How Competitive is the Stock Market?
Theory, Evidence from Portfolios, and Implications for the Rise of Passive Investing*

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Abstract

We develop a framework to theoretically and empirically analyze investor competition on financial markets. The classic view assumes that markets are very competitive: if a group of investors changes its behavior, other investors react such that nothing happens in equilibrium. Our framework quantifies the strength of the competitive response. We estimate a demand system of institutional investors in the US stock market accounting for two layers of equilibrium: how investors compete with each other in setting their strategies and how prices adjust to clear asset markets. We find that investors react to the behavior of others in the market: when an investor is surrounded by less aggressive traders she trades more aggressively. This reaction reduces the equilibrium consequences of changes in individual behavior by 50%. However, it also implies that the stock market is far from the competitive ideal. A consequence of this result is that the large increase in passive investing over the last 20 years has led to substantially more inelastic aggregate demand curves for individual stocks, by 15%.

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1 Introduction

What happens to equilibrium prices when a subset of investors changes its behavior? For example, what are the implications of investors switching to passive strategies, as has been occuring on a large scale over the last few decades?[1] Answering this question relies crucially on how other investors react to this change. In the standard view that "financial markets are competitive", the answer is simple: nothing happens, because other investors pick up any slack left off by those changing their behavior.[2] Casually said: if you stop looking for $20 bills on the floor, someone else will replace you. In this paper, we propose a framework to quantify investor competition in financial markets, combining information from prices and portfolio positions. We implement the framework for the U.S. stock market.

We find that investors react to the behavior of others in the market: when an investor is surrounded by less aggressive traders—that is, with a lower price elasticity of demand—she trades more aggressively. While this reaction strongly mitigates the equilibrium consequences of changes in individual behavior, the stock market is far from the competitive ideal. Our estimates suggest that competition reduces the impact of an increase in passive investing in half. Still, an increase as large as the one of the last 20 years leads to substantially more inelastic aggregate demand curves for individual stocks, by 15%.

To get to these answers, we proceed in three steps. First, we present an equilibrium model of investor competition. Each investor's demand elasticity responds to the elasticity of others in the market; the level of competition is the strength of this response. We demonstrate that while simple, this approach embodies the economics of investor competition across many theories such as those of Grossman and Stiglitz (1980) and Kyle (1989). Second, to take these ideas to the data, we distill the model into a realistic demand system that enriches the

[1]For example, the ICI factbook (ICI, 2020) reports that the total assets of passive mutual funds in the U.S. have increased from $11bn to $2.8tn between 1993 and 2020.

[2]In the same way that efficient markets have a different meaning in the finance literature relative to the remaining of economics, this is not the standard notion of competition. Rather than asking whether investors are price-taker (the focus of Lee and Kyle (2018)), this notion captures how much substitution there is in the strategies of investors. Going back to Kreps and Scheinkman (1983), it is understood that price-taking is not the only aspect shaping competition.

framework of Koijen and Yogo (2019). We show how to overcome the estimation challenges created by equilibrium interactions between investors. Third, we estimate the model using detailed portfolio positions of institutional investors in the U.S. stock market. Using the model, we quantify the impact of a rise in passive investing, and decompose the sources of evolution in the demand for individual stocks.

Our model represents investor competition by its effect on demand elasticities, how much an investor changes her position in an asset as a function of its price. A more elastic demand curve implies more aggressive trading: the investor increases their position a lot when the asset is cheap. Unlike in standard price theory, demand elasticities in financial markets are not only determined by the investor's preferences for the asset but also by the behavior of other investors. Indeed, it matters why the asset is cheap, and decisions of other investors shape the reason behind price movements. For example, asset prices convey information, and depending on this information, the investor should be more or less aggressive in her response to price movements. We assume that each investor's demand elasticity combines an investor-specific component and a response to the aggregate demand elasticity prevalent in the market. The strength of this response is the level of competition. An equilibrium combines two layers. First, the elasticities of all investors must be consistent with each other: the average of all investor elasticities must be equal the aggregate elasticity. Second, the asset price is such that the sum of all demand curves evaluated at this price is equal to the supply of the asset.

This framework is simple because it reduces the problem of investor competition to an equilibrium determination of elasticities. However, this simplicity does not impede its richness: we show that our setting unifies many foundations for competition. For example, we consider a theory in which investors expend resources to obtain more precise information on the assets they invest in, like in Grossman and Stiglitz (1980) — see Veldkamp (2011) for a review. Two elements shape the demand for information: an investor's own ability to gather information and how much information other investors have acquired. If other investors have more information, the value of acquiring information declines, and the investor chooses to obtain less of it. This force is the source of competition between investors, and how easily an investor is able to adjust her information determines how competitive the market is. Because when an investor acquires more information she trades more aggressively, the information equilibrium can be recast exactly as an elasticity equilibrium. Elasticities inherit the strategic substitutability of information choices, giving rise to competition. In contrast, in theories where investors have market power, like in Kyle (1989), it is optimal to trade more aggressively if others are more aggressive as well. In both cases, the equilibrium strategies of investors boil down to a choice of elasticity.

What happens when a group of investors becomes passive? Their investment strategy becomes irresponsive to the price of the asset, hence their demand elasticity becomes zero. This pushes the aggregate elasticity down, which prompts a response from other investors, potentially compensating the direct effect. When the competitive response is strongest, this reaction completely offsets the direct effect, the equilibrium market elasticity is unchanged. This corresponds to the ideal of "competitive financial markets". In the other extreme, if investors do not react, the elasticity provided by the traders who became passive is just lost. We quantify the intensity of competition to obtain the pass-through of a rise in passive investing into aggregate elasticities.

We adapt our basic two-layer equilibrium setting to be able to take it to the data. We implement our framework as a demand system in the style of Koijen and Yogo (2019). The level of demand and the individual-specific component of elasticity are functions of stock characteristics. In addition, demand has a residual component unobservable to the econometrician. The portfolio choice of investors across stocks is represented by a logit system. These choices allow for a rich and realistic specification of demand, and capture heterogeneity across investors. To this set of assumptions we add a second layer: the elasticity equilibrium. The demand elasticity of an investor depends on her individual-specific component and a response to the aggregate elasticity of demand for this stock. In equilibrium, the aggregate demand elasticities must be the average of investor-level elasticities.

To estimate the model we overcome three challenges. First, the interaction between investors through their elasticity decisions introduces a reflection problem (Manski, 1993): a market with high elasticity could be the result of either high individual elasticities or strong positive spillovers. The cross-section of stocks provides a solution to this issue because we observe the same investor in different markets. This investor faces a different mix of competing investors for each stock, therefore a different aggregate demand elasticity. This variation allows us to isolate the spillover from the individual-specific component of elasticity. We face a chicken-and-egg question: to implement this comparison we need to know the elasticity of other investors, which itself has to be estimated in the same way. We prove conditions on the graph of investor-stock connections that solve this issue.

Second, both the price and the aggregate elasticity are equilibrium quantities and therefore depend on portfolio decisions. We construct an instrument for each of these two variables using variation in investment universe across investors. Stocks that more investors can buy naturally have more money chasing them and a higher price. This instrument for the price is introduced in Koijen and Yogo (2019). Because individual components of elasticities are not known a priori, the same idea cannot be directly implemented to craft an instrument for aggregate elasticity. We construct a new model-based instrument combining the variation in investment universe with the estimated individual component of elasticities. This adds a challenge: we have to estimate the instrument and the demand system simultaneously. Still this is valid strategy because model parameters are exogenous by definition.

Third, the inclusion of rich investor heterogeneity, the need to solve for an elasticity equilibrium, and the presence of model-based instrument, all concur to a seemingly intractable estimation. However, we develop a computationally efficient algorithm that estimates the model. The key idea is to isolate low-dimensional fixed-point problems due to the elasticity equilibrium from a larger but standard linear system with high-dimensional investor-specific coefficients.

Our estimates suggest a substantial amount of competition. If the aggregate elasticity for a stock increases by 1, an individual investor decreases her own elasticity of demand by 1.7. This competitive response stabilizes the levels of aggregate elasticity. Intuitively, when a very aggressive investor trades a specific stock, other investors in this stocks adjust by becoming less aggressive. This force implies about 50% less cross-sectional variation in elasticity across stocks than estimates ignoring competitive interactions, highlighting the importance of these interactions.

We use these estimates to assess the impact of a rise in passive investing. To do so, we ask how equilibrium elasticities change when a fraction of investors exogenously switches to be passive. We obtain a simple formula for the pass-through of a change in the fraction of active investor to the aggregate elasticity. This pass-through only depends on the intensity of competition and the initial fraction of active investors. It is decreasing in both quantities. Empirically, we find this pass-trough to be about 0.5. Half of a change in the fraction of active investors translates into a reduction in demand elasticity. Given an estimate that the fraction of active investors has decreased by about 30% over the last 20 years, this yields a decrease in elasticities of 15%. This is a large change: for example, in the context of the information model we studied, it would lead to less informative and more volatile prices. Again, this prediction highlights that while competitive effects are strong, the stock market is far from the competitive ideal. In competitive markets, the pass-through is 0, in which case a rise of passive investing has no impact. On the other hand, without competitive effects, the pass-through is 1, leading to a 30% decrease in elasticity.

The model also provides an account of the actual evolution of the demand for stocks over the last 20 years. The entire cross-sectional distribution of stock-level elasticity has decreased during that period, by 35%. Interestingly, the model attributes about equally this drop to two investor-specific sources of change. First, the fraction of passive investors has increased — the extensive margin. Second, the investor-specific component of the elasticity of active investors has decreased — the intensive margin. This dimension is particularly interesting because developments in computing power and access to big data would have instead suggested that the most aggressive quantitative funds would have increased their elasticities on their own.[3] However, another aspect played an important role: active investors also increased their equilibrium elasticity in response to broad decrease in aggregate elasticities. In a counterfactual exercise in which we shut down the competitive responses, we find that elasticities would have decreased by 62%, while they would have barely moved with strong competition.

Taken together, our results highlight the importance of a finer approach to competition in financial markets. No, it is not the case that "financial markets are competitive" and that all shocks are fully absorbed by other investors. But also no, it does not mean that investors do not interact together at all. This framework is a first step towards quantifying the intensity of competition and its implications. Our estimates suggest that competition played an important role in shaping the response to the rise of passive investment. Competition is likely important for many other questions about investor demand. What happens when a large set of financial institutions must change their trading because of new regulation? What happens when some sophisticated specialized investors get in financial trouble?

Contribution to the existing literature. The idea of competition among investors has a long history in finance. Grossman and Stiglitz (1980) first formalize the notion of competition for information between investors and show it does not lead to informationally efficient markets.[4] Kyle (1989) highlights how market power also creates interaction among investors. These seminal contributions have led to a large theoretical literature pointing out rich ways in which investors react to each other and choose their trading strategy. In the context of the rise of passive investing Subrahmanyam (1991) is an early contribution highlighting liquidity concerns. More recent work includes Bond and García (2018), Malikov (2019), Lee (2020), Buss and Sundaresan (2020), and Kacperczyk, Nosal, and Sundaresan (2020). Farboodi and

[3]Farboodi and Veldkamp (2020) develop a theory of the effect of growth in financial data technology that upends common wisdom.

[4]Coles, Heath, and Ringgenberg (2020) show that an increase in passive investing does not affect price informativeness in this baseline model.

Veldkamp (2020) focus on the choice between information about fundamentals or about demand in the context of the rise in big data. However, with the exception of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), these theories are rarely confronted to portfolio data. Our new approach, summarizing competition through choices of demand elasticity, allows us to bring the theory to the data.

We contribute to a recent literature on estimating demand systems accounting for the large heterogeneity in portfolio holdings, started by Koijen and Yogo (2019). Koijen et al. (2021), Koijen and Yogo (2020), Koijen, Richmond, and Yogo (2020), and Jiang, Richmond, and Zhang (2020) also apply this approach. Balasubramaniam, Campbell, Ramadorai, and Ranish (2021) estimate a factor model of portfolio holdings. Dou, Kogan, and Wu (2020) study how mutual funds change their portfolios in response to common fund flows. Gabaix and Koijen (2020) estimate the aggregate demand for stocks. Our key innovation on that front is to incorporate strategic interactions between investors, a long-theorized feature we find to be quantitatively important.

More broadly our paper relates to a broader literature studying the relation between portfolio quantities and asset prices. De Long et al. (1990) argue that noise trader shocks can affect prices. These ideas have found applications across multiple asset classes: stocks (Shleifer (1986), Warther (1995)), government bonds (Vayanos and Vila (2021), Greenwood and Vayanos (2014), Haddad and Sraer (2020)), options (Gârleanu, Pedersen, and Poteshman (2009)), currency markets (Gabaix and Maggiori (2015), Greenwood et al. (2019), Gourinchas, Ray, and Vayanos (2019)), or corporate bonds (Haddad, Moreira, and Muir (2021)). While our estimates concentrate on the stock market, we bring to the forefront the importance of strategic interactions between investors, which likely also matter in other markets.

Finally our results provide new insights in the debate on the consequences of the long-term rise in passive investing. French (2008) and Stambaugh (2014) provide empirical evidence of a shift towards passive investing. Zooming in on portfolios, we uncover how passive investing is altering how all investors trade and therefore its equilibrium implications. Other work focuses on quasi-natural experiments around index or ETF inclusion such as Chang, Hong, and Liskovich (2014) or Ben-David, Franzoni, and Moussawi (2018). Sammon (2021) studies the response of stock prices around earnings announcements. Bai, Philippon, and Savov (2016), Dávila and Parlatore (2018), and Farboodi et al. (2021) document long term trends in price informativeness.

2 An Equilibrium Model of Financial Markets with Investor Competition

We present our framework for investor competition. The key idea is that there are two layers to an equilibrium in financial markets. First, the price is such that the sum of investor demands equals the supply of the assets. Second, investors compete with each other: they choose how aggressively they trade as a function of how others trade. This aggressiveness is measured by their demand elasticity. We introduce the two layers in turn, then highlight the implications for the rise of passive investing. Table 1 summarizes the model.

2.1 First layer: the asset price clears the market given demand curves

For the sake of simplicity, we focus on the case of one asset in supply S and a continuum of investors indexed by i. In an equilibrium each investor decides how much they buy as a function of the price P of the asset: a demand curve $D_i(P)$, which we can log linearize:

$$d_i = \underline{d}_i - \mathcal{E}_i \times p, \tag{1}$$

where lowercase letters represent log values. The elasticity of this demand curve, $\mathcal{E}_i$, determines how aggressive the investor is. An investor with $\mathcal{E}_i = 0$ does not react to changes in prices, while a trader with large $\mathcal{E}_i$ increases its position a lot when the asset is cheap. Other

Table 1. The 2-layer model of investor competition.

	Individual Decision	Equilibrium Condition
Demand	$d_i = \underline{d}_i - \mathcal{E}_i \times p$	$\int_i D_i(p) = S$
Elasticity	$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \times \mathcal{E}_{agg}$	$\int_i \mathcal{E}_i D_i / S = \mathcal{E}_{agg}$

aspects than the price can also affect the choice of positions. For example an investor could have a preference for environmental, social, and governance (ESG) investing. We pack them up all inside of the constant $\underline{d}_i$; the empirical analysis will be more rigorous about this.

These elasticities play an important role in the determination of equilibrium prices. The aggregate demand curve is $D_{agg}(P) = \int D_i(P)$, and the equilibrium price solves $D_{agg}(P^*) = S$. Aggregate demand has elasticity

$$\mathcal{E}_{agg} = \frac{\int \mathcal{E}_i D_i}{\int D_i}. \tag{2}$$

How strongly aggregate demand for the asset responds to prices is the (position-weighted) average of individual elasticities. This aggregate elasticity shapes the behavior of the equilibrium price. If investors are very aggressive, aggregate demand is perfectly elastic, $\mathcal{E}_{agg} \to \infty$, and prices are pinned down at a fixed level. In such a situation, changes in individual investor characteristics $d_{0,i}$ or in supply S do not affect the price. This is what people sometimes describe as "efficient markets": any deviation of the price from a fundamental value is immediately traded away by aggressive investors. On the other hand, when demand is more inelastic, small changes in the market structure can have a large effect on prices, because investors are unwilling to change their positions. More fleshed-out models such as the one we present in Section 3 show how the aggregate elasticity affects other equilibrium properties such as volatility or price informativeness.[5]

2.2 Second layer: investors set their demand elasticity in response to others

In standard price theory, the elasticity of demand reflects only an individual's preference for a good. In particular, it does not depend on the decisions of other market participants. When deciding how many apples to put in your shopping cart, it does not matter what other shoppers are doing beyond their effect on the price level. However, a dimension that is important in financial markets is missing from the standard theory: it matters why the price is moving. For example, asset prices convey information to investors, and depending on this information, one should be more or less aggressive in its response to price movements.

This relation adds a second layer to the equilibrium. Individual elasticities depend on the structure of price movements; this structure depends on the aggregate demand elasticity. But conversely, the aggregate demand elasticity is an average of individual elasticities. Formally, we can represent this feedback by endogenizing individual demand elasticities as a function of the aggregate demand elasticity:

$$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi\ \mathcal{E}_{agg}. \tag{3}$$

The parameter χ controls the strength of the response to the aggregate elasticity. $\underline{\mathcal{E}}_i$ is a baseline level of elasticity reflecting the investor's own preferences for the asset, for example shaped by her risk aversion, her beliefs about the payoffs. Together, the individual decision equation (2) and the aggregation condition of equation (3) pin down the equilibrium layer of elasticities.

We call χ the competition parameter. If $\chi = 0$, individual investors do not respond at all to the aggregate elasticity. This is a world where financial markets are not competitive: each

[5] See also Gabaix and Koijen (2020) for a discussion of the role of the elasticity of demand in financial markets.

investor follows a strategy that is independent of the actions of other investors. For example, under this view, if a group of sophisticated investors goes bankrupt, nobody else steps in to take advantage of the opportunities that are left untouched. The aggregate elasticity drops sharply. On the other hand if χ is large, financial markets are competitive. If the group of sophisticated investors goes away, other investors pick up the slack by trading more aggressively. This stabilizes the aggregate elasticity and makes the market insensitive to the composition of investors.

Formally, the parameter χ measures the extent of strategic substitutability in demand elasticities.[6] This substitutability is essential to go from individual actions to aggregate equilibrium outcomes, the focus of this paper. The framework does not microfound the decision problems leading to these policies. We make this choice so as to be flexible in measurement without giving up on the presence of strategic interactions. We would not want to convey the idea that theoretical foundations do not exist, quite the opposite. In many theories, demand elasticities are a key feature of investors' strategies and exhibit substitutability or complementarity.[7] Because elasticities are directly measureable in trading and portfolio data, we do not need to take an a priori stand on which theory is correct. Still it is useful to understand what different theories predict for χ.

Section 3 presents a model in the style of Grossman and Stiglitz (1980) where elasticity choices relate to information acquisition. This case provides a foundation for a positive χ. When traders are more informed, they trade more aggressively. This additional information reduces the value of information for others, who respond by acquiring less information and trading less aggressively. Appendix Section B.1 presents a related mechanism, absent information decisions. When traders are more aggressive, they "arbitrage" away more of the effect of noise trading, and prices are more informative. This implies less profits to be

[6]We consider strategic substitutes and complements in the sense of Bulow, Geanakoplos, and Klemperer (1985) and defined in chapter 4 of Veldkamp (2011).

[7]Technically, other aspects of investor decisions may be the source of substitutability (e.g. information acquisition or social interactions). However, because elasticities are directly related to these other decisions, the substitutability manifests itself in the demand elasticity.

made from responding to prices, hence leading to lower individual elasticity. More generally, we can motivate competition from investors' efforts in finding good trading opportunities, with the analogy of dollar bills on the floor. An individual's demand elasticity captures how aggressively they go after investment opportunities, how hard they look for dollar bills. The aggregate elasticity is the total effort of investors in looking for these dollar bills. Therefore the level of competition χ characterizes how much more intensely investors look for dollar bills when others are not.

One can also imagine situations where χ is negative. In the same way that a positive χ materializes substitution between investors' aggressiveness, a negative χ corresponds to complementarity. In Appendix Section B.2, we show that a model of market power in the style of Kyle (1989) yields negative value of χ.[8] This is a common pattern in models with liquidity considerations: when other investors bring more liquidity to the market by being more agressive, it enhances my ability to trade aggressively.[9] Social interactions where investors follow their peers, as in Hong, Kubik, and Stein (2004), could also lead to negative values of χ.

2.3 The effect of a rise in passive investing

We study the effect of a rise in passive investing in a simple version of this structure. For now, consider the following thought experiment. We start from an economy with homogenous investors who, in this initial equilibrium, each have elasticity $\mathcal{E}_i = \mathcal{E}_0$. The aggregate elasticity is therefore also $\mathcal{E}_0$. What happens when a fraction $1 - \alpha$ of these investors becomes passive, that is keep the same holdings, but reduce their elasticity to zero? The answer depends crucially on the level of competition χ.

The direct effect of this change is that now, only a fraction α of investors contribute to the aggregate elasticity. In the new equilibrium, $\mathcal{E}_{agg} = \alpha \mathcal{E}_i$ which decreases the aggregate

[8] We also show that the measure of price impact Kyle's λ is closely related to the inverse of aggregate elasticity.

