Spatial Production Networks*

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The views and opinions expressed are those of the authors alone and do not necessarily reflect those of the Central Bank of Chile.

Motivation

- Production networks geographically complex
  - Fragmented across countries, regions, firms: "Global Value Chains"
  - Key to countries' & regions' economic success (World Bank '19)

- "Macro" and "micro" approaches (Johnson '18, Antras-Chor '21)
  - **Macroeconomics** determined by production network across countries and regions
  - **Microeconomics** of how firms form endogenous production networks
  - Limited understanding of how "macro" and "micro" interact across countries/regions

- Our paper studies endogenous network formation in space & their aggregate implications
  - How do production networks endogenously form across countries/regions from firm decisions?
  - How do networks endogenously respond to macro shocks, and what are aggregate implications?

This Paper

- Microfounded model of spatial production networks with tractable aggregation
  - Firms search and match with suppliers and buyers in the geographic space
  - Characterize aggregate trade flows with gravity equations in extensive and intensive margins
  - Establish existence and uniqueness, counterfactuals, sufficient statistics for welfare
- Apply this model to administrative firm-to-firm transaction level data from Chile
  - Stylized facts about spatial production networks motivating model choices
  - Calibrate to i) observed inter- & intra-national trade and ii) observed responses of production networks to import cost shock
  - Study effects of two counterfactual shocks on domestic networks and welfare
    - (1) international trade shocks on global value chain (2) domestic transportation infrastructure
    - Findings: strong responses of domestic networks, with aggregate and distributional effects

Literature

- "Macro" approach of production networks: Yi (2003, 2009); Johnson-Noguera (2012); Caliendo-Parro (2015); Johnson-Moxnes (2019); Antras-Chor (2019); Huo-Levchenko-Pandalai-Nayar (2020)
- "Micro" approach of production networks: Bernard-Moxnes (2018); Oberfield (2018); Lim (2018); Huneeus (2018); Bernard-Moxnes-Saito (2019); Dhyne-Kikkawa-Mogstad-Tintelnot (2020); Bernard-Dhyne-Magerman-Manova-Moxnes (2020); Zou (2020); Demir-Fieler-Xu-Yang (2021)
- Endogenous production networks in space: Eaton-Kortum-Kramarz (2018); Miyauchi (2021); Panigraphi (2021); Antras-de-Gortari (2020)
- Microfounded gravity trade models and sufficient statistics approach: Eaton-Kortum (2002); Eaton-Kortum-Kramarz (2011); Arkolakis-Costinot-Rodriguez-Clare (2012); Costinot-Rodriguez-Clare (2014); Melitz and Redding (2014, 2015); Ossa (2015)
- Propagation of shocks in production networks: Acemoglu-Carvalho-Ozdaglar-Tahbaz-Salehi (2012); Acemoglu-Akcigit-Kerr (2016); Carvalho-Nirei-Saito-Tahbaz-Salehi (2021); Caliendo-Parro-Rossi-Hansberg-Sarte (2018); Adao-Carrillo-Costinot-Donaldson-Pomeranz (2020)

Outline

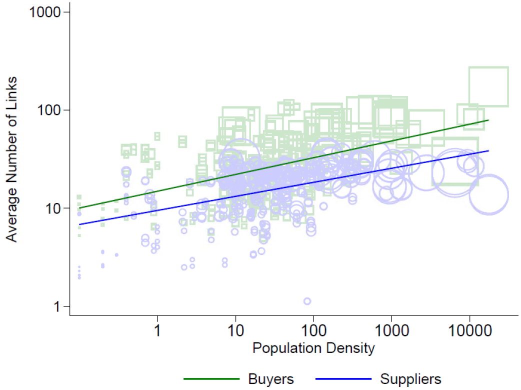
Data and Descriptive Facts

Data

- Domestic firm-to-firm transaction-level dataset in Chile
  - Collected by Internal Revenue Service for value-added tax collection purposes
  - Covers the universe of domestic trade between all firms in Chile regardless of firm size
  - For each transaction, observe dates, seller and buyer firm ID, sales, products, prices, seller's and buyer's municipality

- Linked to various firm data sets:
  - Customs data (for imports and exports)
  - Firm balance sheet characteristics (for total sales)
  - Matched employer-employee dataset (for employment and wages)

