A theoretical framework for environmental and social impact reporting

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Abstract: We provide a theoretical framework for reporting of firms' environmental and social impact (ESI). In our model the characteristics of a ESI report impact managerial efforts related to ESI, cash flows, stock prices, and greenwashing. In particular, we describe the implications of ESI report congruity, whether the report captures ESI inputs or outcomes, and the manager's tradeoffs regarding ESI efforts and reporting bias. Although stock price incentives tend to encourage ESI efforts and greenwashing simultaneously, ESI reports that capture ESI effects on cash flows tend to have a stronger price reactions than ESI reports that capture effects on ESI per se. ESI reports aligned with investors' aggregate preferences provide stronger incentives and lead to more positive outcomes than ESI reports that focus on either ESI or cash flow effects individually.

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1 Introduction

As the awareness about climate change and social inequity becomes more widespread, consumers, investors, and regulators are increasingly responding to firms' environmental and social impacts (ESI).[1] Central to the decisions that the respective stakeholders need to make is the information that they have available. As a result, government and industry bodies are discussing and developing standards for firms' ESI reporting. We add to this discussion by providing a theoretical framework for ESI reporting, highlighting economic forces that affect the impact of ESI reporting.

In this study, we model a firm that reports its ESI to investors, some of whom incorporate ESI into their demand for shares, due to, e.g., altruism, a concern over ESI outcomes, or positive affect (warm glow) from allocating wealth in line with ESI. The firm's ESI and cash flows depend on corporate actions, captured by a manager's multidimensional efforts. The efforts can have heterogeneous effects on ESI and cash flows, so that higher ESI can lead to higher or lower cash flows and the effects can differ across efforts. While the manager is interested in a higher stock price, investors are uncertain about the manager's effort choice because the manager's preference has a stochastic component unobservable to outside investors.

Investors in our model have access to a public report that discloses the aggregate effect of the manager's efforts with noise. The reporting weights on the efforts (and other variables) allow us to nest scenarios in which a report measures the firm's ESI directly and those where a report measures the effects of the firms' ESI choices on its expected cash flows. In our model, the report is also subject to costly biasing by the manager, which captures greenwashing

[1] We use ESI rather than ESG (environmental, social, and governance) or CSR (corporate social responsibility) to focus on environmental and social impacts rather than firms' governance characteristics or responsibilities per se.

whereby firms manipulate the reports of their ESI. Additional noise in the reporting cost function prevents investors from unraveling the biasing, which allows greenwashing to have a material effect on price that does not depend on the manager passing the cost of greenwashing on to investors (as in pure window-dressing models).

Greenwashing, whereby firms present positively-biased portrayals of their ESI or CSR activities or outcomes, is a significant problem. Underpinning the potential for harmful greenwashing are two stylized facts. First, investors are increasingly concerned about firms' environmental and social impacts (ESI). Second, much of the data on ESI is difficult to measure (e.g., quality of jobs provided), or if easy to measure (e.g., smokestack emissions), difficult to translate into dollar equivalents (e.g., effect of smokestack emissions on local and global welfare or cost of avoiding emissions). In our model, we elaborate on various ways firms might engage in greenwashing, and illustrate how parameters affecting different types of greenwashing additionally affect cash flows, ESI efforts and outcomes, stock prices, and stock price reactions to ESI reports.

In particular, we examine ex ante greenwashing related to how the report is set up, as well as ex post greenwashing driven by the manager's reporting and effort choices. Ex ante greenwashing in our model relates to how ESI efforts and stochastic outcomes outside of managers' control are aggregated. Aggregation policies are important because different sensitivities to efforts or stochastic components can affect market responses and managers' incentives (i.e., how ESI scores are aggregated matters). Regarding the ex post greenwashing, managers can directly manipulate their report to exaggerate (or dampen) reported ESI. Additionally, managers can increase efforts that are captured relatively well in the ESI report, and at the same time reduce efforts that have relatively small impacts on the report. Note that while reporting greenwashing happens after the ESI efforts are chosen, it affects the market response and therefore, in equilibrium, can have an effect on the manager's effort choice. ESI reporting features, through this channel, have real effects on ESI activities and outcomes.

Our results provide guidance specifically on how the characteristics of ESI reporting can affect cash flows, the firm's ESI, as well as the extent of greenwashing. For example, when we focus on a single effort (to abstract away from issues involving report incongruence) we show that changes in reporting uncertainty affect the manager's effort and reporting greenwashing in the same direction. Higher reporting uncertainty increases both when ESI efforts have a sufficiently negative impact on cash flows. We also show that reporting ESI inputs (effort) rather than outcomes tends to decrease effort and greenwashing when the effort has a positive impact on both cash flows and ESI. However, when the effort has a sufficiently negative effect on cash flows, reporting ESI inputs rather than outcomes tends to increase effort levels and greenwashing.

Focusing on how the report aggregates the manager's efforts, we investigate reports that are congruent to cash flows, to ESI, and to the average investor's values. In our model, investors learn about the manager's efforts from the report. This implies that a report without noise that aggregates multiple efforts according to their effect on ESI does not perfectly reveal the cash-flow consequences of the manager's efforts and, thus, leaves investors with residual cash flow uncertainty. Following from that, our model shows that a cash-flow congruent report tends to have a stronger price reaction than an ESI-congruent report. This happens because all investors value cash flows, whereas only a fraction of investors value the firm's ESI.[2] We show that, while a cash-flow-congruent report leads to higher expected cash flows, an ESI-congruent report leads to higher expected ESI. Finally, we show that a values-congruent report (cash flow impact plus ESI weighted by the fraction of investors who value ESI) yields the same expected cash flows as the cash-flow-congruent report *and* the same expected ESI as the ESI-congruent report. Unfortunately, the weights of the efforts in the values-congruent report depend on the fraction of ESI investors, a parameter that likely varies over time and across firms and is difficult to measure.

[2] A necessary condition for the ESI-congruent report to have a higher price reaction is that the efforts have a stronger effect on ESI than on cash flows. In this situation, cash-flow congruency perfectly reveals cash flow effects that, in sum, are smaller than the imperfectly revealed ESI effects.

2 Literature Review

We attempt to answer regulators' and researchers' calls on the optimal characteristics of managers' reporting about their strategies to tackle non-financial risks both internally, for firm cash flows, and externally (Christensen et al., 2019; Grewal and Serafeim, 2020). Our model broadly combines three strands of literature. First, similar to the literature on effort-allocation with moral hazard, a firm's manager privately takes multiple actions that affect the firm's outcomes (e.g., Holmstrom and Milgrom, 1991; Datar et al., 2001). Second, as in the literature on earnings management, the firm discloses a report that need not be truthful (e.g., Dye and Sridhar, 2004; Fischer and Verrecchia, 2000). Finally, some investors who receive the report incorporate their beliefs about the firm's ESI when forming their demand, similar to Pástor et al. (2020) and Friedman and Heinle (2016).

We follow Paul (1992) and Feltham and Xie (1994) on their notion of the incongruity of performance measures. We depart from these works in that we introduce an additional (non-financial) dimension of firm performance that is valued by some investors, which, in turn, creates a second type of incongruity - between the social impact's and measure's sensitivities to agent's actions.

Our model extends the literature on earnings management (e.g., Dye and Sridhar, 2008) by allowing a manager to manipulate the report of firms' environmental and social impact, i.e., to greenwash. We analyze the manager's reporting strategy as a function of parameters capturing the manager's incentives, information asymmetry between the manager and investors, and the incongruity of performance measures with social and financial impact.

Several studies provide evidence that individuals value the social impact of their investments. For example, the survey in Krueger et al. (2020) suggests that institutional investors recognize the importance of climate risks for their portfolios' cash flows. Similarly, Bauer et al. (2021) survey members of pension funds and find that two-thirds of respondents are willing to sacrifice some financial benefits to invest in companies whose goals are aligned with sustainable development goals (SDG). Barber et al. (2021) and Bolton and Kacperczyk

(2021) provide further evidence of tradeoffs between ESI and market performance.

The pricing of companies' non-financial performance has received a much recent academic interest. Closely related is Pástor et al. (2020), who show that agents' tastes for green holdings affect asset prices in equilibrium and derive predictions about the returns on a green factor. Zerbib (2020) develops an asset-pricing model where ESG performance is priced due to the impact of two investor groups: those that exclude certain assets from their investment options and those that internalize private costs of externalities in their expected returns. These investors cause two types of premia to occur: taste premia and exclusion premia. Pedersen et al. (2020) analyze an economy where the ESG score contains information related to firm fundamentals and investors have preferences about firms' non-financial performance. They show that in equilibrium, prices of assets satisfy the four-fund separation theorem: each asset is a portfolio of a risk-free asset, tangent portfolio, minimal-variance portfolio, and ESG-tangent portfolio. Chowdhry et al. (2018), Oehmke and Opp (2019), and Friedman and Heinle (2021) derive conditions for impact investment to improve social outcomes when some investors value impact as well as cash flows.

Most of the literature either assumes symmetric information and is silent on the source of the information that investors have about firms' non-financial performance. Lyon and Maxwell (2011) provide a model of greenwashing based on discretionary disclosure of favorable signals (e.g., Jung and Kwon, 1988), in contrast to our model of reporting bias with uncertain costs. Despite the relative paucity of theoretical research, there exists rich empirical evidence for firms' greenwashing or providing inappropriate information on their ESG activities (e.g., Bingler et al., 2021; Basu et al., 2021; Delmas and Burbano, 2011; Marquis et al., 2016), as well as numerous examples from the popular and business press (e.g., Brogger and Marsh, 2021; Kowsmann and Brown, 2021).

A separate literature has focused on the materiality of ESI disclosures (e.g., Khan et al., 2016; Jebe, 2019). Amel-Zadeh and Serafeim (2018) report survey evidence that mainstream investment organizations primarily use ESG information because of its relevance to investment performance, ahead of client demand and ethical considerations. Materiality implies "relevant to investor decision-making," and can be evaluated based either on relevance to fundamentals, i.e., future cash flows or discount rates, or based on investor responses to ESG information releases. The Sustainability Accounting Standards Board (SASB) has promulgated industry-specific sustainability standards focuses on materiality. Our model, by clearly delineating between cash flow relevance, investor response, and effects on ESI allows us to show how focusing on different definitions of materiality in designing ESI reports can affect prices, greenwashing, and corporate ESI efforts.

A related debate to investor-focused materiality is on how trading activity and investor engagement affect firms' social impact. Landier and Lovo (2020) show how the policy of ESG fund forces companies to internalize (at least partially) their externalities. An ESG fund's optimal strategy is to invest in firms with the strongest capital search frictions and the externalities' inefficiency. Green and Roth (2021) derive optimal strategies for social investors to maximize social welfare in an environment of competition between commercial and social investors. De Angelis et al. (2020) show how companies' greenhouse gas emissions can be reduced through the increase in the cost of capital for those companies, that becomes more sensitive to emissions as the share of green investors and environmental stringency increase.

Our contribution to these streams of literature is that we explicitly model reporting of firms' social impact by a manager who can potentially greenwash. We show that in equilibrium, a firm's price is sensitive to environmental report, which is an aggregate signal of a manager's social effort and a part of the firm's environmental outcome that is out of the manager's control. Manager's taste for social effort is priced, and the price decreases as exerting the effort gets costlier. We also analyze how price and its sensitivity to the report varies with two types of a reported measure's incongruity: material and impact. Finally, we derive conditions for the report's sensitivity to the manager's efforts and an uncontrolled part of social impact that maximizes expected price and expected total and value-weighted social output.

3 Model and equilibrium

The manager of a firm chooses two efforts, $\mathbf{e} = (e_1, e_2)$, that can affect both the firm's cash flow and the firm's ESI output.[3] In particular, we assume that the firm has per-share cash flows of $\tilde{x} = \bar{x} - \boldsymbol{\theta}^T\mathbf{e} + \tilde{\varepsilon}_x$ and per-share ESI of $\tilde{y} = \boldsymbol{\eta}^T\mathbf{e} + \tilde{\varepsilon}_y$, where $\tilde{\varepsilon}_x$ and $\tilde{\varepsilon}_y$ are independent, normally distributed random variables with means of 0 and variances of σ_x^2 and σ_y^2, respectively, and where $\boldsymbol{\theta} = (\theta_1, \theta_2) \in \Re^2$ is a constant vector known by all actors. This implies that effort that increases the expected ESI output decreases cash flows when $\theta_i > 0$ and increases cash flows when $\theta_i < 0$.

We assume that there is a continuum of investors with unit mass. A fraction λ of these investors value ESI, while the remaining $1 - \lambda$ value only cash flows. Similar to Pástor et al. (2020), we assume that while all investors are risk averse with respect to cash flows, the ESI-concerned investors are risk-neutral with respect to ESI. The supply of shares is fixed at 1. There is also a risk-free asset (money) with perfectly-elastic supply in which investors can borrow or lend. Let q_i and l_i denote the shares and money (risk-free asset) held by investor i. The type-1 investors who care only about cash flows have utility $u_1 = -\exp\left[-\rho\left(q_1\tilde{x} + l_1\right)\right]$, whereas type-2 investors have a utility of $u_2 = -\exp\left[-\rho\left(q_2\tilde{x} + l_2\right) - q_2 E\left[\tilde{y}|\Omega\right]\right] = -\exp\left[-\rho\left(q_2\tilde{x} + l_2\right)\right]\exp\left[-q_2 E\left[\tilde{y}|\Omega\right]\right]$, where Ω is the investor's information set.

We assume that the manager is interested in maximizing the firm's stock price and has a preference for the firm's ESI output. In particular, we model the manager's preference for the ESI efforts with the cost $\sum_{i\in\{1,2\}} \frac{c_e}{2}\left(e_i - \phi_i\right)^2$, where ϕ_1 and ϕ_2 are realizations of the random vector, $\tilde{\boldsymbol{\phi}} \sim N\left(\bar{\boldsymbol{\phi}}, \Sigma_\phi\right)$, privately observed by the manager. The market does not observe the manager's preferences, but it is common knowledge that $\bar{\phi}_i$ is the manager's

[3] We choose 2 efforts as the minimal number needed to capture issues related to report congruence and effort allocation. The analysis extends straightforwardly to higher-dimension effort vectors.

expected bliss action on effort e_i and

$$\Sigma_\phi = \begin{pmatrix} \sigma^2_{\phi,1} & 0 \\ 0 & \sigma^2_{\phi,2} \end{pmatrix}$$

is the positive definite covariance matrix, which for simplicity we assume is diagonal. We assume all random variables, ε_x, ε_y, and $\boldsymbol{\phi}$ are independent.

The timeline is as follows: at $t = 0$, the manager privately observes $\boldsymbol{\phi} = (\phi_1, \phi_2)^T$ and chooses $\mathbf{e} = (e_1, e_2)^T$. At $t = 1$ investors trade in the shares and establish the stock price, p. At $t = 2$ cash flows are paid out and ESI performance is revealed. Proposition 1 summarizes the equilibrium efforts and price.

