_x)\bar{x} + \phi_x x_t + \sqrt{x_t}\sigma_\eta\eta_{t+1}\right) + C_{(n-1)}^R\left(\phi_z z_t + \sigma_\xi\xi_{t+1} - .5\sigma_\xi^2\right)\right\}\Big],
\end{aligned} \tag{10}$$

which results in the pricing recursions

$$\begin{aligned}
&\implies A_{(n)}^R = a - \rho g + A_{(n-1)}^R + B_{(n-1)}^R(1-\phi_x)\bar{x} \\
&\implies B_{(n)}^R = B_{(n-1)}^R\phi_x + .5B_{(n-1)}^{R^2}\sigma_\eta^2 + .5\sigma_c^2 \\
&\implies C_{(n)}^R = C_{(n-1)}^R\phi_z - \rho.
\end{aligned} \tag{11}$$

The boundary condition from a bond that matures in the current period imposes that $A^R_{(0)} = B^R_{(0)} = C^R_{(0)} = 0$. Any other maturity real bond price can be found in closed form by applying the above recursions. In Figure 6 I plot the coefficients A^R, B^R, and C^R at different maturities.

3.2 Nominal Bond Prices

Let $P^{\pi}_{n,t}$ denote the real price of maturity n nominal bond at time t. As defined above, Π_t denotes the price level. The nominal price, then, of this bond is given by

$$P^N_{n,t} = P^{\pi}_{n,t}\Pi_t. \tag{12}$$

Recall that the SDF $m_{t,t+1}$ discounts real payoffs. Therefore, we can write the pricing equation for the real price of nominal bonds as

$$P^{\pi}_{n,t} = \mathrm{E}\left[\exp\{m_{t,t+1}\}P^{\pi}_{n-1,t+1}\right] \tag{13}$$

Let's multiply the right hand side of the last equation with $\frac{\Pi_{t+1}\Pi_t}{\Pi_{t+1}\Pi_t} = 1$ and rearrange

$$\begin{aligned}
P^{\pi}_{n,t} &= \mathrm{E}\left[\exp\{m_{t,t+1}\}\frac{\Pi_{t+1}\Pi_t}{\Pi_{t+1}\Pi_t}P^{\pi}_{n-1,t+1}\right] \\
\implies P^{\pi}_{n,t}\Pi_t &= \mathrm{E}\left[\exp\{m_{t,t+1}\}\frac{\Pi_t}{\Pi_{t+1}}P^{\pi}_{n-1,t+1}\Pi_{t+1}\right] \\
\implies P^N_{n,t} &= \mathrm{E}\left[\exp\{m_{t,t+1}\}\frac{\Pi_t}{\Pi_{t+1}}P^N_{n-1,t+1}\right].
\end{aligned} \tag{14}$$

Let me again guess and verify that bond prices are affine in the state variables. Like Equation 14 In contrast to the real bond, the nominal bond price will depend on the time-varying rate of expected inflation, y_t. The pricing equation then reads

$$\begin{aligned}
P^N_{n,t} =& \exp\left\{A^N_{(n)} + B^N_{(n)}x_t + C^N_{(n)}z_t + D^N_{(n)}y_t\right\} = \\
&= \mathrm{E}\Big[\exp\left\{a - \rho(g + z_t) - \sqrt{x_t}\sigma_c\epsilon_{t+1}\right\}\exp\{-i - y_t\} \\
&\exp\left\{A^N_{(n-1)} + B^N_{(n-1)}x_{t+1} + C^N_{(n-1)}z_{t+1} + D^N_{(n-1)}y_{t+1}\right\}\Big] \\
&= \mathrm{E}\Big[\exp\left\{a - \rho(g + z_t) - \sqrt{x_t}\sigma_c\epsilon_{t+1}\right\}\exp\{-i - y_t\} \\
&\exp\left\{A^N_{(n-1)} + B^N_{(n-1)}\left((1-\phi_x)\bar{x} + \phi_x x_t + \sqrt{x_t}\sigma_\eta\eta_{t+1}\right)\right\} \\
&\exp\left\{C^N_{(n-1)}\left(\phi_z z_t + \sigma_\xi\xi_{t+1} - .5\sigma^2_\xi\right) + D^N_{(n-1)}\left(\phi_y y_t + \sigma_\chi\chi_{t+1} - .5\sigma^2_\chi\right)\right\}\Big],
\end{aligned} \tag{15}$$

resulting in the recursive solutions for the coefficients of the pricing equation

$$
\begin{aligned}
&\implies A^N_{(n)} = a - \rho g + A^N_{(n-1)} - i + B^N_{(n-1)}(1-\phi_x)\bar{x} \\
&\implies B^N_{(n)} = B^N_{(n-1)}\phi_x + .5B^{N2}_{(n-1)}\sigma^2_\eta + .5\sigma^2_c \\
&\implies C^N_{(n)} = C^N_{(n-1)}\phi_z - \rho \\
&\implies D^N_{(n)} = D^N_{(n-1)}\phi_y - 1.
\end{aligned} \tag{16}
$$

The boundary condition from a nominal bond about to mature results in $A^N_{(0)} = B^N_{(0)} = C^N_{(0)} = D^N_{(0)} = 0$. Any other maturity nominal bond price can be found in closed form by applying the above recursions. In Figure 6 I plot the coefficients A^N, B^N, C^N, and D^N at different maturities.

3.3 Equity Prices

Finally, in order to solve for stock prices let's denote the price of a maturity n dividend strip at time t by $P_{n,t}$. Like fixed income assets, dividend strip prices have to satisfy the recursive pricing equation

$$
P_{n,t} = \mathrm{E}_t\left[\exp\{m_{t,t+1}\}P_{n-1,t+1}\right]. \tag{17}
$$

Multiplying the right-hand-side with $\frac{D_{t+1}D_t}{D_{t+1}D_t} = 1$ and rearranging results in

$$
\frac{P_{n,t}}{D_t} = \mathrm{E}_t\left[\exp\{m_{t,t+1}\}\frac{D_{t+1}}{D_t}\frac{P_{n-1,t+1}}{D_{t+1}}\right]. \tag{18}
$$

Let's guess and verify that the price-dividend ratio of maturity n cash-flow at time t is given by

$$
\frac{P_{n,t}}{D_t} = \exp\left\{A_{(n)} + B_{(n)}x_t + C_{(n)}z_t + D_{(n)}y_t\right\}. \tag{19}
$$

Substituting in the assumed dividend growth process of $D_{t+1}/D_t = \exp\{\Delta d_{t+1}\}$ results in a recursive pricing equation

$$
\begin{aligned}
\frac{P_{n,t}}{D_t} &= \exp\left\{A_{(n)} + B_{(n)}x_t + C_{(n)}z_t + D_{(n)}y_t\right\} \\
&= \mathrm{E}\left[\exp\left\{a - \rho(g+z_t) - \sqrt{x_t}\sigma_c\epsilon_{t+1}\right\}\exp\left\{g + z_t + \theta y_t + \sqrt{x_t}\sigma_d\epsilon_{t+1} - .5\sigma^2_d x_t\right\}\right. \\
&\quad \left.\exp\left\{A_{(n-1)} + B_{(n-1)}x_{t+1} + C_{(n-1)}z_{t+1} + D_{(n-1)}y_{t+1}\right\}\right] \\
&= \mathrm{E}\left[\exp\left\{a - \rho(g+z_t) - \sqrt{x_t}\sigma_c\epsilon_{t+1} + g + z_t + \theta y_t + \sqrt{x_t}\sigma_d\epsilon_{t+1} - .5\sigma^2_d x_t\right.\right. \\
&\quad + A_{(n-1)} + B_{(n-1)}((1-\phi_x)\bar{x} + \phi_x x_t + \sqrt{x_t}\sigma_\eta\eta_{t+1}) \\
&\quad \left.\left.+ C_{(n-1)}(\phi_z z_t + \sigma_\xi\xi_{t+1} - .5\sigma^2_\xi) + D_{(n-1)}(\phi_y y_t + \sigma_\chi\chi_{t+1} - .5\sigma^2_\chi)\right\}\right].
\end{aligned} \tag{20}
$$

Matching coefficients results in the following recursive relationships

$$
\begin{aligned}
&\implies A_{(n)} = a - \rho g + g + A_{(n-1)} + B_{(n-1)}(1-\phi_x)\bar{x} \\
&\implies B_{(n)} = B_{(n-1)}\phi_x - \sigma_c\sigma_d + .5B_{(n-1)}^2\sigma_\eta^2 + .5\sigma_c^2 \\
&\implies C_{(n)} = C_{(n-1)}\phi_z - \rho + 1 \\
&\implies D_{(n)} = D_{(n-1)}\phi_y + \theta.
\end{aligned} \tag{21}
$$

Like with bonds, boundary conditions of an equity strip about to mature imply $A_{(0)} = B_{(0)} = C_{(0)} = D_{(0)} = 0$. In Figure 6 I plot these four coefficients as a function of dividend maturity.

The price dividend ratio of the aggregate dividend claim is just the sum of the price dividend ratios of all the constituent dividend strips.

$$
\frac{P_t^M}{D_t} = \sum_{n=1}^{\infty} \frac{P_{n,t}}{D_t} = \sum_{n=1}^{\infty} \exp\left\{A_{(n)} + B_{(n)}x_t + C_{(n)}z_t + D_{(n)}y_t\right\} \tag{22}
$$

With the aggregate price-dividend ratio I can calculate the aggregate market return from t to $t+1$:

$$
R_{t,t+1} = \frac{P_{t+1}^M + D_{t+1}}{P_t^M} = \left(\frac{P_{t+1}^M}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}\Big/\frac{P_t^M}{D_t}. \tag{23}
$$

3.4 Calibration of the Model

I calibrate the model to match the observed level and volatility in both the 10-year bond beta, correlation, and covariance with respect to stock market returns.

Recall that there is only one shock in the economy that is directly priced: the consumption growth innovation ϵ. The compensation for this risk is determined by the quantity $\sigma_c\sqrt{x_t}$. I choose $\sigma_c = 1.9\%$ and $\bar{x} = 235$, meaning that the average Sharpe ratio of the ϵ shocks is just under .3:

$$
\text{SR} = \sqrt{\bar{x}}\sigma_c = \sqrt{235} \times .019 = .291 \tag{24}
$$

The time variation in stock-bond covariance depends on the volatility of the price of risk process. I pick $\sigma_\eta = 5.8$ and $\phi_x = 87$. The persistence of the price of risk process is identical to the value used in Lettau and Wachter (2007) and results in a price-dividend ratio that has a monthly persistence of .98 which equals the empirical counterpart in the data from 1999 to 2020, as shown in Table 11. The volatility of x_t is picked so that it is proportionally as large with respect to its level as in the calibration of Lettau and Wachter (2007).

The dividend growth shocks parameter σ_d is set at $= 1.4\%$. Note that σ_d does not have the exact interpretation than usual because it appears jointly with the price of risk $\sqrt{x_t}$. The resulting average dividend growth volatility is $\sigma_d\sqrt{235} = 21.5\%$ which matches dividend growth rate volatility estimated from the SP500 stocks in postwar data.

The persistence of the time-varying component of the dividend growth rate shocks is .7, while its the volatility is .17%, both of which are lower than the corresponding values in Lettau and Wachter (2007), .91 and .32%, respectively.

The persistence of the time-varying component of inflation rate is .4 while its volatility is .34%. The volatility matches the value in Lettau and Wachter (2011) while the persistence is lower than the value of .78 used in that calibration.

The parameter ρ corresponds to one over the intertemporal elasticity of substitution (IES). I set $\rho = 1.25$, meaning IES < 1. This choice provides a source of positive stock-bond covariance as IES is less than 1, and also tapers the stock-bond covariance stemming from innovations in the dividend growth rate (the roles of IES and risk aversion in determining stock-bond covariance are discussed in Barsky (1989)). Finally, I set the time discount parameter $a = \ln(.97)$.

In the main calibration I set the inflation rate $i = 2\%$ and growth sensitivity $\theta = .25$. The parameter θ governs the real-nominal relationship in the model. A positive value of θ means that above average expected inflation corresponds to above-average real growth in the aggregate dividend process, according to Equation 4. In the subsequent section I explore the stock-bond covariance as a function of different values of θ.

3.5 Model Discussion

The model directly specifies an SDF that is not derived from any underlying utility function. That said, the way the two key state variables enter the SDF is motivated by equilibrium models. Aggregate consumption growth and volatility both enter the SDF, with coefficients that can be interpreted as a time-varying risk aversion and the reciprocal of the EIS.

In order to motivate the modeling choices, in Appendix 9.2 I calculate prices of stocks and bonds in i.i.d. endowment economy models with CRRA and Epstein-Zin preferences. The Epstein-Zin calculation, in particular, illustrates the motivation behind the SDF used in the model. In the Epstein-Zin model, the risk-free rate is given by

$$\implies r^f = -\ln(\beta) + \rho\mu_c - .5\gamma(\rho + 1)\sigma_c^2,$$

where ρ is the reciprocal of EIS and γ is the risk aversion parameter, and aggregate consumption growth has mean μ_c and volatility σ_c. Just like I have assumed in the model here, the consumption growth rate is multiplied by ρ, and the magnitude of the precautionary savings component of the risk-free rate depends on the risk aversion parameter γ.

In Appendix 9.2 I explore stock-bond covariance as a function of preference parameters. I derive stock and bond prices and consider comparative statics to establish parameter restrictions under which changes in the consumption volatility move stock and bond prices in the opposite direction.

Naturally, there are other channels that could drive the stock-bond covariance. For one, shocks to time discount, represented by the model parameter a, would be a source of positive stock-bond covariance. To the extent the representative investor SDF is a stand-in for an aggregation of individual investors with different time discount parameters, such variation in the representative investor time discount could be a driver of the stock-bond covariance.