1. Number of Domestic Suppliers & Buyers per Firm Relates to Geography



- Robust to controlling for firm sales, which are by themselves strongly correlated with the number of links (Bernard et al '19; '20; Lim '18) Table
- Model supplier & buyer formation decision based on geographic location and productivity

2. Cross-Regional Trade Flows in Extensive & Intensive Margins

- Estimate the following gravity regressions (i, j are municipalities in Chile)

$$\log TradeFlows_{ij} = \beta \log Dist_{ij} + \xi_i + \zeta_j + \epsilon_{ij}$$

	Total Flows		Intensive (Volume per Relationship)		Extensive (Number of Relationships)	
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-1.324		-0.383		-0.941	
	(0.008)		(0.007)		(0.004)	
Log Time Travel		-1.515		-0.441		-1.074
		(0.010)		(0.008)		(0.004)
R^2	0.640	0.639	0.306	0.306	0.822	0.819
Origin Municipality FE	✓	✓	✓	✓	✓	✓
Destination Municipality FE	✓	✓	✓	✓	✓	✓
N	65871	65871	65871	65871	65871	65871

- Model will feature distinct gravity equations in intensive & extensive margins

3. Domestic Production Networks Respond to Import Cost Shocks

- Firm-level impacts of import shocks using shift-share design (Autor-Dorn-Hansen '13)

$$\Delta \log y_{it} = \alpha_0 + \alpha_1 \sum_{c,k} \underbrace{\Delta \log WID_{ckt}}_{c\text{'s export in } k \text{ except Chile}} \times \underbrace{w^D_{ickt_0}}_{\text{import / total input by firm } i} + \epsilon_{it},$$

  - i: firm; t: year; c: country; k: product (6-digit HS code)
  - Results below are long difference from 2007 to 2009 Robustness 2011-2016

	Imports	Exports	Sales	Domestic Suppliers Number	Domestic Suppliers Mean Value	Domestic Buyers Number	Domestic Buyers Mean Value
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Import Shock	0.566	-0.052	0.516	0.253	0.159	0.048	0.251
	(0.206)	(0.497)	(0.167)	(0.093)	(0.160)	(0.144)	(0.250)
Export Shocks	✓	✓	✓	✓	✓	✓	✓
3-digit Industry Fixed Effects	✓	✓	✓	✓	✓	✓	✓
N	9192	4201	27516	27718	27541	19600	19362

- Model will feature responses of domestic production linkages to import cost shocks

Model

Setup

- Space is partitioned by a finite number of locations $i, u, d \in N$
- Continuum of workers of measure L_i in location i (exogenous)
- Two types of goods: intermediate goods and final goods
  - Intermediate goods are traded across locations subject to iceberg trade cost $\tau_{ud} \geq 1$
  - Single final goods for each location, not traded
- Two types of producers:
  - Final goods producers
  - Intermediate goods producers (“firms”)

Production

- Unit cost of production by "firm" ω in location i

$$c^{I}(\omega)=\frac{1}{z(\omega)}w_i^{\beta}\left(\int_{\upsilon\in\Omega_\omega^I}p(\upsilon,\omega)^{1-\sigma}\,d\upsilon\right)^{\frac{1-\beta}{1-\sigma}}$$

  - $z(\omega)$ is productivity of firm ω
  - w_i is local wages
  - Ω_ω^I is the set of suppliers that ω has access to (endogenized by search and matching)
  - $p(\upsilon,\omega)$ is the price charged by supplier υ to ω
  - σ is the elasticity of substitution for intermediate goods

- Continuum of suppliers $\Omega_\omega^I \Rightarrow p(\upsilon,\omega)$ constant markup over marginal cost of υ
- Final goods producers produce using all local intermediate goods (without search frictions) with elasticity of substitution σ under perfect competition

Search and Matching Between Firms: Overview

- Production networks linkage are endogenous under search and matching process
- Firms post advertisements for suppliers and buyers across locations to maximize anticipated profits (Arkolakis '10; Demir-Fieler-Xu-Yang '21)
- Aggregate random matching technology for each pair of locations à la DMP

