# Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Shocks

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# Motivation

- **How and to what extent should fiscal policy be used to mitigate household inequality and risk?**
- **Quantitative answer:** the solution to a Ramsey problem for a model replicating realistically levels of inequality and individual risk.
- The standard incomplete markets (SIM) model has been relatively successful in this front.
- Yet, the Ramsey policy in a quantitative SIM model has been **an open issue** for a long time.

# What do we do?

- Solve the **Ramsey planner's problem**, in a realistically calibrated SIM model, where the planner has access to: (i) linear capital income taxes (ii) linear labor income taxes (iii) lump-sum instrument (iv) government debt.
- Develop a parsimonious method of approximating the fiscal instruments in the time domain. Thus the Ramsey policy is **time-varying** and maximizes welfare along the transition.
- Propose a method of decomposing welfare gains in **non-stationary environments** into (i) level effect (ii) insurance effect (iii) redistributive effect.
- We perturb the optimal policy in several ways to **diagnose** the contribution of each instrument.

# What do we find?

1. Robust features of **Ramsey policy** in the SIM model:
    - Front-loaded, high initial capital income taxes with positive long-run rate.
    - Monotonically increasing labor income taxes.
    - Front-loaded lump-sum transfers and debt issuance in the long-run.

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4. **Variation of policy over time** matters. Optimal, one-time policy change yields only half of the welfare gains.

5. Ramsey policy in the SIM model inherits many features of the **complete-markets**, optimal policy.

# Where do we contribute?

1. Positive long-run capital income taxes and modified golden rule:
   Aiyagari (1995), Acikgoz (2015), Acikgoz, Hagedorn, Holter, and Wang (2018)
2. Optimal policy with heterogeneity:
   - Gottardi, Kajii, and Nakajima (2015), Heathcote, Storesletten, and Violante (2017): analytical characterizations in stylized versions of the SIM model.
   - Itskhoki and Moll (2019), Nuño and Thomas (2016), Acikgoz et al. (2018): Versions of Ramsey problems with heterogeneity.
   - Krueger and Ludwig(2016), Bakis, Kaymak, and Poschke (2015): Optimal, one-time policy change.
3. Gov. debt in incomplete markets:
   Aiyagari and McGrattan (1998), Röhrs and Winter (2017)
4. Ramsey problem in complete markets:
   Judd (1985), Chamley (1986), Straub and Werning (2020), Werning (2007), Greulich, Laczó, and Marcet (2019).
5. Constrained efficiency in the SIM model:
   Davila, Hong, Krusell, and Ríos-Rull (2012)

# The SIM Model

# Environment - Households

- There is a measure one of households.
- Individual states: $a \in A$ - assets, and $e \in E$ - stochastic productivity that follows a Markov process with matrix $\Gamma$.
- Given a sequence of prices and taxes the household solves

$$v_t(a, e) = \max_{c_t, h_t, a_{t+1}} u(c_t, h_t) + \beta \sum_{e_{t+1} \in E} v_{t+1}(a_{t+1}, e_{t+1}) \Gamma_{e, e_{t+1}}$$

subject to

$$\begin{aligned}(1 + \tau^c) c_t(a, e) + a_{t+1}(a, e) = (1 - \tau_t^h) w_t e \, h_t(a, e) + \\ + (1 + (1 - \tau_t^k) r_t) a + T_t\end{aligned}$$

$$a_{t+1}(a, e) \geq \underline{a}$$

# Environment - Firm and Government

- Given prices, in each period, the representative firm solves

$$\max_{K_t, N_t} f(K_t, N_t) - w_t N_t - r_t K_t$$

- Government finances an exogenous stream of expenditure, and lump-sum transfers, with taxes on consumption, labor and capital, or debt

$$G + T_t + r_t B_t = B_{t+1} - B_t + \tau^c C_t + \tau_t^h w_t N_t + \tau_t^k r_t A_t.$$

# Equilibrium

## Definition

Given $K_0$, $B_0$, an initial distribution $\lambda_0$ and a policy $\pi \equiv \{\tau_t^k, \tau_t^h, T_t\}_{t=0}^{\infty}$, a **competitive equilibrium** is a sequence of value functions $\{v_t\}_{t=0}^{\infty}$, an allocation $X \equiv \{c_t, h_t, a_{t+1}, K_{t+1}, N_t, B_{t+1}\}_{t=0}^{\infty}$, a price system $P \equiv \{r_t, w_t\}_{t=0}^{\infty}$, and a sequence of distributions $\{\lambda_t\}_{t=0}^{\infty}$, such that for all $t$:

1. Given $P$ and $\pi$, $c_t(a, e)$, $h_t(a, e)$, and $a_{t+1}(a, e)$ solve the household's problem and $v_t(a, e)$ is the respective value function;
2. Factor prices are set competitively: $r_t = f_K(K_t, N_t)$, $w_t = f_N(K_t, N_t)$;
3. The probability measure $\lambda_t$ is consistent with $\Gamma$ and $a_{t+1}(a, e)$;
4. Government budget constraint holds and debt is bounded;
5. Markets clear,

$$C_t + G + K_{t+1} - K_t = f(K_t, N_t), \quad K_t + B_t = \int_{A \times E} a_t(a, e)\, d\lambda_t.$$

# Ramsey Problem

# Ramsey Problem

## Definition

Given $\lambda_0$, $K_0$, $B_0$ and a welfare function $W$, the **Ramsey problem** is $\max_\pi W(X(\pi))$ subject to $X(\pi)$ being an equilibrium allocation and $\pi$ satisfying $\tau_t^k \leq 1 \ \forall t \geq 0$.

- The benchmark welfare function is utilitarian:

$$W(\pi) = \int_S E_0 \sum_{t=0}^{\infty} \beta^t u(c_t(a, e|\pi), h_t(a, e|\pi)) \, d\lambda_0.$$

- Solving this problem involves searching on the space of sequences $\{\tau_t^k, \tau_t^h, T_t\}_{t=0}^{\infty}$.

# Computational Method

Parameterize the time paths of fiscal instruments as follows:

$$x_t = \left(\sum_{i=0}^{m_{x0}} \alpha_i^x P_i(t)\right) \exp\left(-\lambda^x t\right) + \left(1 - \exp\left(-\lambda^x t\right)\right) \left(\sum_{j=0}^{m_{xF}} \beta_i^x P_i(t)\right)$$

where

- $x_t$ can be $\tau_t^k$, $\tau_t^h$, or $T_t$
- $\{P_i(t)\}_{i=0}^{m_{x0}}$ and $\{P_i(t)\}_{i=0}^{m_{xF}}$ are families of **Chebyshev polynomials**
- $m_{x0}$ and $m_{xF}$ are **orders** of the polynomial approximations for the short-run and long-run dynamics
- $\{\alpha_i^x\}_{i=0}^{m_{x0}}$ and $\{\beta_i^x\}_{i=0}^{m_{xF}}$ are **weights** on the consecutive elements of the family
- $\lambda^x$ controls the **convergence rate** of the fiscal instrument.

# Computational Method - Implementation

$$x_t = \left(\sum_{i=0}^{m_{x0}} \alpha_i^x P_i(t)\right) \exp\left(-\lambda^x t\right) + \left(1 - \exp\left(-\lambda^x t\right)\right) \left(\sum_{j=0}^{m_{xF}} \beta_i^x P_i(t)\right)$$

- Start with small orders and increase them for each instrument until the welfare gains from additional orders are negligible. We arrive at $m_{\tau_k 0} = m_{\tau_n 0} = m_{T0} = 2$, $m_{\tau_k F} = m_{\tau_n F} = 0$, and $m_{TF} = 4$.

- Terminal period at which taxes become constant is endogenous (capped at 100), but transition is computed using 250 periods.

- We end up with the following **17 parameters**:

$$\pi_A = \{\alpha_0^k, \alpha_1^k, \alpha_2^k, \beta_0^k, \lambda^k, \alpha_0^h, \alpha_1^h, \alpha_2^h, \beta_0^h, \lambda^h, \alpha_1^T, \alpha_2^T, \alpha_3^T, \alpha_4^T, \beta_0^T, \beta_1^T, \lambda^T\},$$

Add Flexibility　Global Solver

# Calibration

# Calibration Strategy

- **Three sets of statistics:** (i) time series of macroeconomic data from 1995 to 2007, (ii) cross-sectional, distributional moments on hours worked, wealth, and earnings, and (iii) panel data on the dynamics of labor income.

- In total, we have **38 parameters** in the model and we use **44 targets** to discipline them.

