Spatial Production Networks *

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Abstract

This paper provides a theory and empirical evidence of how production networks are organized in space and how they shape the spatial distribution of economic activity. Consistent with stylized facts from administrative firm-to-firm transaction-level data from Chile, we model firms' decision of forming a network of supplier and buyer relationships depending on their productivity and geographic location. By aggregating these decisions at the regional level, we provide a tractable characterization of the positive and normative properties of the general equilibrium. We calibrate our model to the observed domestic and international trade patterns and to the impacts of international trade shocks on domestic production networks in Chile. Counterfactual simulations of international trade shocks and transportation infrastructure reveal strong endogenous responses in the domestic production network, which significantly contribute to the heterogeneous welfare effects depending on the regions' exposure to the domestic and global production network.

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1 Introduction

One of the most important features of the modern economy is the geographic complexity of production value chains. Production of clothes, automobiles, or smartphones requires a number of production steps fragmented across countries, regions within a country, and firms within a region. Policymakers advocate that successful integration into these global production networks, or "Global Value Chain," is key to countries' and regions' economic success (e.g., World Bank (2019)). Reflecting this importance, a burgeoning academic literature has deepened our understanding on both the microeconomics and the macroeconomics of production networks (see Johnson (2018) and Antràs and Chor (2021) for reviews).[1] However, owing to the complexity of the endogenous production decisions and the limitations in availability of data for production networks across regions and countries, we have limited understanding about how these microeconomic and macroeconomic forces interact within and across country borders. Basic questions are yet to be addressed: How do endogenous production networks form across countries or regions based on firm-level decisions of forming supplier and buyer relationships? How do these networks endogenously respond to macroeconomic shocks and what are the aggregate implications?

This paper studies how production networks are organized in space and how their endogenous formation shapes the spatial distribution of economic activity. We combine rich administrative firm-to-firm transaction-level data from Chile with a microfounded model of endogenous spatial production network formation with tractable aggregation properties. In line with our data, we model firms' decisions to search for suppliers and buyers and to form relationships depending on their productivity and geographic location. By aggregating these decisions at the regional level, we provide a tractable characterization of the positive and normative properties of the general equilibrium. We calibrate our model to the observed domestic and international trade patterns and to the impacts of international trade shocks on domestic production networks in Chile. By undertaking counterfactual simulations of international trade shocks and transportation infrastructure, we find strong endogenous responses in the domestic production network. We also find that these responses significantly contribute to the aggregate and heterogeneous welfare effects depending on the regions' exposure to the domestic and global production network.

We start our analysis by providing a set of descriptive facts about spatial production networks using detailed transaction-level firm-to-firm data of the universe of firms in Chile. We present

[1] The macroeconomic literature focuses on how countries' or regions' macroeconomic conditions are determined given the topography of production networks (e.g., Yi 2003, 2010, Johnson and Noguera 2012, Caliendo and Parro 2015, Johnson and Moxnes 2019, Antràs and De Gortari 2020). The microeconomic literature highlights how firms participate and form production networks, endogenously shaping the topography of production networks (e.g., Bernard, Moxnes, and Saito 2019, Dhyne, Kikkawa, Mogstad, and Tintelnot 2020, Oberfield 2018, Lim 2018, Huneeus 2018, Bernard, Dhyne, Magerman, Manova, and Moxnes 2020, Demir, Fieler, Xu, and Yang 2021).

three facts about the nature of spatial production networks. First, we show that the number of suppliers and buyers per firm is correlated with firms' geographic location and overall size. Second, we show that the cross-regional trade flows increase in the geographic proximity, and this effect is driven to a larger extent by the number of supplier-to-buyer relationships (extensive margin) than the transaction volume per relationship (intensive margin). Third, we find that the domestic supplier-to-buyer linkages respond to international trade shocks depending on the firms' exposure to international markets. These pieces of evidence jointly suggest an important link between the spatial organization of production networks and the spatial distribution of economic activity.

Guided by these descriptive patterns, we develop a microfounded model of endogenous spatial production networks, shaped by heterogeneous firms across different regions. Firms search for suppliers and buyers for each location depending on the anticipated profit and location-pair-specific search costs. These supplier and buyer searches turn into a successful relationship at a certain probability depending on the matching technology and how many suppliers and buyers are searching in each pair of locations. By aggregating these decisions, the model predicts gravity equations of bilateral trade flows in the extensive margin (number of relationships) and in the intensive margin (transaction volume per relationship). These two gravity equations have distinct bilateral resistance terms as a function of search costs, matching efficiency, and iceberg trade costs, hence the model rationalizes different spatial structures of intensive and extensive margins of trade flows as we document from data.

We next embed this endogenous spatial production network formation in general equilibrium and study its positive and normative properties. Despite the complexity of firm-level decisions and their spatial interactions, we show that the equilibrium is characterized by two simple sets of equilibrium conditions corresponding to buyer access and supplier access. These buyer and supplier access conditions are analogous to the ones proposed in existing trade models based on gravity equations (Anderson and Van Wincoop 2003, Redding and Venables 2004, Donaldson and Hornbeck 2016), yet our conditions accommodate the presence of endogenous responses of production network structure. Using this equilibrium characterization, we establish a condition for equilibrium existence and uniqueness, characterize counterfactual equilibrium from an aggregate shock, and provide a sufficient statistics expression for welfare. Welfare, in particular, depends not only on familiar aggregate sufficient statistics omnipresent in the gravity trade models (Arkolakis, Costinot, and Rodríguez-Clare 2012) but also on the additional term that summarizes the endogenous changes in production networks. Furthermore, we show that our model nests a wide class of gravity trade models with intermediate goods as a special case (Eaton and Kortum 2002, Costinot and Rodríguez-Clare 2014), a well-accepted benchmark model used to study macroeconomic implications of exogenous production networks (Antràs and Chor 2021).

In the final section of our paper, we quantitatively assess the implication of endogenous spatial production networks on the spatial organization of economic activity. We calibrate our model by combining cross-sectional patterns of the inter- and intra-national trade in Chile and the responses of domestic production networks on international trade shocks. We also estimate the spatial frictions for production network formation for each pair of locations, and exactly decompose them into the components attributed to physical (iceberg) trade cost and the component attributed to search and matching frictions. We show that both types of frictions are quantitatively important and strongly related to the geographic proximity between the regions.

Armed with the calibrated model, we conclude our paper by studying how international and domestic trade shocks affect the spatial organization of economic activity through two sets of counterfactual simulations. In our first counterfactual simulation, we study the reduction of export and import costs to three major trading partners of Chile: China, Germany, and the United States of America (USA). Using our calibrated model, we find a strong reorganization of the domestic production networks from these international trade shocks. Furthermore, the estimated welfare gains are substantially larger compared to a special case of our model with no extensive margin responses of production networks. These patterns of results indicate that the endogenous responses of domestic production networks amplify the welfare gains. We also find substantial heterogeneity in the welfare gains across regions in Chile, which is shaped not only by the regions' direct exposure to international markets but also by their indirect connections through domestic production networks.

In our second counterfactual simulation, we study an improvement in domestic transportation infrastructure: a large-scale bridge between the mainland of Chile to Chiloé island, the biggest island in Chile. This bridge, planned to open in 2025 as the largest suspension bridge in South America, is expected to shorten the travel time between the mainland to Chiloé island from 35 minutes by ferry to just 2 minutes. By calibrating the expected travel cost reduction from the predicted travel time reduction, we estimate that the opening of the bridge leads to a 0.84 percentage point increase in the aggregate welfare. On the other hand, when we instead use a special case of our model with no extensive margin responses of production networks, we find only a 0.5 percentage point increase in aggregate welfare, which is about 60 percent of the prediction of our baseline model. Therefore, taking into account the endogenous production network formation quantitatively matters for an evaluation of an important domestic policy.

This paper contributes to several strands of literature. First, this paper contributes to the literature on spatial production networks and global value chains. As mentioned earlier, and as surveyed by Johnson (2018) and Antràs and Chor (2021), limited attempts have been made to connect the microeconomics of firms' production network formation in space and the macroeconomics of how countries' and regions' aggregate economic activities are affected by production networks.

An important exception is Eaton, Kortum, and Kramarz (2018), who build a micro-founded model of firm-to-firm trade in space that predicts aggregate gravity equations of cross-regional trade flows. The key distinction from their model is that we assume that each firm matches with a continuum of suppliers, as in Lim (2018), and we incorporate endogenous search intensity following the buyer and supplier search framework of Arkolakis (2010) and Demir, Fieler, Xu, and Yang (2021). We show that this feature of our model leads to tractable characterization of aggregate equilibrium.[2]

Second, this paper contributes to the literature of micro-founded quantitative trade models based on gravity equations, which also serve as a well-accepted benchmark for studying the macroeconomic implication of exogenous spatial production networks (Antràs and Chor 2021). This literature develops tractable multi-location trade models based on Armington models (Anderson (1979)), Ricardian models (Eaton and Kortum (2002)) and models with firm heterogeneity and selective entry into trade (Melitz (2003), Eaton, Kortum, and Kramarz (2011)). More recently, Arkolakis, Costinot, and Rodríguez-Clare (2012) have shown that these models with different micro-foundations have common sufficient statistics expressions for the welfare gains from trade. We contribute to this literature by providing a multi-location model of trade with endogenous production networks that predict similar sets of gravity equations of trade. However, despite this similarity, we show that endogenous production network formation crucially matters for the counterfactual equilibrium outcomes. Furthermore, we provide a modified sufficient statistics expression for the welfare gains that depend on the endogenous response of production networks.

Third, this paper contributes to the literature on the propagation of economic shocks through production networks within and across countries. There is a broad consensus that input-output linkages propagate economic shocks across firms (Carvalho, Nirei, Saito, and Tahbaz-Salehi 2021), sectors (Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi 2012, Acemoglu, Akcigit, and Kerr 2016), and regions (Caliendo, Parro, Rossi-Hansberg, and Sarte 2018). At the same time, a growing number of papers have shown that endogenous responses of firm-level production network formation affect the nature of shock propagation (Dhyne, Kikkawa, Mogstad, and Tintelnot (2020), Lim (2018), Huneeus (2018), Adao, Carrillo, Costinot, Donaldson, and Pomeranz (2020)). We contribute to this literature by providing a theoretical framework that connects firm-level responses of production network formation to aggregate macroeconomic variables across space and by providing empirical evidence on the responses of firm-level and macroeconomic variables to international trade input cost shocks.

[2] As an extension of Eaton, Kortum, and Kramarz (2018), Miyauchi (2021) incorporates dynamic search and matching to study agglomeration economies through increasing returns to scale in matching, and Panigraphi (2021) incorporates multiple dimensions of firm heterogeneity to fit the micro evidence from spatial firm-to-firm trade in India. Antràs and De Gortari (2020) develop a model of sequential production in space, instead of roundabout production, with attractive aggregation properties.

The rest of the paper is organized as follows. Section 2 describes our main data set from Chile and presents descriptive facts about spatial production networks. Section 3 presents our model. Section 4 presents theoretical results on our model's positive and normative predictions. Section 5 calibrates our model using Chilean data and presents counterfactual simulation results. Section 6 concludes.

2 Data and Descriptive Facts

In this section, we describe our main data set, the firm-to-firm transaction data from Chile. We also present a set of salient facts about spatial production networks.

2.1 Data

Our key data source is a firm-to-firm transaction-level data set that covers the universe of domestic trade between firms in Chile. This data set is built on the entire receipts of the transaction between firms and is electronically submitted to the Internal Revenue Service (IRS, or SII for its acronym in Spanish) in Chile. Reporting this information is mandatory for all firms regardless of the firm size. Each receipt includes information of the day that the transaction occurred, the total amount of the transaction, the products involved in the transaction, the price of the transaction, and the seller's and buyer's geographic location at the municipality level (there are 345 municipalities in Chile).[3] Unless otherwise stated, we use the time period of 2018-19.

To study the interaction of domestic production networks with international trade, we merge this data set with customs data. As is usual in other countries, this data set reports the export and import activity of firms, including information of the product being traded, the country of origin or destination, the total nominal flow involved in the transaction, and the unit value of the transaction. We also merge this data set with firm balance sheet information (SII tax form 29) to identify total sales and the main industry of the firm and with matched employer-employee data set (SII tax affidavits 1887 and 1879) to identify labor compensation by the firm. We merge these data sets using unique tax IDs of firms that are common across data sources.[4]

From the entire set of firms in our data set, we drop samples that report no value-added or employment, and samples that report negative value of value-added, sales, or material inputs. After

[3] Note that the seller's and buyer's location may not coincide with firm headquarters for the case of multiple establishment firms. Our empirical results are robust to specifying the unit of firms by firm-municipality pair, instead of firms defined by a unique tax ID.

[4] To secure the privacy of workers and firms, we do not have direct access to individual-level data after merging each of these data sets. In addition, all the results presented in this paper are statistically processed using at least 25 tax IDs under the requirement by the Chilean SII.

imposing these sample restrictions, the data set contains 42 million firm-to-firm-year supplier-to-buyer transactions with 19 million observations of unique firm pairs, which consists of 654 (981) thousand unique supplier-year (buyer-year) observations and 235 (211) thousand unique suppliers (buyers).

Given this paper's focus on the spatial dimension of production networks, we also construct several key geographic variables for our analysis. First, we construct the population size of each municipality in Chile using population census data in 2017.[5] Second, we construct the bilateral travel time and travel distance between all pairs of municipalities in Chile using Google Maps API.

2.2 Descriptive Facts on Spatial Production Networks

In this subsection, we document a number of stylized facts characterizing production networks across space in Chile. We use these patterns to motivate our modeling choices in Section 3. We also calibrate key model parameters using some of these facts in Section 5.

Fact 1. The number of domestic suppliers and buyers per firm is correlated with both firms' geographic location and firm size. We first show that the number of linkages (suppliers and buyers) per firm is strongly related to key geographic variables. Panel A of Figure 1 shows the relationship between the average number of domestic suppliers per firm (conditional on having at least one supplier) and buyers per firm (conditional on having at least one buyer) and the population density at the municipality level. The number of buyers is on average higher than the number of suppliers because there are more firms with positive number of supplier linkages than those with positive number of buyer linkages. Despite these differences in the levels, both variables are strongly positively correlated with the population density.

In Panel B, we show that these relationships are statistically significant at the firm level in a regression framework, conditional on other firm characteristics such as firm sales. Columns 1 and 4 show that the population density is positively and significantly correlated with the number of linkages. Columns 2 and 5, in turn, show that the number of linkages is positively correlated with the logarithm of firm sales. The R-squared of these relationships is particularly high at 0.458 for the number of buyers and lower at 0.197 for the number of suppliers, consistent with the finding Bernard, Dhyne, Magerman, Manova, and Moxnes (2020) that the number of buyers importantly governs the firm-level sales. Despite these strong statistical relationships with firm sales, the number of linkages is statistically significantly related with population density conditional on

[5]See Appendix Figure C.1 for the map and the spatial patterns of population density in Chile.

sales, as evident in Columns 3 and 6.[6]

These facts are consistent with previous findings of Miyauchi (2021), who documents that the number of suppliers per firm and the matching rates with new suppliers are positively correlated with firm density in Japan. These facts are also related to Eaton, Kortum, and Kramarz (2018), who document a strong relationships between the number of bilateral exporting relationships by French exporters and the market size of the destination country.[7] These pieces of evidence support the idea that the geographic location of the firm is correlated with firms' production linkages. Motivated by these findings, we develop a model where the firm linkages and sales are determined by both the geographic factors, on top of the firm-level productivity.

Fact 2. Cross-regional trade flows increase in the geographic proximity, and this effect is driven to a larger extent by the number of supplier-to-buyer relationships (extensive margin) than the transaction volume per relationship (intensive margin). We next discuss the spatial structure of production networks across pairs of municipalities in Chile. In Table 1, we present the results of gravity regressions, where we regress the logarithm of the total transaction volume between a pair of municipalities on the logarithm of the distance, controlling for origin and destination fixed effects. Column 1 shows that the coefficient on the log of distance is significant at -1.324, indicating that a 10% increase of travel time is associated with 13.24% decrease in aggregate trade flows. Column 2 shows that the coefficient on the log of travel time is significant at -1.515. These patterns of spatial decay of the domestic production networks are consistent with the previous findings by Bernard, Moxnes, and Saito (2019) using the number of supplier-to-buyer relationships in Japan and Panigraphi (2021) using the total firm-to-firm transaction volume in India.

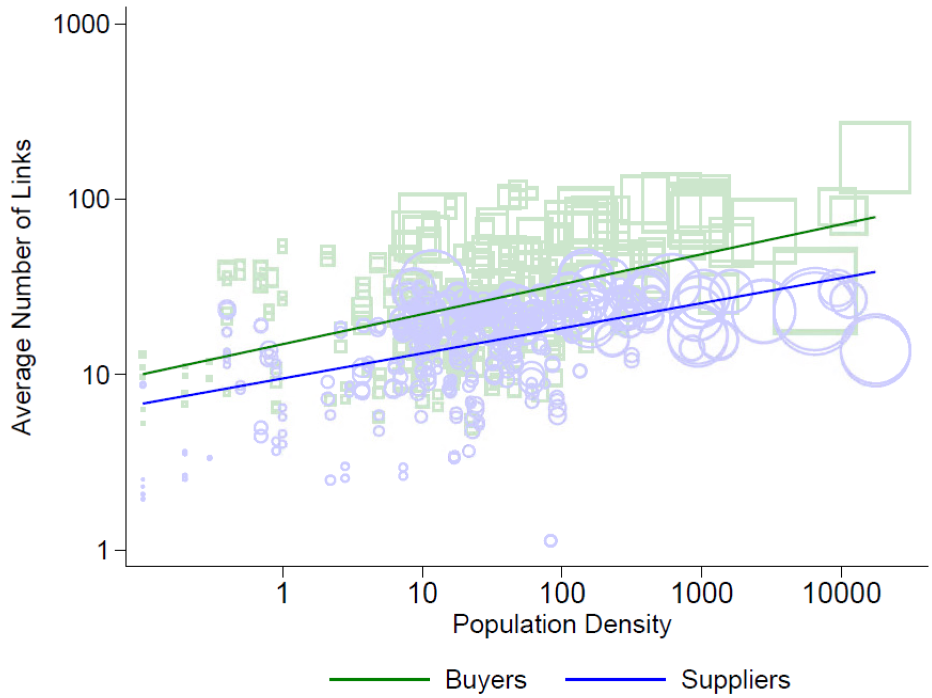
To further understand the nature of these spatial frictions, we decompose the total transaction volumes into the number of supplier-to-buyer relationships (extensive margin) and the transaction volume per relationship (intensive margin) using our detailed firm-to-firm trade data. Columns 3 and 4 present the regression coefficient on the log of distance and travel time on the extensive margin, and Columns 5 and 6 present that on the intensive margin. Mechanically, the sum of the coefficients of Columns 3 and 5 coincides with that of Column 1, and the sum of the coefficients of Columns 4 and 6 coincides with that of Column 2. We find that both extensive and intensive margins are significantly negatively correlated with distance proxies, while the magnitude is substantially larger for the extensive margin (-0.941 for travel distance and -1.074 for travel time) compared to the intensive margin (-0.383 for travel distance and -0.441 for travel

[6]Appendix Table C.1 shows that these relationships are robust to controlling for industry fixed effects (at 6-digit level) and by controlling for the export and import activity of the firm.

[7]Note that they focus on the total number of relationships summed across all firms in the destination location, unlike the average number of relationships per firm at the destination location as shown above.

Figure 1: Number of Domestic Suppliers and Buyers and Geography

(A) Number of Linkages and Population Density



(B) Number of Linkages by Geography and Firm Size

	Buyers			Suppliers		
	(1)	(2)	(3)	(4)	(5)	(6)
Log Density	0.034		0.025	0.115		0.106
	(0.001)		(0.001)	(0.002)		(0.002)
Log Sales		0.422	0.421		0.447	0.445
		(0.001)	(0.001)		(0.001)	(0.001)
R^2	0.011	0.458	0.459	0.018	0.197	0.205
Year FE	✓	✓	✓	✓	✓	✓
State FE	✓	✓	✓	✓	✓	✓
N	380588	380588	380588	381362	381362	381362

Notes: Panel A plots the average number of domestic suppliers and buyers per firm averaged at the municipality level (conditional on having at least one linkage) and population density in 2018 at the municipality level. The size of the circle represents the population size of each municipality. The straight line represents the fit of the linear regressions between the two variables. Panel B presents the regression results at the firm level, where the dependent variable is the number of domestic links per firm (with buyers in Columns 1-3 and suppliers in Columns 4-6). The regression includes year and state fixed effects. There are 15 states in Chile.

time).

Motivated by these findings, we build a model in Section 3 that predict gravity equations in both extensive and intensive margins with different spatial structure, where the difference in the bilateral resistance arises due to different types of spatial frictions.

Table 1: Gravity Regression: Total Trade Flows, Intensive and Extensive Margin

	Total		Intensive		Extensive	
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-1.324		-0.383		-0.941	
	(0.008)		(0.007)		(0.004)	
Log Time Travel		-1.515		-0.441		-1.074
		(0.010)		(0.008)		(0.004)
R^2	0.640	0.639	0.306	0.306	0.822	0.819
Origin Municipality FE	✓	✓	✓	✓	✓	✓
Destination Municipality FE	✓	✓	✓	✓	✓	✓
N	65871	65871	65871	65871	65871	65871

Notes: This table presents the results of the gravity regressions, where we regress the logarithm of the total transaction volume between a pair of municipalities on the logarithm of the distance, controlling for origin and destination fixed effects using SII data from 2018. The dependent variable corresponds to total trade flow, average trade flow (intensive margin), and the number of links between municipalities (extensive margin). Distance (time travel) is measured with kilometers (minutes of time travel) between municipalities using the fastest land or water transportation method available within Chile.

Fact 3. Localized shocks from international markets affect domestic production networks. As a final set of descriptive facts, we study how international trade shocks affect domestic production linkages. In particular, following a similar specification as implemented by Autor, Dorn, and Hanson (2013) and Hummels, Jørgensen, Munch, and Xiang (2014), we study how firms with different import and export exposure respond differently to country-and-product specific import and export shocks.

More concretely, we estimate the following regression model:

$$\Delta \log y_{it} = \alpha_0 + \alpha_1 \Delta Z_{it}^D + \alpha_2 \Delta Z_{it}^S + \epsilon_{it}, \tag{1}$$

where i indexes a firm and t indexes year. y_{it} are outcomes of firm i at year t, including the firms' import, export, total sales, the number of domestic suppliers and buyers, and the average transaction volume per supplier and per buyer. Δx represents the time difference operator of variable x. We mainly consider a long difference specification (the difference between two time periods), hence we do not have to control for time or firm fixed effects. ΔZ_{it}^D and ΔZ_{it}^S are shift-share demand and supply shocks at firm i at year t, respectively. In particular, following Autor, Dorn, and Hanson (2013) and Hummels, Jørgensen, Munch, and Xiang (2014), we measure these shocks as an interaction between firms' exposure to a particular international country and product and the country-and-product specific demand and supply shifters constructed from international trade patterns outside Chile. More concretely, we define ΔZ_{it}^D and ΔZ_{it}^S as:

$$\Delta Z_{it}^D = \sum_{c,k} w_{ickt_0}^D \Delta \log WID_{ckt}, \quad \Delta Z_{it}^S = \sum_{c,k} w_{ickt_0}^S \Delta \log WES_{ckt},$$

where WID_{ckt}, world import demand, is country c's total purchases of product k from the world market minus purchases from Chile in year t. WES_{ckt}, world export supply, is country c's total supply of product k to the world market minus its supply to Chile in year t. The weights for the export shock, $w^D_{ickt_0}$, is given by:

$$w^D_{ickt_0} = \frac{Exports_{ickt_0}}{TotalSales_{it_0}}$$

where $Exports_{ickt_0}$ are total value of export of firm i to country c of product k at baseline year $t = t_0$, and $TotalSales_{it_0}$ is the total sales of firm i at year $t = t_0$. The weight for the import shock, $w^S_{ickt_0}$, is given by:

$$w^S_{ickt_0} = \frac{Imports_{ickt_0}}{WageBill_{it_0} + DomesticPurchase_{it_0} + \sum_{c,k} Imports_{ickt_0}},$$

where $Imports_{ickt_0}$ are total value of import of firm i from country c of product k at baseline year $t = t_0$, $WageBill_{it_0}$ is the total wage bills of firm i in year $t = t_0$, and $DomesticPurchase_{it_0}$ is the total value of domestic sourcing.