Firms' Search Decision

$$\pi_i(z) = \max_{\{n^S_{ui}\}_u, \{n^B_{id}\}_d} \frac{1}{\sigma} \sum_{d \in N} n^B_{id} m^B_{id} D_d (c\tau_{id})^{1-\sigma} - e_i \left\{ \sum_{d \in N} f^B_{id} \frac{\left(n^B_{id}\right)^{\gamma^B}}{\gamma^B} + \sum_{u \in N} f^S_{ui} \frac{\left(n^S_{ui}\right)^{\gamma^S}}{\gamma^S} \right\}$$

$$\text{subject to} \quad c = \frac{w_i^{\beta} \left(\sum_{u \in N} n^S_{ui} m^S_{ui} (C_{ui})^{1-\sigma} \right)^{\frac{1-\beta}{1-\sigma}}}{z}$$

- $\{n^S_{ui}\}_u, \{n^B_{id}\}_d$: number of postings to suppliers and buyers
- m^S_{ui}, m^B_{ui}: matching rates with suppliers and buyers
- e_i: unit price of advertisement services
- $f^B_{id}, f^S_{ui}, \gamma^B, \gamma^S$: exogenous parameters for search cost
- C_{ui}: average cost of suppliers from u to i
- No profits from sales to final goods producers (assume zero bargaining power)

Solution to Firms' Search Problem

- Optimal advertisements:

$$n_{ui}^{S}(z) = a_{ui}^{S} z^{\frac{\delta_1}{\gamma^S}}, \; n_{id}^{B}(z) = a_{id}^{B} z^{\frac{\delta_1}{\gamma^B}}$$

  - $\delta_1 \equiv (\sigma - 1)/(1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S})$
  - a_{ui}^S, a_{id}^B are functions of demand shifter, cost shifter and search costs
  - Geographic factors matter for supplier and buyer linkages on top of z (Fact 1)

- Unit cost:

$$c_i(z) = (C_i^*)\, z^{-\frac{\delta_1}{\gamma^S}\frac{1-\beta}{\sigma-1}-1}; \;\; (C_i^*)^{1-\sigma} \equiv w_i^{\beta(1-\sigma)} \left(\sum_{u \in N} a_{ui}^S m_{ui}^S (C_{ui})^{1-\sigma} \right)^{1-\beta}$$

- Firm revenue:

$$r_i(z) = D_i^* (C_i^*)^{1-\sigma} (z)^{\delta_1}; \quad D_i^* = \sum_d m_{id}^B a_{id}^B D_d^I (\tau_{id})^{1-\sigma}$$

Matching Between Suppliers and Buyers

- Aggregate supplier and buyer postings:

$$\overline{M}^S_{ud} = N_d \int n^S_{ud}(z) dG_d(z), \quad \overline{M}^B_{ud} = N_u \int n^B_{ud}(z) dG_u(z)$$

  - N_i: measure of firms in location i
  - $G_i(\cdot)$: productivity distribution in location i

- Total number of supplier-to-buyer relationships determined by matching function:

$$M_{ud} = \kappa_{ud} \left(\overline{M}^S_{ud}\right)^{\lambda^S} \left(\overline{M}^B_{ud}\right)^{\lambda^B}$$

- Matching probability (intensity):

$$m^S_{ud} = \frac{M_{ud}}{\overline{M}^S_{ud}} \quad m^B_{ud} = \frac{M_{ud}}{\overline{M}^B_{ud}}$$

Gravity Equations of Bilateral Trade Flows: Extensive and Intensive Margin

- Total number of relationships and average transaction volume from u to d :

$$M_{ud} = \chi^E_{ud}\zeta^E_u\xi^E_d \quad (Extensive\ Margin)$$
$$\bar{r}_{ud} = \chi^I_{ud}\zeta^I_u\xi^I_d \quad (Intensive\ Margin)$$

  - $\chi^E_{ud} = \varrho^E \left[\kappa_{ud}\left(f^B_{ud}\right)^{-\frac{\lambda^B}{\gamma^B}}\left(f^S_{ud}\right)^{-\frac{\lambda^S}{\gamma^S}}\left(\tau^{1-\sigma}_{ud}\right)^{\frac{\lambda^B}{\gamma^B}+\frac{\lambda^S}{\gamma^S}}\right]^{\left(1-\frac{\lambda^S}{\gamma^S}-\frac{\lambda^B}{\gamma^B}\right)^{-1}}$, $\chi^I_{ud} = (\tau_{ud})^{1-\sigma}$
  - ζ^E_u and ζ^I_u capture cost shifters ⇒ Supplier effects
  - ξ^E_d and ξ^I_d capture demand shifters ⇒ Buyer effects

