Household Inventory, Temporary Sales, and Price Indices

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Abstract

Large-scale household inventory buildups occurred in Japan five times over the last decade, including those triggered by the Tohoku earthquake in 2011, the spread of COVID-19 infections in 2020, and the consumption tax hikes in 2014 and 2019. Each of these episodes was accompanied by considerable swings in GDP, suggesting that fluctuations in household inventories are one of the sources of macroeconomic fluctuations in Japan. In this paper, we focus on changes in household inventories associated with temporary sales and propose a methodology to estimate changes in household inventories at the product level using retail scanner data. We construct a simple model on household stockpiling and derive equations for the relationships between the quantity consumed and the quantity purchased and between consumption and purchase prices. We then use these relationships to make inferences about quantities consumed, consumption prices, and inventories. Next, we test the validity of this methodology by calculating price indices and check whether the intertemporal substitution bias we find in the price indices is consistent with theoretical predictions. We empirically show that there exists a large bias in the Laspeyres, Paasche, and Törnqvist price indices, which is smaller at lower frequencies but non-trivial even at a quarterly frequency and that intertemporal substitution bias disappears for a particular type of price index if we switch from purchase-based data to consumption-based data.

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1 Introduction

In the first week of March 2020, when the first wave of COVID-19 infections hit Japan, supermarket sales went up more than 20% over the previous year. This was due to hoarding by consumers stemming from an increase in uncertainty regarding the spread of the virus. Similar hoarding occurred during the third wave, which struck Japan in October 2020. Such hoarding has occurred not only during the COVID-19 pandemic but also after the Tohoku earthquake in March 2011 and the subsequent nuclear power plant accident in Fukushima, when residents of Tokyo and other areas that were spared serious damage went on a buying spree for food and other necessities. Consumer hoarding also occurred due to policy shocks: when the consumption tax rate was raised in April 2014 and in October 2019, people hoarded large amounts of goods just before the tax rate was raised, and a prolonged consumption slump occurred thereafter. Each of these episodes was accompanied by considerable swings in GDP, suggesting that fluctuations in household inventories are one of the sources of macroeconomic fluctuations in Japan.

There is a considerable literature on household inventories, addressing a variety of aspects. For instance, Boizot, Robin, and Visser (2001), Griffith et al. (2009), Hendel and Nevo (2006a, 2006b), and Kano (2018) investigate intertemporal patterns of household purchases and consumption from a theoretical and/or empirical perspective. Hendel and Nevo (2013) focus on heterogeneity in households' ability to store goods and show that temporary sales are driven by price discrimination. More recently, the responses of household inventories to various shocks have been studied by Baker, Johnson, and Kueng (forthcoming), Cashin and Unayama (2021), and Hansman et al. (2020).[1] Meanwhile, Baker, Johnson, and Kueng (2020) regard inventories as part of households' non-financial wealth, and Coibion, Gorodnichenko, and Koustas (forthcoming) focus on the increase in US households' stockpiling due to a lower frequency of shopping trips since 1980.

In this study, we focus on changes in household inventories associated with temporary sales. While the five episodes we outlined above are noteworthy for consumer hoarding, such hoarding occurs on a daily basis, especially during temporary sales, which has many important consequences. The objective of this study is twofold. First, we propose a methodology to estimate changes in household inventories at the product level using retail scanner data. Suppose that a temporary sale takes place unexpectedly: a product is sold at the regular price in period 0,

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[1]Baker, Johnson, and Kueng (forthcoming) examine consumers' responses to changes in the sales tax rate in the United States, while Cashin and Unayama (2021) compare the responses of expenditure on durable and nondurable goods as well as storable and nonstorable goods to Japan's consumption tax hike in 1997. Hansman et al. (2020) examine stockpiling by households during disasters such as the COVID-19 pandemic, focusing on the implications of sticky store prices for stockpiling.

at the sale price in period 1, and again at the regular price in period 2. A typical response to this temporary sale would be that the quantity purchased increases in period 1 and exceeds the quantity consumed, resulting in an increase in household inventories. The quantity purchased falls below the quantity consumed in period 2 and subsequent periods, so that inventories are gradually drawn down in these periods. Importantly, the price consumers face in each period also deviates from retailers' selling price. In this process, the quantity and price of purchases can be observed with scanner data, but the quantity consumed, the consumption price, and changes in inventory are not observable. We construct a simple model on household stockpiling and derive equations for the relationships between the quantity consumed and the quantity purchased and between consumption and purchase prices. We then use these relationships to make inference about quantities consumed, consumption prices, and inventories.

The second objective is to empirically test the validity of the methodology described above, focusing on the intertemporal substitution bias in consumer price indices, which has been extensively discussed in the field of price index theory. Existing studies have regarded the intertemporal substitution bias as stemming from changes in household inventories associated with temporary sales (see, for example, Nakamura, Nakamura, and Nakamura 2011). Taking the Törnqvist index, which is known to be a second-order approximation of the cost of living index (COLI), as an example, the weight attached to the price decline from period 0 to period 1 is the sum of the shares of the amount purchased of this product in period 0 and period 1 divided by 2, while the weight attached to the price increase from period 1 to period 2 is the sum of the shares of the amount purchased in the corresponding periods divided by 2. Since the amount purchased in period 2 tends to be smaller than the amount purchased in period 0, the latter exceeds the former. Therefore, the inflation rate from period 0 to period 2 takes a negative value, even though the price has returned to its original level. To eliminate this bias, it is necessary to replace the purchase quantity with the consumption quantity and the purchase price with the consumption price. In this study, we test the validity of our methodology by calculating consumption-based (rather than purchase-based) price indices using the consumption quantity and consumption price estimated based on the model and check whether the intertemporal substitution bias we find in the price indices is consistent with theoretical predictions.

The contributions of this study are as follows. First, we construct a simple partial equilibrium model on stockpiling by consumers, in which goods prices switch between regular and sale prices following a Markov process. Our model is close to that by Hendel and Nevo (2006a) but differs from it in that our model is a quasi-dynamic one. Specifically, we assume that a household consists of a household producer and a household consumer, and that household inventory is held only by the household producer. The household producer is like an inventory trader.

On the other hand, the household consumer holds no inventory: all he/she does is to purchase something today to consume today. While propositions in Hendel and Nevo (2006a) continue to hold in the quasi-dynamic model, this setting allows us to divide household decisions into two parts, making it easier to distinguish between purchasing and consumption. That is, the household consumer has nothing to do with intertemporal optimization, and the optimization problem he/she faces is a static one. All decisions related to intertemporal changes in inventory are made by the household producer. We refer to the price associated with transactions between the household consumer and the household producer as the "consumption price," while the price associated with transactions between the household producer and firms is referred to as the "purchase price." Similarly, the quantity associated with transactions between the household consumer and the household producer is referred to as the "consumption quantity" in our analysis, while the quantity associated with transactions between the household producer and firms is referred to as "purchase quantity." Optimization by the household producer and the household consumer determines the relationships between the consumption price and quantity on the one hand and the purchase price and quantity on the other. Using these relationships, we estimate the price and quantity for consumption.

The second contribution of our study is that we apply our methodology for estimating consumption prices and quantities to the issue of intertemporal substitution bias in price indices stemming from temporary sales.[2] Using our model, we first show theoretically that consumer stockpiling driven by temporary sales results in intertemporal substitution bias and then provide empirical evidence supporting this prediction. Specifically, based on retail scanner data, we show that (1) quantities purchased before, during, and after a temporary sale differ substantially, as predicted by the model; (2) there exists a surprisingly large bias in the Laspeyres, Paasche, and Törnqvist price indices, and the sign and magnitude of this bias are consistent with the predictions of the model. Further, the bias is smaller at lower frequencies but non-trivial even at a quarterly frequency; (3) the intertemporal substitution bias is closely correlated with the frequency and magnitude of sales at the product category level, which is again consistent with the model.

We then replace purchase prices and quantities with the consumption prices and quantities

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[2]Intertemporal substitution bias is often mentioned as one reason for chain drift, as the bias arises typically in chained price indices. Notable empirical studies on chain drift include Frisch (1936), Reinsdorf (1999), Feenstra and Shapiro (2003), de Haan (2008), de Haan and van der Grient (2011), Ivancic, Diewert, and Fox (2011), Nakamura, Nakamura, and Nakamura (2011), de Haan and Krsinich (2014), Goolsbee and Klenow (2018), and Diewert and Fox (2020). To the best of our knowledge, there are no empirical studies investigating whether chain drift is generated by stockpiling by consumers during sales, although Nakamura, Nakamura, and Nakamura (2011) investigate the effect of temporary sales on the bias by comparing price changes when temporary sales are included and when they are excluded.

estimated using our methodology and construct price indices. We empirically show that the intertemporal substitution bias of these price indices is much smaller than in price indices using purchase-based data; however, non-trivial bias still remains. We show that this is also consistent with our model, which predicts that while the consumption price declines at the start of a sale, closely following the purchase price, after the sale ends, the consumption price returns to the regular price level only gradually, even though the purchase price returns to it immediately. Due to this asymmetric evolution of the consumption price, the consumption-based Törnqvist price index deviates downward from the COLI. However, using the model, we show theoretically that intertemporal substitution bias is completely eliminated for the consumption-weighted order r superlative index and confirm this empirically.

The third contribution of our study is that we apply our methodology to a range of other issues, such as the estimation of the elasticity of substitution, the examination of the response of consumer stockpiling to consumption tax hikes, and the analysis of the extent to which stockpiling is affected by business cycle fluctuations. For instance, Hendel and Nevo (2004) show that the response of purchase quantities to a price reduction consists of the consumption effect (i.e., consumers consume more as the good is cheap now) and the stockpiling effect (i.e., consumers stockpile as the good is cheap now), suggesting that estimation based on purchase-based data leads to an upward bias. We empirically compare estimates of the elasticity of substitution based on purchase-based data and those based on consumption-based data and find that the elasticity of substitution is indeed overestimated when it is estimated using purchase-based data.

As for consumers' response to consumption tax hikes, we show that Japan's consumption tax hike in April 2014 led households to increase stockpiling one month before the tax hike, and that stockpiling tended to be more pronounced for goods with longer storability.[3] Finally, regarding the extent to which stockpiling is affected by the business cycle, we empirically find that the extent of stockpiling during a sale depends on labor market conditions as well as financial conditions (which we represent by the interest rate level). Specifically, we find that (1) the extent of stockpiling tends to be smaller when hours worked are longer, implying that, due to longer working hours, consumers have less time to spend on searching for temporary sales, and (2) consumers tend to stockpile less when financial conditions are tighter (interest rates higher), presumably due to the higher costs of financing stockpiling.[4]

Our study is related to three strands of research. The first strand consists of research

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[3]Another study using consumer stockpiling in response to a consumption tax hike in Japan (the hike in April 1997) to estimate the intertemporal elasticity of substitution is that by Cashin and Unayama (2016, 2021).

[4]These results are in line with studies by Klenow and Willis (2007), Sudo et al. (2018), and Kryvtsov and Vincent (2020) showing that there is a statistically significant link between temporary sales and business cycles. Similarly, Sheremirov (2020) shows that temporary sales account for price dispersion.

on household inventories of storable goods. Boizot, Robin, and Visser (2001), Griffith et al. (2009), Hendel and Nevo (2006a), and Kano (2018) empirically document patterns of consumer purchases and consumption. In terms of the theoretical approach, our model is closest to that developed by Hendel and Nevo (2006a) but differs from it in that our model is a quasi-dynamic one with producer households, who hold and sell inventories, and consumer households, who do not hold inventories. The second strand of literature is that on methodologies for eliminating chain drift. Ivancic, Diewert, and Fox (2011), de Haan and van der Grient (2011), and de Haan and Krsinich (2014) propose using the GEKS index (originally proposed by Gini, Elteto, Koves, and Szulc). The third strand of literatur is that on COLIs. Feenstra and Shapiro (2003) and Chevalier and Kashyap (2019) propose a proxy for COLIs to deal with sales in an environment where consumers' optimization problem is static. Osborne (2018) computes a dynamic COLI by calculating the sequence of taxes on or subsidies to households that would keep their period utility constant over time. While Osborne's dynamic COLI is an important contribution to the literature, a dynamic COLI is complex and difficult to calculate and, in addition, hard to interpret.

The remainder of this study is organized as follows. Section 2 develops a quasi-dynamic model on household stockpiling behavior and presents stylized facts on intertemporal substitution bias in chained price indices, on household inventory, and on the relationship between the two. Section 3 discusses our approach to calculating consumption and consumption prices and shows the empirical results, while Section 4 discusses the application of our methodology to other areas involving consumer stockpiling. Section 5 concludes.

2   Model and Stylized Facts

2.1   Quasi-Dynamic Model

In this subsection, we construct a simple partial equilibrium model of household inventory. We assume that product $k \in K_t$ is storable and, for simplicity, that it does not depreciate. Time t is a discrete day.

The novelty of our model is that we assume that households comprise consumers and household producers and distinguish between the two. That is, household producers are a special type of household member that has technology to hold inventories, although there is a cost associated. The household producer provides inventory services consisting of purchasing storable goods from manufacturers (the quantity purchased denoted by x_t^k), holding inventory (denoted by i_t^k), and selling the goods to consumers (the quantity sold denoted by y_t^k). Market entry is free, so that the expected firm value is zero. In this respect, there is no loss of consumer surplus. On the other hand, consumers in a narrow sense cannot hold inventory: their purchases

always equal their consumption, c_t^k. They purchase goods from household producers and/or manufacturers at the *consumption price* r_t^k.

This framework enables us to infer inventories, consumption, and consumption prices, all of which are often unobservable for economists and practitioners at national statistics offices (e.g., in the retailer-side POS scanner data we use) and differ from the quantities purchased and purchase prices, which are observable. Furthermore, it allows us to derive a COLI in a conventional static manner. All we need to know is two variables: c_t^k and r_t^k. We do not need to use a complex dynamic COLI.

Although the economy is hypothetical, many key properties continue to hold. For example, the stylized facts we present in the next subsection can be explained not only by a dynamic model like the one used by Hendel and Nevo (2006a) but also by our quasi-dynamic model. Propositions 1 and 2 in Hendel and Nevo (2006a) continue to hold in the quasi-dynamic model. On the other hand, it would be difficult to explain the stylized facts using a completely static model that ignores stockpiling. A caveat with regard to our framework is that it does not incorporate consumption smoothing by consumers. If the consumption price jumps, consumption also jumps. However, this caveat applies to all conventional frameworks, including those with perishable goods which cannot be stockpiled, that couch consumers' optimization problem in static terms when discussing price indices. To incorporate consumption smoothing, a dynamic model and dynamic COLIs are needed, but as we discussed in the introduction, interpreting short-run movements in dynamic COLIs is often difficult (Osborne 2018, Ueda 2020).