In our baseline analysis, we take the initial period as 2007 and post-period as 2009 and implement the long-difference specification as explained above. These are the time periods when there is a significant economic disturbance in the international trade market. To construct the baseline import and export shares for the firm-specific weights, we take the average of two time periods $t_0 = \{2003, 2004\}$ in order to minimize the measurement error specific to one particular year.[8]

Table 2 presents the results from this analysis. Column 1 shows that firms' import responds significantly to the import shocks constructed above, while they are unaffected by the export shocks. This confirms that the constructed proxies of import shocks indeed increased the imports by Chilean firms. Column 2 shows that the firms' export positively responds to our proxies for the export shocks, while the relationship is statistically insignificant. The lack of statistical insignificance for the export shocks is potentially driven by the fact that a significantly smaller number of firms engage in export than import. Consistent with this interpretation, Column 3 shows that import shocks significantly increase firms' revenue, while we find limited responses of sales on export. Due to the lack of significant effects of export shocks, we focus our subsequent discussion on the responses from import shocks.

Columns 4-7 present how the international trade shocks affect the architecture of domestic production networks. Columns 4 and 5 document the impacts on the number of domestic sup-

[8] We use international trade data of different products traded between countries across the globe for the period of 1996-2018. This information comes from the BACI data from CEPII, that is sourced from Comtrade (United Nations). This data set is merged with the customs data using the product classification and country IDs.

pliers and average transaction volume per domestic supplier, respectively, and Columns 6 and 7 document the impacts on the number of domestic buyers and average transaction volume per domestic buyer, respectively. We find a significant positive response on the number of domestic suppliers (Column 4).[9] This implies that there is gross complementary between imported inputs and domestic sourcing. The average transaction volume, number of buyers, and transaction volume per buyer respond positively, yet they are all statistically insignificant. Strong responses on the number of domestic suppliers imply that import shocks affect other firms and regions indirectly through the endogenous changes of production network structure.

These pieces of evidence provide additional insights on the role of endogenous production network formation on the propagation of international trade shocks to the literature. In particular, by implementing a similar identification design, Dhyne, Kikkawa, Mogstad, and Tintelnot (2020) and Adao, Carrillo, Costinot, Donaldson, and Pomeranz (2020) document that international trade shocks affect sales activity of firms that are indirectly connected to direct importers or exporters in Belgium and Ecuador, respectively. Demir, Fieler, Xu, and Yang (2021) document that these international trade shocks affect the labor compensation by these indirectly connected firms in Turkey. Huneeus (2018) studies similar indirect effects, as well as the changes of supplier-to-buyer linkages, and concludes that the formation of domestic firm-to-firm linkages responds to international trade shocks.

In Section 5, we show that our model can rationalize these responses of domestic production networks to international trade shocks. We also show how these responses of domestic production networks matter for the welfare gains from international trade.

Table 2: International Trade Shocks and Domestic Production Networks

				Suppliers		Buyers	
	Imports	Exports	Sales	Number	Mean Value	Number	Mean Value
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Import Shock	0.566	-0.052	0.516	0.253	0.159	0.048	0.251
	(0.206)	(0.497)	(0.167)	(0.093)	(0.160)	(0.144)	(0.250)
Export Shock	-0.296	0.147	0.125	0.072	0.320	0.054	-0.211
	(0.348)	(0.202)	(0.146)	(0.082)	(0.140)	(0.147)	(0.255)
Industry Fixed Effects	✓	✓	✓	✓	✓	✓	✓
N	9192	4201	27516	27718	27541	19600	19362

Notes: This table presents the results from estimating with OLS the Equation (1). Changes are between 2007 and 2009. The regression includes industry fixed effects at the 2-digit level.

[9] See Furusawa, Inui, Ito, and Tang (2017) for a related discussion and evidence about the substitution between domestic sourcing and imports.

3 Model

In this section, we present a model of endogenous spatial production networks.

We consider an economy that is partitioned by a finite number of locations $i, u, d \in N$. In each location, there is a continuum of workers of measure L_i, which are exogenously given. There are two types of goods in the economy: intermediate goods and final goods. Intermediate goods are traded across regions. Shipping intermediate goods from location u (supplier's production location) to location d (buyer's production location) requires an iceberg trade cost of $\tau_{ud} \geq 1$. Final goods are not traded across regions and are only provided by final goods producers in the region.

3.1 Production

Intermediate goods are produced by intermediate goods producers that we simply refer to as "firms". Each firm produces a distinct variety that is used by other firms for their production. Each firm has a distinct level of productivity denoted by z. Following a long tradition in the international trade of intermediate goods (e.g. Krugman and Venables (1995), Eaton and Kortum (2002)), in order to produce intermediate goods, each firm uses a continuum of intermediate goods. The set of goods that each firm has access to is determined by a search-and-matching process that we describe below. We assume that these intermediate goods are imperfect substitutes with constant elasticity of substitution, while the labor and the composite of intermediate goods have the elasticity of substitution equal to one. Therefore, the unit cost of production for firm ω, $c(\omega)$, is given by

$$c(\omega) = \frac{1}{z(\omega)} w_i^{\beta} \left(\int_{v \in \Omega(\omega)} p(v, \omega)^{1-\sigma} dv \right)^{\frac{1-\beta}{1-\sigma}}, \tag{2}$$

where $z(\omega)$ is firm's productivity; w_i is the wage at firm ω's production location; $\Omega(\omega)$ is the set of intermediate goods producers that firm ω has access to; $p(v, \omega)$ is the intermediate goods price that supplier v charges to firm ω (net of iceberg trade cost); β is the share of labor input ($0 \leq \beta \leq 1$); and σ is the elasticity of substitution across different intermediate goods ($\sigma > 1$). The set of intermediate goods producers $\Omega(\omega)$ are endogenously determined in the equilibrium through search and matching as described below.

Since each firm ω is matched with a continuum of suppliers, and since each supplier is monopolistic, prices charged by the supplier v to firm ω is a constant markup of their marginal cost, inclusive of the iceberg trade cost. Denoting supplier v's location as u and buyer ω's location as i, $p(v, \omega)$ is given by

$$p(v, \omega) = \tilde{\sigma} c(v) \tau_{ui}. \tag{3}$$

where $\tilde{\sigma} = \sigma / (\sigma - 1)$ is the markup ratio under constant elasticity of substitution input demand.

The final goods sector is perfectly competitive. Goods are produced using local intermediate goods with constant elasticity of substitution (CES) production function with the elasticity of substitution σ, which we assume is the same elasticity as those for the production of the intermediate goods.[10] The final goods are not traded across space and are only supplied by the local final goods producers. Under perfect competition, the final goods price index in region i is given by

$$P_i^F = \left(\int_{v \in \Omega_i^F} p^F (v)^{1-\sigma} dv \right)^{\frac{1}{1-\sigma}}, \tag{4}$$

where $p^F (v)$ is the price of the intermediate goods provided by firm v. Unlike the transaction between intermediate goods producers, there are no search and matching frictions between intermediate and final goods producers, and hence Ω_i^F is simply the set of intermediate goods producers in region i.

For simplicity, we assume that final goods producers have all the bargaining power over intermediate goods suppliers. Therefore, the price of intermediate goods by supplier v, $p^F (v)$, is simply the marginal production cost by suppliers, i.e.,[11]

$$p^F (v) = c (v). \tag{5}$$

3.2 Firm Search

There are search and matching frictions in the intermediate goods market. Firms post advertisements to search for buyers and suppliers for each location depending on the anticipated profit and location-pair-specific search costs. These supplier and buyer searches turn into a successful relationship at a certain probability depending on the matching technology and how many suppliers and buyers are searching in each pair of locations. In this section, we discuss firms' search decisions given matching rates, and we discuss how matching rates are determined in the next section.

We first describe firm decisions for searching buyers. In order for firms in region i to acquire buyers in region d, they have to post advertisements. Posting $n_{id}^B \in \mathbb{R}_+$ measure of advertisements requires payment of $e_i f_{id}^B \left(n_{id}^B\right)^{\gamma^B} / \gamma^B$, where e_i is the unit cost of advertisement services

[10] Our argument is broadly unaffected by assuming different elasticity of substitution across intermediate inputs between intermediate goods producers and final goods producers, except that it affects the relevant elasticities for the sufficient statistics expression for welfare in Proposition 3. Similarly, our argument is broadly unaffected by alternatively assuming that final goods production requires labor in addition to intermediate goods.

[11] We abstract from endogenous search and matching and profit from sales to final goods producers to focus on the role of search and matching in the intermediate goods market. One can alternatively introduce consumer search for final goods producers, as in Arkolakis (2010), and our model implications remain broadly the same.

in region i, $\gamma^B > 1$ is a parameter that governs the curvature of the advertisement cost for buyer search, and f_{id}^B is the cost shifter for the location pair i and d. Each of these advertisement turns into a successful match with a random buyer in location d who posts a supplier advertisement to location i at rate m_{id}^B, where m_{id}^B is endogenously determined given matching technology as described in the next section. Once the firm matches with a buyer, they face a monopolistic competition with other firms that also sell to the matched buyer. Therefore, the expected profit by a firm in location i with marginal production cost c per matched buyer in location d is therefore given as follows:

$$\pi_d\left(c\tau_{id}\right) = \frac{1}{\sigma} D_d\left(c\tau_{id}\right)^{1-\sigma} \tag{6}$$

where D_d is the destination-specific intermediate goods demand shifter, which is exogenous to the firm but is endogenously determined in general equilibrium in Section 3.5.

We next describe firm decisions for searching suppliers. In order for firm ω in region i to post $n_{ui}^S \in \mathbb{R}_+$ measure of advertisements for suppliers in region u, the firm has to pay an advertisement cost $e_i f_{ui}^S \left(n_{ui}^S\right)^{\gamma^S} / \gamma^S$, where e_i is the unit cost of advertisement services in region i, $\gamma^S > 1$ is a parameter that governs the curvature of the advertisement cost for supplier search, and f_{ui}^S is the cost shifter for the location pair u and i. Each of these advertisements turns into a successful match with a random supplier in location u who posts a supplier advertisement to location i at rate m_{ui}^S, where m_{ui}^S is endogenously determined given matching technology as described in the next section.[12] We denote the average unit cost of a supplier in location u (net of trade cost) by C_{ui}, which affects the incentive for searching suppliers in location u.

Together, firms' search decision for buyers, $\{n_{id}^B\}_{d\in N}$, and suppliers, $\{n_{ui}^S\}_{u\in N}$, is given below:

$$\begin{aligned}\pi_i(z) = \max_{\{n_{ui}^S\}_u, \{n_{id}^B\}_d} \frac{1}{\sigma} \sum_{d\in N} m_{id}^B n_{id}^B D_i \left(c\tau_{id}\right)^{1-\sigma} - e_i \left\{ \sum_{d\in N} f_{id}^B \frac{\left(n_{id}^B\right)^{\gamma^B}}{\gamma^B} + \sum_{u\in N} f_{ui}^S \frac{\left(n_{ui}^S\right)^{\gamma^S}}{\gamma^S} \right\} \\ \text{subject to} \quad c = \frac{w_i^\beta \left(\sum_{u\in N} n_{ui}^S m_{ui}^S \left(C_{ui}\right)^{1-\sigma}\right)^{\frac{1-\beta}{1-\sigma}}}{z}\end{aligned} \tag{7}$$

The objective function of this problem is the net profit of firms in location i with productivity z. The first term inside the max operator represents the profit from sales to other intermediate goods producers, where $n_{id}^B m_{id}^B$ is the number of successful customers firms are able to match and sell to, and c is the marginal cost of production of this firm. The second term is the advertisement

[12] Whenever the equilibrium variables involve two locations with an upstream and downstream relationships, we adopt the convention of denoting the upstream location first followed by the downstream location in subscripts. For example, n_{ui}^S denotes the supplier advertisement posting by firms in location i to upstream location u, while n_{id}^B denotes the customer advertisement posting by firms in location i to downstream location d.

cost as discussed above. The marginal cost of the firm, c, in turn, depends on wages w_i, number of matched suppliers from location u, $n^S_{ui}m^S_{ui}$, average intermediate production cost of matched suppliers in location u to location i, C_{ui}, and firm productivity, z.

We impose a parameter restriction that $1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}>0$, which guarantees that firms make positive sales and profit. In Appendix A.1, we show that the solution of the optimization takes the following form:

$$n^S_{ui}\left(z\right)=a^S_{ui}z^{\frac{\delta_1}{\gamma^S}};\ n^B_{id}\left(z\right)=a^B_{id}z^{\frac{\delta_1}{\gamma^B}}, \tag{8}$$

where $\delta_1\equiv\frac{\sigma-1}{1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}}>\sigma-1$, and a^S_{ui}, a^B_{id} are given by

$$a^B_{id}=\left(m^B_{id}\frac{D_d}{e_if^B_{id}}\left(\tau_{id}\right)^{1-\sigma}\left(C^*_i\right)^{1-\sigma}\right)^{\frac{1}{\gamma^B-1}}, \tag{9}$$

$$a^S_{ui}=\left(\frac{\left(1-\beta\right)D^*_i}{e_iw_i^{-\frac{\beta}{1-\beta}\left(1-\sigma\right)}f^S_{ui}}m^S_{ui}\left(C^*_i\right)^{\beta\frac{\sigma-1}{1-\beta}}\left(C_{ui}\right)^{1-\sigma}\right)^{\frac{1}{\gamma^S-1}}, \tag{10}$$

where we further define the demand shifter from buyers in all locations by

$$D^*_i=\sum_d m^B_{id}a^B_{id}D_d\left(\tau_{id}\right)^{1-\sigma}, \tag{11}$$

and we define the production cost shifter for firms in location i by

$$\left(C^*_i\right)^{1-\sigma}\equiv w_i^{\beta\left(1-\sigma\right)}\left(\sum_{u\in N}a^S_{ui}m^S_{ui}\left(C_{ui}\right)^{1-\sigma}\right)^{1-\beta}. \tag{12}$$

In expression (8), search intensity, $n^S_{ui}\left(z\right), n^B_{id}\left(z\right)$, depend on location-pair-specific components, a^S_{ui}, a^B_{id}, and the firm-specific component proportional to firm productivity, $z^{\frac{\delta_1}{\gamma^S}}, z^{\frac{\delta_1}{\gamma^B}}$. These expressions are consistent with Fact 1 in Section 2 that the number of suppliers and buyers are related the geographic location of the firm on top of firm-specific component. The location-pair-specific components, a^S_{ui}, a^B_{id}, are determined by the bilateral matching rates, m^S_{ui}, m^B_{ui}, local wages, w_i, unit cost, C_{ui}, demand shifters, D_d, D^*_i, and the search cost shifters, f^S_{ui}, f^B_{id}.

Using these expressions for a^S_{ui}, a^B_{id}, the unit cost of a firm with productivity z, $c_i\left(z\right)$, is given by

$$c_i(z) = (C_i^*)\, z^{-\frac{\delta_1}{\gamma^S}\frac{1-\beta}{\sigma-1}-1}. \tag{13}$$

Note that the unit cost of firms, $c_i(z)$, decays at a faster rate than z^{-1} because more productive firms search suppliers more intensively (equation 10). As a result, firm revenue, $r_i(z)$, and firm profit, $\pi_i(z)$, are also increasing in firm productivity:

$$r_i(z) = D_i^* (C_i^*)^{1-\sigma} (z)^{\delta_1}, \tag{14}$$

$$\pi_i(z) = \frac{1}{\delta_1 \tilde{\sigma}} D_i^* (C_i^*)^{1-\sigma} (z)^{\delta_1}, \tag{15}$$

where again $\tilde{\sigma} \equiv \sigma/(\sigma-1)$. Both these magnitudes increase at a faster rate than $z^{\sigma-1}$ (recall $\delta_1 > \sigma - 1$) because more productive firms search suppliers and buyers more intensively. Furthermore, average costs of intermediate goods form suppliers in region u to firm buyers in region i, C_{ui}, takes the multiplicative form of average costs in region u, $\overline{C}_u$, and the iceberg trade cost, τ_{ui}, such that:

$$C_{ui} = \overline{C}_u \tau_{ui}, \tag{16}$$

where we define $\overline{C}_u$ as the average production cost by firms in location u (weighted by the number of customer advertisement postings).

3.3 Matching between Suppliers and Buyers

We now describe how the matching rates between suppliers and buyers, m_{ui}^S, m_{ui}^B, are determined for each pair of locations.

To do so, we first derive the aggregate measure of supplier and buyer advertisement postings for each pair of locations. The aggregate measure of supplier advertisement posting by customers in location d for suppliers in location u is given by:

$$\overline{M}_{ud}^S = N_d \int n_{ud}^S(z) dG_d(z) = N_d a_{ud}^S \mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right), \tag{17}$$

where N_d is the measure of firms that produces in location d, and $G_d(\cdot)$ is the cumulative distribution function of firm productivity in location d, which we assume to be an arbitrary function of location d. For notational convenience, we denote the integral of the power function of the productivity with respect to the productivity distribution by $\mathbb{M}_d(\chi) = \int z^\chi dG_d(z)$. Similarly, the aggregate measure of customer advertisement posting by suppliers in location u for customers in location d is given by:

$$\overline{M}_{ud}^{B} = N_u \int n_{ud}^{B}(z) dG_u(z) = N_u a_{ud}^{B} \mathbb{M}_u \left(\frac{\delta_1}{\gamma^B} \right). \tag{18}$$

Due to matching frictions, only a fraction of supplier advertisement and buyer advertisement lead to a successful match. Following a long tradition in the literature of labor search and matching (Diamond 1982, Mortensen 1986, Pissarides 1985), we assume that the aggregate number of successful matches between a supplier advertisement in location u and the buyer advertisement in location d is determined by matching technology represented by a Cobb-Douglas matching function:

$$M_{ud} = \kappa_{ud} \left(\overline{M}_{ud}^{S} \right)^{\lambda^S} \left(\overline{M}_{ud}^{B} \right)^{\lambda^B}, \tag{19}$$

where λ^S and λ^B denote the elasticities of total matches created for the pair of regions with respect to the supplier and buyer advertisement postings, respectively, and κ_{ud} is the parameter governing the efficiency of matching technology. We accommodate the possibility of the scale effects of the matching technology, such that $\lambda^S + \lambda^B$ is not necessarily equal to one.[13] Given the number of total supplier-to-buyer matches between bilateral regions, the matching rates m_{ud}^S and m_{ud}^B are now defined by:

$$m_{ud}^{S} = \frac{M_{ud}}{\overline{M}_{ud}^{S}}, \quad m_{ud}^{B} = \frac{M_{ud}}{\overline{M}_{ud}^{B}}. \tag{20}$$

3.4 Aggregate Trade Flows

We now derive the aggregate trade flows between a pair of locations. In particular, we show that both the extensive margin of trade flows (number of supplier-to-buyer relationships) and the intensive margin of trade flows (transaction volume per relationship) follow the form of gravity equations. In this section, we present our main results, and we leave all mathematical derivations in Appendix A.2.

We start with the extensive margin of trade flows. By solving equations (8), (17), (18), (19), and (20), the number of supplier-to-buyer relationships M_{ud} from supplier location u to buyer location d is given the following gravity equation:

$$M_{ud} = \chi_{ud}^{E} \zeta_{u}^{E} \xi_{d}^{E}, \tag{21}$$

[13] See Eaton, Kortum, and Kramarz (2018) and Miyauchi (2021) for the evidence of the increasing returns to scale in matching technology between suppliers and buyers.

where the bilateral resistance term χ^E_{ud} is given by:

$$\chi^E_{ud} = \varrho^E \left[\kappa_{ud} \left(f^B_{ud} \right)^{-\tilde{\lambda}^B} \left(f^S_{ud} \right)^{-\tilde{\lambda}^S} \left(\tau^{1-\sigma}_{ud} \right)^{\tilde{\lambda}^B + \tilde{\lambda}^S} \right]^{\delta_2},$$

where we define $\tilde{\lambda}^S \equiv \lambda^S / \gamma^S$ and $\tilde{\lambda}^B \equiv \lambda^B / \gamma^B$ as the ratio of matching function elasticities and search cost elasticities, and we also define $\delta_2 \equiv \left[1 - \tilde{\lambda}^S - \tilde{\lambda}^B \right]^{-1}$ and $\varrho^E \equiv (1-\beta)^{\tilde{\lambda}^S \delta_2}$. In other words, the bilateral resistance term is a combination of bilateral search, matching, and iceberg frictions, which jointly enter as a shifter for the cost of forming a supplier-to-buyer linkages in each pair of locations. The origin-specific shifter takes the form:

$$\zeta^E_u = \left[\left(N_u \mathbb{M}_u \left(\frac{\delta_1}{\gamma^B} \right) \right)^{\lambda^B \frac{\gamma^B - 1}{\gamma^B}} \left\{ e^{-1}_u \left(C^*_u \right)^{1-\sigma} \right\}^{\tilde{\lambda}^B} \left(\overline{C}_u \right)^{(1-\sigma)\tilde{\lambda}^S} \right]^{\delta_2},$$

which summarizes the capability of location u to generate buyer relationships, which depends on the measure of firms, N_u, productivity, $\mathbb{M}_u \left(\frac{\delta_1}{\gamma^B} \right)$, and cost shifters, C^*_u, $\overline{C}_u$. The destination-specific shifter takes the form:

$$\xi^E_d = \left[\left(N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right) \right)^{\lambda^S \frac{\gamma^S - 1}{\gamma^S}} \left(D_d \right)^{\tilde{\lambda}^B} \left\{ D^*_d e^{-1}_d w_d^{\frac{\beta(1-\sigma)}{1-\beta}} \left(C^*_d \right)^{-\frac{\beta(1-\sigma)}{1-\beta}} \right\}^{\tilde{\lambda}^S} \right]^{\delta_2},$$

which summarizes the capability of location d to generate supplier relationships, which depends on the measure of firms, N_d, productivity, $\mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right)$, and demand shifters, D_d, D^*_d.

We next derive the intensive margin of trade flows. Using equation (6), we can also derive the average volume of bilateral transactions between suppliers in location u and buyers in location d as

$$\overline{r}_{ud} = \chi^I_{ud} \zeta^I_u \xi^I_d, \tag{22}$$

where the bilateral component is only a function of iceberg costs such that

$$\chi^I_{ud} = \left(\tau_{ud} \right)^{1-\sigma},$$

and the origin- and destination-specific shifters are given by:

$$\zeta^I_u = \left(C^*_u \right)^{1-\sigma} \frac{\mathbb{M}_u \left(\delta_1 \right)}{\mathbb{M}_u \left(\frac{\delta_1}{\gamma^B} \right)}, \ \xi^I_d = D_d.$$

The intuition of this gravity equation is as follows. The bilateral resistance term, $\chi^I_{ud} = \left(\tau_{ud} \right)^{1-\sigma}$, captures the iceberg trade cost between the location. The origin location fixed effects, ζ_u^I, capture the average unit cost of location u and depend on the cost index of the origin. Lower cost will increase the firm intensive margin. The destination fixed effects, ξ_d^I, capture the intermediate goods demand per firm in location d.

Equations (21) and (22) show that the gravity equations for extensive and intensive margins have a different spatial structure. Importantly, while the bilateral resistance term of the intensive gravity equation captures only the iceberg trade cost, $(\tau_{ud})^{1-\sigma}$, in Equation (22), the bilateral resistance term of the extensive gravity equation captures the combination of the match efficiency, κ_{ud}, the bilateral search cost shifters, f_{ud}^B, f_{ud}^S, in addition to the iceberg trade cost. This difference gives a structural interpretation of the different spatial decay of the extensive and intensive margin of firm-to-firm trade across regions in Chile as documented in Fact 2 of Section 2.2. In Section 5, we use these model predictions to decompose the component of trade costs into the component attributed to iceberg trade cost and those attributed to search and matching frictions.

