Determinacy without the Taylor Principle

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Outline

The Equilibrium Selection Issue in the NK Model

- Can monetary policy regulate AD by adjusting interest rates?
- Important caveat (e.g., Sargent & Wallace):
  - Same nominal interest rate path consistent with **multiple bounded eq.**
  - Need for equilibrium selection
- Standard approach: **Taylor principle** (raise rates aggressively with inflation)
  - An off-eq. threat to trigger an explosion in π and y (Cochrane)
  - Or a reversion to M regime for large enough deviations (Atkeson, Chari, & Kehoe)
- Alternative: **Fiscal Theory of the Price Level** (Leeper, Sims, Woodford)
  - An off-eq. threat to blow out the government budget (Kocherlakota & Phelan)
  - Or other interpretations of non-Ricardian fiscal policy (Cochrane, Bassetto)
- Eq. selection debate is **a war of "religious beliefs"** (Kocherlakota & Phelan)
  - Cannot be guided by empirical evidence and are inherently untestable

This Paper: Determinacy without the Taylor Principle

- Sunspot eq. artifacts of **perfect intertemporal coordination ("infinite chain")**
  - Current agents respond to "irrelevant" sunspots only if future agents respond in a specific way
  - Future agents respond only if they expect agents further in the future respond; and so on.
- Small perturbations in memory/coordination $\Rightarrow$ breaks the infinite chain $\Rightarrow$ **determinacy**
- **Always selects the standard eq.** (minimum-state-variable eq.)
- Taylor principle perhaps less consequential than previously thought
- No room for FTPL as currently formalized (as an eq. selection device)
  - but **fiscal considerations can matter through the eq. conduct by MP**
- Eases the potential conflict between **stabilization** and **eq. selection**

Pause for Questions

Outline

A Simplified Model

- Dynamic IS ($\bar{E}_t[\cdot] = \int E_{i,t}[\cdot]\, di$ is the average expectation)

$$c_t = -\sigma\left(i_t - \bar{E}_t[\pi_{t+1}]\right) + \bar{E}_t[c_{t+1}] + \rho_t$$

- Phillips curve (static for now, forward looking later)

$$\pi_t = \kappa c_t + \xi_t$$

- Monetary policy

$$i_t = z_t + \phi \pi_t$$

An Equivalent Representation

- Substituting monetary policy and Phillips curve in IS curve $\Rightarrow$

$$c_t = \theta_t + \delta \bar{E}_t [c_{t+1}]$$

where $\{\theta_t\}$ is a function of $\{\rho_t, \xi_t, z_t\}$ and

$$\delta = \delta(\phi) \equiv \frac{1+\kappa\sigma}{1+\phi\kappa\sigma}$$

- **Taylor principle** holds when

$$\phi > 1 \iff \delta < 1$$

- Equivalent formulation

$$\pi_t = \tilde{\theta}_t + \delta \bar{E}_t [\pi_{t+1}]$$

  - this nests the **flexible price case** ($i_t = \bar{E}_t [\pi_{t+1}]$) with $\kappa \to \infty$ ($\delta \to \frac{1}{\phi}$)

Fundamentals, Sunspots, and the Equilibrium Concept

- Fundamentals:

$$\theta_t = \rho \theta_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim_{\text{i.i.d}} \mathcal{N}(0,1)$$

  - In paper: generalization allowing generic state space representations

- Sunspots:

$$\eta_t \sim_{\text{i.i.d}} \mathcal{N}(0,1)$$

- State of nature, or (infinite) history, at t:

$$h^t = \{\theta_{t-k}, \eta_{t-k}\}_{k=0}^{\infty}$$

- Equilibrium concept: **REE (based on potentially limited information about h^t)**

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \gamma_k \theta_{t-k}$$

- Focus on **bounded** eq. ($Var(c_t)$ is finite). Can be justified by escape clauses by ACK.