2.1.1 Setup

We consider the optimization problems of household producers and consumers. The price of storable goods, p_t^k, can take one of two different values, a high (regular) value and a low (sales) value, which are determined stochastically and exogenously.

Consumers

There are a unit mass of consumers. Consumers' cost minimization problem is given by

$$\min_{c_t^k} \left\{ \ \sum_{k \in K_t} r_t^k c_t^k + \lambda_t \left\{ U - \left[\sum_{k \in K_t} b^k \left(c_t^k \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\} \ \right\}, \tag{1}$$

where U is the target utility, $\sigma(> 0)$ denotes the elasticity of substitution, and b^k is a taste or quality parameter for product k in period t.[5] We extensively use the following relation with

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[5] There is no consumer heterogeneity in the model. Many previous studies, such as Boizot, Robin, and Visser (2001) and Hendel and Nevo (2004, 2006a), investigate the frequency of purchases conditional on past purchase behavior. To obtain quantitatively plausible values for this, we need household heterogeneity, because otherwise the frequency of purchases would be either zero or one. In our study, we ignore consumer heterogeneity to focus

respect to the optimal quantity purchased:

$$c_t^k = \left(r_t^k/r_{t'}^k\right)^{-\sigma} c_{t'}^k. \tag{2}$$

Household producers

Household producers maximize their "firm" value:

$$V(i_{t-1}, p_t) = \mathrm{E}_t\left[\sum_{j=0}^{\infty} \beta^j \left\{ \sum_{k \in K_{t+j}} \left(r_{t+j}^k y_{t+j}^k - p_{t+j}^k x_{t+j}^k - C(i_{t+j}^k)\right) \right\} \right], \tag{3}$$

subject to the cost of inventory, $C(0) > 0$, $C(i)' > 0$, $C(i)'' \geq 0$, and the evolution of inventory:

$$i_t^k = i_{t-1}^k - y_t^k + x_t^k. \tag{4}$$

Household producers sell amount y_t^k of product k to consumers at consumption price r_t^k. Furthermore, purchases and inventories must be nonnegative:

$$x_t^k, \ i_t^k \geq 0. \tag{5}$$

Parameter β represents the discount factor.

The first-order conditions with respect to x_t^k and i_t^k are

$$0 = r_t^k - p_t^k + \psi_t^k, \tag{6}$$
$$C'(i_t^k) = \beta \mathrm{E}_t[r_{t+1}^k] - r_t^k + \mu_t^k, \tag{7}$$

where ψ_t^k and μ_t^k represent the Lagrange multipliers with respect to x_t^k and i_t^k, respectively. Note that ψ_t^k is strictly positive when x_t^k is zero, and zero when x_t^k is positive. Likewise, μ_t^k is strictly positive when i_t^k is zero, and zero when i_t^k is positive.

The free entry condition leads to a nonpositive value for an entering household producer with zero inventory holdings: $V(i_{t-1} = 0, p_t) \leq 0$.

Prices

The prices of storable goods follow a Markov process. They take one of the following two values: P_H when there is no sale and P_L $(P_H > P_L)$ during a sale. Moreover:

$$\mathrm{Prob}(P_L|P_H) = \bar{q}$$
$$\mathrm{Prob}(P_L|P_L) = \underline{q}. \tag{8}$$

The unconditional frequency of sales $\mathrm{Prob}(P_L)$ is $f = \bar{q}/(1 + \bar{q} - \underline{q})$.

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on macroeconomic implications rather than individual behavior.

Market clearing

Goods market clearing is given by $\int_0^{N_t} y_{t,j}^k dj + \int_0^{M_t} z_{t,j}^k dj = \int_0^1 c_{t,j}^k dj$, where $z_{t,j}^k$ represents the direct supply of storable product k by manufacturers to consumers. Household consumption c_t equals consumers' purchases from household producers, y_t, and manufacturers, z_t. In the market, there are a unit mass of consumers, N_t represents household producers, and M_t represents manufacturers.

Note that the household producers we consider in the model are still part of the households, although we separate them to simplify our analysis. Thus, the quantity purchased that is recorded in the POS data, X_t, should equal the sum of the quantity purchased by household producers $\int_0^{N_t} x_{t,j}^k dj$ and the quantity purchased directly by consumers $\int_0^{N_t} z_{t,j}^k dj$. Clearly, this is not equal to aggregate consumption $\int_0^1 c_{t,j}^k dj$.

The COLI

As highlighted, consumers' optimization problem is static, so our COLI is identical to the conventional COLI. Consumers' cost minimization problem subject to constant utility yields the following equation for the optimal quantity consumed:

$$c_t^k = \left(r_t^k / b^k \right)^{-\sigma} \lambda_t^\sigma U. \tag{9}$$

The unit cost function, $\lambda_t = \mathcal{C}(r_t)$ for $U = 1$, is given by

$$\mathcal{C}(r_t) = \sum_{k \in K_t} r_t^k c_t^k = \left[\sum_{k \in K_t} \left(b^k \right)^\sigma \left(r_t^k \right)^{1-\sigma} \right]^{1/(1-\sigma)}. \tag{10}$$

Although b^k is unobservable and even if it is time varying, equation (2) tells us that we need to know only two variables for each period, the consumption price r_t^k and the consumption share $r_t^k c_t^k$, to calculate the change in the COLI between period t and period t', $C(r_t)/C(r_{t'})$.[6]

2.1.2 Equilibrium Properties

The left-hand panel of Figure 1 illustrates the pattern of price and quantity changes during a sales event when inventories are held just for one period. The top and bottom panels show prices and quantities, respectively. The sales event takes place in periods $t = 2$ and 3, when the price is lower than during other periods. The bottom panel shows that, in period $t = 2$, quantities purchased by household producers and consumers, X_t, represented by the solid dot,

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[6]Given that household producers form part of households, it would clearly be desirable to incorporate the effect of changes in their "firm" value on consumers' utility and the COLI. However, this effect likely is small, because the free entry condition means that the firm value is close to zero.

increase. The reason is not only that households consume more but also that they stockpile. Thus, X_t is higher than consumption, c_t, represented by the circle, with the difference representing stockpiling. In period $t = 3$, X_t coincides with c_t, since there is no additional need for stockpiling. Then, in period $t = 4$ when the sale ends, household producers sell their inventories to consumers. The consumption price r_t, at which the household producers and consumers transact, lies between P_L and P_H. Since consumption decreases with the consumption price, its level in period $t = 4$ is lower than in periods $t = 2$ and 3 but higher than in periods $t = 1$ and 5. At the beginning of $t = 5$, household producers hold no inventories, so consumers purchase goods at price P_H and consume less. See Appendix A for more discussions on the equilibrium properties of the model.

2.2 Retail and Household Scanner Data

In this and the next subsections, we check the validity of the model by showing that the stylized facts on household inventory are consistent with the equilibrium properties of the model.

In this exercise, we use two sets of scanner data for Japan. The first set consists of retailer-side data, namely, the point-of-sale (POS) scanner data collected by Nikkei Inc. The data include the number of units sold and the sales amount (price times the number of units sold) for each product and retailer on a daily basis. The observation period is around thirty years, running from March 1, 1988 to December 31, 2019. Products recorded consist of processed food and daily necessities, covering 170 of the 588 categories in the CPI and making up about 20 percent of households' expenditure. See Online Appendix B as to how we aggregate variables of interest over days, products, and retailers, and Abe and Tonogi (2010), Sudo, Ueda, and Watanabe (2014), and Sudo et al. (2018) for a detailed description of the data.

The other set consists of household-side data, namely, "*Shoku-map*"[7] scanner data collected by Lifescape Marketing Co. Respondents are mainly a female homemaker, and the data cover about 400 households in each period (about 4,000 households in total). The data record the number of units purchased and the date of consumption for each product and household on a daily basis. Moreover, they record when consumption ends (i.e., when a product is used up or has gone off, etc.) for each product and household. The data cover the period from September 1998 to February 2019. Note that products recorded are food only and there is no information on purchase prices. Another limitation is that there is no information on how much of a product (e.g., in terms of weight) is consumed each time it is consumed. The data record both the number of units purchased and consumed, which is sufficiently useful if products are consumed in discrete units, such as a cup of instant noodles or a can of beer. However, for products like salt, we do not know how much a household uses, although we do know the

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[7] "*Shoku-map*" translates as "food map."

dates when they are used.[8] For example, Figure 2 shows the consumption pattern for salt of a particular household. In the figure, each vertical line represents a consumption flag. The figure indicates that the household purchased salt on day $t = 19$, started using it on day $t = 22$, and used it up on day $t = 144$. The inventory duration thus is $144 - 19 + 1 = 126$ days. See Online Appendix C for detailed explanations about the *Shoku-map* data.

In both sets of data, all products are identified by the Japanese Article Number (JAN) code, which enables us to merge the datasets. Further, we classify products into groups using the 3-digit product categories provided by Nikkei Inc. There are 218 categories in total, such as instant cup noodles, yogurt, beer, and toothbrushes.

In the following analyses using the POS data, we identify temporary sales by employing a sales filter. Specifically, we follow the procedure explained in Nakamura and Steinsson (2010) using their sales filter A with a window of $L = K = J = 42$ days.[9] Product k is classified as being on sale on date t if and only if its price p_t^k deviates from its regular price $\bar{p}_t^k$ by more than two yen.

When no sales are recorded for a particular product at a particular retailer on a particular date, earlier studies usually treated this simply as a missing observation, as if the product disappeared from shelves at the retailer on the date. However, the POS data quite often show no sales records for a particular product, retailer, and date after a sale, suggesting that stockpiling by households during a sale results in zero purchases after the sale ends. Since this conveys important information on household inventory, in this study, we interpolate missing observations by setting $p_t^k = \bar{p}_t^k$ and $x_t^k = 0$ if we have observations after t (i.e., unless the product permanently exits from the market). That is, we set the quantity purchased to zero and the price to the regular price on the nearest past date.[10]

2.3 Stylized Facts on Household Inventory

Using the two datasets, we present four stylized facts that are closely related to the predictions of our model. Note that Facts 1 to 4 correspond to Lemmas 3, 5, 6, and 4, respectively, in Appendix A.

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[8]If a household uses salt N times a day, the data record N, where N is an integer equal to or greater than zero.

[9]The window length is chosen following Eichenbaum, Jaimovich, and Rebelo (2011) and Kehoe and Midrigan (2015). For a detailed examination of the robustness of various filters to identify sales, see Sudo et al. (2018).

[10]If we do not interpolate the price and quantity, the price increase after a sale is completely neglected because the product is not in the matched sample and therefore is excluded from the calculation of the changes in chained price indices. Therefore, without interpolation, the downward bias of the Törnqvist price increases further.

Fact 1: There is asymmetry in the quantity purchased when prices increase and when they decrease.

Using the POS data, we collect the following variables for each product k and sales event s: T denotes the number of days a product is on sale, P_H^1 and X_H^1 denote the price and the quantity purchased just before a sale (say, in period t), respectively, P_L^1 and X_L^1 denote the average price and quantity purchased during the first half of the sale, respectively (i.e., from $t+1$ to $t + \lfloor T/2 \rfloor - 1$), P_L^2 and X_L^2 denote the average price and quantity purchased during the second half of the sale, respectively (i.e., from $t + \lfloor T/2 \rfloor$ to $t+T$), and P_H^2 and X_H^2 denote the price and the quantity purchased just after the sale, respectively (period $t + T + 1$).[11]

Next, we compare the quantities purchased just before a sale and just after a sale (denoted by Q_H), both of which correspond to when a product is not on sale. We calculate the difference of the inverse hyperbolic sine function, that is, $Q_H = \log(X_H^2 + \sqrt{1 + \left(X_H^2\right)^2}) - \log(X_H^1 + \sqrt{1 + \left(X_H^1\right)^2})$ for each product k and sales event s. This function is chosen because X_H may take zero. We then calculate the weighted average of Q_H over k and s within each 3-digit product category, where weights are based on the sales amount, that is, $P_H^1 X_H^1 + P_H^2 X_H^2$. Similarly, we compare the quantities purchased during the first half and the second half of a sale (denoted by Q_L), both of which correspond to when a product is on sale, as $Q_L = \log(X_L^2 + \sqrt{1 + \left(X_L^2\right)^2}) - \log(X_L^1 + \sqrt{1 + \left(X_L^1\right)^2})$. The weighted average is calculated based on sales given by $P_L^1 X_L^1 + P_L^2 X_L^2$. Both Q_H and Q_L indicate that the quantity purchased is asymmetric when the prices of products increase and when they decrease. That is, Q_H represents the asymmetry between just before prices decrease (a sale starts) and just after they increase (the sale ends), both of which correspond to periods when prices are high (i.e., P_H). Similarly, Q_L represents the asymmetry between just after prices decrease (a sale starts) and just before they increase (the sale ends), both of which correspond to periods when prices are low (i.e., P_L).

The left-hand panel of Figure 3 shows Q_H and Q_L for each year. It indicates that Q_H is negative in all years, suggesting that the quantity purchased just after a sale is smaller than that just before. However, while the negative values of Q_H were quite large during the 1990s, they gradually approached zero in the 2010s.

A similar pattern can be observed for Q_L, except that Q_L starts out in negative territory in the 1990s and then gradually shifts into positive territory. The negative values for the 1990s suggest that the quantity purchased was larger at the beginning of a sale than at the end, helping to account for the downward chain drift in the Törnqvist index. Meanwhile,

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[11]Variable T is the smallest integer that is greater than or equal to one and satisfies $|p_{t+T+1} - \overline{p_{t+T+1}}| \leq 2$, where $\overline{p_{t+T+1}}$ represents the regular price.

the positive values for Q_L in the 2010s suggest that the quantity purchased at the beginning of a sale was smaller than that at the end, which works to generate upward chain drift in the Törnqvist index. This can occur when the duration of a sale is known *ex ante* because household producers do not need to stockpile except for the final day of the sale.

In summary, we find that Q_H has been negative and that Q_L has taken both negative and positive values. This finding cannot be explained by a standard model with perishable goods because perishable goods cannot be stockpiled and thus the same price should lead to the same quantity purchased, so that Q_H and Q_L should have mean zero (i.e., there should be no asymmetry). By contrast, our model of stockpiling with storable goods can account for this asymmetry.

Fact 2: When weights are based on purchases, the relationship between the different price indices is given by the following inequality: $\pi^P < \pi^T < 0 < \pi^L$.