[9] For example, such a situation also occurs in Vayanos and Wang (2007).

elasticity. In turn, this fraction of active investors do respond to the change in aggregate elasticity, $\Delta\mathcal{E}_i = -\chi\Delta\mathcal{E}_{agg}$. This response compensates the direct effect when $\chi > 0$. Each active investor responds again to the response of other active investors, until a new equilibrium is found.[10] The new aggregate elasticity is:

$$\mathcal{E}_{NEW} = \underbrace{\alpha\mathcal{E}_0}_{\text{direct effect}} + \underbrace{(1-\alpha)\mathcal{E}_0\frac{\alpha\chi}{1+\alpha\chi}}_{\text{competitive response}} \tag{4}$$

When financial markets are competitive, χ is large and $\mathcal{E}_{NEW} = \mathcal{E}_0$, the aggregate elasticity is unchanged. The drop in elasticity due to the new passive investors is exactly compensated by a greater elasticity of the remaining active investors. On the other hand when investors are insensitive to market conditions, χ close to zero, there is only the direct effect and the elasticity declines by a factor α.

Anticipating our quantitative exercise, we can put numbers in this formula. We estimate a level of competition χ of 1.7. Over the last 20 years, the fraction of active investors has decreased by 30%, so we set $\alpha = 70\%$. This implies that the initial elasticity is multiplied by a factor of $(1.7+1)/(1.7+1/(70\%)) = 0.86$. The rise of passive investing leads to a meaningful drop in elasticity of 14%. This is about half of the direct effect that would have led to a decrease of 30%. However, this is still much more than the 0 predicted by the competitive market ideal.

In Section 4, we fully specify our framework to account for the heterogeneity across investors and stocks, and estimate using holdings data. This allows us to revisit the question of the rise in passive investing in the context of a realistic quantitative model in Section 5.1. Before doing so, we present a foundation of our demand model based on information.

[10] Formally, we do not model this tâtonnement, but rather focus directly on equilibria.

3 An Information-based Foundation for Competition

While there are many justifications for competition and its implications for the choice of demand elasticity, we present a specific one based on information acquisition in financial markets. We write down a fully micro-founded theory in the spirit of Grossman and Stiglitz (1980) and Veldkamp (2011). This theory delivers four insights. First, demand elasticities are connected to the choice of information: when an investor decides to acquire more information, they trade more agressively. Second, individual information acquisition responds to aggregate information, hence individual elasticities respond to the aggregate elasticity. This provides a justification for the structure of equation (3). Third, we show how in this model, the ease of changing information decisions determines the competition parameter χ. Fourth, we highlight the importance of equilibrium elasticities for the behavior of prices: volatility, price informativeness, etc. All derivations are in Appendix A.

3.1 Setting

There is one period and one asset, and a continuum of agents indexed by $i \in I$, where I is a set of measure one. Each agent has CARA preferences with risk aversion ρ_i:

$$U_i = \mathbf{E}_i[-e^{-\rho_i W_i}], \tag{5}$$

and initial wealth $W_{0,i}$. The gross risk-free rate is 1, and the (random) asset payoff is f. The asset is in noisy supply $\bar{x} + x$ with $\bar{x}$ an exogenous fixed parameter and $x \sim \mathcal{N}(0, \sigma_x^2)$.

Initially, each agent is endowed with an independent signal μ_i of the fundamental f, distributed $\mu_i \sim \mathcal{N}(f, \sigma_i^2)$.[11] Each agent can acquire a private signal $\eta_i \sim \mathcal{N}(f, \sigma_{i,\eta}^2)$ at monetary cost $c_i(\sigma_{i,\eta}^{-2} + \sigma_i^{-2})$, with $c_i(.)$ a non-decreasing positive function.[12] That is, obtaining

[11] Following Veldkamp (2011), we assume agents start with a flat prior on f, hence their initial belief is $f \sim \mathcal{N}(\mu_i, \sigma_i^2)$.

[12] This parametrization is without loss of generality relative to a cost function that would only depend on the acquired signal $\sigma_{\eta,i}$.

more precise signals is more costly. The signal being private implies in particular that signal realizations are uncorrelated across agents conditional on the fundamental f.

We study rational expectations equilibria, and among those linear equilibria specifically. These are equilibria in which the price takes the form:

$$p = A + Bf + Cx. \tag{6}$$

An equilibrium is a set of coefficient (A, B, C), information choices $\sigma^2_{i,\eta}$, demand curves $D_i(p|\eta_i)$ such that: (i) each demand function and information choice maximizes expected utility, taking as given the price function; (ii) the market for the asset clears: $\bar{x} + x = \int D_i(p|\eta_i)di$.

Solving for the equilibrium. To solve the model, we proceed in two steps. First we solve for asset demand and the price given information decisions. Later on, we solve for the equilibrium information decision. These two steps reflect the two layers of the equilibrium of Section 2.

In equilibrium, an investor chooses how much of the asset to buy based on the signal and the price, as well as knowledge of the equilibrium price function. Given all this information, her posterior belief is that $f \sim \mathcal{N}(\hat{\mu}_i, \hat{\sigma}^2_i)$, with

$$\hat{\sigma}_i^{-2} = \sigma_i^{-2} + \sigma_{i\eta}^{-2} + \frac{B^2}{C^2}\sigma_x^{-2}, \tag{7}$$

$$\hat{\mu}_i = \hat{\sigma}_i^2 \left(\sigma_i^{-2}\mu_i + \sigma_{i\eta}^{-2}\eta_i + \frac{B^2}{C^2}\sigma_x^{-2}\frac{p-A}{B} \right). \tag{8}$$

In each of the sums, the first term is the initial information, the second term is the acquired signal, and the last one the information about fundamental conveyed by the price.

Given this belief, the individual posts the standard CARA demand:

$$q_i(p) = \frac{1}{\rho_i}\hat{\sigma}_i^{-2}\left(\hat{\mu}_i(p) - p\right), \tag{9}$$

where q_i is a number of shares. The market clearing condition is:

$$\int_I q_i(p)di = \bar{x} + x \tag{10}$$

Using a law of large number for $\hat{\mu}_i$, we solve for A, B, C and we find that:

$$A = -\bar{x}\left[\int_I \frac{1}{\rho_i}\hat{\sigma}_i^{-2}di\right]^{-1}, \quad B = 1, \quad \text{and } C = -\left[\int_I \frac{1}{\rho_i}\left(\sigma_i^{-2} + \sigma_{i\eta}^{-2}\right)di\right]^{-1}. \tag{11}$$

3.2 Equilibrium and Elasticities

We show how this model of information can be cast with investors' demand elasticities. We find that the equilibrium elasticities exhibit the two-layer property of equation (3), and that the information acquisition technology determines the level of competition in the economy.

What determines individual elasticities? The slope of the demand curve characterizes how agressively an investor changes their portfolio when the price moves. This slope is[13]

$$\mathcal{E}_i = -\frac{dq_i}{dp} = \frac{1}{\rho_i}\left(\sigma_i^{-2} + \sigma_{i\eta}^{-2}\right). \tag{12}$$

Two elements shape the agressivity of an investor's trading: their risk aversion and their information. If an investor is more risk averse, higher ρ_i, they trade less agressively. Increasing an asset position to take advantage of a good deal requires taking a larger amount of risk, something they are reluctant to do. The second term in equation (12), $\sigma_i^{-2} + \sigma_{i\eta}^{-2}$, is the precision of the investor's belief about f conditional on their private information: their prior and the acquired signal. If an investor is better informed, they trade more agressively, because they do not see it as that risky. Importantly, information is a *choice* in this model, and therefore we will show that it is not only the primitive characteristics of an investor

[13] For all of this section we study the slope of the demand curve, rather than the elasticity *stricto sensu*, which we still denote $\mathcal{E}_i$ in a small abuse of notation. This approach lends itself to the linearity of the CARA-Normal framework, but is less appealing for empirical applications.

which drive the elasticity, but also the structure of the market they trade in.

The aggregate elasticity shapes the behavior of the equilibrium price. As we have discussed in Section 2, the aggregation of individual demand curve shapes the behavior of prices. Equation (11) characterizes the price. In the theory, the price moves one-to-one with fundamental, $B = 1$, because beliefs average out to f. Still, the price is sensitive to noise trading, $C \neq 0$. The aggregate aggressivity of traders determines how much. Specifically the sensitivity of the price to noise trading is the inverse of the aggregate elasticity:

$$C = -\left(\int_I \mathcal{E}_i di\right)^{-1} = -\mathcal{E}_{agg}^{-1} \tag{13}$$

Taking these two results together we can write $p = A + f - \mathcal{E}_{agg}^{-1}\, x$. If investors are very aggressive, large $\mathcal{E}_{agg}$ and low C, and prices are pinned down at the fundamental f. The price does not respond to shocks to supply or demand which are immediately traded away. On the other hand, when demand is more inelastic, small shocks have a large effect on prices.

This property has important consequences for volatility and price informativeness. In equilibrium the asset return is $f - p = -A + \mathcal{E}_{agg}^{-1}\, x$, such that volatility is larger with more inelastic investors. With a large aggregate elasticity, the price reflect the fundamental: the absolute amount of information in price ($\mathrm{Var}(f|p)^{-1}$) is large. More subtly, the same result holds for the relative price informativeness $\mathcal{I}_i$ — how much does an investor learn from the price above and beyond their signals. We show in Appendix A.6 that

$$\mathcal{I}_i = \frac{\mathrm{Var}(f|\mu_i, \eta_i, p)^{-1}}{\mathrm{Var}(f|\mu_i, \eta_i)^{-1}} = 1 + \mathcal{E}_{agg}\frac{\mathcal{E}_{agg}}{\rho_i \mathcal{E}_i}\sigma_x^{-2}. \tag{14}$$

Holding an investor's characteristics constant, they learn more from prices in a more elastic market.

Individual elasticity responds to aggregate elasticity. This central role of the aggregate elasticity for the behavior of prices makes it relevant for individual information choices.

Let us show this result by deriving the optimal information choice. We show in Appendix A.3 that the utility gain from the signal is proportional to the precision gain about fundamental from observing the signal, including the information from prices. This reduces the optimal information choice to balancing this benefit with the monetary cost of the signal:

$$\max_{\sigma_{i\eta}^{-2}} \frac{1}{2} \log \left(\frac{\sigma_i^{-2} + \sigma_{i\eta}^{-2} + \mathcal{E}_{agg}^2 \sigma_x^{-2}}{\sigma_i^{-2} + \mathcal{E}_{agg}^2 \sigma_x^{-2}} \right) - \rho_i c_i(\sigma_i^{-2} + \sigma_{i\eta}^{-2}) \tag{15}$$

$$\iff \quad \max_{\mathcal{E}_i} \frac{1}{2} \log \left(\rho_i \mathcal{E}_i + \mathcal{E}_{agg}^2 \sigma_x^{-2} \right) - \rho_i c_i(\rho_i \mathcal{E}_i) \tag{16}$$

The total precision of the investor knowledge combines her own information with the one obtained from prices. These two pieces correspond to the investor's individual elasticity and the aggregate elasticity, respectively. The optimal choice of elasticity solving problem (16) therefore includes a response to the aggregate elasticity. This relation gives a foundation to the elasticity equation (3) in our simple model. Closing the model, individual elasticities must aggregate to the aggregate elasticity. The equilibrium of information decisions is equivalent to an equilibrium of elasticity.

The determinants of χ. This theory makes further predictions on the determinants of the competition χ. Approximately, we have $\chi = -\partial \mathcal{E}_i / \partial \mathcal{E}_{agg}$. Competition χ is always positive. When others trade more agressively, the price is more informative. The marginal value of an additional unit of information is lower. It is therefore optimal to acquire less information and trade less agressively, reducing my own elasticity.

The strength of the competitive response depends on how easy it is to adjust my information choice. In Appendix A.5, we show that χ is decreasing in a form of curvature of the information cost function, $c_i''/c_i'^2$. When it is relatively to change how much information investors seek, more competition takes place as they can respond more strongly to others. If information cannot be adjusted, there simply cannot be any competitive response.

The relation between $\mathcal{E}_i$ and $\mathcal{E}_{agg}$ is not linear in general. In Appendix A.3 we find a two-parameter family of simple cost functions under which this relation is exactly linear as in equation (3). Each of the two parameters maps to in closed-form to the value of competition χ and individual elasticity $\mathcal{E}_{0,i}$.

4 Estimating Investor Competition

In this section, we estimate the level of competition χ and investor demand elasticities in the context of the US stock market. First, we enrich our model to account for the heterogeneity of stocks and investors. Then, we design and implement a new identification strategy for demand estimation in presence of a two-layer equilibrium.

4.1 Quantitative Model

Individual decisions. Recall the model of individual decisions developed in Section 2:

$$d_i = \underline{d}_i - \mathcal{E}_i p, \tag{17}$$

$$\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi \mathcal{E}_{agg}. \tag{18}$$

We complete the specification of this model to account for the richness of the data while still being able to estimate competition χ.

In most empirical settings, including ours, agents can invest in many assets. Therefore, an empirical model must make sure that portfolio positions add up to total assets for each investor. In addition, it should also account for the portfolio aspect of financial decisions, that is some form of substitution across assets. Koijen and Yogo (2019) show that a logit framework satisfies both of these requirements. We denote each security by the index k, the total assets of an investor by A_i, and the portfolio share of investor i in security k by w_{ik}. Therefore $d_{ik} = log(w_{ik} A_i) - p_k$. The framework of Koijen and Yogo (2019) corresponds to specifying a log-linear model for relative portfolio shares w_{ik}/w_{i0} instead of the individual demand directly, with index 0 being the outside asset. We follow this approach. For each investor, we take as given total assets under management, A_i, and their investment universe, $\mathcal{K}_i$, that is the set of assets they can invest in.

Second, we need to specify the baseline levels of demand and elasticity $\underline{d}_i$ and $\underline{\mathcal{E}}_i$. We assume that each of those combines potentially distinct sets of asset characteristics using investor specific coefficients. This corresponds to expressing the baseline demand as $\underline{d}_{ik} = \underline{d}_{0i} + \underline{d}'_{1i} X_k^{(d)}$ and the baseline elasticity as $\underline{\mathcal{E}}_{ik} = \underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)}$, where the two vectors of characteristics are $X_k^{(d)}$ and $X_k^{(e)}$. Going back to the setting of Section 3, an interpretation of this assumption is that investors form priors on different assets based on their characteristics. We also account for asset-specific changes in demand by including a shock ϵ_{ik} in $\underline{d}_{ik}$. For example, ϵ_{ik} captures the private signal η and noise trading x of the model of Section 3.

Finally, we estimate the model using only the cross-section in each time-period. Thus, we allow all quantities and parameters of the model to depend on time. For ease of notation we drop the subscript t.

Putting it all together, our model of portfolio demand is:[14]

$$\log \frac{w_{ik}}{w_{i0t}} - p_k = \underline{d}_{0i} + \underline{d}'_{1i} X_k^{(d)} - \mathcal{E}_{ik}\ p_k + \epsilon_{ik}, \tag{19}$$

$$\mathcal{E}_{ik} = \underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi_t\ \mathcal{E}_{agg,k}. \tag{20}$$

Starting from the relative shares $\omega_{ik} = w_{ik}/w_{i0t}$, the actual shares can be obtained by

$$w_{ik} = \frac{\omega_{ik}}{1 + \sum_{k \in \mathcal{K}_i} \omega_{ik}}, \tag{21}$$

$$w_{i0} = \frac{1}{1 + \sum_{k \in \mathcal{K}_i} \omega_{ik}}. \tag{22}$$

Interestingly, the demand system of Koijen and Yogo (2019) is a special case of this frame-

[14]To match equation (19) with equation (1), recall that: $d_{ik} = \log \frac{A_i w_{ik}}{P_k}$. Then $\underline{d}_{ik} = \underline{d}_{0i} + \underline{d}'_{1i} X_k + log(A_i) + \log(w_{0i}) + \epsilon_{ik}$.

work. In their framework, demand elasticities are fixed structural parameters.[15] This corresponds to setting $\underline{\mathcal{E}}_{1i} = 0$ and $\chi = 0$. Therefore, their model implicitly assumes no competition at all. For example, when some investors are removed from the markets, the other ones do not step in with larger elasticities. This is the extreme opposite from the standard hypothesis of perfect competition that would be materialized by $\chi \to \infty$. Our framework lets us quantify how close or far reality is from these extremes.

Passive investors. In addition to this baseline type of investors, we include passive investors. By passive, we mean that these are investors whose demand does not respond to prices. Index funds are a specific example of such investors. Our notion is larger though, because it accommodates arbitrary fixed portfolios. To represent such behavior, we simply replace equation (20) by $\mathcal{E}_{ik} = 0$. Separating out these investors is important, not only because of their low level of elasticity, but also because they do not respond to aggregate trading conditions. We denote the set of active investors for asset k by $Active_k$ and the total assets under management of this group of investors as $|Active_k|$.

Equilibrium prices and elasticities. Going from individual decisions to an equilibrium relies on market clearing. Here, two equilibrium objects play a role in individual decisions: prices, p_k, and aggregate elasticities, $\mathcal{E}_{agg,k}$. Closing the model therefore requires two equilibrium conditions. The aggregate demand for assets is equal to the supply, and individual elasticities add up to aggregate elasticities:

$$\sum_i w_{ik} A_i = p_k, \ \forall k, \tag{23}$$

$$\sum_i \frac{w_{ik} A_i}{p_k} \mathcal{E}_{ik} = \mathcal{E}_{agg,k}, \ \forall k. \tag{24}$$

[15]Technically in the logit model the demand elasticity is $1-(1-w_{ik})(1-\mathcal{E}_{ik})$. For values of w_{ik} that are small relative to one, as in the data, this expression is close to $\mathcal{E}_{ik}$. Hence we refer to $\mathcal{E}_{ik}$ as the demand elasticity throughout the paper.

We normalize the number of shares available to 1 to obtain the market-clearing condition for assets, equation (23). Said otherwise, p_k denotes the log market capitalization.

In the appendix, we present the details of an algorithm to solve for the equilibrium efficiently; we rely on a quasi-Newton method.

4.2 Data

We estimate the model for the U.S. stock market. We use stock level data from CRSP, price, dividend, shares outstanding. We merge the CRSP file with COMPUSTAT for balance sheet information and compute stock level characteristics: book equity, profitability, and investment.

We obtain portfolio holdings data from the 13F filings to the SEC from 2000 to 2016. We use the dataset constructed by Backus, Conlon, and Sinkinson (2019, 2020) from the SEC Edgar website. The SEC requires that every institution with more than \$100m of assets under management file a quarterly report of their stock positions. We find that collectively the holdings reported in the 13F account for 80% of the total stock market capitalization.

We follow Koijen and Yogo (2019) to construct the final panel dataset.

4.3 Identification

To estimate the model described above we have to overcome three difficulties: (i) first, the classic problem of endogenous demand; (ii) a reflection problem induced by the interactions between investor; and (iii) how to implement the estimation given that one of the "regressors", the aggregate elasticity, is unknown.

4.3.1 Identifying demand

Combining equation (19) and (20), the model is similar to a regression equation:

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}'_{1it} X_k^{(d)} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} - \chi \; \mathcal{E}_{agg,k} \right) \; p_k + \epsilon_{ik}. \tag{25}$$

The parameters are $\underline{d}_{0i}$, $\underline{d}_{1i}$, $\underline{\mathcal{E}}_{0i}$, $\underline{\mathcal{E}}_{1i}$, and χ. There are two challenges to identify these parameters: (i) residual demand ϵ_{ik} is unobservable, (ii) aggregate elasticities $\mathcal{E}_{agg,k}$ is itself a function of $\underline{\mathcal{E}}_i$, and the dependent variable w_{ik} as expressed in the equilibrium condition (24). To solve these issues we make identification assumptions.

The simplest possible assumption takes residual demand as exogenous to all other variables to get the moment condition

$$\mathbf{E}\left[\epsilon_{ik}|X_k^{(d)}, X_k^{(e)}, p_k, \mathcal{E}_{agg,k}\right] = 0. \tag{26}$$

Thus we could estimate (25) using ordinary least squares. The independence of ϵ_{ik} from X_k is naturally motivated by taking the supply of assets as exogenous, as in endowment economies (Lucas, 1978). Furthermore, the independence from p_k and $\mathcal{E}_{agg,k}$ relies on the logic that residual demands do not matter for equilibrium outcomes because they "cancel out" in the aggregate. This rules out both the presence of non-atomistic investors and correlated demand shocks — see the equilibrium conditions in equations (23) and (24). Both of these last assumptions are not likely to hold for institutional investors. Therefore we relax these assumptions and propose an alternative identification strategy.

We follow Koijen and Yogo (2019) and assume that the variation in total assets and the investment universe is exogenous to the residual demand. They argue that the investment universe is often determined by mandates, which are predetermined rules on which assets can be held. Similarly assets under management are also predetermined.

Building on this we construct instruments for equilibrium outcomes p_k and $\mathcal{E}_{agg,k}$. The instrument for the price of asset k is

$$\hat{p}_{k,i} = \log\left(\sum_{j\neq i} A_j \frac{\mathbf{1}_{k\in\mathcal{K}_j}}{|\mathcal{K}_j|}\right), \tag{27}$$

where $\mathbf{1}_{k\in\mathcal{K}_j}$ is an indicator variable of when stock k is in investor j investment universe. This instrument corresponds to how much money would flow to stock k if all investors other than

i had an equal-weighted portfolio. Variation in the instrument comes from variation across investors' investment universe. For example, a stock with a large investors has more money flowing towards it. Given our assumption of downward-sloping demand for stocks, a larger exogenous demand generates higher prices that are uncorrelated with residual demand.