- Unit of analysis: **household** rather than an individual. Measure all the relevant statistics in the data at the household level using the equivalence scales from the US Census.

- Household preferences:

$$u(c, h) = \frac{(c^{\gamma}(1-h)^{1-\gamma})^{1-\sigma}}{1-\sigma}$$

# Benchmark Model Parameters

Definition Frisch

| Description | Parameter | Value | |
|---|---|---|---|
| **Preferences and technology** | | | |
| Consumption share | $\gamma$ | 0.510 | **Implied IES: 0.65** |
| Preference curvature | $\sigma$ | 2.069 | **Implied Frisch ($\Psi$): 0.49** |
| Discount factor | $\beta$ | 0.954 | |
| Capital share | $\alpha$ | 0.378* | |
| Depreciation rate | $\delta$ | 0.104 | |
| Borrowing constraint | $\underline{a}$ | $-0.078$ | |
| **Fiscal policy** | | | |
| **Capital income tax (%)** | $\tau^k$ | 41.5* | |
| **Labor income tax (%)** | $\tau^n$ | 22.5* | |
| Consumption tax (%) | $\tau^c$ | 4.7* | |
| Government expenditure | $G$ | 0.069 | |
| Transfers | $T$ | 0.088 | |
| **Labor productivity process** | | | |
| Productivity process curvature | $\eta$ | 1.153 | |

**Persistent shock**

$$\Gamma_P = \begin{bmatrix} 0.994 & 0.002 & 0.004 & 3E{-}5 \\ 0.019 & 0.979 & 0.001 & 9E{-}5 \\ 0.023 & 0.000 & 0.977 & 5E{-}5 \\ 0.000 & 0.000 & 0.012 & 0.987 \end{bmatrix} \quad e_P = \begin{bmatrix} 0.580 \\ 1.153 \\ 1.926 \\ 27.223 \end{bmatrix}$$

**Transitory shock**

$$P_T = \begin{bmatrix} 0.263 \\ 0.003 \\ 0.556 \\ 0.001 \\ 0.001 \\ 0.176 \end{bmatrix} \quad e_T = \begin{bmatrix} -0.574 \\ -0.232 \\ 0.114 \\ 0.133 \\ 0.817 \\ 1.245 \end{bmatrix}$$

# Model Fit to Macro and Panel Data

**(1) Macroeconomic aggregates**

| | **Target** | **Model** |
|---|---|---|
| Intertemporal elasticity of substitution | 0.65 | 0.65 |
| Average hours worked | 0.32 | 0.33 |
| Capital to output | 2.50 | 2.49 |
| Capital income share | 0.38 | 0.38 |
| Investment to output | 0.26 | 0.26 |
| **Transfer to output (%)** | 11.4 | 11.4 |
| **Debt to output (%)** | 61.5 | 61.5 |
| Share of workers (%) | 76.7 | 79.3 |
| Fraction of hhs with negative net worth (%) | 9.7 | 7.9 |
| Correlation between earnings and wealth | 0.43 | 0.43 |

**(2) Statistical properties of labor income**

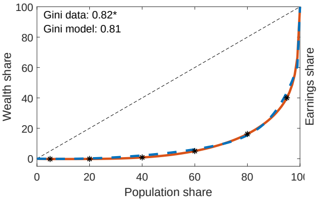
| | | |
|---|---|---|
| **Variance of 1-year growth rate** | 2.33 | 2.32 |
| **Kelly skewness of 1-year growth rate** | $-0.12$ | $-0.13$ |
| **Moors kurtosis of 1-year growth rate** | 2.65 | 2.28 |

**(3) Self-employed status statistics**

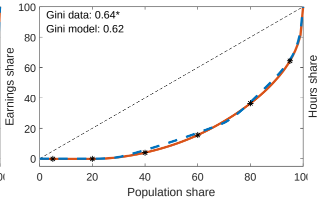
| | | |
|---|---|---|
| **Share in population (%)** | 12.5 | 12.7 |
| Share of wealth (%) | 45.8 | 38.9 |
| Share of earnings (%) | 28.7 | 30.5 |

# Model fit to Inequality Data

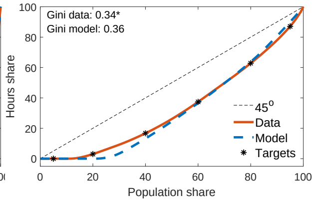
Other Non-targeted moments



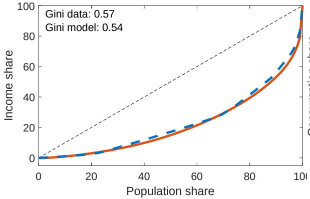

(a) Wealth



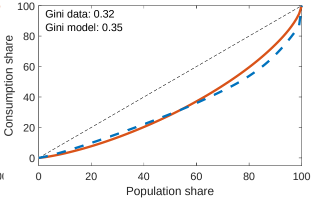

(b) Earnings




(c) Hours




(d) Income

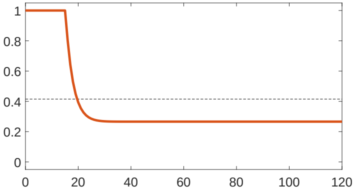



(e) Consumption

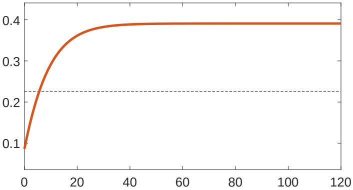
Note: Black dashed lines: initial stationary equilibrium; Red solid curves: optimal transition.

# Results

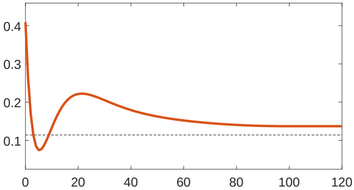
# Optimal Fiscal Policy



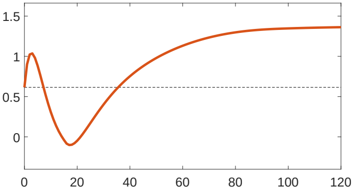

(a) Capital income tax




(b) Labor income tax




(c) Lump-sum/Initial output




(d) Debt/Initial output

Note: Black dashed lines: initial stationary equilibrium; Red solid curves: optimal transition.

# Optimal Fiscal Policy

Welfare Gains:

- Permanent increase in **consumption by 3.52%.**
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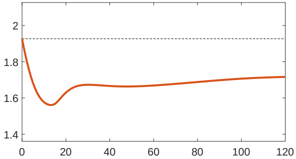
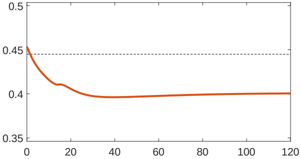
Distributional Effects:

- Reduction in the amount of inequality and risk that households face—**larger and safe transfers.**
- Opposing effects: consumption inequality falls, hours inequality rises. The latter due higher labor supply of productive agents.

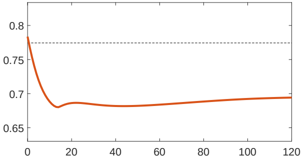
# Optimal Fiscal Policy - Aggregates



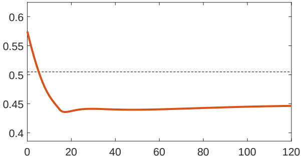

(a) Capital



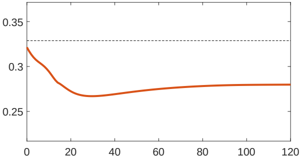

(b) Effective Labor ($N$)



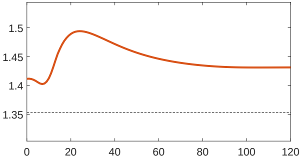

(c) Output




(d) Consumption



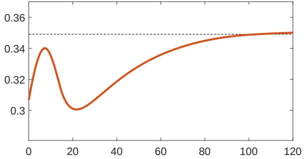

(e) Hours ($H$)



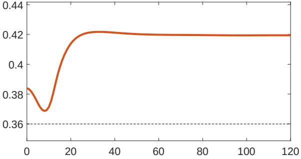

(f) Labor Productivity ($N/H$)

Note: Black dashed lines: initial stationary equilibrium; Red solid curves: optimal transition.