Let us denote the price and quantity purchased for product k in period t by p_t^k and x_t^k. Changes in chained price indices from $t - dt$ to t, π_t^X $(X = L, P, T)$, are defined by

$$\pi_t^L = \sum_{k \in K_{t-dt} \cap K_t} W_{t-dt}^k(K_{t-dt} \cap K_t) \log\left(\frac{p_t^k}{p_{t-dt}^k}\right), \tag{11}$$

$$\pi_t^P = \sum_{k \in K_{t-dt} \cap K_t} W_t^k(K_{t-dt} \cap K_t) \log\left(\frac{p_t^k}{p_{t-dt}^k}\right), \tag{12}$$

$$\pi_t^T = \sum_{k \in K_{t-dt} \cap K_t} \frac{W_{t-dt}^k(K_{t-dt} \cap K_t) + W_t^k(K_{t-dt} \cap K_t)}{2} \log\left(\frac{p_t^k}{p_{t-dt}^k}\right), \tag{13}$$

based on the Laspeyres, Paasche, and Törnqvist approach, respectively.[12] The weight share $W_t^k(K_{t-dt} \cap K_t)$ equals $p_t^k x_t^k / \sum_{k' \in K_{t-dt} \cap K_t} p_t^{k'} x_t^{k'}$, where $k \in K_{t-dt} \cap K_t$ represents a domain of products that exist both in $t - dt$ and t (such a common set is called a *matched sample*). When $dt = 1$, we can construct the chained price indices using the cumulative sum of the past price changes: $P_t^X = \exp\left(\sum_{s=1}^t \pi_s^X\right)$ for $X = L, P, T$.

Using the POS data, we calculate the time series of the price level (P_t^X), normalizing the initial price level to one. The results are striking. The left-hand panel of Figure 4 shows that the price level increases by almost 10^{92} over our 30-year observation period when based on the Laspeyres index, while it decreases to almost 10^{-111} when based on the Paasche index. The price decline is milder but nevertheless remains large when based on the Törnqvist index. The price level still decreases to almost 10^{-9}. Thus, the following inequality holds: $\pi^P < \pi^T < 0 < \pi^L$, which is consistent with Lemma 5.

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[12]There are various types of Laspeyres and Paasche indices. Here, we use the logarithmic Laspeyres and logarithmic Paasche indices.

This bias is caused by the asymmetry in the quantity purchased (Lemma 3), for the following two reasons. First, the quantity purchased before a sale is greater than or equal to that just after the sale. As a result, the share of the product in households' total purchases when the price decreases at the beginning of a sale is larger than that when the price increases at the end of the sale, which decreases the purchase-weighted Törnqvist price index. Second, the quantity purchased during the first half of a sale is greater than or equal to that on the second half of the sale. This means that the share of the product in households' total purchases when the price decreases at the beginning of a sale is larger than that when the price increases at the end of the sale. Consequently, the purchase-weighted Törnqvist price index decreases.

Although the sizes of the changes in the price indices shown above are far too large, this does not necessarily mean that they are biased, since we do not observe a true price index such as a COLI that would allow us to make comparisons. We therefore employ the following test for bias.

Chain Drift Test As argued by Ivancic, Diewert, and Fox (2011) among others, a chained price index should take the same value at $t = 0$ and τ if the prices and quantities for all products at $t = 0$ and τ are equal (i.e., there should be no chain drift). We formulate this argument by defining chain drift $d_{0,\tau,dt}^X$ as

$$d_{0,\tau,dt}^X = \sum_{s=1}^{(\tau-1)/dt} \pi_{(s-1)dt,sdt}^X - \pi_{0,\tau-1}^X, \tag{14}$$

where τ represents the interval over which chain drift is measured, which we set to 365 days, and π_{t_1,t_2}^X represents the change in the chained price index from t_1 to t_2 for $X = L, P, T$.[13] The first term on the right-hand side of the equation represents the change in the chained price index when it is incrementally chained forward by interval dt from 0 to $\tau - 1$. The second term represents the change in the chained price index when we calculate the price change from 0 to $\tau - 1$ all at once. If the chained price index has no chain drift, the second term should completely offset the first term, so that $d_{0,\tau,dt}^X$ should be zero. By definition, $d_{0,\tau,dt}^T$ is zero when $dt = 364$. We calculate $d_{0,\tau,dt}^X$ for each year of the observation period, each 3-digit product category, each price index (X), and each interval (dt).

The right-hand panel of Figure 4 shows the mean of $d_{0,\tau,dt}^X$ over years and 3-digit product categories for each price index (X) and interval (dt). The panel shows that the indices deviate considerably from zero. Further, we find that the deviation is positive for the Laspeyres index and negative for the Törnqvist and Paasche indices, with the negative deviation of the Paasche

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[13] π_{t_1,t_2}^X is the same as $\pi_{t_2}^X$ defined in equations (11) to (13) if $t_1 = t_2 - dt$ and the domain of products is the same. Here, we use the domain of products that exist from $t = 0$ to τ to apply equation (14) to the POS data. This implies that we ignore the effect of product turnover for this calculation.

index being larger, i.e., we find the following relationship: $d^P < d^T < 0 < d^L$. The deviations are larger the smaller is dt. For example, for the Törnqvist index, we find that the deviation is -40% when $dt = 1$ and -1% when $dt = 28$.

After calculating $d^X_{0,\tau,dt}$, we conduct a test for chain drift. The test is a simple sign test for the null hypothesis that the probabilities that $d^X_{0,\tau,dt}$ takes a negative or positive sign are each equal to 0.5. The first column of Table 1 shows that the null hypothesis is rejected for $dt = 1$.

Use of Lower Frequency Data   The finding that the deviations are larger the shorter the interval might suggest that it is preferable to use lower frequency data (such as monthly or quarterly rather than daily data) or fixed base indices. This would help to reduce the bias in chained price indices because inventories of most products, particularly those we analyze in this study, last for less than a month. However, the use of lower frequency data and/or fixed base indices has three shortcomings.

First, as pointed out in many previous studies, the use of lower frequency data alone does not completely eliminate the bias. The right-hand panel shows that chain drift decreases as the interval (dt) increases. However, the drift is still -1% for $dt = 28$ and -0.5% for $dt = 182$ when the Törnqvist index is used, which is not negligible. Further, the chain drift is significantly different from zero for all dt, as the first column of Table 1 shows. Given that in almost all industrial countries the consumer price index (CPI) is published on a monthly basis (around $dt = 28$), this suggests that using a frequency that is sufficiently low to avoid bias is not a realistic solution (also see Ivancic, Diewert, and Fox 2011).

The second shortcoming of using both lower frequency data and fixed base indices is that they ignore some products that are short lived or new. Szulc (1983) points out that a fixed base index is likely to be more biased the more distant the current period from the based period. Also see ILO et al. (2004, 2020) and Ivancic, Diewert, and Fox (2011).

The third shortcoming is that some of the high frequency (e.g., daily) fluctuations in prices and quantities may be closely related to business cycle fluctuations. Specifically, we will show that consumers change their stockpiling behavior depending on the macroeconomic environment such as hours worked and interest rates. This suggests that it may not be a good idea to discarding information from high frequency data in constructing price indices because this will yield biased price indices where the bias changes with the business cycle.[14]

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[14]An analogous issue is whether temporary sales can be ignored in the analysis of monetary policy. Since temporary sales are high frequency events lasting only around a week, most models ignore temporary sales despite the fact that they considerably decrease price stickiness. However, Klenow and Willis (2007), Sudo et al. (2018), and Kryvtsov and Vincent (2020) find statistically significant links between temporary sales and the business cycle and argue that temporary sales decrease the real effect of monetary policy.

Fact 3: Chain drift is associated with household stockpiling during temporary sales.

Household stockpiling and in particular the asymmetry in the quantity purchased are not necessarily the sole reason for the chain drift. Therefore, we next examine whether the chain drift we observe can be explained by the asymmetry in the quantity purchased. To this end, we investigate whether the extent of the asymmetry, as measured by Q_H and Q_L, is correlated with the chain drift. Specifically, we conduct the following three analyses.

First, we examine the time-series correlations between the asymmetry of the quantity purchased (Q_H and Q_L) and the chain drift ($d_{0,\tau,dt=1}^T$) based on the Törnqvist index. The left panel in Figure 3 suggested that Q_H and Q_L trended upward during the observation period. Based on this, one would expect the downward chain drift to have diminished over time, and the right panel of Figure 3 shows that this is indeed what happened: the extent of the downward chain drift has been decreasing. However, the chain drift nevertheless remains negative.

Second, we investigate whether there is a positive correlation between the chain drift and the asymmetry in the quantity purchased. We run a regression for $d_{0,\tau,dt=1}^T$ using Q_H and Q_L at the 3-digit product category level after calculating the temporal mean of each variable. The first column in Table 2 shows the estimation result. The coefficients on both Q_H and Q_L are positive and significant at the 5% level.

Third, we examine the correlation between the chain drift and variables associated with temporary sales. We conduct this exercise because Q_H and Q_L are endogenous and therefore the causality from Q_H and Q_L to the chain drift is ambiguous. We run a regression for $d_{0,\tau,dt=1}^T$ using the following variables associated with sales: q_j, $\overline{q_j}$, $\underline{q_j}$, and $(P_L/P_H)_j$, which respectively represent the probability that product j is on sale, the probability that the product will go on sale on the following day given that it is not currently on sale, the probability that the product will continue to be on sale on the following day given that it is currently on sale, and the ratio of the sale price to the regular price. These are the variables that are more exogenous than Q_H and Q_L and cause the chain drift according to our model.

The second to fourth columns in Table 2 show the estimation results. The coefficient on $(P_L/P_H)_j$ is significantly positive, showing that the size of deflation increases as the size of sale discount is larger. The coefficient on q_j is significantly negative, suggesting that the size of deflation increases as the probability of sales increases. Meanwhile, in column (3), the coefficient on $\overline{q_j}$ is insignificant and that on $\underline{q_j}$ is significantly negative. However, as shown in column (4) this result changes when we control for the degree of stockpiling m, which we will define in the next section. In this case, the coefficient on $\overline{q_j}$ is significantly negative and that on $\underline{q_j}$ is insignificant. These estimation results are consistent with Lemma 7, except for the

insignificant coefficient on $\underline{q}_j$. Conducting further simulations, which we do not show here, we find that an increase in $\underline{q}$ increases the price change but that the size of the effect of $\underline{q}$ is small compared with that of $\bar{q}$ and P_L/P_H. Furthermore, the adjusted R^2 is around 0.5, suggesting that temporary sales account for a considerable portion of the bias in chained price indices.

Fact 4: Consumption tends to decrease as household inventories decrease.

While earlier studies on household inventories often assume that consumption is constant, Hendel and Nevo (2006a) and Kano (2018) highlight that consumption is state dependent. To examine whether consumption is indeed state dependent, we investigate whether consumption decreases until the next purchase as inventories decrease, using the *Shoku-map* data. Suppose that household i uses product k on date $t_l \in t_1, t_2, \cdots, t_{n_{ikt}}$ and t represents the purchase date. If the household uses product k twice on date t_{l_0}, we record t_{l_0} twice. Thus, n_{ikt} represents the number of times product k is used. We define the inventory on date t' $(t \leq t' \leq t_{n_{ikt}})$ as $\lambda_{ikt'} = n_{ikt} - n^*$, where n^* is the maximum integer n that satisfies $t_n < t'$. Note that $\lambda_{ikt'}$ is an integer between 0 and n_{ikt}. For example, $\lambda_{ikt'} = n_{ikt}$ for $t \leq t' \leq t_1$. We further define the sum of inventory Λ_{ijt} for household i in product category j at the beginning of date t as

$$\Lambda_{ijt} = \sum_{k \in j} \lambda_{ikt}. \tag{15}$$

We then estimate the following linear probability model when $\Lambda_{ijt} > 0$:

$$y_{ijt} = c_i + d_j + \alpha \Lambda_{ijt} + \varepsilon_{ijt}, \tag{16}$$

where y_{ijt} is a binary variable for consumption and takes a value of one if household i uses products in product category j on date t and zero otherwise. As an alternative, we also use y_{ijt} defined as the sum of the times that products in product category j are used by household i on date t.

As Table 3 shows, the coefficient on inventories is significantly positive, regardless of which dependent variable is used. This indicates that consumption is state dependent and decreases as inventories decrease.[15] Further, the size of the coefficient is small and significantly lower than one. This suggests that households engage in consumption smoothing, that is, they do not

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[15]For the estimation presented in the table, we did not use instrumental variables but employed ordinary least squares (OLS). This means that the estimates are biased if, for example, $\mathrm{Cor}(\varepsilon_{ijt}, \varepsilon_{ijt-1}) > 0$, that is, if high consumption demand at time $t-1$ not only decreases inventories Λ_{ijt} at time t but also increases consumption at time t. If such endogeneity exists, the coefficient on inventories is underestimated when using OLS. Thus, the fact that we obtained a significantly positive estimate using OLS indicates that the state dependency of consumption continues to hold or is stronger than our estimates suggest. See Online Appendix C for the estimation results at a product category level.

consume as much as they purchase or hold as inventories. As a result, consumption-weighted chained indices provide more stable indicators of price changes than purchase-weighted chained indices.

3 Inference of Consumption and Consumption Prices and Calculation of the COLI

In this section, we propose an approach to calculating consumption and consumption prices using the POS data. We then check the validity of our approach and calculate price indices based on the consumption we infer.

3.1 Methodology

In the previous section, we examined the validity of the model predictions regarding prices and quantities purchased using scanner data for Japan. However, many important variables such as consumption, inventories, and the consumption price are often unobservable, although we can observe the quantity purchased and the posted price using retailer-side POS scanner data. From a practical perspective, consumption and the consumption price are essential for constructing the COLI.[16] For macroeconomists, it is of great interest to see whether any changes in stockpiling behavior can be observed over time and, if so, what the determinants are. Therefore, in this section, we propose a simple and tractable methodology to infer these variables using retailer-side scanner data. See Online Appendix E for details.

The consumption price at $p_t = P_L$ equals P_L. The key variable is the consumption price at $p_t = P_H$ after a sale ends, that is, $r_H(I_{t-1}) \equiv r(I_{t-1}, P_H)$, where $r_H(0) = P_H$. Knowing this variable enables us to obtain consumption c_t after a sale using equation (2).

When $p_t = P_H$, household producers' optimization problem is given by

$$C'(i_H; I_{t-1}) = \beta \{(1 - \overline{q}) r_H(I_t) + \overline{q} P_L\} - r_H(I_{t-1}) + \mu_t \qquad (17)$$

from equation (7). This equation shows that household producers strike a balance between the benefits of a future consumption-price increase (the right-hand side) and the costs of holding inventories (the left-hand side). Note that μ_t is the Lagrange multiplier associated with i_t and equals zero if $i_t > 0$.

We note that when inventory cost function $C(\cdot)$ is written in a certain form, $r_H(I_t) - r_H(I_{t-1})$ becomes a positive constant. In other words, the expectation of a *linear* consumption-price

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[16]How to deal with chain drift including intertemporal substitution bias is an important issue for national statistics offices, especially when they employ high frequency scanner data in constructing consumer price indices. See Konny et al. (2019), Eurostat (2017), and EFTA (2020) for more on this practical aspect of chain drift.

increase prevents household producers from selling all of their inventories instantaneously or from selling none at all. Household producers gradually sell off their inventories to consumers.