- Different spatial structure of "extensive" and "intensive" margins (Fact 2)
  - Eaton-Kortum-Kramarz '18 predict no response of $\bar{r}_{ud}$ on χ^I_{ud} and ζ^I_u due to selection
  - Continuum of suppliers with imperfect substitutes ⇒ intensive margin responds to iceberg costs

General Equilibrium Analysis

General Equilibrium

General equilibrium is defined by:

- Search intensity (a^S_{ui}, a^B_{id})
- Gravity equations ($M_{ud}, \overline{r}_{ud}$)
- Goods market clearing (C^*_u, D_d, D^*_i)
- Labor market clearing / trade balance (w_i)
- Free firm entry (N_i)
- Unit cost of advertisement service (e_i)

$$e_i = A_i (w_i)^{\mu} (C^*_i)^{1-\mu},$$

Characterizing Equilibrium

- Equilibrium reduced to a $2 \times N$ system on wages w_i and cost shifter C_i^*:
  - "Buyer access"

$$w_i = \frac{\vartheta}{L_i} \sum_d X_{id}(\{w\}, \{C^*\}, \{\chi^R\}, \{\chi^N\})$$

where $X_{id} = M_{id}\bar{r}_{id}$

  - "Supplier access"

$$(C_i^*)^{1-\sigma} = w_i^{\beta(1-\sigma)} \left[(\tilde{\sigma})^{\sigma} \mathbb{M}_i \left(\frac{\delta}{\gamma^S} \right) N_i \right]^{\beta-1} \left(\frac{\sum_u X_{ui}}{D_i} \right)^{1-\beta}$$

- Similar to previous literature while incorporating endogenous search and matching
  - Anderson and van Wincoop '03, Reddding and Venables '04, Donaldson and Hornbeck '16

Characterizing Equilibrium

- Rewriting the two equations yields:

$$(w_i)^{1+\tilde{\lambda}^B\delta_2\mu}\left(C_i^*\right)^{(\sigma-1)\delta_2+\tilde{\lambda}^B\delta_2(1-\mu)} = \sum_d K_{id}^D\left(w_d\right)^{\delta_G}\left(C_d^*\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)},$$

$$(w_i)^{1-\delta_G}\left(C_i^*\right)^{-\frac{(\sigma-1)\delta_2}{1-\beta}+\tilde{\lambda}^S\delta_2(1-\mu)} = \sum_u K_{ui}^U\left(w_u\right)^{-\tilde{\lambda}^B\delta_2\mu}\left(C_u^*\right)^{-(\sigma-1)\delta_2-\tilde{\lambda}^B\delta_2(1-\mu)},$$

  - $\delta_G = \left[\tilde{\lambda}^S\mu + \frac{1-\beta\sigma}{1-\beta}\right]\delta_2$; $\delta_2 = \left[1-\tilde{\lambda}^S-\tilde{\lambda}^B\right]^{-1}$
  - K_{id}^D and K_{ui}^U are combination of exogenous parameters, including χ_{ud}^E, χ_{ud}^I, L_i, $G_i(\cdot)$
  - $\{K_{id}^D, K_{ui}^U\}$ and $\{\sigma, \beta, \mu, \tilde{\lambda}^B(=\lambda^B/\gamma^B), \tilde{\lambda}^S(=\lambda^S/\gamma^S)\}$ sufficiently characterize the equilibrium

- Spans canonical gravity trade models with roundabout production (with $\tilde{\lambda}^B = \tilde{\lambda}^S = 0$) but not vice versa (Eaton-Kortum '02, ACR '12; Caliendo-Parro '14 (single-sector); Costinot and Rodriguez-Clare '14,...)
- Provide sufficient conditions for equilibrium existence and uniqueness Details
- Characterize counterfactual equilibrium with $\{X_{id}\}$ and $\{\sigma, \beta, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$ a la DEK Details

Sufficient Statistics for Welfare

Proposition

Proportional changes of welfare are given by:

$$\widehat{\frac{w_i}{P_i^F}} = \Big(\underbrace{\hat{\Lambda}_{ii}}_{\text{own trade share}}\Big)^{-\frac{1}{\sigma-1}\frac{1-\beta}{\beta}} \Big(\underbrace{\hat{M}_{ii}}_{\text{number of linkages within location}}\Big)^{\frac{1}{\sigma-1}\frac{1-\beta}{\beta}}$$

