

# Unbundling Labor\*

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*Preliminary and incomplete.*

## Abstract

This paper presents two new facts on changes in occupational skill requirements and wage inequality in US data and develops a theoretical framework to interpret these facts. First, we show that low-skill occupations have become more similar to one another in terms of skill requirements while high-skill occupations have become more different from one another. Second, we show that residual wage inequality has fallen within low-skill occupations but has risen within high-skill occupations. Our model, a generalization of [Rosen \(1983\)](#) and [Heckman and Scheinkman \(1987\)](#), shows how these facts are naturally linked. Heterogeneous workers supply an *indivisible bundle* of skills when they choose an occupation. In equilibrium, a given skill can command a premium in occupations that are relatively intensive in its use. As occupations become more similar in their skill requirements, the rents earned by workers with high comparative advantage in the skills hitherto most intensively used in that occupation are competed away, decreasing within-occupation wage inequality. Likewise, as occupations become more different in skill requirements, workers with high comparative advantage in intensively used skills earn even larger rents, increasing within-occupation wage inequality. We also document a number of other facts consistent with our framework: relative decreases in low-skill occupation hours premia and experience premia, and increased occupational mobility.

*Keywords:* skill prices, sorting, comparative advantage, Roy model, inequality.

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# 1 Introduction

Can skill-biased technical change generate opposite trends in within-occupation wage inequality for low-skill and high-skill workers? We study this question in a general equilibrium model with heterogeneous workers endowed with multiple skill characteristics. In our model, skill characteristics are indivisibly *bundled* in individual workers, they must be supplied to a single occupation. This gives rise to the possibility that a given skill characteristic is priced differently in different occupations, i.e., that the law of one price for skills may not hold. Changes in within-occupation wage inequality are determined by changes in occupation-specific skill prices. We find that skill-biased technical change can increase wage-inequality in some occupations while at the same time decreasing wage-inequality in other occupations.

Our model of workers with bundled skills follows in the tradition of [Rosen \(1983\)](#) and [Heckman and Scheinkman \(1987\)](#). In equilibrium, the pattern of occupation-specific skill prices is determined by the interactions between the joint distribution of skill characteristics across workers and the different skill intensities of different occupations. If some occupations are sufficiently intensive in a particular skill, the equilibrium features tight bundling constraints and workers are actively *sorted* across occupations according to their comparative advantage. We refer to such an equilibrium as a *bundled equilibrium*. In a bundled equilibrium, a given skill receives a higher price in occupations that use it intensively and a lower price in occupations that do not use it intensively. For brevity, we refer to such deviations from the law of one price for skills as *skill premia*. A skill premium for any one skill does not present an arbitrage opportunity precisely because workers are indivisible bundles of multiple skills. Workers consider the alternative uses of all their skills jointly, not one at a time.

By contrast, if all occupations are sufficiently similar in their skill intensities, the equilibrium features *slack* bundling constraints, and comparative advantage plays no particular role in sorting workers. We refer to such an equilibrium as an *unbundled equilibrium*. In an unbundled equilibrium, the workers are still indivisible bundles of skills but their skills are priced *as if* the workers were divisible. All skill premia are zero and all workers are indifferent between occupations because their skill bundle receives the same compensation everywhere. Workers are heterogeneous, occupations are heterogeneous in their skill intensities, but workers are highly substitutable across occupations in that there is no sorting.

Our model nests the *Roy model* as a special case. In the Roy model workers are endowed with heterogeneous bundles of skills but importantly each occupation uses a single skill.<sup>1</sup> The Roy model is a limiting case of our model where each occupation is maximally intensive in one particular skill. As a consequence, the equilibrium in the Roy model is *always bundled*. The bundled equilibrium of a Roy model features a stark pattern of skill prices: For each occupation, all but one of the occupation-specific skill prices are zero. Only the one skill that is used by that occupation has a positive skill price. The Roy model is an important tool

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<sup>1</sup>As we explain at length below, this single skill can represent an underlying vector of many worker characteristics. The key is that the single skill is a sufficient statistic for this vector.

for thinking about between-occupation inequality, but this stark pattern of skill prices limits its usefulness for thinking about within-occupation inequality. If only one skill contributes to equilibrium wages and the price of that skill is the same for all workers within a given occupation, changes in that skill price change within-occupation wage inequality only through selection effects. Our model allows each occupation to use multiple skills, giving rise to the possibility that changes in skill prices lead to both changes in between-occupation inequality *and* changes in within-occupation inequality. In our model, even within narrowly-defined occupations, different skill prices matter more for some workers than for others.