Outline

The Standard Paradigm

- **FIRE (full information rational expectations)/perfect recall benchmark:**

$$c_t = \theta_t + \delta E_t [c_{t+1}]$$

  - $E_t[\cdot]$ is rational expectation conditional on entire history h^t

- **The MSV (minimum state variable) solution:**

$$c_t = c_t^F \equiv \frac{1}{1-\delta\rho}\theta_t$$

  - guess and verify $c_t = \gamma\theta_t$

- **Is MSV the only solution?**
  - Taylor principle holds when $\phi > 1 \iff \delta < 1$
  - If it does not hold $\delta > 1$, solve backward $\Longrightarrow$ sunspot and backward looking eq.

The Standard Paradigm

Proposition 1. Perfect Recall Benchmark

- When the Taylor principle is satisfied ($|\delta| < 1$), the MSV equilibrium is the unique one
- When this principle is violated $|\delta| > 1$), there exist **a continuum of equilibria**

$$c_t = (1-b)c_t^F + bc_t^B + ac_t^\eta,$$

where

- **Sunspot equilibria** (non-zero solution to $c_t = \delta E_t[c_{t+1}]$)

$$c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k}$$

- **Backward fundamental equilibria**

$$c_t^B \equiv -\sum_{k=1}^{\infty} \delta^{-k} \theta_{t-k}$$

Understanding the Multiplicity

Using the sunspot eq. as an example:

$$c_t^\eta = \delta E_t \left[c_{t+1}^\eta \right]$$

Infinite chain of perfect intertemporal coordination:

- Current agents respond **against their intrinsic interest** because they **expect to be rewarded by future agents**
- Future agents themselves respond based on a similar expectation
- $\cdots$

What's Next: Breaking the Infinite Chain

What's next: two perturbations **breaking the infinite chain of perfect coordination**

Two equivalent representations of the sunspot equilibrium

$$\text{Sequential}: \quad c_t^\eta = \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k}$$

$$\text{Recursive}: \quad c_t^\eta = \delta^{-1} c_{t-1}^\eta + \eta_t$$

- c_t^η needs to **respond to distant-past sunspots** (directly or indirectly)

First perturbation motivated by the sequential representation

- Fading social memory about $\eta_{t-k} \implies$ determinacy

Second perturbation motivated by the recursive representation

- Bounded social memory what drives (a tiny part of) $c_{t-1}^\eta \implies$ determinacy

Outline

The First Perturbation

Memory:

- In each period, a randomly $\lambda \in [0,1]$ of agents are replaced by newborn agents.
- Agents **know fundamentals & sunspots during their lives** but **not before**
- The period-t information set of an agent born s periods ago is given by

$$I_t^s \equiv \{(\theta_t, \eta_t), \ldots, (\theta_{t-s}, \eta_{t-s})\}$$

The First Perturbation

$$I_t^s \equiv \{(\theta_t, \eta_t), ..., (\theta_{t-s}, \eta_{t-s})\}$$

Interpretation:

- **OLG with "fading" social memory**
  - Consistent with perfect individual recall & standard rational expectations solution concept
  - Equivalent behavioral interpretation: agents are infinitely-lived but have bounded recall

Standard paradigm:

- Perfect social memory, nested by $\lambda = 0$

Properties:

- For any $\lambda > 0$, zero mass of agents has *infinite* memory
  - But as $\lambda \to 0$, **almost all agents have *arbitrarily long* memory**
- Prevent direct knowledge about history of endogenous $\{c_{t-k}\}$
  - But as $\lambda \to 0$, **arbitrarily well informed long histories of** $\{c_{t-k}\}$

Determinacy without the Taylor Principle

Proposition 2. Determinacy without the Taylor Principle

With fading social memory, the **unique equilibrium** is the **MSV solution**, $c_t = c_t^F$

- **Regardless** of the value of δ, or **equivalently monetary policy** ϕ.
- No matter how slow the memory decay is (how small λ is).

Proof sketch: focusing on responses to η_0 (a_t).

- "Twin" economy with perfect memory but modified best response:

$$c_t = \theta_t + \delta \bar{E}_t [c_{t+1}] \quad \Longrightarrow \quad c_t = \delta \mu_t E_t [c_{t+1}],$$

  where $\mu_t = (1-\lambda)^t \to 0$ is the proportion of agents remembering η_0 at t.
- But $\delta \mu_t < 1$ eventually, so always determinacy.