Our setting include the additional equilibrium variable $\mathcal{E}_{agg,k}$, for which we develop a new instrument:

$$\hat{\mathcal{E}}_{agg,k} = \frac{1}{1+\chi|Active_k|} \frac{\sum_{j\in Active_k} A_j/|\mathcal{K}_j| \cdot \mathbf{1}_{k\in\mathcal{K}_j} \cdot \underline{\mathcal{E}}_{jk}}{\sum_{j\in Active_k} A_j/|\mathcal{K}_j| \cdot \mathbf{1}_{k\in\mathcal{K}_j}} \tag{28}$$

This instrument is the solution to the elasticity equilibrium defined by equations (20) and (24), where we have replaced the endogenous weights w_{ik} with counterfactual weights under the assumption that each investor holds an equal-weighted portfolio.[16] As a robustness check, we also consider weights proportional to book equity. Here again the variation from the instrument comes from variation across investors' investment universe. We are interested in the response to the interaction of aggregate elasticity with the price (see equation (25)). To isolate this interaction from linear effects, we also include a linear control for the instrument of aggregate elasticity, that is as part of $X_k^{(d)}$.

The two instruments allow us to weaken the moment condition (26) to:

$$\mathbf{E}\left[\epsilon_{ik}|X_k^{(d)}, X_k^{(e)}, \hat{p}_k, \hat{\mathcal{E}}_{agg,k}\right] = 0 \tag{29}$$

The instrument for the aggregate elasticity depends on the model parameters ($\underline{\mathcal{E}}_{0i}$ and $\underline{\mathcal{E}}_{1i}$). This is not an issue for identification as parameters are by definition not endogenous.

[16]Our instrument for aggregate elasticity is the solution to the following problem:

$$\hat{\mathcal{E}}_{ik} = \underline{\mathcal{E}}_{ik} - \chi\, \hat{\mathcal{E}}_{agg,k}; \qquad \sum_j \hat{w}_{jk} A_j/p_k \hat{\mathcal{E}}_{jk} = \hat{\mathcal{E}}_{agg,k},$$

where the counterfactual weights $\hat{w}_{jk}$ are defined as:

$$\hat{w}_{jk} = \frac{A_j/|\mathcal{K}_j|\mathbf{1}_{k\in\mathcal{K}_j}}{\sum_i A_i/|\mathcal{K}_i|\mathbf{1}_{k\in\mathcal{K}_i}}.$$

However this precludes us from using standard methods such as two-stage least squares to estimate the model. Appendix Section C.1 lists the unconditional moments derived from condition (29) that we use for estimation. In Section 4.3.3, we detail our numerical procedure for estimating the model.

Relevance condition. To evaluate the strength of our instruments, we run what would be a first-stage regression in a standard two-stage least square estimation. First, we regress the price onto the instrument and the other characteristics for each manager. For each date, we compute the first and the fifth percentile of the (Kleibergen and Paap, 2006) F-statistics across managers. Figure 1 reports the histogram of these percentiles across all dates. At least 95% of the F-statistics in any given date are above 18 (panel A); furthermore, for all but one date, 99% of the F-statistics are above 9 (panel B). We also confirm the relevance of the elasticity instrument. In the panel, we regress the product of the price interacted with the aggregate elasticity onto their instrumented version and the other characteristics. We represent the histogram of the F-statistic of this regression for each date in panel C; the F-statistic is always above 10.[17]

4.3.2 The reflection problem.

While our instruments provide us with as many moment conditions as parameters, we discuss how the estimation can disentangle the individual component of elasticity from different competitive effects. Individual investor elasticities $\mathcal{E}_i$ depend on an investor specific term, $\underline{\mathcal{E}}_i$ and on the aggregate elasticity $\mathcal{E}_{agg}$ as:

$$\mathcal{E}_{ik} = \underline{\mathcal{E}}_{ik} - \chi \mathcal{E}_{agg,k}. \tag{30}$$

We need to disentangle whether investors are elastic because of their own characteristics or in response to other investors on the market. For example, if we see in a market that all

[17] Appendix Figure IA.4 reports the results for the book-equity weighted instrument.

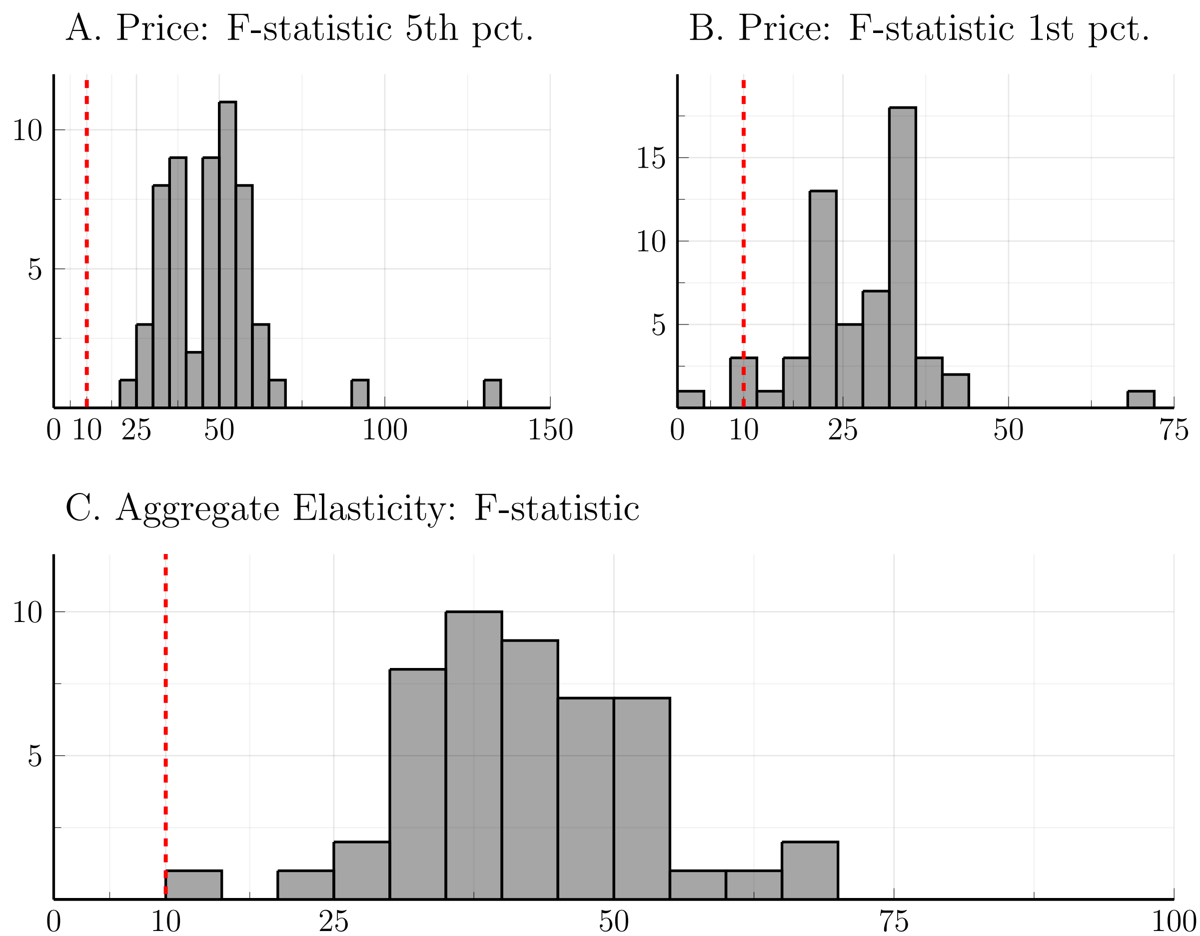

Figure 1. Relevance condition for the price and elasticity instruments. Figure 1 shows the F-statistic of the first-stage for the price and aggregate elasticity variables. For the price, We estimate the F-statistic (Kleibergen-Paap) at the manager level for each year. We summarize these statistics at every date with the 5th percentile (Panel A) and the 1st percentile (Panel B). Panel A shows that 95% of all F-statistics are above 10, and Panel B shows that 99% of all F-statistics across all years but one are above 9. For the aggregate elasticity instrument we regress the elasticity interacted with the price onto their instrumented version and report the F-statistic for each date.

investors behave in a very elastic manner, it could be that each of them if fundamentally very elastic, high $\underline{\mathcal{E}}_i$. But it could also be the consequence of a strong positive feedback where $\chi < 0$. This identification problem is the reflection problem (Manski, 1993).

Two features of our model let us solve the reflection problem. First, there is variation in investor composition across stocks, $\mathcal{K}_i$. Second, we assume that the investor specific

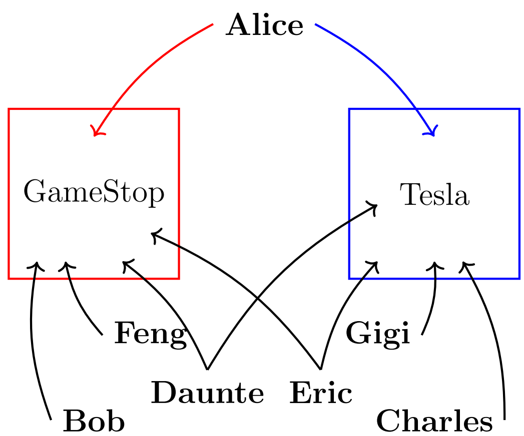

Figure 2. Illustration of Identification Strategy.

component depends on observable asset characteristics, $\underline{\mathcal{E}}_{ik} = \underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}_{1i}' X_k$. To measure the effect of competition, the ideal experiment would be to compare the behavior of the same investor for the same stock with variation in the characteristics of the other investors.

With the second assumption, two stocks with the same characteristics X_k, will elicit the same baseline elasticity $\underline{\mathcal{E}}_i$ for the same investor. In addition because the coefficients on stock characteristics is investor specific, we focus on variation within the same investor across different stocks. Finally to estimate χ, we need variation in $\mathcal{E}_{agg,k}$ across stocks. The different investment universe for different investors guarantees such a source of variation. Figure 2 illustrates this idea: we need to compare how Alice trades differently when facing different groups of other investors, such as for GameStop and Tesla. Last, we need to ensure that the system of equations for all investors and stocks given by (30) and the equilibrium condition for aggregate elasticity (24) has a unique solution. In our example, to estimate Alice's behavior, we also simultaneously need to figure out the elasticity for Bob, Charles, Daunte, etc. The following theorem formalizes the intuition behind the needed identifying variation and proves uniqueness.

Theorem 1. *A decomposition of demand elasticities $\{\mathcal{E}_{ik}\}_{i,k}$ into individual elasticities $\{\underline{\mathcal{E}}_i\}_i$ and the competition parameter χ is unique if:*

(a) The graph $\mathcal{G}$ of investor-stock connections is connected.

(b) *Position-weighted averages of demand elasticities are not constant across stocks: there exists k and k' such that $\sum_{i \in I_k} w_{ik}/p_k A_i \underline{\mathcal{E}}_i \neq \sum_{i \in I_{k'}} w_{ik'}/p_{k'} A_i \underline{\mathcal{E}}_i$.*

We derive and discuss this theorem in Appendix C.2. In particular, we explain that the two conditions for the result to apply are satisfied in our setting.

4.3.3 Implementation

Last, we need to implement the estimation free of the identification issues mentioned above. We cannot estimate (25) using standard methods. We must jointly estimate the competition parameter χ and the aggregate elasticities $\mathcal{E}_{agg,k}$, both parts of the two-layer equilibrium. We find the pair as a fixed point. Starting from a value for the competition parameter χ, we estimate individual level regressions which yield estimates of investor-level elasticities $\mathcal{E}_{ik}$, and of aggregate elasticities $\mathcal{E}_{agg,k}$. We iterate these regressions to reduce the distance between these estimates which are linked through the aggregation of elasticities (see equation (24)).

After iterating over individual regressions, we vary χ according to a Newton scheme and reestimate the individual regressions to minimize the distance between individual and aggregate elasticities. We iterate this last step until convergence. Appendix Section C.3 details this method, which reduces computation time for a given quarter from virtually infinite using naïve methods to about 5 minutes.

4.4 Estimates

We estimate the model for each quarter from 2000Q2 to 2016Q4. Recall that our identification comes from the cross-section, such that for each time period the model is estimated independently.

Competition χ. The average value of the competition parameter is $\chi = 1.7$. We represent an histogram of the estimates of the competition parameter χ for each quarter in Figure 3. The estimates show little variation around their mean. There is only one estimate slightly

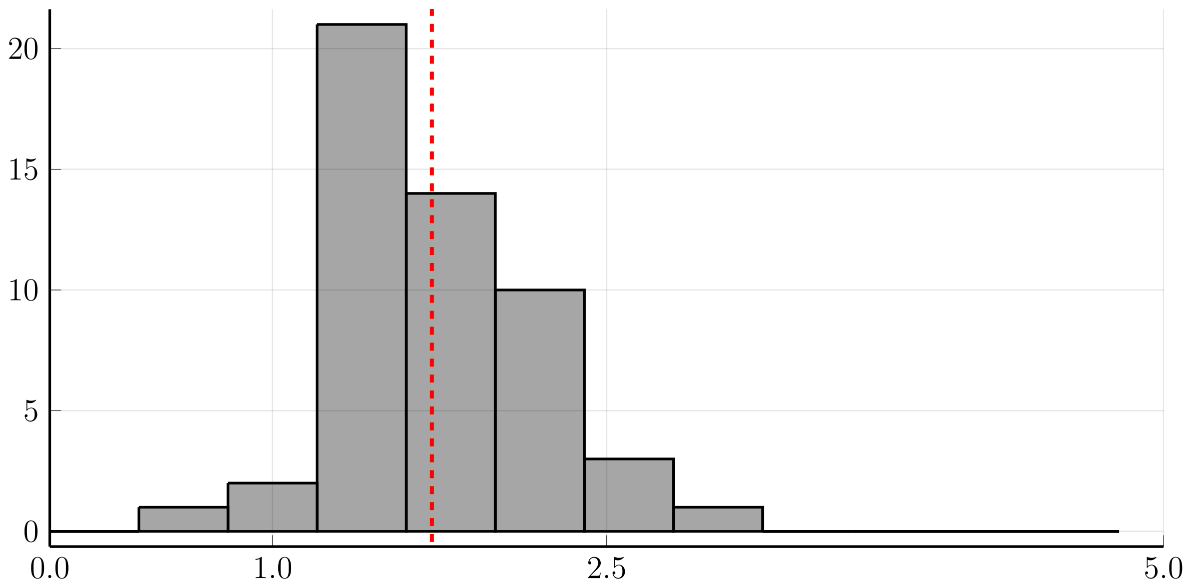

Figure 3. Distribution of the estimates of competition χ across dates. Figure 3 presents an histogram of our estimates of the competition parameter χ for each date between 2000 and 2016. The average estimate over the time-period is $\chi = 1.7$ (dashed redline).

below 1 and only three estimates above slightly 2.5. Further we confirm the estimates are stable over time; Appendix Figure IA.2 reports the time-series of our estimates.

Recall that the perfect competition benchmark corresponds to $\chi \to +\infty$ and the no-competition benchmark to $\chi = 0$. If aggregate elasticity increases by 1, then an atomistic investor would decrease their elasticity by 1.7. This response is substantial at the individual level: competition is present. However, it does not point to an equilibrium behavior in line with perfect competition either. For example our simple calculation in equation (4) shows that we need large values of χ for strong equilibrium effects. Making 50% of investors passive, a value of χ of at least 18 is necessary to compensate 90% of the drop in aggregate elasticity. This is an order of magnitude bigger than our mean estimate of 1.7, and actually than all of our estimates. We investigate the implications of the competition parameter χ for the impact of the rise of passive investing in Section 5.

We also estimate the model without instrumenting for aggregate elasticity. In this case we find an average value of χ of 0.4. This low estimate suggests that it is important to account for the endogeneity of elasticities — because they depend on actual portfolio weights which themselves depend on residual demand. However, even these biased estimates are consistent with a non-negligible individual response, but far from perfect competition. In addition, we confirm the robustness of our conclusion to the choice of the instrument with an alternative pseudo-elasticity using book equity as portfolio weights. We find similar results to our estimates in this case. We report this estimate in Appendix Figure IA.3.

Stock-level Elasticities. The model also delivers estimates of aggregate elasticity, $\mathcal{E}_{agg,k}$, for each stock. Figure 4 represents these elasticities for the date of 2011Q3 as a function of stock market capitalization. Each green dot corresponds to the elasticity estimate of one stock in our model for that date. We compare our estimates to a model where individual level elasticities are fixed, where $\underline{\mathcal{E}}_{1,i} = 0$ and $\chi = 0$. These estimates are represented with red dots in Figure 4.

There is substantial cross-sectional variation in elasticities, lending credence to our ability to identify the competition parameter χ. In both sets of estimates, the demand curve for individual stock is inelastic with average values around 0.3. This magnitude is far from the typical asset-pricing benchmark of perfectly horizontal demand curves with infinite elasticity. However, it is consistent with other empirical estimates, in particular based on portfolio data; see for example the discussion in Chang, Hong, and Liskovich (2014) and Koijen and Yogo (2019).

Figure 4 demonstrates a few ways in which accounting for the endogeneity of demand elasticities is important. First, the full model estimates exhibit less variation than the model with constant elasticities. With constant individual elasticities, variations in investor composition directly translates in variation in aggregate elasticities. However, with positive levels of competition χ, investors react to each other and soften those variations. For example, if an active investor with high-elasticity takes positions in a stock, other investors respond by trading less aggressively. Thus, stocks become more similar to each other.

Second, the full model exhibits a stronger negative relation between the size of a stock and

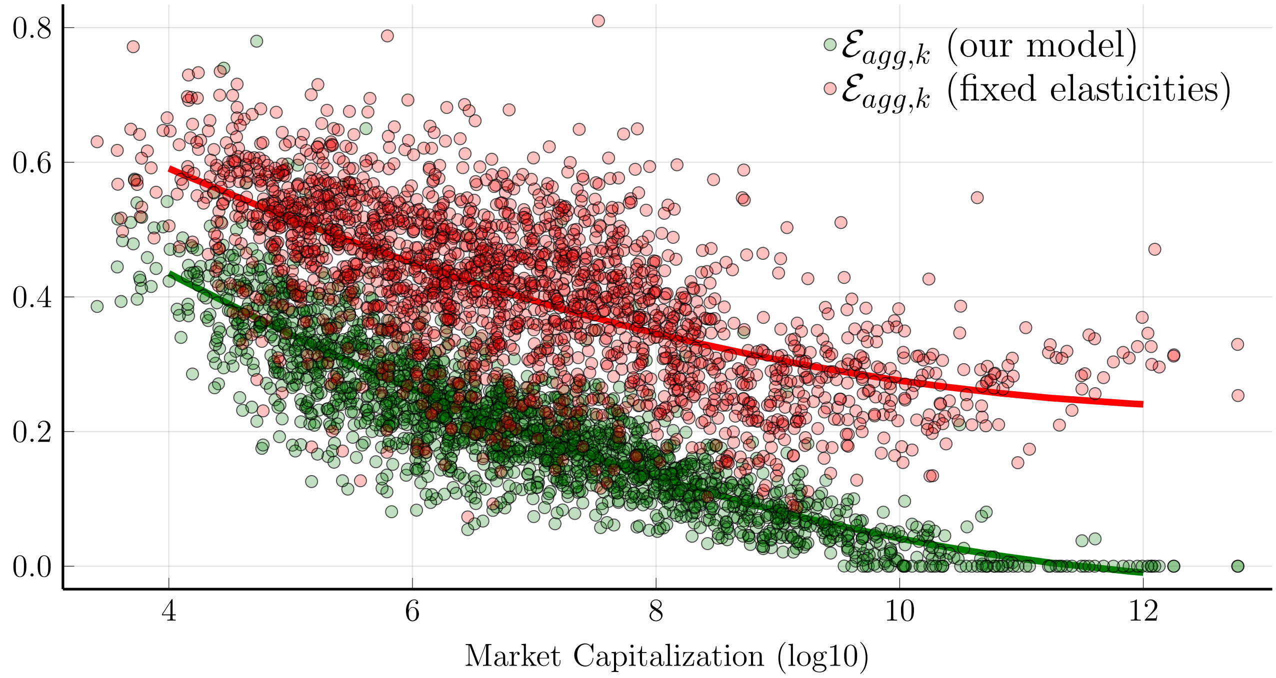

Figure 4. Aggregate elasticity at the stock level: $\mathcal{E}_{agg,k}$. Figure 4 represents estimates of the aggregate elasticity $\mathcal{E}_{agg,k}$, where each stock is represented by a circle, as a function of their market capitalization (in logarithm) for the date 2011Q3. Green circles are our estimates, while red circles correspond to a model where elasticities are fixed.

its elasticity. Koijen and Yogo (2019) point out that large stocks tend to have more inelastic investors overall. Once we allow individual elasticities to respond to stock characteristics and the aggregate elasticity, the data reveals an additional source for this relation: the same investor behaves more inelastically for large stocks than small stocks. This additional source of variation within investor rather than across investors leads to a steeper relation between size and elasticity. We estimate a linear relation between size and elasticity at the investor-level for computational tractability; this linearity leads to extremely low values of elasticity for the very largest stocks.