Whether the equilibrium is bundled or not depends in large part on how intensive each occupation is in different skills. In an extension, we allow firms to choose an *appropriate technology*. Following [Caselli and Coleman \(2006\)](#) we allow firms to choose their skill intensities from a menu of possible intensities that we refer to as the *technology frontier*. In this version of the model, individual firms take skill prices as given and choose the skill intensities on the technology frontier that are optimal for those skill prices. But in turn skill prices depend in equilibrium on the skill intensities of all firms, just as in our benchmark model. We solve the resulting fixed point problem and characterize the effects of such technological choices. Suppose the underlying equilibrium with exogenous technologies is bundled, with positive skill premia. Do firms choose technologies that, in equilibrium, relax the bundling constraints, thereby decreasing skill premia and within occupation inequality? Or do firms choose technologies that, in equilibrium, tighten the bundling constraints, thereby increasing skill premia and within occupation inequality? The answer turns out to depend on whether the skills used by an occupation are *substitutes* or *complements*.

When the skills used by an occupation are substitutes, firms make their technology less intensive in skills that have a high price relative to their marginal product and more intensive in skills that have a low price relative to their marginal product. In other words, firms choose technologies that are less intensive in their ‘weak links’. This makes the technology more specialized, more like a standard Roy model, and tightens the bundling constraints relative to what they would be if the technology was exogenous, thereby increasing skill premia. In turn, the increasing skill premia generates increasing wage inequality, in the form of *polarized* wage gains. There are relatively large wage gains for *specialists*, workers with high endowments of one specific skill, and smaller wage gains (or wage losses) for *generalists*, workers with more evenly-balanced bundles of skills.

By contrast when the skills used by an occupation are complements, firms make their technology more intensive in skills that have a high price relative to their marginal product and less intensive in skills that have a low price relative to their marginal product. In other words, when the skills are complements, firms choose technologies that make weak links *less weak*. This makes the technology less specialized and relaxes the bundling constraints relative to what they would be if the technology was exogenous, thereby decreasing skill premia. In turn, the decreasing skill premia generates decreasing wage inequality. There are relatively

large wage gains for generalists and smaller wage gains (or wage losses) for specialists.

To understand the decrease in wage inequality as a bundled equilibrium becomes increasingly unbundled, it is helpful to decompose a worker's wage in their chosen occupation into (i) their *outside option*, the wage they would obtain in their next best occupation, and (ii) the *rent* that accrues to workers with a higher comparative advantage in their chosen occupation relative to the marginal worker. The marginal worker in a given occupation is simply that worker whose next best occupation offers them the same wage as their current occupation. In a bundled equilibrium, the rent obtained by inframarginal workers can be considerable. As an equilibrium becomes unbundled, these inframarginal rents are dissipated, they are competed away. As the technology becomes less intensive in any particular skill, workers with different skill bundles become effectively more interchangeable. In the limit of a completely unbundled equilibrium, all workers are marginal, there are no inframarginal rents, i.e., no return to comparative advantage, and wage inequality is lower. The same forces operate in reverse if an unbundled equilibrium becomes bundled.

The model gives a new way to interpret the effects of technology adoption in the US labor market. In the complements case a change in the set of available technologies can lead a *bundled equilibrium to unbundle*. Technologies that were closely associated with specific occupations become less differentiated. Workers were initially strongly sorted across occupations. Individuals with relatively high endowments of one skill had a preferred sector, earning rents due to binding bundling constraints. After technology adoption these same workers are essentially indifferent with regards to their occupational choice and within occupation inequality flattens. As we discuss below, we think this is consistent with technology adoption and the evolution of low skill occupations in the US.