Secondly, the long-term bonds priced in the model are risk-free. In practice, even U.S. Treasury prices can reflect a small probability of default. To the extent such default probability co-moves negatively with aggregate dividend shocks, we would again expect to see positive stock-bond covariance stemming from innovations to the default probability.

Thirdly, monetary policy innovations, as well as changes to the monetary policy rule can contribute to the stock-bond covariance. A surprise change in the short-term interest rate tends to move stock and bond prices in the opposite direction. To the extent monetary policy can affect risk premia, or expectations of long term growth rates, monetary policy news can also result in negative stock-bond covariance shocks.

4 Stock-Bond Covariance in the Model

4.1 Simulation Results

In order to study the model implied stock-bond covariance I draw 1,000,000 trading days of model shocks ϵ, η, ξ, and χ and calculate asset prices according to the closed form solutions in Section 3 under the parameter values in Table 2.

The model matches well the principal asset pricing moments from the 1999-2019 sample. In Panel A of Table 11. I show the market excess return, real and nominal 10-year yields, and real and nominal stock-bond covariance, correlation, and stock market beta of the 10-year bond. I compare these estimates with data from 1999 to 2020 as that's the period for which both real and nominal bond prices are available in the US.

The average monthly excess return on the market is .447%, somewhat lower than the data counterpart of .595%. The monthly standard deviation of 6.2% is higher than the data counterpart of 4.6%. The average 10-year real rate is 1.2% while the nominal rate is 3.2%, compared to empirical values of 1.65% and 3.6%. Recall that the time discount rate is a free parameter in the model which can be adjusted to exactly match the level of interest rates.

The model does a great job matching both the mean, median, and standard deviation of real stock-bond covariance, correlation, and beta. The average stock-bond covariance using real bonds is $-.290$ while the model delivers $-.263$. The median values in both the data and the model are close to $-.15$, emphasizing the heavy left tail distribution of the stock-bond covariance both in the data and the model. The standard deviation of both measures is .51.

Similarly, the model fits well the means and medians of real bond stock beta and correlation with the market. The average beta is $-.039$ and $-.085$ in the model and data respectively; the average correlation is $-.072$ and $-.085$ in the model and data respectively.

The model does not fully match the lower level and higher volatility of nominal stock-bond covariance, correlation, and beta. This is mostly due to a small number (four) extreme negative observations with values below -5 in the nominal stock-bond covariance that occurred during the Global Financial Crisis, and during the Covid panic. Without these extreme observations the model fit is considerably better. To that point, note that the median nominal stock-bond covariance in the model ($-.196$) is quite close to the median in the data ($-.333$). Part of this extreme covariance might stem from the specialness of nominal Treasuries in times of crises. (The price gap between nominal and real Treasuries, accounting for the inflation coupon part, is studied in Fleckenstein et al. (2014).)

Just as is the case empirically, the stock-bond covariance is left skewed, while the bond beta is right skewed. In the model this feature stems from the fact that market return volatility tends to be high precisely when the price of risk is high. I show the simulated distribution of 10-year Treasury beta and the stock bond covariance in Figure 3.

Finally, Panel B of Table 11 shows that the price-dividend ratio and the stock-bond covariance have similar persistence in the model and the data. In the data last month's stock-bond covariance explains 49% of the variation in the current covariance, compared to 28% in the model. The one-month autoregressive coefficient in the model is .985, close to the empirical value of .979 in the data.

In the main calibration inflation rate is set to 2% per year and the parameter $\theta = .25$ which indicates that higher-than-average inflation rate tends to coincide with good real dividend growth rate. In order to show that the model can also match the stock-bond covariance prior in the 1980s when high inflation tended to coincide with low growth, I include an "Inflation Calibration". In this calibration the level of inflation is 5 percentage points higher and the parameter θ is set to -2.5, meaning that high inflation periods tend to coincide with low real growth.

Panel C of Table 11 shows the results of this calibration. The levels of the nominal stock-bond covariance, correlation, and bond beta are all considerably higher, in line with data from 1973-1998. All the while, the relationship between price of risk and stock-bond covariance continues to hold true, as I explore below. In the "Inflation Calibration" the covariance between equity risk premium and real yields is still negative for the same reason as in the benchmark model—an increase in the price of risk brings along a larger precautionary savings term. However, the negative correlation between the real growth rate and inflation changes the level of covariance between equity and nominal bond returns.[4]

4.2 Stock-Bond Covariance and State Variables

The simulated data allows me to study the behavior of stock-bond covariance as a function of the state variables. In Table 13 I regress the model-calculated backwards-looking stock-bond covariance on the state variables of the model. As the Table reveals, the stock-bond covariance is increasing in the price dividend ratio, and decreasing in the price of risk. This is expected: because the price of risk follows a CIR process, the price of risk is volatile precisely when it is high.

As described above, high volatility of the price of risk process contributes to a negative stock-bond covariance. The high level of the price of risk, then, corresponds to times when a lot of stock-bond covariance stems from changes to the price of risk. Similarly, when the price of risk is high the price-dividend ratio is low on account of the elevated equity risk premium. Correspondingly, the price-dividend ratio is negatively correlated with the stock-bond covariance. In contrast, the aggregate dividend growth rate has no explanatory power over the variation in the bond beta. Overall, the price of risk can explain 19% of the variation in the stock-bond covariance.

I further explore the model mechanism in Figure 4. Here I show the stock-bond covariance and stock beta of the 1990s bond as a function of the risk price $\sqrt{x_t}$. The top two panels show the real stock-bond covariance and beta, while the bottom two panels show the nominal counterparts, for different values of θ, the parameter controlling the real growth-inflation relationship in the model.

As the top two panels show that the stock-bond covariance and bond beta are both smaller precisely when the price of risk is high. For ease of interpretation I translate the price of risk into instantaneous risk premium by calculating $x_t\sigma_c\sigma_d$. The bottom panel shows that the same negative relationship holds for different levels of θ. Specifically, I show the relationship for $\theta = .25$, the benchmark calibration, for $\theta = 0$ which coincides with the real bond counterpart, and $\theta = -2.5$ which corresponds to the "Inflation Calibration". The finding that the precautionary savings motive can rationalize high-frequency movements in the stock-bond covariance even under high levels of inflation—with the same negative relationship with risk prices—motivates the empirical analysis in the empirical section of this paper.

In Table 12 I document how the model-implied stock-bond covariance changes as a function of a key parameter—the volatility of the price of risk process x_t. In Panel A, I report the sample

[4]There is other empirical evidence for this mechanism. The VAR in Gonçalves (Forthcoming) implies that the covariance between the equity risk premium and nominal yields is negative throughout the postwar sample (see Table IA.5 in that paper).

moments of the nominal stock-bond covariance from nine separate simulations of 1,000,000 trading days. From the top to the bottom row, I increase the value of σ_η—the parameter governing the volatility of the price of risk process—from 0 to 7.8 (the benchmark calibration uses $\sigma_\eta = 5.8$). I report the value in terms of the volatility of instantaneous risk premium, meaning the risk premium on a dividend strip about to mature. The benchmark value of $\sigma_\eta = 5.8$ corresponds to volatility of risk premium of 2.365%. As the Table shows, increasing the volatility of the instantaneous risk premium considerably lowers the mean value of the stock-bond covariance, while increasing its standard deviation, and rendering it more left-skewed. By making the price of risk process more volatile, the model generates a negative stock-bond covariance that is larger in magnitude, and closer to the nominal stock-bond covariance in the data.

4.3 Term Structure of Interest Rates

While the model is designed and calibrated to match the data with respect to the stock-bond covariance, it captures well key characteristics of the yield curve.

Specifically, I show that a two factor structure of safe interest rates emerges in the model, closely reflecting the empirical evidence. To demonstrate this I use the calibrated model to calculate the yield curve for maturities from 1 to 30 years and apply the same principal component decomposition that I used in Section 2.

The safe asset factor loadings display a striking "level" and "slope" factor. The first principal component—level—accounts for 99% of the variation of the yield curve, compared to 95% in the data. The second factor—slope—explains almost all of the residual variance in both the model and the data. Because I observe the price of risk and dividend growth rate in the simulations I can directly estimate how these two state variables affect the term structure. To summarize the state variable's impact on the whole yield curve I estimate regressions with the yield curve factors on the left hand side.

The results are reported in Table 14. The price of risk process is responsible for the majority of changes to the level factor of interest rates. In contrast, the time-varying dividend growth rate is mostly responsible for the variation in the slope factor. The stock-bond covariance—the empirical proxy for the price of risk—explains over 17% of the variation in the level factor and has a weaker explanatory power over the slope factor. Overall, these results are exactly in line with the empirics presented in Section 2. The stock-bond covariance has a positive effect on the level factor, but a negative effect on the slope factor.

I explore the term structure of interest rates further in Figure 5. The top two panels show the behavior the the yield curve as a function of the state variable x_t and z_t, the price of risk and time-varying growth rate, respectively. Compared to the price of risk, the effect of the time-varying dividend growth rate decays faster, reflecting the lower persistence under the calibration parameters shown in Table 2. The bottom two panels summarize the behavior of the entire yield curve by showing the behavior of the first two principal components of yields as a function of the two state variables' values.

4.4 Return Predictability

The model also delivers return predictability in equity prices. As shown in Table 15, the price-dividend ratio predicts the 12-month ahead excess stock returns—it explains 2% of the return variation. Predictability also obtains with the stock-bond covariance though it's weaker still. The model features no predictability in excess bond returns as the growth rate and price of risk shocks are themselves not directly priced by model assumption.

4.5 The Leverage Effect

In the theory section I have assumed a representative firm that actively manages its liabilities to maintain a constant leverage ratio. In reality, market leverage of individual firms, and of the aggregate market changes considerably over time.

Choi et al. (2019) point out that such variation in the capital structure introduces time variation in the stock-bond covariance even if the underlying assets have zero covariance with interest rates. Empirically, Choi et al. (2019) estimate firm-level asset returns by directly measuring returns of equity, bonds, and loans. They demonstrate that the asset-bond covariance is materially different from the usual equity-based calculation.

In the model presented here, the relationship between equity and asset beta depends on the riskiness of the debt. Consider an increase in the price of risk. A firm with Aaa-rated debt would likely see a muted impact on its asset value as the prices of its bonds would increase because of the precautionary savings effect and counter the drop in the value of stocks. In contrast, a firm with non-investment grade debt would likely see firm value drop as the risk premium component of the bond price outweighs the contemporaneous drop in safe rates. The *asset-bond* covariance can therefore be substantially different than the stock-bond covariance. In particular, for firms with safe debt the asset-bond covariance can be higher than the stock-bond covariance.

To illustrate this effect empirically I perform the following aggregate analysis. In each quarter from 1990 to 2019 I calculate the book leverage of firms in the Compustat sample. I then calculate asset returns by assuming all outstanding debt is rated either Aaa, Baa, or High-Yield, and has the maturity of ten years. I am then able to calculate hypothetical asset returns for each level of bond riskiness.

The results are shown in Figure 7. Under the extreme assumption that all debt is Aaa-rated the aggregate asset-bond covariance can occasionally flip sign relative to the stock-bond covariance—see the evidence from 2009 in the bottom panel. In the more realistic cases of Baa or high-yield debt, however, the gap between asset- and equity-based measures is more muted. Importantly, the *correlation* between the asset- and stock-based covariance measures is .87, .69, and .60 for the High-Yield, Baa, and Aaa-based measures, respectively.

5 Literature

As discussed in the Introduction, a number of papers have studied the stock-bond covariance. Closest is recent work in Kozak (2015) proposing an explanation of the time variation in stock-bond covariance using a purely real, production-based model. In this model the time variation in the stock-bond covariance arises from the changing relative share of two production processes in the economy: a relatively safe, bond-like process and a relatively risky, stock-like process. In contrast, in my model the risk exposures of assets are constant, and only the relative share of price of risk and dividend growth shocks is changing. In other recent work, Chernov et al. (2021) generate a time-varying stock-real bond covariance in a model that features distinct regimes that differ in the dynamic relationship between realized and expected consumption growth.

Methodologically the work here is closest to Bekaert and Grenadier (1999) who jointly study the pricing of stocks and bonds in affine economies. They report the stock-bond correlation in the various economies they study, though without addressing the time variation (the recent negative stock-bond covariance period was only emerging at the time of their writing.) Subsequent work in Bekaert et al. (2010) explores empirically the conditional covariance between stock and bond returns. In their calibrated "moody investor" model, bond and stock returns move in the same direction as a function of innovations to the surplus consumption ratio, giving rise to positive stock-bond covariance. Other work that has studied stocks and bonds jointly includes Baele et al. (2010), Baker and Wurgler (2012), Adrian et al. (2015), Koijen et al. (2017), Backus et al. (2018).

As already discussed, Lettau and Wachter (2007) and Lettau and Wachter (2011) use the affine framework to study pricing of equity risk across maturity as well as the term structure of safe rates.

In other related work, Viceira (2012) studies the dynamics of stock-bond covariance, bond beta, and stock-bond correlation and shows that the yield spread can predict the covariance or bond beta. He further uses a VAR to decompose surprise bond returns into cash-flow (inflation) news, risk-free rate news, and bond risk premium news and calculates bond betas with respect to each of these constituent parts of bond returns.

Barsky (1989) studies the role of risk aversion and intertemporal elasticity of substitution in contributing to the comovement of stock and bond prices in general equilibrium.

Campbell et al. (2017) attribute the stock-bond covariance dynamics to a change in the covariance between nominal rates and the real economy. Campbell et al. (2020) explore the implications of a changing covariance between inflation and output using a habits based model of investor preferences. Bilal (2017) shows that the the possibility of the zero lower bound binding can change the stock-bond covariance in a New Keynesian model.