At this juncture, it is worth discussing the difference of our gravity equations with the ones derived by Eaton, Kortum, and Kramarz (2018). In their model, firms have a finite number of tasks that they outsource from a selected set of suppliers. This selection mechanism, together with their assumption of the extreme-value distribution of firm productivity, implies that the expected transaction value does not depend on origin-specific shifters or iceberg shipping costs. Therefore, in their model, the bilateral resistance of the gravity equations are driven entirely by the extensive margin, which is in turn driven by the combination of iceberg costs and search frictions.[14] Our model instead assumes that firms match with a continuum of suppliers that are imperfect substitutes for each other. As a result, our model predicts that intensive margin of trade flow responds to iceberg trade costs and to origin-specific shifters.

3.5 General Equilibrium

We now embed the aforementioned search and matching framework of spatial production network formation in general equilibrium. In particular, we discuss how the advertisement cost, e_i, the average production costs, $\overline{C}_i$, the demand shifters, D_i^* and D_i, firm entry, N_i, and wages, w_i, are determined in general equilibrium. Because these characterizations are relatively standard in the literature of quantitative trade models, we present the key equations in this section and delegate the mathematical derivations to the Appendix A.3.

First, we assume that advertisement service is provided by perfectly competitive advertisement service providers using labor and intermediate goods with Cobb-Douglas production technology. Similarly to final goods producers as discussed in Section 3.1, advertisement service

[14]Similar property of gravity equations holds for Miyauchi (2021) and Panigraphi (2021), which in turn build on Eaton, Kortum, and Kramarz (2018).

providers face no search and matching frictions and access all intermediate goods varieties produced in location i. Therefore, the cost for advertisement service e_i is given by

$$e_i = A_i \left(w_i\right)^{\mu} \left(C_i^*\right)^{1-\mu}, \tag{23}$$

where A_i captures the productivity of the advertisement sector, μ is the input expenditure share for labor, and C_i^* is the cost shifter for the intermediate goods defined by equation (12).

Second, we show that the average cost of intermediate goods sold by suppliers in location i, $\overline{C}_i$, introduced in equation (7), is proportional to the cost index, C_i^*, such that:

$$\overline{C}_i = \left(C_i^*\right)\left(\tilde{\sigma}\right)\left(\frac{\mathbb{M}_i\left(\delta_1\right)}{\mathbb{M}_i\left(\frac{\delta_1}{\gamma^B}\right)}\right)^{1/(1-\sigma)}, \tag{24}$$

where the last component $\left(\mathbb{M}_i\left(\delta_1\right)/\mathbb{M}_i\left(\frac{\delta_1}{\gamma^B}\right)\right)$ captures the advertisement intensity by firms with different productivity.

Third, we characterize the demand shifters using the labor and intermediate goods market clearing conditions. Demand shifters for firms' sales at origin location (defined in equation 11), D_i^*, is given by

$$D_i^* = \frac{1}{\vartheta}\frac{w_i L_i}{\left(C_i^*\right)^{1-\sigma} N_i}\frac{1}{\mathbb{M}_i\left(\delta_1\right)}, \tag{25}$$

where $\vartheta = \beta - \frac{1}{\delta_1\tilde{\sigma}}\left(1-\beta+\mu\frac{\frac{1}{\gamma^B}+\frac{1-\beta}{\gamma^S}}{1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}}\right)$ is the ratio of aggregate firm revenue to aggregate labor compensation. Furthermore, demand shifter for intermediate goods sales at destination location (defined in equation 6), D_d, is given by

$$D_d = \frac{1}{\vartheta\left(\tilde{\sigma}\right)^{\sigma-1}}\frac{L_d}{N_d\mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right)}\left(w_d\right)^{\frac{1-\beta\sigma}{1-\beta}}\left(C_d^*\right)^{\frac{\sigma-1}{1-\beta}}. \tag{26}$$

Note that, except for parameters and exogenous variables, the demand shifters are only a function of wages w_i, cost shifter C_i^*, and firm entry N_i.

Fourth, we characterize firm entry N_i. We follow a long tradition in international trade and spatial economics and assume that in each region, there is a pool of potential entrants of intermediate goods producers (firms) in region i. Once they pay a fixed cost payment F_i in the unit of local labor, each firm stochastically draws productivity z from cumulative distribution function $G_i(\cdot)$, where this productivity distribution can arbitrarily depend on region i. The zero-profit condition for the potential entrants implies that firm entry is proportional to market size, L_i, such that:

$$N_i = \frac{\vartheta}{\delta_1 \tilde{\sigma}} \frac{L_i}{F_i}. \tag{27}$$

Finally, we assume that trade is balanced. Thus, total expenditure in imported intermediate inputs, $\sum_u X_{ui} = \sum_u \bar{r}_{ui} M_{ui}$ equals total intermediate goods sales to other regions; $\sum_d X_{id} = \sum_d \bar{r}_{id} M_{id}$.

We now define the general equilibrium. Recall that gravity equations (21 and 22) and firm search intensity (9, 10) are functions of $e_i, \overline{C}_i, D_i^*, D_i, N_i$, which are in turn functions of the profiles of wages, $\{w_i\}$, and cost shifters, $\{C_i^*\}$, as characterized above by equations (23)-(27) and the trade balancing condition. Therefore, we can define a general equilibrium by the profiles of $\{w_i, C_i^*\}$ that satisfy the above set of equations.

4 Theoretical Analysis

In this section, we establish the theoretical properties of the general equilibrium of our model. In Section 4.1, we show that the equilibrium is characterized by two equations corresponding to buyer and supplier access, and we use this characterization to establish the conditions for the existence and the uniqueness of the equilibrium. In Section 4.2, we characterize the counterfactual equilibrium given a change in exogenous variables. In Section 4.3, we characterize sufficient statistics to evaluate the welfare changes as a response to changes in exogenous variables.

4.1 Equilibrium Characterization

As discussed in Section 3.5, the equilibrium is characterized by the profiles of wages, w_i, and cost shifters, C_i^*. While the equilibrium involves many equations, we show that the equilibrium boils down to two sets of simple equations corresponding to buyer access and supplier access. These two sets of equations are reminiscent of the buyer and supplier access in canonical gravity-based trade models (e.g., Anderson and Van Wincoop (2003) and Redding and Venables (2004)), while we accommodate endogenous search and matching in firm-to-firm trade.

We first discuss how wages are determined by buyer access. Since the total compensation to labor is ϑ fraction of total firm revenue from other intermediate goods producers, we have:

$$w_i = \frac{\vartheta}{L_i} \sum_d X_{id}, \tag{28}$$

where $X_{id} = M_{id} \bar{r}_{id}$ is the aggregate intermediate goods trade flow from i to d. This equation resembles a standard buyer access equation in trade and spatial models: the wage in a location depends on the potential revenue of the location by selling to various other locations. However, unlike these standard models, endogenous search and matching affect the buyer access by endogenously shifting M_{id}.

We next discuss how intermediate cost shifters are determined by supplier access. By combining equation (12) and (24), we have

$$\left(C_i^*\right)^{1-\sigma} = w_i^{\beta(1-\sigma)} \left[(\tilde{\sigma})^{\sigma-1} \mathbb{M}_i \left(\frac{\delta_1}{\gamma^S} \right) N_i \right]^{\beta-1} \left(\frac{\sum_u X_{ui}}{D_i} \right)^{1-\beta}. \tag{29}$$

This equation is reminiscent of a supplier access equation in standard trade models: A better access to intermediate goods ($\sum_u X_{ui}/D_i$) or lower wages (w_i) guarantees that the cost shifter of location i, C_i^*, is lower. However, unlike a standard trade model, endogenous search and matching affect the supplier access by endogenously shifting M_{ui}.

In Appendix A.4, we show that the above buyer access and supplier access equations (28 and 29) are rewritten only in terms of w_i and C_i^* for endogenous variables, such that:

$$\left(w_i\right)^{1+\tilde{\lambda}^B \delta_2 \mu} \left(C_i^*\right)^{(\sigma-1)\delta_2 + \tilde{\lambda}^B \delta_2 (1-\mu)} = \sum_d K_{id}^D \left(w_d\right)^{\delta_G} \left(C_d^*\right)^{\frac{(\sigma-1)\delta_2}{1-\beta} - \tilde{\lambda}^S \delta_2 (1-\mu)}, \tag{30}$$

$$\left(w_i\right)^{1-\delta_G} \left(C_i^*\right)^{-\frac{(\sigma-1)\delta_2}{1-\beta} + \tilde{\lambda}^S \delta_2 (1-\mu)} = \sum_u K_{ui}^U \left(w_u\right)^{-\tilde{\lambda}^B \delta_2 \mu} \left(C_u^*\right)^{-(\sigma-1)\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)}, \tag{31}$$

where δ_G is combination of parameters that summarize the demand effects of downstream locations' wages on upstream locations' economic activity, defined as

$$\delta_G = \left[\tilde{\lambda}^S \mu + \frac{1-\beta\sigma}{1-\beta} \right] \delta_2.$$

In this expression of δ_G, the first term captures the endogenous search decisions by firms in downstream locations, and the second term is the reminiscent of market size effect standard in the literature. Furthermore, K_{id}^D and K_{id}^U are the upstream and downstream connectivity shifters between regions, given by

$$K_{id}^D = \frac{1}{L_i} K_{id}, \; K_{ui}^U = \frac{1}{L_i} K_{ui},$$

where K_{id} is a composite of bilateral resistance terms of gravity equations, χ_{id}^E and χ_{id}^I, productivity distribution, $\mathbb{M}_i(\cdot)$ and $\mathbb{M}_d(\cdot)$, and the population size, L_i and L_d, such that:

$$K_{id} = \varsigma \chi_{id}^E \chi_{id}^I \left[\mathbb{M}_i \left(\frac{\delta_1}{\gamma^B} \right)^{\lambda^B - 1} \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right)^{\lambda^S - 1} \mathbb{M}_i \left(\delta_1 \right)^{\tilde{\lambda}^S} \mathbb{M}_d \left(\delta_1 \right)^{-\tilde{\lambda}^S} L_i^{\lambda^B \frac{\gamma^B - 1}{\gamma^B}} L_d^{\lambda^S \frac{\gamma^S - 1}{\gamma^S}} \right]^{\delta_2} \tag{32}$$

where $\varsigma \equiv \varrho^E \tilde{\sigma}^{\delta_2 \left[(1-\sigma) \left(\left(\tilde{\lambda}^B + 1 \right) \delta_2 + 1 \right) - \left(\lambda^B + \lambda^S - \delta_2^{-1} \right) \right]} \delta_1^{-\delta_2 \left(\lambda^B + \lambda^S - \delta_2^{-1} \right)} \vartheta^{\left(\lambda^S \frac{\gamma^S - 1}{\gamma^S} + \lambda^S \frac{\gamma^S - 1}{\gamma^S} \right) \delta_2 - 1}$ is a composite of model parameters.

There are three important implications about the system of equations (30) and (31). First, it shows that the equilibrium is completely characterized by the upstream and downstream connectivity shifters, K_{id}^D, K_{ui}^U, and the set of structural parameters $\{\sigma, \beta, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$. Conditional on these variables, other exogenous variables such as iceberg costs, τ_{id}, search and matching costs, f_{id}^S, f_{id}^B, matching technology efficiency, κ_{id}, or productivity distributions, $G_i(\cdot)$, are redundant. This feature of the equilibrium characterization is particularly useful for characterizing counterfactual changes of equilibrium as discussed in Section 4.2. Furthermore, given K_{id}^D, K_{ui}^U, matching elasticity, λ^S, λ^B, and search cost elasticity, γ^S, λ^B, matter only in the form of ratios, $\tilde{\lambda}^B \equiv \lambda^B / \gamma^B$ and $\tilde{\lambda}^S \equiv \lambda^S / \gamma^S$. Intuitively, $\tilde{\lambda}^B$ and $\tilde{\lambda}^S$ summarize the changes of the realized matches as a response to the changes in w_i and C_i^* through endogenous search decision, γ^B, γ^S, and the shape of matching technology, λ^B, λ^S. Only the net effects of these two matter for the aggregate equilibrium implications.

The second important remark about the above equilibrium characterization is that it nests a wide class of gravity trade models as a special case. In particular, when $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$, supplier-to-buyer relationships between locations do not respond to endogenous equilibrium variables, hence the production networks are effectively exogenously given. In Appendix A.5, we show that this special case is isomorphic to a canonical multi-country Ricardian model with intermediate goods trade as in Eaton and Kortum (2002), Alvarez and Lucas (2007) when $\sigma - 1$ is instead set as sector productivity dispersion; to the multi-country trade model with firm heterogeneity and selective entry as in Melitz (2003), Eaton, Kortum, and Kramarz (2011) when $\sigma - 1$ is instead set as the shape parameter for the Pareto productivity distribution; and to a broad class of gravity-based trade models as studied in Arkolakis, Costinot, and Rodríguez-Clare (2012), Costinot and Rodríguez-Clare (2014) when $\sigma - 1$ is instead set as trade elasticity. On the other hand, in our general case where $\tilde{\lambda}^S \neq 0$ or $\tilde{\lambda}^B \neq 0$, our model is not isomorphic to these canonical gravity trade models. In Section 5, we quantitatively assess how our model prediction differs to the special case of $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$.

The third important remark is about the existence and uniqueness of the equilibrium. Importantly, the system of equations (30) and (31) follow the same mathematical architecture as the ones that commonly appear in trade and spatial equilibrium models. In particular, using the results from Allen, Arkolakis, and Li (2020), we provide sufficient conditions for the existence and the uniqueness of the equilibrium, summarized by the following proposition:

Proposition 1 *If* $\frac{\beta(\sigma-1)}{1-\beta} \geq (1-\mu)\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)$ *and* $\delta_G \leq 1$ *then the equilibrium always exists and it is unique up-to-scale.*

Proof. See Appendix B.1. ■

These sufficient conditions are intuitive. The first condition ensures that the scale effects of matching technology related to the search cost elasticity, $\tilde{\lambda}^B + \tilde{\lambda}^S$, have to be sufficiently small. The second condition ensures that, δ_G, which summarizes the demand effects of downstream locations' wages on upstream locations' economic activity (equation 30), has to be less than one so that the positive feedback effects from a downstream location do not accumulate to infinity.

4.2 Responses to Shocks

In this subsection, we derive the system of equations for the changes of equilibrium variables as a response to shocks in exogenous variables such as trade costs or productivity shocks.

In particular, we consider how the shocks to connectivity shifters $\hat{K}^D_{id}$ and $\hat{K}^U_{id}$ changes the equilibrium configurations. Here, we adopt the conventional notation to use hat ($\hat{x}$) to denote the proportional changes of x such that $\hat{x} = x'/x$, where x' is the value of x in the presence of the shocks. Note that the changes of $\hat{K}^D_{id}$ and $\hat{K}^U_{id}$ can be induced by the changes in productivity, $G_i(\cdot)$, population, L_i, iceberg trade costs, τ_{id}, search costs, f^B_{id}, f^S_{id}, matching efficiency, κ_{id}, or the population size L_i, following of K^D_{id} and K^U_{id} the expression in equation (32).

Following the exact-hat algebra approach by Dekle, Eaton, and Kortum (2008), we show that the counterfactuals in our model can be determined just by these observed trade flows, X_{id}, and the set of structural parameters $\{\sigma, \beta, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$, as summarized by the following proposition:

Proposition 2 *Given the set of structural parameters* $\{\sigma, \beta, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$ *and the observed bilateral trade flows,* X_{id}*, the counterfactual changes of wages* $\hat{w}_i$ *and intermediate costs* $\hat{C}^*_i$ *from the changes in exogenous variables summarized by* $\hat{K}^D_{id}$ *and* $\hat{K}^U_{id}$ *are given by*

$$\left(\hat{w}_i\right)^{1+\tilde{\lambda}^B\delta_2\mu}\left(\hat{C}^*_i\right)^{(\sigma-1)\delta_2+\tilde{\lambda}^B\delta_2(1-\mu)} = \sum_d \hat{K}^D_{id}\left(\hat{w}_d\right)^{\delta_G}\left(\hat{C}^*_d\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}-\tilde{\lambda}^S\delta_2(1-\mu)}\Psi_{id} \tag{33}$$

$$\left(\hat{w}_i\right)^{1-\delta_G}\left(\hat{C}^*_i\right)^{-\frac{(\sigma-1)\delta_2}{1-\beta}+\tilde{\lambda}^S\delta_2(1-\mu)} = \sum_u \hat{K}^U_{ui}\left(\hat{w}_u\right)^{-\tilde{\lambda}^B\delta_2\mu}\left(\hat{C}^*_u\right)^{-(\sigma-1)\delta_2-\tilde{\lambda}^B\delta_2(1-\mu)}\Lambda_{ui} \tag{34}$$

where we define $\Psi_{id} = X_{id} / \left(\sum_{\ell} X_{i\ell}\right)$ as the observed share of intermediategoods sales of firms in location i to location d, and $\Lambda_{ui} = X_{ui} / \left(\sum_{\ell} X_{\ell i}\right)$ as the observed share of intermediate goods expenditure by firms in location i from location u.

Proof. See Appendix B.2. ■

Similarly to the characterization of baseline equilibrium by equations (30) and (31), the proposition states that the equilibrium is completely characterized by the set of sufficient statistics of structural parameters and exogenous variables. Moreover, compared to (30) and (31), we do not have to know the baseline level of connectivity shifters K_{id}^{D}, K_{ui}^{U}. Instead, we only need to know the baseline level of bilateral trade flows X_{id}. Intuitively, bilateral trade flows X_{id} summarize all the information about the connectivity shifters K_{id}^{D}, K_{ui}^{U}. It is also important to observe that finer microdata about firm-to-firm trade, such as the extensive and intensive margin of trade flows, M_{id} and $\bar{r}_{id}$, are not required for the counterfactual simulation. In other words, endogenous search and matching in spatial production networks affect the counterfactual equilibrium predictions only through the structural parameters, $\tilde{\lambda}^{B}$, $\tilde{\lambda}^{S}$, and μ.

4.3 Sufficient Statistics for Welfare

In this section, we study how productivity or trade shocks affect the residents' welfare in each location. In particular, following the spirit of Arkolakis, Costinot, and Rodríguez-Clare (2012), we derive a sufficient statistics formula for the welfare changes using a small set of elasticities and equilibrium variables. We show that our welfare formula depends not only on familiar aggregate sufficient statistics omnipresent in the gravity trade models (Arkolakis, Costinot, and Rodríguez-Clare 2012) but also on the additional term that summarizes the endogenous changes in production networks.

For simplicity, we consider the welfare changes of residents in location i from shocks summarized by $\hat{K}_{id}^{D}, \hat{K}_{id}^{U}$ that do not involve the changes in the productivity or population in their own location i, $G_i(\cdot)$ and $L_i(\cdot)$, and the within-location iceberg trade costs, search costs, and matching efficiency, τ_{ii}, f_{ii}^{B}, f_{ii}^{S}, κ_{ii}. The following proposition derives a sufficient-statistics expression for the changes of worker welfare, defined by the changes in real wages, $\widehat{w_i / P_i^F}$, as a response to these shocks.

Proposition 3 *Given shocks to the economy summarized by $\hat{K}_{id}^{D}, \hat{K}_{id}^{U}$, the proportional changes of welfare is expressed as:*

$$\frac{\widehat{w_i}}{P_i^F} = \left(\hat{\Lambda}_{ii}\right)^{-\frac{1}{\sigma-1}\frac{1-\beta}{\beta}} \left(\hat{M}_{ii}\right)^{\frac{1}{\sigma-1}\frac{1-\beta}{\beta}} \tag{35}$$

Proof. See Appendix B.3. ■

The proposition shows that the welfare changes are summarized by the responses of only two equilibrium variables. The first variable is $\hat{\Lambda}_{ii}$, the change of the share of intermediate goods expenditure from the producers in their own location. The first term is omnipresent in the analysis of a wide class of trade mode with intermediate input trade as reviewed by Costinot and Rodríguez-Clare (2014) and Antràs and Chor (2021). The exponent is the multiplication of the inverse of trade elasticity, $1/(\sigma-1)$, and the term capturing multiplier effects through input-output linkages, $(1-\beta)/\beta$, as typical in these models.[15] The second variable is $\hat{M}_{ii}$, the change of the number of supplier linkages within their own location. The second term only arises due to the presence of endogenous spatial production networks through search and matching, and it is absent from canonical gravity trade models as discussed above.

A closer examination of the second term conveys more intuition behind the role of endogenous production networks on welfare. In particular, using equations (17) and (19), we have:

$$\hat{M}_{ii} = \hat{a}^S_{ii}\hat{m}^S_{ii}.$$

Therefore, this new margin arises due to the change in production cost by the increased number of suppliers matched per firm, which is, in turn, a combination of the changes of search intensity for suppliers, $\hat{a}^S_{ii}$, and the endogenous matching rates, $\hat{m}^S_{ii}$. The responses of these variables are therefore related to the values of $\tilde{\lambda}^S$ and $\tilde{\lambda}^B$. In particular, in a special case of exogenous supplier and buyer search and matching as discussed above ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$), these terms are all equal to one, giving the same expression for the welfare gains for canonical gravity trade models. Furthermore, the value of μ, the labor share of the advertisement services (equation 23), is also relevant for the welfare gains. When $\mu = 0$, search cost only responds to intermediate goods cost shifter, C^*_i, and when $\mu = 1$, search cost responds only to local wages, w_i. In the next section, we estimate these key parameters for welfare predictions using the observed changes of domestic production networks from international trade shocks.

5 Quantitative Analysis

In this section, we assess our model's quantitative implication by calibrating to firm-to-firm trade data from Chile. In Section 5.1, we discuss our calibration strategy. In Section 5.2, we estimate various sources of frictions in shaping spatial production networks across municipalities in Chile.

[15] See Arkolakis, Costinot, and Rodríguez-Clare (2012), Ossa (2015), Melitz and Redding (2015), Caliendo and Parro (2015), Costinot and Rodríguez-Clare (2014) for the sufficient statistics expressions for trade models with input-output linkages.

In Section 5.3, we undertake counterfactual simulations of international trade shocks and transportation infrastructure in Chile.

5.1 Calibration

In this section, we discuss how we calibrate our model to data.

We first specify the mapping of our model's locations to data. We assume that our model locations consist of a combination of 345 municipalities within Chile and the set of the major international trade partners of Chile. In particular, we include United States, China, and Germany, as three distinct locations in the model, and designate all other countries as a single location in our model.

As discussed in Proposition 2, in order to undertake counterfactual equilibrium simulations, we only need baseline values of the trade flows across locations, X_{ud}, and the set of structural parameters $\{\sigma, \beta, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$. We calibrate the trade flows to exactly match the data. In particular, we obtain cross-regional trade flows X_{ud} by aggregating firm-to-firm trade data across municipalities (when both u and d are municipalities in Chile), by aggregating customs import and export data (when either of u or d is the international country), or by using country-to-country international trade data (when both u and d are international countries).

We calibrate the structural parameters $\{\sigma, \beta, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$ using microdata from Chile. We start by calibrating β, the labor share for producing intermediate goods. In our data, we observe total labor compensation (from employer-employee matched data) and the total intermediate goods expenditure (from firm-to-firm trade data). By taking the share of labor compensation out of the sum of these two, and taking the average of this share across all firms, we obtain the approximate value of $\beta = 0.2$.

We calibrate the remaining parameters $\{\sigma, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$ through indirect inference procedure targeting the responses of import shocks documented as Fact 3 in Section 2.2.[16] More specifically, we use the linearized equilibrium system to obtain the analytical expressions of the regression coefficients of the responses of the firms' domestic supplier and buyer configurations (the number of domestic suppliers and buyers, and the transaction volume per domestic supplier and buyer) on our empirical import shock proxies that we use in Table 2. We then search for parameter configurations $\{\sigma, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$ that minimize the Euclidean distance between the actual regression coefficients and the model-predicted regression coefficients. Appendix D.2 describes further details of this estimation procedure.