Our benchmark model features heterogeneous workers each endowed with an idiosyncratic indivisible bundle of two skills. Workers have identical preferences over a single final consumption good that serves as numeraire. The final good is produced using two occupations. Each occupation is a CES aggregate of the two skills. The occupations are differentially intensive in the different skills. All production technologies have constant returns to scale. We first solve a planning problem to characterize the efficient allocation. We show that the multipliers on bundling constraints reveal the planner's shadow values for skills. We then show how this planning problem can be decentralized, allowing us to interpret these shadow values as the skill prices in a competitive equilibrium. Since this competitive equilibrium corresponds to the solution of a planning problem, it is efficient. We then show how the parameters of the model, in particular the skill intensities in each occupational technology and the joint distribution of skills across workers, determine whether bundling constraints are tight or slack and hence determine whether the equilibrium is bundled or not. We use this characterization to determine the effects of changes in technology on skill premia and hence the distribution of wages within and between occupations. In an extension, we allow firms to choose technologies subject to a CES technology frontier. Since the underlying production

technologies are CES and the technology frontier is CES, the resulting ‘global’ technology is also CES, but with more substitutability (Growiec, 2013, 2018). In this extension with endogenous technology, skill premia become larger if occupations become more specialized and become smaller if occupations become less specialized.

**Bundling and sorting.** We build on the literature beginning with Mandelbrot (1962) that studies the allocation of individuals across sectors and associated skill premia when skills are bundled. Other key contributions include Rosen (1983), Murphy (1986) and Heckman and Scheinkman (1987). The standard Roy model, as in for example Heckman and Sedlacek (1985), is a special case of our model. Recent applications and extensions demonstrate the proliferation of the Roy model Klenow, Hsieh, Hurst and Jones (2019), Adao, Beraja and Pandalai-Nayar (2019), Burstein and Vogel (2017), Dvorkin and Monge-Naranjo (2019), Wolcott (2019), Ado (2016), and Ocampo (2018). In each case within occupation inequality is only driven by selection, and not skill prices.

One of the outcomes of our framework is a flexible notion of whether sorting occurs in equilibrium or not. A large literature on one-to-one matching starting with Becker (1973) considers workers with multi-dimensional skills matching with firms operating technologies that require multi-dimensional skill inputs and heterogeneity in the marginal productivity of different skills. See Lindenlaub (2017) for a prominent recent example of this line of work featuring two dimensional types. Eeckhout and Kircher (2018) consider the many-to-one matching case where firms employ multiple workers. In these frameworks the equilibrium always features sorting, and the question is one of degree. In our framework, for any given distribution of skills, there are regions of the parameter space governing technological intensity for which the equilibrium is unbundled, there is no sorting whatsoever and the distribution of comparative advantage is inconsequential for wages. Yet there can be nearby regions of the parameter space for which the equilibrium is bundled, there is determinate sorting and the distribution of comparative advantage matters for wages.

**Technology choice.** We use the tools developed by Caselli and Coleman (2006), Len-Ledesma and Satchi (2019), and Growiec (2013, 2018) to model the choice of technologies that are appropriate for a given skill endowment. Recent applications of similar ideas to the choice of technology at the firm level include Porzio (2017), Boehm and Oberfield (2018), and Haanwinckel (2018).

**Technological determinants of income inequality.** Our work is also related to recent theoretical models of the technological determinants of income inequality, including Adao et al. (2019), Hsieh and Rossi-Hansberg (2019), and Roys and Seshadri (2014) — a quantitative version of Garicano and Rossi-Hansberg (2006), extended to include human capital accumulation as in Ben-Porath (1967).

## 2 Occupational similarity and wage inequality

In this section we present two new facts on changes in occupational skill requirements and wage inequality in US data. For both facts, we show that low-skill and high-skill occupations have experienced different trends.

**Fact 1.** *Differential trends in occupational similarity.* Low-skill occupations have become more similar to one another in terms of skill requirements, but high-skill occupations have become more distinct from one another.

**Fact 2.** *Differential trends in residual wage inequality.* Within-occupation wage inequality has decreased in low-skill occupations but increases in high-skill occupations. Conditional on observables, workers within a given narrowly defined low skill occupation are getting paid more similarly.

Note that Fact 1 could be used to rationalize differential trends in *between-occupation* wage inequality, i.e., a decrease in between-occupation wage inequality amongst low-skill occupations and an increase in between-occupation wage inequality amongst high-skill occupations. But our emphasis is on the use of Fact 1 to explain Fact 2, the differential trends in *within-occupation* wage inequality. As we show below, our model provides a natural connection between these facts. This is important, because the within-occupation component of wage inequality is substantially larger than the between occupation component.