Recent work in Duffee (2018) argues that shocks to expected inflation are not the primary drivers of long-term bond yields. Subsequent work in Duffee (2021) builds on the same argument to study the impact of inflation innovations on the stock-bond covariance.

As already discussed, Choi et al. (2019) demonstrate that changes in the aggregate leverage of the corporate sector have a material effect on the observed stock-bond covariance.

Also discussed in detail above, Baker and Wurgler (2012) use the cross-section of stock returns to demonstrate that the returns of bond-like stocks are more exposed to bond risk factors. They discuss multiple potential underlying causes, such as cash-flow risk, time-varying discount rates, and investor sentiment.

David and Veronesi (2013) build a learning-based model in which investors seek to understand the joint dynamics of earnings growth, consumption, and inflation. Baele et al. (2010) empirically study the various macroeconomic and financial determinants of the stock-bond covariance. Baele et al. (2020) identify flight-to-quality days in a panel of 23 countries. Xu (2017) studies the correlations between international stock returns, and between international bond returns. She establishes that stock co-movements are counter-cyclical, while bond co-movements are procyclical and attributes this feature to a global risk aversion factor.

Du (2017) goes beyond the representative agent framework and studies the stock-bond covariance in a model with heterogeneous beliefs. In his model heterogeneity in investor beliefs regarding growth and inflation rates delivers a time-varying stock-bond covariance without any effect stemming from risk preferences.

Greenwood and Hanson (2013) study bond returns as a function of market sentiment at issuance time while López-Salido et al. (2017) study the impact of credit market sentiment on economic activity going forward.

Hartzmark (2016) documents the relationship between interest rates on macroeconomic uncertainty. Pflueger et al. (2020) show that the relative valuations of volatile stocks have a strong relationship with the real interest rate. They further demonstrate that positive shocks to the relative valuations of volatile stocks lead to a boom in investment, output, and employment. Their main variable of interest, the difference between average book-to-market ratios of high and low volatility stocks (labeled PSV) is in the recent data strongly correlated (.48) with the stock-bond covariance. This is consistent with the interpretation provided in this paper that episodes of high price of risk see a more negative stock-bond covariance. Pflueger et al. (2020) attribute the variation in PSV to (perhaps behavioral) time variation in investor risk appetite. In this paper the negative stock-bond covariance episodes reveal a high price of risk without appealing to any given structural model: the changes in the price of risk could be driven by disaster risk, habits, behavioral effects, and so on. The stock-bond covariance is likewise positively correlated (.58) with the Bond Market's q measure constructed in Philippon (2009). I show the time series of PSV and the Bond Market's q in Figure 9.

6 Conclusion

This paper studies the stock-bond covariance. I show that a high-frequency variation in the stock-bond covariance arises in a model with time-varying volatility in the price of risk process. The model presented here can be solved in closed form and variation in the stock-bond covariance arises through a purely real channel. Empirically, I show that the stock-bond covariance co-moves with aggregate credit spreads, and can predict excess returns of stock-like bonds, excess returns of risky over safe bonds, currency excess returns, as well as issuance and sectoral holdings of safe assets.

All modern asset pricing models feature time variation in the price of risk. For analytical convenience, these models often abstract from the impact of risk attitudes on safe interest rates. For instance, Campbell and Cochrane (1999) makes the explicit decision to equate the precautionary savings term of the risk free rate with the intertemporal elasticity of substitution term to ensure a constant rate. But the risk dynamics from these models can still be used to study risk-free rates. In particular, I have shown that a price of risk process calibrated with the equity market in mind can match the empirical stock-bond covariance in the recent data, without any contribution from inflation dynamics. Therefore, the time variation in stock-bond covariance can serve as a another asset pricing moment that helps to discipline dynamics of risk prices.

Under the calibration in this paper, the time variation in risk compensation has a first order importance on the safe yield curve. The simulated model exhibits a factor structure closely reflecting the empirical counterpart. The first factor ("level"), responsible for the majority of the term structure variation, is mostly driven by the price of risk process, thus emphasizing the importance of risk prices with respect to understanding the term structure of safe interest rates.

7 Figures

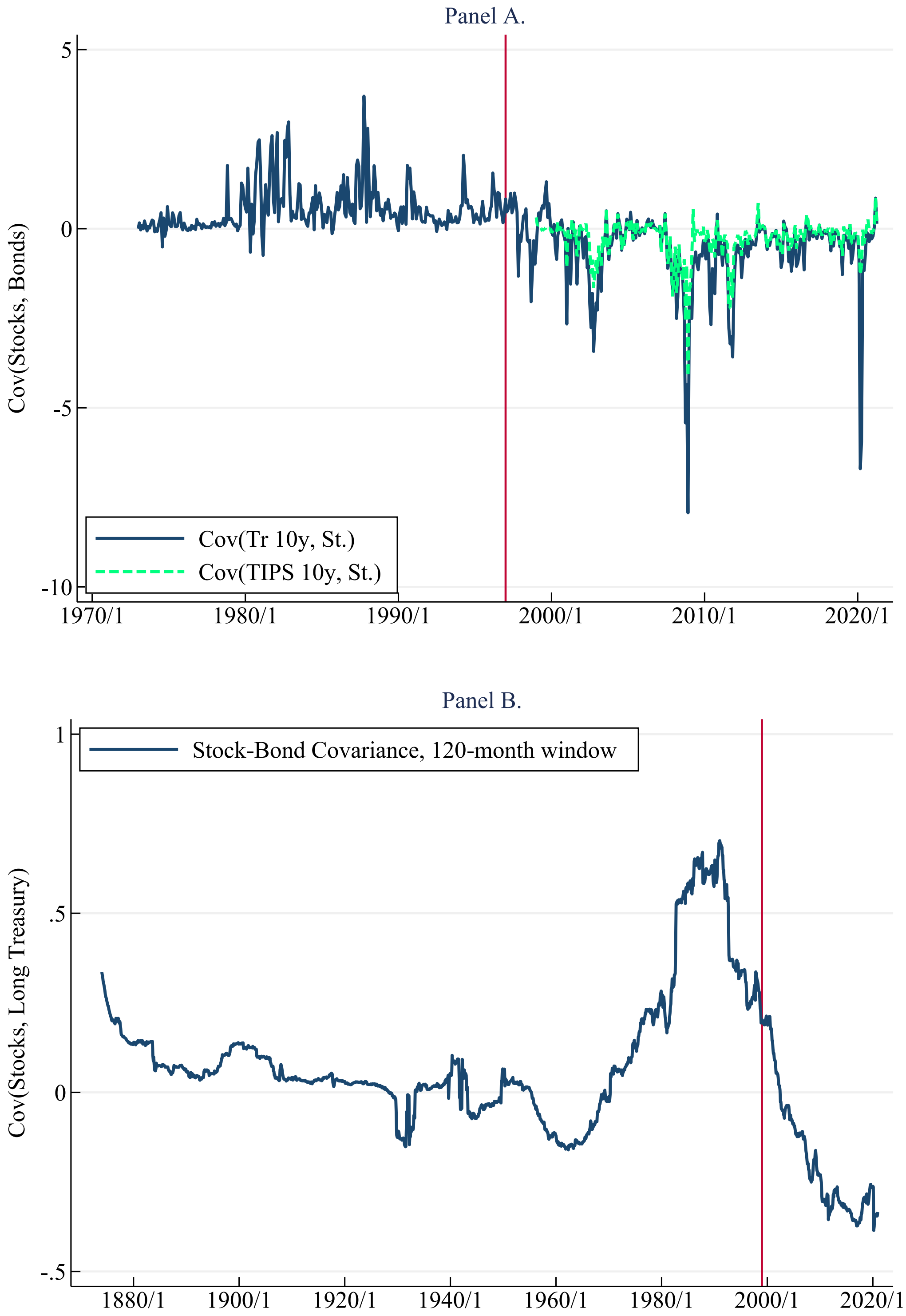

Figure 1: Stock-Bond Covariance. Panel A: Monthly data 1973-2020. Covariances with the market calculated from daily using a 30 trading day rolling window. Plot shows monthly averages. Prices of 10-year Treasury and 10-year TIPS prices from the fitted yield curve of Gürkaynak et al. (2007). Panel B: Monthly data 1870 to 2020. Stock-bond covariance calculated in a rolling 120-month window.

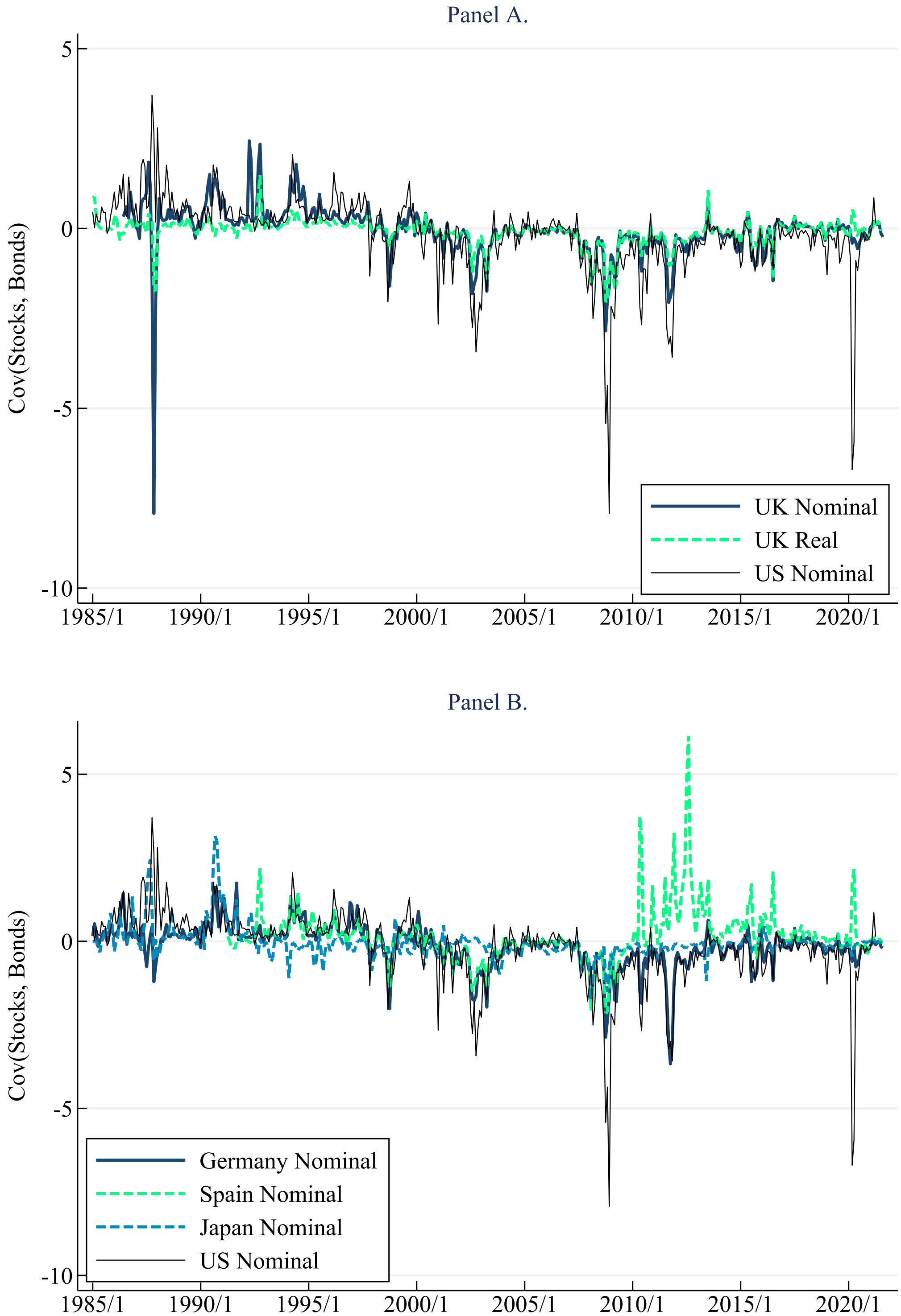

Figure 2: Local Market Stock-Bond Covariance. Monthly data 1985-2020. Covariances calculated using local stock index, and local 10-year constant maturity yield. Daily local stock and bond market data from DataStream. UK real and nominal data from Bank of England.