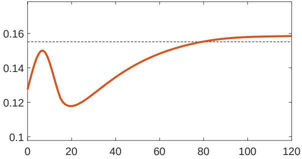
# Optimal Fiscal Policy - Distributional Effects



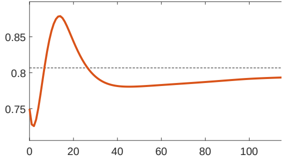

(a) Consumption Gini



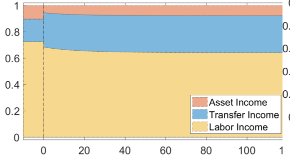

(b) Hours Gini



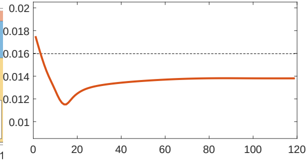

(c) Gini of $c^{\gamma}(1-h)^{1-\gamma}$




(d) Wealth Gini




(e) Income shares




(f) $\mathrm{Var}\left(c^{\gamma}(1-h)^{1-\gamma}\right)$ gr.

Note: Black dashed lines: initial stationary equilibrium; Red solid curves: optimal transition.

# Welfare Decomposition Details

The utilitarian welfare function can increase for three reasons:

1. Reduction in distortions, if the utility of the average agent, $\sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$, increases: **the level effect ($\Delta_L$)**;
2. Transfers from ex-post rich to ex-post poor, if the uncertainty of each individual path $\{c_t, h_t\}_{t=1}^{\infty}$ is reduced: **the insurance effect ($\Delta_I$)**;
3. Transfers from ex-ante rich to ex-ante poor, if the inequality between certainty equivalents for $\{c_t, h_t\}_{t=1}^{\infty}$ is reduced: **the redistribution effect ($\Delta_R$)**.

## Proposition

*If preferences are BGP, then*

$$1 + \Delta = (1 + \Delta_L)(1 + \Delta_I)(1 + \Delta_R).$$

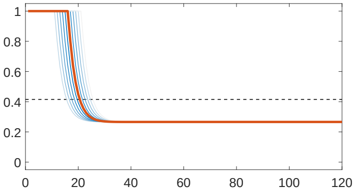
# Welfare Decompositions

| | $\Delta$ | $\Delta_L$ | $\Delta_I$ | $\Delta_R$ |
|---|---|---|---|---|
| **Benchmark** | **3.5** | **0.2** | **1.2** | **2.0** |
| Fixed capital income tax | 0.8 | −0.6 | 1.3 | **0.1** |
| Fixed labor income tax | 2.0 | 0.6 | **-0.3** | 1.7 |
| Constant lump-sum | **3.3** | −0.1 | 1.3 | 2.1 |
| Fixed debt-to-output | **3.1** | −0.2 | 1.4 | 2.0 |

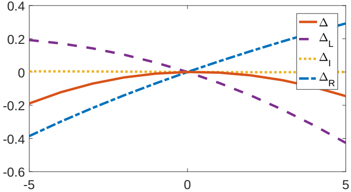
- Almost **60% of welfare gains** from redistribution.
- Capital income taxes: mostly **redistributive tool**, but also loss of the productivity improvements via wealth effects on labor supply.
- Labor income taxes: operate mostly through **insurance margin**.
- Time paths of both lump-sum transfers and government debt contribute marginally to average welfare.

# Perturbations Around Optimal Policy

# Capital Income Taxes at the Upper Bound




(a) Capital income tax

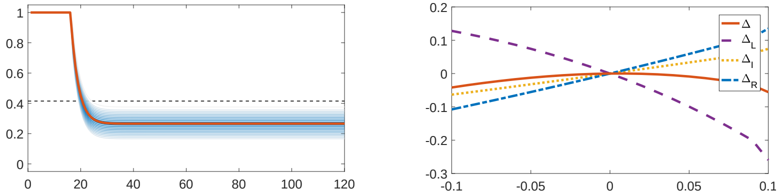



(b) Welfare decomposition

Note (a) Black dashed line: initial stationary equilibrium; Red solid and blue curves: optimal transition and perturbations of it; (b) the $x$-axis represents the movement in number of periods capital income taxes are kept in the upper bound from the optimum.

- **Trade-off:** extra redistribution and negative distortionary effects.
- Effects largely offset each other, hence relatively flat average welfare function.

# Long-run Capital Income Taxes

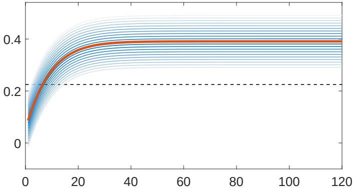



(a) Capital income tax (b) Welfare decomposition

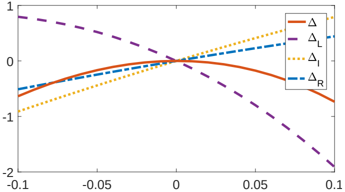
Note (a) Black dashed line: initial stationary equilibrium; Red solid and blue curves: optimal transition and perturbations of it; (b) The $x$-axis represents the movement of long-run capital income taxes away from the optimum.

- **Trade-off:** negative distortionary effects vs. redistribution and insurance.
- Far enough in the future household's dependence on their initial condition fully dissipates, so that changes income taxes have no effect on redistribution, but only on level and insurance.

# Labor Income Taxes




(a) Labor income tax

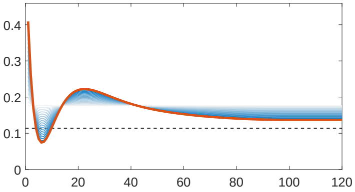



(b) Welfare decomposition

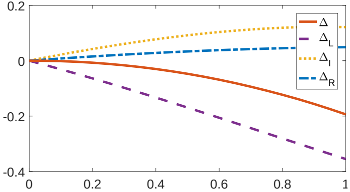
Note (a) Black dashed line: initial stationary equilibrium; Red solid and blue curves: optimal transition and perturbations of it; (b) The $x$-axis represents the movement of labor income taxes away from the optimum.

- **Trade-off:** strong negative distortionary effects vs. insurance.
- Higher labor income tax which is rebated via lump-sum transfers (exactly the experiment here) effectively reduces the labor income risk.

# The Path of Lump-Sum Transfers




(a) Lump-sum/Initial output




(b) Welfare decomposition

Note (a) Black dashed line: initial stationary equilibrium; Red solid and blue lines: optimal transition and perturbations of it; (b) The $x$-axis represents the homotopy parameter between the initial optimal path at $x = 0$ and a flat path at $x = 1$.

- **Trade-off:** Front-loaded lump-sum transfers improve consumption smoothing (level effect) relative to constant pattern.
- Why not smooth front-loading? Severe increase in government debt which adds to crowding out of capital (already dampened due to high initial capital income taxes). Smooth front-loading

Optimal constant lump-sum Capital Levy

# Maximizing Efficiency

# Alternative Welfare Criterion

Consider:

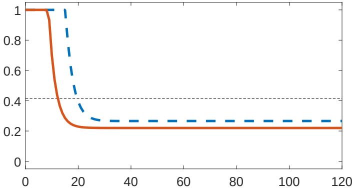
$$W^{\hat{\sigma}} = \left( \int \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right]^{\frac{1-\hat{\sigma}}{1-\sigma}} d\lambda_0 \right)^{\frac{1-\sigma}{1-\hat{\sigma}}},$$

Following Benabou(2002), we refer to $\hat{\sigma}$ as the planner's degree of inequality aversion.

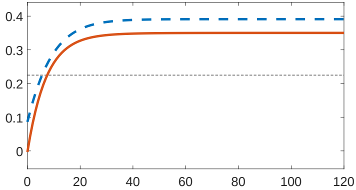
- $\hat{\sigma} = \sigma$, maximizing $W^{\sigma}$ is equivalent to maximizing the utilitarian welfare function
- $\hat{\sigma} \to \infty$, this becomes the Rawlsian welfare function
- $\hat{\sigma} = 0$, then maximizing $W^0$ is equivalent to maximizing efficiency i.e. $(1 + \Delta_L)(1 + \Delta_I)$

Propositions

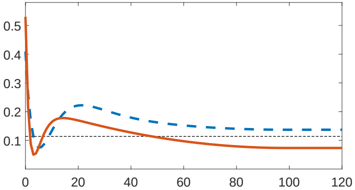
# Optimal Fiscal Policy: Maximizing Efficiency



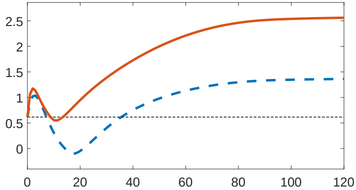

(a) Capital income tax




(b) Labor income tax



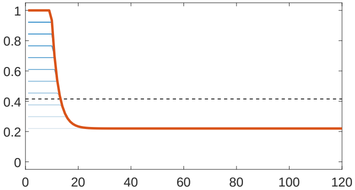

(c) Lump-sum/Initial output



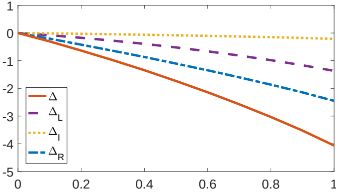

(d) Debt/Initial output

Note: Black dashed line: initial stationary equilibrium; Red solid curve: path that maximizes efficiency optimal transition; Blue dashed curve: path that maximizes the utilitarian welfare function (benchmark results). The welfare gain is 1.8 percent.