- $\tilde{\lambda}^B = \tilde{\lambda}^S = 0 \Rightarrow \hat{M}_{ii} = 1$ as in gravity trade models (ACR '12)
- $\hat{M}_{ii}$ captures changes in productivity through endogenous search and matching

$$\hat{M}_{ii} = \hat{a}_{ii}^S \hat{m}_{ii}^S$$

which is affected by $\tilde{\lambda}^B$, $\tilde{\lambda}^S$, μ

Quantitative Analysis

Calibration

- Locations $\equiv$ 345 municipalities in Chile + China + USA + Germany + "rest of the world"
- Exactly match the bilateral trade flows X_{ud} from domestic firm-to-firm transaction data and customs data
- β: labor share out of total input expenditure (0.2)
- $\{\sigma, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$: indirect inference targeting the responses of import shocks as Fact 3
  - Impose $\tilde{\lambda}^B = \tilde{\lambda}^S$
  - Impose sufficient conditions for equilibrium uniqueness

Estimation and Model Fit

Panel (A) Estimated Parameters

Parameters	Value
β	0.2 (calibrated)
σ	3.07
$\tilde{\lambda}^B = \tilde{\lambda}^S$	0.19
μ	0.74

Panel (B) Model Fit

		Domestic Suppliers		Domestic Buyers	
	Imports	Number	Mean Value	Number	Mean Value
	(1)	(2)	(3)	(4)	(5)
(i) Data					
Import Shock	0.566	0.253	0.159	0.048	0.251
	(0.206)	(0.093)	(0.160)	(0.144)	(0.250)
(ii) Model Prediction					
Import Shock	0.572	0.192	0.199	0.155	0.208

Estimation of Spatial Frictions

- Decompose bilateral trade frictions into "search frictions" and "iceberg cost"

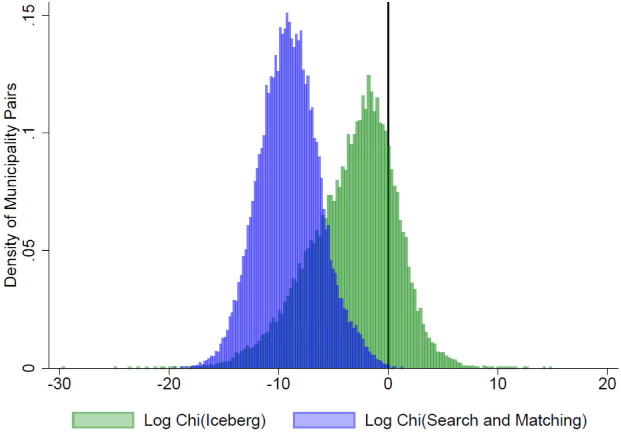
$$\chi_{ud} = \varrho^E \underbrace{\left[\kappa_{ud} \left(f_{ud}^B \right)^{-\tilde{\lambda}^B} \left(f_{ud}^S \right)^{-\tilde{\lambda}^S} \right]^{\delta_2}}_{\equiv \chi_{ud}^{\text{search}}} \underbrace{\left(\tau_{ud}^{1-\sigma} \right)^{\tilde{\lambda}^B + \tilde{\lambda}^S + 1}}_{\equiv \chi_{ud}^{\text{iceberg}}}$$

- Use intensive and extensive margin of bilateral trade flows to estimate these costs relative to within-location trade (Head-Ries '01)

$$\tilde{\chi}_{ud}^{\text{iceberg}} \equiv \frac{\chi_{ud}^{\text{iceberg}}}{\chi_{uu}^{\text{iceberg}}} \frac{\chi_{du}^{\text{iceberg}}}{\chi_{dd}^{\text{iceberg}}} = \left(\frac{\overline{r}_{ud}}{\overline{r}_{uu}} \frac{\overline{r}_{du}}{\overline{r}_{dd}} \right)^{\tilde{\lambda}^B + \tilde{\lambda}^S + 1}, \tilde{\chi}_{ud}^{\text{search}} \equiv \left(\frac{M_{ud}}{M_{uu}} \frac{M_{du}}{M_{dd}} \right) \left(\frac{\overline{r}_{ud}}{\overline{r}_{uu}} \frac{\overline{r}_{du}}{\overline{r}_{dd}} \right)^{-\left(\tilde{\lambda}^B + \tilde{\lambda}^S \right) \delta_2}$$

- Estimate these for all pairs of municipalities in Chile (no M_{ud} and $\overline{r}_{ud}$ from customs data)