### Fact 1. Differential trends in occupational similarity

Our first new fact is differential trends in the skill requirements of US occupations over time: low-skill occupations have become more similar to one another, but high-skill occupations have become more distinct from one another. We show this by combining two sources of data: Department of Labor O\*NET data, and Bureau of Labor Statistics *Occupation and Employment Statistics* (OES) data. We use O\*NET to measure skill requirements of occupations and employment-by-occupation data to appropriately weight these skills across occupations.

We provide a brief overview of our approach, all in the details are in the Appendix. Throughout we split the data into two periods: 2003-2009 and 2010-2018.<sup>2</sup> We rank occupations by the fraction of workers with high school education or less and then split occupations such that half of employment is in low-skill occupations and half is in high-skill occupations.<sup>3</sup> We then employ the following method run separately for low-skill and high-skill occupations in each period:

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<sup>2</sup>O\*NET goes back to 2001, but the questions answered in the first two years are different.

<sup>3</sup>We use David Dorn's harmonized *occ1990* occupation codes <https://www.ddorn.net/data.htm>.

**(1) Aggregation.** O\*NET contains data on more than 250 occupational skill measures at the Standard Occupational Classification (SOC )code level. Using the OES we average these across occupations to the Census occupational code level consistent with the definition of occupations used in constructing Fact 2 below. We then normalize each skill measure by its mean and standard deviation across occupations. This gives data on occupations  $j = 1, \dots, J$  and skills  $m = 1, \dots, M$ .

**(2) Compression.** We adapt the procedure of [Lise and Postel-Vinay \(2020\)](#) to compress these  $M$  skills to a smaller set of  $K = 4$  components. This procedure has two steps:

- (i) **PRINCIPLE COMPONENTS.** First we construct  $K$  principal components. Let  $\mathbf{A}$  be the  $J \times M$  matrix of skill measures by occupation. Then we construct the  $J \times K$  matrix  $\widehat{\mathbf{P}}$  of orthogonal principal components satisfying the following:

$$\mathbf{A} = \widehat{\mathbf{A}}\widehat{\mathbf{P}}' + \mathbf{U}$$

- (ii) **ROTATION AND NAMING.** Second, we take the principal components and in order to ‘name’ the  $K$  skills we rotate these measures such that they satisfy  $K$  exclusion restrictions. We take the following four skills from O\*NET that coincide with those chosen by [Acemoglu and Autor \(2011\)](#), specifically<sup>4</sup>

- (A) O\*NET measure *Analyzing data / information*, representing Non-routine cognitive: Analytical skills
- (B) *Maintaining relationships*, representing Non-routine cognitive: Analytical interpersonal skills
- (C) *Importance of repeating the same task*, representing Routine cognitive skills
- (D) *Controlling machines and processes*, representing Routine cognitive skills

We then determine rotations  $\widetilde{\mathbf{A}}$  and  $\widetilde{\mathbf{P}}$  such that the first column of  $\widetilde{\mathbf{P}}$  places a weight of 1 on skill  $A$ . and zeros on skills  $B, C, D$ , the second column of  $\widetilde{\mathbf{P}}$  places a weight of 1 on skill  $B$ . and zeros on skills  $A, C, D$  and so on. This is achieved by a matrix  $\Psi$  that implements these exclusion restrictions and such that the following holds

$$\mathbf{A} = \underbrace{(\widehat{\mathbf{A}}\Psi')}_{\widetilde{\mathbf{A}}} \underbrace{(\Psi'^{-1}\widehat{\mathbf{P}}')}_{\widetilde{\mathbf{P}}} + \mathbf{U}.$$

The decomposition into  $\widetilde{\mathbf{A}}$  and  $\widetilde{\mathbf{P}}$  therefore explains the same fraction of the variance of the data  $\mathbf{A}$ , as the decomposition into  $\widehat{\mathbf{A}}$  and  $\widehat{\mathbf{P}}$ . We then treat the  $J \times K$  matrix  $\widetilde{\mathbf{A}}$  with entry  $a_{jk}$  as our final measure of skills for each occupation, with these skills named: Non-routine cognitive: Analytical, Non-routine cognitive: Analytical Interpersonal, Routine cognitive, Routine cognitive.