Proposition 1 *In the equilibrium without disclosure, the manager's efforts and the stock market price are given by*

$$\mathbf{e}^\dagger = \boldsymbol{\phi} \text{ and} \tag{1}$$

$$p^\dagger = \bar{x} + \left(\lambda \boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) \bar{\boldsymbol{\phi}} - \rho \left(\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} + \sigma_x^2\right). \tag{2}$$

As Proposition 1 shows, the manager chooses efforts equal to their bliss point, $e_i = \phi_i$. As a result, the expected efforts are given by their expected bliss points, $\bar{\phi}$. Because investors receive no additional information, the firm's price is given by expected cash flows, $E[x] = \bar{x} - \boldsymbol{\theta}^T \bar{\boldsymbol{\phi}}$, minus a risk premium for the uncertainty in cash flows, $Var[x] = \rho \left(\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} + \sigma_x^2\right)$, plus the weighted expected ESI output, $\lambda E[y] = \lambda \boldsymbol{\eta}^T \bar{\boldsymbol{\phi}}$. These results follow prior literature where ESI output is random and a λ-fraction of investors has a risk neutral warm glow from holding shares in a firm that provides this ESI output. In our model, we allow for a correlation in cash flows and ESI, an incremental unit of effort e_i increases ESI output by η_i and decreases cash flows by θ_i. As a result, an additional unit of effort e_i changes price by $\lambda\eta_i - \theta_i$. In addition, because investors are risk averse with respect to cash flows, the uncertainty about the effort effect on cash flows increases the risk premium. That is, the risk premium arises in response to both the fundamental risk in cash flows, σ_x^2, as well as the variation in cash flows that arise from the manager's effort choice, $\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} = \left(\theta_1^2 + \theta_2^2\right) \sigma_\phi^2$.

3.1 ESI reporting

In the baseline model above, investors receive no information about the manager's effort or the other random components in cash flows or ESI output. In what follows, we extend the model to include a firm's report about it's aggregate ESI output. In particular, After observing $\boldsymbol{\phi}$ and choosing $\mathbf{e}$, the manager releases a report about the expected value of the firm's ESI output y.

To incorporate greenwashing, we assume that the report r need not be the manager's truthful expectation of y but can be biased. In particular, we assume that the manager has an an unknown cost of greenwashing the report, similar to reporting bias in Dye and Sridhar (2004). The unknown cost prevents unraveling or undoing of the greenwashing effect in pricing, and could alternatively be incorporated via a mechanism as in Fischer and Verrecchia (2000) with uncertain incentives.

To analyze the effect of different reporting regimes, we allow efforts and ε_y to have differential effects on the ESI report. Specifically, we assume that the manager's cost of providing report r is $\frac{c_r}{2}\left(r - \zeta_1 e_1 - \zeta_2 e_2 - \nu\varepsilon_y - \varepsilon_r\right)^2$. Here, ε_r, is normally distributed and independent of the other random variables, with $\tilde{\varepsilon}_r \sim N\left(0, \sigma_r^2\right)$. As in Dye and Sridhar (2004, p. 56), ε_r "reflects idiosyncratic circumstances that influence the [manager's] misreporting costs." The non-random reporting cost term, c_r, is a positive constant. The ζ_i terms capture sensitivities of the report (costs) to efforts, while ν captures the sensitivity of the report (cost) the uncontrolled component of ESI, ε_y.For example, when $\varepsilon_r = 0$, the manager can avoid any reporting costs by choosing $r = \zeta_1 e_1 + \zeta_2 e_2 + \nu\varepsilon_y$. As such we can interpret these parameters as the weights of different elements defined in the ESI reporting regulation. These reporting parameters allow us to capture whether the report reflects ESI inputs (i.e., efforts) or ESI outcomes (i.e., efforts and the uncontrollable ε_y) as well as two types of reporting incongruity. The first, which we might call (cash flow) materiality incongruity, is the degree to which $\frac{\zeta_1}{\zeta_2} \neq \frac{\theta_1}{\theta_2}$. With materiality incongruity, the ESG report fails to proportionately capture the relative impacts of e_1 and e_2 on expected cash flows, $E[x]$. The second, which we can call impact incongruity, is the degree to which $\frac{\zeta_1}{\zeta_2} \neq \frac{\eta_1}{\eta_2}$. Impact incongruity implies that the ESG report, r, fails to proportionately capture the relative impacts of e_1 and e_2 on expected ESI, $E[y]$.

The manager's objective is to maximize the above utility subject to reporting costs,

$$u_m = p - \frac{c_e}{2}(\mathbf{e}-\boldsymbol{\phi})^T(\mathbf{e}-\boldsymbol{\phi}) - \frac{c_r}{2}\left(r - \boldsymbol{\zeta}^T\mathbf{e} - \nu\varepsilon_y - \varepsilon_r\right)^2,$$

where $\boldsymbol{\zeta} = (\zeta_1, \zeta_2)^T$.

The adjusted timeline is as follows: The timeline is as follows at $t = 0$, the manager privately observes $\boldsymbol{\phi} = (\phi_1, \phi_2)^T$ and chooses $\mathbf{e} = (e_1, e_2)^T$. At $t = 1$ the manager observes ε_r and provides a report r to the market, investors trade in the shares and establish price p. At $t = 2$ cash flows are paid out and ESI performance is revealed. Proposition 1 summarizes the equilibrium efforts and price.

3.2 Equilibrium with ESI reporting

The following Proposition summarizes the equilibrium efforts and price.

Proposition 2 *In the equilibrium with disclosure, the manager's efforts, the disclosed report, and the stock market price are given by*

$$\mathbf{e}^* = \boldsymbol{\phi} + \frac{\psi}{c_e}\boldsymbol{\zeta}, \tag{3}$$

$$r^* = \frac{\psi}{c_r} + \boldsymbol{\zeta}^T\mathbf{e}^* + \nu\varepsilon_y + \varepsilon_r, \text{ and} \tag{4}$$

$$\begin{aligned} p^* = {} & \bar{x} + \left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\left(\frac{\psi}{c_e}\boldsymbol{\zeta} + \bar{\boldsymbol{\phi}}\right) + \psi\left(r^* - \left(\frac{\psi}{c_r} + \frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta} + \boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}\right)\right) \\ & -\rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} + \sigma_x^2 - \frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2}\right), \end{aligned} \tag{5}$$

where $\psi = \frac{dp^*}{dr} = \frac{(\lambda\boldsymbol{\eta}-\boldsymbol{\theta})^T\Sigma_\phi\boldsymbol{\zeta}+\lambda\nu\sigma_y^2}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}$.

Proposition 2 shows that disclosure of the ESI report to the market affects the manager's effort incentives. In addition to the bliss point, $\boldsymbol{\phi}$, the manager also considers the marginal impact of effort on price through the disclosed report. In particular, the $\frac{\psi}{c_e}\boldsymbol{\zeta}$ term in (3) incorporates the how price changes in the report, ψ, the sensitivity of the report to efforts, $\boldsymbol{\zeta}$, and effort costs, c_e, determine the manager's of deviation from the bliss efforts. When, for example, the report increases in e_1 but decreases in e_2, $\zeta_1 > 0 > \zeta_2$, then the manager will increase e_1 and decrease e_2 relative to the equilibrium without reporting as long as stock price increases in the report, $\psi > 0$. This is a type of real greenwashing where the manager increases ESI efforts that are pronounced in disclosed reports. For example, while a public report may stress a firm's carbon emissions, that same report may not measure the emission of hazardous chemicals (such as PFAS and PFOS). In this situation, real greenwashing occurs when the manager shifts the efforts towards the reduction of carbon emissions and away from avoiding other emissions.

The stock-price sensitivity to the disclosed report also drives another form of greenwashing. This can be seen in the $\frac{\psi}{c_r}$ term in the equilibrium report in equation (4). The stronger the reaction to the report, the more the manager will bias the report away from the measured efforts, $\boldsymbol{\zeta}^T\mathbf{e}^*$, the measured random component in ESI, $\nu\varepsilon_y$, and the reporting incentives, ε_r. In the Dye and Sridhar (2004) framework, the random reporting incentives ε_r become similar to measurement noise from the perspective of investors. In equilibrium, the manager places a weight of 1 on the random term and simply adds it to the report.

The equilibrium price in Proposition 2 shows that price continues to be the sum of expected cash flows, weighted expected ESI output, and a risk premium. However, because of the disclosed information, investors use the report (adjusted for the expected report, $E[r] = \frac{\psi}{c_r} + \frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta} + \boldsymbol{\zeta}^T\overline{\boldsymbol{\phi}}$) to update their beliefs about the firm's performance. This causes a reduction in the risk premium. As such, the last term in the risk premium, $\frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}$, denotes the information content of the report about the firm's cash flows. Here it is easy to see that, from the perspective of measuring cash flows, random variation in ESI and reporting incentives in the disclosed report ($\nu^2\sigma_y^2$ and σ_r^2, respectively) reduce the information content of the ESI report.

Corollary 3 emphasizes the effects of a setting with ESI disclosure, relative to a setting without.

Corollary 3 *The price impact of disclosure is*

$$\begin{aligned} p^* - p^\dagger &= \left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\frac{\psi}{c_e}\boldsymbol{\zeta} \\ &+ \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}_r + \nu^2\sigma_y^2 + \sigma_r^2}\left(\boldsymbol{\zeta}^T\left(\boldsymbol{\phi} - \bar{\boldsymbol{\phi}}\right) + \nu\varepsilon_y + \varepsilon_r\right) \\ &+ \rho\frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2}. \end{aligned} \quad (6)$$

The first term in eqn. (6) is the weighted incentive effect of disclosure, $\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)E\left[\mathbf{e}^* - \mathbf{e}^\dagger\right]$. Because the firm issues r, the firm's price responds to the manager's efforts and, thus, provides effort incentives to the manager. Note that the weights $\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)$ show up twice in this term, first as a leading coefficient that scales the two expected efforts and second through the price sensitivity to the report, ψ. Because price weights the two efforts by their effects on cash flows and, partly, ESI output, the manager has corresponding incentives to provide effort.

The second term is the price impact of the updating on the firm's expected outcomes, both cash and ESI. Note that the market only observes a measure that aggregates three types of random variables, these are related to the manager's efforts, the firm's ESI output, and the reporting incentives. Reporting greenwashing reduces the information that investors can glean (about cash flows and ESI output) because price will be moved away from the true information that the manager obtains.

The third term is the price impact of reducing cash flow uncertainty because investors learn about the manager's choice of effort. Because investors are risk averse with respect to cash flows, they demand a lower risk premium when they learn about the manager's efforts.

We next elaborate on the equilibrium greenwashing. Prior literature has provided multiple definitions for greenwashing. Delmas and Burbano (2011, p. 65) "define greenwashing as the intersection of two firm behaviors: poor environmental performance and positive communication about environmental performance." Implicit in their definition is a divergence between performance and communication. Lyon and Maxwell (2011, p. 9) capture greenwashing in a selective disclosure model, and define greenwashing as "selective disclosure of positive information about a company's environmental or social performance, without full disclosure of negative information on these dimensions, so as to create an overly positive corporate image." Lyon and Maxwell's definition explicitly incorporates both E and S dimensions of ESI. However, they focus on partial disclosure, which in our model may be related either to the properties of the report $(\boldsymbol{\zeta}, \nu)$ or the manager's ex post reporting incentives, ε_r. It is reasonably straightforward to interpret variation in $\boldsymbol{\zeta}$, ν, or ε_r as outcomes of an unmodeled partial disclosure subgame in the sense of reducing market participants' ability to infer y from r.

In our setting, total greenwashing, in equilibrium, is the difference between the report, r^*, and the ESI outcome, y^*:

$$\begin{aligned} G^* &= r^* - y^* = \left(\frac{\psi}{c_r} + \boldsymbol{\zeta}^T \mathbf{e}^* + \nu\varepsilon_y + \varepsilon_r\right) - \boldsymbol{\eta}^T \mathbf{e}^* - \varepsilon_y \\ &= (\boldsymbol{\zeta} - \boldsymbol{\eta})^T \mathbf{e}^* + (\nu - 1)\,\varepsilon_y + \frac{\psi}{c_r} + \varepsilon_r. \end{aligned} \tag{7}$$

The total greenwashing in (7) can be decomposed into ex ante components associated with the design of the report and ex post components associated with equilibrium efforts. Ex ante components include: ESI incongruence reflecting the degree to which the sensitivity of the report to efforts differs from the sensitivity of ESI to efforts, captured by the $(\boldsymbol{\zeta} - \boldsymbol{\eta})^T$ term multiplying $\mathbf{e}^*$; and whether ESI outcomes outside of the manager's control, ε_y, are captured by the report, reflected in the $(\nu - 1)$ term multiplying ε_y. We refer to this latter source of ex ante greenwashing as whether the report captures inputs (i.e., efforts such as amounts spent on carbon mitigation) or outcomes (i.e., actual emissions as reflected in y). The ex post components of equilibrium greenwashing are related to the manager's equilibrium choices regarding efforts and reporting. Real greenwashing is based on the manager's efforts, $\mathbf{e}^*$, can reflect both levels and allocations of effort, and interacts with ex ante greenwashing captured by ESI incongruence. From the expression for equilibrium effort in (3), it is clear that real greenwashing will further depend on the manager's stochastic preferences as well as properties of the report. Finally, reporting greenwashing, captured by $\frac{\psi}{c_r} + \varepsilon_r$, captures greenwashing driven by the manager's reporting choice. Reporting greenwashing depends on the price responsiveness to the report, ψ, the non-random cost of reporting bias, c_r, and the idiosyncratic component of the manager's reporting objective, ε_r. Expected reporting greenwashing is given by $\frac{\psi}{c_r}$.

4 Analysis

4.1 Comparative statics

In this section we develop comparative statics related to the equilibrium objects of interest: price, p^*; efforts, $\mathbf{e}^*$; and the report, r^*. The comparative statics are useful in helping to clarify the relevant economic forces in our setting.

4.1.1 Costs

We start with comparative statics related to the costs of reporting discretion and effort, c_r and c_e, respectively. With respect to c_r, we have

$$\frac{dp^*}{dc_r} = 0, \; \frac{d\mathbf{e}^*}{dc_r} = \mathbf{0}, \text{ and } \frac{dr^*}{dc_r} = -\frac{\psi}{c_r^2}.$$

An increase in the misreporting cost tends to counterbalance the firm's misreporting incentive. When $\psi > 0$ ($\psi < 0$), an increase in the misreporting cost pushes the report down (up), by reducing expected reporting greenwashing. Because the expected greenwashing is unwound by investors and does not affect their ability to learn from the report about either actions or the ESI outcome, a change in the cost of reporting discretion, c_r, does not affect equilibrium efforts or prices.

With respect to c_e:

$$\frac{dp^*}{dc_e} = -\left(\lambda\boldsymbol{\eta} - \boldsymbol{\theta}\right)^T \boldsymbol{\zeta}\frac{\psi}{c_e^2}, \ \frac{d\mathbf{e}^*}{dc_e} = -\frac{\psi}{c_e^2}\boldsymbol{\zeta}, \text{ and } \frac{dr^*}{dc_e} = -\frac{\psi}{c_e^2}\boldsymbol{\zeta}^T\boldsymbol{\zeta}.$$

A higher cost of effort tends to push efforts towards zero, proportional to the degree to which they affect the report ($\boldsymbol{\zeta}$) and the effect of the report on price (ψ). This has a knock-on effect on the report and firm price, through a difference in the effect of expected efforts on values-weighted output, $-\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\frac{\psi}{c_e^2}\boldsymbol{\zeta}$ and the report, $-\frac{\psi}{c_e^2}\boldsymbol{\zeta}^T\boldsymbol{\zeta}$.