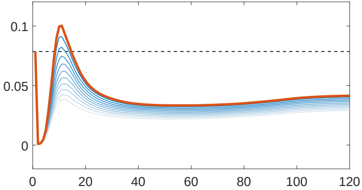
# Redistribution Leads to Efficiency Gains



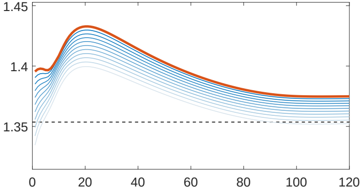

(a) Capital income tax




(b) Welfare decomposition




(c) Prop. with negative assets




(d) Labor productivity

Note (a,c,d) Black dashed line: initial stationary equilibrium; Red solid and blue curves: path that maximizes efficiency and variations upon it; (b) the $x$-axis represents the homotopy parameter between the initial optimal path at $x = 0$ and a flat path at $x = 1$.

# Transitional Effects

# Transitional Effects and Time-Variation are Important

| | $\tau^k$ | $\tau^h$ | $T/Y$ | $B/Y$ | $K/Y$ | $\Delta$ | $\Delta_L$ | $\Delta_I$ | $\Delta_R$ |
|---|---|---|---|---|---|---|---|---|---|
| Initial equil. | 41.5 | 22.5 | 11.4 | 61.5 | 2.49 | – | – | – | – |
| Stat. equil. (SE) | – | 36.4 | 18.8 | −265.1 | 3.53 | 14.8 | 8.1 | 0.7 | 5.5 |
| SE no debt | −7.2 | 27.1 | 9.1 | 61.5 | 2.85 | 1.2 | 2.8 | 0.0 | −1.5 |
| **Constant policy** | 67.5 | 27.9 | 19.7 | 53.9 | 2.02 | 1.7 | −0.7 | 0.8 | 1.6 |
| **Benchmark** | 26.7 | 39.1 | 15.1 | 154.3 | 2.48 | 3.5 | 0.2 | 1.2 | 2.1 |

Note: All values, except for $K/Y$, are in percentage points.

- SE no debt policy, once transition is accounted for, would actually lead to a welfare *loss* equivalent to an **11.7%** permanent reduction in consumption.
- **Constant policy:** weighted average of the time-varying instruments from our benchmark results with more weight on the short-run levels. Yields **only 48% of welfare gains** of the time-varying policy.

# Other Results in the Paper

1. Two Period Model: analytical characterization of the optimal fiscal policy. Details
2. Long Run Optimality Conditions. Details
3. The Role of Incomplete Markets. Details
4. Alternative Calibrations and Robustness Checks. Details
5. Comparison with backward iteration method by Acikgoz, Hagedorn, Holter, and Wang (2018).

# Conclusions

- Quantitatively characterize the solution to the Ramsey problem in the SIM model.
- Capital income taxes are an effective way to provide redistribution, which leads to a considerably more efficient allocation of labor via wealth effects on labor supply.
- Time variation of policy and transitional effects are quantitatively important.
- Our solution method and welfare decomposition can be applied to a broad range of economies.

# ADDITIONAL SLIDES

# Computational Method - Global Solver

Still, a formidable computational task. Need thorough procedure:

- **Global stage:** draw from a quasi-random sequence a very large number of policies in the domain of $\boldsymbol{\pi}_A$. We compute transition and evaluate welfare $W(\boldsymbol{\pi}_A)$. Select the ones that yield the highest levels of welfare.
- **Clustering:** similar policies in terms of welfare are placed in the same cluster.
- **Local stage:** for each cluster run a derivative-free optimizer based on an algorithm designed by Powell (2010).
- **Stopping criterion:** Bayesian rule detecting the number of local minima.

Parallelized and run on 1200 cores on Niagara cluster at the University of Toronto.

Back

# Average Welfare Gain

- Consider a policy reform and denote by $\{c_t^j, h_t^j\}$ the equilibrium consumption and labor paths of a household with and without the reform, with $j = R$ or $j = NR$ respectively.

- The average welfare gain, $\Delta$ is then

$$\int \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u \left( (1+\Delta) c_t^{NR}, h_t^{NR} \right) \right] d\lambda_0 = \int \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u \left( c_t^{R}, h_t^{R} \right) \right] d\lambda_0, \tag{1}$$

where $\lambda_0$ is the initial distribution over states $(a_0, e_0)$.

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# Level Effect

- Let the aggregate level of $c_t$ and $h_t$ at each $t$ be

$$C_t^j \equiv \int c_t^j d\lambda_t^j, \quad \text{and} \quad H_t^j \equiv \int h_t^j d\lambda_t^j,$$

  where $\lambda_t^j$ is the distribution over $(a_0, e^t)$ conditional on whether or not the reform is implemented.

- **The level effect, $\Delta_L$,** is then given by

$$\sum_{t=0}^{\infty} \beta^t u\left((1+\Delta_L) C_t^{NR}, H_t^{NR}\right) = \sum_{t=0}^{\infty} \beta^t u\left(C_t^R, H_t^R\right). \tag{2}$$

Back

# Insurance effect

- Let $\{\bar{c}_t^j(a_0, e_0), \bar{h}_t^j(a_0, e_0)\}$ denote a certainty-equivalent sequence of consumption and labor conditional on a household's initial state that satisfies

$$\sum_{t=0}^{\infty} \beta^t u\left(\bar{c}_t^j(a_0, e_0), \bar{h}_t^j(a_0, e_0)\right) = \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t^j, h_t^j\right)\right]. \tag{3}$$

- Let $\bar{C}_t^j$ and $\bar{H}_t^j$ denote aggregate certainty equivalents, that is

$$\bar{C}_t^j = \int \bar{c}_t^j(a_0, e_0)\, d\lambda_0, \quad \text{and} \quad \bar{H}_t^j = \int \bar{h}_t^j(a_0, e_0)\, d\lambda_0, \quad \text{for } j = R, NR. \tag{4}$$

- **The insurance effect, $\Delta_I$,** is defined by

$$1+\Delta_I \equiv \frac{1 - p_{risk}^R}{1 - p_{risk}^{NR}}, \quad \text{where} \sum_{t=0}^{\infty} \beta^t u\left((1 - p_{risk}^j)\, C_t^j, H_t^j\right) = \sum_{t=0}^{\infty} \beta^t u\left(\bar{C}_t^j, \bar{H}_t^j\right). \tag{5}$$

Here, $p_{risk}^j$ is the welfare cost of risk.

Back

# Redistribution effect

- **The redistribution effect, $\Delta_R$**, can be defined as

$$1 + \Delta_R \equiv \frac{1 - p^R_{ineq}}{1 - p^{NR}_{ineq}}, \tag{6}$$

where

$$\sum_{t=0}^{\infty} \beta^t u\left((1 - p^j_{ineq})\bar{C}^j_t, \bar{H}^j_t\right) = \int \sum_{t=0}^{\infty} \beta^t u\left(\bar{c}^j_t(a_0, e_0), \bar{h}^j_t(a_0, e_0)\right) d\lambda_0. \tag{7}$$

- Analogously to $p^j_{risk}$, $p^j_{ineq}$ denotes the cost of inequality. Redistribution, according to this definition, is also a type of insurance but with respect to the ex-ante risk a household faces concerning which initial condition $(a_0, e_0)$ they will receive.