Table IA.1 shows that these conclusions hold not only for this specific date, but across our sample. We report the distribution across dates of various statistics of the cross-section of $\mathcal{E}_{agg}$. In particular, we confirm that our estimates have less variation in elasticity across stocks (Panel B), by about 50%, and a steeper relation between elasticity and stock size (Panel C), by about 25%.

Table 2.
Properties of Aggregate Elasticity $\mathcal{E}_{agg}$

	Panel A: Statistics of average elasticity across stocks			
	Average	25th pct.	Median	75th pct.
Elasticity $\mathcal{E}_{agg}$	0.193	0.159	0.181	0.215
Fixed elasticity	0.492	0.467	0.486	0.503
	Panel B: Statistics of in the cross-section of the elasticity within dates			
	Average	25th pct.	Median	75th pct.
Elasticity $\mathcal{E}_{agg}$	0.107	0.0905	0.103	0.115
Fixed elasticity	0.156	0.135	0.144	0.179
	Panel C: Regression coefficient (by dates) of elasticity on size			
	Average	25th pct.	Median	75th pct.
Elasticity $\mathcal{E}_{agg}$	−0.0569	−0.0603	−0.053	−0.0466
Fixed elasticity	−0.0432	−0.0475	−0.0442	−0.0412

Table IA.1 presents statistics of the aggregate elasticity $\mathcal{E}_{agg}(k,t)$. We estimate the elasticity in our model accounting for competition χ and without competition as in Koijen and Yogo (2019) (denoted by fixed elasticity). Panel A has summary statistics of the average elasticity by date. Panel B has summary statistics of the cross-sectional standard deviation by date. Panel C has summary statistics of the coefficient β_t of the regression $\mathcal{E}_{agg}(k,t) = \alpha_t + \beta_t size_{k,t} + \varepsilon_{k,t}$. The sample period is 2000 to 2016.

The negative relation between size and elasticity might appear surprising given the evidence on prices suggesting that large stocks are more informationally efficient.[18] However there are good reasons to think that institutions are more reluctant to change their positions for large stocks than for small stocks. Mechanically, the largest stocks occupy a larger share of portfolios. As of July 2021, the five largest corporations in the U.S. stock market account for about 18% of total market capitalization.[19] As a consequence, a large change in portfolio weight would have a large effect on an institution's portfolio return. Many institutions are

[18] See Lo and MacKinlay (1990), Jegadeesh and Titman (1993), Lakonishopk, Shleifer, and Vishny (1994), and Hong, Lim, and Stein (2000).

[19] The total market capitalization of Apple, Microsoft, Amazon, Alphabet (Google), and Facebook amount to \$8.8tn for total U.S. market capitalization of \$49tn.

either benchmarked to the index or have hard dollar limits on how much they can trade a given stock, and hence they would be unwilling to take on such large changes. As an illustration, Figure 5 decomposes trading activity—the sum of squared relative change in portfolio position—across percentiles of portfolio weights; Appendix Section D.1 details this calculation. There is much less trading activity for the larger portfolio positions: the top 50% of portfolio positions only account for 9% of trading activity. As such, the interpretation of our results is not so much that large stocks experience more mispricing but rather that high investor elasticity cannot be the explanation for the evidence on their returns.[20]

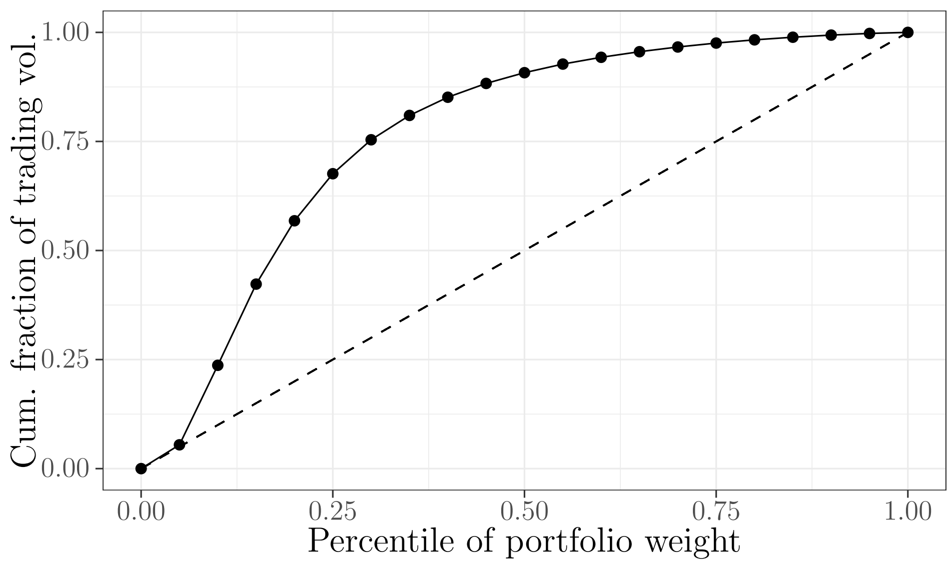

Figure 5. Trading activity across portfolio positions. Figure 5 presents the cumulative share of trading activity (defined in equation (IA.97)) by quantiles of investor portfolio weights.

5 Implications

[20]In the model of Section 3, both elasticity and the quantity of noise trading determine price informativeness. Farboodi et al. (2021) use a richer structural model to decompose informativeness into data, growth, and volatility.

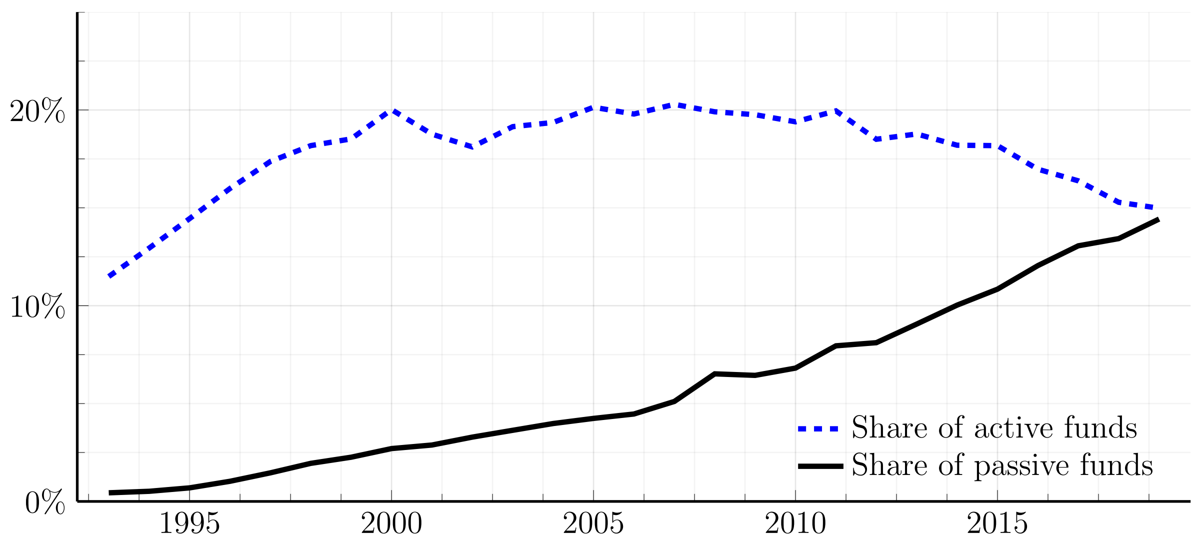

Figure 6. Share of Passive and Active Funds. Figure 6 shows the share of domestic mutual funds and ETFs as a fraction of the US stock market capitalization for passive funds (black solid line) and active funds (blue dashed line); Source ICI (2020).

5.1 The Rise of Passive Investing

The last 20 years have seen a large increase in passive investing, a fact documented in French (2008). More recently Stambaugh (2014) shows that both the fraction of mutual funds that are actively managed (at the extensive margin) and the active share of the portfolio of active equity mutual funds (at the intensive margin) have declined. We update and confirm these trends in Figure 6. The share of passive funds of the US stock market has grown from nearly zero at the beginning of the 1990s to more than 15% in 2019. Concurrently, the share of active funds topped out at the end of the 1990s and has declined from 20% to 15% from 2000 to 2019.[21] Our model takes a more comprehensive view of who are the passive investors, not restricting ourselves to mutual funds. With this approach we find that the share of passive strategies has grown by 20 percentage points over the last 20 years (see Appendix Figure IA.6).

Has the shift to passive portfolios impacted the behavior of prices? Understanding how

[21]We report the dollar numbers in Figure IA.5. Net assets of passive funds has grown from virtually zero to 5.4$tn in 2019, whereas the net assets of active funds only increased from 600$bn in 1993 to 5.5$tn in 2019.

investors react to changes in the behavior of other investors is crucial to answer this question. In the standard view of competitive markets, when some investors stop looking for profitable trading opportunities, some other investors step in to replace them; prices do not change. In contrast, if investors do not respond to others, the demand for stock becomes more inelastic wich strongly affects the behavior of prices. For example in the theory of Section 3, more inelastic demand leads to prices that are more volatile and less informative. Our model, and in particular the parameter χ, accounts for the strength of this reaction. We use the estimated parameters to quantify the impact of the rise in passive investing on aggregate demand elasticities.

Starting with the demand system from Section 4, we impose an exogenous change in the fraction of active investors and compute the new equilibrium elasticities. Of course the rise of passive investing is not a purely exogenous phenomemon. However, most plausible explanations of this phenomemon are independent from the rest of the demand system. For example, the development of financial technology made it cheaper to pursue passive strategies: fees on passive funds have dropped dramatically and ETFs became available. Or, one subset of investors, maybe after listening to finance professors, realized they were making mistakes when pursuing active strategies. We show in Appendix **??** that such shocks are equivalent to exogenous change in the fraction of passive investors in the micro-founded model of Section 3.

Computing the effect of the rise of passive investing corresponds to the calculation of equation (4), accounting for heterogeneous investors. Combining the individual demand elasticity $\mathcal{E}_{ik}$ in equation (20) with the equilibrium condition of (24), we have

$$\mathcal{E}_{agg,k} = \sum_{i \in Active_k} \frac{w_{ik} A_i}{\sum_{j \in Active_k} w_{jk} A_j} \cdot \underline{\mathcal{E}}_{ik} \times |Active_k| \times \frac{1}{1 + \chi \, |Active_k|}. \tag{31}$$

The aggregate elasticity is the product of three terms: (i) the average baseline elasticity among active investors, weighted by their respective positions; (ii) the fraction of the asset held by active investors, $|Active_k|$; and (iii) an adjustment for the response of active investors to the aggregate elasticity, which depends on χ.

From this expression we obtain the effect of a change in the fraction of active investing. If we change $|Active_k|$ while holding everything else constant corresponds to the assumption that the set of active investors that become passive are a representative sample of the active population. This leads to a simple formula:

$$\frac{d \log \mathcal{E}_{agg}}{d \log |Active_k|} = \frac{1}{1 + \chi \ |Active_k|}. \tag{32}$$

The pass-through from a rise in active investment to aggregate elasticity is determined by two numbers: the competition parameter χ and the fraction of active investors. In competitive financial markets, when χ is large, aggregate elasticity does not respond to shift in passive investing, the pass-through is zero. At the opposite end, when $\chi = 0$ such that investors do not respond to market conditions, the pass-through is 100%. An increase in the fraction of passive investors translates into a one-to-one decrease in aggregate demand elasticity. Further, because only active investors change their elasticities in response to others (passive investors always have an elasticity of zero), starting with a larger fraction of active investors leads to a smaller pass-through.

We can readily compute the pass-through: it solely depends on two observable quantities, χ and $|Active_k|$. In Section 4, we estimated the competition parameter and found that $\chi = 1.7$. Recall we measure the total quantity of passive investors as investors with an elasticity of zero in the Koijen-Yogo demand system. Not surprisingly, we find a trend down from 78% in 2000 to 58% in 2016. Taking the average across dates for the share of active investors, 65%, and for the competition parameter, $\chi = 1.7$, we find a value of the pass-through of[22]

$$\frac{1}{1 + \chi \ |Active_k|} = \frac{1}{1 + 1.7 \times 0.65} = 47.5\%. \tag{33}$$

[22] When the share of active investors is at 78% as in 2000 the pass-through is 43%, while when it is towards its lowest value of 58% at the end of the sample it is 50%.

This implies that competition is strong enough to compensate about half of the direct effect of a rise in passive investing. It is still far from the full cancellation of the competitive market view.

We multiply this pass-through by the rise in the proportion of passive investing to obtain the total effect on elasticity. We consider different takes for the size of the exogenous change. First we use our comprehensive measure of passive investing. The decline from 78% to 58% corresponds to a 30% drop, leading to elasticities lowered by $47.5\% \times 30\% = 14.25\%$. Second we look at a more narrow measure of the rise in passive investing centered around the assets under management of passive mutual funds and ETFs. Their fraction of total market capitalization has increased by 15 percentage points in the last 30 years. Starting from a baseline of 78% of active investors, this represents a 21% drop in the total fraction of active investors. With our pass-through of 0.47, this reduces elasticities by 10%.

5.2 Decomposing the evolution of the demand for stocks

In the previous exercise, we isolated the causal effect of a change in passive investing on equilibrium demand elasticities. Next, we propose a positive account of the data: we decompose the actual changes in elasticity over the last twenty years in light of our model.

5.2.1 The downward trend in aggregate elasticity

Figure 7 presents the time series of the distribution of equilibrium elasticities across stocks. For each date, we compute quantiles of the cross-section of aggregate elasticities, $\mathcal{E}_{agg,k}$. We find a striking downward trend in equilibrium elasticities across the whole distribution of stocks. The average elasticity (bold solid line) goes from 0.23 to 0.16, a 35% drop. The tails of the distribution also decrease. The 90th percentile (upper dashed line) drops from 0.38 to 0.27 (a 33% decrease). The 10th percentile (lower dashed line) also drops from 0.085 to 0.05. The downward trend in equilibrium elasticities affects the whole distribution of stocks. We further our understanding of what is behind the decline in the next section through a simple decomposition.

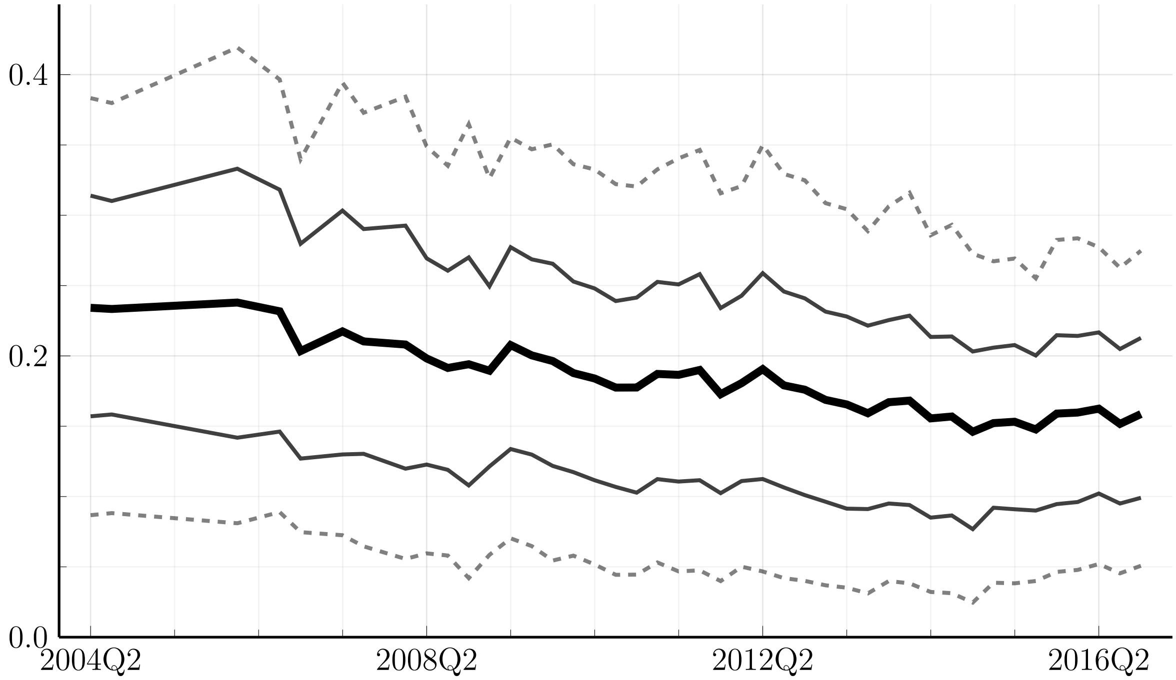

Figure 7. Distribution of aggregate elasticity across stocks.
Figure 7 traces out the distribution of aggregate elasticity $\mathcal{E}_{agg,k}$ over time. The bold line represents the average elasticity across stocks for each year. The solid lines represents the 25th and 75th percentile and the dashed lines the 10th and 90th percentile.

5.2.2 Sources of change in elasticity

In Section 4, we estimated the demand elasticities for each investor-stock in each quarter from 2000 to 2016. While our identification strategy is purely cross-sectional, we can use the time-series dimension of our estimates as a description of the evolution of the demand for stocks over time. To make parameters such as the investor-specific demand elasticity $\underline{\mathcal{E}}_i$ comparable across periods, we reestimate the model under the assumption that the competition parameter is constant over time. We set χ equal to its average value of 1.7. This is a small departure from the original estimation since the estimates are not very dispersed, and exhibit no trend in the time series.

We decompose changes in elasticity from year to year into three components using equation (31). We denote by $\langle \underline{\mathcal{E}}_{ik} \rangle$ the position-weighted average of the individual-specific component of the elasticity of active investors, $\underline{\mathcal{E}}_{ik}$; this corresponds to the first term in equation (31). We derive the effect of a change in investor composition,

$$\underbrace{d\mathcal{E}_{agg,k}}_{\text{Change in aggregate elasticity}} = \underbrace{|Active_k| \cdot d\langle \underline{\mathcal{E}}_{ik} \rangle}_{\text{Individual elasticity of active investors}} + \underbrace{\langle \underline{\mathcal{E}}_{ik} \rangle \cdot d|Active_k|}_{\text{Share of active investors}} + \\ + \underbrace{\left(\underbrace{\frac{-\chi|Active_k|}{1+\chi|Active_k|} \cdot d(\langle \underline{\mathcal{E}}_{ik} \rangle |Active_k|)}_{\text{Response to direct changes}} + \underbrace{\frac{-|Active_k|}{(1+\chi|Active_k|)^2} \langle \underline{\mathcal{E}}_{ik} \rangle \cdot d|Active_k|}_{\text{Fraction responding}} \right)}_{\text{Competitive response}} . \tag{34}$$

The first component corresponds to the average individual-level component elasticity of active investors; how their own characteristics contribute to the elasticity. The second component accounts for changes in the share of active investors over time and their ultimate effect on the elasticities. These forces correspond respectively to the intensive and extensive margin of individual elasticities. The last component corresponds to the equilibrium forces and are largely shaped by the parameter χ. It has two parts. First, the equilibrium response itself wich dampens the total effect of both the extensive and intensive margins ($d(\langle \underline{\mathcal{E}}_{ik} \rangle Active_k)$). When competition is low and $\chi = 0$, this term is zero and there is no equilibrium response; when competition is high and $\chi \to +\infty$, the equilibrium coefficient goes to -1 and there is full compensation of the direct effect on aggregate elasticity. The second part represents the change of the equilibrium effect itself when the number of active investors changes; only active investors' demands respond to others and the more there are, the larger the equilibrium response will be.