In the following analysis, we assume this linearity holds. While this admittedly is a restrictive assumption, it can be interpreted as an intermediate of the following two scenarios. The first is when $C'' \to 0$. In this scenario, $r_H(I_t) - r_H(I_{t-1})$ increases in t. Put differently, if inventory costs are not convex, household producers require a greater consumption-price increase as time goes by, because they discount the future ($\beta < 1$) and expect another sale to come at some point ($\bar{q} > 0$). As for the second scenario, suppose $\beta = 1$ and $\bar{q} = 0$. Then $r_H(I_t) - r_H(I_{t-1})$ decreases in t. The cost of holding inventories decreases as household producers' inventories decrease because of $C'' > 0$, which makes household producers require a smaller consumption-price increase as time goes by.

The benefit of this linearity assumption is that it greatly simplifies our analysis. Given the path of $r_H(I_{t-1})$, we compute consumption c_t as $(r_H(I_{t-1})/P_L)^{-\sigma} c_L^*$, where c_L^* represents consumption during a sale. Briefly put, c_L^* is the lower value of the quantity purchased (observable in the POS data) during the first and second half of a sale because according to the model, consumption during a sale is equal to the lowest amount of purchases during the sale. Furthermore, we can calculate the degree of stockpiling, m. More precisely, we define m to denote how long inventories last after a sale ends. In continuous time, we can derive the following equation:

$$m_{cont} = \frac{P_H - P_L}{P_L} \frac{\sigma - 1}{1 - (P_H/P_L)^{-\sigma+1}} \frac{I_L}{c_L^*}. \tag{18}$$

Simply put, the equation can be derived because cumulative consumption for m_{cont} periods equals the initial inventories outstanding just after a sale ends, I_L, and the consumption price linearly increases from P_L to P_H in m_{cont} periods. Inventories I_L equal the cumulative amount of purchases during a sale minus the cumulative amount of consumption, Tc_L^*, in the same period. Because the variables P_H, P_L, c_L^*, σ, and I_L are observable/estimable from purchase data, we can estimate m_{cont}, the path of consumption prices, and the path of quantities consumed after a sale ends. In the next subsection, we apply our methodology to infer consumption from retail scanner data and verify the methodology empirically by showing that the consumption-weighted Törnqvist price index continues to have, albeit small, downward bias, while the consumption-weighted order r superlative index has no bias, as shown in Lemmas 5 and 8 in Appendix A.

The right-hand panel of Figure 1 illustrates the pattern of price and quantity changes in the case of $m = 5$. The consumption price, depicted by the circles in the top panel, increases at a constant rate from $t = 3$ to 8. The bottom panel shows that while the quantity purchased falls to zero from $t = 4$ to 7, consumption does not fall to zero, but decreases gradually.

In Online Appendix F, we employ numerical simulations to depict a typical path of the price

and quantity and calculate biases in chained price indices. The simulations show that the COLI and the chained order r superlative index do not have any bias and asymptotically return to their original level. The chained consumption-weighted Törnqvist index has a downward bias, while the purchase-weighted price indices have a much larger bias. The size of the bias in the purchase-weighted Törnqvist index is comparable to the actual size of the bias in the Törnqvist index for Japan.

It should be noted that the fact that goods are storable means that price indices will always be subject to the intertemporal substitution bias. This is the case regardless of whether a simplifying assumption such as the one we introduce is employed. In fact, the model in the previous section succeeded in qualitatively explaining the bias in various chained price indices, including its sign, without relying on this simplifying assumption.

However, additional assumptions are needed for quantitative purposes such as investigating the size of the bias. Since the linearity assumption is only one of many possible assumptions, in Online Appendix G, we examine how much the size of the chain drift changes in response to different assumptions about the path of the consumption price after a sale ends. We conclude that the alternative approaches leave our quantitative results more or less unchanged.

In what follows, we take the following four steps. First, we identify all sales events for each product and retailer using the sales filter. Second, we record the price and the quantity purchased just before and after a sale, as well as the average price and the quantity purchased in the first and the second half of a sale. This enables us to obtain consumption prices and the quantities consumed just before and during the sale (but not after the sale). Third, we calculate the elasticity of substitution, σ, for each 3-digit product category. We use the consumption prices and the quantities consumed before and during the sale obtained in the previous step to estimate σ from equation (2). We will come back to this point in Section 4.1. Finally, we calculate $r_t = r_H(I_{t-1})$, c_t, c_L^*, m, and I_L for each sales event for each product at each retailer. We employ the methodology discussed in the previous subsection to calculate m. Once we have obtained m, we can calculate the path of consumption prices $r_t = r_H(I_{t-1})$ and consumption c_t after a sale ends. A detailed explanation of how we calculate these variables is provided in Online Appendix E.

3.2 The Degree of Stockpiling

We apply the approach explained in the previous subsection to Japanese POS scanner data. Figure 5 presents a histogram of the degree of stockpiling m_{cont} at the 3-digit product category level (hereafter we simplify the notation by dropping $cont$ from m_{cont}). The distribution of m ranges from one to five, with the mode being around two. Table 4 lists the top and bottom five product categories with the largest and smallest m. The top three categories are instant

cup noodles, diluted beverages, and frozen meals, in that order. These products can indeed be stored for a long time. However, the storability of products does not necessarily imply a high degree of stockpiling. In fact, of the five bottom categories, razors, cosmetic accessories (e.g., hand cream and sunscreen lotion), home medical supplies, and batteries are also highly storable. What distinguishes them from storable goods such as cup noodles is that once they are opened, they can continue to be used for a long time. That is, whereas cup noodles, for example, are consumed more or less immediately after they are opened, razors, cosmetic accessories, and batteries can often be used for longer than a week. As a result, consumers tend not to purchase more than two units even when they are on sale. This suggests that the degree of stockpiling, m, may be negatively correlated with the length that a product lasts once it is opened.[17]

3.3 Validity Checks of Our Approach

3.3.1 Comparison of Inflation Rates: Data and Simulation

To check the validity of our approach, we first examine whether the simulated bias in the chained indices is comparable to the actual price change. On the one hand, we calculate the actual price change defined as the average of price changes based on the purchase-weighted Törnqvist index for each 3-digit product category j (denoted by π_j) over the observation period. On the other hand, we calculate $\overline{q_j}$, $\underline{q_j}$, $(P_L/P_H)_j$, and m_j to obtain the simulated bias in the chained indices for each 3-digit product category. Then, using these values, we simulate the model and calculate the average price change in the purchase-weighted Törnqvist index (see Online Appendix F for the simulation method). The elasticity of substitution σ is set to 3 for all product categories.

Figure 6 shows that our approach does a reasonably good job in explaining the actual change in chained price indices. In the figure, each circle represents a product category. The figure shows that product categories that have a large price change in the data tend to exhibit a large bias in the simulation as well. The circles tend to lie below the 45 degree line, suggesting that the actual price change is smaller than the simulated bias.[18]

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[17]The degree of stockpiling is highly heterogeneous even within products belonging to the same product category. See Online Appendix F.

[18]Column (4) in Table 2 shows that the chain drift is negatively correlated with m_j. In Online Appendix F, using simulations, we investigate how the inflation rate, π_j, depends on m_j, $\overline{q_j}$, $\underline{q_j}$, and $(P_L/P_H)_j$. We find that π_j depends negatively on m_j.

3.3.2 Comparison with the *Shoku-map* Data

To further check the validity of our approach, we compare the degree of stockpiling (m) obtained from the POS data with related variables in the *Shoku-map* data. Specifically, we look at the duration of inventories defined as the difference between the date of purchase (t_p) and the date on which the item is used up (t_l). Using the cross-sectional dispersion of m at the 3-digit product category level, we examine if there is any significant correlation. The left-hand panel of Figure 7 shows that there is no significant correlation between $\log(m)$ in the POS data and $\log(t_l - t_p + 1)$ in the *Shoku-map* data for the period of $1988 - 2013$. This suggests that m does not necessarily represent storability.

If m captures stockpiling, it should incorporate not only storability but also the quantity purchased. For this reason, the average quantity purchased in the *Shoku-map* data, $\log(q)$, may correlate with $\log(m)$ in the POS data. The right-hand panel of Figure 7 shows that this is indeed the case. A positive correlation is observed between $\log(q)$ and $\log(m)$ for the average of the entire observation period with the correlation coefficient of $+0.32$.

3.4 Price Indices

Using the POS data, we calculate the time-series of the price level based on the following three definitions. The first is the Törnqvist index based on the purchase weight. This is the conventional approach and we showed the results in Figure 4. The second definition is the Törnqvist index based on the consumption weight. Here, we use the consumption price as well as consumption to calculate the price index. The third definition is the order r superlative index, where we use the estimated elasticity of substitution σ at the category level (see Appendix A for the definition).

Table 1 presents the average chain drift and inflation rate based on the three price indices. It shows that the chain drift in the consumption-weighted Törnqvist index is significantly negative for $dt \leq 52$. By contrast, the chain drift in the consumption-weighted order r superlative index for $dt = 1$ is insignificant. This result shows that our approach using the consumption-weighted order r superlative index is effective in eliminating chain drift, although the chain drift is significantly negative for intermediate values of dt (i.e., 7 or 14 days).

The bottom row shows that the averages of the annualized inflation rates are negative and still sizable for all three indices. However, the size of measured deflation decreases the more household stockpiling is taken into account; that is, it is -46% for the purchase-weighted Törnqvist index, -13% for the consumption-weighted Törnqvist index, and -11% for the consumption-weighted order r superlative index. Online Appendix F compares developments

in the inflation rates over time.[19]

4   Further Issues Regarding Household Inventories

Household inventories matter not only for price indices. In this section, we first investigate the implications of household inventories for the measurement of the elasticity of substitution. Further, we then examine how household stockpiling changes in response to a preannounced consumption tax hike and the business cycle.

4.1   The Elasticity of Substitution

4.1.1   Our Approach to Measuring the Elasticity of Substitution

Our approach is also useful for obtaining the value of the elasticity of substitution, σ.[20] Note that the estimate of the elasticity of substitution is biased unless we properly take stockpiling into account. As argued by Hendel and Nevo (2004), price reductions influence the quantity purchased not only via the consumption effect (consumption is price sensitive) but also via the stockpiling effect (i.e., consumers stockpile for future consumption). Owing to the latter effect, which is often larger than the former, the elasticity of substitution is likely to be overestimated when we ignore stockpiling.

Our approach makes it possible to obtain the value of σ. For each 3-digit product category, for each sales event of product k and retailer r, we collect the records of the log ratio of the quantity consumed during a sale to the quantity consumed when the product is sold at the regular price divided by the log ratio of the sale price to the regular price, that is, $\Gamma \equiv -\log\left(c_L/c_H\right)/\log\left(r_L/r_H\right)$. Equation (2) suggests that Γ equals σ on average.[21] We calculate

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[19]Other factors may contribute to the bias in the chained price indices. The most important factor likely is product turnover. Ueda, Watanabe, and Watanabe (2019) show that in Japan product prices tend to decline over the life-span of a product. In this case, price indices that use only matched samples and ignore price differences between old and new products will be biased downward. This downward bias is indeed consistent with the result shown in Table 1 that the inflation rate based on the Törnqvist index is 6% lower than the chain drift in the POS data. Another possible reason for bias is cross-sectional substitution if, for example, the timing of sales is correlated across retailers or products. Such cross-sectional substitution may amplify or reduce household stockpiling, which would affect the bias in chained price indices. See Ivancic, Diewert, and Fox (2011).

[20]The elasticity we measure in this study is the short-run intertemporal own price elasticity of consumption. By this we mean the following. First, by "short-run intertemporal elasticity" we mean that the elasticity we focus on incorporates stockpiling (see Ogaki and Reinhart 1998). Second, by "own price elasticity" we mean that we focus on the elasticity in response to changes in the price of the product in question, not of other products that are substitutes, which would be the cross-price elasticity.

[21]It can be assumed that prices are exogenous for households, especially over such a short time horizon. Thus, any bias from endogeneity is unlikely to be a major problem.

the unweighted average of Γ across sales events, products, and retailers for each 3-digit product category, which we define as σ.

For comparison, a simple calculation of σ is possible if we ignore storability. Hendel and Nevo (2004) argue that neglecting stockpiling leads to an overestimation of σ by a factor of two to six. To confirm this, we obtain σ by simply calculating the slope of $-\Delta \log X_t / \Delta \log p_t$ using the observed series of purchases X_t and posted prices p_t, where Δ is the difference from the previous date. In this simple calculation, we use all observations as long as the posted price changes by more than two yen from the previous date. Note that, according to equation (2), the simple calculation would be valid if c_t and r_t equal X_t and p_t, respectively.

4.1.2 Results

Figure 8 shows the value of the elasticity of substitution σ for 3-digit product categories, calculated from $\Gamma = -\log\left(c_L/c_H\right)/\log\left(r_L/r_H\right)$. The left-hand panel displays the histogram of σ and shows that σ is distributed smoothly around the mode of three and most of the values are positive.

The dotted line in the left-hand panel shows the histogram of the simple estimate of σ, which is distributed to the right of the solid line (our estimate). This result is in line with Hendel and Nevo's (2004) result, suggesting that σ is overestimated if we ignore storability. The right-hand panel shows the scatter plot of the value of σ, where each dot represents a 3-digit product category. In most categories, the simple estimates are larger than our estimates. Nevertheless, the two measures are not independent of each other and exhibit a positive correlation. Finally, it should be noted that the estimates of σ are negative for some product categories (although they are not necessarily significant), whereas from a theoretical perspective we would expect them to be positive. We aggregate the variables of interest at the 3-digit category level only when σ is greater than zero.

4.2 Effect of the Consumption Tax Rate Hike

On April 1st, 2014, the consumption tax rate in Japan was raised from 5% to 8%. This was a preannounced event, which prompted households to stockpile before the tax hike.[22] To examine the effect of the consumption tax hike, we calculate the change in $\log(m)$ in March 2014 from the same month in the previous year.

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[22]In Japan, increases in the consumption tax rate—from 3 to 5% in April 1997, from 5 to 8% in April 2014, and from 8 to 10% in October 2019—have always been a major political and economic event, because they have been accompanied by large demand increases before the tax hike and persistent weak demand (recessions) after the hike. For this reason, Prime Minister Shinzo Abe postponed the latest hike, from 8% to 10% twice, once in 2014 and once in 2016.