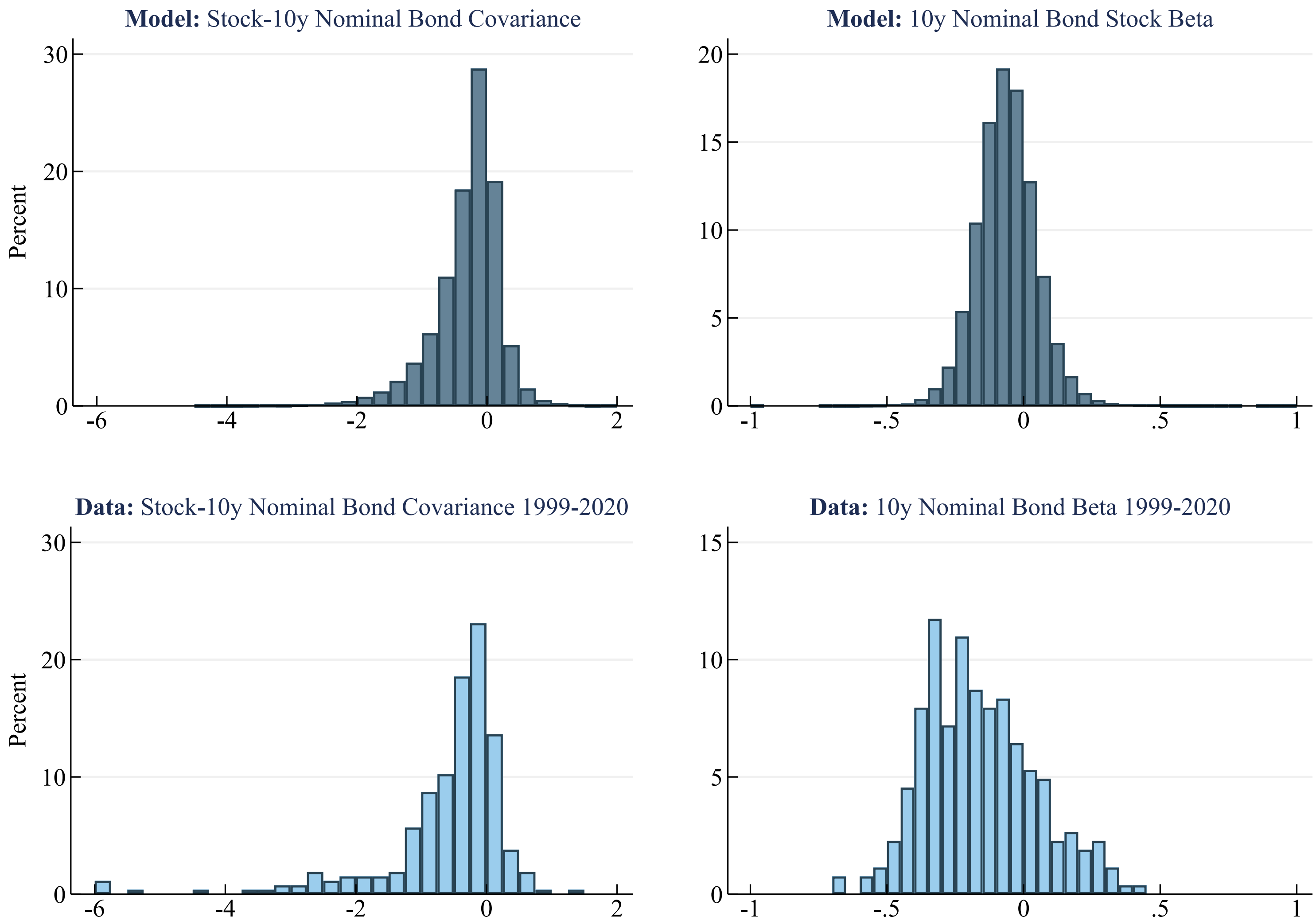

Figure 3: 10-year Treasury Bond Market Beta. Covariance of 10-year Treasury Bonds and Stocks. Distributions in the Model and in the Data. Top panel shows the model distribution, bottom panel shows the empirical distribution. The bond beta distribution is right skewed, while the covariance distribution is left skewed, just like in the data. Nominal stock-bond covariance has two values below -6 which are plotted at -6. 1,000,000 trading days of simulated data. Empirical distribution from monthly data 1999-2020.

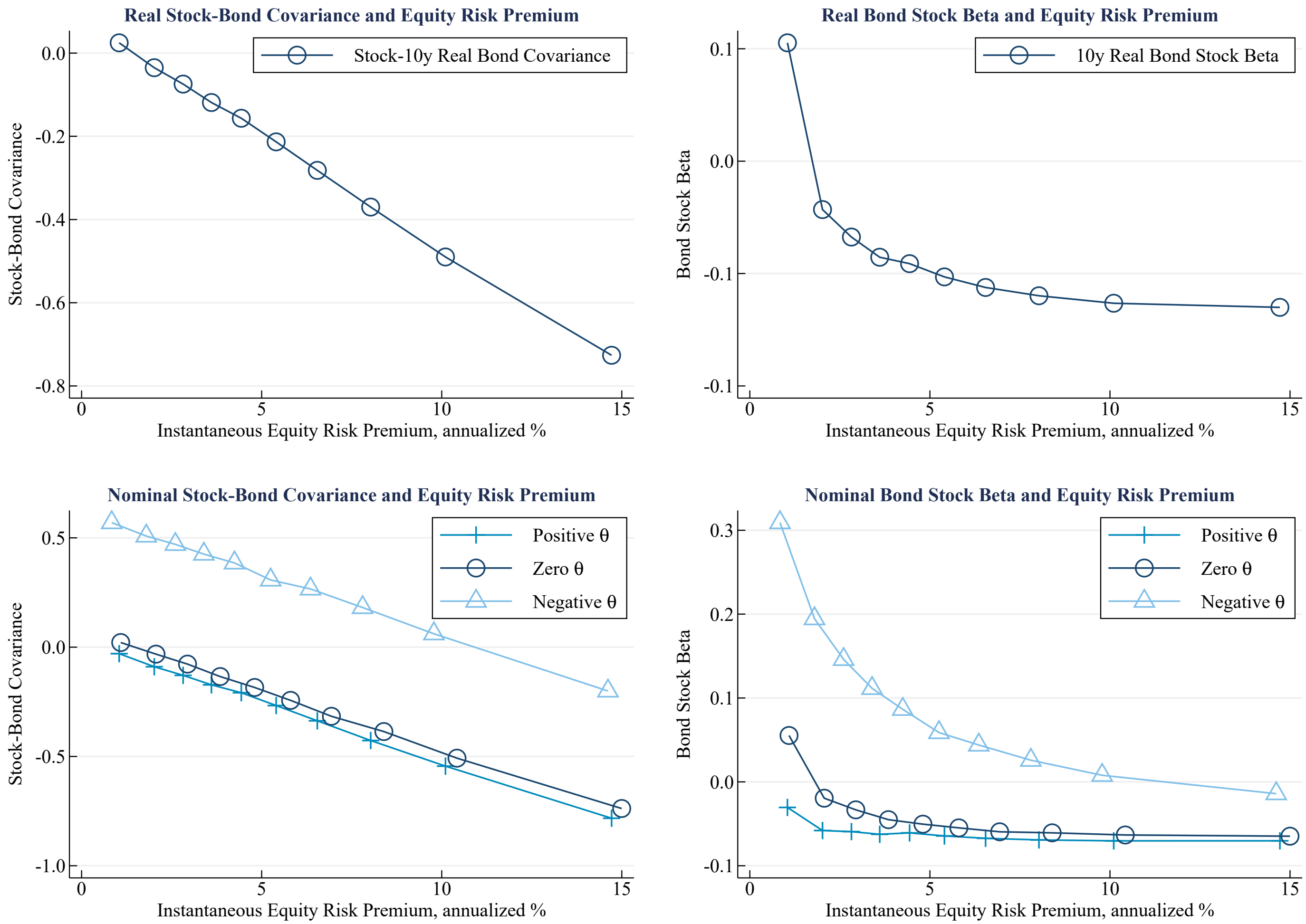

Figure 4: Covariance and Beta as a Function of Risk Prices. The top two panels show the stock-bond covariance and bond market beta for real bonds as a function of the risk price. For ease of interpretation I report the price of risk as an annualized risk premium. Bottom two panels show the same quantity, but additionally as a function of the parameter θ that governs the relationship between time-varying inflation and the real growth rate. The positive value of $\theta = .25$ corresponds to the benchmark calibration that seeks to capture recent data from 1999 to 2020. The negative value of $\theta = -2.5$ corresponds to the "inflation calibration," seeking to match the data from 1980s.

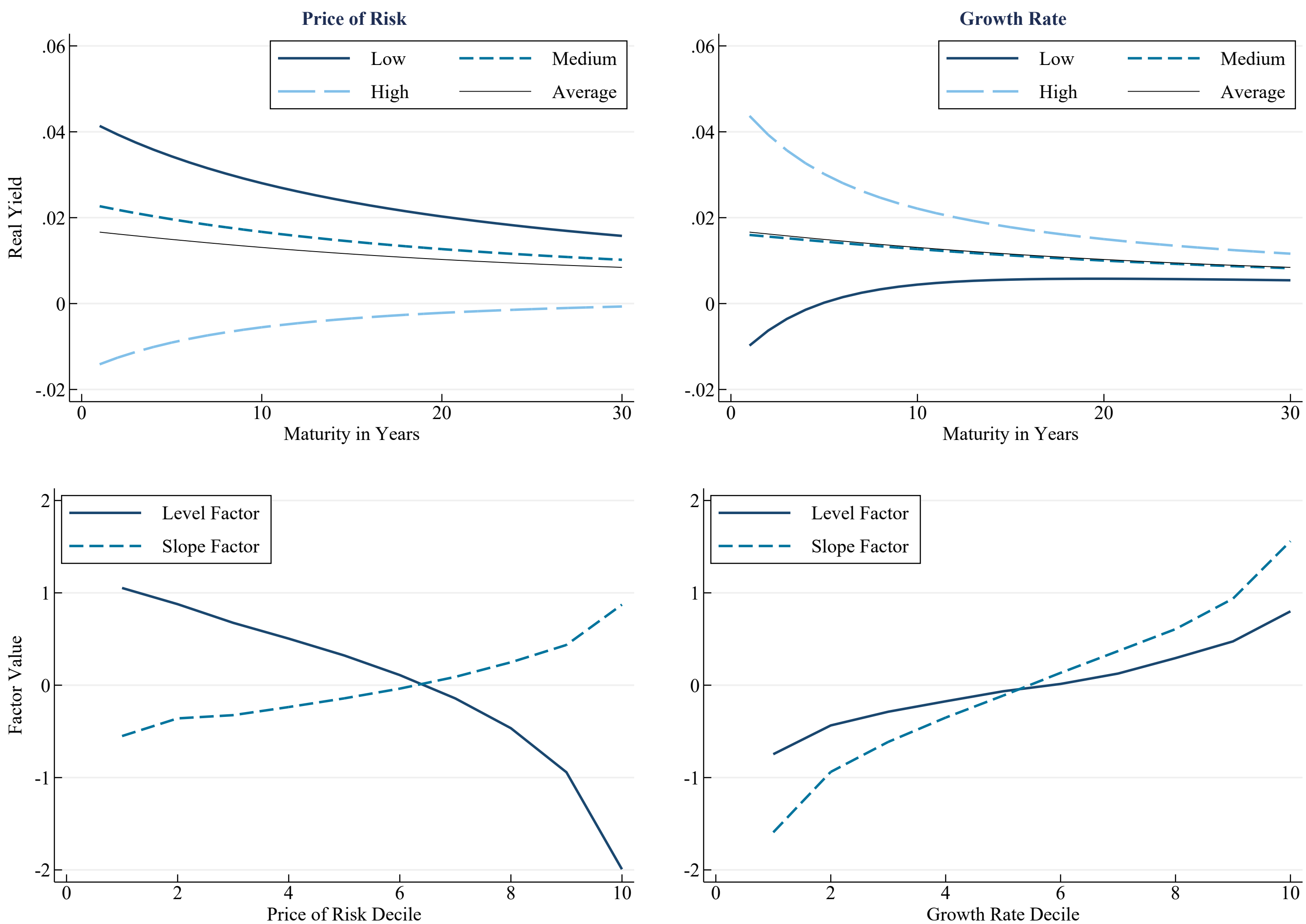

Figure 5: Yield Curve Factors in the Model and Data. Yield Curve in the Model as a Function of Price of Risk x_t and Dividend Growth Rate $g + z_t$. Top two panels show the yield curve as a function of state variable terciles. Bottom two panels show the value of yield curve principal factors as a function of state variable deciles.

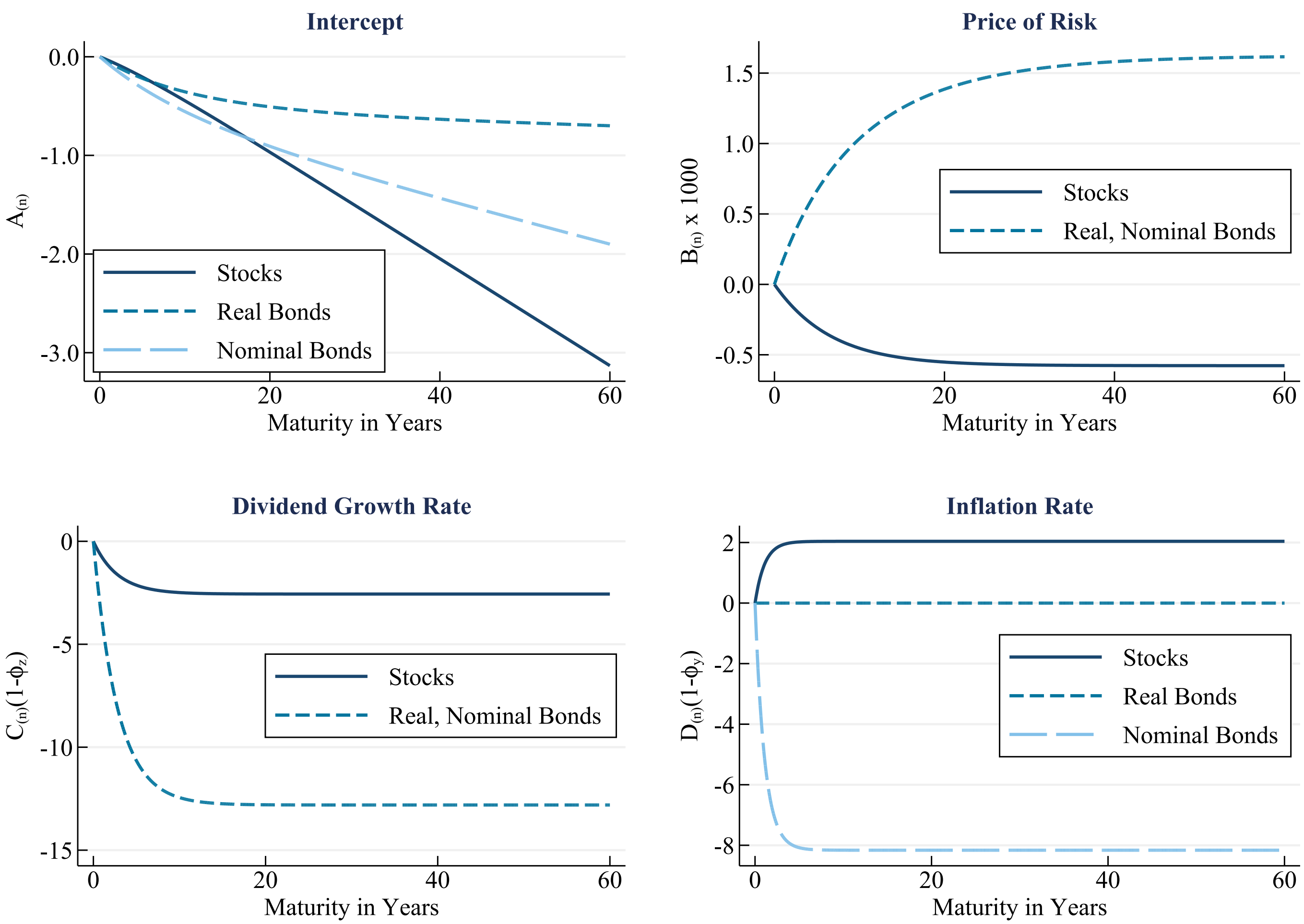

Figure 6: Risky and Safe Asset Prices in the Model. Pricing function coefficients for stocks, real bonds, and nominal bonds.