4.1.2 Congruence

To evaluate the effects of $\boldsymbol{\zeta}$ on the equilibrium objects of interests, we first calculate the sensitivity of the price response to $\boldsymbol{\zeta}$

$$\begin{aligned} \frac{d\psi}{d\boldsymbol{\zeta}} &= \frac{d}{d\boldsymbol{\zeta}} \frac{\left(\lambda\boldsymbol{\eta} - \boldsymbol{\theta}\right)^T \Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2} \\ &= \frac{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)\left(\Sigma_\phi\left(\lambda\boldsymbol{\eta} - \boldsymbol{\theta}\right)\right) - 2\left(\left(\lambda\boldsymbol{\eta} - \boldsymbol{\theta}\right)^T \Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2\right)\Sigma_\phi\boldsymbol{\zeta}}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)^2}. \end{aligned} \tag{8}$$

To facilitate intuition, we write equilibrium price again, separating out the components:

$$
\begin{aligned}
p^* &= \bar{x} + (\lambda\boldsymbol{\eta} - \boldsymbol{\theta})^T \left(\frac{\psi}{c_e}\boldsymbol{\zeta} + \bar{\boldsymbol{\phi}}\right) && (9)\\
&\quad + \frac{(\lambda\boldsymbol{\eta} - \boldsymbol{\theta})^T \Sigma_\phi \boldsymbol{\zeta}}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2} r && (10)\\
&\quad + \frac{\lambda\nu\sigma_y^2}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta}_r + \nu^2\sigma_y^2 + \sigma_r^2} r && (11)\\
&\quad - \frac{(\lambda\boldsymbol{\eta} - \boldsymbol{\theta})^T \Sigma_\phi \boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2} \left(\frac{\psi}{c_r} + \frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta} + \boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}\right) && (12)\\
&\quad - \rho\left(\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} + \sigma_x^2 - \frac{\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\zeta}\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\theta}}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2}\right) && (13)
\end{aligned}
$$

The first line, (9), shows the expected cash flows, $\bar{x}$, plus the effect of expected effort, $E[\mathbf{e}^*] = \frac{\psi}{c_e}\boldsymbol{\zeta}$, on the values-weighted output, where the values-weighting is given by $(\lambda\boldsymbol{\eta} - \boldsymbol{\theta})^T$. The second line in (10) is a term that captures the use of the report to learn about efforts, which is equivalent to using the report to learn about the random bliss actions, $\boldsymbol{\phi}$, given $\mathbf{e}^* = \frac{\psi}{c_e}\boldsymbol{\zeta} + \boldsymbol{\phi}$. Because the effect comes through efforts, the learning is weighted by the values-weights, $(\lambda\boldsymbol{\eta} - \boldsymbol{\theta})^T$. The term in the third line, (11), captures the use of r to learn about $\tilde{\varepsilon}_y$, which is relevant to investors but outside of the manager's control. The fourth line, (12), captures the adjustments for the expected report, $E[r^*] = \frac{\psi}{c_r} + \frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta} + \boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}$, inherent in using the report to learn about $\mathbf{e}^*$ and r^* in lines (10) and (11), respectively. The last line captures the risk premium, which is based on the ex ante expected variance of cash flows conditional on equilibrium effort, $\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} + \sigma_x^2$, net of the amount learned about the $\boldsymbol{\phi}$-driven randomness in effort, $\frac{\Sigma_\phi \boldsymbol{\zeta}\boldsymbol{\zeta}^T \Sigma_\phi}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta}_r + \nu^2\sigma_y^2 + \sigma_r^2}$, scaled quadratically by the effects of efforts on cash flows, $\boldsymbol{\theta}$.[4]

[4] We can set $E[\varepsilon_r] = \frac{\psi}{c_r} + \frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta} + \boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}$, which implies $E[r^*] = 0$ to get rid of the term in line (12). Technically, we can set $\bar{\boldsymbol{\phi}}$ such that $E[\mathbf{e}^*] = 0$, but this is a knife-edge case that limits our ability to talk about the effects of expected managerial preferences.

Using the price decomposition in (9)-(13), and the expression for $\frac{d\psi}{d\boldsymbol{\zeta}}$ in (8) we have

$$
\begin{aligned}
\frac{dp^*}{d\boldsymbol{\zeta}} &= \left(\frac{1}{c_e}\frac{d\psi}{d\boldsymbol{\zeta}}\boldsymbol{\zeta}^T+\frac{\psi}{c_e}\mathbf{I}\right)(\lambda\boldsymbol{\eta}-\boldsymbol{\theta}) &(14)\\
&+\frac{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)\left(\Sigma_\phi\left(\lambda\boldsymbol{\eta}-\boldsymbol{\theta}\right)\right)-2\left(\left(\lambda\boldsymbol{\eta}-\boldsymbol{\theta}\right)^T\Sigma_\phi\boldsymbol{\zeta}\right)\Sigma_\phi\boldsymbol{\zeta}}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)^2}r &(15)\\
&-\lambda\nu\sigma_y^2\left(\frac{\mathbf{2}\Sigma_\phi\boldsymbol{\zeta}}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)^2}\right)r &(16)\\
&-\frac{d\psi}{d\boldsymbol{\zeta}}\left(\frac{\psi}{c_r}+\frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta}+\boldsymbol{\zeta}^T\boldsymbol{\phi}\right)-\psi\left(\frac{1}{c_r}\frac{d\psi}{d\boldsymbol{\zeta}}+\frac{d\psi}{d\boldsymbol{\zeta}}\frac{\boldsymbol{\zeta}^T\boldsymbol{\zeta}}{c_e}++\frac{2\psi}{c_e}\boldsymbol{\zeta}+\frac{d}{d\boldsymbol{\zeta}}\bar{\boldsymbol{\phi}}\right) &(17)\\
&+\rho\left(\frac{2\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)\Sigma_\phi\boldsymbol{\theta}\boldsymbol{\theta}^T\boldsymbol{\Sigma}_\phi-2\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}\right)\Sigma_\phi}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)^2}\boldsymbol{\zeta}\right) &(18)
\end{aligned}
$$

In line (14), we have the effect on the expected action. Line (15) shows the effects of $\boldsymbol{\zeta}$ on using r to learn about actions. There are both numerator and denominator effects. The numerator effect is driven by the effect of $\boldsymbol{\zeta}$ on the sensitivity of r to the manager's efforts. The denominator effect is driven by the variance in r due to the manager's ex ante random preferences, $\boldsymbol{\phi}$. Line (16) shows the effect of $\boldsymbol{\zeta}$ on using r to learn about uncontrollable ESI. Here, only the denominator effect from (15) is present, as greater sensitivity to efforts makes r a worse signal about ε_y. Line (17) is inconsequential, as it affects the price level via the expected report, but it is the deviation between the realized and expected report, $r - E[r]$, on which learning from the report is based (see equation (5)). The risk premium, in (18) also shows both numerator and denominator effects, similar to (15).

Congruence between the report and other objects of interest can be captured by the cosine distance between it and another object, say η, as $\frac{\boldsymbol{\eta}^T\boldsymbol{\zeta}}{|\boldsymbol{\eta}||\boldsymbol{\zeta}|}$. If the vectors' lengths are fixed, then congruence is captured by the dot product, e.g., $\boldsymbol{\eta}^T\boldsymbol{\zeta}$. Congruence can affect investors' ability to use reports to learn about the allocation of efforts, reflected in the $(\lambda\boldsymbol{\eta}-\boldsymbol{\theta})^T\Sigma_\phi\boldsymbol{\zeta}$ term in (15), and this allocation is plausibly relevant to translating efforts into cash flow and ESI effects. Congruence with cash flow effects, $\boldsymbol{\theta}$, can help investors reduce the risk premium associated with cash flows, as shown in line (18). However, the general takeaway from $\frac{d}{d\boldsymbol{\zeta}}p^*$ in

(14)-(18) is that there is not an overall price-increasing effect of having reports be congruent with ESI, via $\boldsymbol{\zeta} = \boldsymbol{\eta}$, or with cash flows, via $\boldsymbol{\zeta} = -\boldsymbol{\theta}$. Indeed, a price-maximizing capital markets perspective requires concern with both how congruity affects the representation of efforts in ESI reports and how it affects learning about other features of interest to investors, e.g., cash flows.[5]

To derive the effect of report-effort sensitivities on expected price, $\frac{d}{d\boldsymbol{\zeta}} E[p^*]$, note that taking the expectation causes lines (15)-(17) in (14)-(18) to cancel out, via the law of iterated expectations, or $E[r - E[r]] = 0$.

The effects on efforts and reports are:

$$\frac{d\mathbf{e}^*}{d\boldsymbol{\zeta}} = \frac{1}{c_e}\frac{d\psi}{d\boldsymbol{\zeta}}\boldsymbol{\zeta}^T + \frac{\psi}{c_e}\mathbf{I}$$

and

$$r^* = \frac{1}{c_r}\frac{d\psi}{d\boldsymbol{\zeta}} + \mathbf{I}\mathbf{e}^* + \frac{d\mathbf{e}^*}{d\boldsymbol{\zeta}}\boldsymbol{\zeta}.$$

Both of these depend on $\boldsymbol{\zeta}$ through $\boldsymbol{\zeta}$'s effect on ψ, the sensitivity of price to the report.

4.1.3 Reporting of ESI outcomes versus inputs

Recall that ν affects whether the report captures ESI outcomes, ε_y, in addition to ESI inputs given by managerial efforts. To show how ν affects the equilibrium objects of interest, we again start with the price response, ψ:

$$\begin{aligned}\frac{d\psi}{d\nu} &= \frac{d}{d\zeta}\frac{(\lambda\boldsymbol{\eta} - \boldsymbol{\theta})^T \Sigma_\phi \boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2} \\ &= \frac{\lambda\sigma_y^2\left(\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} - \nu^2\sigma_y^2 + \sigma_r^2\right)\right) - 2\nu\sigma_y^2(\lambda\boldsymbol{\eta} - \boldsymbol{\theta})^T\Sigma_\phi\boldsymbol{\zeta}}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)^2}.\end{aligned}$$

[5]Setting an optimal $\boldsymbol{\zeta}$ based on either value-relevance ($\frac{d}{dr}p^*$) or expected price maximization ($\max_{\boldsymbol{\zeta}} E[p^*]$) seems to require significant information about investor prefences, (λ and ρ), the effects of various types of ESI efforts on cash flows and ESI outcomes ($\boldsymbol{\eta}$ and $\boldsymbol{\theta}$), and the degree of uncontrollable factors affecting ESI outcomes of interest, σ_y^2. Many of these parameters are the subject of active inquiry.

which has a positive effect related to learning about ε_y and a negative effect due to ν making it harder to learn about efforts, $\mathbf{e}$.

$$\frac{dp^*}{d\nu} = (\lambda\boldsymbol{\eta} - \boldsymbol{\theta})^T \left(\frac{d\psi}{d\nu}\frac{1}{c_e}\boldsymbol{\zeta} + \bar{\boldsymbol{\phi}}\right) \tag{19}$$

$$-(\lambda\boldsymbol{\eta} - \boldsymbol{\theta})^T \Sigma_\phi \boldsymbol{\zeta} \frac{2\nu\sigma_y^2}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)^2} r \tag{20}$$

$$+\frac{\lambda\sigma_y^2\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}_r - \nu^2\sigma_y^2 + \sigma_r^2\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}_r + \nu^2\sigma_y^2 + \sigma_r^2\right)^2} r \tag{21}$$

$$-\left(2\frac{d\psi}{d\nu}\left(\frac{\psi}{c_r} + \frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta}\right) + \frac{d\psi}{d\nu}\boldsymbol{\zeta}^T\boldsymbol{\phi}\right) \tag{22}$$

$$-\rho\left(\frac{2\nu\sigma_y^2\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)^2}\right) \tag{23}$$

In line (19), we have the effect on the expected action. Line (21) shows the effects of ν on using r to learn about uncontrolled ESI, ε_y. There are both numerator and denominator effects. The numerator effect is driven by the effect of ν on the sensitivity of r to ε_y. The denominator effect is driven by the variance in r due to the degree to which r captures ε_y. Line (20) shows the effect of ν on using r to learn about the manager's efforts. Here, only the denominator effect from (21) is present, as greater sensitivity to uncontrollable ESI makes r a worse signal about managerial efforts, $\mathbf{e}$. Line (22) is inconsequential, as was line (17) in the expression of $\frac{dp^*}{d\boldsymbol{\zeta}}$. The risk premium, in (23) in contrast to (18), shows only a denominator effect because, as with (20), reports that are more sensitive to ε_y provide relatively less information about the manager's efforts, which are relevant to cash flows. Thus, the risk premium effect causes price to be lower when the ESI report is more sensitive to uncontrollable ESI. Had we assumed a correlation between ε_y and ε_x, then information about ε_y reflected in r would be informative about ε_x and would cause an increase in ν to have a positive effect on price via a negative effect on the risk premium.[6] Additionally, had we assumed investors were risk averse with respect to ESI, then information about ε_y would

[6] We derive the equilibrium with $Cov(\varepsilon_y, \varepsilon_x) \neq 0$ in the Appendix.

reduce the posterior variance of y and this would be priced.

As can be seen from the expressions for equilibrium price and price impact in (5) and (6), respectively, and the comparative statics derived above, there are several factors that affect outcomes of interest, even in our relatively parsimonious model. The factors interact, furthermore, to make signing effects concisely infeasible. To facilitate the development of intuition around the main frictions present in the setting, we divide much of the subsequent analyses into two parts. The first part uses a single-effort setting to focus on the effects of investor preferences, λ, reporting and effort costs, c_r and c_e, and ex ante ESI-related variances, σ_ϕ^2, σ_ε^2, and σ_r^2. The second part focuses on measurement congruence, using multi-dimensional efforts while limiting the influence of random terms only indirectly related to effort provision. Taking ESI reporting sensitivities $(\boldsymbol{\zeta}, \boldsymbol{\nu})$ as design parameters, we consider how these and the exogenous parameters interact to affect greenwashing, efforts that affect ESI and cash flow performance, expected stock prices, and the sensitivity of prices to the report.

4.2 Single-dimension

This section limits the manager to a single effort dimension, $\mathbf{e} = e$, which allows for scalar representations that focus on the implications of investor preferences, effort costs, and ESI-related sources of uncertainty.

In a single-dimension effort setting, where $\boldsymbol{\eta} = \eta$, $\boldsymbol{\theta} = \theta$, $\boldsymbol{\zeta} = \zeta$, $\Sigma_\phi = \sigma_\phi^2$, $\boldsymbol{\phi} = \phi$, and

$\bar{\boldsymbol{\phi}} = \bar{\phi}$, we can write (3)-(5) as

$$\begin{aligned}
e_I^* &= \phi + \frac{\psi_I}{c_e}\zeta, && (24)\\
r_I^* &= \frac{\psi_I}{c_r} + \zeta e^* + \nu\varepsilon_y + \varepsilon_r && (25)\\
&= \frac{\psi_I}{c_r} + \frac{\psi_I}{c_e}\zeta^2 + \zeta\phi + \nu\varepsilon_y + \varepsilon_r, \text{ and} && (26)\\
p_I^* &= \bar{x} + (\lambda\eta - \theta)\left(\frac{\psi_I}{c_e}\zeta + \bar{\phi}\right) + \psi_I\left(\boldsymbol{\zeta}\left(\phi - \bar{\phi}\right) + \nu\varepsilon_y + \varepsilon_r\right)\\
&\quad -\rho\left(\frac{\theta^2\sigma_\phi^2\left(\nu^2\sigma_y^2 + \sigma_r^2\right)}{\zeta^2\sigma_\phi^2 + \nu^2\sigma_y^2 + \sigma_r^2} + \sigma_x^2\right) && (27)
\end{aligned}$$

where I indicates the one-dimensional effort setting and

$$\psi_I = \frac{dp^*}{dr} = \frac{(\lambda\eta - \theta)\,\sigma_\phi^2\zeta + \lambda\nu\sigma_y^2}{\sigma_\phi^2\zeta^2 + \nu^2\sigma_y^2 + \sigma_r^2}. \tag{28}$$

Corollary 4 *When ESI efforts are one-dimensional, and $\zeta, \eta, \psi_I > 0$: the price response to the report (ψ_I), expected effort (e_I^*), and expected report bias,($\frac{\psi_I}{c_r}$), are increasing in the fraction of ESI-concerned investors (λ), the sensitivity of ESI to managerial effort (η), and the sensitivity of cash flows to managerial effort ($-\theta$).*

Corollary 4 follows directly from comparative statics on (24) and (28). Increases in the fraction of ESI-concerned investors (λ), the sensitivity of ESI to managerial effort (η), and the sensitivity of cash flows to managerial effort ($-\theta$), each enhance the effect of managerial effort on the firms' output, value-weighted by average investor preferences ($\lambda\eta - \theta$). Because the report is informative about the manager's effort, for any $\zeta > 0$, an increase in the relevance of the manager's effort to investors causes the price response to the report, ψ_I, to increase. ESI effort, e_I^*, and expected reporting greenwashing, given by $E\left[\frac{\psi_I}{c_r} + \varepsilon_r\right] = \frac{\psi_I}{c_r}$, are both linear functions of the sensitivity of price to the report because the manager's incentives to engage in ESI effort or reporting bias stem from the degree to which either of these is reflected in stock price.