These regression coefficients intuitively capture the information about each of our target parameters. The impacts of import shocks on domestic sourcing (both extensive and intensive mar-

[16] We target import shocks, instead of export shocks, because we do not observe significant effects of our proxy of export shocks on the actual export in Table 2.

gin) are informative about the value of σ because it captures the degree of substitution between imported intermediate goods and domestic intermediate goods. The average responses of extensive margin (number of suppliers and buyers) relative to intensive margin (volume of transaction per supplier and per buyer) are informative about the value of $\tilde{\lambda}^S$ and $\tilde{\lambda}^B$, which are key elasticities summarizing the models' endogenous responses of production networks through search and matching. Recall that, in our special case of $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$, the model predicts no responses on these extensive margins. Since $\tilde{\lambda}^S$ and $\tilde{\lambda}^B$ are difficult to separately identify in practice, we impose a restriction that $\tilde{\lambda}^S = \tilde{\lambda}^B$. Lastly, the average responses of extensive margin are also informative about μ, because μ governs which factor prices (wages or composite intermediate goods) matter more for the incentive of search. In particular, a higher value of μ indicates a more positive response on the number of suppliers and buyers from import shocks, because import shocks directly decrease the cost shifter C_i^*.[17]

Table 3 presents our calibration results. Panel A reports our estimated parameters. Panel B reports the model fit, in which we present the regression coefficients of import shocks using actual data (in Panel B (i), reproduced from Table 2) and the same regression coefficients using model prediction under the estimated parameters (in Panel B (ii)). We find a moderately small elasticity of substitution across intermediate inputs $\sigma = 3.07$, implying a relatively small degree of substitution between domestic and international intermediate goods. Consistent with this interpretation, positive import shocks (increased attractiveness of imported intermediate goods) have positive impacts on domestic sourcing, both in the data and in the model prediction, as evident in Column 2 and 3 of Panel B. We find positive values of $\tilde{\lambda}^B = \tilde{\lambda}^S = 0.19$, indicating that endogenous responses of extensive margin of production networks are important. In particular, the model replicates the strong positive responses in the number of suppliers as found in Column 2 of Panel B. Lastly, we find $\mu = 0.74$, indicating that search costs are more strongly influenced by the composite intermediate goods costs than wages.

5.2 Unpacking Spatial Frictions in Production Network Formation

In this section, we estimate the spatial frictions for production network formation for each pair of locations. In particular, we use our model structure to exactly decompose these frictions into iceberg trade costs and search and matching frictions. We also study how each of these margins is related to the geographic proximity between regions in Chile, which we use for the counterfactual

[17] In addition to the four parameters discussed above, we also introduce and estimate the parameter that translates the empirical proxy for import shocks (as constructed in Section 2.2) to the structural import shock proxy K_{ui}^U, targeting the changes of import values. See Appendix D.2 for the details. To ensure that we obtain stable results for the counterfactual simulation, we impose constraints in parameters so that they satisfy sufficient conditions for equilibrium uniqueness as derived in Proposition 1. We find that our estimated parameters are at the boundary of $\delta_G = 1$.

Table 3: Parameter Estimates and Model Fit

Panel (A) Estimated Parameters

Parameters	Value
β	0.2 (calibrated)
σ	3.07
$\tilde{\lambda}^B = \tilde{\lambda}^S$	0.19
μ	0.74

Panel (B) Model Fit

		Suppliers		Buyers	
	Imports	Number	Mean Value	Number	Mean Value
	(1)	(2)	(3)	(4)	(5)
(i) Data					
Import Shock	0.566	0.253	0.159	0.048	0.251
	(0.206)	(0.093)	(0.160)	(0.144)	(0.250)
(ii) Model Prediction					
Import Shock	0.572	0.192	0.199	0.155	0.208

Notes: This table reports the estimation results (Panel A) and the model fit of the structural parameters (Panel B). In Panel B, we present the regression coefficients of import shocks using actual data (in Panel B (i), reproduced from Table 2) and the same regression coefficients using model prediction under the estimated parameters (in Panel B (ii)). See Appendix D.2 for the details of the estimation procedure.

simulation for transportation infrastructure in Section 5.3.

To start the analysis, we first reformulate the gravity equations of the aggregate trade volume from u to d, X_{ud}. By noting that X_{ud} is the multiplication of extensive margin (number of relationships), M_{ud}, and the intensive margin (transaction volume per relationship), $\bar{r}_{ud}$, both of which follow gravity equations (Equations 21 and 22), X_{ud} is expressed as:

$$X_{ud} = M_{ud}\bar{r}_{ud} = \chi_{ud}\zeta_u\xi_d,$$

where $\chi_{ud} = \chi^E_{ud}\chi^I_{ud}$, $\zeta_u = \zeta^E_u\zeta^I_u$, and $\xi_d = \xi^E_d\xi^I_d$. In particular, the bilateral resistance term of the extensive margin gravity equation, $\chi^E_{ud} = \varrho^E\left[\kappa_{ud}f^{B\ -\tilde{\lambda}^B}_{ud}f^{S\ -\tilde{\lambda}^S}_{ud}\left(\tau^{1-\sigma}_{ud}\right)^{\tilde{\lambda}^B+\tilde{\lambda}^S}\right]^{\delta_2}$, is affected by both search and matching frictions and iceberg trade cost, while the bilateral resistance of the intensive margin gravity equation, $\chi^I_{ud} = (\tau_{ud})^{1-\sigma}$, is only affected by the iceberg trade cost. Therefore, we can exactly decompose the bilateral resistance of total trade flows, χ_{ud}, into the component that is related to search and matching frictions and that related to iceberg trade costs:

$$\chi_{ud} = \chi^{\text{search}}_{ud}\chi^{\text{iceberg}}_{ud},$$
$$\chi^{\text{search}}_{ud} = \left[\kappa_{ud}\left(f^B_{ud}\right)^{-\tilde{\lambda}^B}\left(f^S_{ud}\right)^{-\tilde{\lambda}^S}\right]^{\delta_2},\ \chi^{\text{iceberg}}_{ud} = \left(\tau^{1-\sigma}_{ud}\right)^{\tilde{\lambda}^B+\tilde{\lambda}^S+1}. \quad (36)$$

Note that we define $\chi_{ud}^{\text{search}}$ by the combination of the matching efficiency, χ_{ud}, and search costs shifters, f_{ud}^{B} and f_{ud}^{S}. In our model, these objects always appear in the form of $\chi_{ud}^{\text{search}}$, and hence we do not have to separately identify these components.

We now proceed to estimate $\chi_{ud}^{\text{search}}$ and $\chi_{ud}^{\text{iceberg}}$. As is standard in gravity-based trade models, we cannot separately identify the bilateral resistance terms, χ_{ud}, from origin and destination shifters, ζ_u and ξ_d. Therefore, we follow Head and Ries (2001) to construct the proxies for bilateral trade costs relative to within-location trade. More specifically, by combining the expressions for the gravity equations in extensive and intensive margins (Equations 21 and 22), we have:

$$\tilde{\chi}_{ud}^{\text{iceberg}} \equiv \frac{\chi_{ud}^{\text{iceberg}}}{\chi_{uu}^{\text{iceberg}}} \frac{\chi_{du}^{\text{iceberg}}}{\chi_{dd}^{\text{iceberg}}} = \left(\frac{\overline{r}_{ud}}{\overline{r}_{uu}} \frac{\overline{r}_{du}}{\overline{r}_{dd}} \right)^{\tilde{\lambda}^B + \tilde{\lambda}^S + 1} \tag{37}$$

$$\tilde{\chi}_{ud}^{\text{search}} \equiv \frac{\chi_{ud}^{\text{search}}}{\chi_{uu}^{\text{search}}} \frac{\chi_{du}^{\text{search}}}{\chi_{dd}^{\text{search}}} = \left(\frac{M_{ud}}{M_{uu}} \frac{M_{du}}{M_{dd}} \right) \left(\frac{\overline{r}_{ud}}{\overline{r}_{uu}} \frac{\overline{r}_{du}}{\overline{r}_{dd}} \right)^{-\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)\delta_2}$$

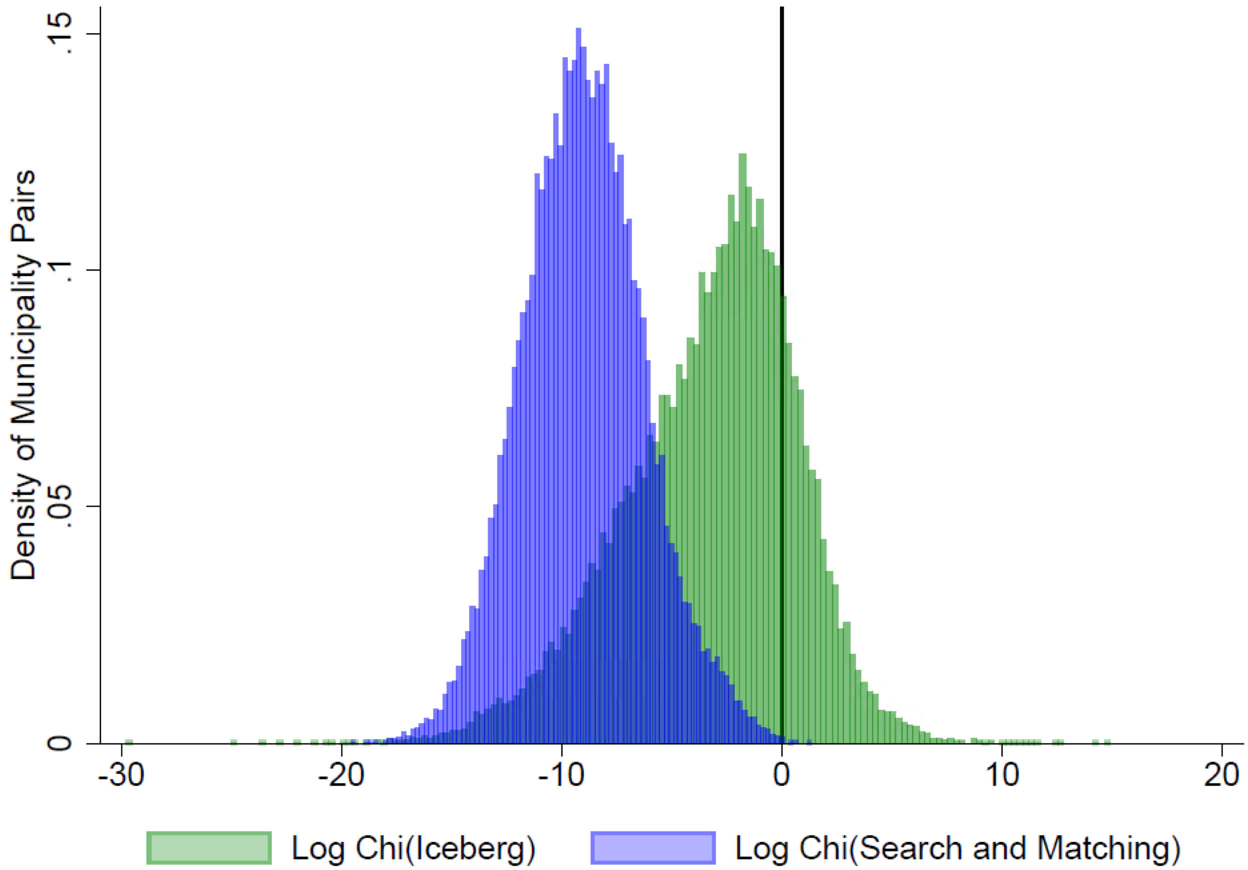
Note that these proxies can be constructed with the extensive and intensive margin trade flows between regions using domestic firm-to-firm trade data in Chile and the estimated parameters of $\tilde{\lambda}^B$ and $\tilde{\lambda}^S$.[18]

Figure 2 presents the probability distribution functions of the estimated $\log(\tilde{\chi}_{ud}^{\text{iceberg}})$ and $\log(\tilde{\chi}_{ud}^{\text{search}})$ across pairs of municipalities in Chile. We find that both $\log(\tilde{\chi}_{ud}^{\text{search}})$ and $\log(\tilde{\chi}_{ud}^{\text{iceberg}})$ are on average in the negative range, while $\log(\tilde{\chi}_{ud}^{\text{search}})$ is on average larger in absolute values than $\log(\tilde{\chi}_{ud}^{\text{iceberg}})$, indicating that search and matching frictions are more relevant frictions than iceberg costs. At the same time, both $\log(\tilde{\chi}_{ud}^{\text{search}})$ and $\log(\tilde{\chi}_{ud}^{\text{iceberg}})$ have a wide dispersion, indicating that both types of frictions are relevant in shaping the heterogeneity of frictions in spatial production network formation across regions.

To further understand the nature of these frictions, we investigate how these two types of trade costs are related to the geographic proximity of the municipality pairs. To do so, we express both iceberg costs and search frictions as functions of geographic proximity up to idiosyncratic factors, such that $\chi_{ud}^{\text{iceberg}} = T_{ud}^{\nu^i} \exp(\epsilon_{ud}^i)$ and $\chi_{ud}^{\text{search}} = T_{ud}^{\nu^s} \exp(\epsilon_{ud}^s)$, where T_{ud} is the proxy for the geographic proximity between u and d (travel time or distance), and ν^s and ν^i are the elasticities of the trade cost with respect to T_{ud}, and ϵ_{ud}^i and ϵ_{ud}^s are idiosyncratic factors. By applying the similar transformation as in Equation (37) and taking log, we have our estimating

[18] Since the customs data does not report the identity of the counterpart of international trade by Chilean firms, we can construct these proxies only for the pairs of municipalities in Chile, but not between Chilean municipalities and international countries.

Figure 2: Distribution of $\log(\tilde{\chi}_{ud}^{\text{iceberg}})$ and $\log(\tilde{\chi}_{ud}^{\text{search}})$



Notes: Probability distribution functions of log of the Head and Ries (2001)-proxy for the iceberg cost shifters $\log(\tilde{\chi}_{ud}^{\text{iceberg}})$ and search and matching friction shifters $\log(\tilde{\chi}_{ud}^{\text{search}})$, estimated using Equation (37) with $\tilde{\lambda}^B = \tilde{\lambda}^S = 0.19$ and the data of firm-to-firm transactions from the SII from 2018.

equations for ν^s and ν^i:

$$\begin{aligned} \log \tilde{\chi}_{ud}^{\text{search}} &= \nu^s \log \tilde{T}_{ud} + \log \tilde{\epsilon}_{ud}^s, \\ \log \tilde{\chi}_{ud}^{\text{iceberg}} &= \nu^i \log \tilde{T}_{ud} + \log \tilde{\epsilon}_{ud}^i, \end{aligned} \tag{38}$$

where $\tilde{T}_{ud} = \frac{T_{ud}}{T_{uu}} \frac{T_{du}}{T_{dd}}$, $\tilde{\epsilon}_{ud}^s = \frac{\epsilon_{ud}^s}{\epsilon_{uu}^s} \frac{\epsilon_{du}^s}{\epsilon_{dd}^s}$, and $\tilde{\epsilon}_{ud}^i = \frac{\epsilon_{ud}^i}{\epsilon_{uu}^i} \frac{\epsilon_{du}^i}{\epsilon_{dd}^i}$, and $\tilde{\chi}_{ud}^{\text{iceberg}}$ and $\tilde{\chi}_{ud}^{\text{search}}$ are as constructed in Equation (37).

Table 4 presents the estimation results of the regression equations (38) by ordinary least squares (OLS) estimators. Columns 1 and 2 presents the results using the distance of kilometers between municipalities in Chile. Columns 3 and 4 present the results using travel time between municipalities. The table shows that, while there is a strong negative correlation of both frictions with geographic proximity proxies (longer travel time or distance imply for greater frictions), the regression coefficients for the search-matching frictions (Columns 1 and 2) is significantly larger in absolute value than iceberg frictions (Columns 3 and 4). In other words, bilateral search-matching costs increase significantly more with longer travel distance and travel time than iceberg cost frictions. Furthermore, the R^2 of the regression of search and matching friction on log distance is significantly larger (0.278) than the regression of iceberg trade cost on log distance (0.049). Thie finding reinforces the interpretation that geographic proximity matters more for search and matching frictions than for iceberg trade costs. These findings are in line with the recent evidence that search and matching frictions are relevant for the spatial trade structure.[19]

Table 4: Decomposition of Spatial Frictions in Production Network Formation

	Iceberg		Search and Matching	
	(1)	(2)	(3)	(4)
Log Distance	-0.376		-0.633	
	(0.007)		(0.004)	
Log Time Travel		-0.436		-0.682
		(0.008)		(0.005)
R^2	0.049	0.053	0.278	0.257
N	53956	53956	53956	53956

Notes: This table presents the regression results of the bilateral frictions in iceberg cost ($\tilde{\chi}_{ud}^{\text{iceberg}}$) and search-matching frictions ($\tilde{\chi}_{ud}^{\text{search}}$) on travel time and travel distance at the bilateral location-level using SII data from 2018. Distance (time travel) is measured with kilometers (minutes of time travel) between municipalities using the fastest land or water transportation method available within Chile. p<0.1.

5.3 Counterfactual Simulations

In this section, we present two sets of counterfactual simulations: international trade shocks from major trading partners, and a planned domestic transportation infrastructure that connects different regions in Chile.

5.3.1 International Trade Shocks

In this section, we study how international trade shocks affect domestic economic activity in the presence of endogenous spatial production networks formation. In particular, using our model,

[19]See Chaney (2014), Allen (2014), Brancaccio, Kalouptsidi, and Papageorgiou (2020), Dasgupta and Mondria (2018), Eaton, Jinkins, Tybout, and Xu (2016), Lenoir, Martin, and Mejean (2020), Krolikowski and McCallum (2021), Startz (2021), Miyauchi (2021) for recent theory and evidence of search frictions in international and intranational trade. In particular, Eaton, Kortum, and Kramarz (2018) provide a similar decomposition of trade frictions into iceberg cost and search frictions using a different theoretical framework, and reach a similar conclusion about the importance of search and matching frictions.

we simulate the equilibrium responses from the changes in the trade costs between Chile and three major international trade partners, China, Germany, and the USA. We show that the aggregate and distributional implications of these international trade shocks are substantially influenced by incorporating the endogenous formation of spatial production networks.

More concretely, we consider a 10% reduction of iceberg trade cost between Chile and the three countries (China, Germany, and the USA), in both directions of exports and imports. From Equations (21) and (22), this change of iceberg trade cost is isomorphic to the change of bilateral resistance shifters $\hat{\chi}_{ud} = \hat{\chi}^E_{ud}\hat{\chi}^I_{ud} = 1.35$ under our calibrated parameters of $\tilde{\lambda}^B$ and $\tilde{\lambda}^S$. Note that, because different municipalities in Chile have different import and export exposures to each of the three countries, these simulated international trade shocks differently affect different municipalities in Chile. Furthermore, these trade shocks can have indirect effects on municipalities in Chile through domestic production networks, even if the regions themselves are not directly exposed to international markets.

To benchmark our results, we also simulate the equilibrium by hypothetically shutting down the endogenous responses of domestic production networks. More concretely, simulate the equilibrium by hypothetically setting $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$, instead of our baseline calibration of $\tilde{\lambda}^S = \tilde{\lambda}^B = 0.19$. As discussed in Section 4.1, in this special case, the extensive margin of spatial production networks do not respond to aggregate shocks. Therefore, the difference between our baseline model ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0.19$) and this special case ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$) is informative about the role of endogenous spatial production network formation in the economic effects of international trade shocks. To make sure that these differences are solely attributed to the endogenous responses of production networks, but not to the magnitudes of the international trade shocks, we set the same value of the changes in bilateral resistance shifters between municipalities in Chile and the three international countries as the baseline, such that $\hat{\chi}_{ud} = 1.35$.

We start by presenting the aggregate welfare effects from these international trade shocks. In Table 5, we present the percentage point changes of the aggregate welfare, measured by the population-weighted average of welfare changes across municipalities. We find that the trade shock to China increases welfare by 3.65%, a larger value compared to those to Germany (0.40%) and the United States (2.55%). These differences in the magnitudes reflect the different levels of direct trade exposures between Chile and the three countries, as well as the indirect trade exposures through domestic production linkages within Chile, as we further discuss below.[20] At the same time, under the alternative model with no endogenous extensive margin response ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$), welfare gain decreases relative to the baseline model by 2.11, 0.10 and 1.19 percentage

[20]In 2018, exports to China constitute about 32% of overall exports, and imports from China constitute about 22% of overall imports, which correspond to about 9% and 6% of Chile's GDP, respectively. In contrast, the exports from Germany are significantly smaller, with 0.4% of GDP for exports and 1% of GDP for imports.

points, respectively. Thus, ignoring the endogenous extensive margin response underestimates the welfare responses significantly.

Table 5: Aggregate Welfare Gains from International Trade Shocks (%)

	China	Germany	USA
Baseline	3.65	0.40	2.55
No Extensive	1.54	0.30	1.37
Baseline - No Extensive	2.11	0.10	1.19

Notes: This table presents the aggregate welfare effects from the international trade shocks from the three countries: China, Germany, and the USA. The welfare gains are measured by the percentage point increase in the population-weighted average of welfare changes across municipalities. “Baseline” and “No Extensive” correspond to our simulation results using our baseline calibration of $\tilde{\lambda}^S = \tilde{\lambda}^B = 0.19$ and our model's special case of no extensive margin response $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$, respectively. For all cases of China, Germany, and the USA, and for both model specifications, we set the magnitudes of trade cost reduction as $\hat{\chi}_{ud} = 1.35$.

These aggregate numbers of welfare changes mask significant heterogeneity across different municipalities in Chile. In Panel A of Figure 3, we plot the welfare gains for each municipality in Chile against the proxy of the direct exposure to international trade, measured by the sum of the import and export share to each of the international countries. Furthermore, we decompose the welfare changes into direct effects (the components of the welfare changes attributed to the changes of trade cost from and to the region) and the indirect effects (the components of the welfare changes attributed to the changes of wages and intermediate goods costs in other regions), using the linearized model discussed in Appendix D.1.[21]

Panel A of Figure 3 shows that there is a strong and positive correlation between the direct welfare effects and direct exposure to international trade. The variation of the direct effects tends to be larger than the indirect effects, indicating that direct effects importantly govern the regions' welfare gains. At the same time, indirect effects are also non-negligible, both in terms of the aggregate values and in terms of the variation across regions. Note that the indirect effects can be both positive and negative, depending on the equilibrium responses of other regions.[22] On average, we find that indirect effects tend to be positive, as evident from the positive values of indirect effects for regions with zero direct exposure to international trade. Interestingly, the indirect effects are relatively flat across different levels of direct international trade exposure. These patterns of results indicate that the geography of domestic networks matters for the spatial dis-

[21]The decomposition of the general equilibrium effects to direct and indirect effects is similar in spirit with Adao, Arkolakis, and Esposito (2019). We find that the gap between the prediction of the linearized model and our original nonlinear model is limited, confirming the tight approximation of the linearized model.

[22]For example, indirect effects can be positive if the locations' main domestic sourcing destinations decrease production costs or the locations' main sales destinations increase demand. Conversely, indirect effects can be negative if the locations' main domestic sourcing destinations increase production costs or the locations' main sales destinations decrease demand.

tribution of welfare gains on top of the direct exposures to international trade, again emphasizing the important role of production networks within a country.

As further robustness of our results about the heterogeneity of direct and indirect effects of international trade shocks, in Panel B of Figure 3, we present the results of the variance decomposition of the total welfare effects across municipalities in Chile into the components attributed to direct effects, indirect effects, and the covariance term, separately for each international country of shocks. The results indicate that the direct effects account for the majority of heterogeneity across municipalities, with some heterogeneity across international countries of shocks. At the same time, the indirect effects are also relevant for the regional variation of welfare gains (e.g., 11% for the USA shock to 26% for the China shock). Interestingly, the covariance terms are positive for China shock and negative for Germany and USA shocks, highlighting that shocks from different international countries have different spatial propagation patterns.[23]

5.3.2 Transportation Infrastructure

In our second counterfactual simulation, we study how planned large-scale transportation infrastructure affects the shape of the domestic spatial production networks, and how it leads to the welfare gains of residents across different regions in Chile.