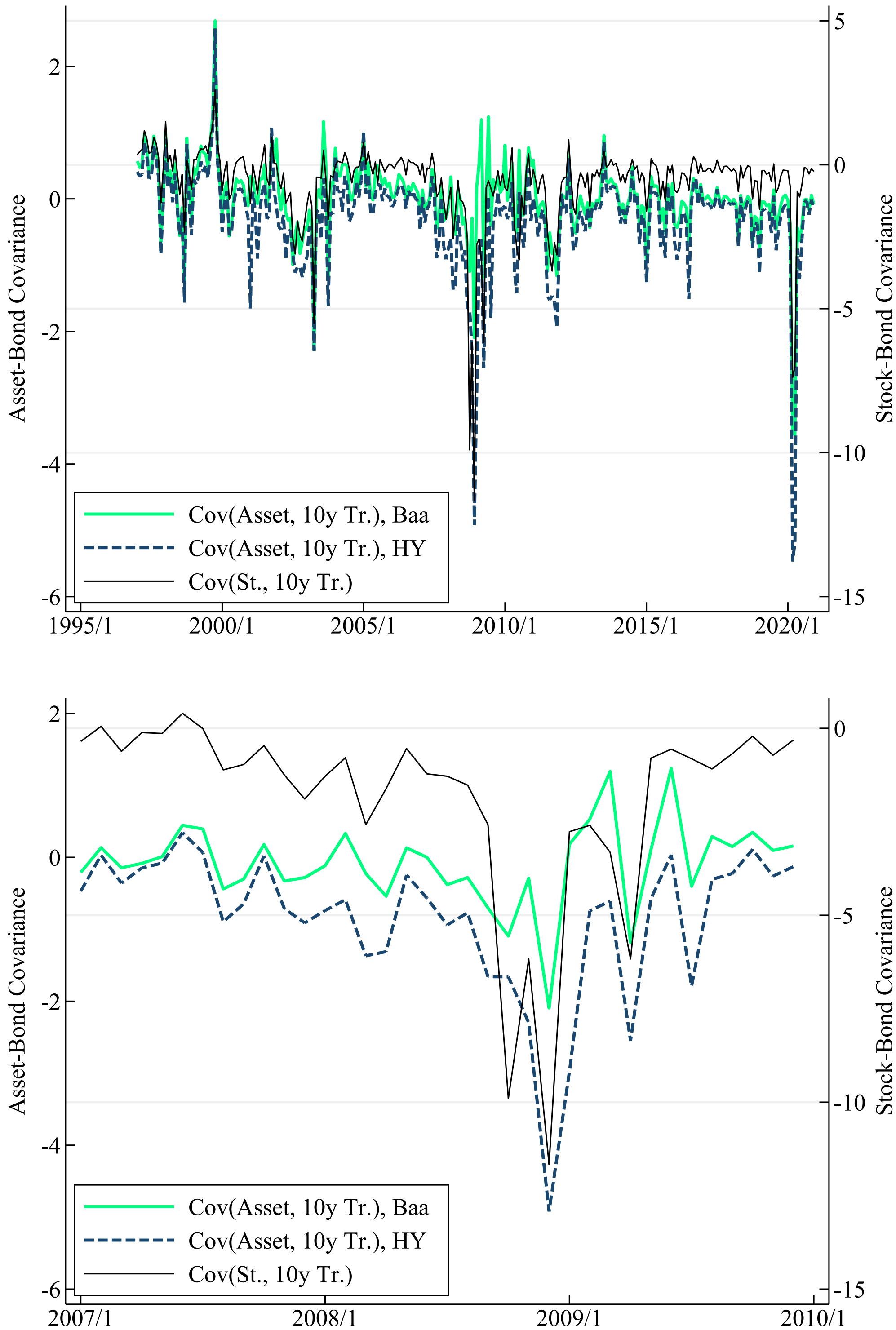

Figure 7: Asset-Bond Covariance. Asset returns are calculated using Compustat book leverage, and assuming that the debt has 10-year maturity and the credit rating indicated in the figure. Bottom panel shows the years 2007-2009 only.

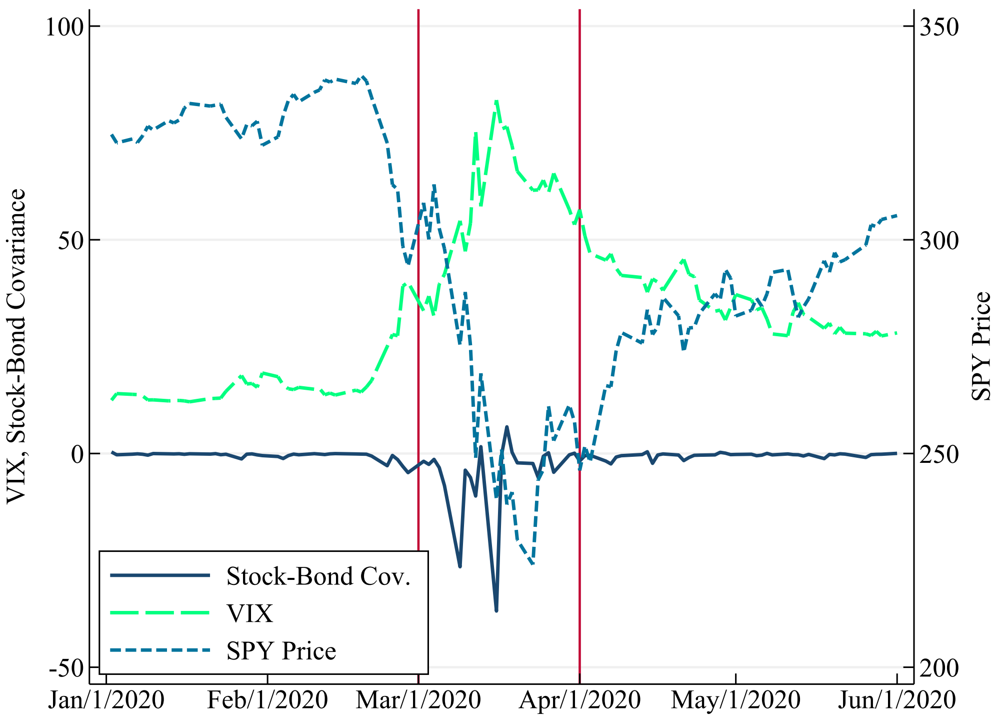

Figure 8: Stock-Bond Covariance During the Pandemic. Covariance estimated using the SPY and IEF ETF returns with 15-minute increments in a two day look-back window.

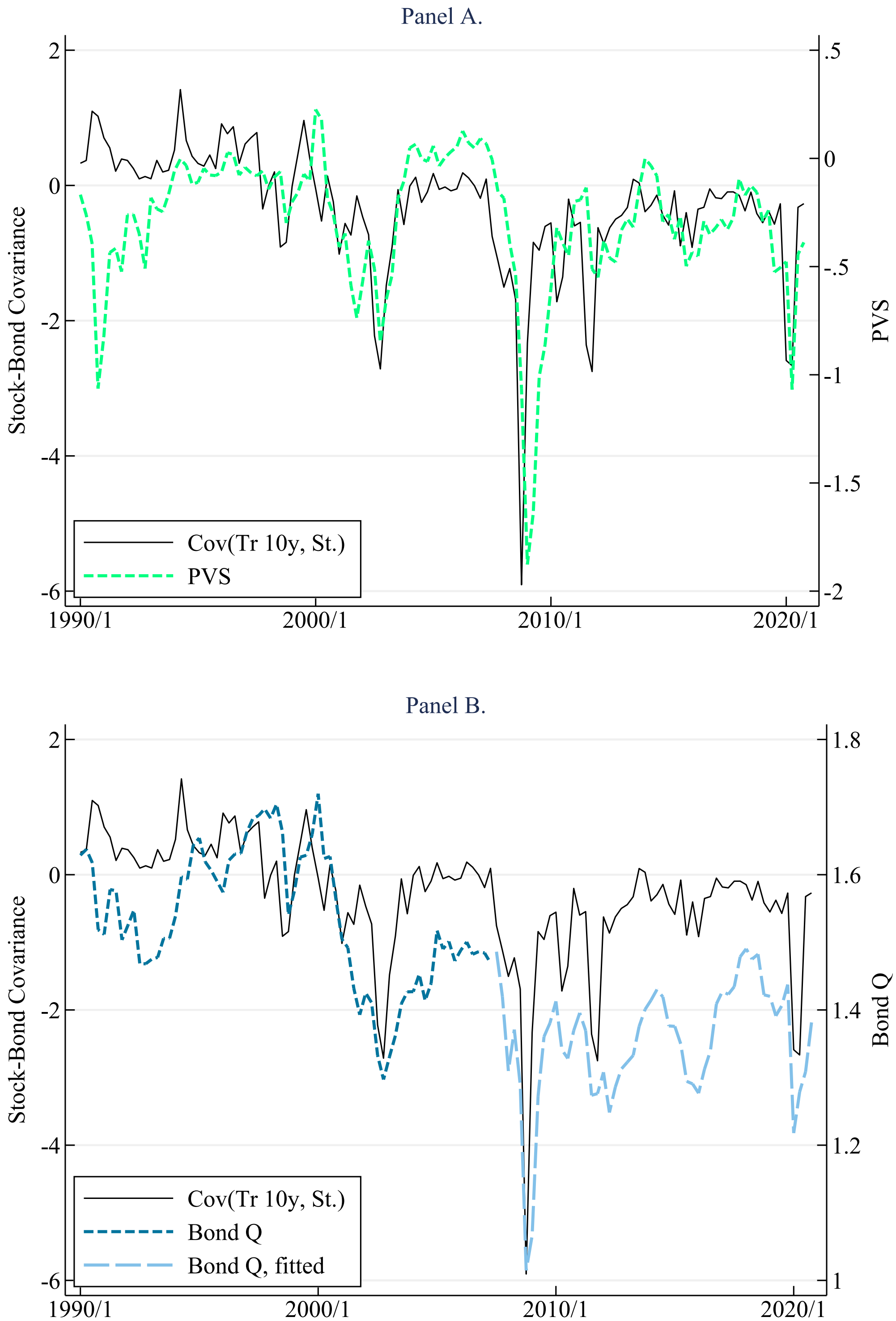

Figure 9: Stock-Bond Covariance. PVS measure from Pflueger et al. (2020). Bond Market's Q from Philippon (2009). Quarterly data 1990-2020.

8 Tables

Variable	Mean	Median	S.D.	p10	p90
Panel A. US and UK, 1999 to 2020.					
United States Nominal Cov.	-0.624	-0.316	1.088	-1.795	0.172
United States Real Cov.	-0.294	-0.150	0.535	-0.976	0.141
United Kingdom Nominal Cov.	-0.287	-0.171	0.474	-0.828	0.107
United Kingdom Real Cov.	-0.201	-0.110	0.380	-0.629	0.107
Panel B. Select Advanced Economies, 1999 to 2020.					
Australia Nominal Cov.	-0.275	-0.127	0.535	-0.938	0.196
Canada Nominal Cov.	-0.303	-0.165	0.567	-0.811	0.102
Denmark Nominal Cov.	-0.217	-0.130	0.412	-0.775	0.127
Germany Nominal Cov.	-0.381	-0.235	0.590	-0.941	0.096
Japan Nominal Cov.	-0.192	-0.131	0.265	-0.473	0.005
Norway Nominal Cov.	-0.325	-0.170	0.583	-0.829	0.064
Sweden Nominal Cov.	-0.406	-0.231	0.618	-1.081	0.096
Switzerland Nominal Cov.	-0.196	-0.088	0.392	-0.518	0.062
Panel C. US and UK, data up to 1999					
United States Nominal Cov. 1973-1999	0.501	0.353	0.671	-0.021	1.402
United Kingdom Nominal Cov., 1985-1999	0.358	0.311	0.893	-0.029	1.066
United Kingdom Real Cov., 1985-1999	0.080	0.081	0.291	-0.112	0.300

Table 1: Stock-Bond Covariance in the US, UK, and Other Advanced Economies. Monthly data. Covariances calculated using local daily data from DataStream on local equity market returns and local 10-year constant maturity yields.