The left-hand panel of Figure 9 shows that the changes in $\log(m)$ in March 2014 are positive for most product categories. Furthermore, the correlation coefficient between this variable and $\log(t_l - t_p + 1)$ based on the *Shoku-map* data is high at $+0.48$, although the correlation was insignificant for the period of 1988–2013. This evidence supports our interpretation that m captures the degree of stockpiling. That is, the degree of stockpiling increases in response to an anticipated price increase, and more storable goods tend to be stockpiled more. Furthermore, the right-hand panel of Figure 9 shows that a positive correlation continues to be observed between the average quantity purchased in the *Shoku-map* data, $\log(q)$, and $\log(m)$ in March 2014: the correlation coefficient is $+0.398$.

4.3 Developments in Households' Stockpiling Behavior

4.3.1 Changes in Stockpiling Behavior

The degree of stockpiling m is not only heterogeneous but also time-varying. The line with dots in Figure 10 shows the time-series developments in aggregate $\log(m_t)$, given by the unweighted mean of $\log(m_j)$ for 3-digit product category j, from January 1989 to December 2018. The line indicates that there has been a secular decrease in the last two decades.

It should be noted that m can change as a result of changes not only in households' intrinsic behavior but also in prices, which are exogenous to households. To show this formally, we use equation (7) when $p_t = P_L$. Household producers optimize their inventories during a sale to satisfy

$$C'(i_L; I_{t-1}) = \beta \left\{ (1 - \underline{q}) r_H(I_t) + \underline{q} P_L \right\} - P_L + \mu_t. \tag{19}$$

If a sale ends in period t, the consumption price increases in period $t+1$. This provides household producers with a profit if they hold inventories, as the right-hand side of the equation shows. However, household producers incur a cost when holding inventories, as shown in the left-hand side of the equation. Suppose $C'(i_t; I_{t-1}) = C > 0$ and linear consumption-price increases. Then we have $C = \beta(1 - \underline{q}) \left\{ \frac{1}{m} \frac{P_H - P_L}{P_L} + 1 \right\} P_L - (1 - \beta \underline{q}) P_L$, and, in turn,

$$m = \beta(1 - \underline{q}) \frac{P_H - P_L}{P_L} \left(1 - \beta + \frac{C}{P_L} \right)^{-1}. \tag{20}$$

This equation suggests that the degree of stockpiling m negatively depends on the probability that a sale will continue to occur at $t+1$ given that a sale occurs at t, positively depends on the size of the sale discount, and negatively depends on the cost of holding inventories. In other words, more stockpiling occurs the sooner a sale is expected to end and the larger the sale discount is. More generally, $\overline{q}$ is also likely to influence r_H and, in turn, m. Thus, $\overline{q}$, $\underline{q}$, and $(P_H - P_L)/P_H$ should constitute explanatory variables for m.

Therefore, we regress the following equation: $\log(m_{jt}) = c_j + d_t + AX_{jt} + \varepsilon_{jt}$, using the vector of explanatory variables $X_{jt} = \{\log(1 - \overline{q_{jt}}), \ \log\left(1 - \underline{q_{jt}}\right), \ \log((P_H - P_L)/P_H)_{jt}\}$, where j and t represent the 3-digit product category and the month (January 1989 to December 2018), respectively. Time fixed effect d_t captures the aggregate, demand-side, time-varying component of m_{jt}. Some of the variables we use are persistent and close to an I(1) process. Thus, to avoid spurious regression, we also estimate the above equation using the time differences of the variables such as $\Delta \log(m_{jt}) \equiv \log(m_{jt}) - \log(m_{jt-1})$.

Table 5 presents the estimation results. In the table, column (1) shows the result when we use $\log(m_{jt})$ as the dependent variable and do not include the time fixed effect. Column (2) shows the result when we use $\log(m_{jt})$ as the dependent variable and include the time fixed effect. Column (3) shows the result when we use the time difference of $\log(m_{jt})$ as the dependent variable and do not include the time fixed effect.

The coefficient on $\log((P_H - P_L)/P_H)$ is significantly positive in all columns. The positive relationship between $(P_H - P_L)/P_H$ and m is consistent with the relationship derived from equation (20). The coefficient on $\log(1 - \overline{q})$ is insignificant at the five percent level in columns (1) and (2) but significantly negative in the regression that uses time differences (column (3)).

The coefficient on $\log\left(1 - \underline{q}\right)$ is significantly negative in all columns, even though equation (20) suggests it should be positive. One possible reason is that $\underline{q}$ might be endogenous. It should be noted that in this regression, the cost of inventories is not controlled for. For product categories with low inventory costs, m is likely to be high. If firms hold longer sales (high $\underline{q}$) for these products, we would expect to observe a negative coefficient on $\log\left(1 - \underline{q}\right)$ rather than a positive one. Another reason is that in equation (18) m is proportional to I_L, which tends to increase as the duration of a sale T increases. Owing to this construction, our measure of m tends to increase as $\underline{q}$ increases.

The solid line in Figure 10 shows the time-series of time fixed effect d_t (column 2 in the table), which represents changes in the degree of stockpiling after controlling for the effects of price changes. Developments in d_t differ from those in aggregate $\log(m_t)$. Specifically, d_t exhibits an increase in the 2000s, while aggregate $\log(m_t)$ does not.[23]

4.3.2 Effects of Macroeconomic Variables on Stockpiling Behavior

What brought about the secular decrease in household stockpiling behavior (m_t and d_t) in the 1990s and then the reversal in the 2000s (d_t, but not m_t)? Household stockpiling behavior likely is influenced by a number of factors, which we consider in this subsection.

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[23]We find that this deviation is explained by both the decrease in the probability of sales ($\underline{q}$) and the decrease in the size of sale discounts ($\log((P_H - P_L)/P_H)$). For the detailed changes in these variables associated with sales, see Sudo, Ueda, and Watanabe (2014).

Possible Channels   First, according to equation (20), an increase in β increases stockpiling because households put greater weight on future consumption. Possible factors that may bring about changes in β are preference shocks, which are often incorporated in dynamic stochastic general equilibrium models as part of demand shocks and also cause changes in real interest rates. Specifically, preference shocks generate a negative relationship between stockpiling and real interest rates. Furthermore, it is also thought that higher inflation expectations promote stockpiling, which also yields a negative correlation between real interest rates and stockpiling. Thus, investigating developments in real interest rates should provide a clue as to how and why stockpiling has changed in Japan.[24] Over the last two decades, Japan has seen successive waves of monetary accommodation, leading to a decline in nominal interest rates, although, due to the zero lower bound on nominal interest rates, it is debatable whether real interest rates have declined as a result of monetary policy.

Second, stockpiling behavior may be influenced by factors that change the cost of holding inventories C. According to equation (20), an increase in C decreases the incentive for stockpiling. One possible factor that may influence C is an interest rate. For example, a higher interest rate increases borrowing costs, which may prevent households from stockpiling.[25]

The third factor concerns labor market conditions. Consider the following two opposing hypotheses. Suppose that labor market conditions are unfavorable for households, that is, low labor demand brings about high unemployment, low hours worked, and low income. One hypothesis is that households face stricter financial (liquidity) constraints and are therefore unable to purchase as much as they would like when prices are low. In that case, unemployment has a negative effect on stockpiling, while hours worked have a positive effect. The other hypothesis is that when unemployment is high and hours worked are low, households have more time for shopping, which allows them to find products that are on sale and stockpile inventories. Also, a decrease in income may make households more price-sensitive. In that case, unemployment has a positive effect on stockpiling and hours worked have a negative effect.

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[24]Similarly, interest rates and the cost of holding inventories are also important determinants of firm inventory investment (see, e.g., Kahn, 2016).

[25]Another possible factor is the size of houses and Japan's demographic structure (population aging). However, looking at data for the size of houses from the Housing and Land Survey conducted by the Ministry of Internal Affairs and Communications every five years shows that there has been a steady increase in both the average housing area and the average housing area per household member from 1993 to 2013 from $88.4m^2$ to $93.0m^2$ and from $29.8m^2$/person to $38.5m^2$/person, respectively. This suggests that C should have decreased monotonically and d_t increased monotonically. However, such a monotonic decrease did not occur, as shown in Figure 10. Population aging is also a monotonic development in Japan.

Regression   Bearing these factors in mind, we examine whether the degree of stockpiling m_t depends on the macroeconomic environment. We estimate the following equation: $\Delta \log (m_{jt}) = c_j + B\Delta Z_t + A\Delta X_{jt} + \mu_{jt}$ or $\Delta \log (m_{jt}) = c_j + B\Delta Z_t + A\widetilde{\nu_{jt}} + \mu_{jt}$. Here, ΔX_t is the time difference of the price variables used above from month $t-1$ to t, and ΔZ_t is the time difference of exogenous variables consisting of the unemployment rate, log hours worked, and the real interest rate from month $t-1$ to t and $t-2$ to $t-1$. The real interest rate in period t is defined as the overnight call rate in period t minus the actual inflation rate based on the CPI from t to $t+12$ (all in percent). We estimate the equation using the time difference to avoid spurious regression. Lagged variables for Z_t are added to incorporate the possibility that it takes time for labor market conditions and the real interest rate to influence household stockpiling behavior.[26]

In the first regression, we estimate the degree of stockpiling using ΔX_t as the independent variable. In the second regression, we use $\widetilde{\nu_{jt}}$, the residuals of the estimated equation of $\Delta X_{jt} = e_j + D\Delta Z_t + \nu_{jt}$, as the independent variable. Labor market conditions and the real interest rate ΔZ_t likely influence firms' pricing ΔX_{jt} as well. Since $\widetilde{\nu_{jt}}$ is orthogonal to ΔZ_t, by using $\widetilde{\nu_{jt}}$, we aims to evaluate the overall effect of ΔZ_t on $\Delta \log (m_{jt})$, which incorporates the indirect effect through ΔX_{jt}.

Table 6, particularly column (5), shows the main estimation results, while column (1) shows the estimation results when we simply use ΔX_{jt}. The effect of the unemployment rate on $\Delta \log (m_{jt})$ is small, because the two coefficients on the unemployment rate at t and $t-1$ more or less cancel each other out. The two coefficients on hours worked at t and $t-1$ are significantly negative, suggesting that longer hours worked decrease the degree of stockpiling. This result supports the hypothesis that longer hours worked decrease households' time for shopping, which prevents them from stockpiling inventories during sales, rather than the hypothesis focusing on households' financial constraints and predicting the opposite effect. The coefficient on the real interest rate at t is significantly negative, suggesting that a higher real interest rate decreases the degree of stockpiling. This is consistent with the reasoning mentioned above.

Columns (2) to (4) show that, in response to longer hours worked, firms change their pricing so that the frequency of sales decreases (low $\bar{q}$), the duration of sales increases (high $\underline{q}$), and the size of sale discounts increases (high $(P_H - P_L)/P_H$). The effects of the changes in the unemployment rate and the real interest rate on pricing are unclear, however.

These results suggest that using lower frequency data to calculate price indices has the risk of ignoring the effects of business cycle fluctuations. This may yield biased price indices where the bias changes with the business cycle. Online Appendix H provides a quantitative examination of the implications of this endogeneity of stockpiling.

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[26]Note that m_{jt} is not likely to influence Z_t because the former is a variable at the product category level.

5 Conclusion

Goods storability, especially stockpiling during temporary sales, causes a large degree of bias when price indices are based on purchases, since consumers tend to stockpile when prices are low (i.e., during a sale) and purchases exceed consumption. To deal with this issue, we constructed a model to explain the stylized facts, proposed a tractable approach to infer consumption and consumption prices using data on purchases and purchase prices, and applied the approach to Japanese data. We showed that consumers' stockpiling behavior can be conveniently summarized by a single variable: the degree of stockpiling during a sale, which expresses how long inventories last after a sale ends. Applying the approach to POS data for Japan, we found that our approach of using a consumption-weighted index succeeds in eliminating chain drift. Furthermore, we showed that the degree of stockpiling depends on the business cycle.

Tasks for the future include, first, a more careful consideration of heterogeneity at the product and household levels. We found that there exists sizable heterogeneity in the degree of stockpiling across products. A more detailed investigation might shed new light on household inventory behavior. Equally important is the heterogeneity at the household level. Considering the possibility that stockpiling behavior depends on the size of the family and home, income, age, etc., could provide new insights.

Second, we should apply our approach to a wider range of product categories than those covered in our data, processed food and daily necessities, which make up only about 20 percent of households' expenditure. For instance, prices for some storable goods (e.g., gasoline and fresh food), durable goods (e.g., clothing and personal computers), and services (e.g., travel) occasionally change substantially just like in a temporary sale, which seems to cause demand fluctuations similar to stockpiling. It is worth testing whether our approach is useful for the analysis of these product categories.

Appendix A. Equilibrium Properties

In this appendix, we discuss the equilibrium properties of the model. In the following discussion, we omit the superscript for product k for simplicity. Further, we denote aggregate inventories at the end of period $t - 1$, by $I_{t-1} \equiv \int_0^{N_t} i_{t-1,j} dj$.

A.1 Consumption Price

The first lemma states the property of the consumption price, the price at which consumers make their purchase, r_t. When p_t equals P_L (during a sale), r_t equals P_L. That is, consumers purchase goods directly from manufacturers at $r_t = P_L$. When $p_t = P_H$ (during a non-sale),

consumers may purchase goods from household producers at a price below P_H. Price r_t is lower the larger inventories I_{t-1} are.

Lemma 1 *Consumption price r_t satisfies $P_L \leq r_t \leq P_H$. When $p_t = P_L$, $r_t = P_L$. Furthermore, $r_t = r(I_{t-1}, p_t, b)$ is nondecreasing in p_t and b and nonincreasing in I_{t-1}.*

The proofs of this and the lemmas that follow are provided in Online Appendix D.

A.2 Stockpiling by Household Producers

The next lemma states the stockpiling behavior of household producers. Only when $p_t = P_L$ do household producers purchase goods and hold inventories with the aim of selling the goods at a higher price after the sale has ended.

Lemma 2 *If $p_t = P_H$, household producers do not purchase goods, that is, $x_t = 0$. If $p_t = P_L$, household producers purchase goods and hold inventories. Inventories are independent of i_{t-1}, I_{t-1}, and b.*

A.3 Asymmetry in the Quantity Purchased (Intertemporal Substitution)

The next lemma shows that the change in quantities purchased in response to a price increase and a price decrease is asymmetric.

Lemma 3 *The quantity purchased by household producers and consumers just before a sale is greater than or equal to that just after a sale.*

The quantity purchased by household producers and consumers on the first day of a sale is greater than or equal to that on the final day of a sale.

This asymmetry reflects intertemporal substitution. Household producers stockpile when the price of a product is low, so that on the day after a sale, households do not need to purchase as much of the product as before the sale.