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# The Role of Incomplete Markets

# Role of Market Incompleteness

Using an approach similar to Werning (2007), we characterize analytically the solution for the following simpler economies (with borrowing constraints substituted for No-Ponzi conditions):

- Economy 1: Representative Agent ($\Gamma = I$, $e = 1$, $a_0 = \bar{a}$)
- Economy 2: Asset Heterogeneity ($\Gamma = I$, $e = 1$)
- Economy 3: Productivity Heterogeneity ($\Gamma = I$, $a_0 = \bar{a}$)
- Economy 4: Heterogeneity in Both ($\Gamma = I$)

◂ Back

# Optimal Taxes: Characterization

## Proposition

*There exist a finite integer $t^*$ and a constant $\Theta$ such that the optimal tax system is given by $\tau_t^k = 1$ for $0 \leq t < t^*$; while for $t \geq t^*$ $\tau_t^k$ follows*

$$\frac{1 + (1 - \tau_{t+1}^k) r_{t+1}}{1 + r_{t+1}} = \frac{1 - N_t}{1 - N_{t+1}} \frac{1 - \tau_{t+1}^h}{1 - \tau_t^h} \frac{\tau_t^h + \tau^c}{\tau_{t+1}^h + \tau^c}; \tag{8}$$

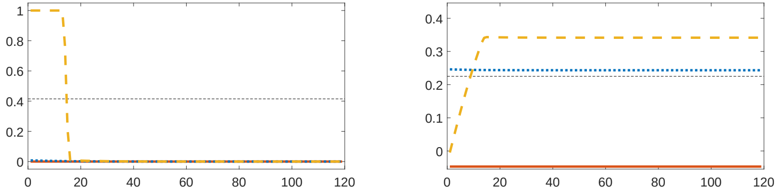
*for $0 \leq t \leq t^*$, $\tau_t^h$ evolves according to*

$$\frac{1 + (1 - \tau_{t+1}^k) r_{t+1}}{1 + r_{t+1}} = \frac{\Theta + \sigma (1 - N_{t+1})^{-1}}{\Theta + \sigma (1 - N_t)^{-1}} \frac{1 - \tau_{t+1}^h}{1 - \tau_t^h} \frac{1 + \tau^c + \alpha (\sigma - 1) \left(\tau^c + \tau_t^h\right)}{1 + \tau^c + \alpha (\sigma - 1) \left(\tau^c + \tau_{t+1}^h\right)}; \tag{9}$$

*and for all $t > t^*$, $\tau_t^h$ is determined by*

$$\tau_t^h (N_t) = \frac{(1 + \tau^c)}{(1 - N_t) \Theta + \alpha + \sigma (1 - \alpha)} - \tau^c. \tag{10}$$

# Optimal Taxes: Complete Markets Build-up



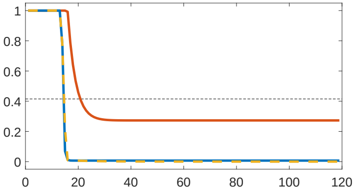

(a) Capital income tax



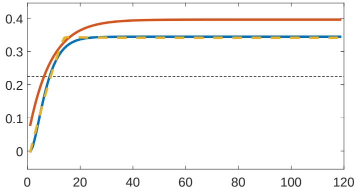
(b) Labor income tax

- **Black dashed line:** initial taxes
- **Red solid curve:** optimal taxes for representative economy
- **Blue solid curve:** optimal taxes with only labor-income inequality
- **Yellow dashed curve:** optimal taxes with labor-income and wealth inequality

# Optimal Taxes: Complete Markets vs. SIM model




(a) Capital income tax



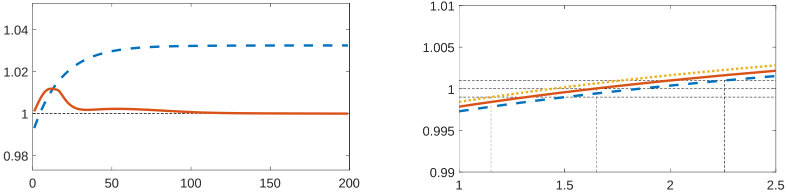

(b) Labor income tax

- **Black dashed line:** initial taxes
- **Red solid curve:** optimal taxes from Benchmark SIM model
- **Blue solid curve:** optimal taxes calculated using the same parameterized paths used in the Benchmark experiment
- **Yellow dashed curve:** optimal taxes calculated using Proposition 2

Back

# Long-run Optimality Conditions

# Long-Run Optimality Conditions




(a) $\beta(1 + f_K(K, N))$ (b) Debt to GDP vs. MGR

Note: Red solid curve: benchmark experiment; Dashed blue curve: optimal transition with constant policy.

Aiyagari(1995):

- Ramsey planner's decision to move aggregate resources across time is risk-free, in the long run, implies the modified golden rule (rationalizes positive long-run capital income taxes).

Acikgoz, Hagedorn, Holter, and Wang (2018):

- Derive long-run optimality conditions for the Ramsey planner in the SIM model—we verify they hold for our Ramsey allocation.

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# Mechanism: Two-Period Economy

Why use distortive capital and labor income taxes
when non-distortive lump-sum taxes are available?

# Two-Period Economy

- Continuum of ex-ante identical households receive $\omega$ in period 1.
- In period 2, they have random productivity levels:

$$e_L = 1 - \frac{\varepsilon}{\pi}, \qquad e_H = 1 + \frac{\varepsilon}{1-\pi}.$$

- No insurance market: only risk-free asset, $a$, available.
- Households solve

$$\max_{a, h_L, h_H} u(\omega - a, \bar{h}) + \beta \left[\pi_L u(c_L, h_L) + \pi_H u(c_H, h_H)\right]$$

$$\text{s.t.} \quad c_i = (1-\tau^h) w e_i h_i + (1-\tau_R^k) Ra + T, \quad i = L, H.$$

- In period 2, firms choose $K$ and $N$ to maximize profits given a CRS production function $f(K, N)$, and prices $w$ and $r$.

Back

# Two-Period Economy

## Definition

The Ramsey problem is to choose $\tau^k$, $\tau^h$, and $T$ to maximize welfare (the expected utility of the agents) subject to the economy being in equilibrium.

- BGP preferences:

$$u(c, h) = \frac{(c^\gamma (1-h)^{1-\gamma})^{1-\sigma}}{1-\sigma}$$

  - $\gamma$ controls the preference between consumption and leisure.
  - $\sigma$ controls the preference for risk and over-time smoothness.

Back

# The Effect of Risk

## Proposition

*The optimal tax system is such that*

$$\tau^h = \frac{\Omega}{1 - N + \gamma\Omega}, \quad \textit{and} \quad \tau_R^k = \frac{(1-\gamma)\tau^h}{1-\gamma\tau^h},$$

*where*

$$\Omega \equiv \frac{\pi_L(1-e_L)u_{c,L} + \pi_H(1-e_H)u_{c,H}}{\pi_L u_{c,L} + \pi_H u_{c,H}} \geq 0.$$

- $\Omega$ can be interpreted as a measure of planner's distaste for risk.
- **Insurance**: $\tau^h$ increases in the amount of risk.

Back

# The Effect of Inequality

- Let $e_L = e_H = 1$
- The initial endowment: $\omega_L$ for a proportion $p_L$ of hhs and $\omega_H > \omega_L$ for the rest. Let $\bar{\omega}$ be the average endowment.

## Proposition

*If $\sigma = 1$, then the optimal tax system is such that*

$$\tau_R^k = \frac{\gamma + \beta}{\beta} \frac{\Lambda}{\bar{\omega} - K + \Lambda}, \quad \textit{and} \quad \tau^h = 0,$$

*where*

$$\Lambda \equiv \frac{p_L(K - a_L)u_{c,L} + p_H(K - a_H)u_{c,H}}{p_L u_{c,L} + p_H u_{c,H}} \geq 0.$$

- $\Lambda$ can be interpreted as a measure of planner's distaste for inequality.
- **Redistribution**: $\tau_R^k$ reduces the proportion of household income that depends on unequal asset income.

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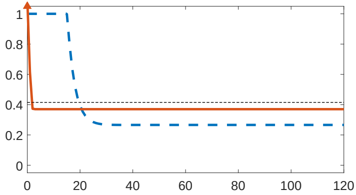
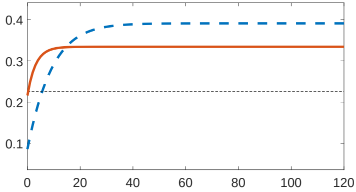
# Capital Levy and Constant Transfers

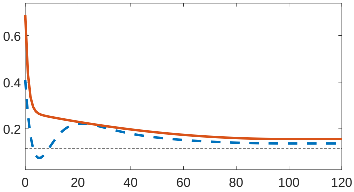
# Optimal Fiscal Policy: Capital Levy



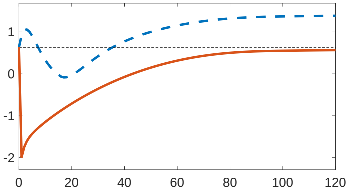

(a) Capital income tax




(b) Labor income tax




(c) Lump-sum/Initial output




(d) Debt/Initial output

Notes: Black dashed line: initial stationary equilibrium; Red solid curve: path that maximizes the utilitarian welfare function allowing for capital income taxes to move in period 0 (though the tax level at $t = 0$ is not plotted since it is equal to $(1 + r_0)/r_0 = 21.96$); Blue dashed curve: optimal transition (benchmark).