We present the three terms of this decomposition over time in Figure 8 and we summarize the total effects in Table 3. Over time we estimate that the aggregate stock-level elasticity has decreased by 35% on average. Consistent with the importance of the rise in passive investing discussed in Section 5.1, we find that the direct effect of the decrease in the fraction

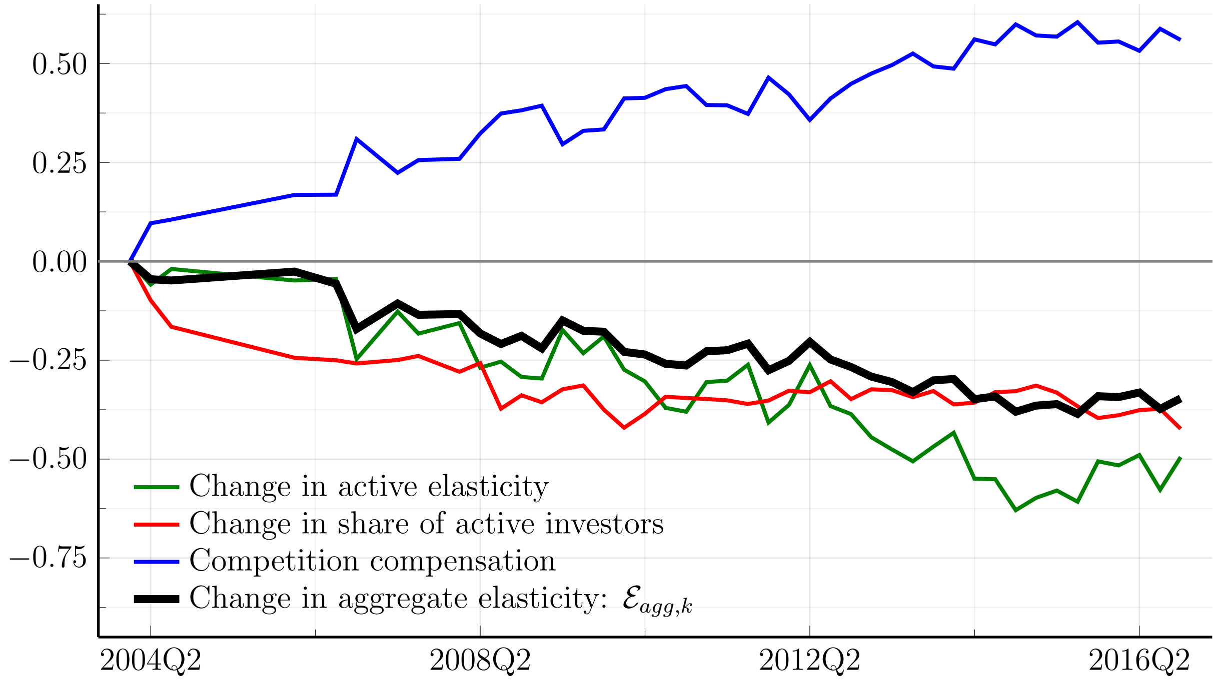

Figure 8. Decomposition of the change in aggregate elasticity.
Figure 8 shows the decomposition derived in equation (34) over time. We compute each term of the decomposition for each date and accumulate the changes over time.

of active investors contributes a drop of 49%. Interestingly, investors also decrease their own elasticities at the intensive margin. This second direct force furthers adds 42% to the drop. However, the competitive response strongly mitigates these individual changes in equilibrium. Competition reverses more than half of the decline, leading to the total change in aggregate elasticity of −35%.

5.2.3 Individual elasticity trends across investors

Our decomposition reveals that the change in the individual-specific component of the elasticity of active investors, $\underline{\mathcal{E}}_{ik}$ is a key contributor to the decline in aggregate elasticities. We zoom in on the properties of this component across investors. We evaluate the quantiles of individual elasticities $\underline{\mathcal{E}}_{ik}$ at the stock level across investors for each date. We draw the mean of these statistics across stocks in Figure 9. The decline we found on average extends to the

Table 3. Decomposition of the change in aggregate elasticity $\mathcal{E}_{agg}$

Aggregate elasticity	Decomposition		
Total change (2004-2016)	Active share	Active elasticity	Competition
−35%	−49%	−42%	55%

Table 3 reports the total change in aggregate elasticity and its decomposition, as derived in equation (34). We compute each term of the decomposition for each date and accumulate the changes over time. We report each term as a fraction of the total change in elasticity.

whole distribution of investors. The least elastic investors did become more inelastic: the individual elasticity of the 10th percentile goes from 0.25 at the end of 2003 to 0.05 at the end of our sample in 2016. Strikingly, the most elastic investors, at the top of the distribution, also experienced a significant decline from 1.2 to to 0.95. This dimension is particularly interesting because developments in computing power and access to big data would have instead suggested that the most aggressive quantitative funds would have increased their elasticities on their own. Farboodi and Veldkamp (2020) propose a more nuanced view of the effects of data improvements.

5.2.4 The cross-section of the stock-level evolution of elasticities

Our results so far show that, on average, both the rise of passive investing and a decrease in individual-level elasticity at the intensive margin contribute to the secular decline in individual stock elasticity. It is tempting to think that changes in these two components originate from the same forces. Zooming in on the cross-section of stocks, we can assess whether the comovement of these two trends also holds at the stock level.

We regress annual log changes in stock-level elasticity on changes in the fraction of active investors:

$$\log(\mathcal{E}_{agg,k,t}) - \log(\mathcal{E}_{agg,k,t-1}) = \beta \log(|Active_{k,t}|) - \log(|Active_{k,t-1}|) + \alpha_k + \gamma_t + e_{k,t}. \quad (35)$$

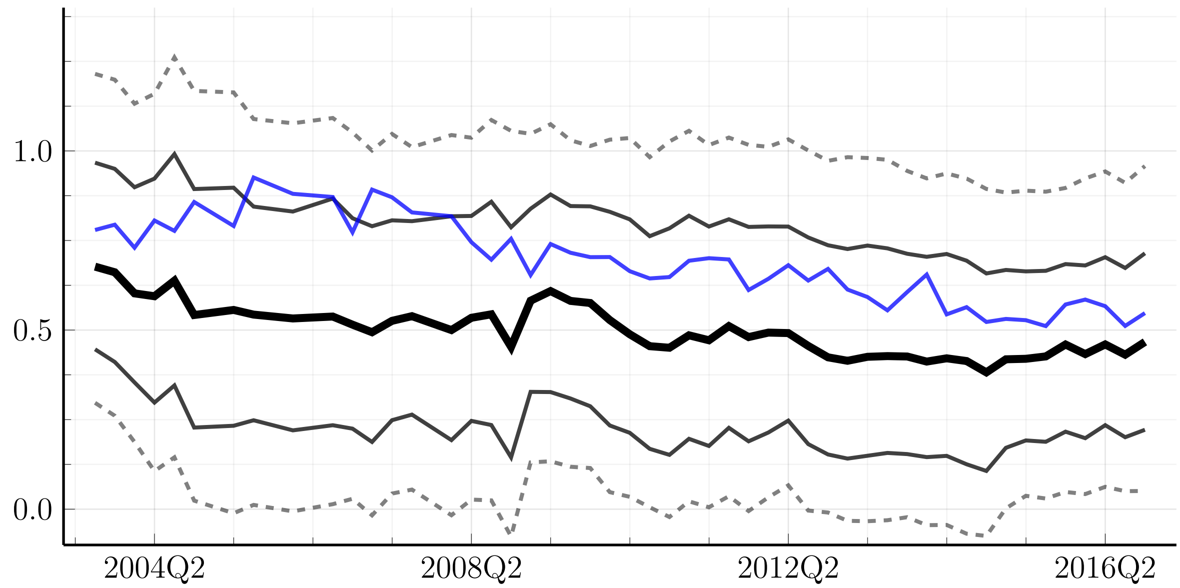

Figure 9. Distribution of individual-specific elasticities $\underline{\mathcal{E}}_{ik}$.
Figure 9 shows the quantiles of the distribution of individual elasticities $\underline{\mathcal{E}}_{ik}$ across investors for each stock and each date. We average the quantiles for each date to plot their time series. The black bold line is the average across investors. The two think grey lines represent the 25th and 75th percentiles. The two dashed grey lines represent the 10th and 90th percentiles. And the solid blue line represent the average individual elasticities of the household investor.

The inclusion of time and stock fixed effects allow to focus on variation independent of the average variation. A benchmark value for the coefficient β is the pass-through from equation (32), about 50%. However, if changes in individual-level elasticities or other types of changes in investor composition, are correlated with the active share, this would push β away from the theoretical pass-through. So effectively, we are assessing whether other trends in investor behavior are correlated with change in passive investing beyond through the competition effect.

Table 4 presents the result, using the unconstrained cross-sectional model estimates. Column 1 is a univariate regression; column 2 and 3 add date then stock fixed effects. Throughout we find a coefficient of about 0.7, close to the theoretical pass-through.[23] This result suggests that the two aggregate-level trends, rise of passive investing and decrease in elasticity

[23]Statistical significance is not completely meaningful in this setting, because the left-hand-side of the regression is model-generated.

Table 4. Change in aggregate stock-level elasticity $\mathcal{E}_{agg,k}$ on the active share

	Log Change in Elasticity		
	(1)	(2)	(3)
Change in Active share	0.696***	0.721***	0.700***
	(0.142)	(0.103)	(0.099)
Date Fixed Effects		Yes	Yes
Stock Fixed Effects			Yes
N	25,947	25,947	25,360
R^2	0.014	0.886	0.894

Table 4 reports a panel regression of annual log change in stock level elasticity $\mathcal{E}_{agg,k}$ on the annual log change in the active share $|Active_k|$. Column 2 adds date fixed effects. Column 3 adds stock fixed effects. Standard errors are 2-way clustered by date and stock.

of active investors, are distinct phenomena. They do not occur for the same stocks. Furthermore, because our model estimates are only based on cross-sectional evidence, this result from including the time series dimension provides additional support for our theory. Going in this direction, in Appendix Table IA.2, we confirm that the regression results are mostly unchanged when using the estimates that impose a constant value of χ through time.

5.2.5 Evolution under counterfactual levels of competition

Finally we can ask how the changes in the individual components of investor demand would have affected the aggregate elasticities under different competition regimes. We start from the equilibrium levels of demand elasticity at the beginning of our sample (2003Q4). We feed into the model the two direct components highlighted above: how individual elasticities, $\underline{\mathcal{E}}_{ik}$, change over time and who becomes passive. However we make different assumptions on how investors react to changes in the behavior others. We show the time series of the results in Figure 10. The black line represents the actual evolution of the average aggregate elasticity across stocks; the colored lines show the counterfactual results.

We first consider the case of competitive investors, corresponding to $\chi \to +\infty$. In this

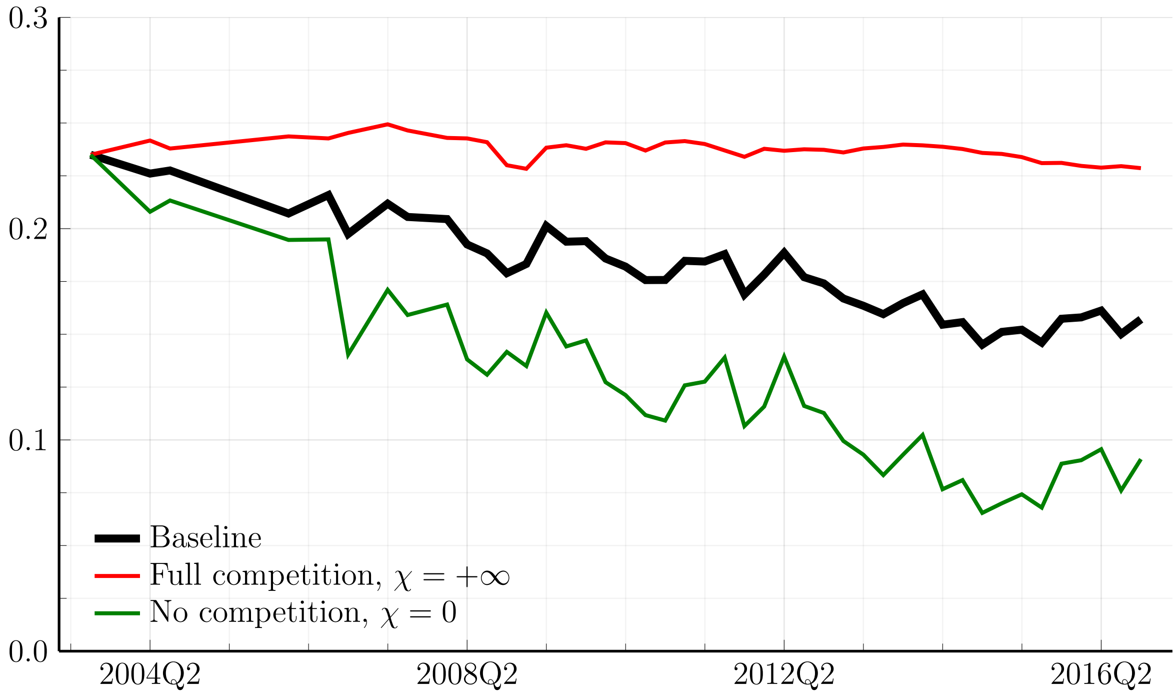

Figure 10. The evolution of aggregate elasticity under alternative competition regimes.
Figure 10 shows the evolution of aggregate elasticity $\mathcal{E}_{agg,k}$ under alternative competition regimes. The bold black line presents our baseline estimate. The top red line shows the elasticity with fully competitive investors ($\chi \to \infty$). The bottom green line shows the elasticity with no competition ($\chi = 0$).

situation any change in individual behavior is completely counteracted by other investors. The aggregate elasticities for each stock are pinned down at their initial level. The only source of variation in the average elasticity over time are changes in the composition of the universe of stocks. This is the red line in Figure 10, which experiences very little change over our sample. This result also confirms that the trends that we have documented of a decline in aggregate elasticities are not the consequence of changes in which stocks are traded.

The other extreme is the situation where investors do not react to others at all and $\chi = 0$. Then, all the changes in individual investor behavior directly feed into aggregate elasticities. This leads to a more dramatic drop in elasticities overtime than our baseline estimates. This is the green line in Figure 10. We observe a strong decrease, 75% larger than the baseline.

Overall these results confirm that changes in the behavior of investors have profoundly changed the aggregate demand curves for individual stocks. Competition among investors played an important role in mitigating the total impact of those changes. However, competition was not strong enough to fully negate the course of a downward trend in aggregate elasticities.

6 Conclusion

The idea that investors compete with each other is fundamental in financial markets. In theories of financial decisions, how others trade has a large impact on how you trade. We put forward a framework to measure competition and analyze its impact on equilibrium outcomes. Our framework is simple yet encompasses many theories of investor competition. In the US stock market we find evidence that investors do react to each other but also that this response is much weaker than anticipated by classic views. The effects of changes in the composition of investors on the demand for stock is reduced by 50%. This implies for example that the rise in passive investing leads to substantially more inelastic markets.

The presence of measurable competition bears on many other important issues in finance. To assess the impact of financial regulation on some market participants, for example the Basel III leverage constraint on banks, one cannot ignore how other institutions will respond. Likewise to understand how the distress of some financial institutions creates fire-sale spillovers, one must realize that other investors will step up. Our framework measures how many actually will. Recent work in international finance emphasizes the importance of cross-border flows and global imbalances. What happens if a large sovereign institution stops investing in one market, like China with US treasuries? Again the competition among investors will be a crucial input in determining the final impact of such momentous shift. Last, the rise and availability of big data promises to change the face of institutional investing.

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A Model of Information Acquisition

A.1 Setup

There is one period and one asset, and a continuum of agents indexed by $i \in [0, 1]$. Each agent has CARA preferences with risk aversion ρ_i:

$$U_i = \mathbf{E}_i[-e^{-\rho_i W_i}], \tag{IA.1}$$

and initial wealth W_i. The gross risk-free rate is 1, and the (random) asset payoff is f. The asset is in noisy supply $\bar{x} + x$ with $\bar{x}$ an exogenous fixed parameter and $x \sim \mathcal{N}(0, \sigma_x^2)$.

Each agent has a prior that $f \sim \mathcal{N}(\mu_i, \sigma_i^2)$. Following Veldkamp (2011), agents start with a flat prior on f and receive signal μ_i such that the signal is distributed $\mu_i \sim \mathcal{N}(f, \sigma_i^2)$. Each agent can acquire a private signal $\eta_i \sim \mathcal{N}(f, \sigma_{i,\eta}^2)$ at cost $c_i(\sigma_i^{-2} + \sigma_{i,\eta}^{-2})$, with $c_i(.)$ a non-decreasing positive function. That is, obtaining more precise signals is more costly. The signal being private implies in particular that signal realizations are uncorrelated across agents conditional on the fundamental f.

We focus on rational expectations equilibria, and among those linear equilibria specifically. These are equilibria in which the price takes the form:

$$p = A + Bf + Cx. \tag{IA.2}$$

An equilibrium is a set of coefficient (A, B, C), information choices $\sigma_{i,\eta}^2$, demand curves $D_i(p|\eta_i)$ such that:

(a) Each demand function and information choice maximizes expected utility, taking as given the price function.

(b) The market for the asset clears: $\bar{x} + x = \int D_i(p|\eta_i)di$.

To solve the model, we process in three steps: first we solve for the price given information decisions; then we will maximize utility within the model with respect to the information choice, keeping separation between information decision and portfolio allocation.

A.2 Solving Prices given Information

We are going to solve for the price function $p = A + Bf + Cx$. First we solve for allocations given the information choice and finally we use market clearing to pin down the price.

Agents form posterior on the fundamental f based on their prior μ_i, signal η_i, and from prices. The signal agents can extract from prices about f is:

$$s(p) = \frac{p - A}{B} = f + \frac{C}{B}x. \tag{IA.3}$$

Given the three signals, we are able to derive the posterior belief about f which will be distributed as $\mathcal{N}(\hat{\mu}_i, \hat{\sigma}_i^2)$ as follows:

$$\hat{\sigma}_i^{-2} = \sigma_i^{-2} + \sigma_{i\eta}^{-2} + \frac{B^2}{C^2}\sigma_x^{-2} \tag{IA.4}$$

$$\hat{\mu}_i = \hat{\sigma}_i^2 \left(\sigma_i^{-2}\mu_i + \sigma_{i\eta}^{-2}\eta_i + \frac{B^2}{C^2}\sigma_x^{-2}s(p) \right) \tag{IA.5}$$

Asset Demand. Abstracting from the cost of acquiring information, the utility function is for a given asset holding q_i:

$$\begin{aligned} U_i(q_i) &= -\mathbf{E}\left[\exp\left(-\rho_i\left(fq_i - pq_i\right)\right)\right] && \text{(IA.6)} \\ &= -\exp\left(-\rho_i q_i \left(\mathbf{E}[f] - p\right) + \frac{\rho^2}{2} q_i^2 \operatorname{Var}[f] \right). && \text{(IA.7)} \end{aligned}$$

The first order condition with respect to q_i gives us immediately:

$$\begin{aligned} & -\rho_i \left(\mathbf{E}[f] - p\right) + \rho^2 q_i \operatorname{Var}[f] = 0 \\ \iff \quad & q_i = \frac{1}{\rho_i \operatorname{Var}[f]} \left(\mathbf{E}[f] - p\right) \\ \iff \quad & q_i = \frac{1}{\rho_i} \hat{\sigma}_i^{-2} \left(\hat{\mu}_i - p\right) && \text{(IA.8)} \end{aligned}$$

Market Clearing. The market clearing condition reads:

$$\int q_i di = \bar{x} + x. \tag{IA.9}$$

Given asset demand this translates into:

$$\int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} \left(\hat{\mu}_i - p\right) di = \bar{x} + x \tag{IA.10}$$

The goal now is to find (A, B, C), which we identify directly from the market clearing condition. First we replace the expressions for the price function and the posteriors mean and variances in the market clearing equation:

$$\int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} \left[f + \frac{B}{C} \frac{\hat{\sigma}_i^2}{\sigma_x^2} x \right] di - \int \frac{1}{\rho_i} \hat{\sigma}_i^{-2} \left[A + Bf + Cx\right] di = \bar{x} + x \tag{IA.11}$$

First, we identify all the terms that are linear in f and find that $B = 1$. Next, we group the terms that are linear in x we have:

$$\begin{aligned}
& \int \frac{1}{\rho_i}\frac{1}{C}\sigma_x^{-2} di - \int \frac{1}{\rho_i}\hat{\sigma}_i^{-2} C di = 1 \\
\iff \quad & \int \frac{1}{\rho_i}\left[\frac{1}{C}\sigma_x^{-2} - C\hat{\sigma}_i^{-2} - C\sigma_{i\eta}^{-2} - \frac{1}{C}\sigma_x^{-2}\right] di = 1 \\
\iff \quad & C = -\left[\int \frac{1}{\rho_i}\left(\sigma_i^{-2} + \sigma_{i\eta}^{-2}\right) di\right]^{-1}. \qquad \text{(IA.12)}
\end{aligned}$$

Where we used the expression of the posterior found above to substitute into the second equation. Last we gather the constant terms to find A and

$$A = -\bar{x}\left[\int \frac{1}{\rho_i}\hat{\sigma}_i^{-2} di\right]^{-1} \qquad \text{(IA.13)}$$

A.3 Optimal Information

Computing Expected Utility Conditional on the signal and the price, expected utility is:

$$\begin{aligned}
U_i(q_i) &= -\mathbf{E}\left[\exp\left(-\rho_i\left(f q_i - p q_i\right)\right)|p, \eta\right] && \text{(IA.14)} \\
&= -\exp\left(-\rho_i q_i\left(\mathbf{E}[f|p, \eta] - p\right) + \frac{\rho^2}{2} q_i^2 \operatorname{Var}[f|p, \eta]\right) && \text{(IA.15)} \\
&= -\exp\left(-\frac{1}{2}\frac{\left(\mathbf{E}[f|p, \eta] - p\right)^2}{\operatorname{Var}[f|p, \eta]}\right), && \text{(IA.16)}
\end{aligned}$$

where the last line is derived using standard properties of quadratic functions.[24]

We can write:

$$\mathbf{E}[f|p, \eta] - p = \underbrace{\left(\mathbf{E}[f|p, \eta] - \mathbf{E}[f|p]\right)}_{z} + \left(\mathbf{E}[f|p] - p\right) \qquad \text{(IA.17)}$$

Conditional on p, z has mean 0 and its variance σ_z^2 can be obtained from:

$$\underbrace{f - \mathbf{E}[f|p]}_{\text{variance: } (\sigma_i^{-2} + \sigma_x^{-2}/C^2)^{-1}} = \underbrace{\left(f - \mathbf{E}[f|p, \eta]\right)}_{\text{variance: } \hat{\sigma}_i^2} + z \qquad \text{(IA.18)}$$

[24]For a function $f(x) = ax^2 + bx$, the maximum is reached for $x^* = -b/(2a)$ and its value is $f(x^*) = -b^2/(4a)$.