Decomposition of Spatial Frictions



- Search and matching costs are larger than iceberg costs

Decomposition of Spatial Frictions

	Iceberg		Search and Matching	
	(1)	(2)	(3)	(4)
Log Distance	-0.376		-0.633	
	(0.007)		(0.004)	
Log Time Travel		-0.436		-0.682
		(0.008)		(0.005)
R^2	0.049	0.053	0.278	0.257
N	53956	53956	53956	53956

- Search and matching costs is more sensitive to geographic distance than iceberg trade cost
- Consistent with recent literature on search and matching frictions in trade (Chaney '14, Allen '14, Eaton-Kortum-Kramarz '18, Brancaccio-Kalouptsidi-Papageorgiou '20, Lenoir-Martin-Mejean '20, Krolikowski-McCallum '21, Startz '21, Miyauchi '21)
- Use these estimates for a counterfactual of transportation improvement

Counterfactual Simulations

- Undertake two counterfactual simulations
  1. International Trade: Effects of shocks on global value chain surrounding Chile
  2. Domestic Transportation Infrastructure: Effects of Chiloe island mega-bridge
- Two scenarios for both counterfactual simulations
  1. Baseline ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0.19$)
  2. No Endogenous Responses in Extensive Margin ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$)

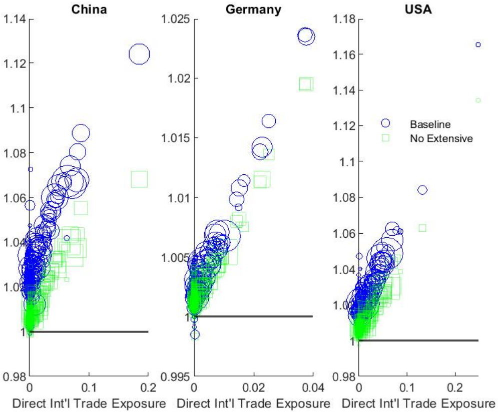
1. International Trade: Effects of Shocks on Global Value Chain of Chile

- Consider a 10% reduction of iceberg trade costs for baseline model
  - $\hat{\chi}_{ud} = 1.35$ for $u, d \in China, Germany, USA$
  - Give the same shock $\hat{\chi}_{ud}$ in no extensive margin case ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$)
- Average welfare gains (percentage points):

	China	Germany	USA
Baseline	3.65	0.40	2.55
No Extensive	1.54	0.30	1.37
Baseline - No Extensive	2.11	0.10	1.19

- Ignoring endogenous extensive margin substantially underestimates welfare gains

Heterogeneous Effects by Direct International Exposure



- Direct international trade exposure (export + import share) strongly correlates with welfare gains
- Baseline model predicts larger indirect effects, as evident from higher intercepts
- Different patterns across countries due to relative importance of export and import and different position in domestic production networks

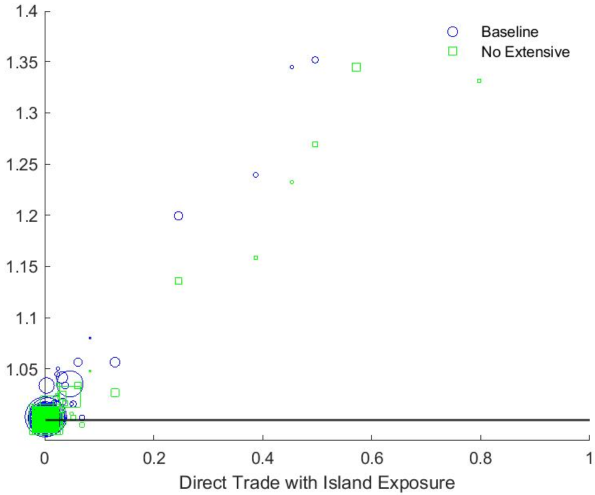
2. Transportation Infrastructure: Effects of Chiloe Island Mega-Bridge

- Planned to open in 2025 as the largest suspension bridge in South America
  - Will shorten travel time to mainland from 35 minutes (by ferry) to just 2 minutes
- Simulate the reduction of bilateral trade costs proportional to travel time reduction
  - Use travel time elasticities of trade and search costs from cross-section data
- Average welfare gains:

	New Bridge
Baseline	0.84
No Extensive	0.50
Baseline - No Extensive	0.34

- Ignoring endogenous extensive margin substantially underestimates welfare gains

Substantial Heterogeneous Welfare Effects from the Bridge



Conclusion

Conclusion

- Provide a tractable micro-founded model of production networks in space
  - Establish existence and uniqueness, counterfactuals, sufficient statistics for welfare
- Apply our model to firms' domestic and foreign transaction data from Chile
  - Presents stylized facts about spatial production networks consistent with our model
  - In counterfactuals, we find strong responses of domestic networks, which affects aggregate and distributional implications
- Framework can also be used for international production networks across countries

Appendix

Number of Linkages by Geography and Firm Size

Return

- Firm-level regression of the log number of domestic buyers and suppliers on population density and firm sales

	Buyers			Suppliers		
	(1)	(2)	(3)	(4)	(5)	(6)
Log Density	0.034		0.025	0.115		0.106
	(0.001)		(0.001)	(0.002)		(0.002)
Log Sales		0.422	0.421		0.447	0.445
		(0.001)	(0.001)		(0.001)	(0.001)
R^2	0.011	0.458	0.459	0.018	0.197	0.205
Year FE	✓	✓	✓	✓	✓	✓
State FE	✓	✓	✓	✓	✓	✓
N	380588	380588	380588	381362	381362	381362

3. Production Networks Respond to Import Cost Shocks: 2011-2016 Return

- Firm-level impacts of import shocks using shift-share design (Autor et al '13)

$$\Delta \log y_{it} = \alpha_0 + \alpha_1 \sum_{c,k} \underbrace{\Delta \log WID_{ckt}}_{c\text{'s export in } k \text{ except Chile}} \times \underbrace{w^D_{ickt_0}}_{\text{import / total input}} + \epsilon_{it},$$

  - i: firm; t: year; c: country; k: product (6-digit HS code)
  - Results below are long difference from 2011 to 2016

				Domestic Suppliers		Domestic Buyers	
	Imports	Exports	Sales	Number	Mean Value	Number	Mean Value
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Import Shock	0.917	-0.197	0.842	0.226	0.549	0.667	0.395
	(0.243)	(0.533)	(0.201)	(0.115)	(0.198)	(0.698)	(0.611)
Export Shocks	✓	✓	✓	✓	✓	✓	✓
Industry Fixed Effects	✓	✓	✓	✓	✓	✓	✓
N	10420	3737	29613	27142	27052	5602	5533

Existence and Uniqueness Return

- Mathematical structure commonly appears in trade and spatial models (Allen, Arkolakis, Li '21):

Proposition

If $\frac{\beta(\sigma-1)}{1-\beta} \geq (1-\mu)\left(\tilde{\lambda}^{B} + \tilde{\lambda}^{S}\right)$ *and* $\delta_G \leq 1$ *then the equilibrium always exists and it is unique up-to-scale.*

Responses to Shocks Return

- Denote observed import and export share by $\Psi_{id} = \frac{X_{id}}{\sum_{\ell} X_{i\ell}}$ and $\Lambda_{ui} = \frac{X_{ui}}{\sum_{\ell} X_{\ell i}}$
- Consider counterfactual changes in $\hat{K}_{id}^{D}$ and $\hat{K}_{id}^{U}$ ($\hat{x} \equiv x'/x$)

Proposition

The counterfactual changes of wages $\hat{w}_i$ and intermediate cost shifter $\hat{C}_i^*$ are solved by

$$(\hat{w}_i)^{1+\tilde{\lambda}^B\delta_2\mu} \left(\hat{C}_i^*\right)^{(\sigma-1)\delta_2+\tilde{\lambda}^B\delta_2(1-\mu)} = \sum_d \hat{K}_{id}^{D} (\hat{w}_d)^{\delta_G} \left(\hat{C}_d^*\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)} \Psi_{id}$$

$$(\hat{w}_i)^{1-\delta_G} \left(\hat{C}_i^*\right)^{-\frac{(\sigma-1)\delta_2}{1-\beta}+\tilde{\lambda}^S\delta_2(1-\mu)} = \sum_u \hat{K}_{ui}^{U} (\hat{w}_u)^{-\tilde{\lambda}^B\delta_2\mu} \left(\hat{C}_u^*\right)^{-(\sigma-1)\delta_2-\tilde{\lambda}^B\delta_2(1-\mu)} \Lambda_{ui}$$