Description	Variable	Value
Time Discount	a	-0.0305
Consumption Growth Rate	g	0.0200
Inflation rate	i	0.0200
Price of Risk Squared Persistence	ϕ_x	0.8700
Inflation Rate Persistence	ϕ_y	0.4000
Dividend Growth Rate Shock Persistence	ϕ_z	0.7000
One over IES	ρ	1.2500
Consumption Growth Rate Volatility	σ_c	0.0190
Inflation Growth Rate Volatility	σ_χ	0.0035
Dividend Growth Volatility	σ_d	0.0140
Price of Risk Squared Volatility	σ_η	5.8000
Dividend Growth Rate Volatility	σ_ξ	0.0016
Growth sensitivity of inflation rate	θ	0.2500
Average Price of Risk Squared	$\bar{x}$	235.0000

Table 2: Model Parameters. Preference parameters are unitless. All other parameters reported in annualized terms. The model is simulated daily.

Panel A.

	Yields		Principal Components	
	D.yield 1y	D.yield 10y	D.Level	D.Slope
D.Cov(Tr 10y, St.)	0.0845*	0.0480*	0.0714*	-0.00981
	(2.28)	(2.25)	(2.30)	(-0.77)
Constant	-0.0109	-0.00966	-0.0115	-0.000848
	(-0.57)	(-0.74)	(-0.71)	(-0.15)
R^2	0.018	0.012	0.017	0.002

Panel B.

	Yields		Principal Components	
	D.yield 1y	D.yield 10y	D.Level	D.Slope
D.Cov(Tr 10y, St.)	0.0750	0.0392	0.0637	-0.0160
	(1.64)	(1.62)	(1.80)	(-1.13)
D.Vol(St.)	-0.329	-0.307	-0.267	-0.215
	(-0.85)	(-1.32)	(-0.85)	(-1.44)
Constant	-0.0109	-0.00966	-0.0115	-0.000846
	(-0.57)	(-0.74)	(-0.71)	(-0.15)
Observations	574	574	574	574
R^2	0.020	0.015	0.018	0.006

Table 3: Treasury Yield Curve and Stock-Bond Covariance. Forward curve factors estimated using principal component analysis from the fitted daily yield curve in Gürkaynak et al. (2007). Stock-bond covariance is calculated using CRSP value-weighted aggregate return and the return on 10-year constant-maturity Treasury. The look-back window is 30 trading days and the data is averaged to a monthly frequency. Vol(St.) refers to the market volatility over the same look-back window. Yields measured from the fitted yield curve in Gürkaynak et al. (2007). Heteroskedasticity robust standard errors. Monthly data 1973-2020, except for the 30-year yield which starts in 1985.

Panel A.

	D.Aaa-Tr	D.Baa-Tr	D.HY-Tr	D.GZ Spread	D.EBP
D.Cov(Tr 10y, St.)	-0.0425***	-0.0816***	-0.476***	-0.112**	-0.0894**
	(-3.45)	(-4.02)	(-5.39)	(-2.74)	(-3.27)
Constant	0.00159	0.00226	0.00205	0.00356	0.000837
	(0.32)	(0.25)	(0.04)	(0.35)	(0.08)
Observations	569	569	288	565	565
R^2	0.057	0.092	0.322	0.128	0.063

Panel B.

	D.Aaa-Tr	D.Baa-Tr	D.HY-Tr	D.GZ Spread	D.EBP
D.Cov(Tr 10y, St.)	-0.0241**	-0.0515***	-0.152	-0.0693**	-0.0720**
	(-3.00)	(-3.71)	(-1.49)	(-3.21)	(-2.63)
D.Vol(St.)	0.642***	1.054***	5.254***	1.749***	0.711*
	(6.37)	(4.68)	(5.27)	(3.74)	(2.16)
Constant	0.00149	0.00211	0.00290	0.00228	0.000317
	(0.31)	(0.25)	(0.07)	(0.25)	(0.04)
Observations	569	569	288	565	565
R^2	0.126	0.174	0.435	0.300	0.085

Table 4: Corporate Spreads and Stock-Bond Covariance. HY refers to High-Yield bonds. GZ refers to the Gilchrist and Zakrajšek (2012) corporate spread. Aaa and Baa bond spread data from Moody's, HY from Bank of America. Monthly data 1973-2020, except HY which is available from 1997. Heteroskedasticity robust standard errors.

Panel A. Excess Treasury Returns.

	1973-2020		1997-2020	
	10y 12m ER	30y 12m ER	10y 12m ER	30y 12m ER
Cov(Tr 10y, St.)	0.00000870	0.0235	0.00519	0.0540**
	(0.00)	(1.10)	(0.99)	(3.12)
Constant	0.0334**	0.0489	0.0497***	0.108***
	(2.65)	(1.42)	(4.17)	(3.76)
Observations	568	568	281	281
R^2	0.000	0.006	0.006	0.093

Panel B. Excess Stock Returns.

	1973-2020		1997-2020	
	ER 12m	ER 60m	ER 12m	ER 60m
Cov(Tr 10y, St.)	0.0221	0.0550	-0.00463	-0.140***
	(1.26)	(1.18)	(-0.25)	(-6.77)
Constant	0.103***	0.537***	0.0744**	0.240*
	(5.64)	(6.62)	(2.86)	(2.30)
Observations	563	515	276	228
R^2	0.019	0.027	0.001	0.225

Table 5: Aggregate Bond and Stock Return Predictability. Newey-West standard errors with lags corresponding to overlap in return horizon. Monthly data 1973-2020.

Panel A.

	1973-2020		1997-2020		
	Aaa-Tr. 12m	Baa-Tr. 12m	Aaa-Tr. 12m	Baa-Tr. 12m	HY-Tr. 12m
Cov(Tr 10y, St.)	-0.0135	-0.0250	-0.0264**	-0.0631*	-0.0642***
	(-1.52)	(-1.00)	(-2.68)	(-2.40)	(-3.59)
Constant	-0.00325	-0.00411	-0.0158	-0.0395	-0.0391
	(-0.35)	(-0.23)	(-1.01)	(-1.57)	(-1.56)
Observations	556	556	245	245	271
R^2	0.025	0.026	0.086	0.121	0.166

Panel B.

	1973-2020		1997-2020		
	Aaa-Tr. 12m	Baa-Tr. 12m	Aaa-Tr. 12m	Baa-Tr. 12m	HY-Tr. 12m
Cov(Tr 10y, St.)	-0.0116	-0.0246	-0.0268**	-0.0579**	-0.0595***
	(-1.31)	(-0.98)	(-2.78)	(-2.59)	(-3.31)
CP Factor	0.386	1.675	1.218	0.595	0.844
	(0.61)	(1.55)	(0.92)	(0.25)	(0.43)
BW Sentiment	-0.0210*	-0.0310*	-0.0250	-0.0817*	-0.0609
	(-2.35)	(-2.39)	(-1.08)	(-2.20)	(-1.89)
Constant	-0.00658	-0.0201	-0.0236	-0.0395	-0.0386
	(-0.56)	(-0.84)	(-1.14)	(-1.14)	(-1.29)
Observations	551	551	240	240	265
R^2	0.070	0.069	0.142	0.213	0.234

Table 6: Relative Bond Return Predictability. 12-month excess returns of risky corporate bonds over equivalent maturity Treasury bonds. Newey-West standard errors with 12 lags. Bottom panel controls for the Cochrane and Piazzesi (2005) predictive factor as constructed by Hodrick et al. (2021) and the Baker and Wurgler (2006) sentiment factor. Monthly data 1973-2020.

Panel A.

	AGE	DBE	ME	EBE	EFA	GS	RVOL	AGG
L.Cov(Tr 10y, St.)	0.135***	0.0979	-0.00543	0.208**	0.0722*	0.103	0.150***	0.109***
	(3.79)	(1.85)	(-0.11)	(2.87)	(2.06)	(1.71)	(4.58)	(3.87)
Constant	0.0392	-0.0494	-0.400***	0.109	0.396***	0.146	0.284*	0.0748
	(0.52)	(-0.56)	(-3.76)	(0.72)	(5.96)	(1.54)	(1.97)	(1.12)
Observations	503	503	503	503	503	503	503	503
R^2	0.150	0.080	0.000	0.112	0.087	0.067	0.069	0.148

Panel B.

	AGE	DBE	ME	EBE	EFA	GS	RVOL	AGG
L.Cov(Tr 10y, St.)	0.112***	0.0687	-0.0289	0.159**	0.0674*	0.0996	0.0981*	0.0821**
	(3.88)	(1.60)	(-0.57)	(3.06)	(1.96)	(1.75)	(2.26)	(3.16)
L.CP Factor	-3.205	-1.507	3.078	8.878	-1.263	-4.252	-1.266	0.0664
	(-1.42)	(-0.50)	(1.12)	(1.53)	(-0.69)	(-1.39)	(-0.29)	(0.03)
L.BW Sentiment	0.216***	0.231***	0.111	0.192	0.0557	0.0994**	0.381*	0.184***
	(4.43)	(3.35)	(1.17)	(1.49)	(1.31)	(2.70)	(2.57)	(3.41)
Constant	0.0709	-0.0343	-0.430***	0.0219	0.408***	0.188	0.297*	0.0745
	(1.35)	(-0.48)	(-4.08)	(0.20)	(5.26)	(1.94)	(2.53)	(1.56)
Observations	503	503	503	503	503	503	503	503
R^2	0.439	0.417	0.097	0.269	0.125	0.123	0.420	0.491

Table 7: Relative Equity Return Predictability. 60-month excess returns for long-short portfolios of bond-like stocks, constructed after Baker and Wurgler (2012). See text for details. EFA, GS and RVOL returns multiplied with -1 so that all coefficients on Cov(Tr 10y, St.) are predicted to be positive. The last column repeats the analysis using an equal-weighted portfolio Panels B controls for the Cochrane and Piazzesi (2005) predictive factor as constructed by Hodrick et al. (2021) and the Baker and Wurgler (2006) sentiment factor. Monthly data 1973-2020.

Panel A.

	12-month Excess Returns over USD. 1988-2020			
	AUD	CAD	CHF	EUR
Cov(Tr 10y, St.)	-0.0409**	-0.0163	-0.0158	-0.0203
	(-3.07)	(-1.92)	(-1.37)	(-1.53)
Constant	0.00451	-0.00378	-0.00278	-0.0164
	(0.27)	(-0.36)	(-0.19)	(-0.86)
Observations	368	368	368	247
R^2	0.111	0.047	0.023	0.037
	GBP	JPY	NZD	AGG
Cov(Tr 10y, St.)	0.00421	-0.0137	-0.0359*	-0.0187*
	(0.39)	(-1.10)	(-2.52)	(-1.99)
Constant	0.00349	0.00388	0.0167	0.00257
	(0.27)	(0.25)	(0.94)	(0.23)
Observations	368	368	368	368
R^2	0.002	0.016	0.079	0.054

Panel B.

	12-month Excess Returns over USD. 1997-2020			
	AUD	CAD	CHF	EUR
Cov(Tr 10y, St.)	-0.0561***	-0.0226*	-0.0202	-0.0203
	(-3.74)	(-2.27)	(-1.93)	(-1.53)
Constant	-0.0166	-0.0104	-0.0120	-0.0164
	(-0.78)	(-0.74)	(-0.82)	(-0.86)
Observations	271	271	271	247
R^2	0.174	0.074	0.051	0.037
	GBP	JPY	NZD	AGG
Cov(Tr 10y, St.)	-0.00358	-0.0191	-0.0523**	-0.0281**
	(-0.31)	(-1.52)	(-3.24)	(-2.80)
Constant	-0.00850	-0.00498	-0.00691	-0.0113
	(-0.53)	(-0.28)	(-0.30)	(-0.79)
Observations	271	271	271	271
R^2	0.002	0.035	0.138	0.107

Table 8: FX Return Predictability. 12-month excess returns for seven major currency pairs over USD. Newey-West standard errors with 12 lags. The column labeled "AGG" repeats the analysis for an equal-weighted portfolio of the seven individual portfolios. Monthly data 1988-2020.

	Cov(Treasury 10y, Stocks)				
Implied Volatility 10y	-0.125	0.0940	-0.646***	-0.365***	-0.220**
	(-0.87)	(0.98)	(-4.96)	(-3.71)	(-2.83)
Jump Probability Calls (LT)		-2.962***		-2.387***	-1.811**
		(-3.75)		(-3.45)	(-3.08)
Jump Probability Puts (RT)			1.983***	1.586***	1.460***
			(7.68)	(6.29)	(5.82)
Vol(St.)					-5.315**
					(-3.24)
Constant	0.766	-0.376	3.099***	1.712**	1.472**
	(0.79)	(-0.61)	(3.82)	(3.26)	(2.87)
Observations	335	335	335	335	335
R^2	0.045	0.295	0.289	0.442	0.558

Table 9: Implied Volatility of Treasury Prices. Jump Probabilities of Treasury Prices. Treasury implied volatility and jump probabilities estimated from options prices on the 10-year Treasury futures contract according to Equations 1 and 2. Options data from the CME Group. Monthly data 1997-2014.

Panel A.

	Issuance			Holdings		
	D.IG Ratio	D.A share	D.IG Fin/GDP	D.B-D	D.HH	D.MMF
D.Cov(Tr 10y, St.)	-0.0289*	-0.0281	-0.204	-0.302***	-0.00515	-0.191***
	(-2.28)	(-1.53)	(-1.79)	(-3.37)	(-0.03)	(-3.98)
Constant	-0.000304	-0.00191	0.000101	-0.000149	0.00685	0.0317
	(-0.11)	(-0.61)	(0.01)	(-0.00)	(0.09)	(1.01)
Observations	355	355	356	189	189	189
R^2	0.018	0.012	0.018	0.099	0.000	0.088

Panel B.