Using Veldkamp equation (7.32), this maps to:[25]

$$F = -\frac{1}{2}\frac{1}{\hat{\sigma}_i^2} \tag{IA.19}$$

$$G = (\mathbf{E}[f|p] - p)\,\frac{1}{\hat{\sigma}_i^2} \tag{IA.20}$$

$$H = -\frac{1}{2}\,(\mathbf{E}[f|p] - p)^2\,\frac{1}{\hat{\sigma}_i^2} \tag{IA.21}$$

So expected utility conditional on the price is:

$$\begin{aligned}
U_0|_p &= -(1 - 2\sigma_z^2 F)^{-1/2} \exp\left(\frac{1}{2} G^2 \left(1 - 2\sigma_z^2 F\right)^{-1} \sigma_z^2 + H\right) \\
&= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp\left(\frac{1}{2}\frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma_i}^2}\left[(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1}\frac{1}{\hat{\sigma}_i^2}\sigma_z^2 - 1\right]\right) \\
&= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp\left(\frac{1}{2}\frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma_i}^2}\left[(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1}(\frac{1}{\hat{\sigma}_i^2}\sigma_z^2 - 1 - \frac{\sigma_z^2}{\hat{\sigma}_i^2})\right]\right) \\
&= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp\left(-\frac{1}{2}\frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma_i}^2}\left[(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1}\right]\right) \\
U_0|_p &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2} \exp\left(-\frac{1}{2}\frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma_i}^2 + \sigma_z^2}\right).
\end{aligned} \tag{IA.22}$$

Expected utility is:

$$\begin{aligned}
\mathbf{E}\left[U_0|_p\right] &= -(1 + \frac{\sigma_z^2}{\hat{\sigma}_i^2})^{-1/2}\mathbf{E}\exp\left(-\frac{1}{2}\frac{(\mathbf{E}[f|p] - p)^2}{\hat{\sigma_i}^2 + \sigma_z^2}\right) \\
&= -\left[\frac{\sigma_i^{-2} + \sigma_x^{-2}/C^2}{\sigma_i^{-2} + \sigma_{i\eta}^{-2} + \sigma_x^{-2}/C^2}\right]^{1/2} \cdot \mathbf{E}\left[\exp\left(-\frac{1}{2}\frac{(\mathbf{E}[f|p] - p)^2}{(\sigma_i^{-2} + \sigma_x^{-2}/C^2)^{-1}}\right)\right]
\end{aligned} \tag{IA.23}$$

where we use (IA.18) in the second equality.

Optimal information. To derive the optimal information choice, we trade off utility with the cost of acquiring information which translates in utility terms:

$$U_0^{(c)} = U_0 \cdot \exp\left(\rho_i c_i(\sigma_i^{-2} + \sigma_{i\eta}^{-2})\right) \tag{IA.24}$$

[25]There is a general formula for the mean of the exponential of the quadratic of a normal variable. If we take the multivariate normal $z \sim \mathcal{N}(0, \Sigma)$:

$$\mathbf{E}\left[\exp(z'Fz + G'z + H)\right] = |I - 2\Sigma F|^{-1/2} \exp\left(\frac{1}{2}G'(I - 2\Sigma F)^{-1}\Sigma G + H\right)$$

$c_i(\cdot)$ is a function that is increasing in the signal precision and can depend on the i. The first order condition that determines the information choice is (maximizing $-\log(-U)$):

$$
\begin{aligned}
& \max_{\sigma_{i\eta}^{-2}} -\log(-U_0) - \rho_i c_i(\sigma_i^{-2} + \sigma_{i\eta}^{-2}) \\
\iff\; & \max_{\sigma_{i\eta}^{-2}} -\log(-U_0) - \rho_i c_i(\rho_i \mathcal{E}_i) \\
\iff\; & \frac{1}{2}\frac{1}{\sigma_i^{-2} + \sigma_{i\eta}^{-2} + \sigma_x^{-2}/C^2} = \rho_i c_i'(\sigma_i^{-2} + \sigma_{i\eta}^{-2}). \\
\iff\; & \frac{1}{2}\frac{1}{\rho_i \mathcal{E}_i + \mathcal{E}_{agg}^2 \sigma_x^{-2}} = \rho_i c_i'(\rho_i \mathcal{E}_i). \qquad \text{(IA.25)}
\end{aligned}
$$

Example 1: linear cost function. We consider the case of a constant marginal cost for any information acquired past the initial endowment: $c_i(x) = c_{1,i} \max\left(x - \sigma_i^{-2}, 0\right)$. Note that in this case not all agents acquire information since $\sigma_{i\eta}^{-2} > 0$, so the actual precision is:

$$
\sigma_{i\eta}^{-2} = \max\left(\frac{1}{2\rho_i c_{1,i}} - \sigma_i^{-2} - C^{-2}\sigma_x^{-2}, 0\right) \qquad \text{(IA.26)}
$$

This demand for information and the formula for the coefficient C characterizes the equilibrium. We recall that:

$$
C^{-2} = \left[\int \frac{1}{\rho_j}(\sigma_j^{-2} + \sigma_{j\eta}^{-2}) dj\right]^2 \qquad \text{(IA.27)}
$$

Example 2: linear response to aggregate elasticity. To relate to the model of Section 2, we ask if there is a reasonable family of cost functions that give rise exactly to equation (3). We are looking for a cost function such that $\mathcal{E}_i = \alpha - \beta \mathcal{E}_{agg}$. Equivalently, this corresponds to $\mathcal{E}_{agg} = \frac{1}{\beta}(\alpha - \mathcal{E}_i)$. Plugging in the first order condition, this gives:

$$
\begin{aligned}
2\rho_i^2 c_i'(\rho_i \mathcal{E}_i) &= \frac{1}{\mathcal{E}_i + \frac{\sigma_x^{-2}}{\rho_i \beta^2}(\alpha - \mathcal{E}_i)^2} && \text{(IA.28)} \\
=_{def}\; \tilde{c}_i'(\mathcal{E}_i) &= \frac{1}{\frac{\sigma_x^{-2}}{\rho_i \beta^2}\mathcal{E}_i^2 + \left(1 - 2\frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2}\right)\mathcal{E}_i + \frac{\alpha^2 \sigma_x^{-2}}{\rho_i \beta^2}} && \text{(IA.29)}
\end{aligned}
$$

The denominator of the righ-hand-side is a second degree polynomial, we solve for its roots. The discriminant is:

$$
\begin{aligned}
\Delta &= \left(1 - 2\frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2}\right)^2 - 4\frac{\sigma_x^{-2}}{\rho_i \beta^2}\frac{\alpha^2 \sigma_x^{-2}}{\rho_i \beta^2} && \text{(IA.30)} \\
&= 1 - 4\frac{\alpha \sigma_x^{-2}}{\rho_i \beta^2} && \text{(IA.31)}
\end{aligned}
$$

Let us assume $\Delta < 0$. This is equivalent to $\rho_i \beta^2 < 4\alpha \sigma_x^{-2}$. In this case, we have, using standard results on the primitive of the inverse of a polynomial:

$$\tilde{c}_i(\mathcal{E}_i) = \frac{2\arctan\left(\frac{2\frac{\sigma_x^{-2}}{\rho_i\beta^2}\mathcal{E}_i + \left(1 - 2\frac{\alpha\sigma_x^{-2}}{\rho_i\beta^2}\right)}{\sqrt{4\frac{\alpha\sigma_x^{-2}}{\rho_i\beta^2} - 1}}\right)}{\sqrt{4\frac{\alpha\sigma_x^{-2}}{\rho_i\beta^2} - 1}} + K \tag{IA.32}$$

The cost function is convex as long as the argument of the arctangent is negative, so:

$$\mathcal{E}_i \leq \alpha - \frac{\rho_i\beta^2}{2\sigma_x^{-2}}. \tag{IA.33}$$

We can see that if the right-hand-side of this condition is positive, the condition of $\Delta < 0$ is automatically satisfied.

After rescaling, $2\rho_i c_i(\rho_i\mathcal{E}_i) = \tilde{c}_i(\mathcal{E}_i)$, or equivalently $c_i(x) = \frac{1}{2\rho_i}\tilde{c}_i(x/\rho_i)$ we have:

$$c_i(x) = \frac{1}{\rho_i}\frac{1}{\sqrt{2\alpha\tilde{\beta} - 1}}\arctan\left(\frac{\tilde{\beta}\frac{x}{\rho_i} + (1 - \alpha\tilde{\beta})}{\sqrt{2\alpha\tilde{\beta} - 1}}\right) + \tilde{K} \tag{IA.34}$$

with $\tilde{\beta} = 2\sigma_x^{-2}/(\rho_i\beta^2)$, and the condition $\alpha - \rho_i\beta^2/(2\sigma_x^{-}2) \geq 0$ becomes $\alpha\tilde{\beta} \geq 1$. We can collect these results in a proposition.

Proposition 2. *For any $a > 0$ and $b > 0$ so that $ab > 1$, assume the information cost follows the function:*

$$\begin{aligned}
&c_i(x) = 0, \text{ if } \ x < 0, \\
&c_i(x) = \frac{1}{\rho_i}\frac{1}{\sqrt{2ab - 1}}\arctan\left(\frac{b\frac{x}{\rho_i} + (1 - ab)}{\sqrt{2ab - 1}}\right) + K, \text{ if } 0 \leq x/\rho_i \leq a - b^{-1} \\
&c_i(x) = +\infty, \text{ if } \ x/\rho_i \geq a - b^{-1},
\end{aligned} \tag{IA.35}$$

where K is such that $c_i(0) = 0$. This cost function is increasing and convex. Then the optimal elasticity is:

$$\mathcal{E}_i = \mathcal{E}_{0,i} - \chi\mathcal{E}_{agg}, \tag{IA.36}$$

with $\mathcal{E}_{0,i} = a$ and $\chi = \sqrt{(2\sigma_x^{-2})/(\rho_i b)}$.

A.4 Demand Elasticity

We recall the demand schedule for agent i:

$$\begin{aligned}
q_i &= \frac{1}{\rho_i}\hat{\sigma}_i^{-2}\left(\hat{\mu}_i - p\right) \\
&= \frac{1}{\rho_i}\hat{\sigma}_i^{-2}\left(\hat{\sigma}_i^2\left[\sigma_i^{-2}\mu_i + \sigma_{i\eta}^{-2}\eta_i + C^{-2}\sigma_x^{-2}s(p)\right] - p\right) \\
&= \frac{1}{\rho_i}\left(\sigma_i^{-2}\mu_i + \sigma_{i\eta}^{-2}\eta_i + C^{-2}\sigma_x^{-2}(p-A) - \hat{\sigma}_i^{-2}p\right) \\
&= \frac{1}{\rho_i}\left(\sigma_i^{-2}\mu_i + \sigma_{i\eta}^{-2}\eta_i + \left(C^{-2}\sigma_x^{-2} - \hat{\sigma}_i^{-2}\right)p - C^{-2}\sigma_x^{-2}A\right). \qquad \text{(IA.37)}
\end{aligned}$$

We can read the demand elasticity as:

$$\mathcal{E}_i = -\frac{dq_i}{dp} = -\frac{1}{\rho_i}\left(C^{-2}\sigma_x^{-2} - \hat{\sigma}_i^{-2}\right) = \frac{1}{\rho_i}\left(\sigma_i^{-2} + \sigma_{i\eta}^{-2}\right). \qquad \text{(IA.38)}$$

In the model the regression of q_i on p would not give us the proper elasticity. There is a bias in the regression because p is correlated with μ_i and η_i. It is still possible to recover the elasticty using an instrument; for example the supply shock x covaries with p but is uncorrelated with μ_i and η_i.

We can express the equilibrium in terms of demand elasticities. We define the aggregate demand elasticity as:

$$\mathcal{E} = \int \mathcal{E}_j dj = -C^{-1}. \qquad \text{(IA.39)}$$

We can rewrite the choice of information as a choice elasticity:

$$\mathcal{E}_k = \underbrace{\frac{1}{2\rho_i^2 c_{1,i}}}_{\text{Investor characteristics}} - \frac{\sigma_x^{-2}}{\rho_i} \underbrace{\mathcal{E}^2}_{\text{market elasticity}} \qquad \text{(IA.40)}$$

A.5 Flexibility in Information Acquisition

We turn to the study of flexibility in the acquisition of information. First we look at how $\sigma_{i\eta}$ changes as we vary the aggregate elasticity C^{-2} We take (IA.25) and using the implicit function theorem:

$$\begin{aligned}
&\frac{2}{\rho_i}\left(\sigma_i^{-2} + \sigma_{i\eta}^{-2} + C^{-2}\sigma_x^{-2}\right) - \frac{1}{c_i'(\sigma_{i\eta}^{-2})} = 0 \\
&\chi = -\frac{d\sigma_{i\eta}^{-2}}{dC^{-2}} = \frac{2/\rho_i\sigma_x^{-2}}{2/\rho_i + c_i''/c_i'^2} = \frac{\sigma_x^{-2}}{1 + \frac{\rho_i}{2}\frac{c_i''}{c_i'^2}} \qquad \text{(IA.41)}
\end{aligned}$$

The response depends on the curvature of the information acquisition cost function. If the curvature is zero (as is the case in our linear cost example), then the response is highest.

Whereas a large curvature will elicit a weaker response.

A.6 Price Informativeness

We define price informativeness for investor i as the ratio of the precision of their belief about the fundamental when they condition on their private information and on the price and the precision of their belief using private information only:

$$\begin{aligned}\mathcal{I}_i &= \frac{\mathrm{Var}(f|\mu_i,\eta_i,p)^{-1}}{\mathrm{Var}(f|\mu_i,\eta_i)^{-1}} = \frac{\sigma_i^{-2}+\sigma_{i\eta}^{-2}+\mathcal{E}_{agg}^2\,\sigma_x^{-2}}{\sigma_i^{-2}+\sigma_{i\eta}^{-2}} \\ &= 1+\mathcal{E}_{agg}\frac{\mathcal{E}_{agg}}{\rho_i\mathcal{E}_i}\sigma_x^{-2}\end{aligned} \tag{IA.42}$$

We also define the absolute price informativeness of the price as

$$\mathcal{I}_{abs} = \mathrm{Var}(f|p)^{-1} = \mathcal{E}_{agg}^2\sigma_x^{-2}. \tag{IA.43}$$

B Other foundations for competition χ

B.1 Learning from Prices

We consider a model in which agents can learn from prices which highlights a distinct mechanism from that of the previous section. Two main assumptions differ: agents cannot acquire information, and there is residual uncertainty about the asset payoff that cannot be learned. This setting leads to a new determinant of demand elasticity, beyond risk aversion and prior information. When many traders are aggressive, prices are more informative. How should one react? On the one hand, the extra information implies that price variation are less indicative of future return, and that pushes the investor to trade less aggressively. On the other hand, the extra information implies that returns appear less risky, and that pushes the investor to trade more aggressively. Increased price informativeness reveals relatively more about the fundamental than the payoff risk, exactly because of the presence of residual uncertainty.[26] Therefore the first effect dominates: the investor responds by being less aggressive, $\chi > 0$. This response is stronger when residual uncertainty is higher.

B.1.1 Setup

The asset trades at endogenous price p and pays off $f + \epsilon$, with $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. There is a continuum of mass 1 of agents indexed by i. Each agent has CARA preferences with risk aversion ρ_i. Each agent has a flat prior on f and receives an independent signal μ_i such that $\mu_i \sim \mathcal{N}(f, \sigma_i^2)$. The asset is in noisy supply $\bar{x} + x$ with $\bar{x}$ a constant and $x \sim \mathcal{N}(0, \sigma_x^2)$.

We look for a rational expectations equilibrium, with:

$$p = A + Bf + Cx \tag{IA.44}$$

B.1.2 Equilibrium

Learning from the price. After observing the price, agent i's posterior belief about the fundamental f is $\mathcal{N}\left(\hat{\mu}_i, \hat{\sigma}_i^2\right)$, with:

$$\hat{\sigma}_i^{-2} = \sigma_i^{-2} + \frac{B^2}{C^2}\sigma_x^{-2} \tag{IA.45}$$

$$\hat{\mu}_i = \hat{\sigma}_i^2 \left(\sigma_i^{-2}\mu_i + \frac{B^2}{C^2}\sigma_x^{-2}s(p)\right) \tag{IA.46}$$

where the signal from the price is:

$$s(p) = \frac{p - A}{B} = f + \frac{C}{B}x \tag{IA.47}$$

Taking the average over agents of type i (we will use a law of large numbers in the

[26] In the model of the previous section, the two effects exactly cancelled out. The response was coming from changes in information acquired, which is shut down here.

population), we have

$$E_i\left[\hat{\mu}_i\right] = \hat{\sigma}_i^2\left(\sigma_i^{-2}f + \frac{B^2}{C^2}\sigma_x^{-2}\left[f + \frac{C}{B}x\right]\right) \tag{IA.48}$$

$$= f + \frac{B}{C}\frac{\hat{\sigma_i}^2}{\sigma_x^2}x \tag{IA.49}$$

Asset Demand. Asset demand q_i is given by the standard optimum portfolio choice:

$$q_i = \frac{1}{\rho_i}\frac{E\left[f+\epsilon|\mu_i,p\right]-p}{var\left[f+\epsilon|\mu_i,p\right]} \tag{IA.50}$$

$$= \frac{1}{\rho_i}\frac{\hat{\mu}_i - p}{\hat{\sigma_i}^2+\sigma_\epsilon^2} \tag{IA.51}$$

Market Clearing. The total demand for the asset must equal its supply:

$$\int q_i di = \bar{x} + x, \tag{IA.52}$$

$$\int \frac{1}{\rho_i}\frac{1}{\hat{\sigma_i}^2+\sigma_\epsilon^2}\left[f + \frac{B}{C}\frac{\hat{\sigma_i}^2}{\sigma_x^2}x - A - Bf - Cx\right] = \bar{x} + x. \tag{IA.53}$$

This gives:

$$B = 1, \quad \text{(terms in } f) \tag{IA.54}$$

$$\int \frac{1}{\rho_i}\frac{1}{\hat{\sigma_i}^2+\sigma_\epsilon^2}\left[\frac{B}{C}\frac{\hat{\sigma_i}^2}{\sigma_x^2} - C\right]di = 1. \quad \text{(terms in } x) \tag{IA.55}$$

Plugging in the definition of $\hat{\sigma_i}^2$, we obtain

$$\int \frac{1}{\rho_i}\frac{1}{\hat{\sigma_i}^2+\sigma_\epsilon^2}\left[\frac{1}{C^2}\frac{\hat{\sigma_i}^2}{\sigma_x^2} - 1\right]di = C^{-1}, \tag{IA.56}$$

$$\int \frac{1}{\rho_i}\frac{1}{\hat{\sigma_i}^2+\sigma_\epsilon^2}\left[\frac{1}{C^2}\sigma_x^{-2}\frac{1}{\sigma_i^{-2}+\frac{1}{C^2}\sigma_x^{-2}} - 1\right]di = C^{-1}, \tag{IA.57}$$

$$\int \frac{1}{\rho_i}\frac{\hat{\sigma_i^2}}{\hat{\sigma_i}^2+\sigma_\epsilon^2}\left[\frac{1}{C^2}\sigma_x^{-2} - \sigma_i^{-2} - \frac{1}{C^2}\sigma_x^{-2}\right]di = C^{-1}. \tag{IA.58}$$

Therefore we have:

$$C^{-1} = -\int \frac{1}{\rho_i}\frac{\hat{\sigma_i}^2}{\hat{\sigma}_i^2+\sigma_\epsilon^2}\frac{1}{\sigma_i^2}di, \tag{IA.59}$$

$$-C^{-1} = \int \frac{1}{\rho_i}\frac{1}{1+\sigma_\epsilon^2\left(\sigma_i^{-2}+\frac{1}{C^2}\sigma_x^{-2}\right)}\frac{1}{\sigma_i^2}di. \tag{IA.60}$$

Define $\tilde{C} = -C$, which is positive. We can rewrite:

$$\tilde{C}^{-1} = \int \frac{1}{\rho_i} \frac{1}{1 + \sigma_\epsilon^2 \left(\sigma_i^{-2} + \frac{1}{\tilde{C}^2}\sigma_x^{-2}\right)} \frac{1}{\sigma_i^2} di. \tag{IA.61}$$

The left-hand-side of this equation is decreasing in $\tilde{C}$. The right-hand-side is increasing in $\tilde{C}$. If $\tilde{C} \to 0$, the left-hand-side goes to ∞ and the right-hand-side goes to 0. If $\tilde{C} \to \infty$, the left-hand-side goes to 0 and the right-hand-side has a finite positive limit. Therefore, there is a unique solution to the equation, and a unique linear equilibrium.