Corollary 5 *When ESI efforts are one-dimensional and $\zeta, \eta, \psi_I > 0$: an increase in the sensitivity of the report to managerial effort, ζ, makes the stock price more responsive to the report (higher ψ_I) if and only if*

$$(\lambda\eta - \theta)\left(\nu^2\sigma_y^2 + \sigma_r^2 - \sigma_\phi^2\zeta^2\right) - 2\lambda\nu\sigma_y^2\zeta > 0. \tag{29}$$

When the condition in (29) is satisfied, an increase in ζ also leads to an increase in expected report bias ($\frac{\psi_I}{c_r}$). The condition in (29) is sufficient but not necessary for expected effort to be increasing in ζ.

Investors, in aggregate, use the report to learn about the manager's effort, which is unknown ex ante due to uncertainty about her bliss action, ϕ, as well as the uncontrollable portion of the ESI outcome ε_y. An increase in the sensitivity of the report to the manager's effort makes the report more reflective of effort but less reflective of ε_y. If the ESI and cash flow effects of effort offset, i.e., $(\lambda\eta - \theta) \approx 0$, then the condition in (29) will tend to be negative precisely because higher ζ makes the report less informative about ε_y, which are valued by the λ portion of type-2 investors. When the effect of effort on values-weighted outcomes is large, i.e., $(\lambda\eta - \theta) >> 0$, then learning about effort is important. However, $\frac{d\psi_I}{d\zeta}$ can still be negative, particularly if $\sigma_\phi^2\zeta^2$ is large relative to $\nu^2\sigma_y^2 + \sigma_r^2$. This latter effect occurs because the variance of r is quadratic in ζ (with coefficient σ_ϕ^2), while the use of r to learn about effort is driven by the covariance between efforts and expected outcomes ($E[x]$ and $E[y]$), which is linear in ζ.

In the subsections below, we exploit the expressions in (24)-(28) to explore initial questions related to ex post greenwashing via managerial actions and ex ante greenwashing related to whether the report, r, should capture ESI inputs, e, or outputs, y. We examine greenwashing related to incongruence in Section 4.3.

4.2.1 Greenwashing with 1-dimensional effort and an ESI-congruent report

Let $\zeta = \eta$ and $\nu = 1$, such that the equilibrium report becomes

$$r_I^\dagger = \frac{\psi^\dagger}{c_r} + \eta e_I^\dagger + \varepsilon_y + \varepsilon_r = y_I^\dagger + \frac{\psi^\dagger}{c_r} + \varepsilon_r \tag{30}$$

where $\psi^\dagger = \frac{(\lambda\eta-\theta)\sigma_\phi^2\eta+\lambda\sigma_y^2}{\sigma_\phi^2\eta^2+\sigma_y^2+\sigma_r^2}$. From (30), we can characterize the equilibrium report as reflecting the firm's ESI performance, plus a bias reflecting expected greenwashing, $\psi^\dagger/c_r$, plus a noise term capturing the random component, from investors' perspective, of reporting greenwashing, ε_r. For instance, ε_r could capture opportunities for greenwashing or ex post concerns about either cash flow or ESI reports that are ex ante random.

When considering reporting greenwashing in this section, there are two main parameters of interest. The first is the manager's reporting cost, c_r, which encourages the manager to provide a report closer to $y + \varepsilon_r$. The second is the variance in ε_r, σ_r^2, which introduces ex ante randomness into the manager's cost of providing a given report.

Corollary 6 *When ESI efforts are one-dimensional and* $(\zeta, \nu) = (\eta, 1)$*:*

1. *An increase in the manager's reporting cost level,* c_r*, decreases the expected greenwashing,* $E\left[r_I^\dagger - y_I^\dagger\right]$*, when* $\psi^\dagger > 0$ *and increases it otherwise.*

2. *An increase in the manager's reporting cost variance,* σ_r^2*: (i) increases the variance of the report,* $r_I^\dagger$*; (ii) makes the stock price less responsive to the report, i.e.,* $\frac{d\psi^\dagger}{d\sigma_r^2} \propto -\psi^\dagger$*; (iii) decreases (increases) the expected greenwashing,* $E\left[r_I^\dagger - y_I^\dagger\right]$*, when* $\psi^\dagger > 0$ *(*$\psi^\dagger < 0$*); and (iv) decreases (increases) the manager's effort level,* $e_I^\dagger = \phi + \frac{\psi^\dagger\eta}{c_e}$*, when* $\psi^\dagger > 0$ *(*$\psi^\dagger < 0$*).*

The effects of reporting cost levels and greenwashing depend on the manager's incentives with regard to the ESI report. For clarity, we focus our discussion on settings in which the parameters are such that price responds positively to the report, $\psi^\dagger > 0$, i.e., enough investors care about ESI and the effects of ESI effort on cash flows are not too negative, $\lambda > \frac{\theta\sigma_\phi^2\eta}{\sigma_\phi^2\eta^2+\sigma_y^2}$. In this case, the manager, who benefits from a higher stock price, tends to bias the report by a positive amount, $\frac{\psi^\dagger}{c_r}$, in expectation. A higher reporting cost level makes this bias more costly, and thus tends to reduce it. In expectation, this reduces expected greenwashing.

Reporting cost variance, σ_r^2, affects greenwashing through two separate channels. First, it directly affects the expected divergence between the report and true ESI performance, via higher magnitudes of ε_r. From investors' perspectives, this makes the report more variable or noisy with respect to underlying ESI and its cash flow implications, which both types of investors care about. The noisier report has a weaker effect on price, which leads to the second channel: the manager has weaker incentives to engage in greenwashing, in expectation, when price is less responsive to the report. That is, expected greenwashing, $E\left[r_I^\dagger - y_I^\dagger\right] = \frac{\psi^\dagger}{c_r}$, is lower when the report is noisier due to an increase in the manager's reporting cost variance.

Because prices are less responsive to the report, $\frac{d\psi^\dagger}{d\sigma_r^2} \propto -\psi^\dagger$, higher reporting cost variance reduces the manager's incentives to exert efforts. This is captured in part 2(iv) of Corollary 6. This effect depends on the price response to the report, $\psi^\dagger$, as well as the sensitivity of the report to the manager's action, ζ (set to η in this analysis), and the manager's effort cost level, c_e. As in much of the real effects literature in accounting, the properties of reports influence real efforts that managers undertake.

Price, in the one-dimensional case with $(\zeta, \nu) = (\eta, 1)$, is

$$p_I^\dagger = \bar{x} + (\lambda\eta - \theta)\left(\frac{\psi^\dagger}{c_e}\eta + \bar{\phi}\right) + \psi^\dagger\left(\eta\left(\phi - \bar{\phi}\right) + \varepsilon_y + \varepsilon_r\right) - \rho\left(\frac{\theta^2\sigma_\phi^2\left(\sigma_y^2 + \sigma_r^2\right)}{\zeta^2\sigma_\phi^2 + \sigma_y^2 + \sigma_r^2} + \sigma_x^2\right). \quad (31)$$

Note that the price is not affected by the manager's reporting cost level, c_r, as this only affects the expected greenwashing, $\frac{\psi^\dagger}{c_r}$, which is anticipated and unwound when investors use the report to learn about x and y. However, the reporting cost variance has non-trivial effects on stock price.

Corollary 7 *When ESI efforts are one-dimensional and* $(\zeta, \nu) = (\eta, 1)$*, the effect of reporting cost variance on stock price,* $\frac{dp^{\boxplus}}{d\sigma_r^2}$*, is given by the following expression*

$$\frac{dp_I^\dagger}{d\sigma_r^2} = \frac{d\psi^\dagger}{d\sigma_r^2}\left(\eta\left(\phi - \bar{\phi}\right) + \varepsilon_y + \varepsilon_r\right) + (\lambda\eta - \theta)\frac{d\psi^\dagger}{d\sigma_r^2}\frac{\eta}{c_e} - \rho\left(\frac{\theta^2\zeta^2\sigma_\phi^4}{\left(\zeta^2\sigma_\phi^2 + \sigma_y^2 + \sigma_r^2\right)^2}\right) \tag{32}$$

The reporting cost variance affects the firm's stock price through three channels. First, an increase in reporting cost variance attenuates the responsiveness of price to the report. This is captured by the first term in (32), $\frac{d\psi^\dagger}{d\sigma_r^2}\left(\eta\left(\phi - \bar{\phi}\right) + \varepsilon_y + \varepsilon_r\right)$. This makes price reflect less relevant information, i.e., less of the manager's ex ante uncertain effort incentives reflected in ϕ and the uncontrolled ESI output, ε_y, while also making the price less reflective of the variable portion of the manager's reporting costs, ε_r. Second, because prices are less responsive to the report, the manager has weaker incentives to exert efforts, as reflected in the second term in (32), $\frac{d\psi^\dagger}{d\sigma_r^2}\frac{(\lambda\eta-\theta)\eta}{c_e}$. This effect captures the effect on managerial effort from part 2(iv) of Corollary 6, $\frac{de_I^\dagger}{d\sigma_r^2} = \frac{d\psi^\dagger}{d\sigma_r^2}\frac{\eta}{c_e}$, multiplied by investors' values-weighted preferences, $\lambda\eta - \theta$, which in the one-dimensional effort model reflects the values-weighted effect of effort on ESI and cash flows. Third, a higher reporting cost variance decreases the amount that investors can learn from r about the firm's cash flows. Thus, higher σ_r^2 tends to decrease price via an increases in the risk premium, captured by the third term in (32), $-\frac{\rho\theta^2\zeta^2\sigma_\phi^4}{\left(\zeta^2\sigma_\phi^2+\sigma_y^2+\sigma_r^2\right)^2}$. Note that taking the expectation (32) yields the effect of reporting cost variance on the firm's expected price. The first term drops out but the second and third remain. Thus, reporting cost variance affects expected price via its effects on the manager's effort incentives and the risk premium demanded by investors after observing the report.

4.2.2 Should r capture ESI inputs outcomes?

In this section we use our single-dimension model to examine the implications of ESI reports that focus on inputs, such as carbon mitigation efforts, versus ESI reports that focus on outputs or outcomes, such as smokestack emissions. We capture the former case by setting

$\nu = 0$ and the latter by setting $\nu = 1$. We have already derived the case with $\nu = 1$ above, which yields $r_I^\dagger = y_I^\dagger + \frac{\psi^\dagger}{c_r} + \varepsilon_r$. With $\nu = 0$, we have

$$\begin{aligned}
e_I^\blacklozenge &= \phi + \frac{\psi^\blacklozenge}{c_e}\eta, \\
r_I^\blacklozenge &= \frac{\psi^\blacklozenge}{c_r} + \eta e_I^\blacklozenge + \varepsilon_r \\
&= \frac{\psi^\blacklozenge}{c_r} + \eta\left(\phi + \frac{\psi^\blacklozenge}{c_e}\eta\right) + \varepsilon_r, \\
\psi^\blacklozenge &= \frac{dp_I^\blacklozenge}{dr} = \frac{(\lambda\eta - \theta)\,\sigma_\phi^2\eta}{\sigma_\phi^2\eta^2 + \sigma_r^2}, \text{ and} \\
p_I^\blacklozenge &= \bar{x} + (\lambda\eta - \theta)\left(\frac{\psi^\blacklozenge}{c_e}\eta + \bar{\phi}\right) + \psi^\blacklozenge\left(\eta\left(\phi - \bar{\phi}\right) + \varepsilon_r\right) - \rho\left(\frac{\theta^2\sigma_\phi^2\sigma_r^2}{\zeta^2\sigma_\phi^2 + \sigma_r^2} + \sigma_x^2\right).
\end{aligned}$$

Investors use the report to learn about cash flows and ESI. These, in turn, can be decomposed into using the report to learn about the ex ante unknown determinants of the manager's effort, ϕ, and the uncontrollable portion of ESI performance, ε_y. When $\nu = 1$, the report captures ε_y, but this makes it harder to use the report to learn about efforts. In contrast, when $\nu = 0$, the report is more useful with regard to learning about effort, but no longer reflects the component of the ESI outcome that ESI-investors care about but that the manager cannot control.

Corollary 8 *When ESI efforts are one-dimensional and $\zeta = \eta$, price is more positively responsive to a report that captures ESI inputs ($\nu = 0$) than a report that captures ESI outcomes ($\nu = 1$) if and only if $\psi^\blacklozenge > \psi^\dagger \Leftrightarrow -\frac{\theta\eta\sigma_\phi^2}{\sigma_r^2} > \lambda$.*

The $-\frac{\theta\eta\sigma_\phi^2}{\sigma_r^2} > \lambda$ inequality in Corollary 8 captures the relevant factors that affect the difference in price responsiveness across settings with ESI input reports versus ESI outcome reports. ESI outcomes are relevant to type-2 investors who intrinsically care about ε_y. ESI inputs, while valuable to both types of investors, are valuable to type-1 investors only because they also have cash flow implications, via θ. When the cash flow implications of ESI efforts are more positive (i.e., θ is more negative), price will tend to be more responsive to reports that capture only ESI inputs. Note further that the dependence of $-\frac{\theta\eta\sigma_\phi^2}{\sigma_r^2} > \lambda$ on η is driven by our assumption that $\eta = \zeta$. That is, the η in the numerator of the left hand side of the inequality reflects the sensitivity of the report to managerial effort, ζ, which we have set to η so that the ESI report captures the effect of effort, e, on the ESI outcome, y. Furthermore, learning about effort is more important when ex ante uncertainty about the manager's effort (incentive) is more uncertain, captured by a higher σ_ϕ^2. Finally, a higher variance of reporting costs, σ_r^2, is more relevant, proportionally, when investors are focusing on learning about efforts only rather than outcomes, as the variance of the outcome, y, incorporates the variance of ε_y as well as the variance of effort, e.

For ESI efforts that have positive effects on expected ESI and negative effects on expected cash flows, i.e., $\eta, \theta > 0$, the condition in Corollary 8 simplifies to $-\frac{\sigma_\phi^2}{\sigma_r^2} > \lambda$, which is false for any $\lambda > 0$. Thus, with $\eta, \theta > 0$, price is more responsive to a report that captures ESI outcomes rather than ESI efforts. For ESI efforts that have positive effects on both expected ESI and cash flows, i.e., $\eta > 0 > \theta$, the condition in Corollary 8 simplifies to $\frac{\sigma_\phi^2}{\sigma_r^2} > \lambda$, which can be interpreted as a comparison between a signal-to-noise ratio on the left to the fraction of ESI-interested investors on the right.