To study the impacts of transportation infrastructure in a policy-relevant context, we focus on a new bridge planned to open in 2025 that connects the mainland of Chile and Chiloé, the biggest island in Chile.[24] Chiloé is populated by approximately 1% of Chile's population. As of 2021, the only available transportation mode to access Chiloé island from the mainland is through a ferry crossing the Chacao Channel, which takes about 35 minutes (including average waiting time) over around 2 kilometers of sea travel. To promote the economic development and growth of the island, the government has implemented a plan to construct a new bridge. The bridge is planned as a suspension bridge of around 2.6 kilometers, the largest of such bridges in South America. The new bridge is estimated to reduce the time of crossing the Chacao channel to just 2 minutes.

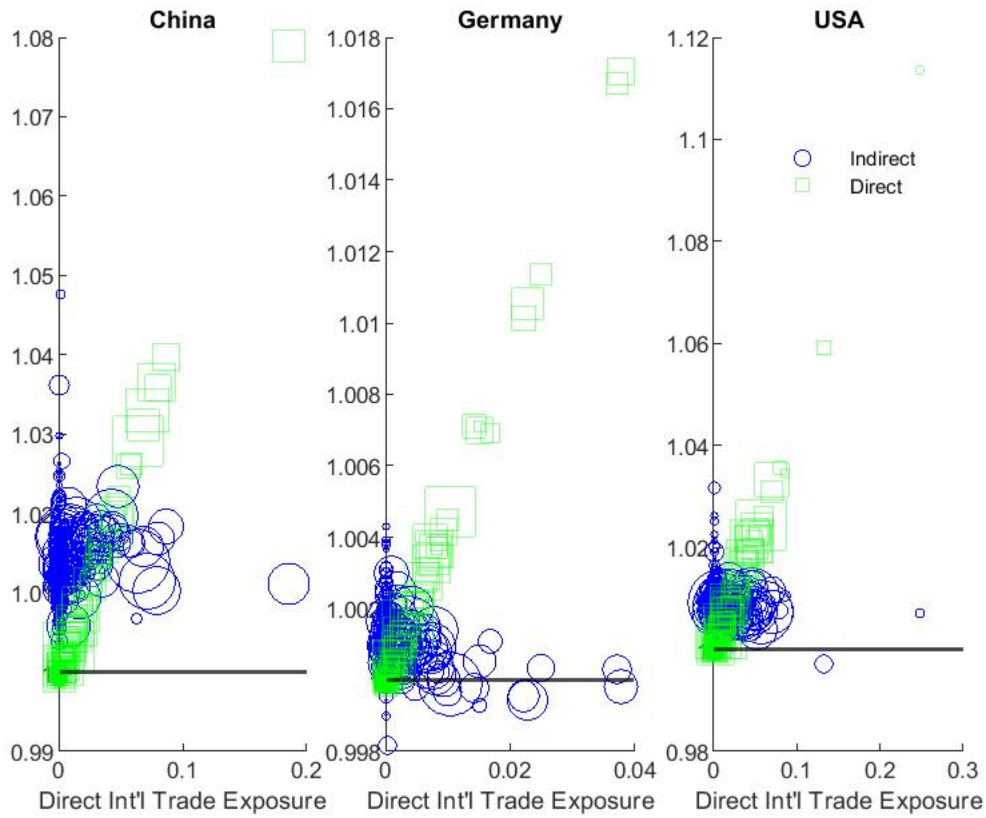
We use our calibrated model to study the welfare implications of the new bridge. Undertaking this counterfactual simulation requires an assumption about how the new bridge affects the trade cost across regions in Chile. For simplicity, we assume that the reduction of trade costs is proportional to the expected changes in travel time. More concretely, using the same assumption as in Section 5.2, we assume that the changes of bilateral residence of trade cost attributed to the

[23]In Appendix Table D.1 and D.2, we present how the patterns of heterogeneous welfare gains differ between our baseline model and the version of our model by shutting endogenous responses in the extensive margin of production networks ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$). We show that the contribution of the indirect effects is substantially smaller in the model with no extensive margin adjustment (Table D.2), indicating that endogenous formation of domestic production networks is particularly relevant for welfare gains through indirect effects.

[24]See Figure C.1 for the geographic location of the future bridge.

Figure 3: Heterogeneous Welfare Gains from International Trade Shocks

(A) Welfare Gains from Trade Shocks and Direct International Trade Exposures



(B) Variance Decomposition of Welfare Gains

	China	Germany	USA
Direct	65	105	97
Indirect	26	19	15
Covariance	9	-23	-12
Total	100	100	100

Notes: This figure documents the heterogeneous effects across municipalities of trade shocks to different countries. Panel A shows the correlation between the proxy for the direct international trade exposure and the welfare gains from trade shocks as discussed in Section 5.3.1. Direct international trade exposure is measured as the sum of the share of imports and exports to regional total expenditure: $\Lambda_{country,i} + \Psi_{i,country}$. The figure also decomposes the welfare changes into direct effects (the components of the welfare changes attributed to the changes of trade cost from and to the region) and the indirect effects (the components of the welfare changes attributed to the changes of wages and intermediate goods costs in other regions), using the linearized model discussed in Appendix D.1. Each municipality circle is weighted by population. Panel B shows the variance decomposition of the same welfare gains across regions into the direct welfare effects, the indirect welfare effects, and the covariance term between the two.

iceberg trade costs are given by $\hat{\chi}_{ud}^{\text{iceberg}} = \hat{T}_{ud}^{\nu^i}$, and that attributed to search and matching frictions is given by $\hat{\chi}_{ud}^{\text{search}} = \hat{T}_{ud}^{\nu^s}$, where $\hat{T}_{ud}$ is the proportional travel time change between u and d due to the new bridge, and we use the estimated values of ν^i and ν^s from the cross-sectional data in Table 4.[25] These assumptions of trade cost reduction, of course, are an approximation, because the new bridge may affect other dimensions of trade costs than travel time. Therefore, the goal of this counterfactual is not to provide an accurate prediction about the impacts of the new bridge. Instead, the goal of this exercise is to highlight how the endogenous spatial network formation matter for the welfare assessment of domestic transportation infrastructure.

We start by presenting our prediction on the aggregate welfare effects. Table 6 presents the estimated aggregate welfare effect from the new bridge, measured by the population-weighted average of welfare changes across municipalities. We find that the bridge increases welfare by 0.84 percentage points. While the effects are small in aggregates, these effects are concentrated around the population of the Chiloé islands and the surrounding regions, as further discussed below.

Similarly to Section 5.3.1, we also undertake the same counterfactual simulation by hypothetically assuming that there are no endogenous responses of the extensive margin of production networks, such that $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$. Under this alternative scenario, we find that the aggregate welfare gains of the bridge are 0.50 percentage points. This number is more than a third less than the baseline model. Thus, ignoring the endogenous changes of production networks leads to a significant underestimation of welfare gains from transportation infrastructure. Intuitively, the model abstracting endogenous production formation rules out the productivity gains through the increased extensive margin of production linkages, as discussed in Proposition 3.

Table 6: Aggregate Welfare Gains from the Bridge to Chiloé Island

	New Bridge
Baseline	0.84
No Extensive	0.50
Baseline - No Extensive	0.34

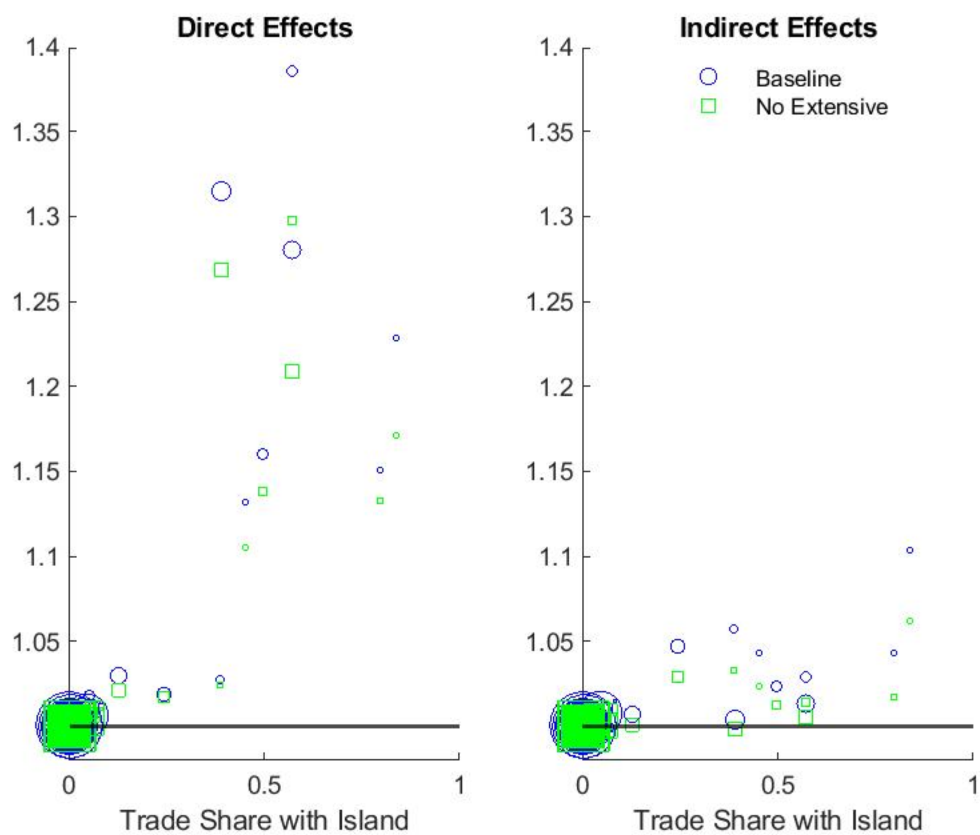
Notes: This table presents the aggregate welfare effects of the new bridge. The welfare gains are measured by the percentage point increase in the population-weighted average of welfare changes across municipalities. "Baseline" and "No Extensive" correspond to our simulation results using our baseline calibration of $\tilde{\lambda}^S = \tilde{\lambda}^B = 0.19$ and our model's special case of no extensive margin response $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$, respectively.

We conclude this section by further studying the sources of heterogeneity of the welfare gains from the bridge. In Figure 4, we plot the relationship between the direct and indirect welfare gains against the trade shares of each municipality to and from the Chiloé island, where we follow the

[25] We estimate $\hat{T}_{u,island} = (T^0_{u,island} - 35)/T^0_{u,island}$, where T^0_{ud} is the minutes it takes from travelling from location u to the island of Chiloé before the construction of the bridge using the fastest land or water transportation method available within Chile. 35 minutes correspond to the time travel saved by the construction of the bridge in traveling to the island. We calibrate the changes of travel time from the island $\hat{T}_{island,d}$ similarly as above.

same decomposition of direct and indirect effects as discussed in Section 5.3.1. These figures show a substantial heterogeneity of welfare gains across regions. In particular, regions that are highly connected with the island in baseline (higher value of the horizontal axis) tend to benefit more.[26] Furthermore, indirect effects also tend to correlate with trade shares with the island. Therefore, abstracting indirect effects through endogenous production networks not only underestimate the welfare gains, but also the distributional gains to regions close to Chiloé island, the intended beneficiary from this policy.

Figure 4: Heterogeneous Welfare Gains from the Bridge to Chiloé Island



Notes: This figure documents the heterogeneous effects across municipalities of transportation infrastructure, in particular the new bridge to the main island Chiloé. The figure shows the correlation across municipalities between the exposure to trade with the island and the direct (left-hand side graph) and indirect welfare effects (right-hand side graph) for both the baseline model and the model without extensive margin adjustment, that is when $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$. Exposure to trade is measured as the sum of the exposure to suppliers and buyers of the island implied by the model: $\Lambda_{island,i} + \Psi_{i,island}$. Direct and indirect welfare effects are derived in Appendix D.1. Each municipality circle is weighted by population.

[26]Naturally, these locations with higher trade exposure to Chiloé island is geographically close to Chiloé island, as further discussed by Appendix D.4.

6 Conclusion

In this paper, we study how production networks are organized in space and how their endogenous formation shapes the spatial distribution of economic activity. Using rich administrative firm-to-firm transaction-level data from Chile, we document that production networks are related to geography; geographic proximity affects trade flows both in the extensive margin (number of supplier-to-buyer relationships) and in the intensive margin (transaction volume per relationship); and international trade shocks affect the shape of domestic production networks. Guided by these pieces of evidence, we build a microfounded model of spatial production network formation based on firms' decisions to search for suppliers and buyers and to form relationships depending on their productivity and geographic location. By aggregating these decisions at the regional level, we provide a tractable characterization of the positive and normative properties of the general equilibrium. We calibrate our model to the observed domestic and international trade patterns and to the impacts of international trade shocks on domestic production networks in Chile. By undertaking counterfactual simulations of international trade shocks and transportation infrastructure, we find strong endogenous responses in the domestic production network. We also find that these responses significantly contribute to the aggregate and heterogeneous welfare effects depending on the regions' exposure to the domestic and global production network.

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Online Appendix for "Spatial Production Networks"

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A Mathematical Derivations

In this appendix, we describe the details of the mathematical derivations of our model presented in Section 3 and 4.

A.1 Firm Search

In this appendix, we solve for firms' search problem (7) in Section 3.2. We first note that firms' search problem (7) is a strictly convex optimization problem when $\gamma^B > 1$ and $\gamma^S > 1$. Therefore, there is a unique solution to the problem and the first order conditions are necessary and sufficient for the solution. The first-order condition of (7) with respect to n^B_{id} , and n^S_{ui} are given by:

$$e_i f^B_{id} \left(n^B_{id}\right)^{\gamma^B - 1} = \frac{1}{\sigma} m^B_{id} D_d \left(\tau_{id}\right)^{1-\sigma} \frac{w_i^{\beta(1-\sigma)} \left(\sum_{u \in N} n^S_{ui} m^S_{ui} \left(C_{ui}\right)^{1-\sigma}\right)^{1-\beta}}{z^{1-\sigma}} \tag{A.1}$$

$$\begin{aligned} e_i f^S_{ui} \left(n^S_{ui}\right)^{\gamma^S - 1} = & \frac{1}{\sigma} \left\{ \sum_{d \in N} n^B_{id} m^B_{id} D_d \left(\tau_{id}\right)^{1-\sigma} \right\} (1-\beta) \\ & \times \frac{w_i^{\beta(1-\sigma)} \left(\sum_{u' \in N} n^S_{u'i} m^S_{u'i} \left(C_{u'i}\right)^{1-\sigma}\right)^{-\beta}}{z^{1-\sigma}} m^S_{ui} \left(C_{ui}\right)^{1-\sigma} \end{aligned} \tag{A.2}$$

Now, we conjecture that the solutions take the form of (8), replicated here:

$$n^S_{ui}(z) = a^S_{ui} z^{\frac{\delta_1}{\gamma^S}};\ n^B_{id}(z) = a^B_{id} z^{\frac{\delta_1}{\gamma^B}} \tag{A.3}$$

where we define $\delta_1 \equiv (\sigma - 1) / \left\{1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S}\right\} > 0$ and a^S_{ui}, a^B_{id} are unknown constants. Plugging these equations into (A.1), and (A.2), we obtain the expressions for a^S_{ui}, a^B_{id} as stated in equations (9), and (10) in the main paper. Because the solution is unique, this is the only possible solution.

By plugging these equations into the cost function (the constraint of equation 7), the unit cost of a firm with productivity z is given by

$$c_i(z) = \frac{w_i^{\beta} \left(\sum_{u \in N} a^S_{ui} z^{\frac{\delta_1}{\gamma^S}} m^S_{ui} \left(C_{ui}\right)^{1-\sigma}\right)^{\frac{1-\beta}{1-\sigma}}}{z} = \left(C^*_i\right) z^{\frac{\delta_1}{\gamma^S}\frac{1-\beta}{1-\sigma} - 1} \tag{A.4}$$

Furthermore, the revenue of a firm with productivity z is given by

$$r_i(z) = \left\{ \sum_{d \in N} n^B_{id} m^B_{id} D_d \left(\tau_{id}\right)^{1-\sigma} \right\} \left(c_i(z)\right)^{1-\sigma} = D^*_i \left(C^*_i\right)^{1-\sigma} (z)^{\delta_1} \tag{A.5}$$

which coincide with equations (13) and (14), respectively. Lastly, by plugging the first-order conditions into the optimal firm profit (7), we obtain the profit equation expressed in terms of firm productivity

$$
\begin{aligned}
\pi_i(z) =& \frac{1}{\sigma} D_i^* (c)^{1-\sigma} - e_i \left\{ \sum_{d\in N} f_{id}^B \frac{\left(n_{id}^B\right)^{\gamma^B}}{\gamma^B} + \sum_{u\in N} f_{ui}^S \frac{\left(n_{ui}^S\right)^{\gamma^S}}{\gamma^S} \right\} \\
=& \frac{1}{\sigma} D_i^* (c)^{1-\sigma} - \frac{1}{\sigma} \frac{\sum_{d\in N} n_{id}^B m_{id}^B D_d \left(\tau_{id}\right)^{1-\sigma}}{\gamma^B} (c)^{1-\sigma} - \frac{1}{\sigma}\frac{1}{\gamma^S} D_i^* (c)^{1-\sigma} (1-\beta) \\
=& \frac{1}{\sigma} \left\{ 1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S} \right\} D_i^* \left(C_i^*\right)^{1-\sigma} (z)^{\delta_1} \\
=& \frac{1}{\delta_1 \tilde{\sigma}} D_i^* \left(C_i^*\right)^{1-\sigma} (z)^{\delta_1},
\end{aligned} \tag{A.6}
$$

where we define $\tilde{\sigma} \equiv \frac{\sigma}{\sigma-1}$, and this expression corresponds to equation (15) of our main paper. Lastly, the demand for advertisement services is given by

$$
h_i(z) = \frac{1}{\sigma} \left\{ \frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S} \right\} D_i^* \left(C_i^*\right)^{1-\sigma} (z)^{\delta_1} = \frac{\frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S}}{1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S}} \pi_i(z). \tag{A.7}
$$

A.2 Aggregate Trade Flows

Extensive margin. We use equations (8), (17), (18), (19), and (20) to solve for the aggregate number of successful supplier-to-buyer matches, M_{ud}, aggregate number of advertisement postings, $\overline{M}_{ud}^S$,$\overline{M}_{ud}^B$, and the matching probabilities, m_{ud}^S,m_{ud}^B. First, by combining equation (20), (17), and (10), we have:

$$
M_{ud} = m_{ud}^S N_d a_{ud}^S \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right) = \left(m_{ud}^S \right)^{\frac{\gamma^S}{\gamma^S - 1}} \tilde{a}_{ud}^S, \tag{A.8}
$$

where

$$
\begin{aligned}
\tilde{a}_{ud}^S &= N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right) \left(\frac{1}{e_d f_{ud}^S} D_d^* (1-\beta) w_d^{\beta(1-\sigma)} \left(\sum_{\ell \in N} a_{\ell d}^S m_{\ell d}^S \left(\overline{C}_\ell \tau_{\ell d} \right)^{1-\sigma} \right)^{-\beta} \left(\overline{C}_u \tau_{ud} \right)^{1-\sigma} \right)^{\frac{1}{\gamma^S - 1}} \\
&= N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right) \left(\frac{1}{e_d f_{ud}^S} D_d^* (1-\beta) w_d^{\frac{\beta(1-\sigma)}{1-\beta}} \left(C_d^* \right)^{-\frac{\beta(1-\sigma)}{1-\beta}} \left(\overline{C}_u \tau_{ud} \right)^{1-\sigma} \right)^{\frac{1}{\gamma^S - 1}},
\end{aligned} \tag{A.9}
$$

where the second transformation uses equation (12). Similarly, by combining equation (20), (18), and (9), we have:

$$
M_{ud} = m_{ud}^B N_u a_{ud}^B \mathbb{M}_u \left(\frac{\delta_1}{\gamma^B} \right) = \left(m_{ud}^B \right)^{\frac{\gamma^B}{\gamma^B - 1}} \tilde{a}_{ud}^B, \tag{A.10}
$$

where

$$\tilde{a}^B_{ud} = N_u \mathbb{M}_u \left(\frac{\delta_1}{\gamma^B}\right) \left(\frac{1}{e_u f^B_{ud}} D_d \left(\tau_{ud}\right)^{1-\sigma} \left(C^*_u\right)^{1-\sigma}\right)^{\frac{1}{\gamma^B-1}}. \tag{A.11}$$

Lastly, from equations (A.8) and (A.10), we have

$$\left(m^S_{ud}\right)^{\frac{\gamma^S}{\gamma^S-1}} = \left(m^B_{ud}\right)^{\frac{\gamma^B}{\gamma^B-1}} \left(\frac{\tilde{a}^B_{ud}}{\tilde{a}^S_{ud}}\right) \tag{A.12}$$

Now, by plugging (20) into equation (19), and using equations (A.8) and (A.12), we have

$$\begin{aligned}
&\left(m^S_{ud}\right)^{\lambda^S} \left(m^B_{ud}\right)^{\lambda^B} = \kappa_{ud} M_{ud}^{\lambda^S+\lambda^B-1} \\
\iff &\left(\left(m^B_{ud}\right)^{\frac{\gamma^B}{\gamma^B-1}} \left(\frac{\tilde{a}^B_{ud}}{\tilde{a}^S_{ud}}\right)\right)^{\lambda^S \frac{\gamma^S-1}{\gamma^S}} \left(m^B_{ud}\right)^{\lambda^B} = \kappa_{ud} \left(\left(m^S_{ud}\right)^{\frac{\gamma^S}{\gamma^S-1}} \tilde{a}^S_{ud}\right)^{\lambda^S+\lambda^B-1} \\
\iff &\left(\left(m^B_{ud}\right)^{\frac{\gamma^B}{\gamma^B-1}} \left(\frac{\tilde{a}^B_{ud}}{\tilde{a}^S_{ud}}\right)\right)^{\lambda^S \frac{\gamma^S-1}{\gamma^S}} \left(m^B_{ud}\right)^{\lambda^B} = \kappa_{ud} \left(\left(m^B_{ud}\right)^{\frac{\gamma^B}{\gamma^B-1}} \tilde{a}^B_{ud}\right)^{\lambda^S+\lambda^B-1} \\
\iff &\left(m^B_{ud}\right)^{\frac{\gamma^B}{\gamma^B-1}\left\{\frac{\lambda^S}{\gamma^S}+\frac{\lambda^B}{\gamma^B}-1\right\}} = \kappa^{-1}_{ud} \left(\tilde{a}^B_{ud}\right)^{1-\frac{\lambda^S}{\gamma^S}-\lambda^B} \left(\tilde{a}^S_{ud}\right)^{-\lambda^S \frac{\gamma^S-1}{\gamma^S}}
\end{aligned}$$

By plugging this equation into equation (A.10), we have

$$M_{ud} = \left[\kappa_{ud} \left(\tilde{a}^B_{ud}\right)^{\lambda^B \frac{\gamma^B-1}{\gamma^B}} \left(\tilde{a}^S_{ud}\right)^{\lambda^S \frac{\gamma^S-1}{\gamma^S}}\right]^{\delta_2},$$

where $\delta_2 = \left[1 - \tilde{\lambda}^S - \tilde{\lambda}^B\right]^{-1}$, $\tilde{\lambda}^S = \lambda^S/\gamma^S$, and $\tilde{\lambda}^B = \lambda^B/\gamma^B$ as defined in our main paper.