	Issuance			Holdings		
	D.IG Ratio	D.A share	D.IG Fin/GDP	D.B-D	D.HH	D.MMF
D.Cov(Tr 10y, St.)	0.00421	-0.0263	-0.324*	-0.240**	0.0592	-0.131*
	(0.25)	(-1.21)	(-2.37)	(-2.98)	(0.36)	(-2.39)
D.Vol(St,)	0.718**	0.0395	-2.607*	1.479*	1.549	1.441**
	(3.02)	(0.17)	(-2.38)	(2.05)	(1.18)	(2.65)
Constant	-0.000282	-0.00191	0.000369	-0.00102	0.00593	0.0308
	(-0.11)	(-0.61)	(0.02)	(-0.02)	(0.08)	(1.01)
Observations	355	355	356	189	189	189
R^2	0.064	0.012	0.030	0.117	0.008	0.127

Table 10: Issuance and Holdings. Monthly bond issuance data 1990-2020 constructed from Mergent FISD data. Quarterly sectoral holdings data from 1973 to 2020 from Financial Accounts of the US. B-D stands for Broker-Dealers, HH for Households, and MMF for Money Market Funds.

Variable	Model Mean	Model Median	Model S.D.	Data Mean	Data Median	Data S.D.
Panel A. Main Calibration. Data from 1999-2020.						
Monthly market excess return	0.522	0.303	6.174	0.595	1.175	4.550
Real 10y yield	1.308	1.614	1.799	1.473	1.396	1.342
Nominal 10y yield	3.315	3.615	1.831	3.566	3.644	1.464
Stock 10y nominal bond covariance	-0.299	-0.196	0.520	-0.621	-0.333	1.058
Stock 10y real bond covariance	-0.244	-0.143	0.492	-0.290	-0.148	0.510
10y nominal bond stock beta	-0.061	-0.063	0.113	-0.163	-0.186	0.191
10y real bond stock beta	-0.039	-0.045	0.108	-0.085	-0.079	0.135
Stock 10y nominal bond correlation	-0.112	-0.115	0.181	-0.298	-0.358	0.313
Stock 10y real bond correlation	-0.072	-0.074	0.155	-0.085	-0.079	0.135
Panel B. Monthly AR(1) Coefficients. Data from 1999-2020.						
Price-Dividend Ratio AR(1)	0.985			0.979		
Cov(Tr. 10y, St.) AR(1)	0.525			0.670		
Cov(TIPS 10y, St.) AR(1)	0.536			0.575		
Panel C. "Inflation" Calibration. Data from 1973-1998.						
Monthly market excess return	0.516	0.237	6.834	0.608	0.860	4.669
Nominal 10y yield	8.379	8.639	1.812	8.497	7.850	2.123
Stock 10y nominal bond covariance	0.298	0.374	0.564	0.502	0.356	0.640
10y nominal bond stock beta	0.097	0.084	0.128	0.293	0.273	0.264
Stock 10y nominal bond correlation	0.198	0.180	0.254	0.336	0.365	0.251

Table 11: Model Moments. 1,000,000 days of data simulated according to parameter values in Table 2. One "month" in the model consists of 21 trading days. Data in Panel B simulated according to the same parameter values, except $i = .07$ and $\theta = -2.5$.

Panel A. Model:					
Equity Risk Prem.		Stock-Bond Covariance			
R.P.	σ(R.P.)	Mean	Median	Skew	S.D.
6.251	0.000	0.058	0.056	0.052	0.324
6.251	0.326	0.055	0.051	0.024	0.327
6.251	0.734	0.027	0.029	-0.008	0.344
6.251	1.142	-0.021	-0.013	-0.131	0.361
6.251	1.550	-0.089	-0.066	-0.357	0.409
6.251	1.957	-0.178	-0.123	-0.949	0.485
6.251	2.365	-0.292	-0.189	-1.401	0.584
6.251	2.773	-0.447	-0.261	-1.747	0.753
6.251	3.181	-0.521	-0.268	-2.280	0.852
Panel B. Data from 1999 to 2020.:					
	Nominal Cov.:	-0.624	-0.316	-3.123	1.088
	Real Cov.:	-0.294	-0.150	-2.637	0.535

Table 12: Stock-Bond Covariance and Volatility of the Risk Premium. Each row in Panel A represents one simulation of 1,000,000 days according to parameter values in Table 2 with the exception of the parameter σ_η. The second column reports the value $\sqrt{\bar{x}}\sigma_\eta\sigma_c\sigma_d$, which corresponds to the conditional volatility of instantaneous equity risk premium in the model. The first column reports the unconditional instantaneous equity risk premium in the model. The third to sixth columns report nominal stock-bond covariance moments. For reference, Panel B includes the empirical moments from the 1999-2020 sample.

	Cov(Stocks, 10y Bond) Model			
Price Dividend Ratio	0.180***			
	(66.09)			
Price of Risk		-0.0425***		-0.0425***
		(-73.91)		(-73.93)
Dividend Growth Rate			2.383*	-0.195
			(2.02)	(-0.18)
Constant	-3.556***	0.350***	-0.244***	0.350***
	(-69.04)	(52.05)	(-107.95)	(52.08)
Observations	47620	47620	47620	47620
R^2	0.189	0.193	0.000	0.193

Table 13: Stock-Bond Covariance in the Model. The Stock-Bond covariance as a function of price dividend ratio, price of risk, and dividend growth rate. 1,000,000 trading days of simulated data, collapsed to monthly frequency. One “month” in the model consists of 21 trading days.

	Level Factor			Slope Factor		
Cov(Stocks, 10y Tr.)	3.944***			-0.267***		
	(51.45)			(-23.54)		
Price of Risk		-0.928***			0.0628***	
		(-126.94)			(34.56)	
Dividend Growth Rate			1.243***			0.252***
			(33.80)			(61.04)
Constant	1.178***	12.95***	-0.118	-0.0798***	-0.876***	-0.0239**
	(16.14)	(118.95)	(-1.63)	(-7.41)	(-32.38)	(-2.95)
Observations	47620	47620	47620	47620	47620	47620
R^2	0.143	0.757	0.183	0.034	0.180	0.392

Table 14: Accounting for Term Structure Factors. Level and Slope refer to the first two principal components of the simulated term structure. 1,000,000 trading days of simulated data collapsed to monthly frequency. One "month" consists of 21 trading days.

	1-year Market Returns			1-year 10y Bond Returns		
Price Dividend Ratio	-0.0327***			0.00284*		
	(-13.26)			(2.54)		
Price of Risk		0.00858***			-0.000612*	
		(14.95)			(-2.34)	
Stock-10y Bond Covariance			-0.0369***			0.00261
			(-10.97)			(1.73)
Constant	0.663***	-0.0578***	0.0552***	-0.0485*	0.0123**	0.00425**
	(14.66)	(-6.62)	(17.40)	(-2.36)	(3.09)	(2.99)
Observations	47620	47620	47620	47608	47608	47608
R^2	0.027	0.034	0.007	0.001	0.001	0.000

Table 15: Stock and Bond Return Predictability in the Model. 60-month excess returns on the equity market and 10-year bond. 1,000,000 days of simulated data collapsed to monthly frequency. One "month" consists of 21 trading days. Newey-West standard errors.

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9 Appendix

9.1 Model Simulation Details

In this section I provide more details on simulating the model.

The model has three state variables and four shocks. The state variables are x_t, the square of the aggregate price of risk, z_t, time-varying component of dividend growth rate, and y_t, the time-varying component of expected inflation. There are four shocks, η_{t+1}, ξ_{t+1}, χ_{t+1} and ϵ_{t+1}. The first three of these shocks represent innovations to the three state variables, in the order I listed them above, and the fourth represents the realized dividend growth shock in period $t+1$.

The model is solved in closed form by recursively applying the pricing relationships is Equations (11, 14, and 21) combined with the parametrization in Table 2. All the parameters with a time component, such as the time discount, growth rates of consumption, dividends, and inflation are reported in annualized terms. Likewise, the persistence and volatility of the time-varying component of these variables are reported in annualized terms. To use these parameters in calculating asset prices at a monthly resolution I divide the growth rates by 12, the volatilities by $\sqrt{12}$, and transform the persistence parameters by taking them to the power of $\frac{1}{12}$.

In contrast, the preference parameters ρ (one over the IES), and the aggregate price of risk $\sqrt{x_t}$ are both unitless and their levels require no adjustment. However, simulating the path of x_t the persistence and volatility parameters are adjusted in the same manner.

With the parameters for a monthly calculation in hand I calculate asset prices. I plot the solution in Figure 6. These four sets of coefficients allow me to calculate the price of any maturity equity, or fixed income cash-flow.

I simulate the model at a "daily" frequency, meaning that I split each year into 252 periods. In order to simulate the model I draw 1,000,000 realizations of the four shocks, where the volatilities are given by the annualized number reported in Table 2, divided by $\sqrt{252}$. To calculate the state variables' evolution I again need to transform the growth rate and persistence parameters in Table 2 into daily counterparts. I divide the growth rates by 252, and the persistence parameters are transformed by taking them to the power of $\frac{1}{252}$. With the state variables' evolution in hand, I can calculate asset prices at any maturity according to

$$P^R_{n,t} = \exp\left\{A^R_{(n)} + B^R_{(n)}x_t + C^R_{(n)}z_t\right\} \tag{25}$$

$$P^N_{n,t} = \exp\left\{A^N_{(n)} + B^N_{(n)}x_t + C^N_{(n)}z_t + D^N_{(n)}y_t\right\} \tag{26}$$

$$\frac{P_{n,t}}{D_t} = \exp\left\{A_{(n)} + B_{(n)}x_t + C_{(n)}z_t + D_{(n)}y_t\right\}. \tag{27}$$

As already shown in the body of the paper, the price-dividend ratio of the aggregate market can be calculated by summing up all the individual dividend payments:

$$\frac{P_t^M}{D_t} = \sum_{n=1}^{\infty} \frac{P_{n,t}}{D_t} = \sum_{n=1}^{\infty} \exp\left\{A_{(n)} + B_{(n)} x_t + C_{(n)} z_t + D_{(n)} y_t\right\}. \tag{28}$$

Note that because I calculated asset prices at a monthly horizon, this calculation results in a price-dividend ratio with respect to the monthly dividend amount. In other words, it needs to be divided by 12 to transform into a standard, annual price dividend ratio.

Finally, I calculate the daily stock market return by re-expressing the definition of a return in terms of stationary variables

$$\begin{aligned} R_{t+1} &= \frac{P_{t+1} + D_{t+1}}{P_t} \\ &= \left(\frac{P_{t+1}}{D_{t+1}} + 1\right) \frac{D_{t+1}}{P_t} \frac{D_t}{D_t} \\ &= \left(\frac{P_{t+1}}{D_{t+1}} + 1\right) \frac{D_{t+1}}{D_t} \Big/ \frac{P_t}{D_t}. \end{aligned} \tag{29}$$

Note that the span of the return calculation determines the period over which the denominator in the price-dividend ratio is calculated. In order to use the price-dividend ratio calculated above, I need to transform the monthly dividend amount into a daily amount by dividing with $252/12 = 21$. With the price-dividend ratio in hand I calculate daily market returns by calculating:

$$R_{t,t+1} = \frac{P_{t+1}^M + D_{t+1}}{P_t^M} = \left(\frac{P_{t+1}^M}{D_{t+1}} + 1\right) \frac{D_{t+1}}{D_t} \Big/ \frac{P_t^M}{D_t}. \tag{30}$$

where D_{t+1}/D_t is the daily realized dividend growth rate.

9.2 Stock-Bond Covariance in i.i.d. Models with CRRA or Epstein-Zin Preferences

The following section explores the the impact of the precautionary savings term on stock and bond prices in an i.i.d. model under both CRRA and Epstein-Zin preferences. I use these derivations to calculate preference parameter restrictions under which a change in endowment volatility, or a change in the risk aversion parameter, move stock and bond prices in the opposite direction.

This section is inspired by the calculations in Barsky (1989) and has benefited from the derivations in Gourio (2010).

9.2.1 CRRA Preferences

The SDF under CRRA preferences is given by

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{31}$$

Let me consider an i.i.d. economy. The log consumption growth process follows

$$\ln \left(\frac{C_{t+1}}{C_t} \right) = \Delta c_{t+1} = \mu_c - .5\sigma_c^2 + \sigma_c \epsilon_{t+1}, \tag{32}$$

where $\epsilon_{t+1} \sim N(0,1)$. Therefore the log SDF is given by

$$m_{t,t+1} = \ln(\beta) - \gamma\mu_c + .5\gamma\sigma_c^2 - \gamma\sigma_c\epsilon_{t+1}. \tag{33}$$

Note the inclusion of the $-5.\sigma_c^2$ term in the consumption growth process. This term ensures that expected growth in consumption does not have a mechanical dependence on growth volatility, making it easier to interpret the impact of changes in consumption volatility on asset prices.

Let me calculate prices of safe and risky assets under this SDF. The risk-free rate is given by the reciprocal of the expectation of the SDF

$$\begin{aligned}
\frac{1}{R^f} &= \mathrm{E}_t \left[\exp\{m_{t,t+1}\} \right] = \mathrm{E}_t \left[\exp\{\ln(\beta) - \gamma\mu_c + .5\gamma\sigma_c^2 - \gamma\sigma_c\epsilon_{t+1}\} \right] \\
&= \exp\{\ln(\beta) - \gamma\mu_c + .5\gamma\sigma_c^2 + .5\gamma^2\sigma_c^2\} \\
\implies \ln(R^f) = r^f &= -\ln(\beta) + \gamma\mu_c - .5\gamma\sigma_c^2 - .5\gamma^2\sigma_c^2 \\
&= -\ln(\beta) + \gamma\mu_c - .5\gamma(\gamma+1)\sigma_c^2.
\end{aligned} \tag{34}$$

The risk-free rate is comprised of three terms: the time discount $\ln(\beta)$, the term corresponding to intertemporal substitution $\gamma\mu_c$, and the precautionary savings term $-.5\gamma(\gamma+1)\sigma_c^2$.