It should be noted that whether the quantity purchased on the first day of a sale is smaller than that purchased on the final day of a sale depends on whether prices are stochastic. In the model, prices are stochastic, and neither household producers nor consumers can accurately predict future prices. This is why household producers hold inventories at the end of the first day of a sale. Suppose instead that the duration of a sale is known *ex ante*. In this case, there is no incentive for household producers to stockpile except for the final day of the sale. Thus, the quantity purchased by household producers and consumers on the first day of a sale should be smaller than or equal to that on the final day of a sale.[27]

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[27]Feenstra and Shapiro (2003) document upward chain drift for the Törnqvist index and conjecture that

A.4 State-dependent Consumption

Consumption c_t is state dependent; specifically, it depends on consumption price r_t.

Lemma 4 *c_t decreases in r_t. After a sale ends, r_t and c_t are nondecreasing and nonincreasing over time, respectively, until the next sale begins.*

A.5 Intertemporal Substitution Bias in Chained Price Indices

Using equations (11) to (13), we calculate changes in purchase-weighted chained price indices from $t - dt$ to t, π_t^X ($X = L, P, T$) based on the Laspeyres, Paasche, and Törnqvist approach, respectively. Further, we define the change in the consumption-weighted chained Törnqvist price index as π^{T*}, for which we use consumption quantity c_t^k and price r_t^k instead of x_t^k and p_t^k, and the change in the COLI as π^{COLI}.

Moreover, we introduce the degree of stockpiling $m(\geq 0)$, which indicates how many days' worth of inventories remain in the hands of household producers. If $m = 0$, no stockpiling occurs during a sale and hence no bias (i.e., no chain drift) due to sales arises. Therefore, what we are interested in is the case where $m > 0$. In the next subsection, we will specify how m is determined. In Online Appendix D, we discuss how equilibrium is determined in the special case of $m = 1$, that is, when inventories are cleared in just one period after a sale.

Lemma 5 *Consider one sales event for product k such that $p_t = p_{t+T+1} = P_H$ and $p_{t+j} = P_L$ for $j = 1, \cdots, T$ ($T \geq 1$). Suppose that the prices and quantities of other goods remain unchanged; $\sum_{k' \in K_0 \cap K_t} p_t^{k'} x_t^{k'} = 1$; and the price before and after the sale is P_H for a sufficiently long duration (i.e., $p_{t-j} = p_{t+T+1+j} = P_H$ for $j = 0, 1, \cdots, T_H$, where T_H is sufficiently large compared with m).*

If $\sigma > (<)1$ and $m > 0$, the cumulative sum of changes in the chained price index from t to $t + T + 1 + T_H$ satisfies $\pi^{COLI} = 0$; $\pi^L > (<)0$; $\pi^P < (>)0$; $\pi^T < 0$; and $\pi^{T*} < (>)0$.

If $\sigma = 1$ or $m = 0$, then $\pi^{COLI} = \pi^L = \pi^P = \pi^{T*} = 0$ and $\pi^T < 0$.

If $\sigma > 1$, $\pi^P < \pi^T < 0 < \pi^L$. If $\sigma > 1$ and $m = 1$, $\pi^T < \pi^{T*} < 0$.

This lemma shows that, when $\sigma > 1$, purchase-weighted chained price indices entail biases, with the Paasche and Törnqvist indices having a downward and the Laspeyres index having an upward bias. It is well known that the Törnqvist index is a good approximation of the COLI up to the second order (Diewert 1976). However, as pointed out by Feenstra and Shapiro

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temporary sales "attract high purchases only when they are accompanied by advertising, and this tends to occur in the final weeks of a sale." If such advertisement informs households of when the sale ends, there is no incentive for household producers to stockpile except for the final day of the sale. Our model shows that in this case chain drift may potentially (but not necessary) go in an upward direction.

(2003) and Ivancic, Diewert, and Fox (2011) among others, the purchase-weighted Törnqvist price index entails a bias, which stems from using quantities purchased rather than quantities consumed.[28]

The above lemma also suggests that using purchase weights is not the sole reason for the bias. Using consumption weights does not eliminate the bias from the Törnqvist index (i.e., $\pi^{T*} < 0$ if $\sigma > 1$) because the path of consumption prices is asymmetric. Specifically, the model shows that the consumption price decreases quickly when the purchase price drops at the start of a sale but then increases again only gradually when the purchase price returns to the regular price after the sale ends, as a result of household inventories. Such asymmetry is not a coincidence but a natural outcome of the non-negative constraints on inventories and purchases. This asymmetric response of the consumption price and the fact that a temporary price change almost always consists of a decrease (i.e., a sale) mean that although the Törnqvist price index is a good approximation of the COLI up to the second order, the third-order approximation error cannot be eliminated if $\sigma > 1$, so that the Törnqvist price index continues to entail a downward bias even when it is constructed based on consumption weights.

The elasticity of substitution σ influences the sign of the chain drift in purchase-weighted indices. When $\sigma < 1$, the sign of the chain drift changes for the Paasche and Laspeyres indices changes: the Paasche index has a positive bias, while the Laspeyres index has a negative bias. Interestingly, the sign of the bias in the Törnqvist index is negative irrespective of σ.

A.6 The Sign and Magnitude of the Bias in Chained Price Indices

The next lemma shows how the bias in the purchase-weighted Törnqvist index depends on variables associated with temporary sales. We consider price changes over a sufficiently long period (e.g., 30 years), which we denote by $\overline{\pi^T}$. The following lemma shows that $\overline{\pi^T}$ depends on the probability of sales and the ratio of sale prices to regular prices, that is, $\overline{\pi^T} = \overline{\pi^T}(\overline{q}, \underline{q}, P_L/P_H)$.

Lemma 6 *Suppose $\sigma > 1$ and $m > 0$. Then $\overline{\pi^T} = \overline{\pi^T}(\overline{q}, \underline{q}, P_L/P_H)$ is decreasing in $\overline{q}$, increasing in $\underline{q}$, and increasing in P_L/P_H.*

The result that $\overline{\pi^T}$ increases in $\underline{q}$ may appear counterintuitive. The reason for this is that

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[28]Chain drift is frequently discussed in the context of a nonlinear correlation between changes in prices and changes in quantities purchased (price/quantity bouncing), which implies that the weight of a product in a price index differs at the time of a positive price change and at the time of a negative price change. Our model shows that stockpiling causes such price/quantity bouncing as a result of intertemporal substitution. A similar argument has been made by, for example, Triplett (2003), de Haan and van der Grient (2011), and von Auer (2019). Also note that the relationship between the Laspeyres and Paasche indices that $\pi^P < 0 < \pi^L$ holds even when no stockpiling occurs. One could therefore argue that the chain drift does not necessarily occur due to stockpiling. However, the bias in the Törnqvist price index, $\pi^T < 0$, cannot be explained without stockpiling.

an increase in $\underline{q}$ lengthens the duration of a sale. This decreases the number of sales events and therefore increases $\overline{\pi^T}$.

The following lemma shows that the sign of the bias in the Törnqvist index can be positive in the following case.

Lemma 7 *Suppose that households know when a sale will end once it begins. Then π^T is positive if m exceeds a certain threshold and negative otherwise.*

As noted, the bias in chained price indices is caused by the asymmetry in the quantity purchased. When the asymmetry has the opposite sign, that is, when the quantity purchased on the first day of a sale is smaller than or equal to that on the final day of a sale, the sign of the bias in the Törnqvist index can be positive.

A.7 COLI

To resolve the bias in chained indices and derive a better approximation of the COLI, we need a superlative index that takes the elasticity of substitution σ into account. One candidate is the order r superlative index, where we define P_r as

$$P_r(r_0, r_1, c_0, c_1) = \frac{\left\{\sum_{k \in K_0 \cap K_1} s_0^k \left(r_1^k/r_0^k\right)^{(1-\sigma)}\right\}^{1/\{2(1-\sigma)\}}}{\left\{\sum_{k \in K_0 \cap K_1} s_1^k \left(r_0^k/r_1^k\right)^{(1-\sigma)}\right\}^{1/\{2(1-\sigma)\}}}, \tag{21}$$

where s_t^k represents the consumption share of product k in period t.

The following lemma shows that P_r serves as a COLI if the unit cost function is expressed as

$$\mathcal{C}(r_t) = \left[\sum_{i \in K_0 \cap K_1} \sum_{k \in K_0 \cap K_1} \alpha^{ik} \left(r_t^i\right)^{(1-\sigma)} \left(r_t^k\right)^{(1-\sigma)}\right]^{1/\{2(1-\sigma)\}} \tag{22}$$

where $\alpha^{ik} = \alpha^{ki}$. This cost function is based on a more generalized form of utility than that given by equation (1).[29]

Lemma 8 *Given the unit cost function of (22), P_r equals $\mathcal{C}(r_1)/\mathcal{C}(r_0)$.*

References

[1] Abe, Naohito, and Akiyuki Tonogi (2010) "Micro and Macro Price Dynamics in Daily Data," *Journal of Monetary Economics*, 57(6), 716–728.

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[29]Another index that can serve as a COLI is the Lloyd–Moulton Index. See ILO et al. (2004).

[2] Baker, Scott R., Stephanie Johnson, and Lorenz Kueng (forthcoming) "Shopping for Lower Sales Tax Rates," *American Economic Journal: Macroeconomics*.

[3] Baker, Scott R, Stephanie G Johnson, and Lorenz Kueng (2020) "Financial Returns to Household Inventory Management," NBER Working Paper 27740.

[4] Blattberg, Robert, and Scott Neslin (1989) "Sales Promotion: The Long and the Short of It," *Marketing Letters*, 1(1), 81–97.

[5] Boizot, Christine, Jean-Marc Robin, and Michael Visser (2001) "The Demand for Food Products. An Analysis of Interpurchase Times and Purchased Quantities," *Economic Journal*, 111, 391–419.

[6] Cashin, David, and Takashi Unayama (2016) "Measuring Intertemporal Substitution in Consumption: Evidence from a VAT Increase in Japan," *Review of Economics and Statistics*, 98(2), 285–297.

[7] Cashin, David, and Takashi Unayama (2021) "The Spending and Consumption Response to A VAT Rate Increase," *National Tax Journal*, 74(2), 313–346.

[8] Chevalier, Judith, and Anil Kashyap (2019) "Best Prices: Price Discrimination and Consumer Substitution," *American Economic Journal: Economic Policy*, 11(1), 126–159.

[9] Coibion, Olivier, Yuriy Gorodnichenko, and Dmitri Koustas (forthcoming) "Consumption Inequality and the Frequency of Purchases," *American Economic Journal: Macroeconomics*.

[10] de Haan, Jan (2008) "Reducing Drift in Chained Superlative Price Indexes for Highly Disaggregated Data," presented at the EMG Workshop.

[11] Diewert, W. Erwin (1976) "Exact and Superlative Index Numbers," *Journal of Econometrics*, 4(2), 115–145.

[12] Diewert, W. Erwin and Kevin J. Fox (2020) "Substitution Bias in Multilateral Methods for CPI Construction," *Journal of Business & Economic Statistics*, forthcoming.

[13] Eichenbaum, Martin, Nir Jaimovich, and Sergio Rebelo (2011) "Reference Prices, Costs and Nominal Rigidities," *American Economic Review* , 101, 234–262.

[14] Erdem, Tulin, Susumu Imai, and Michael Keane (2003) "A Model of Consumer Brand and Quantity Choice Dynamics under Price Uncertainty," *Quantitative Marketing and Economics*, 1(1), 5–64.

[15] European Free Trade Association (EFTA) (2020) "Transaction Data in the Consumer Price Index," https://www.efta.int/Statistics/news/Transaction-data-consumer-price-index-518411.

[16] Eurostat (2017) "Practical Guide for Processing Supermarket Scanner Data," https://circabc.europa.eu/sd/a/8e1333df-ca16-40fc-bc6a-1ce1be37247c/Practical-Guide-Supermarket-Scanner-Data-September-2017.pdf.

[17] Feenstra, Robert, and Matthew Shapiro (2003) "High-Frequency Substitution and the Measurement of Price Indexes," in Robert Feenstra and Matthew Shapiro (eds.), *Scanner Data and Price Indexes*, University of Chicago and NBER.

[18] Frisch, Ragnar (1936) "Annual Survey of General Economic Theory: The Problem of Index Numbers," *Econometrica*, 4(1), 1–38.

[19] Goolsbee, Austan D., and Peter J. Klenow. (2018) "Internet Rising, Prices Falling: Measuring Inflation in a World of E-Commerce," *AEA Papers and Proceedings*, 108, 488–492.

[20] Gowrisankaran, Gautam, and Marc Rysman (2012) "Dynamics of Consumer Demand for New Durable Goods," *Journal of Political Economy*, 120, 1173–1219.

[21] Griffith, Rachel, Aviv Nevo, Ephraim Leibtag, and Andrew Leicester (2009) "Consumer Shopping Behavior: How Much Do Consumers Save?" *Journal of Economic Perspectives*, 23, 99–120.

[22] de Haan, Jan, and Heymerik A. van der Grient (2011) "Eliminating Chain Drift in Price Indexes Based on Scanner Data," *Journal of Econometrics*, 161, 36–46.

[23] de Haan, Jan, and Frances Krsinich (2014) "Scanner Data and the Treatment of Quality Change in Nonrevisable Price Indexes," *Journal of Business & Economic Statistics*, 32(3), 341–358.

[24] Hansman, Christopher, Harrison Hong, Aureo de Paula, and Vishal Singh (2020) "A Sticky-Price View of Hoarding," NBER Working Paper No. 27051.

[25] Hendel, Igal, Alessandro Lizzeri, and Nikita Roketskiy (2014) "Nonlinear Pricing of Storable Goods," *American Economic Journal: Microeconomics*, 6(3), 1–34.

[26] Hendel, Igal, and Aviv Nevo (2003) "The Post-Promotion Dip Puzzle: What Do the Data Have to Say?" *Quantitative Marketing and Economics*, 1, 409–424.

[27] Hendel, Igal, and Aviv Nevo (2004) "Intertemporal Substitution and Storable Products," *Journal of the European Economic Association*, 2(2/3), 536–547.

[28] Hendel, Igal, and Aviv Nevo (2006a) "Sales and Consumer Inventory," *RAND Journal of Economics*, 37, 543–561.

[29] Hendel, Igal, and Aviv Nevo (2006b) "Measuring the Implications of Sales and Consumer Inventory Behavior," *Econometrica*, 74, 1637–1673.

[30] Hendel, Igal, and Aviv Nevo (2013) "Intertemporal Price Discrimination in Storable Goods Markets," *American Economic Review*, 103(7), 2722–2251.

[31] Hong, Pilky, Preston McAfee, and Ashish Nayyar (2002) "Equilibrium Price Dispersion with Consumer Inventories," *Journal of Economic Theory*, 105, 503–517.

[32] ILO, IMF, OECD, Eurostat, United Nations, and World Bank (2004) "Consumer Price Index Manual: Theory and Practice," ILO Publications, Geneva.