Plugging $\tilde{a}^B_{ud}$ and $\tilde{a}^S_{ud}$ from equations (A.9) and (A.11) in the equation above,

$$\begin{aligned}
M_{ud} = &(1-\beta)^{\tilde{\lambda}^S \delta_2} \\
&\times \left[\kappa_{ud} \left(f^B_{ud}\right)^{-\tilde{\lambda}^B} \left(f^S_{ud}\right)^{-\tilde{\lambda}^S} \left(\tau^{1-\sigma}_{ud}\right)^{\tilde{\lambda}^B+\tilde{\lambda}^S}\right]^{\delta_2} \\
&\times \left[\left(N_u \mathbb{M}_u \left(\frac{\delta_1}{\gamma^B}\right)\right)^{\lambda^B \frac{\gamma^B-1}{\gamma^B}} \left\{e^{-1}_u \left(C^*_u\right)^{1-\sigma}\right\}^{\tilde{\lambda}^B} \left(\overline{C}_u\right)^{(1-\sigma)\tilde{\lambda}^S}\right]^{\delta_2} \\
&\times \left[\left(N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S}\right)\right)^{\lambda^S \frac{\gamma^S-1}{\gamma^S}} \left(D_d\right)^{\tilde{\lambda}^B} \left\{D^*_d e^{-1}_d w_d^{\frac{\beta(1-\sigma)}{1-\beta}} \left(C^*_d\right)^{-\frac{\beta(1-\sigma)}{1-\beta}}\right\}^{\tilde{\lambda}^S}\right]^{\delta_2},
\end{aligned}$$

which corresponds to the extensive margin gravity equation (21).

Intensive margin We now derive the average volume of transactions between suppliers in location u and buyers in location d, $\overline{r}_{ud}$. Using equation (6), $\overline{r}_{ud}$ is expressed as

$$\overline{r}_{ud} = \frac{\int D_d \left(c_i(z)\tau_{ud}\right)^{1-\sigma} a_{ud}^B z^{\frac{\delta_1}{\gamma^B}} m_{ud}^B dG_i(z)}{M_{ud}},$$

where the numerator is the total transaction volume from u to d, and the denominator is the number of realized matches from u to d.

The numerator is rewritten as

$$\begin{aligned}
&\int_{z_u^*} D_d \left(c_i(z)\tau_{ud}\right)^{1-\sigma} a_{ud}^B z^{\frac{\delta_1}{\gamma^B}} m_{ud}^B dG_i(z) \\
&= a_{ud}^B m_{ud}^B D_d \left(\tau_{ud}\right)^{1-\sigma} \int \left(C_u^*\right)^{1-\sigma} z^{\frac{\delta_1}{\gamma^S}(1-\beta)+(\sigma-1)} z^{\frac{\delta_1}{\gamma^B}} dG_u(z) \\
&= a_{ud}^B m_{ud}^B D_d \left(\tau_{ud}\right)^{1-\sigma} \left(C_u^*\right)^{1-\sigma} \mathbb{M}_u\left(\delta_1\right),
\end{aligned}$$

where the last transformation used the fact that $\delta_1 = \frac{\delta_1}{\gamma^S}(1-\beta) + (\sigma - 1) + \frac{\delta_1}{\gamma^B}$. The denominator is rewritten as

$$M_{ud} = \int a_{ud}^B z^{\frac{\delta_1}{\gamma^B}} m_{ud}^B dG_i(z) = a_{ud}^B m_{ud}^B \mathbb{M}_u\left(\frac{\delta_1}{\gamma^B}\right).$$

Therefore,

$$\overline{r}_{ud} = D_d \left(\tau_{ud}\right)^{1-\sigma} \left(C_u^*\right)^{1-\sigma} \frac{\mathbb{M}_u\left(\delta_1\right)}{\mathbb{M}_u\left(\frac{\delta_1}{\gamma^B}\right)},$$

which corresponds to the intensive margin gravity equation (22).

A.3 General Equilibrium

In this appendix, we analytically solve for the average intermediate goods cost, $\overline{C}_i$, and demand shifters, D_i^*, D_i, as discussed in Section 3.5..

A.3.1 Average Intermediate Goods Prices ($\overline{C}_i$)

The average intermediate goods price by firms in location i before trade cost payment (weighted by customer advertisement postings) $\overline{C}_i$ is given by

$$\overline{C}_i^{1-\sigma} \tau_{id}^{1-\sigma} = \frac{\int p_i\left(z\right)^{1-\sigma} \tau_{id}^{1-\sigma} n_{id}^B\left(z\right) dG_i(z)}{\int n_{id}^B\left(z\right) dG_i(z')},$$

for any d, where $p_i(z)$ is the prices charged by the firm before trade cost payment, given by

$$p_i(z)^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{w_i^{\beta}}{z}\right)^{1-\sigma}\left(\sum_{u\in N} a_{ui}^{S} m_{ui}^{S} z^{\frac{\delta_1}{\gamma^S}}\left(\tau_{ui}\overline{C}_u\right)^{1-\sigma}\right)^{1-\beta}.$$

From the above equation, we have

$$\overline{C}_i^{1-\sigma} = \frac{\int p_i(z)^{1-\sigma} z^{\frac{\delta_1}{\gamma^B}} dG_i(z)}{\int z^{\frac{\delta_1}{\gamma^B}} dG_i(z')}.$$

The numerator of $\overline{C}_i^{1-\sigma}$ is given by

$$\begin{aligned}
&\int_{z_i^*} p_i(z)^{1-\sigma} z^{\frac{\delta_1}{\gamma^B}} dG_i(z)\\
&= \int_{z_i^*}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\frac{w_i^{\beta}}{z}\right)^{1-\sigma}\left(\sum_{u\in N} a_{ui}^{S} m_{ui}^{S} z^{\frac{\delta_1}{\gamma^S}}\left(\tau_{ui}\overline{C}_u\right)^{1-\sigma}\right)^{1-\beta} z^{\frac{\delta_1}{\gamma^B}} dG_i(z)\\
&= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(C_i^*\right)^{1-\sigma}\mathbb{M}_u\left(\frac{\delta_1}{\gamma^S}(1-\beta)+(\sigma-1)+\frac{\delta_1}{\gamma^B}\right)\\
&= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(C_i^*\right)^{1-\sigma}\mathbb{M}_u\left(\delta_1\right),
\end{aligned}$$

where we use equation (12) in the second equality. Therefore, we obtain equation (24),

$$\overline{C}_i^{1-\sigma} = (\tilde{\sigma})^{1-\sigma}\left(C_i^*\right)^{1-\sigma}\frac{\mathbb{M}_i(\delta_1)}{\mathbb{M}_i\left(\frac{\delta_1}{\gamma^B}\right)}.$$

A.3.2 Origin-Location-Specific Demand Shifter (D_i^*)

We derive the origin-specific demand shifter, D_i^* using the labor market clearing condition. There are two sources of labor demand: intermediate goods producers and advertisement service providers. The aggregate revenue and profit by intermediate goods producers in location i is given by

$$R_i = N_i\int r_i(z)G_i(z)dz = N_i D_i^*\left(C_i^*\right)^{1-\sigma}\mathbb{M}_i(\delta_1), \tag{A.13}$$

$$\Pi_i = N_i\int \pi_i(z)\,G_i(z)dz = \frac{1}{\delta_1\tilde{\sigma}}N_i D_i^*\left(C_i^*\right)^{1-\sigma}\mathbb{M}_i(\delta_1) = \frac{1}{\delta_1\tilde{\sigma}}R_i,$$

where above transformation use the expression for firm revenue from other intermediate producers, $r_i(z)$, and firm profit, $\pi_i(z)$, in equations (14) and (15). In the free entry equilibrium, firm profit is completely offset by the fixed cost payment for labor. Therefore, the total labor compensation by intermediate goods producers is given by $\beta\left(R_i - \Pi_i\right) + \Pi_i$, where the first term is the component of the marginal cost, and the second term is the component of the fixed cost.

Second, we derive the labor demand by advertisement intermediaries. From equation (A.7), the revenue of the advertisement sector is $\frac{\frac{1}{\gamma^B}+\frac{1-\beta}{\gamma^S}}{1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}}$ times aggregate profit, Π_i.

Under labor market clearing condition, the total labor supply, L_i, must be equal to labor demand. Therefore, we have:

$$w_i L_i = \left[\beta\left(R_i - \Pi_i\right) + \Pi_i\right] + \frac{\frac{1}{\gamma^B}+\frac{1-\beta}{\gamma^S}}{1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}}\Pi_i \equiv \vartheta R_i$$

where $\vartheta = \beta - \frac{1}{\delta_1\tilde{\sigma}}\left(1-\beta+\mu\frac{\frac{1}{\gamma^B}+\frac{1-\beta}{\gamma^S}}{1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}}\right)$ is the ratio of aggregate firm revenue from other intermediate producers to labor compensation. Combining this equation with (A.13), we have

$$D_i^* = \frac{1}{\vartheta}\frac{w_i L_i}{\left(C_i^*\right)^{1-\sigma} N_i}\frac{1}{\mathbb{M}_i\left(\delta_1\right)},$$

which is the same equation as equation (25).

A.3.3 Destination-Location-Specific Demand Shifter (D_i)

In this subsection, we derive the destination-speicific demand shifter (D_i). Note that the aggregate intermediate goods demand Y_i is the same as aggregate intermediate goods sales under trade balancing. Therefore,

$$Y_i = R_i = \frac{1}{\vartheta}w_i L_i,$$

where ϑ is again the ratio of aggregate firm revenue from other intermediate producers to labor compensation. Noting the definition of D_i in equation (6) as the demand shifter per matched buyer, intermediate goods market clearing implies that:

$$\begin{aligned}
\frac{1}{\vartheta}w_d L_d &= \sum_u N_u \int (\tau_{ud})^{1-\sigma} D_d (c_d(z))^{1-\sigma} n^B_{ud}(z)\, m^B_{ud} dG_u(z) \\
&= D_d \sum_u N_u (\tau_{ud})^{1-\sigma} \int (C^*_u)^{1-\sigma} a^B_{ud} m^B_{ud} z^{\frac{\delta_1}{\gamma^B}+\frac{\delta_1}{\gamma^S}(1-\beta)+(\sigma-1)} dG_u(z) \\
&= D_d \sum_u (\tau_{ud})^{1-\sigma} (C^*_u)^{1-\sigma} N_u a^B_{ud} m^B_{ud} \mathbb{M}_u(\delta_1) \\
&= D_d \sum_u (\tau_{ud})^{1-\sigma} (C^*_u)^{1-\sigma} N_d a^S_{ud} m^S_{ud} \frac{\mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right)}{\mathbb{M}_u\left(\frac{\delta_1}{\gamma^B}\right)} \mathbb{M}_u(\delta_1) \\
&= D_d (\tilde{\sigma})^{\sigma-1} N_d \mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right) \sum_u (\tau_{ud})^{1-\sigma} \overline{C}_u^{1-\sigma} a^S_{ud} m^S_{ud} \quad \text{(from equation 24)} \\
&= D_d (\tilde{\sigma})^{\sigma-1} N_d \mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right) \left[w_d^{-\beta} C^*_d\right]^{\frac{1-\sigma}{1-\beta}} \quad \text{(from equation 12)}
\end{aligned}$$

Therefore,

$$\begin{aligned}
D_d &= \frac{w_d L_d}{\vartheta (\tilde{\sigma})^{\sigma-1} N_d \mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right) \left[w_d^{-\beta(1-\sigma)} (C^*_d)^{1-\sigma}\right]^{\frac{1}{1-\beta}}}. \\
&= \frac{1}{\vartheta (\tilde{\sigma})^{\sigma-1}} \frac{L_d}{N_d \mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right)} (w_d)^{\frac{1-\beta\sigma}{1-\beta}} (C^*_d)^{\frac{\sigma-1}{1-\beta}}
\end{aligned}$$

which correspond to equation (26).

A.4 Deriving System of Equations in Wages (w_i) and Intermediate Costs (C^*_i)

In this section, we derive the equations (30) and (31) that characterize the general equilibrium in Section 4.1. We first derive equation (30), which corresponds to the buyer access equation. By plugging gravity equations (21) and (22) into the buyer access condition (28), we have

$$w_i = \vartheta \frac{1}{L_i} \zeta^E_i \zeta^I_i \sum_d \chi^E_{id} \chi^I_{id} \xi^E_d \xi^I_d \tag{A.14}$$

Now, the origin-specific shifters of the gravity equations, $\zeta_i^E \zeta_i^I$, is given by

$$
\begin{aligned}
\zeta_i^E \zeta_i^I &= \left[\left(N_i \mathbb{M}_i \left(\frac{\delta_1}{\gamma^B} \right) \right)^{\lambda^B \frac{\gamma^B - 1}{\gamma^B}} \left\{ w_i^{-\mu} C_i^{*-(1-\mu)} C_i^{*1-\sigma} \right\}^{\tilde{\lambda}^B} \left(\tilde{\sigma}^{1-\sigma} C_i^{*1-\sigma} \frac{\mathbb{M}_i (\delta_1)}{\mathbb{M}_i \left(\frac{\delta_1}{\gamma^B} \right)} \right)^{\tilde{\lambda}^S} \right]^{\delta_2} \\
&\quad \times C_i^{*1-\sigma} \frac{\mathbb{M}_i (\delta_1)}{\mathbb{M}_i \left(\frac{\delta_1}{\gamma^B} \right)} \qquad \text{(A.15)} \\
&= K_i^{\zeta} w_i^{-\tilde{\lambda}^B \delta_2 \mu} \left(C_i^* \right)^{-(\sigma-1)\left\{ \left[\tilde{\lambda}^B + \tilde{\lambda}^S \right] \delta_2 + 1 \right\} - \tilde{\lambda}^B \delta_2 (1-\mu)} \\
&= K_i^{\zeta} w_i^{-\tilde{\lambda}^B \delta_2 \mu} \left(C_i^* \right)^{-(\sigma-1)\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)}, \qquad \text{(A.16)}
\end{aligned}
$$

where we define

$$
\begin{aligned}
K_i^{\zeta} &= (\tilde{\sigma})^{(1-\sigma)\delta_2} (N_i)^{\lambda^B \frac{\gamma^B-1}{\gamma^B} \delta_2} \left(\mathbb{M}_i \left(\frac{\delta_1}{\gamma^B} \right) \right)^{\left(\lambda^B \frac{\gamma^B - 1}{\gamma^B} - \tilde{\lambda}^S \right) \delta_2 - 1} \left(\mathbb{M}_i (\delta_1) \right)^{\tilde{\lambda}^S \delta_2} \\
&= (\tilde{\sigma})^{(1-\sigma)\delta_2} \left(\frac{\vartheta}{\delta_1 \tilde{\sigma}} \frac{L_i}{F_i} \right)^{\lambda^B \frac{\gamma^B-1}{\gamma^B} \delta_2} \left(\mathbb{M}_i \left(\frac{\delta_1}{\gamma^B} \right) \right)^{\left(\lambda^B - 1 \right) \delta_2} \left(\mathbb{M}_i (\delta_1) \right)^{\tilde{\lambda}^S \delta_2} \\
&= (\tilde{\sigma})^{(1-\sigma)\delta_2 - \lambda^B \frac{\gamma^B-1}{\gamma^B} \delta_2} \delta_1^{-\delta_2 \lambda^B \frac{\gamma^B-1}{\gamma^B}} (\vartheta)^{\lambda^B \frac{\gamma^B-1}{\gamma^B} \delta_2} \left(\mathbb{M}_i \left(\frac{\delta_1}{\gamma^B} \right) \right)^{\left(\lambda^B - 1 \right) \delta_2} \left(\mathbb{M}_i (\delta_1) \right)^{\tilde{\lambda}^S \delta_2} (L_i)^{\lambda^B \frac{\gamma^B-1}{\gamma^B} \delta_2}.
\end{aligned}
$$

Similarly, the destination-specific shifters of the gravity equations, $\xi_d^E \xi_d^I$, is given

$$
\begin{aligned}
\xi_d^E \xi_d^I &= \left[\left(N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right) \right)^{\lambda^S \frac{\gamma^S-1}{\gamma^S}} (D_d)^{\tilde{\lambda}^B} \left\{ D_d^* \left((w_d)^{\mu} \left(C_d^* \right)^{1-\mu} \right)^{-1} w_d^{\frac{\beta(1-\sigma)}{1-\beta}} \left(C_d^* \right)^{-\frac{\beta(1-\sigma)}{1-\beta}} \right\}^{\tilde{\lambda}^S} \right]^{\delta_2} D_d \\
&= \left(N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right) \right)^{\lambda^S \frac{\gamma^S-1}{\gamma^S} \delta_2} w_d^{\tilde{\lambda}^S \delta_2 \left(\frac{\beta(1-\sigma)}{1-\beta} - \mu \right)} (D_d)^{\tilde{\lambda}^B \delta_2 + 1} \left\{ D_d^* \right\}^{\tilde{\lambda}^S \delta_2} \left(C_d^* \right)^{\tilde{\lambda}^S \delta_2 \left(-\frac{\beta(1-\sigma)}{1-\beta} - (1-\mu) \right)} \\
&= \left(N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right) \right)^{\lambda^S \frac{\gamma^S-1}{\gamma^S} \delta_2} w_d^{\tilde{\lambda}^S \delta_2 \left(\frac{\beta(1-\sigma)}{1-\beta} - \mu \right)} \left(\frac{1}{\vartheta (\tilde{\sigma})^{\sigma-1}} \frac{L_d}{N_d \mathbb{M}_d \left(\frac{\delta_1}{\gamma^S} \right)} (w_d)^{\frac{1-\beta\sigma}{1-\beta}} \left(C_d^* \right)^{\frac{\sigma-1}{1-\beta}} \right)^{\tilde{\lambda}^B \delta_2 + 1} \\
&\quad \times \left\{ \frac{1}{\vartheta} \frac{w_d L_d}{\left(C_d^* \right)^{1-\sigma} N_d} \frac{1}{\mathbb{M}_d (\delta_1)} \right\}^{\tilde{\lambda}^S \delta_2} \left(C_d^* \right)^{\tilde{\lambda}^S \delta_2 \left(-\frac{\beta(1-\sigma)}{1-\beta} - (1-\mu) \right)} \\
&= K_d^{\xi} \left(C_d^* \right)^{\frac{\sigma-1}{1-\beta} \left(\tilde{\lambda}^B \delta_2 + 1 \right) + (\sigma-1) \tilde{\lambda}^S \delta_2 + \tilde{\lambda}^S \delta_2 \left(-\frac{\beta(1-\sigma)}{1-\beta} - (1-\mu) \right)} (w_d)^{\tilde{\lambda}^S \delta_2 \left(\frac{\beta(1-\sigma)}{1-\beta} - \mu \right) + \frac{1-\beta\sigma}{1-\beta} \left(\tilde{\lambda}^B \delta_2 + 1 \right) + \tilde{\lambda}^S \delta_2} \\
&= K_d^{\xi} \left(C_d^* \right)^{\frac{\sigma-1}{1-\beta} \left(\tilde{\lambda}^B \delta_2 + \tilde{\lambda}^S \delta_2 + 1 \right) - (1-\mu) \tilde{\lambda}^S \delta_2} (w_d)^{\tilde{\lambda}^S \delta_2 \left(\frac{\beta(1-\sigma)}{1-\beta} - \mu \right) + \frac{1-\beta\sigma}{1-\beta} \left(\tilde{\lambda}^B \delta_2 + 1 \right) + \tilde{\lambda}^S \delta_2} \\
&= K_d^{\xi} \left(C_d^* \right)^{\frac{(\sigma-1)\delta_2}{1-\beta} - (1-\mu) \tilde{\lambda}^S \delta_2} (w_d)^{\delta_G} \qquad \text{(A.17)}
\end{aligned}
$$

where we define $\delta_G = \tilde{\lambda}^S \delta_2 \left(\mu + \frac{1-\beta\sigma}{1-\beta}\right) + \left(\tilde{\lambda}^B \delta_2 + 1\right) \frac{1-\beta\sigma}{1-\beta} = \tilde{\lambda}^S \delta_2 \mu + \frac{1-\beta\sigma}{1-\beta}\delta_2$, and

$$
\begin{aligned}
K_d^{\xi} &= (\tilde{\sigma})^{(1-\sigma)\left(\tilde{\lambda}^B \delta_2 + 1\right)} \vartheta^{-\delta_2} (N_d)^{\delta_2\left(\lambda^S \frac{\gamma^S - 1}{\gamma^S} - 1\right)} \left(\mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right)\right)^{\lambda^S \frac{\gamma^S-1}{\gamma^S}\delta_2 - \tilde{\lambda}^B \delta_2 - 1} (\mathbb{M}_d(\delta_1))^{-\tilde{\lambda}^S \delta_2} (L_d)^{\delta_2} \\
&= (\tilde{\sigma})^{(1-\sigma)\left(\tilde{\lambda}^B \delta_2 + 1\right)} \vartheta^{-\delta_2} \left(\frac{\vartheta}{\delta_1 \tilde{\sigma}} \frac{L_d}{F_d}\right)^{\delta_2\left(\lambda^S \frac{\gamma^S - 1}{\gamma^S} - 1\right)} \left(\mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right)\right)^{\lambda^S \frac{\gamma^S-1}{\gamma^S}\delta_2 - \tilde{\lambda}^B \delta_2 - 1} (\mathbb{M}_d(\delta_1))^{-\tilde{\lambda}^S \delta_2} (L_d)^{\delta_2} \\
&= (\tilde{\sigma})^{(1-\sigma)\left(\tilde{\lambda}^B \delta_2 + 1\right) - \delta_2\left(\lambda^S \frac{\gamma^S - 1}{\gamma^S} - 1\right)} \delta_1^{-\delta_2\left(\lambda^S \frac{\gamma^S - 1}{\gamma^S} - 1\right)} \vartheta^{\delta_2 \lambda^S \frac{\gamma^S - 1}{\gamma^S}} F_d^{-\delta_2\left(\lambda^S \frac{\gamma^S - 1}{\gamma^S} - 1\right)} \\
&\quad \times \left(\mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right)\right)^{\left(\lambda^S - 1\right)\delta_2} (\mathbb{M}_d(\delta_1))^{-\tilde{\lambda}^S \delta_2} L_d^{\delta_2 \lambda^S \frac{\gamma^S - 1}{\gamma^S}}.
\end{aligned}
$$

Furthermore, the multiplication of the constant terms, $K_i^{\zeta} K_d^{\xi}$, is given by:

$$
\begin{aligned}
K_i^{\zeta} K_d^{\xi} = (\tilde{\sigma})^{(1-\sigma)\left(\left(\tilde{\lambda}^B + 1\right)\delta_2 + 1\right)\delta_2 - \delta_2\left(\lambda^B + \lambda^S - \delta_2^{-1}\right)} \delta_1^{-\delta_2\left(\lambda^B + \lambda^S - \delta_2^{-1}\right)} (\vartheta)^{\left(\lambda^S \frac{\gamma^S - 1}{\gamma^S} + \lambda^S \frac{\gamma^S - 1}{\gamma^S}\right)\delta_2} \\
\times \left[\left(\mathbb{M}_i\left(\frac{\delta_1}{\gamma^B}\right)\right)^{\lambda^B - 1} \left(\mathbb{M}_d\left(\frac{\delta_1}{\gamma^S}\right)\right)^{\lambda^S - 1} (\mathbb{M}_i(\delta_1))^{\tilde{\lambda}^S} (\mathbb{M}_d(\delta_1))^{-\tilde{\lambda}^S} (L_i)^{\lambda^B \frac{\gamma^B - 1}{\gamma^B}} L_d^{\lambda^S \frac{\gamma^S - 1}{\gamma^S}}\right]^{\delta_2}.
\end{aligned}
$$

By plugging these equations into the buyer access equation (A.14),

$$
\begin{aligned}
(w_i)^{1+\tilde{\lambda}^B \delta_2 \mu} \left(C_i^*\right)^{(\sigma-1)\delta_2 + \tilde{\lambda}^B \delta_2 (1-\mu)} &= \varrho^E \vartheta \frac{1}{L_i} \sum_d \chi_{id}^E \chi_{id}^I K_i^{\zeta} K_d^{\xi} \left(C_d^*\right)^{\frac{(\sigma-1)\delta_2}{1-\beta} - \tilde{\lambda}^S \delta_2 (1-\mu)} (w_d)^{-\delta_G} \\
&= \varsigma \frac{1}{L_i} \sum_d K_{id} \left(C_d^*\right)^{\frac{(\sigma-1)\delta_2}{1-\beta} - \tilde{\lambda}^S \delta_2 (1-\mu)} (w_d)^{-\delta_G}
\end{aligned}
$$

which corresponds to equation (30) by defining ς and K_{id} as in the main paper.