Corollary 9 *When ESI efforts are one-dimensional and $\zeta = \eta$, expected managerial effort and expected greenwashing are more positive when the report captures ESI inputs ($\nu = 0$) than when the report captures ESI outcomes ($\nu = 1$) if and only if $\psi^\blacklozenge > \psi^\dagger \Leftrightarrow -\frac{\theta\eta\sigma_\phi^2}{\sigma_r^2} > \lambda$.*

Corollary 9 is a consequence of expected effort and greenwashing (in the sense of Delmas and Burbano, 2011) being directly responsive to the price sensitivity to the report, ψ, and Corollary 8. Interestingly, this implies that there are situations where allowing the report to reflect ESI outcomes, which type-1 investors view as noise, can lead to more positive efforts and potentially greater expected cash flows. Similarly, allowing the report to capture ESI outcomes can change incentives related to ESI inputs and lead to more negative expected ESI performance. As before, there are real effects of the measurement regime that can be ex ante counterintuitive. Regarding expected greenwashing, it suffices to note that this is equal to the price responsiveness to the report, scaled by the level of reporting cost, c_r. Thus, with $\eta, \theta > 0$ ($\eta > 0 > \theta$), efforts and greenwashing are larger (smaller) when the report captures ESI outcomes rather than ESI efforts alone.

4.3 Congruence

In this section, we go back to the baseline model in which the manager takes two actions. However, to eliminate the effect of noise (from the perspective of any investor) in the reporting system, we assume that $\sigma_y^2 = \sigma_r^2 = 0$. In particular, when $\sigma_r^2 = 0$, there is no uncertainty in the manager's bias. This allows investors to unravel the manager's reporting strategy to infer the linear combination of efforts implied by $\boldsymbol{\zeta}^T\mathbf{e}$. Furthermore, when $\sigma_y^2 = 0$, ESI outcomes are determined entirely by the manager's efforts. Consequently, the report only captures variation in ESI that is relevant for cash flows. As a result, the parameter ν has no impact on the equilibrium. Primarily, we set $\sigma_y^2 = \sigma_r^2 = 0$ to focus on the effect of congruence in the reporting system. In what follows, we will examine three potential reporting system choices, cash-flow congruence ($\boldsymbol{\zeta} = -\boldsymbol{\theta}$), ESI congruence ($\boldsymbol{\zeta} = \boldsymbol{\eta}$), and values congruence ($\boldsymbol{\zeta}_\theta = \lambda\boldsymbol{\eta} - \boldsymbol{\theta}$). We assume that cash-flow and ESI impacts are not perfectly aligned, i.e., there is no constant κ such that $\boldsymbol{\eta} = \kappa\boldsymbol{\theta}$.

4.3.1 Comparing cash-flow congruence and ESI congruence

When $\sigma_y^2 = \sigma_r^2 = 0$ and the report is cash-flow congruent, $\boldsymbol{\zeta} = -\boldsymbol{\theta}$, the equilibrium is given by

$$\mathbf{e}_\theta^* = \boldsymbol{\phi} - \frac{\psi_\theta^*}{c_e}\boldsymbol{\theta},\ r_\theta^* = \frac{\psi_\theta^*}{c_r} - \boldsymbol{\theta}^T\mathbf{e}_\theta^*, \tag{33}$$

$$\psi_\theta^* = 1 - \lambda\frac{\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}}, \text{ and} \tag{34}$$

$$p_\theta^* = \bar{x} + \left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\left(-\frac{1}{c_e}\psi_\theta^*\boldsymbol{\theta} + \bar{\boldsymbol{\phi}}\right) - \psi_\theta^*\boldsymbol{\theta}^T\left(\boldsymbol{\phi} - \bar{\boldsymbol{\phi}}\right) - \rho\sigma_x^2. \tag{35}$$

The last term in equation (35) is the risk premium based only on $\rho\sigma_x^2$. The risk premium term shows that when there is no measurement noise and the report aggregates the efforts in the same way that they affect cash flows, investors can perfectly infer any variation in cash flows that stems from the manager's efforts. However, equation (34) suggests that investors cannot perfectly infer the impact of the manager's efforts on the ESI output. In particular, the market response coefficient in (34) is equal to 1, for the cash-flow relevant news, minus a term, $\lambda\frac{\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}}$, that shows that ESI interested investors are not able to infer all ESI variation because the efforts are not aggregated for that purpose. The numerator in this term captures incongruence between ESI and cash flow implications via $\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\theta}$, weighted by the $\boldsymbol{\phi}$ covariance matrix.

When $\sigma_y^2 = \sigma_r^2 = 0$ and the report is ESI congruent, $\boldsymbol{\zeta} = \boldsymbol{\eta}$, the equilibrium is given by

$$\mathbf{e}_\eta^* = \boldsymbol{\phi} + \frac{\psi_\eta^*}{c_e}\boldsymbol{\eta},\ r_\eta^* = \frac{\psi_\eta^*}{c_r} + \boldsymbol{\eta}^T\mathbf{e}_\eta^*, \tag{36}$$

$$\psi_\eta^* = \lambda - \frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\eta}}{\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\eta}}, \text{ and} \tag{37}$$

$$\begin{aligned} p_\eta^* &= \bar{x} + \left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\left(\frac{\psi_\eta^*}{c_e}\boldsymbol{\eta} + \bar{\boldsymbol{\phi}}\right) \\ &\quad + \psi_\eta^*\boldsymbol{\eta}^T\left(\boldsymbol{\phi} - \bar{\boldsymbol{\phi}}\right) - \rho\left(\sigma_x^2 + \boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} - \frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\eta}\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\eta}}\right) \end{aligned} \tag{38}$$

Here, the risk premium in (38) shows that when the efforts are aggregated according to their influence on the firm's ESI output, investors are not able to infer the entire variation in cash flows that stems from the manager's efforts. We can write the risk premium as

$$\rho\left(\sigma_x^2 + \boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} - \frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\eta}\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\eta}}{\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\eta}}\right) = \rho\left(\sigma_x^2 + \frac{\boldsymbol{\theta}^T\Sigma_\phi\left(\boldsymbol{\theta}\boldsymbol{\eta}^T - \boldsymbol{\eta}\boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\eta}}{\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\eta}}\right)$$

where $\left(\boldsymbol{\theta}\boldsymbol{\eta}^T - \boldsymbol{\eta}\boldsymbol{\theta}^T\right)$ captures the incongruence between ESI and cash flow effects, and goes to zero as $\boldsymbol{\eta} \to \kappa\boldsymbol{\theta}$, for some scalar κ. In contrast to the response coefficient ψ_θ^* in (34), ψ_η^* in (37) is equal to λ (capturing the fraction of investors who value the ESI relevant information in the report) minus a term, $\frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\eta}}{\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\eta}}$, that shows the inability to infer the cash flow effects from the ESI-congruent report. As above, this term captures incongruence between ESI and cash flow implications, weighted by the $\boldsymbol{\phi}$ covariance matrix, via $\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\eta}$.

The following corollary compares the response coefficients across the two regimes, which allows us to discuss differences in expected efforts and bias.

Corollary 10 *When $\sigma_y^2 = \sigma_r^2 = 0$ and $\sigma_{\phi,1}^2 = \sigma_{\phi,2}^2$, (i) the response coefficient for an ESI-congruent report is only higher than that for a cash-flow congruent report when the firm's ESI output is more responsive to the manager's effort than the firm's cash flows, i.e., $\psi_\eta^* > \psi_\theta^*$ iff $\lambda\left(1 + \frac{\theta_1\eta_1+\theta_2\eta_2}{\theta_1^2+\theta_2^2}\right) > 1 + \frac{\theta_1\eta_1+\theta_2\eta_2}{\eta_1^2+\eta_2^2}$; (ii) the firm's expected ESI output (the firm's expected cash flow) is weakly higher (weakly lower) for an ESI-congruent report than for a cash-flow congruent report, i.e., $\boldsymbol{\eta}^T\left(\mathbf{e}_\eta^* - \mathbf{e}_\theta^*\right) \geq 0$ and $-\boldsymbol{\theta}^T\left(\mathbf{e}_\eta^* - \mathbf{e}_\theta^*\right) \geq 0$; and (iii) expected price is higher for an ESI-congruent report than for a cash-flow congruent report if and only if $\lambda^2 > \frac{\theta_1^2+\theta_2^2}{\eta_1^2+\eta_2^2}\left(1 + c_e\rho\sigma_\phi^2\right)$.*

In Corollary 10, part (i) shows that because all investors care about cash flows, the market response to a report that focuses on cash-flow implications tends to be higher. In order for the market response to a report that focuses on ESI outputs to be higher, the efforts that show up in the report need to have a stronger impact on the ESI outputs. That is, in a scenario without reporting noise, the response coefficient is largely dictated by three components: how much there is to learn about each effort ($\sigma_{\phi,i}^2$); how congruent the report is to the interest of the respective investor; and how important the respective task is. In turn, this implies that when the vectors $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ are of equal length ($|\boldsymbol{\theta}| = |\boldsymbol{\eta}|$), then $\sigma_y^2 = \sigma_r^2 = 0$ and $\sigma_{\phi,1}^2 = \sigma_{\phi,2}^2$ imply that the market response is stronger to a cash-flow oriented ESI report.

Part (ii) of Corollary 10 effectively echoes the adage "you get what you measure." In particular, when the response coefficient is positive, the manager has incentives to report a higher aggregate report. This implies that when the report is congruent to the firm's ESI output, then the manager has incentives to allocate more effort to a task that has a higher impact on the ESI output. ESI output is the same in both reporting systems when $\lambda = 0$. That is, when ESI is priced only for its effects on cash flows, then different reporting systems do not yield different expected ESI outcomes. Furthermore, ESI and cash flow converge under both reporting systems as $\frac{\theta_1}{\theta_2} \to \frac{\eta_1}{\eta_2}$, that is when the relative sensitivities of cash flows and ESI to the manager's efforts converge or, alternatively, when $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ become congruent such that $\boldsymbol{\theta} \to \kappa\boldsymbol{\eta}$ for scalar κ.

Finally, part (iii) shows that the firm's expected stock price is higher under an ESI-congruent reporting system when the fraction of investors that value ESI is sufficiently large. Corollary 10 part (iii) also shows that the threshold for λ increases in the risk aversion and the variance of the effort incentives. As we discuss above, investors in our model are risk averse with respect to cash flows, but risk neutral with respect to ESI. For this reason, the risk premium embedded in price is lower when the report is cash-flow congruent, as investors can use the cash-flow congruent report to infer the effort-induced variation in cash flows. A lower investor risk aversion (or effort uncertainty), lowers the price benefit of a cash-flow congruent report. Furthermore, part (iii) shows that the threshold for λ increases in the length of the vector $\boldsymbol{\theta}$ but decreases in the length of the vector $\boldsymbol{\eta}$. When cash flow is relatively more sensitive to the efforts measured in the ESI report (high value for $\theta_1^2 + \theta_2^2 = \boldsymbol{\theta}^T\boldsymbol{\theta} = |\boldsymbol{\theta}|^2$), a cash-flow congruent report has a more beneficial effect on cash flows, which makes it more likely that a cash-flow congruent reporting system leads to higher expected prices.

For the regulation of ESI reporting, Corollary 10 (ii) implies that when the goal is to motivate stronger ESI performance, the report should focus directly on the ESI outcome and should define materiality in terms of the ESI outcome, rather than in terms of the firm's cash flows. Put another way, ex ante ESI congruence discourages ex post real greenwashing. However, disclosure standards that are ex ante ESI congruent will come at a cost to cash flows. From part (iii) of Corollary 10, managers who seek to maximize expected stock price may nevertheless advocate for ESI-congruent reporting standards when there are enough ESI sensitive investors in the market.

4.3.2 Values congruence

The discussion so far shows that congruence of the report to the firm's ESI output and to cash flows comes with different costs. However, both of these reporting systems are incongruent to the interests of an average, representative investor who values efforts according to $\lambda\boldsymbol{\eta}-\boldsymbol{\theta}$. In what follows, we investigate a values congruent reporting system ($\boldsymbol{\zeta}_V = \lambda\boldsymbol{\eta}-\boldsymbol{\theta}$), maintaining our assumptions of $\sigma_y^2 = \sigma_r^2 = 0$ and $\sigma_{\phi,1}^2 = \sigma_{\phi,2}^2 = \sigma_\phi^2$, to focus on congruity and effort-sensitivity issues. Here, the equilibrium is given by

$$\mathbf{e}_V^* = \boldsymbol{\phi}+\frac{1}{c_e}\left(\lambda\boldsymbol{\eta}-\boldsymbol{\theta}\right), \; r_V^* = \frac{1}{c_r}+\left(\lambda\boldsymbol{\eta}-\boldsymbol{\theta}\right)^T\mathbf{e}^*, \tag{39}$$

$$\psi_V^* = \frac{dp^*}{dr} = 1, \text{ and} \tag{40}$$

$$p_V^* = \bar{x}+\left(\lambda\boldsymbol{\eta}-\boldsymbol{\theta}\right)^T\left(\frac{1}{c_e}\left(\lambda\boldsymbol{\eta}-\boldsymbol{\theta}\right)+\bar{\boldsymbol{\phi}}\right)+\left(\lambda\boldsymbol{\eta}-\boldsymbol{\theta}\right)^T\left(\boldsymbol{\phi}-\bar{\boldsymbol{\phi}}\right) -\rho\left(\sigma_x^2+\sigma_\phi^2\frac{\lambda^2\left(\theta_1\eta_2-\theta_2\eta_1\right)^2}{\left(\lambda\eta_1-\theta_1\right)^2+\left(\lambda\eta_2-\theta_2\right)^2}\right). \tag{41}$$

Because the report is not congruent with cash flows (for $\lambda > 0$ and $\theta_1\eta_2 \neq \theta_2\eta_1$), the report does not eliminate investors' uncertainty about the effects of effort on cash flows, leaving a higher risk premium in (41) than with cash-flow congruent reporting in (35). However, a values congruent report leads to an even higher risk premium than with an ESI-congruent report when $2\lambda > \frac{\theta_1^2+\theta_2^2}{\theta_1\eta_1+\theta_2\eta_2}$. Notice that when one effort increases cash flows ($\theta_i < 0$) and the other effort decreases cash flows ($\theta_{-i} > 0$), while both tasks increase ESI output ($\eta_1, \eta_2 > 0$) the values congruent report will receive most of its variation from the effort that increases cash flows. As a result, investors will not be able to learn about the effort that decreases cash flows and may, thus, end up with a higher posterior uncertainty.

The following corollary compares outcomes for the values congruent reporting system with those from the cash-flow and ESI congruent reporting systems.

Corollary 11 *When $\sigma_y^2 = \sigma_r^2 = 0$ and $\sigma_{\phi,1}^2 = \sigma_{\phi,2}^2 = \sigma_\phi^2$, a values congruent report yields: (i) weakly higher expected ESI output and cash flows than either an ESI-congruent or a* *cash-flow-congruent report, i.e.,* $\boldsymbol{\eta}^T\left(\mathbf{e}_V^* - \mathbf{e}_\eta^*\right) = 0$, $\boldsymbol{\eta}^T\left(\mathbf{e}_V^* - \mathbf{e}_\theta^*\right) \geq 0$, $\boldsymbol{\theta}^T\left(\mathbf{e}_V^* - \mathbf{e}_\eta^*\right) \geq 0$, *and* $\boldsymbol{\theta}^T\left(\mathbf{e}_V^* - \mathbf{e}_\theta^*\right) = 0$*; (ii) a higher price than an ESI-congruent report when* $\frac{1}{c_e}\frac{(\theta_1\eta_2-\theta_2\eta_1)^2}{\eta_1^2+\eta_2^2} + \rho\sigma_\phi^2\left(\frac{(\theta_1\eta_2-\theta_2\eta_1)^2}{\eta_1^2+\eta_2^2} - \frac{\lambda^2(\theta_1\eta_2-\theta_2\eta_1)^2}{(\lambda\eta_1-\theta_1)^2+(\lambda\eta_2-\theta_2)^2}\right) > 0$*; and (iii) a higher price than a cash-flow-congruent report when* $\frac{1}{c_e}\lambda^2\frac{(\theta_1\eta_2-\theta_2\eta_1)^2}{\theta_1^2+\theta_2^2} - \rho\sigma_\phi^2\left(\frac{\lambda^2(\theta_1\eta_2-\theta_2\eta_1)^2}{(\lambda\eta_1-\theta_1)^2+(\lambda\eta_2-\theta_2)^2}\right) > 0$.