[33] ILO, IMF, OECD, Eurostat, United Nations, and World Bank (2020) "Consumer Price Index Manual: Concepts and Methods," draft version (January), https://www.imf.org/en/Data/Statistics/cpi-manual.

[34] Ivancic, Lorraine, W. Erwin Diewert, and Kevin J. Fox (2011) "Scanner Data, Time Aggregation and the Construction of Price Indexes," *Journal of Econometrics*, 161, 24–35.

[35] Kahn, James A. (2016) "Inventory Investment," *New Palgrave Dictionary of Economics*, Palgrave Macmillan, https://doi.org/10.1057/978-1-349-95121-5_2283-1.

[36] Kano, Kazuko (2018) "Consumer Inventory and Demand for Storable Goods: New Evidence from a Consumer Survey," *Japanese Economic Review*, 69(3), 284–305.

[37] Kehoe, Patrick J., and Virgiliu Midrigan (2015) "Prices Are Sticky After All," *Journal of Monetary Economics*, 75, 35–53.

[38] Klenow, Peter J., and Jonathan L. Willis (2007) "Sticky Information and Sticky Prices," *Journal of Monetary Economics*, 54(Supplement), 79–99.

[39] Könus A. A. (1924) "The Problem of the True Index of the Cost of Living," in *Economic Bulletin of the Institute of Economic Conjuncture* (in Russian), 9–10, 64–71; published in English (1939) in *Econometrica*, 7, 10–29.

[40] Konny, Crystal G., Brendan K. Williams, and David M. Friedman (2019) "Big Data in the U.S. Consumer Price Index: Experiences and Plans," NBER Chapters, in *Big Data for Twenty-First Century Economic Statistics*, NBER.

[41] Kryvtsov, Oleksiy, and Nicolas Vincent (2020) "The Cyclicality of Sales and Aggregate Price Flexibility," *Review of Economic Studies*, forthcoming.

[42] Nakamura, Alice O., Emi Nakamura, and Leonard I. Nakamura (2011) "Price Dynamics, Retail Chains and Inflation Measurement," *Journal of Econometrics*, 161, 47–55.

[43] Nakamura, Emi, and Jon Steinsson (2010) "More Facts about Prices," Columbia University. https://eml.berkeley.edu/~jsteinsson/papers/fivefactssupplement.pdf. Accessed April 30, 2020.

[44] Neslin, Scott A., and Linda G. Schneider Stone (1996) "Consumer Inventory Sensitivity and the Postpromotion Dip," *Marketing Letters*, 7, 77–94.

[45] Ogaki, Masao, and Carmen M. Reinhart (1998) "Measuring Intertemporal Substitution: The Role of Durable Goods," *Journal of Political Economy*, 106(5), 1078–1098.

[46] Osborne, Matthew (2018) "Approximating the Cost-of-Living Index for a Storable Good," *American Economic Journal: Microeconomics*, 10(2), 286–314.

[47] Reinsdorf, Marshall B. (1999) "Using Scanner Data to Construct CPI Basic Component Indexes," *Journal of Business and Economic Statistics*, 17(2), 152–160.

[48] Reis, Ricardo (2009) "A Dynamic Measure of Inflation," Working Paper.

[49] Salop, Steven C., and Joseph E. Stiglitz (1982) "The Theory of Sales: A Simple Model of Equilibrium Price Dispersion with Identical Agents," *American Economic Review*, 72(5), 1121–1130.

[50] Sheremirov, Viacheslav (2020) "Price Dispersion and Inflation: New Facts and Theoretical Implications," *Journal of Monetary Economics*, 114, 59–70.

[51] Sudo, Nao, Kozo Ueda, and Kota Watanabe (2014) "Micro Price Dynamics during Japan's Lost Decades," *Asian Economic Policy Review*, 9(1), 44–64.

[52] Sudo, Nao, Kozo Ueda, Kota Watanabe, and Tsutomu Watanabe (2018) "Working Less and Bargain Hunting More: Macroimplications of Sales during Japan's Lost Decades," *Journal of Money, Credit and Banking*, 50(2–3), 449–478.

[53] Szulc, Bohdan J. (1983) "Linking Price Index Numbers," In *Price Level Measurement*, W. E. Diewert and C. Montmarquette (eds). Ottawa: Statistics Canada.

[54] Triplett, Jack E. (2003) "Using Scanner Data in the Consumer Price Index: Some Neglected Conceptual Considerations," in Robert Feenstra and Matthew Shapiro (eds.), *Scanner Data and Price Indexes*, University of Chicago and NBER.

[55] Ueda, Kozo, Kota Watanabe, and Tsutomu Watanabe (2019) "Product Turnover and the Cost of Living Index: Quality vs. Fashion Effects," *American Economic Journal: Macroeconomics*, 11(2), 310–347.

[56] Ueda, Kozo (2020) "Dynamic Cost of Living Index for Storable Goods," *Economics Letters*, 189, 109013.

[57] von Auer, Ludvig (2019) "The Nature of Chain Drift," presented at the Ottawa Group Meeting.

[58] Wang, Emily Yucai (2013) "The Effect of More Accurate Cost-of-Living Indexes on Welfare Recipients," Working Paper.

[59] Watanabe, Kota, and Tsutomu Watanabe (2014) "Estimating Daily Inflation Using Scanner Data: A Progress Report," CARF Working Paper F342, University of Tokyo.

[60] Zhang, Li-Chun, Ingvild Johansen, and Ragnhild Nygaard (2019) "Tests for Price Indices in a Dynamic Item Universe," *Journal of Official Statistics*, 35(3), 683–697.

Table 1: Chain Drift and Inflation Rate

dt	Price indices		
	Törnqvist (purchase-weighted)	Törnqvist (consumption-weighted)	Order r superlative (consumption-weighted)
	Annual chain drift		
1	-40.44***	-5.69***	-1.20
7	-7.24***	-2.28***	-1.23***
14	-2.43***	-1.10***	-0.59***
28	-0.97***	-0.46***	-0.04
52	-0.66***	-0.20**	0.23
91	-0.61***	0.06	0.17
182	-0.49***	0.01	0.14
	Annualized inflation rate		
1	-46.34	-13.06	-10.71

Note: The figures for chain drift show the averages for the period from 1989 to 2019. See the main text for an explanation of the calculation of the chain drift, $d_{0,365,dt}$, where dt represents the interval. We transform $d_{0,365,dt}$, which is on a daily basis, to an annualized rate as follows: $\exp(365 \times d_{0,365,dt}) - 1$. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively, when we apply the sign test for the null hypothesis that the probabilities that the chain drift is positive or negative are each equal to 0.5. The annualized inflation rate is calculated as $\exp(365 \times \overline{x_{dt=1}}) - 1$, where $\overline{x_{dt=1}}$ is the mean of daily log inflation from 1990 to 2018.

Table 2: Relationship between Chain Drift and Stockpiling

	(1)	(2)	(3)	(4)
		Dependent variable:		
		Chain drift of purchase-weighted Törnqvist		
Q_H	0.698**			
	(0.315)			
Q_L	1.781***			
	(0.463)			
q		−1.916***		
		(0.483)		
$\overline{q}$			−0.807	−2.190**
			(1.039)	(1.089)
$\underline{q}$			−0.708***	0.014
			(0.154)	(0.259)
$\log(P_L/P_H)$		3.979***	3.929***	2.899***
		(0.644)	(0.618)	(0.673)
$\log m$				−0.344***
				(0.101)
Constant	−0.320***	0.178***	0.418***	0.223**
	(0.039)	(0.062)	(0.075)	(0.093)
Adjusted R^2	0.102	0.400	0.440	0.470
Observations	189	195	195	195

Note: Figures in parentheses represent standard errors. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. The explanatory variables are as follows: Q_H and Q_L represent the degree of asymmetry in the quantity purchased when the price of a product increases and when it decreases, q is the probability that a product is on sale, $\overline{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $\underline{q}$ is the probability that a product will continue to be on sale on the following day given that it is currently on sale, P_L/P_H is the ratio of the sale price to the regular price, and m is the degree of stockpiling.

Table 3: State-dependent Consumption

Dependent variable	1 if household uses product	Number of times a product is used
Inventory	0.0006895***	0.00410***
	(0.0000171)	(0.000901)
Observations	90,545,020	90,545,020
No. of households	3,602	3,602
R^2	0.12	0.12
Fixed effects	household/category	household/category

Note: Figures in parentheses represent robust standard errors. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 4: Top and Bottom Five Categories with Regard to the Degree of Stockpiling

	Product category	m		Product category	m
Top 5	Instant cup noodles	4.98	Bottom 5	Razors	1.15
	Diluted beverages	3.74		Prepared bread meals	1.19
	Frozen meals	3.58		Cosmetic accessories	1.26
	Packaged instant noodles	3.23		Home medical supplies	1.27
	Coffee beverages	3.08		Batteries	1.30

Note: The degree of stockpiling m is inferred using the POS data.

Table 5: Regression of the Degree of Stockpiling

	(1)	(2)		(3)
	$\log(m)$	$\log(m)$		$\Delta\log(m)$
$\log(1-\overline{q})$	0.6254	-0.9757*	$\Delta\log(1-\overline{q})$	-2.4682***
	(0.428)	(0.547)		(0.607)
$\log(1-\underline{q})$	-0.6270***	-0.7785***	$\Delta\log(1-\underline{q})$	-0.5104***
	(0.035)	(0.034)		(0.036)
$\log(1-P_L/P_H)$	0.3511***	0.3057***	$\Delta\log(1-P_L/P_H)$	0.2546***
	(0.044)	(0.046)		(0.032)
Fixed effects	category	category/month		category
Observations	54,000	54,000		53,850
Within R^2	0.650	0.753		0.283
No. of categories	150	150		150

Note: Variable m is the degree of stockpiling, $\overline{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $\underline{q}$ is the probability that a product will continue to be on sale on the following day given that it is currently on sale, and P_L/P_H is the ratio of the sale price to the regular price. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 6: Effects of Macroeconomic Variables on Stockpiling Behavior

|	(1)	(2)	(3)	(4)	(5)
	$\Delta\log(m)$	$\Delta\log(1-\overline{q})$	$\Delta\log(1-\underline{q})$	$\Delta\log(1-P_L/P_H)$	$\Delta\log(m)$
Δ(unemp rate)	0.0026	-0.0017***	-0.0338***	0.0277***	0.0312***
	(0.004)	(0.000)	(0.003)	(0.003)	(0.003)
Δ(unemp rate(-1))	-0.0106***	0.0007***	0.0244***	-0.0300***	-0.0327***
	(0.004)	(0.000)	(0.003)	(0.003)	(0.004)
$\Delta\log$(hours worked)	-0.2940***	0.0210***	0.1341***	0.1989***	-0.3612***
	(0.043)	(0.002)	(0.030)	(0.033)	(0.043)
$\Delta\log$(hours worked(-1))	-0.6677***	0.0077***	-0.6106***	0.4365***	-0.2602***
	(0.046)	(0.001)	(0.044)	(0.040)	(0.044)
Δ(real r)	-0.0133***	0.0009***	0.0147***	-0.0052***	-0.0243***
	(0.003)	(0.000)	(0.002)	(0.002)	(0.002)
Δ(real r(-1))	-0.0036*	-0.0002***	-0.0086***	0.0037**	0.0024
	(0.002)	0.000	(0.002)	(0.002)	(0.002)
$\Delta\log(1-\overline{q})$	-2.4013***				-2.4013***
	(0.615)				(0.615)
$\Delta\log(1-\underline{q})$	-0.5117***				-0.5117***
	(0.036)				(0.036)
$\Delta\log(1-P_L/P_H)$	0.2604***				0.2604***
	(0.032)				(0.032)
Fixed effects	category	category	category	category	category
Observations	53,700	53,700	53,700	53,700	53,700
Within R^2	0.289	0.008	0.008	0.005	0.289
No. of categories	150	150	150	150	150

Note: Variable m is the degree of stockpiling, $\overline{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $\underline{q}$ is the probability that a product will continue to be on sale on the following day given that it is currently on sale, P_L/P_H is the ratio of the sale price to the regular price, and real r is the real interest rate. In column (5), the explanatory variables corresponding to $\Delta\log(1-\overline{q})$, $\Delta\log(1-\underline{q})$, and $\Delta\log(1-P_L/P_H)$ are the residuals of the estimation for columns (2), (3), and (4), respectively. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Figure 1: Pattern of Price and Quantity Changes during a Sales Event
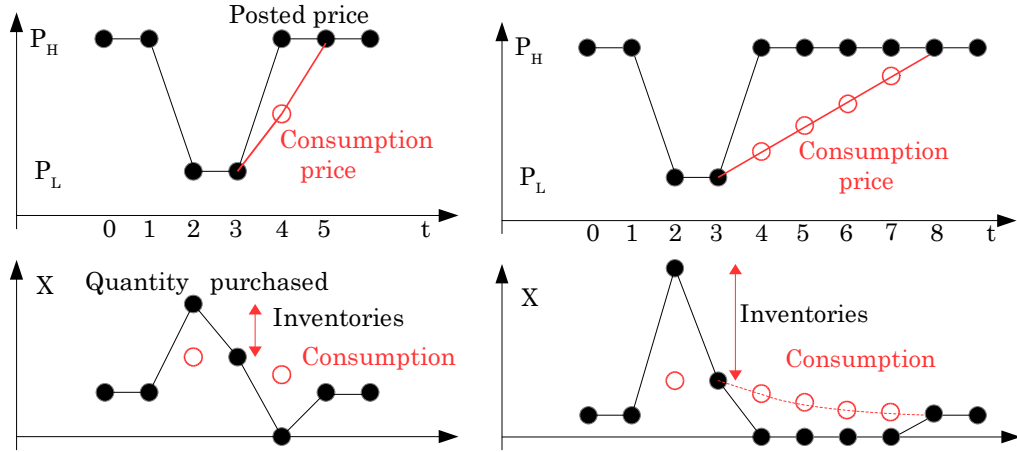
Note: The solid dots represent observable posted prices (top) and quantities purchased (bottom). The circles represent unobservable consumption prices (top) and quantities consumed (bottom).

Figure 2: Consumption Pattern of Salt
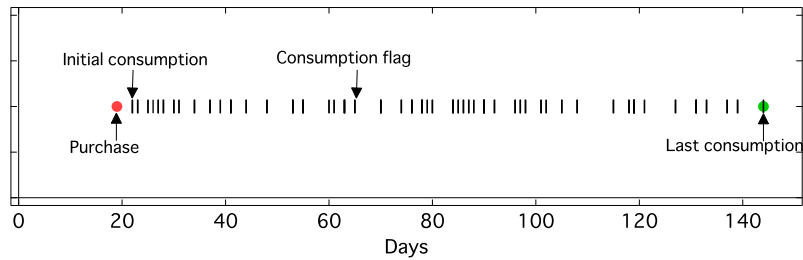
Note: The figure shows the consumption pattern for salt of a particular household in the *Shoku-map* data. Each vertical line represents a consumption flag.