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<sup>4</sup>[Acemoglu and Autor \(2011\)](#) rank occupations on these four measures from O\*NET, rather than the principal components approach taken here.

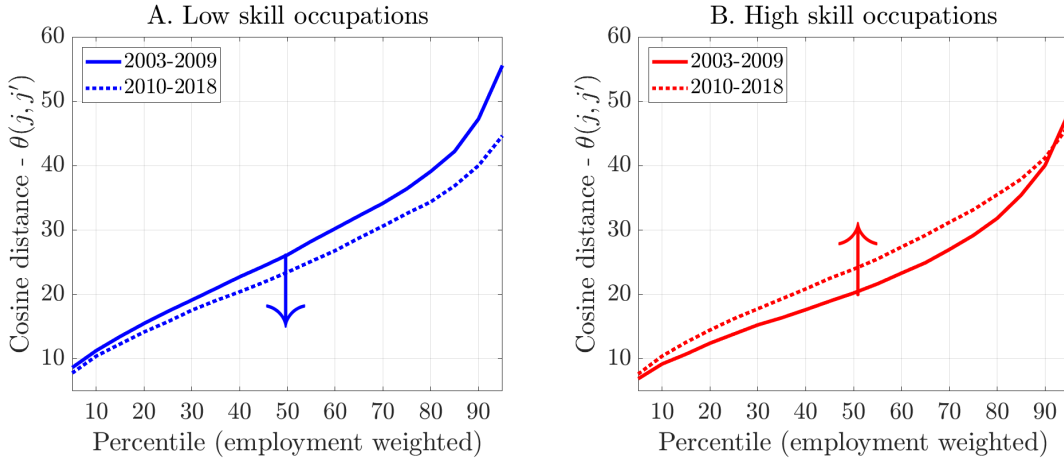


Figure 1: Cumulative employment weighted distribution of the distance between low and high skill occupations in the US: 2000-2019.

Notes Plots the cumulative distribution of the cosine distance  $\varphi(\mathbf{a}_j, \mathbf{a}_{j'})$  between all pairs of occupations  $j$  and  $j'$ . Panel A shows that for *low skill occupations*, the distribution for 2000-2010 first order stochastically dominates the distribution for 2011-2019, which demonstrates that occupations have become *more similar*. Panel B shows that for *high skill occupations*, the distribution for 2011-2019 first order stochastically dominates the distribution for 2000-2010, which demonstrates that occupations have become *more different*.

**(3) Distance.** Following [Gathmann and Schnberg \(2010\)](#), we compute the distance between pairs of occupations  $j, j'$  as the cosine distance between their skill vectors

$$\varphi(\mathbf{a}_j, \mathbf{a}_{j'}) = \cos^{-1} \left( \frac{\mathbf{a}_j^\top \mathbf{a}_{j'}}{\|\mathbf{a}_j\| \cdot \|\mathbf{a}_{j'}\|} \right) \in \left[ 0, \frac{\pi}{2} \right]$$

where  $\mathbf{a}_j$  denotes the  $K \times 1$  vector of skills for occupation  $j$  taken from  $\tilde{\mathbf{A}}$ . This gives us a  $J \times (J - 1)$  matrix of cosine distances.

**(4) Weighting.** We weight by employment. Each pair of occupations is weighted by  $\omega_{j,l}$  which is given by the product of the OES fraction of employment in each occupation:

$$\omega_{j,l} = \frac{n_j}{\sum_{j'} n_{j'}} \times \frac{n_l}{\sum_{l'} n_{l'}}.$$

Our key result is then clearly visible from [Figure 1](#), which plots the cumulative distribution of  $\varphi(\mathbf{a}_j, \mathbf{a}_{j'})$  for low and high skill occupations in the early and later period. As shown in Panel A, at all quantiles of the distribution of distances between occupations, low-skill occupations have become more similar to one another over this relatively short span of time. The distribution of distances between occupations in the earlier period first order stochastically dominates the distribution in the later period. The opposite is true for high-skill occupations, as shown in Panel B. At almost all quantiles of the distribution of distances between occupations, high-skill occupations have become more different from one another.