We next derive equation (31). From the definition of C_i^* in equation (12),

$$
\begin{aligned}
(C_i^*)^{1-\sigma} &= w_i^{\beta(1-\sigma)} \left(\sum_{u \in N} a_{ui}^S m_{ui}^S \left(C_{ui}\right)^{1-\sigma} \right)^{1-\beta} \\
&= w_i^{\beta(1-\sigma)} \left(\sum_{u \in N} \frac{M_{ui}}{N_i \mathbb{M}_i \left(\frac{\delta_1}{\gamma^S}\right)} \left(\tau_{ui}\right)^{1-\sigma} \left(\tilde{\sigma}\right)^{1-\sigma} \left(C_u^*\right)^{1-\sigma} \frac{\mathbb{M}_u\left(\delta_1\right)}{\mathbb{M}_u\left(\frac{\delta_1}{\gamma^B}\right)} \right)^{1-\beta} \\
&= w_i^{\beta(1-\sigma)} \left(\sum_{u \in N} \frac{M_{ui}}{N_i \mathbb{M}_i \left(\frac{\delta_1}{\gamma^S}\right)} \frac{\bar{r}_{ui}}{D_i} \left(\tilde{\sigma}\right)^{1-\sigma} \right)^{1-\beta} \\
&= w_i^{\beta(1-\sigma)} \left(\frac{\left(\tilde{\sigma}\right)^{1-\sigma}}{N_i \mathbb{M}_i \left(\frac{\delta_1}{\gamma^S}\right)} \frac{1}{D_i} \right)^{1-\beta} \left(\sum_{u \in N} M_{ui} \bar{r}_{ui} \right)^{1-\beta} \\
&= w_i^{\beta(1-\sigma)} \left(\frac{\left(\tilde{\sigma}\right)^{1-\sigma}}{N_i \mathbb{M}_i \left(\frac{\delta_1}{\gamma^S}\right)} \frac{1}{\frac{1}{\vartheta(\tilde{\sigma})^{\sigma-1}} \frac{L_i}{N_i \mathbb{M}_i\left(\frac{\delta_1}{\gamma^S}\right)} \left(w_i\right)^{\frac{1-\beta\sigma}{1-\beta}} \left(C_i^*\right)^{\frac{\sigma-1}{1-\beta}}} \right)^{1-\beta} \left(\varrho^E \xi_i^E \xi_i^I \sum_u \zeta_u^E \zeta_u^I \chi_{ui}^E \chi_{ui}^I \right)^{1-\beta} \\
&= (\vartheta)^{1-\beta} w_i^{\beta(1-\sigma)} \left(\frac{1}{L_i \left(w_i\right)^{\frac{1-\beta\sigma}{1-\beta}} \left(C_i^*\right)^{\frac{\sigma-1}{1-\beta}}} \right)^{1-\beta} \left(K_i^\zeta \left(C_i^*\right)^{\frac{(\sigma-1)\delta_2}{1-\beta} - (1-\mu)\tilde{\lambda}^S \delta_2} \left(w_i\right)^{-\delta_G} \right)^{1-\beta} \\
&\quad \times \left(\varrho^E \sum_u \chi_{ui}^E \chi_{ui}^I K_u^\zeta w_u^{-\tilde{\lambda}^B \delta_2 \mu} \left(C_u^*\right)^{-(\sigma-1)\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)} \right)^{1-\beta} \\
&= (\vartheta)^{1-\beta} w_i^{\beta(1-\sigma)-(1-\beta\sigma)-\delta_G(1-\beta)} \left(C_i^*\right)^{(\sigma-1)\delta_2 - (1-\mu)\tilde{\lambda}^S \delta_2 (1-\beta)} \\
&\quad \times \left(\varrho^E \frac{1}{L_i} K_i^\zeta \sum_u \chi_{ui}^E \chi_{ui}^I K_u^\zeta w_u^{-\tilde{\lambda}^B \delta_2 \mu} \left(C_u^*\right)^{-(\sigma-1)\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)} \right)^{1-\beta} \iff
\end{aligned}
$$

$$
\begin{aligned}
w_i^{1+\delta_G} \left(C_i^*\right)^{-\frac{\sigma-1}{1-\beta}\delta_2 + \tilde{\lambda}^S \delta_2 (1-\mu)} &= \varrho^E \vartheta \frac{1}{L_i} K_i^\zeta \sum_u \chi_{ui}^E \chi_{ui}^I K_u^\zeta w_u^{-\tilde{\lambda}^B \delta_2 \mu} \left(C_u^*\right)^{-(\sigma-1)\delta_2 - (1-\mu)\tilde{\lambda}^S \delta_2} \\
&= \varsigma \sum_u K_{ui} w_u^{-\tilde{\lambda}^B \delta_2 \mu} \left(C_u^*\right)^{-(\sigma-1)\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu)}
\end{aligned}
$$

which corresponds to equation (31).

A.5 Isomorphism to Gravity Trade Models when $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$

In this section, we discuss that our model comes down to be isomorphic to canonical gravity trade models in the literature when we set $\tilde{\lambda}^S = \tilde{\lambda}^B = 0$, $\delta_2 = 1$ and $\delta_G = \frac{1-\beta\sigma}{1-\beta}$. Under these parameter values, the equilibrium conditions (30) and (31) come down to the following set of equations:

$$(w_i)\left(C_i^*\right)^{(\sigma-1)} = \sum_d K_{id}^D \left(w_d\right)^{-\frac{\beta\sigma-1}{1-\beta}} \left(C_d^*\right)^{\frac{\sigma-1}{1-\beta}}, \tag{A.18}$$

$$(w_i)^{1+\frac{\beta\sigma-1}{1-\beta}} \left(C_i^*\right)^{-\frac{\sigma-1}{1-\beta}} = \sum_u K_{ui}^U \left(C_u^*\right)^{-(\sigma-1)}. \tag{A.19}$$

To see the isomorphism to canonical gravity trade models more closely, we redefine the cost shifter C_i^* such that

$$C_i^* = w_i^\beta \tilde{C}_i^{1-\beta}$$

Using the newly defined $\tilde{C}_i$, the first equation (A.18) is rewritten as

$$\begin{aligned}
(w_i)^{1+\beta(\sigma-1)} \tilde{C}_i^{(1-\beta)(\sigma-1)} &= \sum_d K_{id}^D \left(w_d\right)^{-\frac{\beta\sigma-1}{1-\beta}+\frac{\beta(\sigma-1)}{1-\beta}} \left(\tilde{C}_d\right)^{\sigma-1} \iff \\
(w_i)^{1+\beta(\sigma-1)} \tilde{C}_i^{(1-\beta)(\sigma-1)} &= \sum_d K_{id}^D \left(w_d\right)^{1} \left(\tilde{C}_d\right)^{\sigma-1},
\end{aligned}$$

and the second equation (A.19) is rewritten as

$$\begin{aligned}
(w_i)^{1+\frac{\beta\sigma-1}{1-\beta}-\frac{\sigma-1}{1-\beta}\beta} \left(\tilde{C}_i\right)^{-(\sigma-1)} &= \sum_u K_{ui}^U w_i^{-\beta(\sigma-1)} \tilde{C}_i^{-(1-\beta)(\sigma-1)} \iff \\
\left(\tilde{C}_i\right)^{-(\sigma-1)} &= \sum_u K_{ui}^U w_i^{-\beta(\sigma-1)} \tilde{C}_i^{-(1-\beta)(\sigma-1)}
\end{aligned}$$

The first and second equations correspond to equation (3.10 + 3.14) and (3.8) in Alvarez and Lucas (2007) with $\theta = 1/(\sigma-1)$ without taxes, respectively. Furthermore, the first and second equations correspond to (45) and (42-45) in Eaton, Kortum, and Kramarz (2011) with $\theta = \sigma - 1$ without taxes, respectively.

B Proofs

B.1 Proof of Proposition 1

We will use Theorem 1 (ii) in Allen, Arkolakis, and Li (2020) and express the system in terms of their notation. Notice that the matrices $K_{id}^D, K_{id}^U > 0$. Define the matrices

$$\Gamma = \begin{bmatrix} 1+\tilde{\lambda}^B\delta_2\mu & (\sigma-1)\,\delta_2+\tilde{\lambda}^B\delta_2\,(1-\mu) \\ 1-\delta_G & -\frac{(\sigma-1)\delta_2}{1-\beta}+\tilde{\lambda}^S\delta_2\,(1-\mu) \end{bmatrix} = \begin{bmatrix} 1+c_1 & c_2 \\ 1-\delta_G & -c_3 \end{bmatrix}$$

and

$$B = \begin{bmatrix} \delta_G & \frac{(\sigma-1)\delta_2}{1-\beta} - \tilde{\lambda}^S \delta_2 (1-\mu) \\ -\tilde{\lambda}^B \delta_2 \mu & -(\sigma-1)\delta_2 - \tilde{\lambda}^B \delta_2 (1-\mu) \end{bmatrix} = \begin{bmatrix} \delta_G & c_3 \\ -c_1 & -c_2 \end{bmatrix}$$

where

$$c_1 = \tilde{\lambda}^B \delta_2 \mu$$
$$c_2 = (\sigma-1)\delta_2 + \tilde{\lambda}^B \delta_2 (1-\mu)$$
$$c_3 = \frac{(\sigma-1)\delta_2}{1-\beta} - \tilde{\lambda}^S \delta_2 (1-\mu)$$

where $c_1 > 0$ and $c_2 > 0$ under our model parameter assumptions. We will provide a sufficient condition that the spectral radius of $A = |B\Gamma^{-1}|$ is equal to 1 and thus the equilibrium is unique according to the theorem. To show this, note that:

$$\begin{aligned} B\Gamma^{-1} &= \frac{1}{-c_3(1+c_1) - c_2(1-\delta_G)} \begin{bmatrix} \delta_G & c_3 \\ -c_1 & -c_2 \end{bmatrix} \begin{bmatrix} -c_3 & -c_2 \\ -(1-\delta_G) & 1+c_1 \end{bmatrix} \\ &= \frac{1}{-c_3(1+c_1) - c_2(1-\delta_G)} \begin{bmatrix} c_3 & -\delta_G c_2 + c_3(1+c_1) \\ c_1 c_3 + (1-\delta_G) c_2 & -c_2 \end{bmatrix} \end{aligned}$$

We now show that, when $\delta_G \leq 1$ and $\frac{\beta(\sigma-1)}{1-\beta} > (1-\mu)\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)$ as assumed in Proposition 1, the largest eigenvalue of $B\Gamma^{-1}$ is less than one. From second condition, we have $c_3 > 0$ and $c_3 > c_2$. Furthermore, $-\delta_G c_2 + c_3(1+c_1) \geq c_1 c_3 + (1-\delta_G) c_2 > 0$. Therefore, the absolute value of $B\Gamma^{-1}$ is given by

$$|B\Gamma^{-1}| = \frac{1}{c_3(1+c_1) + c_2(1-\delta_G)} \begin{bmatrix} c_3 & -\delta_G c_2 + c_3(1+c_1) \\ c_1 c_3 + (1-\delta_G) c_2 & c_2 \end{bmatrix}$$

Note that the sum of the rows for the first column and second column are both one. Therefore, from Collatz–Wielandt Formula (see Remark 5 in Allen, Arkolakis, and Li (2020)), the largest eigenvalue of $B\Gamma^{-1}$ is one under this condition. Therefore, when $\delta_G < 1$ and $\frac{\beta(\sigma-1)}{1-\beta} > (1-\mu)\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)$, the equilibrium exists and it is unique up to scale.

B.2 Proof of Proposition 2

In the main text, we define $\Psi_{id} = \frac{X_{id}}{\sum_\ell X_{i\ell}}$ as the observed share of intermediate goods sales by firms in location i to location d, and $\Lambda_{ui} = \frac{X_{ui}}{\sum_\ell X_{\ell i}}$ is the observed share of intermediate goods expenditure by firms in location i from location u. Following a similar manipulations as in Appendix A.4, we have

$$\Psi_{id} = \frac{X_{id}}{\sum_\ell X_{i\ell}} = \frac{\chi^E_{id}\chi^I_{id}\xi^E_d\xi^I_d}{\sum_\ell \chi^E_{i\ell}\chi^I_{i\ell}\xi^E_\ell\xi^I_\ell} = \frac{K^D_{id}(w_d)^{\delta_G}(C^*_d)^{\frac{(\sigma-1)\delta_2}{1-\beta}}}{\sum_\ell K^D_{i\ell}(w_\ell)^{\delta_G}(C^*_\ell)^{\frac{(\sigma-1)\delta_2}{1-\beta}}}$$

$$\Lambda_{ui} = \frac{X_{ui}}{\sum_\ell X_{\ell i}} = \frac{\zeta_u^E \zeta_u^I \chi_{ui}^E \chi_{ui}^I}{\sum_\ell \zeta_\ell^E \zeta_\ell^I \chi_{\ell i}^E \chi_{\ell i}^I} = \frac{K_{ui}^U (w_u)^{-\tilde{\lambda}^B \delta_2} (C_u^*)^{-(\sigma-1)\delta_2}}{\sum_\ell K_{\ell i}^U (w_\ell)^{-\tilde{\lambda}^B \delta_2} (C_\ell^*)^{-(\sigma-1)\delta_2}}$$

where K_{id}^D and K_{ui}^U correspond to the definitions of equations (30) and (31). Now, by denoting the variable x in the new equilibrium by x' (with a prime) and the ratio change of x as $\hat{x} = x/x'$, we can rearrange equation (30) as

$$\begin{aligned}
(\hat{w}_i)^{1+\tilde{\lambda}^B \delta_2} (\hat{C}_i^*)^{(\sigma-1)\delta_2} &= \frac{\sum_d K_{id}^{D'} \left(w_d'\right)^{\delta_G} \left(C_d^{*'}\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}}}{\sum_\ell K_{id}^D (w_d)^{\delta_G} (C_d^*)^{\frac{(\sigma-1)\delta_2}{1-\beta}}} \\
&= \sum_d K_{id}^{D'} \left(w_d'\right)^{\delta_G} \left(C_d^{*'}\right)^{\frac{(\sigma-1)\delta_2}{1-\beta}} \frac{\Psi_{id}}{K_{id}^D (w_d)^{\delta_G} (C_d^*)^{\frac{(\sigma-1)\delta_2}{1-\beta}}} \\
&= \sum_d \hat{K}_{id}^D (\hat{w}_d)^{\delta_G} (\hat{C}_d^*)^{\frac{(\sigma-1)\delta_2}{1-\beta}} \Psi_{id}
\end{aligned}$$

which corresponds to equation (33) of Proposition 2. Similarly, we have

$$(\hat{w}_i)^{1+\delta_G} (\hat{C}_i^*)^{-\frac{(\sigma-1)\delta_2}{1-\beta}} = \sum_u \hat{K}_{ui}^U (\hat{w}_u)^{-\tilde{\lambda}^B \delta_2} (\hat{C}_u^*)^{-(\sigma-1)\delta_2} \Lambda_{ui},$$

which corresponds to equation (34) of Proposition 2.

B.3 Proof of Proposition 3

To derive the sufficient statistics for welfare, we first obtain the analytical expression for the final goods price index, P_i^F. Under the assumptions of production functions and perfect competition of final goods producers (equation 4), P_i^F is given by:

$$\left(P_i^F\right)^{1-\sigma} = \int \{c_i(z)\}^{1-\sigma} G_i(z)\, dz = (C_i^*)^{1-\sigma} \mathbb{M}_i \left(-\frac{\delta_1}{\gamma^S}\frac{1-\beta}{\sigma-1} - 1\right). \tag{B.1}$$

Therefore, the final goods price index is proportional to the cost index C_i^*.

Next, we derive the expression of the changes of cost shifter C_i^*. To do so, first note that the share of intermediate goods expenditure from suppliers in location u is given by:

$$\Lambda_{ui} = \frac{a_{ui}^S m_{ui}^S \left(\overline{C}_u \tau_{ui}\right)^{1-\sigma}}{\sum_{\ell \in N} a_{\ell i}^S m_{\ell i}^S \left(\overline{C}_\ell \tau_{\ell i}\right)^{1-\sigma}}, \tag{B.2}$$

where this expression comes from the solution to the firms' search problem (7). By combining this expression with the definition of C_i^* in equation (12), and because $\hat{C}_u^* = \hat{\overline{C}}_u$ from equation

(24), we have

$$
\begin{aligned}
\left(\hat{C}_u^*\right)^{1-\sigma} &= \hat{w}_u^{\beta(1-\sigma)} \left(\hat{a}_{uu}^S \hat{m}_{uu}^S \left(\hat{\bar{C}}_u\right)^{1-\sigma} \hat{\Lambda}_{uu}^{-1}\right)^{1-\beta} \\
&= \hat{w}_u^{\beta(1-\sigma)} \left(\hat{a}_{uu}^S \hat{m}_{uu}^S \left(\hat{C}_u^*\right)^{1-\sigma} \hat{\Lambda}_{uu}^{-1}\right)^{1-\beta} \\
&= \hat{w}_u^{\beta(1-\sigma)} \left(\hat{M}_{uu} \left(\hat{C}_u^*\right)^{1-\sigma} \hat{\Lambda}_{uu}^{-1}\right)^{1-\beta} \quad \text{(from equations 17 and 20)} \\
&= \hat{w}_u^{(1-\sigma)} \left(\hat{M}_{uu}\right)^{\frac{1-\beta}{\beta}} \hat{\Lambda}_{uu}^{-\frac{1-\beta}{\beta}}
\end{aligned}
$$

Together, the changes of welfare is given by

$$
\widehat{\frac{w_i}{P_i^F}} = \left(\hat{\Lambda}_{ii}\right)^{-\frac{1}{\sigma-1}\frac{1-\beta}{\beta}} \left(\hat{M}_{ii}\right)^{\frac{1}{\sigma-1}\frac{1-\beta}{\beta}},
$$

which corresponds to the expression in Proposition 3.

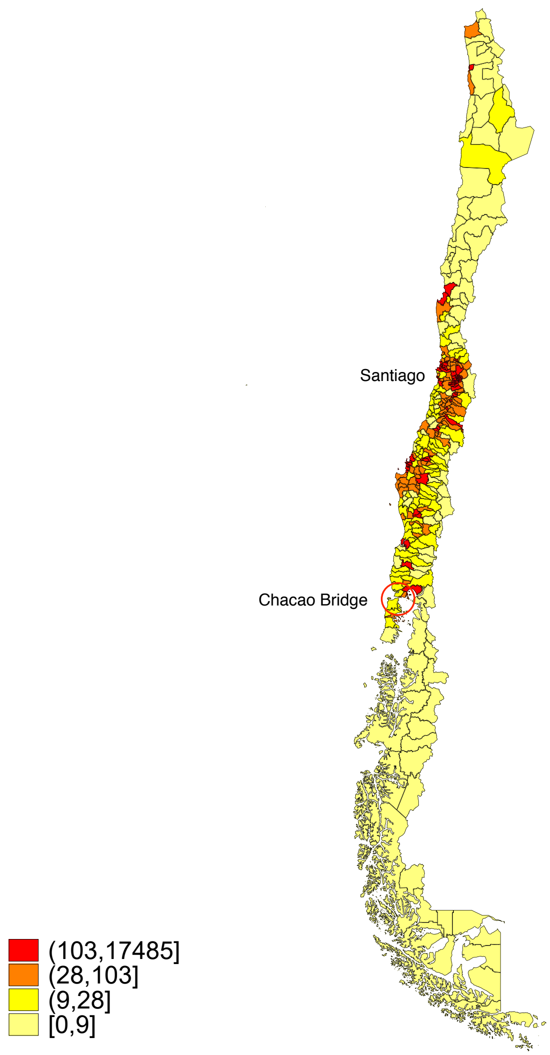
C Additional Figures and Tables for Descriptive Facts

In this section, we provide additional figures and tables for Section 2 of the paper.

In Figure C.1 we plot the population density of municipalities in Chile. We show that density is concentrated near Santiago and in the south of Santiago. We also show the location of Chacao Bridge, the planned bridge that we study in one of our counterfactual simulations in Section 5.3.2.

In Table C.1 we document robustness exercises of Fact 2 from Section 2.2. The Table presents the results of the same regression as Panel B of Figure 1, where we further control for additional firm-level characteristics, such as narrow industry classification and international trade activity (both exports and imports). We find that the coefficients on the population density remain positive and significant, providing further support to the fact that geographic proxies are strongly correlated with the number of domestic suppliers and buyers.

Figure C.1: Map of Chile with Population Density



Notes: This figure shows the map of Chile at the municipality level. Darker color indicates a higher population per squared kilometers (density). The map shows the location of the capital city of Chile, Santiago, and the new Chacao Bridge, which is planned to connect the mainland with the largest island of Chile, Chiloé, by 2025.

Table C.1: Number of Suppliers and Buyers and Geography: Robustness

	Customers			Suppliers		
	(1)	(2)	(3)	(4)	(5)	(6)
Log Density	0.028^{***}		0.017^{***}	0.062^{***}		0.051^{***}
	(0.001)		(0.001)	(0.002)		(0.002)
Log Sales		0.422^{***}	0.422^{***}		0.414^{***}	0.413^{***}
		(0.001)	(0.001)		(0.001)	(0.001)
R^2	0.188	0.537	0.537	0.290	0.414	0.415
Year FE	✓	✓	✓	✓	✓	✓
Industry FE	✓	✓	✓	✓	✓	✓
State FE	✓	✓	✓	✓	✓	✓
Other Controls	✓	✓	✓	✓	✓	✓
N	380588	380588	380588	381362	381362	381362

Notes: This table documents the firm relationship between the number of links with population density using SII data. It shows a regression analysis at the firm level where the left-hand-side variable is the number of links per firm (with buyers in Columns 1-3 and suppliers in Columns 4-6). The regression includes a year, state and industry fixed effects. There are 15 states in Chile. It includes also controls for dummies of export and import status, as well as export and import intensities of firms engaged in international trade.

D Quantitative Section Appendix

D.1 Linearized Equilibrium System

In this section, we provide a linearized equilibrium system. We use this system for the calibration of structural parameters in Section 5.1 and for the decomposition of direct and indirect shocks in Section 5.3.