Figure 3: Asymmetry in the Quantity Purchased When the Price Increases and When It Decreases
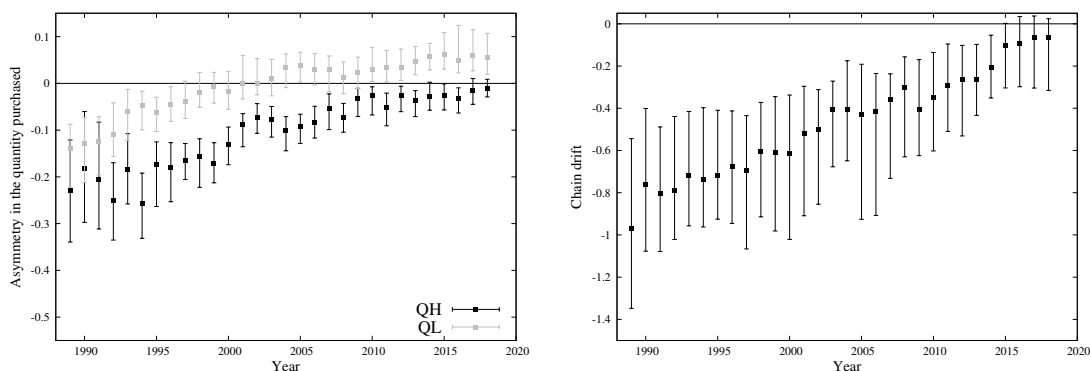
Note: In the left-hand panel, Q_H and Q_L represent the degree of asymmetry in the quantity purchased when the price increases and when it decreases. Specifically, Q_H indicates the asymmetry in the quantity purchased between just before a sale starts and just after the sale ends, while Q_L indicates the asymmetry in the quantity purchased between just after a sale starts and just before the sale ends. The black squares and the horizontal bars at the end of the vertical lines represent the median as well as the 25th and 75th percentiles of products and sale events in each year. The right-hand panel shows the degree of chain drift, $d_{0,365,1}$, for each year.

Figure 4: Price Changes in Chained Indices
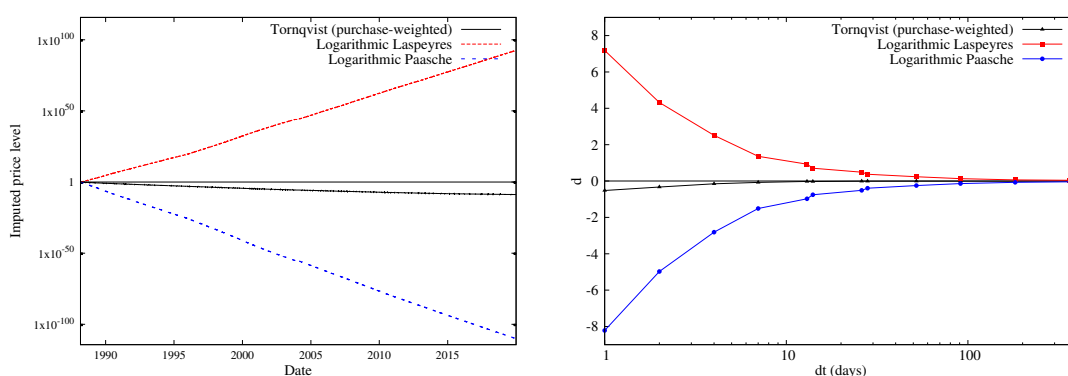
Note: The left-hand panel shows the time-series of price levels based on the purchase-weighted Laspeyres, Paasche, and Törnqvist indices using the POS data. The initial price level is normalized to one. The right-hand panel shows the degree of chain drift, $d_{0,365,dt}$, based on the purchase-weighted Laspeyres, Paasche, and Törnqvist indices, where we employ different intervals dt from 1 day to 364 days.
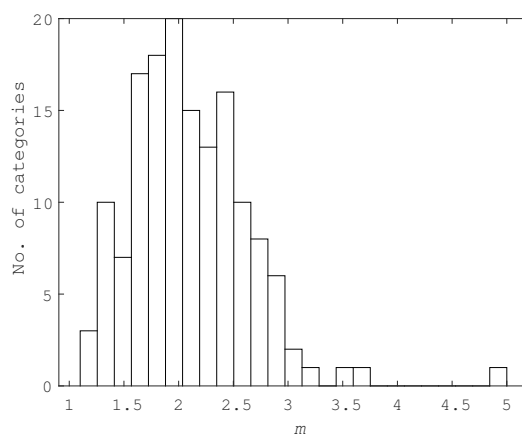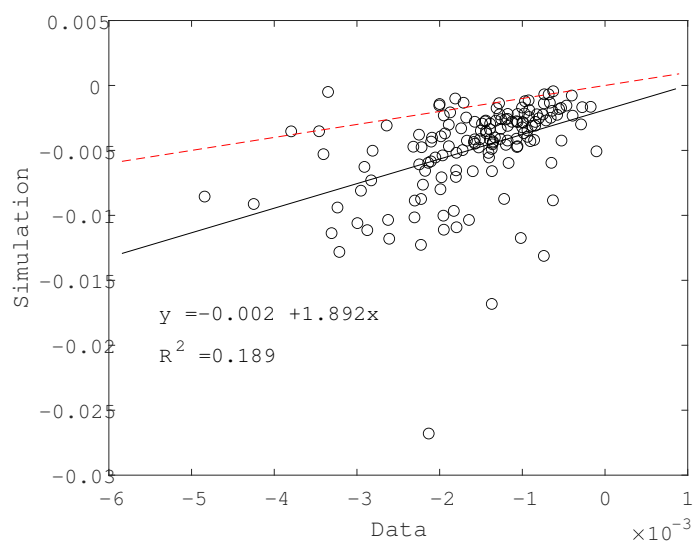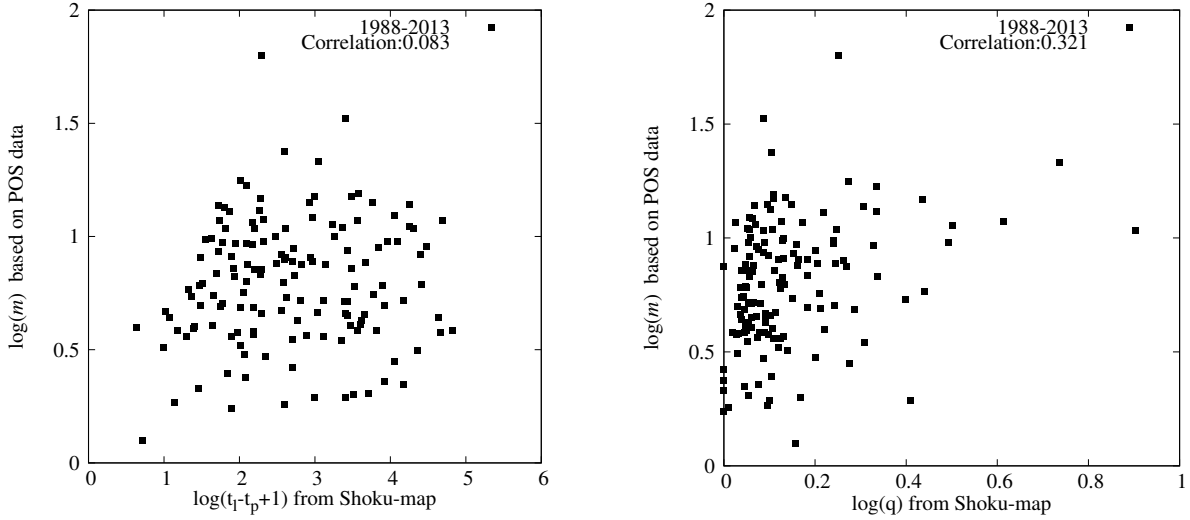
Figure 5: Histogram of the Degree of Stockpiling

Figure 6: Inflation Rates Based on the Törnqvist Index: Data and Simulation

$$y = -0.002 + 1.892x$$
$$R^2 = 0.189$$

Note: Each circle represents a 3-digit product category. The inflation rates are the daily averages and are based on the purchase-weighted Törnqvist index. The red dashed line represents the 45 degree line.

Figure 7: Correlation Between Length of Time Until Product is Consumed/Quantity Purchased and Degree of Stockpiling
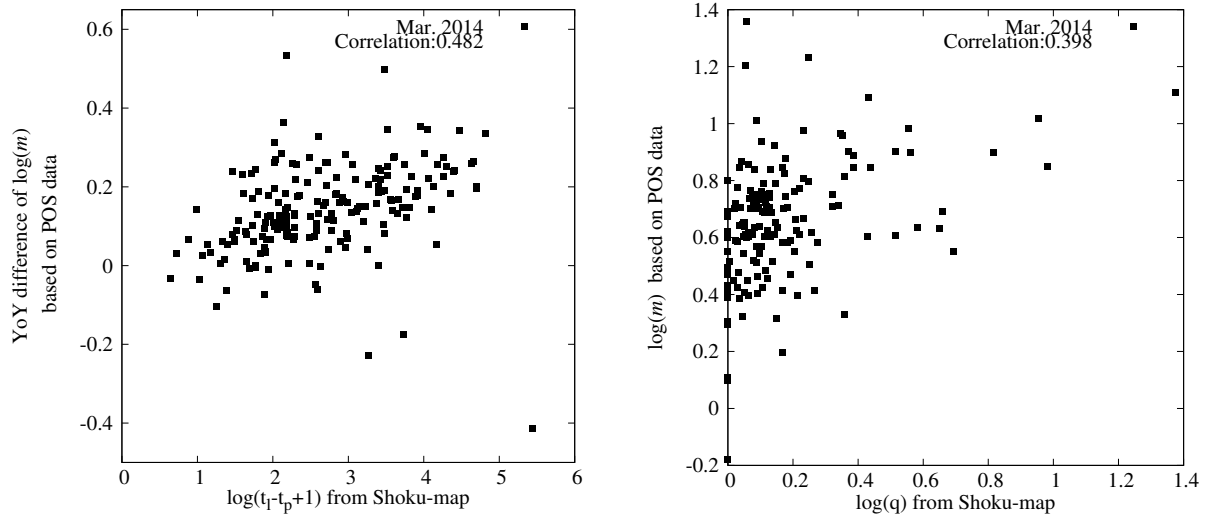

Note: Each dot represents a 3-digit product category. In the left-hand panel, the horizontal axis represents the log of the length of time until a product is used up, which is defined as the difference between the date of purchase (t_p) and the date the household finishes the product (t_l) plus one. In the right-hand panel, the horizontal axis represents the log of the quantity purchased.

Figure 8: Elasticity of Substitution
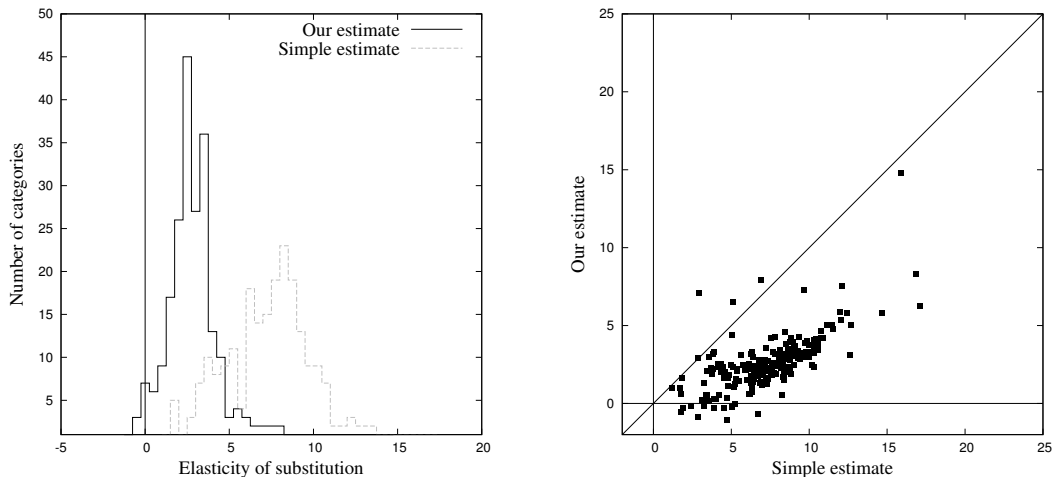
Note: "Our estimate" represents our calculation of the elasticity of substitution σ from $\Gamma \equiv -\log\left(c_L/c_H\right)/\log\left(r_L/r_H\right)$ using the inferred series of consumption c and consumption price r. "Simple estimate" represents the calculation of σ from $-\Delta\log X_t/\Delta\log p_t$, where X and p represent the quantity purchased and the posted price, respectively. The left-hand panel shows the histogram of the values of σ for 3-digit product categories, while the right-hand panel shows a scatter plot where each dot represents a 3-digit product category.

Figure 9: Correlation Between Length of Time Until Product is Consumed/Quantity Purchased and Degree of Stockpiling: One Month Before the Consumption Tax Increase

Note: Each dot represents a 3-digit product category. In the left-hand panel, the horizontal axis represents the log of the length of time until a product is used up, which is defined as the difference between the date of purchase (t_p) and the date the household finishes the product (t_l) plus one. In the right-hand panel, the horizontal axis represents the log of the quantity purchased.

Figure 10: Aggregate Time-Series of the Degree of Stockpiling
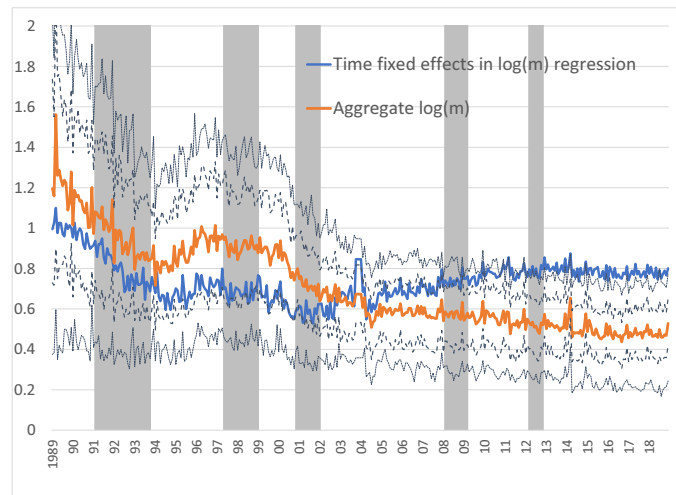
Note: The solid thick line denoted as "Aggregate log(m)" represents the mean of $\log(m)$ for all 3-digit product categories, while the thin lines represent the 10th, 25th, 75th, and 90th percentiles of $\log(m)$. The shaded areas indicate recession periods as identified by the Cabinet Office.