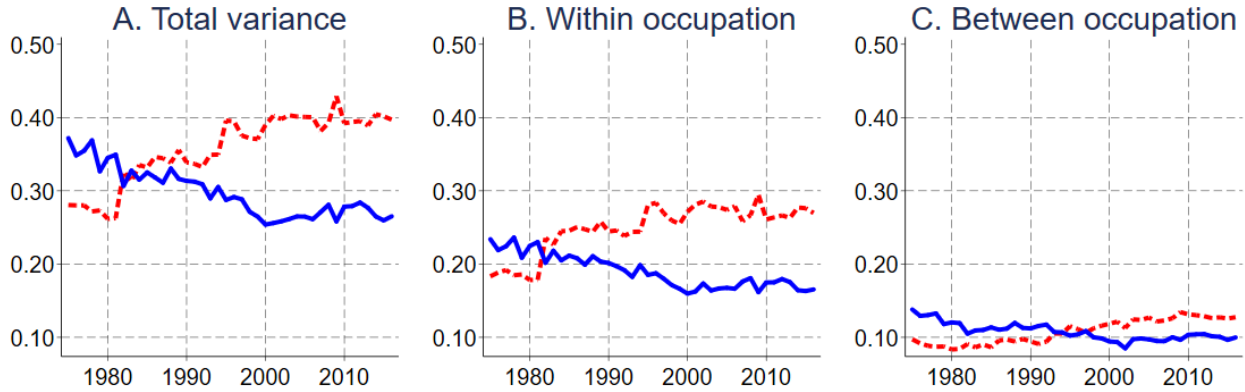


Figure 2: Residual wage inequality in low and high skill occupations in the US

Notes Plots the decomposition of residual wage inequality into within- and between-occupation components. The occupational categories *occskill1* (blue line) and *occskill2* (red line) are respectively *low* and *high* skill occupations, determined by whether an occupation has a fraction of individuals employed with a high school degree or less, that is above or below the median.

Our goal in this paper is to link the implications of these differential trends in occupational similarity to differential trends in wage inequality within occupations.

## Fact 2. Differential trends in residual wage inequality

While it is well-known that wage inequality has risen substantially in the US and many other countries, it is perhaps less well-known that wage inequality within low-skill occupations has fallen at the same time as overall wage inequality has risen. To document this, we use measures of residual wage inequality from the March CPS. We use the sample restrictions of Heathcote, Perri and Violante (2010) to construct measures of earnings and hours and then form the residualized log wages using standard regression controls (education, industry, experience, race) and controls for log hours. We estimate these specifications separately for low and high skill occupations, and separately in 5 year rolling windows. Further details are given in the Appendix.

Figure 2 shows the total variance of residualized log wages, its decomposition into within- (3-digit) occupation and between-occupation components, and the contributions to each of these components from low-skill occupations (blue) and high-skill occupations (red).<sup>5</sup> As Figure 2 shows, the total variance of residualized wages has risen for high-skill workers but fallen for low-skill workers. For both high- and low-skill workers the within-occupation component of total wage inequality is more than twice as large as the between-occupation component and the changes in within-occupation inequality account for most of the changes in total inequality. We will interpret the decline in wage inequality within low-skill occupations as evidence suggestive of low-skill workers becoming increasingly ‘commodified,’ that low-skill workers are becoming paid ‘more similarly’ over time.

<sup>5</sup>Recall that we rank occupations by the fraction of workers with high school education or less and then split occupations such that half of employment is in low- and half is in high-skill occupations.

**Differential trends in hours premia, experience premia and occupational mobility.** In [Section 7](#) below we describe in detail an additional body of new empirical facts that we try to understand through the lens of our model. We briefly outline these additional facts here, all of which our model will interpret similarly to Fact 2 — i.e., low skill workers are becoming increasingly commodified, with idiosyncratic skill differences mattering less for compensation and occupational choice.

These facts are as follows. First, in the wage regressions used to residualize wages for [Figure 2](#), the regression coefficients on various labor market characteristics have fallen for workers in low-skill occupations while remaining relatively stable for workers in high-skill occupations. The premia that workers in low-skill occupations historically received for being more experienced or for working longer hours have largely vanished ([Figure 10](#) and [Figure 11](#)). The corresponding premia for workers in high-skill occupations have not. Second, the rate of occupational mobility has increased substantially for workers in low skill occupations while remaining stable for workers in high skill occupations ([Figure 12](#) and [Figure A2](#)).