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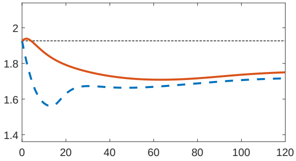
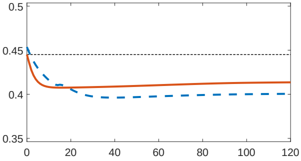
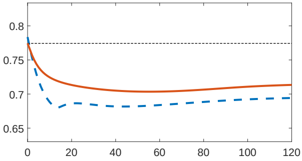
# Aggregates: Capital Levy



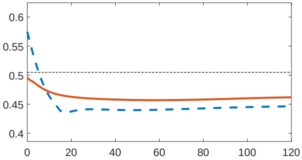

(a) Capital



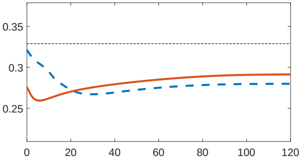

(b) Effective Labor ($N$)



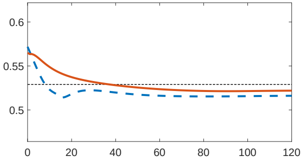

(c) Output




(d) Consumption




(e) Hours ($H$)




(f) Avg. $c^{\gamma}(1-h)^{1-\gamma}$

Note: Black dashed lines: initial stationary equilibrium; Red solid curves: optimal transition. Thick dashed line: benchmark results.

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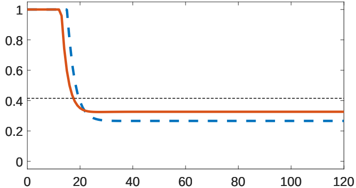
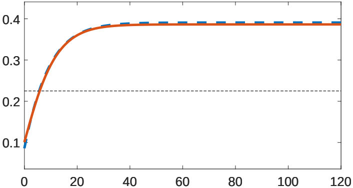
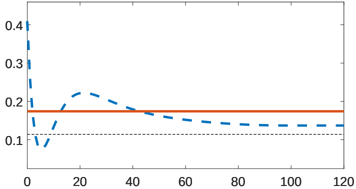
# Optimal Fiscal Policy: Constant Lump-Sum Transfers



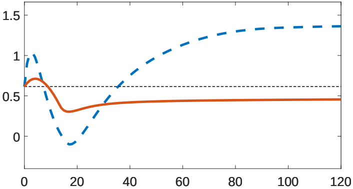

(a) Capital income tax




(b) Labor income tax



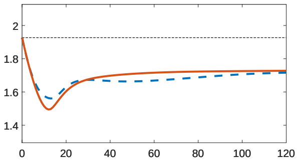

(c) Lump-sum/Initial output



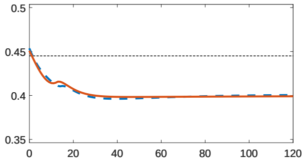

(d) Debt/Initial output

Notes: Thin dashed line: initial stationary equilibrium; Solid line: path that maximizes the utilitarian welfare function with the added restriction that lump-sum transfers are not allowed to vary over time after the initial change; Thick dashed line: benchmark results.

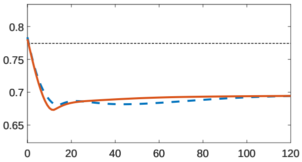
# Aggregates: Constant Lump-Sum Transfers



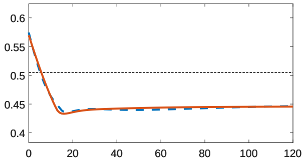

(a) Capital



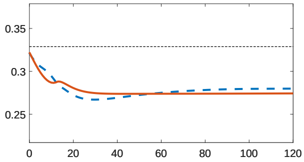

(b) Effective Labor ($N$)



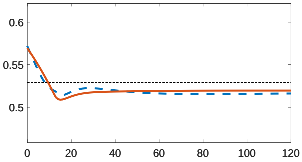

(c) Output




(d) Consumption




(e) Hours ($H$)




(f) Labor Productivity ($N/H$)

Note: Black dashed lines: initial stationary equilibrium; Red solid curves: optimal transition. Thick dashed line: benchmark results.

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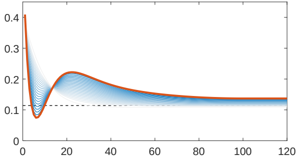
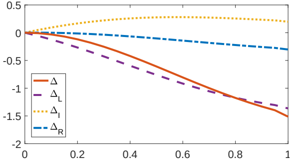
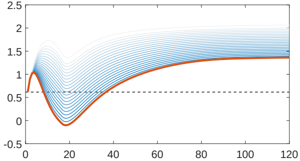
# Smooth front-loading



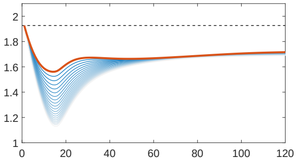
(a) Lump-sum/Initial output



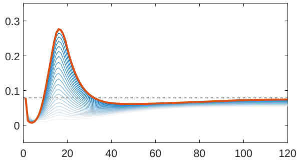

(b) Welfare decomposition



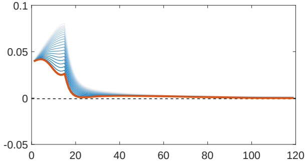
(c) Debt/Initial output



(d) Capital



(e) Prop. w/ Neg. Assets



(f) Intertemporal Wedge

Note (a) Black dashed line: initial stationary equilibrium; Red solid and blue lines: optimal transition and perturbations of it; (b) The $x$-axis represents the homotopy parameter between the initial optimal path at $x = 0$ and a flat path at $x = 1$.

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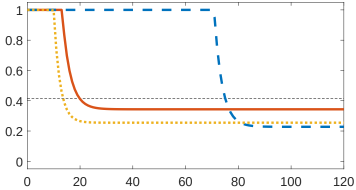
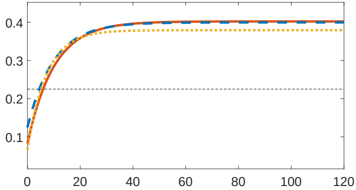
# Robustness Analysis: IES and Frisch

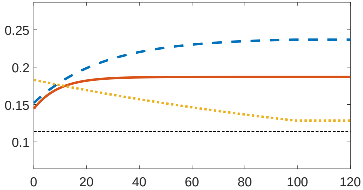
# Optimal Fiscal Policy: Robustness with respect to IES



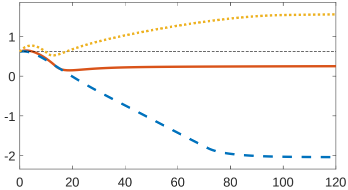

(a) Capital income tax




(b) Labor income tax



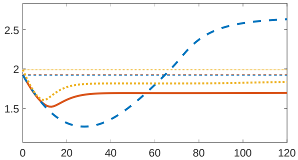

(c) Lump-sum/Initial output



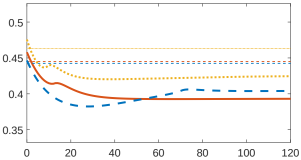

(d) Debt/Initial output

Notes: Black dashed line: initial stationary equilibrium; Red solid curve: optimal transition with benchmark IES of 0.65; Blue dashed curve: optimal transition with IES equal to 0.5; Yellow dotted curve: optimal transition with IES equal to 0.8.

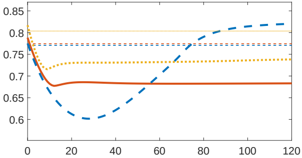
# Aggregates: Robustness with respect to IES



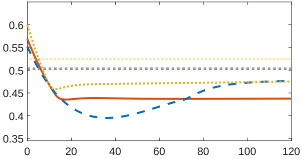

(a) Capital



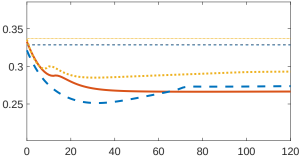

(b) Effective Labor ($N$)



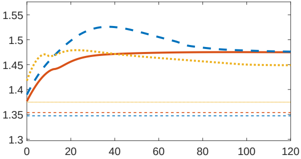

(c) Output




(d) Consumption




(e) Hours ($H$)




(f) Labor Productivity ($N/H$)

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# Definition of the average Frisch elasticity

- Household-level Frisch elasticities depend on the household's labor supply.