Logic

- I can see the current sunspot very clearly
- It would make sense to react if all future agents will keep responding to it **in perpetuity**
- But I worry that agents **far in the future will fail to do so**
  - either because they will have forgotten it
  - or because they may worry that agents further into the future will not react to it
- It therefore makes sense to ignore the sunspot

Outline

A Micro-funded NK Model

- A micro-founded IS curve robust to incomplete information

$$c_t = -\beta\omega\sigma \left\{ \sum_{k=0}^{+\infty} (\beta\omega)^k \bar{E}_t \left[i_{t+k} - \pi_{t+k+1} \right] \right\} + (1-\beta\omega) \left\{ \sum_{k=0}^{+\infty} (\beta\omega)^k \bar{E}_t \left[c_{t+k} \right] \right\} + \rho_t$$

  - $\omega = 1 - \lambda$ is the survival probability (as the OLG structure above)
  - embeds individual optimality + market clearing + budgets
  - reduces to the RA Euler equation (plus transversality) when $\overline{E}_t[\cdot] = E_t[\cdot]$

- Standard dynamic NKPC

$$\pi_t = \kappa c_t + \beta E_t \left[\pi_{t+1} \right] + \xi_t$$

- Monetary policy

$$i_t = z_t + \phi_c c_t + \phi_\pi \pi_t$$

The Generalized Model and Nesting

- The generalized model

$$c_t = \theta_t + \bar{E}_t \left[\sum_{k=0}^{+\infty} \delta_k c_{t+k} \right]$$

  - only requires that the sum $\sum_{k=0}^{\infty} |\delta_k|$ is finite

- Nests the previous micro-founded NK with

$$\delta_k = (1 - \beta\omega - \beta\omega\sigma\phi_c)(\beta\omega)^k + \omega\sigma\kappa \left(-\phi_\pi\beta + (1 - \omega\phi_\pi\beta) \frac{1 - \omega^k}{1 - \omega} \right) \beta^k.$$

The Generalized Results mu

Proposition 3. Fading Memory Rules out Sunspot Volatility

With fading social memory ($\lambda > 0$), the equilibrium is unique and is given by the MSV solution.

Proof sketch: focusing on response to η_0 (a_t).

- "Twin" economy with perfect memory but modified best response:

$$c_t = \theta_t + \bar{E}_t \left[\sum_{k=0}^{+\infty} \delta_k c_{t+k} \right] \implies c_t = \mu_t E_t \left[\sum_{k=0}^{+\infty} \delta_k c_{t+k} \right],$$

  where $\mu_t \to 0$ is the proportion of agents remembering η_0 at t.
- But $\mu_t \left(\sum_{k=0}^{\infty} |\delta_k| \right) < 1$ eventually, so always determinacy
- Effective complementary < 1, uniquely pinned down by iterating of best responses

Outline

Observing Past Outcomes

- Baseline: preclude *direct* observation of past outcomes, such as c_{t-1}

- But note: agents have *almost perfect* knowledge of past outcomes
  - for any T, almost all agents learn $\{c_{t-1}, ..., c_{t-T}\}$ nearly perfectly as $\lambda \to 0$

- Still, what if perfectly observing past outcomes?
  - Could **long memory of sunspots and past fundamentals** be efficiently "stored" in **short memory of past outcomes**?

- For example, the recursive formulation of the sunspot equilibrium (turn off θ_t briefly)

$$c_t = \eta_t + \delta^{-1} c_{t-1}$$

- Perfect memory of c_{t-1} suffice as the memory of the history of sunspots
  - sunspot equilibria strike back?