### 3 Model

Our key theoretical contribution is to show how changes in within-occupation wage inequality can emerge endogenously in response to changes in skill requirements across occupations. In this section we present our model and show how the equilibrium distribution of wages within and across occupations is determined by the underlying distribution of worker skills and occupational technologies. [Section 3.1](#) sets up the model. [Section 3.2](#) solves a planning problem to characterize the efficient allocation. [Section 3.3](#) decentralizes this problem to show that the competitive equilibrium is efficient. This lets us interpret the equilibrium price of each skill in terms of the shadow prices in the planning problem. In particular, we show how equilibrium skill premia can be interpreted in terms of the tightness of the bundling constraints facing the planner. Combining these results gives us a straightforward mapping between the occupational technologies and equilibrium wages.

#### 3.1 Setup

There is a unit mass of workers with heterogeneous endowments of two *skills*,  $X$  and  $Y$ . A homogeneous final good is produced using two *tasks* or *occupations* that are differentially intensive in the different skills.

**Skill endowments.** Worker  $i \in [0, 1]$  has endowments  $x(i) > 0$  and  $y(i) > 0$  of the two skills. We refer to a worker’s  $(x, y)$  pair as their *type*. Let  $H(x, y) = \text{Prob}[x(i) \leq x, y(i) \leq y]$

denote the distribution of skill types. The associated aggregate endowments are

$$\bar{X} := \int_0^1 x(i) di, \quad \text{and} \quad \bar{Y} := \int_0^1 y(i) di \quad (1)$$

**Occupational technologies.** Output of task  $j = 1, 2$  is a constant-returns-to-scale CES aggregate of the two skills

$$C_j = F_j(X_j, Y_j) = Z_j \left[ \alpha_j X_j^\sigma + (1 - \alpha_j) Y_j^\sigma \right]^{1/\sigma}, \quad 0 < \alpha_j < 1, \quad \sigma < 1 \quad (2)$$

The aggregates of skill  $X$  and skill  $Y$  in task  $j$  are

$$X_j = \int_{i \rightarrow j} x(i) di, \quad \text{and} \quad Y_j = \int_{i \rightarrow j} y(i) di \quad (3)$$

where the notation  $\{i \rightarrow j\}$  denotes the set of workers  $i$  assigned to task  $j$ . We will also say that workers doing task  $j$  have *occupation*  $j$ . The aggregate quantities  $X_j, Y_j$  of skills in task  $j$  are the sum of the individual skills of workers assigned to occupation  $j$ , i.e., the skills of individual workers are perfect substitutes *within* a given occupation.

**Final good and preferences.** The final good is a constant-returns-to-scale aggregate  $U(C_1, C_2)$  of output from the two tasks. Each worker has linear utility over the final good.

**Comparative advantage.** To simplify the exposition, we assume that occupation 1 is relatively intensive in skill  $X$  and occupation 2 is relatively intensive in skill  $Y$ , that is

$$\alpha_1 \geq \frac{1}{2} \geq \alpha_2 \quad (4)$$

Workers with high  $x(i)$  and will tend to have a comparative advantage in occupation 1 while workers with high  $y(i)$  will tend to have a comparative advantage in occupation 2.

**Individual-level bundling constraints.** Workers must supply both of their skills *to the same occupation*. If worker  $i$  chooses occupation  $j$  then worker  $i$  supplies both  $x(i)$  and  $y(i)$  to occupation  $j$ . Workers cannot supply  $x(i)$  to one occupation and  $y(i)$  to the other. We refer to this collection of restrictions as the *individual-level bundling constraints*, since they reflect the fact that the skills  $x(i), y(i)$  are physically bundled together in worker  $i$ .

We now want to understand the implications of the underlying distribution of skills  $H(x, y)$  and occupational technologies  $F_j(X, Y)$  for equilibrium wages in our economy. To do this, we first solve a planning problem to characterize the efficient allocation and then show that the competitive equilibrium in our model is efficient so that we can interpret equilibrium wages using the solution to the planning problem.

### 3.2 Efficient allocation

The *efficient allocation* is an allocation of workers to occupations that maximizes final output subject to the production technology and the feasibility constraints, including the constraint that each worker's skills  $x(i), y(i)$  must be allocated to the same occupation  $j$ .