Let me now price a risky asset with the dividend payout growth given by

$$\ln\left(\frac{D_{t+1}}{D_t}\right) = \Delta d_{t+1} = \mu_d - .5\sigma_d^2 + \sigma_d \epsilon_{t+1}. \tag{35}$$

Note the asset payout loads on the same i.i.d. shock than the consumption growth rate. To price this asset, let's start with the asset pricing equation and substitute in the definition of a return from Equation (30).

$$\begin{aligned} 1 =& \mathrm{E}_t\left[M_{t,t+1} R_{t,t+1}\right] \\ \implies 1 =& \mathrm{E}_t\left[M_{t,t+1}\left(\frac{P_{t+1}}{D_{t+1}}+1\right)\frac{D_{t+1}}{D_t}\Big/\frac{P_t}{D_t}\right]. \end{aligned} \tag{36}$$

Note that because the SDF and cash-flow processes are both i.i.d., the price-dividend ratio is a constant. Let's denote it by v. Because it's a constant we can pull it out of the expectation operator and re-express Equation (36) as:

$$\frac{v}{v+1} = \mathrm{E}_t\left[M_{t,t+1}\frac{D_{t+1}}{D_t}\right]. \tag{37}$$

Substituting in the SDF and cash-flow process yields in

$$\begin{aligned} \frac{v}{v+1} =& \mathrm{E}_t\left[\exp\left(\ln(\beta) - \gamma\mu_c + .5\gamma\sigma_c^2 - \gamma\sigma_c\epsilon_{t+1}\right)\exp\left(\mu_d - .5\sigma_d^2 + \sigma_d\epsilon_{t+1}\right)\right] \\ =& \exp\left(\ln(\beta) - \gamma\mu_c + .5\gamma\sigma_c^2 + .5\gamma^2\sigma_c^2 + \mu_d - \gamma\sigma_c\sigma_d\right) \\ =& \exp\left(-r^f + \mu_d - \gamma\sigma_c\sigma_d\right). \end{aligned} \tag{38}$$

We can therefore calculate the price-dividend ratio as

$$v = \frac{\exp(-r^f + \mu_d - \gamma\sigma_c\sigma_d)}{1 - \exp(-r^f + \mu_d - \gamma\sigma_c\sigma_d)}. \tag{39}$$

Recalling the definition of return in Equation (30) allows us to calculate expected returns

$$\begin{aligned} \mathrm{E}_t[R_{t+1}] =& \mathrm{E}_t\left[\frac{v+1}{v}\frac{D_{t+1}}{D_t}\right] \\ =& \mathrm{E}_t\left[\exp\left(r^f - \mu_d + \gamma\sigma_c\sigma_d\right)\exp\left(\mu_d - .5\sigma_d^2 + \sigma_d\epsilon_{t+1}\right)\right] \\ =& \exp\left(r^f + \gamma\sigma_c\sigma_d\right). \end{aligned} \tag{40}$$

And therefore the log equity premium is given by

$$\mathrm{RP} = \ln\left(\frac{\mathrm{E}_t[R_{t+1}]}{R^f}\right) = \gamma\sigma_c\sigma_d. \tag{41}$$

Finally, let me express the stock's cash-flow volatility σ_d as a multiple κ of consumption volatility, where κ represents leverage, like in Abel (1999)

$$\sigma_d = \kappa \sigma_c. \tag{42}$$

Let's now consider the impact of a change in consumption volatility, σ_c^2, on the risk-free rate, the expected return on the risky asset with loading σ_d on consumption shocks, and the risk premium of such a risky asset. Taking derivatives with respect to σ_c^2 results in

$$\frac{\partial r^f}{\partial \sigma_c^2} = -.5\gamma^2 - .5\gamma = -.5\gamma(\gamma + 1) \tag{43}$$

$$\frac{\partial \, \mathrm{E}_t[R_{t+1}]}{\partial \sigma_c^2} = -.5\gamma^2 - .5\gamma + \gamma\kappa = -.5\gamma(\gamma - 1 + 2\kappa) \tag{44}$$

$$\frac{\partial \mathrm{RP}}{\partial \sigma_c^2} = \gamma\kappa. \tag{45}$$

(Note that these comparative statics correspond to Equations (12), (13), and (14) in Barsky (1989).)

As the first of the above three equations shows, for any value $\gamma > 0$ an increase in consumption growth volatility σ_c^2 increases the price of the risk-free asset. As the third equation shows, for any value $\gamma > 0$ an increase in consumption growth volatility increases the risk premium of a risky asset, as long as the asset doesn't load negatively on the consumption growth shocks.

The impact of consumption volatility on the expected return of the risky asset depends both on the leverage parameter κ and risk aversion γ. As long as $\gamma < 2\kappa - 1$, an increase in σ_c^2 will increase the required return on equity. In contrast, if $\gamma > 2\kappa - 1$ an increase in consumption growth volatility increases the risk premium, but decreases the risk-free by more, and so the expected return on the risky asset goes down.

In the case where the asset in question is the aggregate consumption stream the parameter $\kappa = 1$ and the critical value becomes $\gamma = 1$. In other words, expected returns increase in consumption volatility only if $\gamma < 1$. For assets that are levered claims on the underlying consumption process, such as the aggregate equity claim, the upper bound of risk aversion under which an increase in σ_c^2 increases expected returns is higher.

In sum, as long as $\gamma < 2\kappa - 1$ an increase in consumption growth volatility affects stock and bond prices in opposite directions. The literature has used values around 3 which would result in an upper bound of 5 on the risk aversion parameter.

Finally, note that in the parameter range where an increase in σ_c^2 moves stock and bond prices in the opposite direction, a marginal increase in the risk aversion parameter will likewise move stock and bond prices in the opposite direction, like in the model presented in this paper.

9.2.2 Epstein-Zin Preferences

Let's now repeat the exercise using Epstein-Zin preferences.

The SDF is given by

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \left(\frac{V_{t+1}}{\mathrm{E}_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}, \tag{46}$$

where ρ is the reciprocal of the Intertemporal Elasticity of Substitution (IES) and γ is the risk aversion parameter. Let consumption and asset payouts be specified as in the CRRA section above. Because both are i.i.d. the value function normalized by consumption is a constant.

Let's again rewrite the SDF in terms of stationary quantities.

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \left(\frac{C_{t+1}}{C_t}\right)^{\rho-\gamma} \left(\frac{C_{t+1}}{C_t}\right)^{\gamma-\rho} \left(\frac{V_{t+1}}{\mathrm{E}_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} \tag{47}$$

$$= \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{\left(\frac{V_{t+1}}{C_{t+1}}\right)^{\rho-\gamma}}{\mathrm{E}_t\left[\left(\frac{V_{t+1}}{C_{t+1}}\right)^{1-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]^{\frac{\rho-\gamma}{1-\gamma}}}. \tag{48}$$

Recognizing the value-to-consumption ratio is a constant because of the i.i.d. assumption, we can pull it out of the expectation and the SDF becomes:

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{1}{\mathrm{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]^{\frac{\rho-\gamma}{1-\gamma}}}. \tag{49}$$

Like before, the risk-free rate is given by the reciprocal of the expectation of the SDF. Let's first evaluate the new term in the SDF with respect to the CRRA calculation above by plugging in the consumption process

$$\begin{aligned}
\mathrm{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]^{\frac{\rho-\gamma}{1-\gamma}} &= \left\{\exp\{(1-\gamma)\mu_c - (1-\gamma).5\sigma_c^2 + (1-\gamma)^2.5\sigma_c^2\}\right\}^{\frac{\rho-\gamma}{1-\gamma}} \\
&= \exp\{(\rho-\gamma)\mu_c - (\rho-\gamma).5\sigma_c^2 + (\rho-\gamma)(1-\gamma).5\sigma_c^2\} \\
&= \exp\{(\rho-\gamma)\mu_c - \gamma(\rho-\gamma).5\sigma_c^2\}.
\end{aligned} \tag{50}$$

The risk-free rate is therefore given by

$$
\begin{aligned}
\frac{1}{R^f} &= \mathrm{E}_t\left[\exp\{m_{t,t+1}\}\right] = \frac{\mathrm{E}_t\left[\exp\{\ln(\beta)-\gamma\mu_c+.5\gamma\sigma_c^2-\gamma\sigma_c\epsilon_{t+1}\}\right]}{\mathrm{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]^{\frac{\rho-\gamma}{1-\gamma}}} \\
&= \frac{\exp(\ln(\beta)-\gamma\mu_c+.5\gamma\sigma_c^2+.5\gamma^2\sigma_c^2)}{\exp\{(\rho-\gamma)\mu_c-\gamma(\rho-\gamma).5\sigma_c^2\}} \\
&= \exp\{\ln(\beta)-\rho\mu_c+.5\rho\sigma_c^2+.5\gamma\rho\sigma_c^2\} \\
\implies r^f &= -\ln(\beta)+\rho\mu_c-.5\gamma(\rho+1)\sigma_c^2
\end{aligned}
$$

This expression is reminiscent of the risk-free rate under CRRA preferences (Equation (34). The differences are that now the consumption growth rate μ_c is now multiplied by ρ, the inverse of the IES, and the precautionary savings term depends on both of the preference parameters.

To price an equity claim we can proceed just like we did under CRRA preferences. Let's substitute in $M_{t,t+1}$ and $\frac{D_{t+1}}{D_t}$ into Equation (37)

$$
\begin{aligned}
\frac{v}{v+1} =& \mathrm{E}_t\left[\frac{\beta\exp\left(-\gamma\mu_c+.5\gamma\sigma_c^2-\gamma\sigma_c\epsilon_{t+1}\right)}{\mathrm{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]^{\frac{\rho-\gamma}{1-\gamma}}}\exp\left(\mu_d-.5\sigma_d^2+\sigma_d\epsilon_{t+1}\right)\right] \\
=&\beta\exp\left(-\rho\mu_c+.5\gamma(\rho+1)\sigma_c^2+\mu_d-\gamma\sigma_c\sigma_d\right) \\
=&\exp\left(-r^f+\mu_d-\gamma\sigma_c\sigma_d\right).
\end{aligned}
$$

Like before, we can therefore calculate the price-dividend ratio as

$$
v = \frac{\exp(-r^f+\mu_d-\gamma\sigma_c\sigma_d)}{1-\exp(-r^f+\mu_d-\gamma\sigma_c\sigma_d)}. \tag{51}
$$

The expected return on the risky asset is

$$
\begin{aligned}
\mathrm{E}_t[R_{t+1}] =& \mathrm{E}_t\left[\frac{v+1}{v}\frac{D_{t+1}}{D_t}\right] \\
=& \mathrm{E}_t\left[\exp\left(r^f-\mu_d+\gamma\sigma_c\sigma_d\right)\exp\left(\mu_d-.5\sigma_d^2+\sigma_d\epsilon_{t+1}\right)\right] \\
=& \exp\left(r^f+\gamma\sigma_c\sigma_d\right)
\end{aligned}
$$

Just like with CRRA preferences, the risk premium on the risky asset is given by $\gamma\sigma_c\sigma_d$

$$
\mathrm{RP} = \ln\left(\frac{\mathrm{E}_t[R_{t+1}]}{R^f}\right) = \gamma\sigma_c\sigma_d.
$$

We can again calculate the derivative of the risk-free rate, expected return on the risky asset, and the risk premium with respect to changes in the consumption volatility σ_c^2. Let me again express the risky asset loading on the ϵ_{t+1} shocks as $\sigma_d = \kappa\sigma_c$.

$$\frac{\partial r^f}{\partial \sigma_c^2} = -.5\gamma(\rho+1) \tag{52}$$

$$\frac{\partial \, \mathrm{E}_t[R_{t+1}]}{\partial \sigma_c^2} = -.5\gamma(\rho+1) + \gamma\kappa = -.5\gamma(\rho+1-2\kappa) \tag{53}$$

$$\frac{\partial \mathrm{RP}}{\partial \sigma_c^2} = \gamma\kappa \tag{54}$$

(Note these calculations correspond to Equations (23), (24), and (25) in Barsky (1989).)

The risk-free rate is again unambiguously decreasing in volatility σ_c. The magnitude of this precautionary term of the risk-free rate depends on both preference parameters, and an increase in either γ or ρ makes the effect stronger. Just like before, the risk premium is increasing in consumption growth volatility, as long as the asset in question loads on the consumption shocks with a positive sign.

More interesting is the behavior of the expected return of risky assets as a function of changes in σ_c^2. The direction of the effect depends on the inverse of IES parameter ρ and on the leverage parameter of the asset, κ, but not on the risk aversion parameter γ. As long as $\rho + 1 - 2\kappa < 0$ an increase in consumption growth volatility increases the expected returns on risky assets. In this range, an increase in σ_c^2 decreases the risk-free rate, but increases the risk premium by more, and so the expected return on the asset goes up.

If the asset in question happens to be a claim on the aggregate equity, $\kappa = 1$, and the cutoff value is $\rho = 1$. If $\rho < 1$, meaning EIS > 1, an increase in σ_c^2 increases expected returns (this is a parameter restriction emphasized in the Long Run Risk literature). However, if the asset in question has leverage above 1, the critical value of ρ under which equity expected returns decrease with σ_c^2 is higher.

The risk aversion parameter γ does not determine the direction of this effect, but it does modulate it's strength. This motivates the modeling choice in the paper under which the precautionary savings terms moves stock and bond prices in the opposite direction.

Finally, note that in the parameter region where a change in σ_c^2 moves stock and bond prices in the opposite direction, a small increase in γ also moves stock and bond prices in the opposite direction.