B.1.3 Equilibrium Elasticities

We now derive demand elasticities. We show how individual demand elasticities respond to the aggregate elasticity. Demand is given by:

$$q_i = \frac{1}{\rho_i} \frac{\hat{\mu}_i - p}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \tag{IA.62}$$

$$= \frac{1}{\rho_i} \frac{\hat{\sigma}_i^2 \left(\sigma_i^{-2}\mu_i + \frac{B^2}{C^2}\sigma_x^{-2} s(p)\right) - p}{\hat{\sigma}_i^2 + \sigma_\epsilon^2}. \tag{IA.63}$$

Therefore the slope of the demand curve is:

$$\mathcal{E}_i = -\frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left(\frac{1}{C^2}\sigma_x^{-2} - \hat{\sigma}_i^{-2}\right) \tag{IA.64}$$

$$= -\frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \left(\frac{1}{C^2}\sigma_x^{-2} - \sigma_i^{-2} - \frac{1}{C^2}\sigma_x^{-2}\right) \tag{IA.65}$$

$$= \frac{1}{\rho_i} \frac{\hat{\sigma}_i^2}{\hat{\sigma}_i^2 + \sigma_\epsilon^2} \frac{1}{\sigma_i^2}. \tag{IA.66}$$

Here, we observe clearly the intuition for the role of price informativeness. When prices are more informative, low $\hat{\sigma}_i^2$, expected returns respond less to the price, the numerator of the first fraction. However, the perceived risk of the asset also decreases, the denominator of the first fraction. Because of residual uncertainty σ_ϵ^2, the effect on the asset risk is weaker than the effect on expected return: the ratio decreases and the trader becomes less aggressive.

More aggregate elasticity leads to more informative prices, so this mechanism will lead to a negative response of individual elasticity to aggregate elasticity. Formally, note that $\int_i \mathcal{E}_i = \mathcal{E}_{agg} = \tilde{C}^{-1}$. Plugging in, we obtain:

$$\mathcal{E}_i = \frac{1}{\rho_i} \frac{1}{1 + \sigma_\epsilon^2 \left(\sigma_i^{-2} + \mathcal{E}_{agg}^2 \sigma_x^{-2}\right)} \frac{1}{\sigma_i^2} \tag{IA.67}$$

$$= \frac{1}{\rho_i} \frac{1}{\sigma_i^2 + \sigma_\epsilon^2 + \sigma_i^2 \sigma_\epsilon^2 \sigma_x^{-2} \mathcal{E}_{agg}^2} \tag{IA.68}$$

Clearly, the individual elasticity $\mathcal{E}_i$ is decreasing in the aggregate elasticity $\mathcal{E}_{agg}$. Linearizing this expression, we obtain the counterpart of competition $\chi > 0$:

$$\chi = -\frac{d\mathcal{E}_i}{d\mathcal{E}_{agg}} \tag{IA.69}$$

$$= \frac{1}{\rho_i} \frac{2\sigma_i^2 \sigma_\epsilon^2 \sigma_x^{-2} \mathcal{E}_{agg}}{\left(\sigma_i^2 + \sigma_\epsilon^2 + \sigma_i^2 \sigma_\epsilon^2 \sigma_x^{-2} \mathcal{E}_{agg}^2\right)^2}. \tag{IA.70}$$

B.2 Price Impact

We now consider a model in which investors have non-negligible price impact and take it into account when making trading decisions in the style of Kyle (1989). This leads to a foundation for a negative competition parameter χ. Intuitively, when other traders are aggressive, I face a very elastic residual supply curve when sending orders to the market. This implies that my trades will not have a large price impact, hence I can also trade aggressively.

B.2.1 Setup

There are I investors indexed by i. Each agent has CARA preferences with risk aversion ρ_i:

$$U_i = \mathbf{E}_i[-e^{-\rho_i W_i}], \tag{IA.71}$$

and initial wealth W_i. The gross risk-free rate is 1, and the random asset payoff f is distributed $\mathcal{N}(\mu, \sigma^2)$. The asset is in noisy supply $\bar{x} + x$ with $\bar{x}$ an exogenous fixed parameter and $x \sim \mathcal{N}(0, \sigma_x^2)$.

As in Kyle (1989) we are interested in rational expectation equilibria with imperfect competition. We look for a linear pricing rule $p = A + Cx$. We solve for the individual demand strategies and look for linear strategies of the form:

$$d_i = \underline{d}_i - \mathcal{E}_i p \tag{IA.72}$$

B.2.2 Solving for optimal demand strategies

Investor i maximizes their profit taking as given the residual demand from other investors' demand schedule. We use market clearing to find the residual supply curve:

$$\sum_i d_i = \bar{x} + x$$

$$d_i = \bar{x} + x - \sum_{k \neq i} \underline{d}_k + \left(\sum_{k \neq i} \mathcal{E}_k\right) p$$

$$p(d_i) = \underbrace{\left(\sum_{k \neq i} \mathcal{E}_k\right)^{-1}}_{\lambda_{-i}} d_i + \underbrace{\left(\sum_{k \neq i} \mathcal{E}_k\right)^{-1} \cdot \left(\sum_{k \neq i} \underline{d}_k - \bar{x} - x\right)}_{p_{-i}}. \tag{IA.73}$$

To find the optimal demand of investor i for the asset, we write their program[27]

$$\max_d \mathbf{E}\{f - p(d)|p_{-i}\}d - \frac{\rho_i}{2}\operatorname{Var}\{f - p(d)|p_{-i}\}d^2 \tag{IA.74}$$
$$\max_d (\mu - p_{-i})d - \lambda_{-i}d^2 - \frac{\rho_i}{2}d^2\sigma^2.$$

The first order condition gives us:

$$d_i = \frac{\mu - p}{\rho_i\sigma^2 + \lambda_{-i}}. \tag{IA.75}$$

We can already see that stronger λ_{-i} leads to less aggressive trading because of a larger price impact. Remember that λ_{-i} is the aggregate of demand elasticities of other investors, a quantity closely related to aggregate elasticity. We now close the equilibrium to show this relation more clearly.

B.2.3 Solving for aggregate demand elasticity

Given our original demand $d_i = \underline{d}_i - \mathcal{E}_i p$, we are able to identify the linear terms as:

$$\underline{d}_i = \frac{\mu}{\rho_i\sigma^2 + \lambda_{-i}}; \qquad \mathcal{E}_i = \frac{1}{\rho_i\sigma^2 + \lambda_{-i}} = \frac{1}{\rho_i\sigma^2 + (\mathcal{E}_{agg} - \mathcal{E}_i)^{-1}}, \tag{IA.76}$$

where we define the aggregate elasticity:

$$\mathcal{E}_{agg} = \sum_i \mathcal{E}_i. \tag{IA.77}$$

Next we show that there is a unique solution for the aggregate elasticity. From the expression in equation (IA.76), we remark that $\mathcal{E}_i$ solves a quadratic. We rule out the larger of the two roots and the solution is[28]

$$\mathcal{E}_i = \frac{1}{2}\left(\frac{2}{\rho_i\sigma^2} + \mathcal{E}_{agg} - \sqrt{\left(\frac{2}{\rho_i\sigma^2}\right)^2 + \mathcal{E}_{agg}^2}\right) \tag{IA.78}$$

To show that there is a unique stable equilibrium we consider the fixed point problem $F(x) = x$, with F defined by:

$$f_i(x) = \frac{1}{2}\left(\frac{2}{\rho_i\sigma^2} + x - \sqrt{\left(\frac{2}{\rho_i\sigma^2}\right)^2 + x^2}\right), \tag{IA.79}$$

$$F(x) = \sum_i f_i(x). \tag{IA.80}$$

[27]Note that expectation and variances are conditional on the residual demand curve p_{-i} which is equivalent to conditioning on p

[28]The larger root is such that $\mathcal{E}_i > \mathcal{E}_{agg}$ which violates $\sum_i \mathcal{E}_i = \mathcal{E}_a gg$.

The function F is positive, increasing, and concave. Moreover $F(0) = 0$, $F'(0) = I/2$, and $\lim_{x\to+\infty} F'(x) = 0$, we conclude that there is a unique non-zero solution for $\mathcal{E}_{agg}$ as long as $I \geq 3$.

The relation derived in (IA.78) between $\mathcal{E}_i$ and $\mathcal{E}_{agg}$ is not linear. We can approximate this equation linearly by $\mathcal{E}_i = \underline{\mathcal{E}}_i - \chi\mathcal{E}_{agg}$ with

$$\chi = -\frac{1}{2}\left(1 - \frac{\mathcal{E}_{agg}}{\sqrt{\mathcal{E}_{agg}^2 + \left(\frac{2}{\gamma_i\sigma^2}\right)^2}}\right) < 0 \tag{IA.81}$$

This expression gives bounds on the value of χ: $-1/2 \leq \chi < 0$.

C Identification Strategy

C.1 Moment Conditions

We estimate the model using the method of moments. All of the moment conditions derive from the identifying assumption of equation (29). We list these moments here:

$$\mathbf{E}\left[\epsilon_{jk}\mathbf{1}_{\{j=i\}}\right] = 0, \forall i \tag{IA.82}$$

$$\mathbf{E}\left[\epsilon_{jk}X_k^{(d)}\mathbf{1}_{\{j=i\}}\right] = 0, \forall i \tag{IA.83}$$

$$\mathbf{E}\left[\epsilon_{jk}\hat{p}_{k,i}1_{\{j=i\}}\right] = 0, \forall i \tag{IA.84}$$

$$\mathbf{E}\left[\epsilon_{jk}X_k^{(e)}\hat{p}_{k,j}1_{\{j=i\}}\right] = 0, \forall i \tag{IA.85}$$

$$\mathbf{E}\left[\epsilon_{jk}\hat{\mathcal{E}}_{agg,k}\hat{p}_{k,j}\right] = 0 \tag{IA.86}$$

There are exactly as many moment conditions as model parameters.

C.2 Solving the Reflection Problem

One challenge for identification is the reflection problem. How can we separate the individual component of demand elasticity from the competitive response to other investors. We show that the presence of variation in investor population across stocks allows to solve this problem. To isolate this argument from other identification concerns, we assume that we observe individual elasticities, $\mathcal{E}_{ik}$. For exposition purposes, we focus on a simplified version of the model in which $\underline{\mathcal{E}}_i$ does not depend on asset characteristics.

We provide sufficient conditions for the uniqueness of a decomposition of the individual elasticities into investor specific elasticities $\underline{\mathcal{E}_i}$ and the competitive response controlled by χ. After proving this result, we come back to the economic content and the empirical relevance of these conditions.

Before stating the theorem, we introduce a few notations. We define the undirected graph $\mathcal{G}$ of investor-stock connections. The vertices (the nodes) are the investors i and the stocks k. There is an edge between i and k if and only if $i \in I_k$. There are no edges between two investors or two stocks.

Theorem 3. *A decomposition of demand elasticities $\{\mathcal{E}_{ik}\}_{i,k}$ into individual elasticities $\{\underline{\mathcal{E}}_i\}_i$ and the competition parameter χ is unique if:*

(a) The graph $\mathcal{G}$ of investor-stock connections is connected.

(b) Position-weighted averages of demand elasticities are not constant across stocks: there exists k and k' such that $\sum_{i \in I_k} w_{ik}/p_k A_i \underline{\mathcal{E}}_i \neq \sum_{i \in I_{k'}} w_{ik'}/p_{k'} A_i \underline{\mathcal{E}}_i$.

Proof. Let us assume that there exist two distinct decompositions $(\{\underline{\mathcal{E}}_i^{(1)}\}_i, \chi^{(1)}) \neq (\{\underline{\mathcal{E}}_i^{(2)}\}_i, \chi^{(2)})$ and the two conditions *(a)* and *(b)* hold. Each decomposition for $l \in \{1,2\}$

satisfies the two conditions of the elasticity layer

$$\mathcal{E}_{agg,k} = \sum_{i \in I_k} w_{ik} A_i / p_k \mathcal{E}_{ik}, \qquad \text{for all } k \in \mathcal{K} \tag{IA.87}$$

$$\mathcal{E}_{ik} = \underline{\mathcal{E}}_i^{(l)} - \chi^{(l)} \mathcal{E}_{agg,k}, \qquad \text{for all } k \in \mathcal{K} \text{ and } i \in I_k. \tag{IA.88}$$

We subtract the decomposition of $\mathcal{E}_{ik}$ for $l = 1$ from the decomposition for $l = 2$ and obtain:

$$\left(\chi^{(2)} - \chi^{(1)}\right) \mathcal{E}_{agg,k} = \underline{\mathcal{E}}_i^{(2)} - \underline{\mathcal{E}}_i^{(1)}, \qquad \text{for all } k \in \mathcal{K} \text{ and } i \in I_k. \tag{IA.89}$$

Here we see immediately that if $\chi^{(1)} = \chi^{(2)}$, then for all i, $\underline{\mathcal{E}}_i^{(1)} = \underline{\mathcal{E}}_i^{(2)}$, thus violating the initial assumption of distinct decompositions. Hence, we focus on the case of $\chi^{(1)} \neq \chi^{(2)}$.

We define the function:

$$f(x) = \begin{cases} (\chi^{(2)} - \chi^{(1)}) \mathcal{E}_{agg,x} & \text{for } x \in \mathcal{K} \\ \underline{\mathcal{E}}_x^{(2)} - \underline{\mathcal{E}}_x^{(1)} & \text{for } x \in I. \end{cases} \tag{IA.90}$$

We restate the equality of equation (IA.89) as:

$$f(x) = f(x'), \quad \text{if and only if there is an edge between } x \text{ and } x' \text{ on } \mathcal{G}. \tag{IA.91}$$

Therefore, since the graph $\mathcal{G}$ is connected: $\forall x, x', f(x) = f(x')$, and f is a constant. We write the constant $f = a$, and plug in the constant in the aggregation of individual elasticities:

$$\mathcal{E}_{agg,k} = \sum_{i \in I_k} w_{ik} A_i / p_k \mathcal{E}_{ik} = \sum_{i \in I_k} w_{ik} A_i / p_k \underline{\mathcal{E}}_i^{(1)} - \chi^{(1)} \sum w_{ik} A_i / p_k \mathcal{E}_{agg,k} \tag{IA.92}$$

$$\iff (1 + \chi^{(1)}) \mathcal{E}_{agg,k} = \sum_{i \in I_k} w_{ik} A_i / p_k \underline{\mathcal{E}}_i^{(1)} \tag{IA.93}$$

$$\iff (1 + \chi^{(1)}) \frac{a}{\chi^{(2)} - \chi^{(1)}} = \sum_{i \in I_k} w_{ik} A_i / p_k \underline{\mathcal{E}}_i^{(1)} \qquad \text{for all } k, \tag{IA.94}$$

where we use $\mathcal{E}_{agg,k} = a/(\chi^{(2)} - \chi^{(1)})$. Equation (IA.94) violates assumption *(b)*, which concludes the proof. ∎

The intuition behind theorem 3 is that identification relies on comparing the behavior of one investor for two different stocks with different populations of investors. If this investor trades less agressively when surrounded by more agressive investors, we conclude that competition χ is positive. A challenge to implement this comparison is that we already need to know the elasticity of these other investors. This is a chicken-and-egg question. The ability to find a unique solution to this problem relies on being able to cycle through investors with enough variation in composition: this is the essence of conditions (a) and (b).

To better understand why these conditions are important, we show examples of how the model is not identified when either (a) or (b) is violated. Starting with (a), let us consider the case where each stock has its own non-overlapping population of investors. In this case, there is no identification. Because a given investor only invests in one stock, it is not possible to tell if this investor is agressive because of her own characteristics or in response to the other investors. As an example that violates condition (b), consider the case in which all investors have the same size and relative portfolio positions such that: $\forall k, k', w_{ik}A_i/p_k = w_{ik'}A_i/p_{k'}$. Investor composition is the same for all stocks and therefore there is no information in comparing different stocks. Relatedly, we could also consider a violation of (b) where all individual elasticities are identical across investors: $\underline{\mathcal{E}}_i = \underline{\mathcal{E}}$. Then, for all k we have $\sum_{i \in I_k} w_{ik}/p_k A_i \underline{\mathcal{E}}_i = \underline{\mathcal{E}}$: the aggregate elasticity for all stocks is identical. Intuitively, even though there is variation in investor composition across stocks, all investors behave the same way in terms of elasticity. This is equivalent to having a single investor, and we cannot separate individual elasticities from the response to other investors.

How can we assess these conditions empirically? The graph $\mathcal{G}$ of investors-stocks connections can be observed directly in our data and we can assess immediately that condition (a) is satisfied using known algorithms such as depth-first-search. Condition (b) is potentially more challenging because it relies on parameter estimates $\underline{\mathcal{E}}_i$. However, the inspecting the condition shows it holds generically. Condition (b) stipulates the equality of K linear forms applied to the vector $(\underline{\mathcal{E}}_i)_i$. It is violated if and only if $(\underline{\mathcal{E}}_i)_i \in \bigcap_{k>1}(w_k - w_1)^{\perp}$, a set of measure 0 for almost all combinations of w_k. In addition, there is still the possibility of verifying whether the condition is satisfied empirically, once the econometrician has found a set of parameter estimates.

C.3 Numerical Procedure

The main numerical challenge is that we need to solve for the equilibrium elasticity $\mathcal{E}_{agg,k}$ at the same time as we are estimating the model, and in particular the competition parameter χ. We describe a tractable approach to do so. We solve a series of nested problems.

Step 1. Given a guess for χ and $\{\mathcal{E}_{agg,k}\}_k$, we can estimate all remaining model parameters by two-stage least squares regression investor by investor. This corresponds to the run for each investor i the regression:

$$\log \frac{w_{ik}}{w_{i0}} - p_k - \chi\ \mathcal{E}_{agg,k}\ p_k = \underline{d}_{0i} + \underline{d}'_{1it} X_k^{(d)} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}'_{1i} X_k^{(e)} \right)\ p_k + \epsilon_{ik}, \tag{IA.95}$$

where p_k and $X_k^{(e)} p_k$ are instrumented by $\hat{p}_{k,i}$ and $X_k^{(e)} \hat{p}_{k,i}$. Running these regressions is equivalent to solving moment conditions (IA.82) to (IA.85).

Step 2. Given a guess for χ, we look for equilibrium values of $\{\mathcal{E}_{agg,k}\}_k$. We start from the aggregate elasticities implied by the model of (Koijen and Yogo, 2019). We run step 1 above. With the newly estimated $\underline{\mathcal{E}}_i$ and the parameter χ, we solve explicitly for the equilibrium elasticity they imply by solving the linear system of equations (20) and (24). We update our guessed aggregate elasticity by taking a weighted average of the previous iteration and these new implied values with weights of 75% and 25%. We repeat this updating process until the values of $\{\mathcal{E}_{agg,k}\}_k$ converge. This step ensures that our estimated model satisfies the 2-layer equilibrium.

Step 3. We estimate χ. We start from a guess for χ and run step 2 to find the aggregate elasticities it implies. With these values, we estimate the pooled regression of equation (25):

$$\log \frac{w_{ik}}{w_{i0}} - p_k = \underline{d}_{0i} + \underline{d}_{1it}' X_k^{(d)} - \left(\underline{\mathcal{E}}_{0i} + \underline{\mathcal{E}}_{1i}' X_k^{(e)} - \chi \ \mathcal{E}_{agg,k} \right) \ p_k + \epsilon_{ik}, \tag{IA.96}$$

using two-stage least squares with all the instruments of investor-level regression and $\hat{\mathcal{E}}_{agg,k} \hat{p}_{k,i}$. This is a very large scale regression with many fixed effects and investor-specific coefficients. We speed up the estimation of this large-scale regression tremendously by taking advantage of the Frisch-Waugh-Lovell theorem. We absorb all individual-level variables using investor-specific regressions, and are left with only the coefficient χ to estimate in the pooled data.

The pooled regression gives us a new guess for χ. We use a univariate Newton method to find a fixed point for χ. With such a fixed point, we are sure that our estimates satisfy simultaneously all the moment conditions of Appendix Section C.1 and the 2-layer equilibrium.

D Additional Empirical Results

D.1 Trading big and small stocks

We investigate whether firms trade big and small stocks differently. Our estimates of elasticities by stocks suggest that the demand for large stocks is more inelastic (see Figure 4). To explain this result, one hypothesis is that large stocks mechanically tend to receive a high portfolio weight and that investors are unwilling to adjust their largest positions. For example, a 10% relative increase in portfolio weight would create much larger tracking error to the index for large positions than for small positions. Also, the granular nature of large stocks imply that they have fewer substitutes.

To complement our structural results and investigate this hypothesis, we compare the trading activity of investors across the distribution of their portfolio. For a given investor-quarter, we compute for each stock the squared relative change in the number of shares:

$$\text{Trading Activity}_{i,k,t} = \left[\left(\frac{A_{i,t} w_{ik,t}}{p_{k,t}} - \frac{A_{i,t-1} w_{ik,t-1}}{p_{k,t-1}} \right) / \frac{A_{i,t} w_{ik,t}}{p_{k,t}} \right]^2 \tag{IA.97}$$

We sort positions by portfolio weights, and compute the ratio of the cumulative sum of trading activity to the total sum. This gives us a relation between the percentile of portfolio weight and the cumulative share of total trading activity. We average this relation within size groups of investors and present our results in Figure IA.1 for various dates.

If trading activity for all portfolio weights, this curve should coincide with the 45 degree line. Instead, we see the curve is always above the 45 degree line and particularly flat along the largest investor positions. This implies there is relatively less trading activity for the largest stocks. In addition, we observe that this pattern is more pronounced for the largest investors (panel D) than for small investors (panel A). Because larger investors are more important for the biggest stocks, this will amplify the lack of trading activity for the biggest stocks.