- We measure the intensive-margin aggregate Frisch elasticity with the unweighted average of household-level Frisch elasticities for employed households:

$$\Psi \equiv \int_{h(a,e)\,\geq\,\underline{h}} \left(\gamma + (1-\gamma)\frac{1}{\sigma}\right) \frac{1-h(a,e)}{h(a,e)}\, d\lambda_0(a,e).$$

where following Heathcote, Perri, and Violante (2010) we consider a household to be employed if they work more than five hours per week, that is, if $h \geq \underline{h} \equiv 0.05 = 260/52000$.

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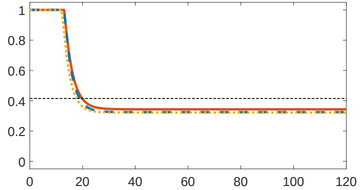
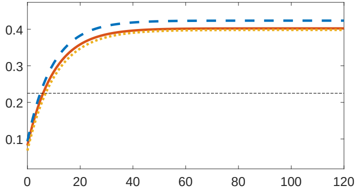
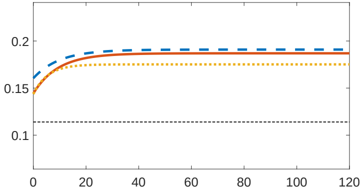
# Optimal Fiscal Policy: Robustness w.r.t. Frisch



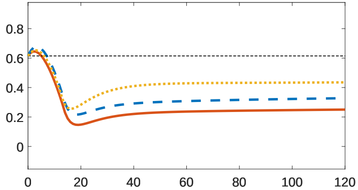

(a) Capital income tax




(b) Labor income tax




(c) Lump-sum/Initial output




(d) Debt/Initial output

Notes: Black dashed line: initial stationary equilibrium; Red solid curve: optimal transition with benchmark Frisch of 0.5; Blue dashed curve: optimal transition with Frisch equal to 0.35; Yellow dotted curve: optimal transition with Frisch equal to 0.65

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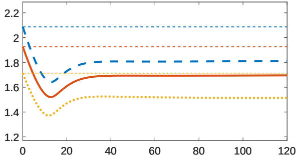
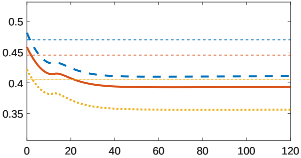
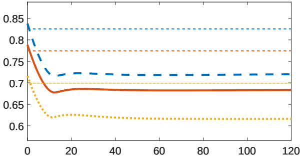
# Aggregates: Robustness w.r.t. Frisch



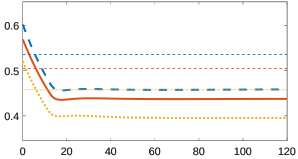

(a) Capital



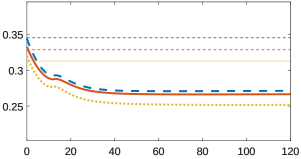

(b) Effective Labor ($N$)



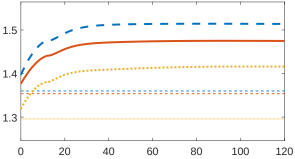

(c) Output




(d) Consumption




(e) Hours ($H$)




(f) Labor Productivity ($N/H$)

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# Non-targeted Moments

# Income Sources by Quintile of Income

| **Quintile** | **Model** | | | **US data** | | |
|---|---|---|---|---|---|---|
| | Labor | Asset | Transfer | Labor | Asset | Transfer |
| 1st | 80.1 | 0.2 | 19.7 | 83.6 | 0.4 | 16.1 |
| 2nd | 77.0 | 2.6 | 20.4 | 86.5 | 1.1 | 12.3 |
| 3rd | 74.1 | 5.3 | 20.5 | 85.6 | 1.9 | 12.5 |
| 4th | 74.8 | 9.4 | 15.8 | 84.1 | 3.8 | 12.2 |
| 5th | 63.1 | 31.2 | 5.7 | 70.4 | 21.4 | 8.2 |
| All | 70.4 | 16.7 | 12.9 | 77.3 | 12.3 | 10.4 |

Note: Table summarizes the pre-tax total income decomposition. We define the asset income as the sum of income from capital and business. Data come from the 2007 Survey of Consumer Finances.

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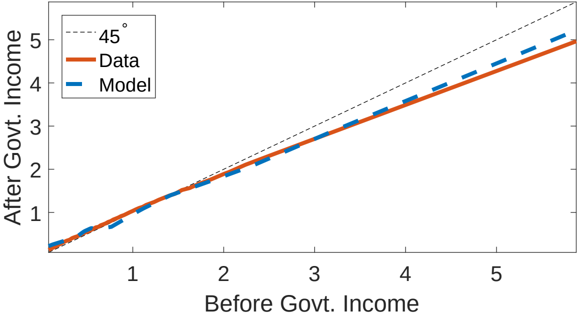
# Income tax schedule




Notes: The axis units are income relative to the mean.

- The tax rates are calibrated to match effective tax rates. However, we also approximate well the actual income tax schedule (data from Heathcote, Storesletten & Violante (2014)).

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# Maximizing Efficiency

# Maximizing Efficiency

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## Assumption

*The certainty equivalents display parallel patterns if* $\bar{c}_t^j(a_0, e_0) = \eta^j(a_0, e_0)\tilde{C}_t^j$, *and* $1 - \bar{h}_t^j(a_0, e_0) = \eta^j(a_0, e_0)(1 - \tilde{H}_t^j)$, *for some function* $\eta^j(a_0, e_0)$ *and paths* $\{\tilde{C}_t^j\}$, *and* $\{\tilde{H}_t^j\}$.

## Proposition

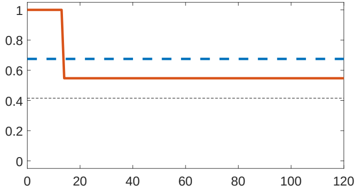
*For balanced-growth-path preferences if the certainty equivalents satisfy Assumption, then the components* $\Delta_L$, $\Delta_I$, *and* $\Delta_R$ *are independent of the paths* $\{\tilde{C}_t^j\}$, *and* $\{\tilde{H}_t^j\}$.

## Proposition

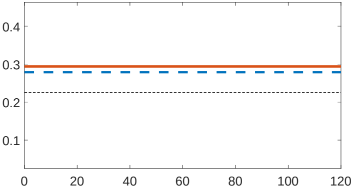
*If the certainty equivalents satisfy Assumption, then, maximizing* $W^0$ *is equivalent to maximizing* $(1 + \Delta_L)(1 + \Delta_I)$.

# Adding Flexibility in the Time Domain

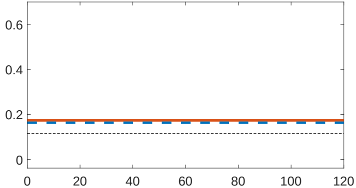
# Number of Parameters: 2 to 3



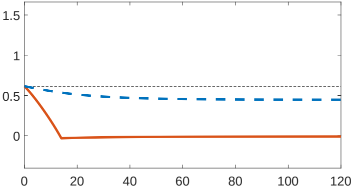

(a) Capital income tax




(b) Labor income tax




(c) Lump-sum/Initial output




(d) Debt/Initial output

Notes: Black dashed line: initial stationary equilibrium; Blue dashed curve: optimal policy with 2 parameters $\left(\tau^k, \tau^h\right)$; Red solid curve: optimal policy with 3 parameters $\left(t^*, \tau^k_F, \tau^h\right)$.

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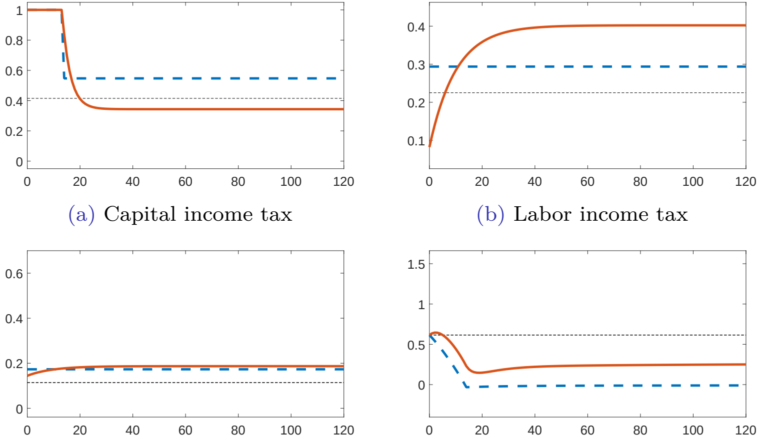
# Number of Parameters: 3 to 8




(a) Capital income tax



(b) Labor income tax



(c) Lump-sum/Initial output



(d) Debt/Initial output

Notes: Black dashed line: initial stationary equilibrium; Blue dashed curve: optimal policy with 3 parameters; Red solid curve: optimal policy with 8 parameters $\left(\alpha_0^k, \beta_0^k, \lambda^k, \alpha_0^h, \beta_0^h, \lambda^h, \beta_0^T, \lambda^T\right)$.