Corollary 11 (i) shows the strong result that ESI output and cash flows are at least as high with the values congruent report than with either of the other two reports. Parts (ii) and (iii) show that the effect on the risk premium can outweigh the real effect on efforts from the reporting system. In particular, when the risk aversion and the effort uncertainty are sufficiently large (such that the conditions in the corollary are violated) and when the condition $2\lambda > \frac{\theta_1^2+\theta_2^2}{\theta_1\eta_1+\theta_2\eta_2}$ holds, then the values congruent report leads to lower prices due to the loss of cash-flow relevant information.

5 Conclusion

In our model ESI reporting is of interest to investors for multiple reasons, some investors inherently care about the firm's ESI and all investors care about the potential cash-flow consequences of the firm's ESI efforts. Because ESI reporting informs investors about the firm's outcomes and reduces their perceived risks, it provides firms with incentives to manipulate investors' perceptions. As such, our model speaks to a number of frictions in the ESI reporting space. In particular, we capture several dimensions of congruity, i.e., whether the report reflects the firm's ESI, the firm's ESI efforts, or the impacts of ESI actions on cash flows. We show how the properties of the report affect the firm's outcomes, both in terms of cash flows and ESI. We also investigate how firms change their reports and efforts in response to capital market incentives that originate from investors' use of the ESI report.

Our model allows us to make several empirical predictions about the price impact and equilibrium consequences of ESI reporting. Because our model features effects on both cash flows and ESI, the predictions from our model could help to analyze whether and how much investors value ESI, over and above its impact on cash flows. The theoretical framework we develop suggests that the characteristics of ESI reporting standards determine the impact of the respective reports. In other words, reporting standards that lead to higher cash flows need not lead to higher stock prices when investors directly value ESI. As a result, the primary objective of defining reporting standards determines the optimal characteristics of these standards.

6 Appendix

6.1 Proof of Proposition 2

Below, we derive a linear equilibrium in which the risky asset's price is a linear function of the report: $p = a + br$. Proceeding via backward induction, we start with the competitive market for firm shares in period 2, conditional on the report, r, provided to the market. All investors observe r. We conjecture that the manager's choice of effort e_i is linear in ϕ, i.e., $e_i = \alpha_{ei} + \beta_{ei1}\phi_1 + \beta_{ei2}\phi_2$, $\mathbf{e} = \boldsymbol{\alpha}_e + \boldsymbol{\beta}_e\boldsymbol{\phi}$, where $\boldsymbol{\alpha}_e = (\alpha_{e1}, \alpha_{e2})^T$ and $\boldsymbol{\beta}_e = \begin{pmatrix} \beta_{e11} & \beta_{e12} \\ \beta_{e21} & \beta_{e21} \end{pmatrix}$. Then the expected efforts are $\bar{\mathbf{e}} = \boldsymbol{\alpha}_e + \boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}$ and the ex ante covariance of efforts is $Cov(\mathbf{e}) = \boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T$, with $\Sigma_\phi = Cov(\boldsymbol{\phi}) = \begin{pmatrix} \sigma^2_{\phi,1} & 0 \\ 0 & \sigma^2_{\phi,2} \end{pmatrix}$. For simplicity, we set $\sigma^2_{\phi,1} = \sigma^2_{\phi,2} = \sigma^2_\phi$ in the remainder.

The report is linear in e, ε_y, ε_r, i.e., $r = \alpha_r + \beta_{r1}e_1 + \beta_{r2}e_2 + \gamma_y\nu\varepsilon_y + \gamma_r\varepsilon_r = \alpha_r + \boldsymbol{\beta}_r^T\mathbf{e} + \gamma_y\nu\varepsilon_y + \gamma_r\varepsilon_r$, where $\boldsymbol{\beta}_r = (\beta_{r1}, \beta_{r2})^T$. Substituting, we have

$$r = \alpha_r + \beta_{r1}(\alpha_{e1} + \beta_{e11}\phi_1 + \beta_{e12}\phi_2) + \beta_{r2}(\alpha_{e2} + \beta_{e21}\phi_1 + \beta_{e22}\phi_2) + \gamma_y\nu\varepsilon_y + \gamma_r\varepsilon_r$$

which has expectation and covariance:

$$\begin{aligned}
\bar{r} &= \alpha_r + \boldsymbol{\beta}_r^T\bar{\mathbf{e}} = \alpha_r + \boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\
Cov(r) &= Cov\left(\boldsymbol{\beta}_r^T\mathbf{e} + \gamma_y\nu\varepsilon_y + \gamma_r\varepsilon_r\right) = Cov\left(\boldsymbol{\beta}_r^T\mathbf{e}\right) + Cov\left(\gamma_y\nu\varepsilon_y\right) + Cov(\gamma_r\varepsilon_r) \\
&= Cov\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\boldsymbol{\phi}\right) + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2 = \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2 \\
&= (\beta_{r1}, \beta_{r2})\begin{pmatrix} \beta_{e11} & \beta_{e12} \\ \beta_{e21} & \beta_{e21} \end{pmatrix}\begin{pmatrix} \sigma^2_\phi & 0 \\ 0 & \sigma^2_\phi \end{pmatrix}\begin{pmatrix} \beta_{e11} & \beta_{e12} \\ \beta_{e21} & \beta_{e21} \end{pmatrix}^T(\beta_{r1}, \beta_{r2})^T + \\
&\quad \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2
\end{aligned}$$

The relevant outputs can be written as $\tilde{x} = \bar{x} - \theta_1e_1 - \theta_2e_2 + \tilde{\varepsilon}_x$ and $\tilde{y} = \eta_1e_1 + \eta_2e_2 + \tilde{\varepsilon}_y$, where $\tilde{\varepsilon}_x \sim N\left(0, \sigma_x^2\right)$, and $\tilde{\varepsilon}_y \sim N\left(0, \sigma_y^2\right)$. We can write these as

$$\begin{aligned}
\tilde{x} &= \bar{x} - \boldsymbol{\theta}^T \mathbf{e} + \tilde{\varepsilon}_x \text{ and} \\
\tilde{y} &= \boldsymbol{\eta}^T \mathbf{e} + \tilde{\varepsilon}_y
\end{aligned}$$

From these, we have

$$\begin{aligned}
E[x] &= \bar{x} - \boldsymbol{\theta}^T \bar{\mathbf{e}} = \bar{x} - \boldsymbol{\theta}^T \boldsymbol{\alpha}_e - \boldsymbol{\theta}^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \\
Cov[x] &= \boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\theta} + \sigma_x^2 \\
Cov[x, r] &= Cov\left[\bar{x} - \boldsymbol{\theta}^T \mathbf{e} + \tilde{\varepsilon}_x, \alpha_r + \boldsymbol{\beta}_r^T \mathbf{e} + \gamma_y \nu \tilde{\varepsilon}_y + \gamma_r \tilde{\varepsilon}_r\right] \\
&= Cov\left[-\boldsymbol{\theta}^T \mathbf{e}, \boldsymbol{\beta}_r^T \mathbf{e}\right] = -\boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r \\
E[y] &= \boldsymbol{\eta}^T \boldsymbol{\alpha}_e + \boldsymbol{\eta}^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \\
Cov[y] &= \boldsymbol{\eta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\eta} + \sigma_y^2 \\
Cov[y, r] &= Cov\left[\boldsymbol{\eta}^T \mathbf{e} + \tilde{\varepsilon}_y, \alpha_r + \boldsymbol{\beta}_r^T \mathbf{e} + \gamma_y \nu \tilde{\varepsilon}_y + \gamma_r \tilde{\varepsilon}_r\right] \\
&= Cov\left[\boldsymbol{\eta}^T \mathbf{e}, \boldsymbol{\beta}_r^T \mathbf{e}\right] + Cov\left[\tilde{\varepsilon}_y, \gamma_y \nu \varepsilon_y\right] \\
&= \boldsymbol{\eta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y \nu \sigma_y^2
\end{aligned}$$

Taking the manager's strategy $\mathbf{e} = \boldsymbol{\alpha}_e + \boldsymbol{\beta}_e \boldsymbol{\phi}$ as given, we have the following joint distributions:

$$\begin{pmatrix} x \\ r \end{pmatrix} \sim N\left(\begin{pmatrix} \bar{x} - \boldsymbol{\theta}^T \boldsymbol{\alpha}_e - \boldsymbol{\theta}^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \\ \alpha_r + \boldsymbol{\beta}_r^T \boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \end{pmatrix}, \begin{pmatrix} \boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\theta} + \sigma_x^2 & -\boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r \\ -\boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r & \boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y^2 \nu^2 \sigma_y^2 + \gamma_r^2 \sigma_r^2 \end{pmatrix}\right)$$

and

$$\begin{pmatrix} y \\ r \end{pmatrix} \sim N\left(\begin{pmatrix} \boldsymbol{\eta}^T \boldsymbol{\alpha}_e + \boldsymbol{\eta}^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \\ \alpha_r + \boldsymbol{\beta}_r^T \boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \end{pmatrix}, \begin{pmatrix} \boldsymbol{\eta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\eta} + \sigma_y^2 & \boldsymbol{\eta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y \nu \sigma_y^2 \\ \boldsymbol{\eta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y \nu \sigma_y^2 & \boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y^2 \nu^2 \sigma_y^2 + \gamma_r^2 \sigma_r^2 \end{pmatrix}\right)$$

These yield the following conditional distributions:

$$\begin{aligned}
\mu_x &= E[x|r] = \bar{x} - \boldsymbol{\theta}^T\boldsymbol{\alpha}_e - \boldsymbol{\theta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} - \left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1} \\
&\quad \left(r - \left(\alpha_r + \boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}\right)\right) \\
&= \bar{x} - \boldsymbol{\theta}^T\boldsymbol{\alpha}_e - \boldsymbol{\theta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} + \left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1}\left(\alpha_r + \boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}\right) \\
&\quad - \left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1} r \\
\Sigma_x &= Var[x|r] = \boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\theta} + \sigma_x^2 - \\
&\quad \left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1}\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r\right) \\
\mu_y &= E[y|r] = \boldsymbol{\eta}^T\boldsymbol{\alpha}_e + \boldsymbol{\eta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} + \left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_y^2\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1} \\
&\quad \left(r - \left(\alpha_r + \boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}\right)\right) \\
&= \boldsymbol{\eta}^T\boldsymbol{\alpha}_e + \boldsymbol{\eta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\
&\quad - \left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_y^2\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1}\left(\alpha_r + \boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}\right) \\
&\quad + \left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_y^2\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1} r
\end{aligned}$$

For now, let $\mu_x = E[x|r]$, $\Sigma_x = Var[x|r]$, and $\mu_y = E[y|r]$.

Investor demands conditional on price are given by

$$\begin{aligned}
q_1 &= \frac{\mu_x - p}{\rho\Sigma_x} = \text{ and} \\
q_2 &= \frac{\mu_x + \mu_y - p}{\rho\Sigma_x}
\end{aligned}$$

Market clearing implies

$$\begin{aligned}
\lambda q_2 + (1-\lambda) q_1 &= 1 \\
\lambda\frac{\mu_x + \mu_y - p}{\rho\Sigma_x} + (1-\lambda)\frac{\mu_x - p}{\rho\Sigma_x} &= 1
\end{aligned}$$

Solving for p gives

$$p = \mu_x + \lambda\mu_y - \rho\Sigma_x$$

such that price is a linear function of the report, r.

Next, we derive the manager's reporting strategy conditional on the market pricing function and a chosen e:

$$\begin{aligned} r^* &\in \arg\max_r p - \frac{c_e}{2}\sum_{i\in\{1,2\}}(e_i-\phi_i)^2 - \frac{c_r}{2}\left(r-\zeta_1 e_1-\zeta_2 e_2-\nu\varepsilon_y-\varepsilon_r\right)^2 \\ &= \arg\max_r \mu_x + \lambda\mu_y - \rho\Sigma_x - \frac{c_r}{2}\left(r-\zeta_1 e_1-\zeta_2 e_2-\nu\varepsilon_y-\varepsilon_r\right)^2 \end{aligned}$$

Note from above that μ_x and μ_y are linear in r, while λ and $\rho\Sigma_x$ are independent of r. The FOC implies that :

$$\begin{aligned} 0 &= \frac{d\mu_x}{dr} + \lambda\frac{d\mu_y}{dr} - c_r\left(r^*-\zeta_1 e_1-\zeta_2 e_2-\nu\varepsilon_y-\varepsilon_r\right) \\ \Rightarrow r^* &= \frac{\frac{d\mu_x}{dr}+\lambda\frac{d\mu_y}{dr}}{c_r} + \zeta_1 e_1+\zeta_2 e_2+\nu\varepsilon_y+\varepsilon_r \\ &= \frac{-\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r\right)+\lambda\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y\nu\sigma_y^2\right)}{c_r\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y^2\nu^2\sigma_y^2+\gamma_r^2\sigma_r^2\right)} + \boldsymbol{\zeta}^T\mathbf{e}+\nu\varepsilon_y+\varepsilon_r \end{aligned}$$

and the SOC is satisfied for any $c_r > 0$.

Next we need to solve out for the assumed parameters in the manager's reporting function via matching coefficients. Recall that the linear assumption was $r = \alpha_r + \beta_{r1}e_1 + \beta_{r2}e_2 + \gamma_y\nu\varepsilon_y + \gamma_r\varepsilon_r = \alpha_r + \boldsymbol{\beta}_r^T\mathbf{e} + \gamma_y\nu\varepsilon_y + \gamma_r\varepsilon_r$, where $\boldsymbol{\beta}_r = (\beta_{r1}, \beta_{r2})^T$. Matching coefficients implies $\boldsymbol{\beta}_r = \boldsymbol{\zeta}$, $\gamma_r = \gamma_y = 1$, and $\alpha_r = \frac{-\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r\right)+\lambda\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\nu\sigma_y^2\right)}{c_r\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y^2\nu^2\sigma_y^2+\gamma_r^2\sigma_r^2\right)} = \frac{-\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}\right)+\lambda\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu\sigma_y^2\right)}{c_r\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)}$, so

$$r^\dagger = \frac{\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\lambda\nu\sigma_y^2}{c_r\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)} + \boldsymbol{\zeta}^T\mathbf{e}+\nu\varepsilon_y+\varepsilon_r$$

Plugging these into the pricing function gives

$$p = \mu_x^{\dagger} + \lambda \mu_y^{\dagger} - \rho \Sigma_x^{\dagger},$$

where

$$\begin{aligned}
\mu_x^{\dagger} &= \bar{x} - \boldsymbol{\theta}^T \boldsymbol{\alpha}_e - \boldsymbol{\theta}^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \\
&+ \frac{\left(\boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2\right)} \left(\frac{\left(\lambda \boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \lambda \nu \sigma_y^2}{c_r \left(\boldsymbol{\zeta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2\right)} + \boldsymbol{\zeta}^T \boldsymbol{\alpha}_e + \boldsymbol{\zeta}^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \right) \\
&- \frac{\left(\boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2\right)} r \\
\Sigma_x^{\dagger} &= \boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\theta} + \sigma_x^2 - \frac{\left(\boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta}\right)\left(\boldsymbol{\theta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2\right)} \\
\mu_y^{\dagger} &= \boldsymbol{\eta}^T \boldsymbol{\alpha}_e + \boldsymbol{\eta}^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \\
&- \frac{\left(\boldsymbol{\eta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \nu \sigma_y^2\right)}{\left(\boldsymbol{\zeta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2\right)} \left(\frac{\left(\lambda \boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \lambda \nu \sigma_y^2}{c_r \left(\boldsymbol{\zeta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2\right)} + \boldsymbol{\zeta}^T \boldsymbol{\alpha}_e + \boldsymbol{\zeta}^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \right) \\
&+ \frac{\left(\boldsymbol{\eta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \nu \sigma_y^2\right)}{\left(\boldsymbol{\zeta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2\right)} r
\end{aligned}$$