To do so, we start by log-linearizing our equilibrium system in equation (33) and (34). We follow Adao, Arkolakis, and Esposito (2019) in computing the direct and the indirect effects of the general equilibrium system and in expressing the full upstream and downstream effect of the trade shocks. By denoting $\tilde{x} = \log\left(x'/x\right)$ where x' is the new equilibirum under the shock, the log-linearized system is given by:

$$\left(1 + \tilde{\lambda}^B \delta_2 \mu\right)\left(\tilde{w}_i\right) + \bar{c}_B\left(\tilde{C}_i^*\right) = \sum_d \Psi_{id}\left(\tilde{K}_{id}^D + \delta_G \tilde{w}_d + \bar{c}_S \tilde{C}_d^*\right), \tag{D.1}$$

$$\left(1 - \delta_G\right)\left(\tilde{w}_i\right) - \bar{c}_S\left(\tilde{C}_i^*\right) = \sum_u \Lambda_{ui}\left(\tilde{K}_{ui}^U - \tilde{\lambda}^B \delta_2 \mu \tilde{w}_u - \bar{c}_B \tilde{C}_u^*\right), \tag{D.2}$$

where

$$\bar{c}_S \equiv \delta_2\left[\frac{(\sigma-1)}{1-\beta} - \tilde{\lambda}^S\left(1-\mu\right)\right], \bar{c}_B \equiv \delta_2\left[(\sigma-1) + \tilde{\lambda}^B\left(1-\mu\right)\right].$$

We rewrite the above system of equations in the matrix form. The first equation is rewritten as:

$$\left(\bar{c}_B \bar{\boldsymbol{I}} - \bar{c}_s \bar{\boldsymbol{\Psi}}\right)\boldsymbol{C}^* = -\left(\left(1 + \tilde{\lambda}^B \delta_2 \mu\right)\bar{\boldsymbol{I}} - \delta_G \bar{\boldsymbol{\Psi}}\right)\boldsymbol{w} + \bar{\boldsymbol{\eta}}^{\Psi} \implies$$

$$\boldsymbol{C}^* = -\left(\bar{c}_B\, \bar{\boldsymbol{I}} - \bar{c}_s \bar{\boldsymbol{\Psi}}\right)^{-1}\left(\left(1 + \tilde{\lambda}^B \delta_2 \mu\right)\bar{\boldsymbol{I}} - \delta_G \bar{\boldsymbol{\Psi}}\right)\boldsymbol{w} + \left(\bar{c}_B\, \bar{\boldsymbol{I}} - \bar{c}_s \bar{\boldsymbol{\Psi}}\right)^{-1}\bar{\boldsymbol{\eta}}^{\Psi} \tag{D.3}$$

where we denote $\bar{\boldsymbol{\Psi}}$ is the matrix of Ψ_{id} with rows for the upstream location (i) and columns for the downstream location (d), $\bar{\boldsymbol{\eta}}^{\Psi}$ is a vector of $\hat{\eta}_i^{\Psi} \equiv \sum_d \Psi_{id}\left(\hat{K}_{id}^D\right)$, $\boldsymbol{C}^*$ and $\boldsymbol{w}$ are the vector of $\tilde{C}_i^*$ and $\tilde{w}_i$, respectively, and $\bar{\boldsymbol{I}}$ is identity matrix. Similarly, the second equation is rewritten as

$$\left(\bar{c}_S \bar{\boldsymbol{I}} - \bar{c}_B \bar{\boldsymbol{\Lambda}}\right)\boldsymbol{C}^* = \left(\left(1 - \delta_G\right)\bar{\boldsymbol{I}} - \tilde{\lambda}^B \delta_2 \mu \bar{\boldsymbol{\Lambda}}\right)\boldsymbol{w} - \bar{\boldsymbol{\eta}}^{\Lambda} \implies$$

$$\boldsymbol{C}^* = \left(\bar{c}_S \bar{\boldsymbol{I}} - \bar{c}_B \bar{\boldsymbol{\Lambda}}\right)^{-1}\left[\left(\left(1 - \delta_G\right)\bar{\boldsymbol{I}} + \tilde{\lambda}^B \delta_2 \mu \bar{\boldsymbol{\Lambda}}\right)\boldsymbol{w} - \bar{\boldsymbol{\eta}}^{\Lambda}\right] \tag{D.4}$$

where $\bar{\boldsymbol{\Lambda}}$ is the matrix of Λ_{ui} with rows for the upstream location (u) and columns for the downstream location (i), and $\mathbf{J}^{\Lambda}$ is a vector of $\eta_i^{\Lambda} \equiv \sum_d \Lambda_{di}\left(\hat{K}_{di}^U\right)$. By combining equations (D.3) and (D.4), we have:

$$\left(\bar{c}_S \bar{\boldsymbol{I}} - \bar{c}_B \bar{\boldsymbol{\Lambda}}\right)^{-1}\left(\left(1 - \delta_G\right)\bar{\boldsymbol{I}} + \tilde{\lambda}^B \delta_2 \mu \bar{\boldsymbol{\Lambda}}\right)\boldsymbol{w} - \left(\bar{c}_S \bar{\boldsymbol{I}} - \bar{c}_B \bar{\boldsymbol{\Lambda}}\right)^{-1}\bar{\boldsymbol{\eta}}^{\Lambda}$$
$$= -\left(\bar{c}_B\, \bar{\boldsymbol{I}} - \bar{c}_s \bar{\boldsymbol{\Psi}}\right)^{-1}\left[\left(\left(1 + \tilde{\lambda}^B \delta_2 \mu\right)\bar{\boldsymbol{I}} - \delta_G \bar{\boldsymbol{\Psi}}\right)\boldsymbol{w} - \bar{\boldsymbol{\eta}}^{\Psi}\right] \iff$$

$$
\begin{aligned}
&\left[\left(\bar{c}_S\bar{\boldsymbol{I}}-\bar{c}_B\bar{\boldsymbol{\Lambda}}\right)^{-1}\left(\left(1-\delta_G\right)\bar{\boldsymbol{I}}+\tilde{\lambda}^B\delta_2\mu\bar{\boldsymbol{\Lambda}}\right)+\left(\bar{c}_B\,\bar{\boldsymbol{I}}-\bar{c}_s\bar{\boldsymbol{\Psi}}\right)^{-1}\left(\left(1+\tilde{\lambda}^B\delta_2\mu\right)\bar{\boldsymbol{I}}-\delta_G\bar{\boldsymbol{\Psi}}\right)\right]\boldsymbol{w}\\
&=\underbrace{\left(\bar{c}_S\bar{\boldsymbol{I}}-\bar{c}_B\bar{\boldsymbol{\Lambda}}\right)^{-1}\bar{\boldsymbol{\eta}}^{\Lambda}}_{\text{upstream shock}}+\underbrace{\left(\bar{c}_B\,\bar{\boldsymbol{I}}-\bar{c}_s\bar{\boldsymbol{\Psi}}\right)^{-1}\bar{\boldsymbol{\eta}}^{\Psi}}_{\text{downstream shock}},
\end{aligned}
\tag{D.5}
$$

which solves the wages $\boldsymbol{w}$. Given $\boldsymbol{w}$, $\mathbf{C}^*$ is solved using equation (D.4). Furthermore, since final goods prices P_i^F is proportional to C_i^*, the welfare changes are given by $\boldsymbol{w}-\mathbf{C}^*$. Lastly, in our special case where we shut down endogenous responses of production network formation ($\tilde{\lambda}^S=\tilde{\lambda}^B=0$), the above equations hold the same with $\bar{c}_S=\frac{(\sigma-1)}{1-\beta}$, $\bar{c}_B=\sigma-1$, $\delta_2=1$, and $\delta_G=\frac{1-\beta\sigma}{1-\beta}$.

Definition of direct and indirect effects. In Section 5.3, we separate the welfare effects of shocks into direct and indirect effects using above linearized system. We define direct effects by the effects of shocks that are directly attributed to the changes of $\hat{K}_{id}^D$, but not through the changes of wages $w_{i'}$ and cost shifter $C_{i'}^*$ outside the location i. Formally, we define the direct effects of the shocks to $\tilde{w}_i$ and $\tilde{C}_i^*$ that satisfy the following set of equations:

$$
\left(1+\tilde{\lambda}^B\delta_2\mu\right)\left(\tilde{w}_i\right)+\bar{c}_B\left(\tilde{C}_i^*\right)=\Psi_{ii}\left(\tilde{K}_{ii}^D+\delta_G\tilde{w}_i+\bar{c}_S\tilde{C}_i^*\right)+\sum_{d\neq i}\Psi_{id}\tilde{K}_{id}^D,
\tag{D.6}
$$

$$
\left(1-\delta_G\right)\left(\tilde{w}_i\right)-\bar{c}_S\left(\tilde{C}_i^*\right)=\Lambda_{ii}\left(\tilde{K}_{ii}^D+\delta_G\tilde{w}_i+\bar{c}_S\tilde{C}_i^*\right)+\sum_{u\neq i}\Lambda_{ui}\tilde{K}_{ui}^U,
\tag{D.7}
$$

where the difference from (D.3) and (D.4) is the omission of $\tilde{w}_{i'}$ and $\tilde{C}_{i'}^*$ for $i'\neq i$ from the left hand side.

Similarly to the full effect, we can express the direct effects on $\tilde{w}_i$ and $\tilde{C}_i^*$ by the matrix form as:

$$
\boldsymbol{C}^*=\left(\left(\bar{c}_S\bar{\boldsymbol{I}}-\bar{c}_B\bar{\boldsymbol{\Lambda}}\right)\otimes\bar{I}\right)^{-1}\left[\left(\left(1-\delta_G\right)\bar{\boldsymbol{I}}+\tilde{\lambda}^B\delta_2\mu\bar{\boldsymbol{\Lambda}}\right)\boldsymbol{w}-\bar{\boldsymbol{\eta}}^{\Lambda}\right],
\tag{D.8}
$$

$$
\begin{aligned}
&[((\bar{c}_S\bar{\boldsymbol{I}}-\bar{c}_B\bar{\boldsymbol{\Lambda}})\otimes\bar{I})^{-1}\left(\left(1-\delta_G\right)\bar{\boldsymbol{I}}+\tilde{\lambda}^B\delta_2\mu\bar{\boldsymbol{\Lambda}}\otimes\bar{I}\right)\\
&\quad+((\bar{c}_B\,\bar{\boldsymbol{I}}-\bar{c}_s\bar{\boldsymbol{\Psi}})\otimes\bar{I})^{-1}\left(\left(1+\tilde{\lambda}^B\delta_2\mu\right)\bar{\boldsymbol{I}}-\delta_G\bar{\boldsymbol{\Psi}}\otimes\bar{I}\right)]\boldsymbol{w}\\
&=((\bar{c}_S\bar{\boldsymbol{I}}-\bar{c}_B\bar{\boldsymbol{\Lambda}})\otimes\bar{I})^{-1}\bar{\boldsymbol{\eta}}^{\Lambda}+((\bar{c}_B\,\bar{\boldsymbol{I}}-\bar{c}_s\bar{\boldsymbol{\Psi}})\otimes\bar{I})^{-1}\bar{\boldsymbol{\eta}}^{\Psi}.
\end{aligned}
\tag{D.9}
$$

where $\otimes$ indicates element-by-element multiplication.

Using the direct effects characterized by (D.8) and (D.9), indirect effects are defined by subtracting these direct effects from the full effects characterized by (D.4) and (D.5).

D.2 Indirect Inference Procedure

In this section, we describe the indirect inference procedure for parameters (σ, μ, $\tilde{\lambda}^B$, $\tilde{\lambda}^S$) as discussed in Section 5.1. The basic idea is to use the linearized equilibrium system to obtain the analytical expressions of the regression coefficients on the proxies for the import shocks. We then search parameter configurations $\{\sigma, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$ that minimize the Euclidean distance between the regression coefficients on imports, the number of suppliers, average transaction volume per supplier, number of buyers, average transaction volume per buyer, and the change of import value, from data in Table 2 and from the model prediction.

More precisely, we first use the linearized system (equations D.4 and D.5) as derived in Appendix D.1 to obtain the expression of $\tilde{w}_i$ and $\tilde{C}_i^*$ as a linear function of parameters and the vector of import and export shocks ($\bar{\boldsymbol{\eta}}^\Lambda$ and $\bar{\boldsymbol{\eta}}^\Psi$). Given the derived $\tilde{w}_i$ and $\tilde{C}_i^*$, the changes of the origin and destination components of the gravity equations are given by:

$$\begin{bmatrix} \tilde{\zeta}_i^I \\ \tilde{\zeta}_i^E \\ \tilde{\xi}_i^I \\ \tilde{\xi}_i^E \end{bmatrix} = D \begin{bmatrix} \tilde{w}_i \\ \tilde{C}_i^* \end{bmatrix}, \quad D = \begin{bmatrix} 0 & (1-\sigma) \\ -\tilde{\lambda}^B \delta_2 \mu & -(\sigma-1)(\delta_2 - 1) - \tilde{\lambda}^B \delta_2 (1-\mu) \\ \frac{1-\beta\sigma}{1-\beta} & \frac{\sigma-1}{1-\beta} \\ \delta_G - \frac{1-\beta\sigma}{1-\beta} & \frac{(\sigma-1)}{1-\beta}(\delta_2 - 1) - \tilde{\lambda}^S \delta_2 (1-\mu) \end{bmatrix}. \tag{D.10}$$

Using these $\tilde{\zeta}_i^I, \tilde{\zeta}_i^E, \tilde{\xi}_i^I, \tilde{\xi}_i^E$, we predict the changes of the number of suppliers by firms in location i, $\overline{M}_i^S$, and the average purchases from suppliers, $\tilde{\bar{r}}_i^S$, by the following expressions:

$$\overline{M}_i^S = \sum_u \frac{M_{ui}}{\sum_\ell M_{\ell i}} \tilde{\zeta}_u^E + \tilde{\xi}_i^E, \quad \tilde{\bar{r}}_i^S = \sum_u \frac{\bar{r}_{ui}}{\sum_\ell \bar{r}_{\ell i}} \tilde{\zeta}_u^I + \tilde{\xi}_i^I \tag{D.11}$$

where $\overline{M}_i^S$ and $\tilde{\bar{r}}_i^S$ correspond to the outcome variables documented in Fact 3 in Section 2.2. Similarly, the changes of the number of buyers by firms in location i, $\overline{M}_i^B$, and the average sales to buyers, $\tilde{\bar{r}}_i^B$, is given by

$$\overline{M}_i^B = \sum_d \frac{M_{id}}{\sum_\ell M_{i\ell}} \tilde{\xi}_d^E + \tilde{\zeta}_d^E, \quad \tilde{\bar{r}}_i^B = \sum_d \frac{\bar{r}_{id}}{\sum_\ell \bar{r}_{i\ell}} \tilde{\xi}_d^E + \tilde{\zeta}_d^E \tag{D.12}$$

where $\overline{M}_i^B$ and $\tilde{\bar{r}}_i^B$ correspond to the outcome variables documented in Fact 3 in Section 2.2. Using these outcome variables as outcome variables, we can run the same regressions as defined in Fact 3 in Section 2.2, where the dependent variable is the proxies for import and export shocks as discussed in Section 2.2. We define $\beta(\Theta)$ as these regression coefficients under model parameter Θ.

We now finally define the indirect inference estimator. We first assume that our proxy of import shocks in Section 2.2, $\tilde{K}_{Ri}^{*U}$, is a linear function of the import shock, $\tilde{K}_{Ri}^U$, such that $\tilde{K}_{Ri}^U = \psi^U \tilde{K}_{Ri}^{*U}$ and $\psi^U \geq 0$ is some parameter. Similarly, we assume that our proxy of export shocks, $\tilde{K}_{iR}^{*D}$, is a linear function of the import shock, $\tilde{K}_{iR}^{*D}$, such that $\tilde{K}_{iR}^D = \psi^D \tilde{K}_{iR}^{*D}$ and $\psi^D \geq 0$ is some parameter. Denoting the combination of parameters $\Theta \equiv \{\sigma, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S, \psi^U, \psi^D\}$, the indirect inference estimator $\hat{\Theta}$ is defined by the minimizer of the Euclidean distance between the model-predicted regression coefficients and the regression coefficients estimated using actual data:

$$\hat{\Theta} = \min_\Theta \sum ||\beta(\Theta) - \hat{\beta}||^2 / Var(\hat{\beta})$$

where $\hat{\beta}$ is the shift-share regression coefficients from data, $Var(\hat{\beta})$ is the variance of these regression coefficients, and $\beta(\Theta)$ is again the predicted regression coefficients under model parameter Θ.

Note that we target the shift-share regression coefficients of the impacts of import shocks at the firm level (in Table 2), instead of at the location level. Our model predicts that these two are identical because the import shares are identical across firms within a location. Therefore, the model-predicted regression coefficients are identical at the firm-level and location-level.

To ensure that we obtain stable results for the counterfactual simulation, we impose constraints in parameters so that they satisfy sufficient conditions for equilibrium uniqueness as derived in Proposition 1.

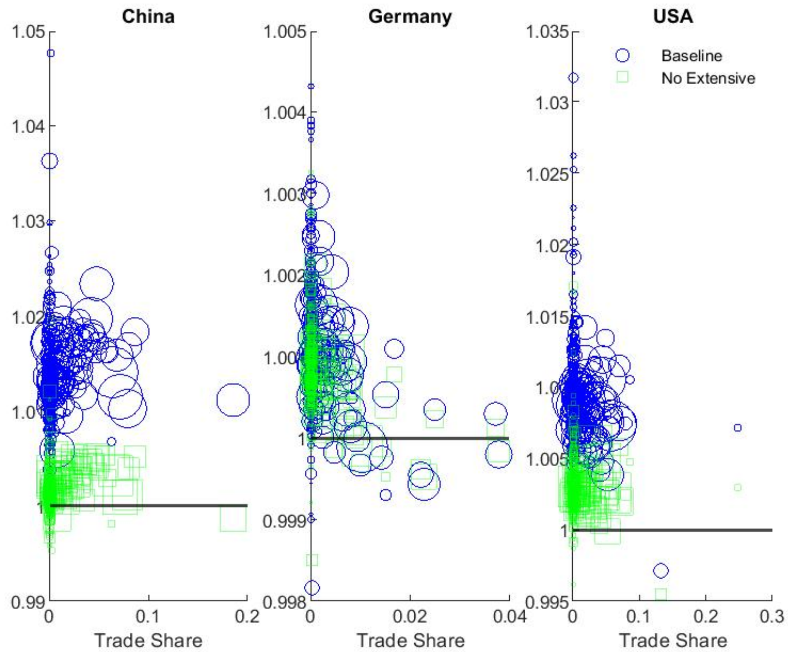
D.3 Additional Results for International Trade Shocks Counterfactuals

In this section, we provide additional results of our counterfactual simulation of international trade shocks as presented in Section 5.3.1.

In Figure D.1, we plot the welfare gains for each municipality in our baseline model ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0.19$) and by shutting down extensive margin responses of production network formation ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$), against our proxy of the direct international trade exposure as defined in Figure 3. Interestingly, we find that the differences in the welfare gains from these two models are overall similar across different levels of direct international trade exposure.

To further understand these patterns, in Figure D.2, we decompose the welfare gains predicted by our model of extensive margin responses ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$) into direct and indirect effects, similarly for our baseline model in Figure 3. In Panel A, we find a similar positive correlation between international trade shares and the direct effects as in our baseline model. However, compared to our baseline model, we find a smaller contribution of the indirect effects. Relatedly, in Panel B of Figure D.2, we show that the contribution of the indirect effects to the overall variation of regional welfare gains is significantly smaller by assuming away extensive margin responses ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$) compared to our baseline model in Panel B of Figure 3. For example, in the baseline model, the indirect effects account for 26 percent of the variance of welfare effect to trade shocks from China, whereas in the model with no extensive margin, it accounts only for 13 percent. These pieces of evidence jointly indicate that endogenous formation of domestic production networks is particularly relevant for welfare gains through indirect effects.

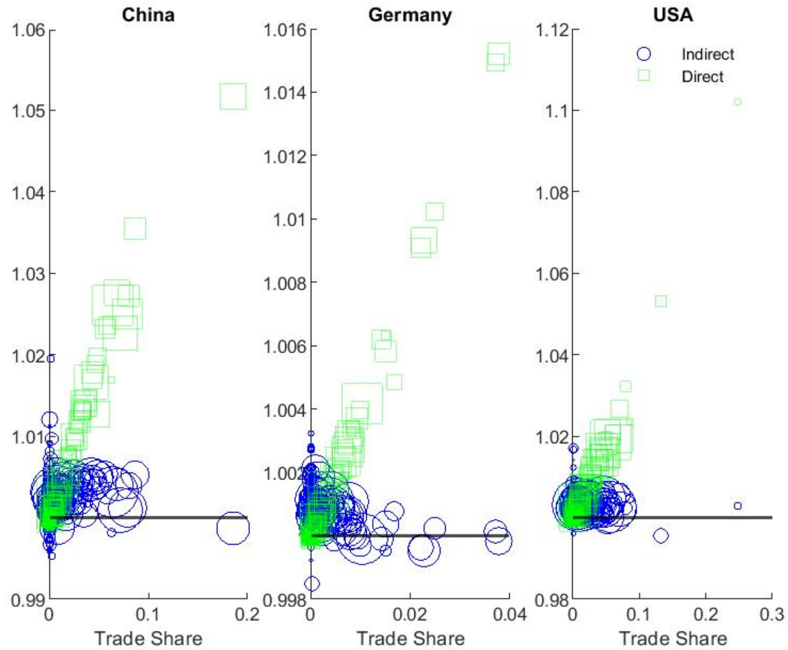
Figure D.1: Heterogeneous Indirect Welfare Effect of International Trade Shocks: Baseline versus No Extensive Margin Model



Notes: This figure shows the results of the counterfactual simulation as presented in Figure 3 except that we conduct simulation by shutting down endogenous extensive margin responses ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$), unlike Figure 3 where we allow for extensive margin responses with baseline calibration ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0.19$). The figure plots the total welfare gains for each region in the two models induced by the shocks to each of the three international countries. See the footnote of Figure3 for the definitions of the figures.

Figure D.2: Heterogeneous Welfare Gains from International Trade Shocks: No Extensive Margin Model

(A) Welfare Gains from Trade Shocks and Direct International Trade Exposures



(B) Variance Decomposition of Welfare Gains

	China	Germany	USA
Direct	75	111	98
Indirect	13	12	8
Covariance	11	-23	-5
Total	100	100	100

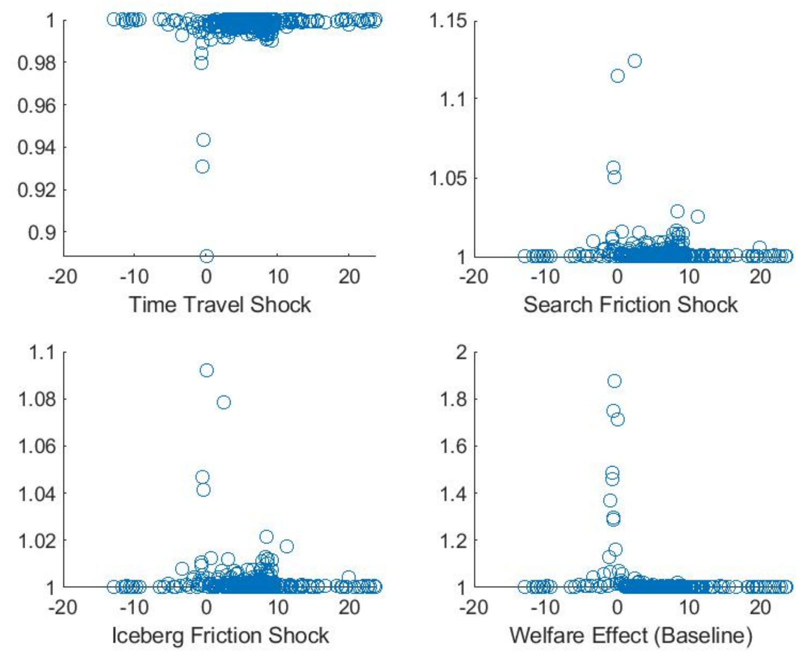
Notes: This figure shows the results of the counterfactual simulation as presented in Figure 3, except that we conduct simulation by shutting down endogenous extensive margin responses ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0$), unlike Figure 3 where we allow for extensive margin responses with baseline calibration ($\tilde{\lambda}^S = \tilde{\lambda}^B = 0.19$). See the footnote of Figure3 for the definitions of the figures.

D.4 Additional Results for Transportation Infrastructure Counterfactuals

In this section, we provide additional results of our counterfactual simulation of domestic transportation infrastructure as presented in Section 5.3.2.

Figure D.3 presents how the magnitude of the shocks and welfare gains implied by the new bridge varies by the geographic proximity to the bridge. In the figure, the horizontal axis corresponds to the relative latitudes of the municipality. Zero in the horizontal axis corresponds to the municipality that is positioned to the same latitude as the bridge. Note that Chile spans north to south, and hence latitude approximates the geographic position of the location. We find that, naturally, regions around the new bridge face the largest reduction of travel time; the average travel time to all other municipalities decreases by around 10%. Under our calibration described in Section 5.3.2, this change increases $\hat{\chi}_{ud}^{iceberg}$ and $\hat{\chi}_{ud}^{search}$ by slightly more than 10% in most affected areas. Lastly, most affected areas near the bridge face welfare gains of up to 80% in locations, while this benefit decays sharply as a function of the geographic distance to the bridge.

Figure D.3: Travel Time Changes, Trade Frictions, and Welfare Effects by the Relative Geographic Position to Chacao Bridge



Notes: This figure plots the average changes in time travel to all other locations in Chile, $\hat{\chi}_{ud}^{iceberg}$, $\hat{\chi}_{ud}^{search}$, and welfare gains predicted by our counterfactual in Section 5.3.2, against the relative latitudes of the municipality. Zero in the horizontal axis corresponds to the municipality that is positioned to the same latitude as the bridge. Note that Chile spans north to south, and hence latitude approximates the geographic position of the location.