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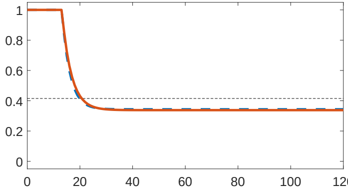
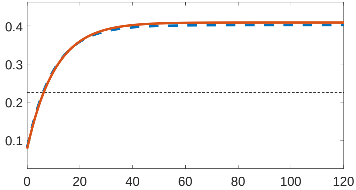
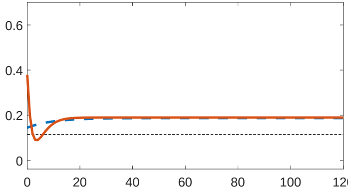
# Number of Parameters: 8 to 11



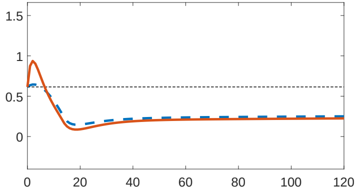

(a) Capital income tax




(b) Labor income tax




(c) Lump-sum/Initial output




(d) Debt/Initial output

Notes: Black dashed line: initial stationary equilibrium; Blue dashed curve: optimal policy with 8 parameters; Red solid curve: optimal policy with 11 parameters $\left(\alpha_0^k, \alpha_1^k, \beta_0^k, \lambda^k, \alpha_0^h, \alpha_1^h, \beta_0^h, \lambda^h, \alpha_1^T, \beta_0^T, \lambda^T\right)$.

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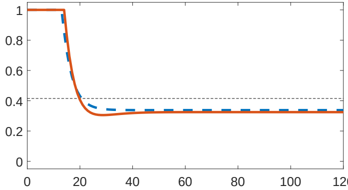
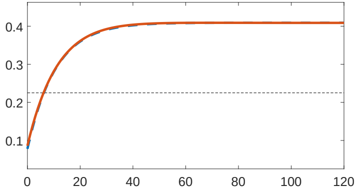
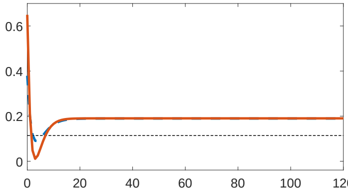
# Number of Parameters: 11 to 14



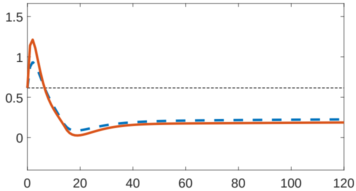

(a) Capital income tax




(b) Labor income tax




(c) Lump-sum/Initial output




(d) Debt/Initial output

Notes: Black dashed line: initial stationary equilibrium; Blue dashed curve: optimal policy with 11 parameters; Red solid curve: optimal policy with 14 parameters $\left(\alpha_0^k, \alpha_1^k, \alpha_2^k, \beta_0^k, \lambda^k, \alpha_0^h, \alpha_1^h, \alpha_2^h, \beta_0^h, \lambda^h, \alpha_1^T, \alpha_2^T, \beta_0^T, \lambda^T\right)$.

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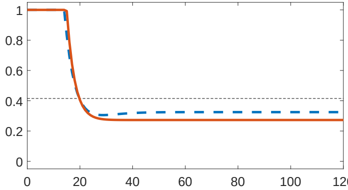
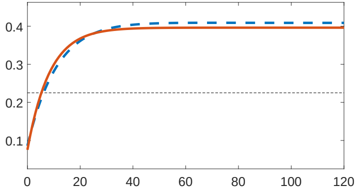
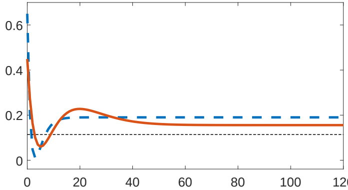
# Number of Parameters: 14 to 16



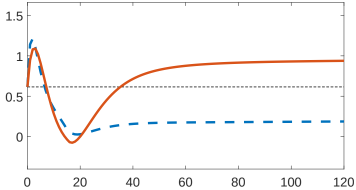

(a) Capital income tax




(b) Labor income tax




(c) Lump-sum/Initial output




(d) Debt/Initial output

Notes: Black dashed line: initial stationary equilibrium; Blue dashed curve: optimal policy with 14 parameters; Red solid curve: optimal policy with 16 parameters $\left(\alpha_0^k, \alpha_1^k, \alpha_2^k, \beta_0^k, \lambda^k, \alpha_0^h, \alpha_1^h, \alpha_2^h, \beta_0^h, \lambda^h, \alpha_1^T, \alpha_2^T, \alpha_3^T, \alpha_4^T, \beta_0^T, \lambda^T\right)$.

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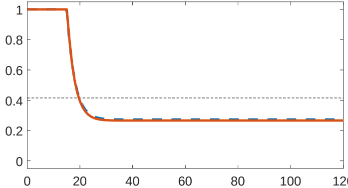
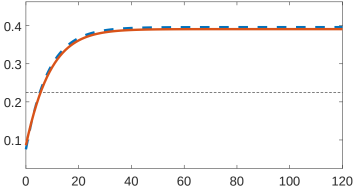
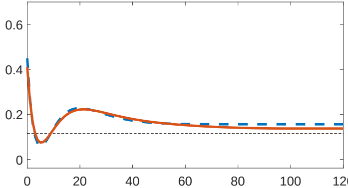
# Number of Parameters: 16 to 17



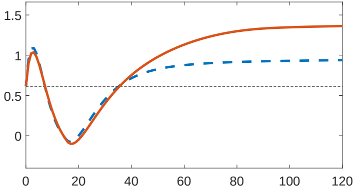

(a) Capital income tax




(b) Labor income tax




(c) Lump-sum/Initial output




(d) Debt/Initial output

Notes: Black dashed line: initial stationary equilibrium; Blue dashed curve: optimal policy with 16 parameters; Red solid curve: optimal policy with 17 parameters $\left(\alpha_0^k, \alpha_1^k, \alpha_2^k, \beta_0^k, \lambda^k, \alpha_0^h, \alpha_1^h, \alpha_2^h, \beta_0^h, \lambda^h, \alpha_1^T, \alpha_2^T, \alpha_3^T, \alpha_4^T, \beta_0^T, \beta_1^T, \lambda^T\right)$, with $\beta_2^T$ chosen such that the derivative of $T_t$ at $t = 100$ is equal to zero.

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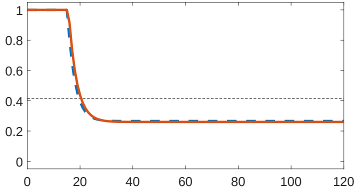
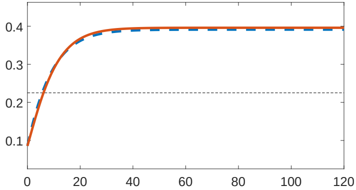
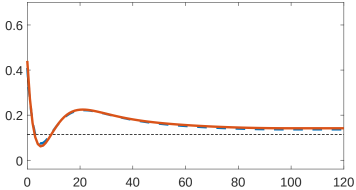
# Number of Parameters: 17 to 20



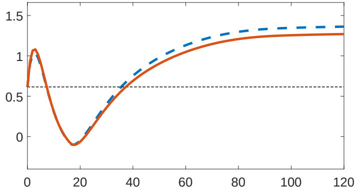

(a) Capital income tax




(b) Labor income tax




(c) Lump-sum/Initial output




(d) Debt/Initial output

Notes: Black dashed line: initial stationary equilibrium; Blue dashed curve: optimal policy with 17 parameters; Red solid curve: optimal policy (local search) with 20 parameters $\left(\alpha_0^k, \alpha_1^k, \alpha_2^k, \alpha_3^k, \beta_0^k, \lambda^k, \alpha_0^h, \alpha_1^h, \alpha_2^h, \alpha_3^h, \beta_0^h, \lambda^h, \alpha_1^T, \alpha_2^T, \alpha_3^T, \alpha_4^T, \alpha_5^T, \beta_0^T, \beta_1^T, \lambda^T\right)$, with $\beta_2^T$ chosen such that the derivative of $T_t$ at $t = 100$ is equal to zero.

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