Finally, we solve for the manager's choice of $\mathbf{e}$.

$$\begin{aligned}
\mathbf{e}^* &\in \arg\max_{\mathbf{e}} E\left[\mu_x^{\dagger} + \lambda \mu_y^{\dagger} - \rho \Sigma_x^{\dagger} - \frac{c_e}{2} \sum_{i \in \{1,2\}} (e_i - \phi_i)^2 - \frac{c_r}{2} \left(r - \zeta_1 e_1 - \zeta_2 e_2 - \nu \varepsilon_y - \varepsilon_r\right)^2\right] \\
&= \arg\max_{\mathbf{e}} E\left[\mu_x^{\dagger} + \lambda \mu_y^{\dagger} - \rho \Sigma_x^{\dagger} - \frac{c_e}{2} \sum_{i \in \{1,2\}} (e_i - \phi_i)^2 - \frac{c_r}{2} \left(\frac{\left(\lambda \boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \lambda \nu \sigma_y^2}{c_r \left(\boldsymbol{\zeta}^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2\right)} \right)^2\right]
\end{aligned}$$

The FOC is given by

$$\begin{aligned}
0 &= \frac{d}{d\mathbf{e}}\frac{\partial}{\partial r}E\left[\mu_x^\dagger + \lambda\mu_y^\dagger - \rho\Sigma_x^\dagger\right] - \frac{c_e}{2}\frac{d}{d\mathbf{e}}\left(\mathbf{e}^* - \boldsymbol{\phi}\right)^T\left(\mathbf{e}^* - \boldsymbol{\phi}\right) \\
0 &= \frac{\partial}{\partial r}E\left[\mu_x^\dagger + \lambda\mu_y^\dagger\right]\frac{dr}{d\mathbf{e}} - c_e\left(\mathbf{e}^* - \boldsymbol{\phi}\right) \\
0 &= \left(-\frac{\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)} + \frac{\lambda\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu\sigma_y^2\right)}{\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right)\boldsymbol{\zeta} - c_e\left(\mathbf{e}^* - \boldsymbol{\phi}\right) \\
&\Rightarrow \mathbf{e}^* = \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta} + \boldsymbol{\phi}
\end{aligned}$$

which, from $\mathbf{e} = \boldsymbol{\alpha}_e + \boldsymbol{\beta}_e\boldsymbol{\phi}$ implies $\boldsymbol{\beta}_e$ is a 2×2 identity matrix and $\boldsymbol{\alpha}_e = \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta} = \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta}$. Therefore, in equilibrium, we have

$$p^* = \mu_x^* + \lambda\mu_y^* - \rho\Sigma_x^*,$$

where

$$\begin{aligned}
\mu_x^* &= \bar{x} - \boldsymbol{\theta}^T\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta} - \boldsymbol{\theta}^T\bar{\boldsymbol{\phi}} \\
&\quad + \frac{\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_r\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)} + \boldsymbol{\zeta}^T\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta} + \boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}\right) \\
&\quad - \frac{\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}r \\
\Sigma_x^* &= \boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} + \sigma_x^2 - \frac{\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)} \\
\mu_y^* &= \boldsymbol{\eta}^T\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta} + \boldsymbol{\eta}^T\bar{\boldsymbol{\phi}} \\
&\quad - \frac{\left(\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu\sigma_y^2\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_r\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)} + \boldsymbol{\zeta}^T\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta} + \boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}\right) \\
&\quad + \frac{\left(\boldsymbol{\eta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu\sigma_y^2\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}r
\end{aligned}$$

Gathering coefficients, we have p^*

$$= \bar{x} + \frac{1}{c_e}\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right)\left(\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) - \left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right)\boldsymbol{\zeta}^T\right)\boldsymbol{\zeta} \tag{42}$$

$$+\left(\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) - \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta}^T\right)\bar{\boldsymbol{\phi}} \tag{43}$$

$$-\frac{1}{c_r}\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right)^2 \tag{44}$$

$$-\rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} + \sigma_x^2 - \frac{\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right) \tag{45}$$

$$+\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right) r \tag{46}$$

The terms in p^* are as follows:

- In line (42), we have the expected cash flow, $\bar{x}$, and a term that goes to zero if either $c_e \to \infty$ or $\boldsymbol{\zeta}$ goes to zero.
- In line (43), we have a term that captures the contribution from the manager's bliss actions, which goes to zero if $\bar{\boldsymbol{\phi}} \to \mathbf{0}^2$, where, abusing notation slightly, $\mathbf{0}^2 = (0, 0)^T$.
- In line (44), we have a term that captures a loss from a lack of costs that discipline reporting. This term is negative, but goes to zero as $c_r \to \infty$, i.e., as misreporting relative to what the manager observed gets prohibitively costly.
- In line (45), we have the risk premium term that goes to zero as investors become risk-neutral in cash flows, i.e., $\rho \to 0$. Recall that investors are risk-neutral with respect to impact, y.
- In line (46), we have a term that captures the sensitivity of price to the report. This term goes to zero as reporting noise gets large, i.e., $\sigma_r^2 \to \infty$.

Let

$$\psi = \frac{dp^*}{dr} = \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2}.$$

Then we can write p^* as

$$
\begin{aligned}
p^* &= \bar{x} + \left(\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right) - \psi\boldsymbol{\zeta}^T\right)\left(\frac{\psi}{c_e}\boldsymbol{\zeta} + \bar{\boldsymbol{\phi}}\right) - \frac{\psi^2}{c_r} + \psi r^* \\
&\quad -\rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} + \sigma_x^2 - \frac{\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right) \\
&= \bar{x} + \left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\left(\frac{\psi}{c_e}\boldsymbol{\zeta} + \bar{\boldsymbol{\phi}}\right) + \psi\left(\boldsymbol{\zeta}^T\left(\boldsymbol{\phi} - \bar{\boldsymbol{\phi}}\right) + \nu\varepsilon_y + \varepsilon_r\right) \\
&\quad -\rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta} + \sigma_x^2 - \frac{\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\right)}{\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right)
\end{aligned}
$$

Rearranging slightly gives the expression for equilibrium price in (5).

Substituting from above, for efforts and the report, equilibrium expressions in (3) and (4) are:

$$
\begin{aligned}
\mathbf{e}^* &= \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_e\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\boldsymbol{\zeta} + \boldsymbol{\phi} = \frac{\psi}{c_e}\boldsymbol{\zeta} + \boldsymbol{\phi} \text{ and} \\
r^* &= \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta} + \lambda\nu\sigma_y^2}{c_r\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)} + \boldsymbol{\zeta}^T\mathbf{e} + \nu\varepsilon_y + \varepsilon_r = \frac{\psi}{c_r} + \boldsymbol{\zeta}^T\mathbf{e} + \nu\varepsilon_y + \varepsilon_r \\
&= \frac{\psi}{c_r} + \frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta} + \boldsymbol{\zeta}^T\boldsymbol{\phi} + \nu\varepsilon_y + \varepsilon_r.
\end{aligned}
$$

6.2 Correlated ε_x and ε_y

In this section/extension, suppose $Cov\left(\varepsilon_x, \varepsilon_y\right) = \Sigma_{xy} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}$, with determinant: $\sigma_x^2\sigma_y^2 - \sigma_{xy}^2 > 0$ such that Σ_{xy} is positive definite. Below, we derive a linear equilibrium in which the risky asset's price is a linear function of the report: $p = a + br$.

Proceeding via backward induction, we start with the competitive market for firm shares in period 2, conditional on the report, r, provided to the market. All investors observe r. We conjecture that the manager's choice of effort e_i is linear in ϕ, i.e., $e_i = \alpha_{ei} + \beta_{ei1}\phi_1 + \beta_{ei2}\phi_2$, $\mathbf{e} = \boldsymbol{\alpha}_e + \boldsymbol{\beta}_e\boldsymbol{\phi}$, where $\boldsymbol{\alpha}_e = \left(\alpha_{e1}, \alpha_{e2}\right)^T$ and $\boldsymbol{\beta}_e = \begin{pmatrix} \beta_{e11} & \beta_{e12} \\ \beta_{e21} & \beta_{e21} \end{pmatrix}$. Then the expected efforts are $\bar{\mathbf{e}} = \boldsymbol{\alpha}_e + \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}}$ and the ex ante covariance of efforts is $Cov(\mathbf{e}) = \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T$, with $\Sigma_\phi = Cov(\boldsymbol{\phi}) = \begin{pmatrix} \sigma_\phi^2 & 0 \\ 0 & \sigma_\phi^2 \end{pmatrix}$.

The report is linear in e, ε_y, ε_r, i.e., $r = \alpha_r + \beta_{r1} e_1 + \beta_{r2} e_2 + \gamma_y \nu \varepsilon_y + \gamma_r \varepsilon_r = \alpha_r + \boldsymbol{\beta}_r^T \mathbf{e} + \gamma_y \nu \varepsilon_y + \gamma_r \varepsilon_r$, where $\boldsymbol{\beta}_r = (\beta_{r1}, \beta_{r2})^T$. Substituting, we have

$$r = \alpha_r + \beta_{r1} (\alpha_{e1} + \beta_{e11} \phi_1 + \beta_{e12} \phi_2) + \beta_{r2} (\alpha_{e2} + \beta_{e21} \phi_1 + \beta_{e22} \phi_2) + \gamma_y \nu \varepsilon_y + \gamma_r \varepsilon_r$$

which has expectation and covariance:

$$\begin{aligned}
\bar{r} &= \alpha_r + \boldsymbol{\beta}_r^T \bar{\mathbf{e}} = \alpha_r + \boldsymbol{\beta}_r^T \boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \bar{\boldsymbol{\phi}} \\
Cov(r) &= Cov\left(\boldsymbol{\beta}_r^T \mathbf{e} + \gamma_y \nu \varepsilon_y + \gamma_r \varepsilon_r\right) = Cov\left(\boldsymbol{\beta}_r^T \mathbf{e}\right) + Cov\left(\gamma_y \nu \varepsilon_y\right) + Cov(\gamma_r \varepsilon_r) \\
&= Cov\left(\boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \boldsymbol{\phi}\right) + \gamma_y^2 \nu^2 \sigma_y^2 + \gamma_r^2 \sigma_r^2 = \boldsymbol{\beta}_r^T \boldsymbol{\beta}_e \Sigma_\phi \boldsymbol{\beta}_e^T \boldsymbol{\beta}_r + \gamma_y^2 \nu^2 \sigma_y^2 + \gamma_r^2 \sigma_r^2 \\
&= (\beta_{r1}, \beta_{r2}) \begin{pmatrix} \beta_{e11} & \beta_{e12} \\ \beta_{e21} & \beta_{e21} \end{pmatrix} \begin{pmatrix} \sigma_\phi^2 & 0 \\ 0 & \sigma_\phi^2 \end{pmatrix} \begin{pmatrix} \beta_{e11} & \beta_{e12} \\ \beta_{e21} & \beta_{e21} \end{pmatrix}^T (\beta_{r1}, \beta_{r2})^T + \gamma_y^2 \nu^2 \sigma_y^2 + \gamma_r^2 \sigma_r^2
\end{aligned}$$

The relevant outputs can be written as $\tilde{x} = \bar{x} - \theta_1 e_1 - \theta_2 e_2 + \tilde{\varepsilon}_x$ and $\tilde{y} = \eta_1 e_1 + \eta_2 e_2 + \tilde{\varepsilon}_y$, where $\tilde{\varepsilon}_x \sim N(0, \sigma_x^2)$, and $\tilde{\varepsilon}_y \sim N\left(0, \sigma_y^2\right)$. We can write these as

$$\begin{aligned}
\tilde{x} &= \bar{x} - \boldsymbol{\theta}^T \mathbf{e} + \tilde{\varepsilon}_x \text{ and} \\
\tilde{y} &= \boldsymbol{\eta}^T \mathbf{e} + \tilde{\varepsilon}_y
\end{aligned}$$

From these, we have

$$
\begin{aligned}
E\left[x\right] &= \bar{x}-\boldsymbol{\theta}^T\bar{\mathbf{e}} = \bar{x}-\boldsymbol{\theta}^T\boldsymbol{\alpha}_e-\boldsymbol{\theta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\
Cov\left[x\right] &= \boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\theta}+\sigma_x^2 \\
Cov\left[x,r\right] &= Cov\left[\bar{x}-\boldsymbol{\theta}^T\mathbf{e}+\tilde{\varepsilon}_x, \alpha_r+\boldsymbol{\beta}_r^T\mathbf{e}+\gamma_y\nu\tilde{\varepsilon}_y+\gamma_r\tilde{\varepsilon}_r\right] \\
&= Cov\left[-\boldsymbol{\theta}^T\mathbf{e}, \boldsymbol{\beta}_r^T\mathbf{e}\right]+Cov\left[\tilde{\varepsilon}_x, \gamma_y\nu\tilde{\varepsilon}_y\right] = -\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y\nu\sigma_{xy} \\
E\left[y\right] &= \boldsymbol{\eta}^T\boldsymbol{\alpha}_e+\boldsymbol{\eta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\
Cov\left[y\right] &= \boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\eta}+\sigma_y^2 \\
Cov\left[y,r\right] &= Cov\left[\boldsymbol{\eta}^T\mathbf{e}+\tilde{\varepsilon}_y, \alpha_r+\boldsymbol{\beta}_r^T\mathbf{e}+\gamma_y\nu\tilde{\varepsilon}_y+\gamma_r\tilde{\varepsilon}_r\right] \\
&= Cov\left[\boldsymbol{\eta}^T\mathbf{e}, \boldsymbol{\beta}_r^T\mathbf{e}\right]+Cov\left[\tilde{\varepsilon}_y, \gamma_y\nu\varepsilon_y\right] = \boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y\nu\sigma_y^2
\end{aligned}
$$

Taking the manager's strategy $\mathbf{e}=\boldsymbol{\alpha}_e+\boldsymbol{\beta}_e\boldsymbol{\phi}$ as given, we have the following joint distributions:

$$
\begin{pmatrix} x \\ r \end{pmatrix} \sim N\left(\begin{pmatrix} \bar{x}-\boldsymbol{\theta}^T\boldsymbol{\alpha}_e-\boldsymbol{\theta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\ \alpha_r+\boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e+\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \end{pmatrix}, \begin{pmatrix} \boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\theta}+\sigma_x^2 & -\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y\nu\sigma_{xy} \\ -\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y\nu\sigma_{xy} & \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y^2\nu^2\sigma_y^2+\gamma_r^2\sigma_r^2 \end{pmatrix}\right)
$$

and

$$
\begin{pmatrix} y \\ r \end{pmatrix} \sim N\left(\begin{pmatrix} \boldsymbol{\eta}^T\boldsymbol{\alpha}_e+\boldsymbol{\eta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\ \alpha_r+\boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e+\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \end{pmatrix}, \begin{pmatrix} \boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\eta}+\sigma_y^2 & \boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y\nu\sigma_y^2 \\ \boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y\nu\sigma_y^2 & \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r+\gamma_y^2\nu^2\sigma_y^2+\gamma_r^2\sigma_r^2 \end{pmatrix}\right)
$$

These yield the following conditional distributions:

$$\begin{aligned}
\mu_x &= E[x|r] = \bar{x} - \boldsymbol{\theta}^T\boldsymbol{\alpha}_e - \boldsymbol{\theta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\
&\quad - \left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_{xy}\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1}\left(r - \left(\alpha_r + \boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}\right)\right) \\
&= \bar{x} - \boldsymbol{\theta}^T\boldsymbol{\alpha}_e - \boldsymbol{\theta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\
&\quad + \left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_{xy}\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1}\left(\alpha_r + \boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}\right) \\
&\quad - \left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_{xy}\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1} r \\
\Sigma_x &= Var[x|r] = \boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\theta} + \sigma_x^2 \\
&\quad - \left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_{xy}\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1}\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_{xy}\right) \\
\mu_y &= E[y|r] = \boldsymbol{\eta}^T\boldsymbol{\alpha}_e + \boldsymbol{\eta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\
&\quad + \left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_y^2\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1}\left(r - \left(\alpha_r + \boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}\right)\right) \\
&= \boldsymbol{\eta}^T\boldsymbol{\alpha}_e + \boldsymbol{\eta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\
&\quad - \left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_y^2\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1}\left(\alpha_r + \boldsymbol{\beta}_r^T\boldsymbol{\alpha}_e + \boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}\right) \\
&\quad + \left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_y^2\right)\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)^{-1} r
\end{aligned}$$

For now, let $\mu_x = E[x|r]$, $\Sigma_x = Var[x|r]$, and $\mu_y = E[y|r]$.