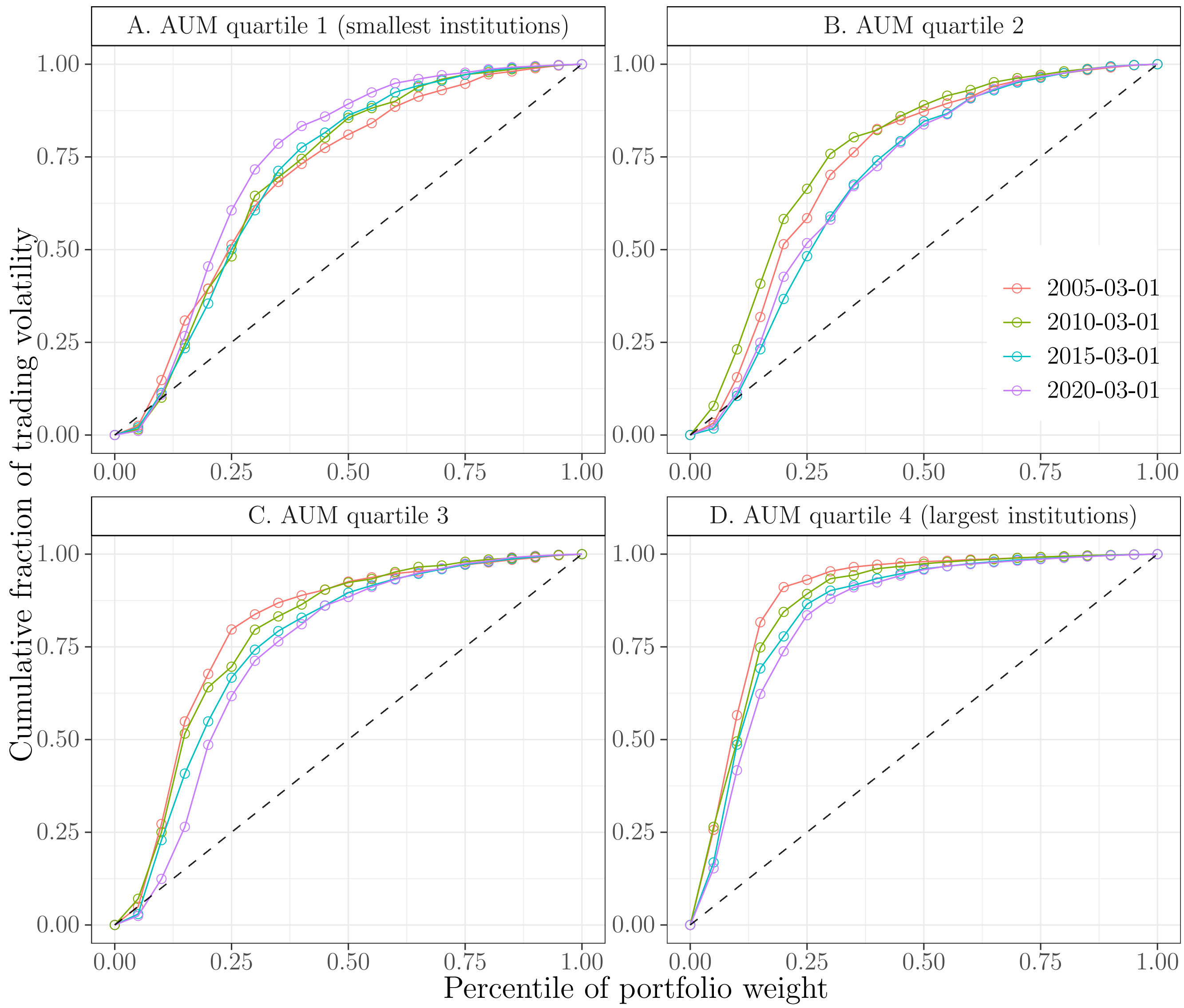

Figure IA.1. Trading activity across portfolio positions. Figure IA.1 presents the cumulative share of trading activity (defined in equation (IA.97)) by quantiles of investor portfolio weights. We aggregate the statistics by date and quartiles of assets under management.

E Appendix Tables

Table IA.1.
Summary Statistics of Aggregate Elasticity $\mathcal{E}_{agg}$ with book-equity weighted instrument

	Average	25th pct.	Median	75th pct.
	Panel A: Statistics of average elasticity across stocks			
Elasticity $\mathcal{E}_{agg}$	0.264	0.24	0.279	0.297
Fixed elasticity	0.501	0.467	0.487	0.552
	Panel B: Statistics of in the cross-section of the elasticity within dates			
	Average	25th pct.	Median	75th pct.
Elasticity $\mathcal{E}_{agg}$	0.133	0.124	0.141	0.152
Fixed elasticity	0.157	0.135	0.144	0.179
	Panel C: Regression coefficient (by dates) of elasticity on size			
	Average	25th pct.	Median	75th pct.
Elasticity $\mathcal{E}_{agg}$	-0.0686	-0.0763	-0.071	-0.0664
Fixed elasticity	-0.0431	-0.0481	-0.0441	-0.0408

Table IA.1 presents statistics of the aggregate elasticity $\mathcal{E}_{agg}(k,t)$. We estimate the elasticity in our model accounting for competition χ and without competition as in Koijen and Yogo (2019) (denoted by fixed elasticity). Panel A has summary statistics of the average elasticity by date. Panel B has summary statistics of the cross-sectional standard deviation by date. Panel C has summary statistics of the coefficient β_t of the regression $\mathcal{E}_{agg}(k,t) = \alpha_t + \beta_t size_{k,t} + \varepsilon_{k,t}$. The sample period is 2000 to 2016

Table IA.2. Change in aggregate stock-level elasticity $\mathcal{E}_{agg,k}$ on the active share using estimates from the model with a constant χ

	Log Change in Elasticity		
	(1)	(2)	(3)
Change in Active share	0.678***	0.655***	0.624***
	(0.066)	(0.034)	(0.035)
Date Fixed Effects		Yes	Yes
Stock Fixed Effects			Yes
N	27,725	27,725	27,121
R^2	0.128	0.288	0.338

Table IA.2 reports a panel regression of annual log change in stock level elasticity $\mathcal{E}_{agg,k}$ on the annual log change in the active share $|Active_k|$. We use the estimates from the model with a constant value of χ over time. Column 2 adds date fixed effects. Column 3 adds stock fixed effects. Standard errors are 2-way clustered by date and stock.

F Appendix Figures

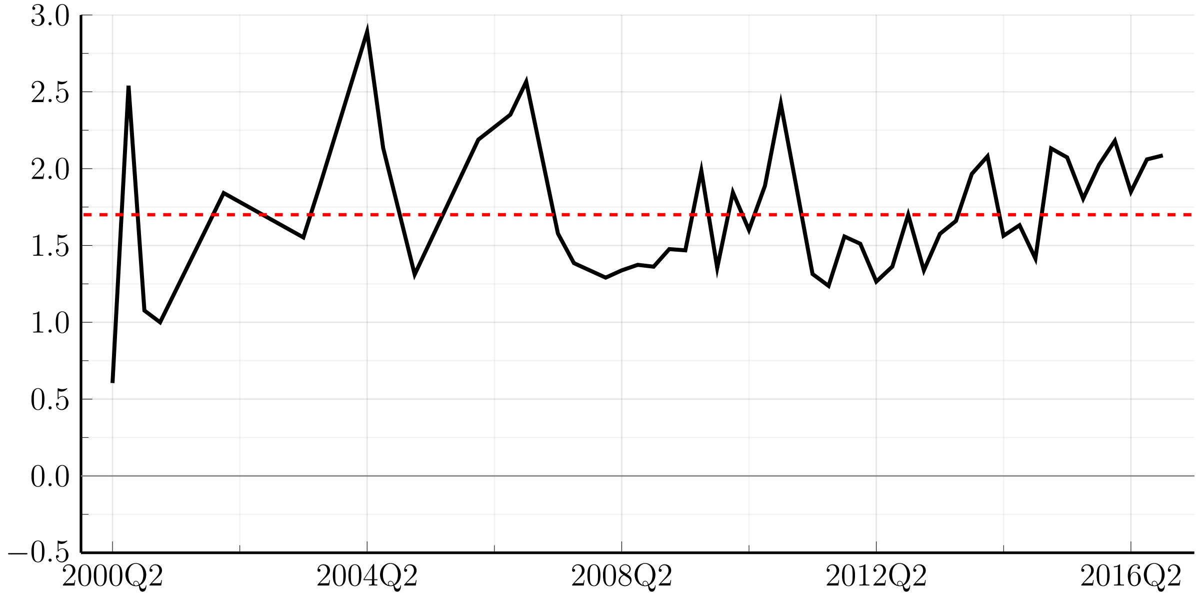

Figure IA.2. Time-series of competition χ.
Figure IA.2 shows the time-series of the estimates for the competition parameter χ.

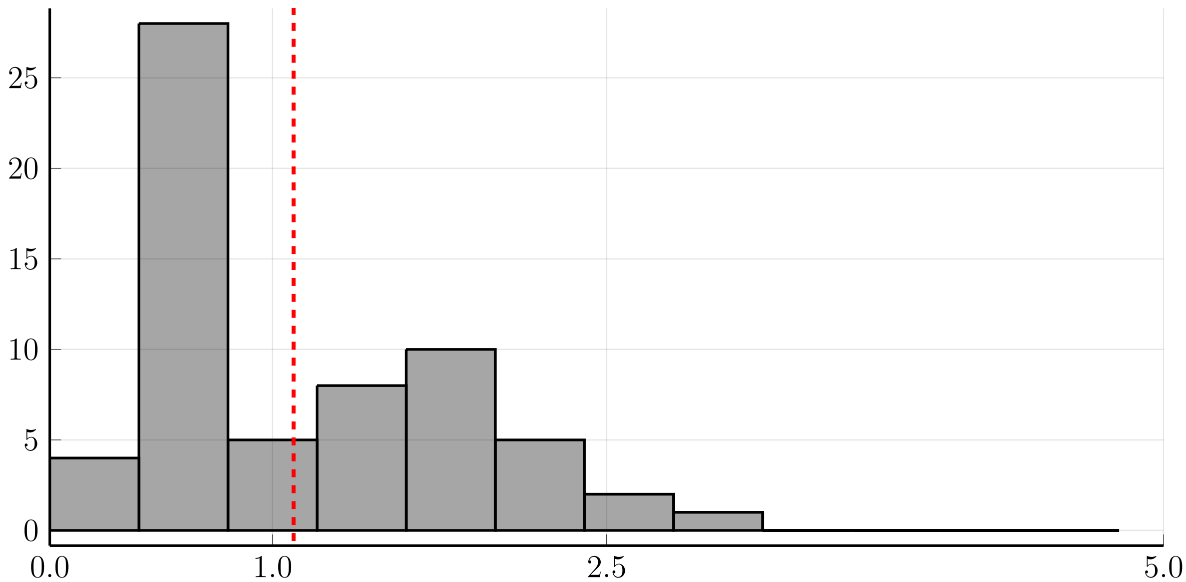

Figure IA.3. Estimates of χ using the book-equity weighted instrument.
Figure IA.3 presents an histogram of our estimates of the competition parameter χ where the instrument for aggregate elasticity weights portfolios by book equity, for each date between 2000 and 2016. The average estimate over the time-period is $\chi = 1.1$ (dashed redline).

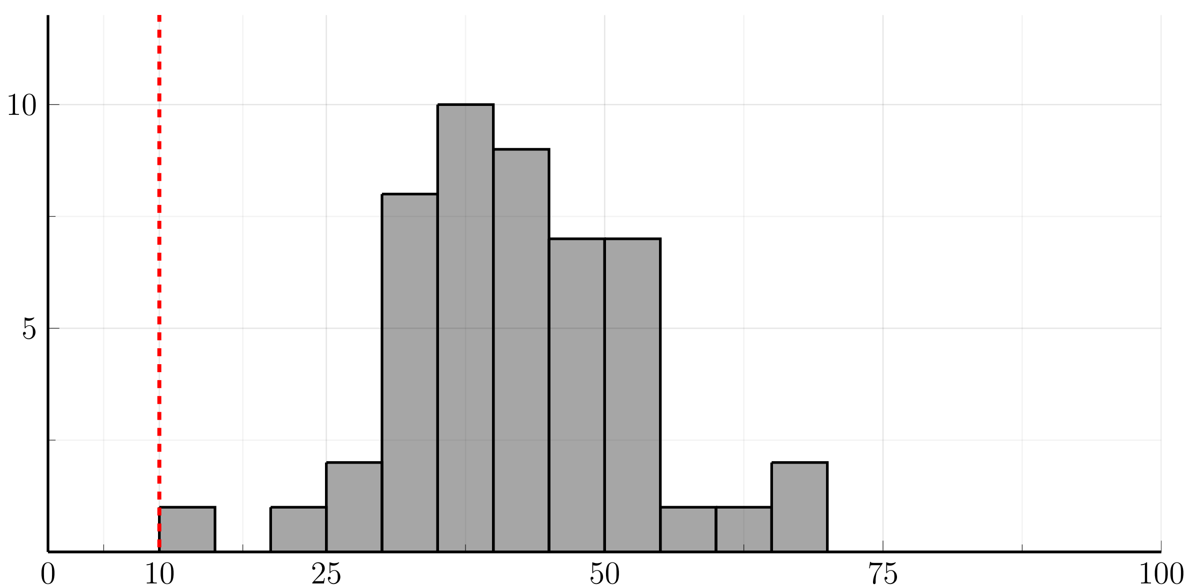

Figure IA.4. Relevance condition for the the book-equity weighted elasticity instrument.
Figure IA.4 shows the first-stage F-statistics (Kleibergen-Paap) when the instrument of aggregate elasticity weights portfolio by book equity.

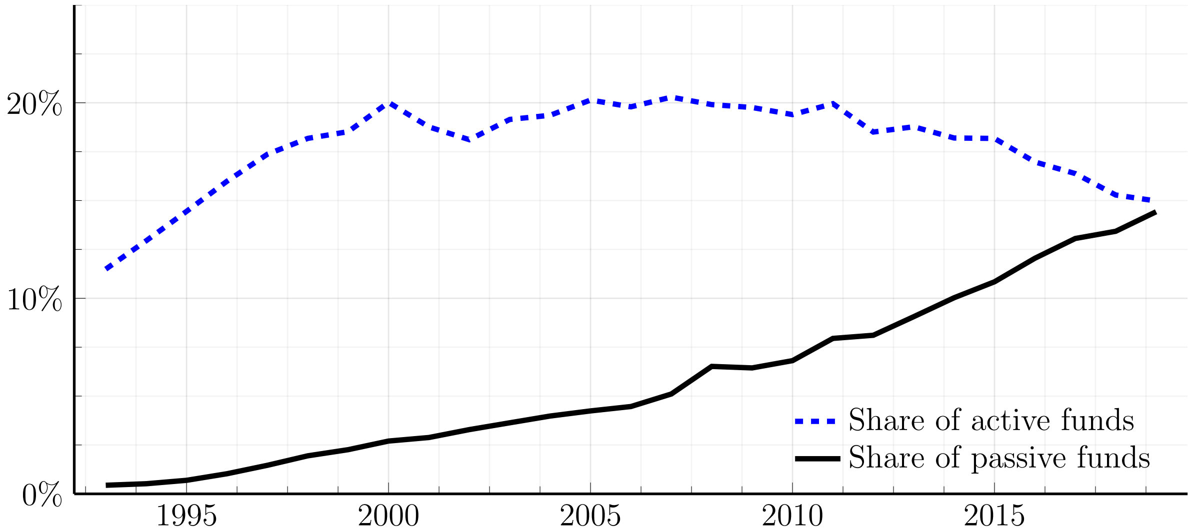

Figure IA.5. Net Assets of Passive and Active Funds.
Figure IA.5 shows the net assets of domestic mutual funds and ETFs in trillions of dollars (year-end) for passive funds (black solid line) and active funds (blue dashed line); Source ICI (2020).

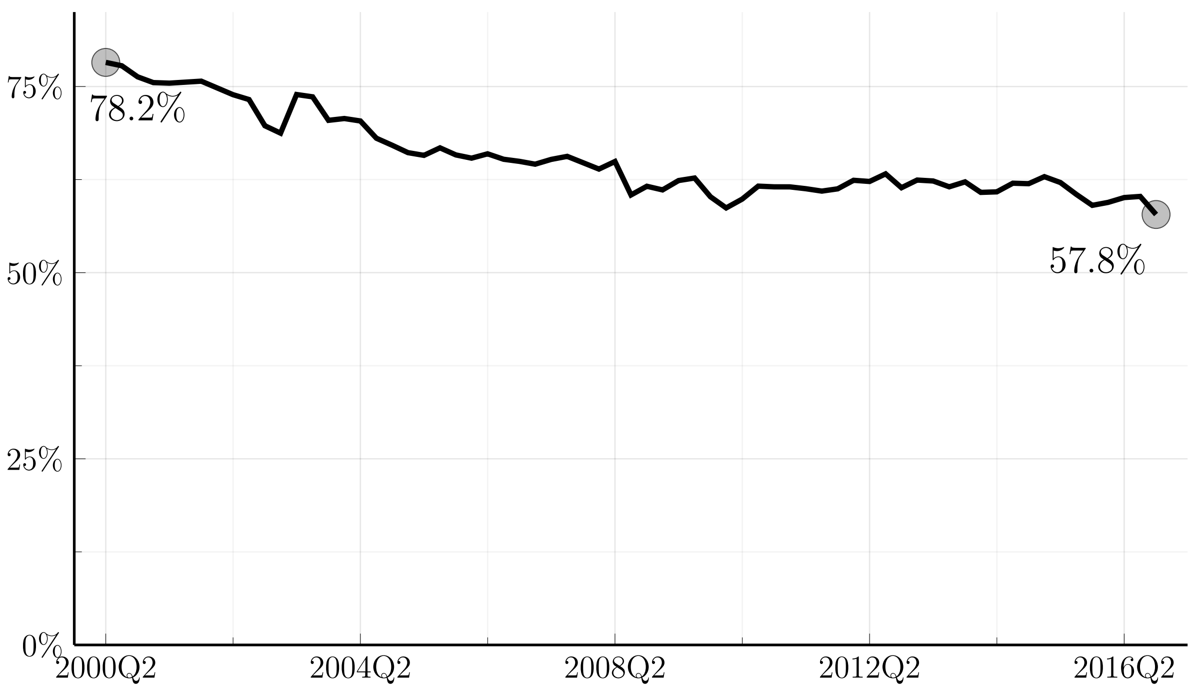

Figure IA.6. Fraction of Active Investors.
Figure IA.6 reports the fraction of active investors according to our model. For each stock, we compute the ratio of total position of active investors and the market capitalization. We report the average across stocks.

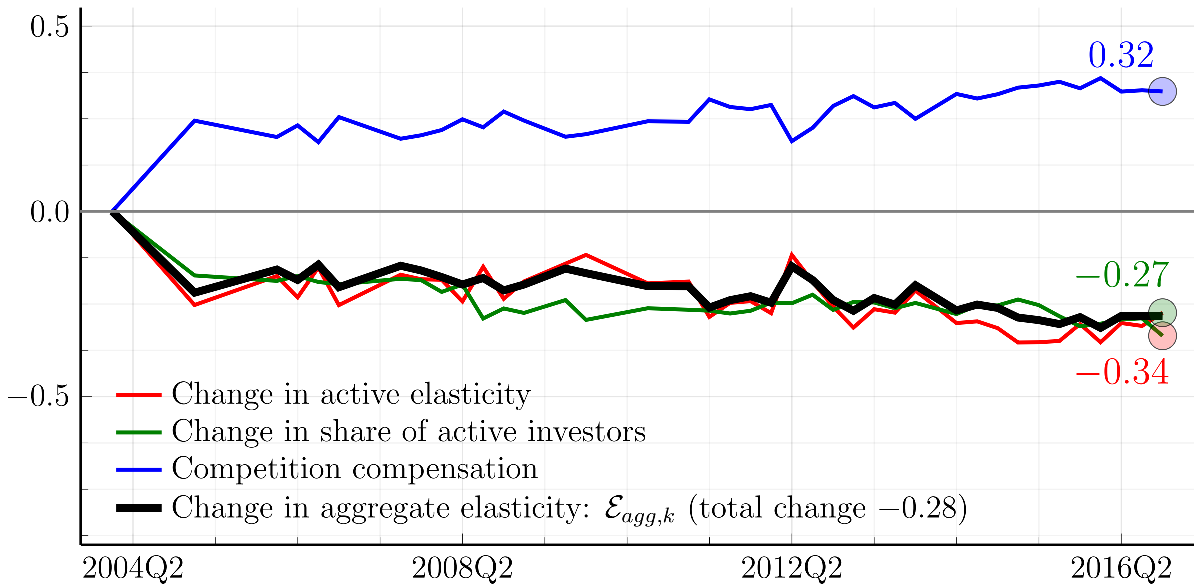

Figure IA.7. Distribution of aggregate elasticity.
Figure IA.7 traces out the distribution of aggregate elasticity $\mathcal{E}_{agg,k}$ over time. The bold line represents the average elasticity across stocks for each year. The solid lines represents the 25th and 75th percentile and the dashed lines the 10th and 90th percentile.