Storing Memory in Endogenous Outcomes

- Still takes a strong, **fragile**, form of **intertemporal coordination**
  - Current agents respond because they expect future **respond in a perfect way**
  - Infinite chain of coordination $\cdots$
- Add i.i.d. fundamental shocks $\zeta_t \in [-\varepsilon, \varepsilon]$ (arbitrarily small) known only to t

$$c_t = \zeta_t + \delta \bar{E}_t [c_{t+1}]$$

- For a sunspot eq, requires **perfect knowledge of ζ_t at $t+1$**

$$c_{t+1} = \eta_{t+1} + \delta^{-1} (c_t - \zeta_t)$$

- But if ζ_t unknown to agents at $t+1$, the sunspot equilibrium collapses

The Second Perturbation

- Bring back fundamentals θ_t with arbitrarily small. i.i.d. perturbations $\zeta_t \in [-\varepsilon, \varepsilon]$

$$c_t = \theta_t + \zeta_t + \delta \mathbb{E}[c_{t+1} | I_t]$$

- A representative agent in each period, with info set

$$I_t = \{\zeta_t\} \cup \{\theta_t \ldots, \theta_{t-K}\} \cup \{\eta_t \ldots, \eta_{t-K}\} \cup \{c_{t-1}, \cdots, c_{t-K}\}$$

  - Long memory of past sunspots, fundamentals, & outcomes **for arbitrarily large but finite K**
  - But knowledge of only current ζ_t & no memory of past ζs

Proposition 5. Storing Memory in Endogenous Outcomes

With above info. structure, regardless of δ, there is a **unique equilibrium** and is given by $c_t = c_t^F + \zeta_t$, where c_t^F is the same **MSV solution** as before.

- Break the infinite chain $\Longrightarrow$ MSV as the unique eq

Outline

Fiscal Theory of Price Level (FTPL)

- Essence of the FTPL: **non-Ricardian fiscal policy**
  - primary surplus do respond enough to public debt level
  - An off-equilibrium threat to blow out the government budget (Kocherlakota & Phelan)
  - Or other interpretations (Cochrane, Bassetto)
- Standard paradigm: FTPL perfectly logical with "passive MP" ($\phi < 1$)
  - concur with **passive-monetary and active-fiscal regime in Leeper (1991)**
- Our contribution: no need/space for eq selection from FTPL
  - **underscores the fragility of existing formalization of FTPL**
  - but allow fiscal considerations to matter on eq. through conduct of MP

Feedback Rules and the Ramsey Implementation

- Consider the **Ramsey optimum**. How can monetary policy uniquely implement it?

- If the monetary authority **observes the underlying shocks**, uniquely implemented with:

$$i_t = i_t^o + \phi(\pi_t - \pi_t^o),$$

  where i_t^o and π_t^o are rates and inflation in the optimum and $\phi > 1$.

- What if the monetary authority **does not observe the underlying shocks?**
  - implemented through feedback rules?

$$i_t = \phi \pi_t$$

- Two conflicting roles
  - **Stabilization** ($\phi < 1$ possible in the Ramsey optimum)
  - **Eq. selection** ($\phi > 1$ necessary in the standard paradigm)

- Here: Liberates the **stabilization role** of monetary policy from **its eq. selection role**

Alternative Boundedly-Rational Solution Concepts

- Group 1: **relax REE but maintain a "fix point" between expectations & actual eq.**
  - e.g., Cognitive discounting in Gabaix (20); Diagnostic expectations in Bordalo et. al (20)
  - may shrink the determinacy region but the indeterminacy problem remains
- Group 2: completely **shuts down the "fix point"**
  - e.g. level-k thinking (Garcia-Schmidt & Woodford, 19; Farhi & Werning, 19)
  - produces a unique solution but opens a new issue
  - whenever $\phi < 1$, **Level-k solution becomes infinitely sensitive to Level-0 behavior**

Outline

Conclusion

- Main lesson: NK indeterminacy/FTPL hinge on **strong info assumptions**
- **A small friction in memory & intertemporal coordination** can result in **determinacy**
- Taylor principle perhaps less consequential than previously thought
  - more crucial: boundedness (commitment to rule out large deviations)
- No room for FTPL as currently formalized (as an eq. selection device)
  - but **fiscal considerations can matter if internalized by MP**
  - Model MP-FP interaction as a game of between monetary & fiscal authority?