Investor demands conditional on price are given by

$$\begin{aligned}
q_1 &= \frac{\mu_x - p}{\rho\Sigma_x} = \text{ and} \\
q_2 &= \frac{\mu_x + \mu_y - p}{\rho\Sigma_x}
\end{aligned}$$

Market clearing implies

$$\begin{aligned}
\lambda q_2 + (1-\lambda) q_1 &= 1 \\
\lambda\frac{\mu_x + \mu_y - p}{\rho\Sigma_x} + (1-\lambda)\frac{\mu_x - p}{\rho\Sigma_x} &= 1
\end{aligned}$$

Solving for p gives

$$p = \mu_x + \lambda\mu_y - \rho\Sigma_x$$

such that price is a linear function of the report, r.

Next, we derive the manager's reporting strategy conditional on the market pricing function and a chosen e:

$$\begin{aligned} r^* &\in \arg\max_r p - \frac{c_e}{2}\sum_{i\in\{1,2\}}(e_i - \phi_i)^2 - \frac{c_r}{2}(r - \zeta_1 e_1 - \zeta_2 e_2 - \nu\varepsilon_y - \varepsilon_r)^2 \\ &= \arg\max_r \mu_x + \lambda\mu_y - \rho\Sigma_x - \frac{c_r}{2}(r - \zeta_1 e_1 - \zeta_2 e_2 - \nu\varepsilon_y - \varepsilon_r)^2 \end{aligned}$$

Note from above that μ_x and μ_y are linear in r, while λ and $\rho\Sigma_x$ are independent of r. The FOC implies that :

$$\begin{aligned} 0 &= \frac{d\mu_x}{dr} + \lambda\frac{d\mu_y}{dr} - c_r(r^* - \zeta_1 e_1 - \zeta_2 e_2 - \nu\varepsilon_y - \varepsilon_r) \\ \Rightarrow r^* &= \frac{\frac{d\mu_x}{dr} + \lambda\frac{d\mu_y}{dr}}{c_r} + \zeta_1 e_1 + \zeta_2 e_2 + \nu\varepsilon_y + \varepsilon_r \\ &= \frac{-\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r - \gamma_y\nu\sigma_{xy}\right) + \lambda\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y\nu\sigma_y^2\right)}{c_r\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)} + \boldsymbol{\zeta}^T\mathbf{e} + \nu\varepsilon_y + \varepsilon_r \end{aligned}$$

and the SOC is satisfied for any $c_r > 0$.

Next we need to solve out for the assumed parameters in the manager's reporting function via matching coefficients. Recall that the linear assumption was $r = \alpha_r + \beta_{r1}e_1 + \beta_{r2}e_2 + \gamma_y\nu\varepsilon_y + \gamma_r\varepsilon_r = \alpha_r + \boldsymbol{\beta}_r^T\mathbf{e} + \gamma_y\nu\varepsilon_y + \gamma_r\varepsilon_r$, where $\boldsymbol{\beta}_r = (\beta_{r1}, \beta_{r2})^T$. Matching coefficients implies $\boldsymbol{\beta}_r = \boldsymbol{\zeta}$, $\gamma_r = \gamma_y = 1$, and $\alpha_r = \frac{-\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r - \nu\sigma_{xy}\right) + \lambda\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \nu\sigma_y^2\right)}{c_r\left(\boldsymbol{\beta}_r^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\beta}_r + \gamma_y^2\nu^2\sigma_y^2 + \gamma_r^2\sigma_r^2\right)} = \frac{-\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} - \nu\sigma_{xy}\right) + \lambda\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu\sigma_y^2\right)}{c_r\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}$, so

$$r^\dagger = \frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu\left(\lambda\sigma_y^2 + \sigma_{xy}\right)}{c_r\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)} + \boldsymbol{\zeta}^T\mathbf{e} + \nu\varepsilon_y + \varepsilon_r$$

Plugging these into the pricing function gives

$$p = \mu_x^\dagger + \lambda\mu_y^\dagger - \rho\Sigma_x^\dagger,$$

where

$$\begin{aligned}
\mu_x^\dagger &= \bar{x} - \boldsymbol{\theta}^T\boldsymbol{\alpha}_e - \boldsymbol{\theta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\
&\quad + \frac{\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu\sigma_{xy}\right)}{\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \lambda\nu\sigma_y^2 + \nu\sigma_{xy}}{c_r\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)} + \boldsymbol{\zeta}^T\boldsymbol{\alpha}_e + \boldsymbol{\zeta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}\right) \\
&\quad - \frac{\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} - \nu\sigma_{xy}\right)}{\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}r \\
\Sigma_x^\dagger &= \boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\theta} + \sigma_x^2 - \frac{\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu\sigma_{xy}\right)\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu\sigma_{xy}\right)}{\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)} \\
\mu_y^\dagger &= \boldsymbol{\eta}^T\boldsymbol{\alpha}_e + \boldsymbol{\eta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}} \\
&\quad - \frac{\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu\sigma_y^2\right)}{\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \lambda\nu\sigma_y^2 + \nu\sigma_{xy}}{c_r\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)} + \boldsymbol{\zeta}^T\boldsymbol{\alpha}_e + \boldsymbol{\zeta}^T\boldsymbol{\beta}_e\bar{\boldsymbol{\phi}}\right) \\
&\quad + \frac{\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu\sigma_y^2\right)}{\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}r
\end{aligned}$$

Finally, we solve for the manager's choice of $\mathbf{e}$.

$$\begin{aligned}
\mathbf{e}^* &\in \arg\max_{\mathbf{e}} E\left[\mu_x^\dagger + \lambda\mu_y^\dagger - \rho\Sigma_x^\dagger - \frac{c_e}{2}\sum_{i\in\{1,2\}}\left(e_i - \phi_i\right)^2 - \frac{c_r}{2}\left(r - \zeta_1 e_1 - \zeta_2 e_2 - \nu\varepsilon_y - \varepsilon_r\right)^2\right] \\
&= \arg\max_{\mathbf{e}} E\left[\mu_x^\dagger + \lambda\mu_y^\dagger - \rho\Sigma_x^\dagger - \frac{c_e}{2}\sum_{i\in\{1,2\}}\left(e_i - \phi_i\right)^2 - \frac{c_r}{2}\left(\frac{\left(\lambda\boldsymbol{\eta}^T - \boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \lambda\nu\sigma_y^2 + \nu\sigma_{xy}}{c_r\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta} + \nu^2\sigma_y^2 + \sigma_r^2\right)}\right)^2\right]
\end{aligned}$$

The FOC is given by

$$
\begin{aligned}
0 &= \frac{d}{d\mathbf{e}}\frac{\partial}{\partial r}E\left[\mu_x^\dagger+\lambda\mu_y^\dagger-\rho\Sigma_x^\dagger\right]-\frac{c_e}{2}\frac{d}{d\mathbf{e}}\left(\mathbf{e}^*-\boldsymbol{\phi}\right)^T\left(\mathbf{e}^*-\boldsymbol{\phi}\right)\\
0 &= \frac{\partial}{\partial r}E\left[\mu_x^\dagger+\lambda\mu_y^\dagger\right]\frac{dr}{d\mathbf{e}}-c_e\left(\mathbf{e}^*-\boldsymbol{\phi}\right)\\
0 &= \left(-\frac{\left(\boldsymbol{\theta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}\right)-\nu\sigma_{xy}}{\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)}+\frac{\lambda\left(\boldsymbol{\eta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu\sigma_y^2\right)}{\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)}\right)\boldsymbol{\zeta}-c_e\left(\mathbf{e}^*-\boldsymbol{\phi}\right)\\
&\Rightarrow\ \mathbf{e}^*=\frac{\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\lambda\nu\sigma_y^2+\nu\sigma_{xy}}{c_e\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)}\boldsymbol{\zeta}+\boldsymbol{\phi}
\end{aligned}
$$

which, from $\mathbf{e}=\boldsymbol{\alpha}_e+\boldsymbol{\beta}_e\boldsymbol{\phi}$ implies $\boldsymbol{\beta}_e$ is a 2×2 identity matrix and

$$
\begin{aligned}
\boldsymbol{\alpha}_e &= \frac{\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\lambda\nu\sigma_y^2+\nu\sigma_{xy}}{c_e\left(\boldsymbol{\zeta}^T\boldsymbol{\beta}_e\Sigma_\phi\boldsymbol{\beta}_e^T\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)}\boldsymbol{\zeta}\\
&= \frac{\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta}+\lambda\nu\sigma_y^2+\nu\sigma_{xy}}{c_e\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2\right)}\boldsymbol{\zeta}.
\end{aligned}
$$

Let

$$
\psi=\frac{dp^*}{dr}=\frac{\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta}+\lambda\nu\sigma_y^2+\nu\sigma_{xy}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}.
$$

Then, in equilibrium, we have

$$
\begin{aligned}
p^* &= \bar{x}+\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\left(\frac{\psi}{c_e}\boldsymbol{\zeta}+\bar{\boldsymbol{\phi}}\right) && (47)\\
&\quad+\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\Sigma_\phi\boldsymbol{\zeta}\frac{1}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}r && (48)\\
&\quad+\frac{\lambda\nu\sigma_y^2}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}_r+\nu^2\sigma_y^2+\sigma_r^2}r+\frac{\nu\sigma_{xy}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}_r+\nu^2\sigma_y^2+\sigma_r^2}r+ && (49)\\
&\quad-\psi\left(\frac{\psi}{c_r}+\frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta}+\boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}\right) && (50)\\
&\quad-\rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}+\sigma_x^2-\frac{\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu\sigma_{xy}\right)\left(\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}+\nu\sigma_{xy}\right)}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}\right) && (51)
\end{aligned}
$$

The first line, (47), shows the expected cash flows, $\bar{x}$, plus the effect of expected effort, $E\left[\mathbf{e}^*\right]=\frac{\psi}{c_e}\boldsymbol{\zeta}$, on the values-weighted output, where the values-weighting is given by

$\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)$. The second line in (48) is a term that captures the use of the report to learn about efforts, which is equivalent to using the report to learn about the random bliss actions, $\boldsymbol{\phi}$, given $\mathbf{e}^*=\frac{\psi}{c_e}\boldsymbol{\zeta}+\boldsymbol{\phi}$. Because the effect comes through efforts, the learning is weighted by the values-weights, $\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)$. The term in the third line, (49), captures the use of r to learn about $\tilde{\varepsilon}_y$and $\tilde{\varepsilon}_x$, which is relevant to investors but outside of the manager's control. The fourth line, (50), captures the adjustments for the expected report, $E\left[r^*\right]=\frac{\psi}{c_r}+\frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta}+\boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}$, inherent in using the report to learn about $\mathbf{e}^*$ and r^* in lines (48) and (49), respectively. The last line captures the risk premium, which is based on the ex ante expected variance of cash flows conditional on equilibrium effort, $\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}+\sigma_x^2$, net of the amount learned about the $\boldsymbol{\phi}$-driven randomness in effort, $\frac{\Sigma_\phi\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}_r+\nu^2\sigma_y^2+\sigma_r^2}$, scaled quadratically by the effects of efforts on cash flows, $\boldsymbol{\theta}$, and the uncontrolled randomness in cash flows, σ_x^2,. We can set $E\left[\varepsilon_r\right]=\frac{\psi}{c_r}+\frac{\psi}{c_e}\boldsymbol{\zeta}^T\boldsymbol{\zeta}+\boldsymbol{\zeta}^T\bar{\boldsymbol{\phi}}$, which implies $E\left[r^*\right]=0$ to get rid of the term in line (50). Technically, we can set $\bar{\boldsymbol{\phi}}$ such that $E\left[\mathbf{e}^*\right]=0$, but this is a knife-edge case that limits our ability to talk about the effects of expected managerial preferences.

Expected price can straightforwardly be written as

$$\begin{aligned} E\left[p^*\right] &= \bar{x}+\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\left(\frac{\psi}{c_e}\boldsymbol{\zeta}+\bar{\boldsymbol{\phi}}\right)-\rho\left(\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}+\sigma_x^2-\frac{\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}\right) \\ &= \bar{x}+\left(\lambda\boldsymbol{\eta}^T-\boldsymbol{\theta}^T\right)\left(\frac{\psi}{c_e}\boldsymbol{\zeta}+\bar{\boldsymbol{\phi}}\right)-\rho\left(\frac{\boldsymbol{\theta}^T\Sigma_\phi\left(\boldsymbol{\theta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}-\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}\right)+\left(\nu^2\sigma_y^2+\sigma_r^2\right)\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}+\sigma_x^2\right) \end{aligned}$$

The sensitivity of p^* to r, ψ, affect expected price only through its effect on expected effort. However, the sensitivity of the report to effort, $\boldsymbol{\zeta}$, also affects the degree to which investors can use r to learn about $\mathbf{e}^*$ and, consequently, reduce the posterior variance of cash flows. Rewriting the risk premium as

$$\frac{\boldsymbol{\theta}^T\Sigma_\phi\left(\boldsymbol{\theta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}-\boldsymbol{\zeta}\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\theta}\right)+\left(\nu^2\sigma_y^2+\sigma_r^2\right)\boldsymbol{\theta}^T\Sigma_\phi\boldsymbol{\theta}}{\boldsymbol{\zeta}^T\Sigma_\phi\boldsymbol{\zeta}+\nu^2\sigma_y^2+\sigma_r^2}+\sigma_x^2$$

shows how the first term in the numerator goes to zero as the report becomes congruent with cash flows, i.e., as $\boldsymbol{\zeta} \to \boldsymbol{\theta}$, we have

$$\frac{\boldsymbol{\theta}^T \Sigma_\phi \left(\boldsymbol{\theta}\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} - \boldsymbol{\zeta}\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\theta}\right) + \left(\nu^2 \sigma_y^2 + \sigma_r^2\right) \boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta}}{\boldsymbol{\zeta}^T \Sigma_\phi \boldsymbol{\zeta} + \nu^2 \sigma_y^2 + \sigma_r^2} + \sigma_x^2 \to \frac{\left(\nu^2 \sigma_y^2 + \sigma_r^2\right) \boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta}}{\boldsymbol{\theta}^T \Sigma_\phi \boldsymbol{\theta} + \nu^2 \sigma_y^2 + \sigma_r^2} + \sigma_x^2$$

This does not necessarily minimize the risk premium, which in turn goes to $\left(\nu^2 \sigma_y^2 + \sigma_r^2\right) + \sigma_x^2$ if $\boldsymbol{\zeta} \to \boldsymbol{\theta} \to \infty$.

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