Let  $\phi(i) \in [0, 1]$  denote the assignment of worker  $i$  with  $\phi(i) = 1$  if worker  $i$  is assigned to occupation 1 and  $\phi(i) = 0$  if worker  $i$  is assigned to occupation 2. A *direct approach* to the efficient allocation is to solve:

DIRECT PLANNING PROBLEM. Choose assignment  $\phi(i) \in [0, 1]$  for each  $i$  to maximize

$$U(F_1(X_1, Y_1), F_2(X_2, Y_2)) \tag{5}$$

subject to

$$X_1 = \int_0^1 x(i)\phi(i) di, \quad X_2 = \bar{X} - \int_0^1 x(i)\phi(i) di \tag{6}$$

$$Y_1 = \int_0^1 y(i)\phi(i) di, \quad Y_2 = \bar{Y} - \int_0^1 y(i)\phi(i) di \tag{7}$$

where  $X_j$  and  $Y_j$  denote the aggregate skill capacities assigned to occupation  $j$ .

Let  $\lambda_{jk}$  denote the shadow price of skill  $k = X, Y$  in occupation  $j = 1, 2$ . The planner's first order conditions include

$$\lambda_{jk} = U_j F_{jk} > 0 \tag{8}$$

so that at the efficient allocation the shadow price of skill  $k$  in occupation  $j$  is the social marginal product of skill  $k$  in occupation  $j$ . Since the Lagrangian associated with the planner's problem is linear in  $\phi(i)$ , the assignment of workers to occupations is generally at a corner. The planner sets  $\phi(i) = 1$  and assigns worker  $i$  to occupation 1 whenever

$$\lambda_{1X} x(i) + \lambda_{1Y} y(i) \geq \lambda_{2X} x(i) + \lambda_{2Y} y(i)$$

or, equivalently, whenever worker  $i$  has relative endowment

$$\frac{x(i)}{y(i)} \geq \frac{\lambda_{2Y} - \lambda_{1Y}}{\lambda_{1X} - \lambda_{2X}} \tag{9}$$

To understand the role of the bundling constraints in this setting, first consider a *relaxed problem* where the planner can unbundle a given worker's skill capacities and allocate them to different occupations, say  $\phi_X(i) = 1$  if  $x(i)$  is assigned to  $j = 1$  and  $\phi_Y(i) = 1$  if  $y(i)$  is assigned to  $j = 1$ . Now with the ability to separately assign skills, the planner's optimality conditions give  $\lambda_{1X} = \lambda_{2X}$  and  $\lambda_{1Y} = \lambda_{2Y}$  so that we obtain the usual tangency conditions  $U_1 F_{1X} = U_2 F_{2X}$  and  $U_1 F_{1Y} = U_2 F_{2Y}$ . At the solution of this relaxed problem the planner values skill  $X$  the same in either occupation and values skill  $Y$  the same in either occupation.

At the solution, the planner is indifferent to the assignment of any given worker to any occupation — any given worker has the same value in either occupation. We call the allocation associated with the solution of this relaxed problem an *unbundled allocation*.

To recover our original problem we can then add the constraints  $\phi_X(i) = \phi_Y(i)$  so that the skill capacities must be assigned to the same occupation. This approach introduces a continuum of constraints and a continuum of multipliers, one for each  $i \in [0, 1]$ . Instead of working with this continuum of individual-level bundling constraints, however, it turns out to be helpful to work instead with a single *aggregate bundling constraint* that summarizes the features of the individual-level bundling constraints that are relevant for the planning problem. This reduces the dimension of the problem, letting us state the planning problem only in terms of the of the skill aggregates  $X_j, Y_j$  for  $j = 1, 2$ .

**Aggregate bundling constraint.** Let  $\mathcal{E} = [0, \bar{X}] \times [0, \bar{Y}]$  denote the *Edgeworth box* with the understanding that if  $(X_1, Y_1) \in \mathcal{E}$  then  $X_2 = \bar{X} - X_1$  and  $Y_2 = \bar{Y} - Y_1$ . When skills are indivisibly bundled in workers, the feasible set for the planner is not  $\mathcal{E}$  but rather a subset  $\mathcal{B} \subseteq \mathcal{E}$  constructed so as to respect the individual-level bundling constraints.

Without loss of generality we can order workers in terms of their relative endowment  $x(i)/y(i)$ .<sup>6</sup> Suppose workers are ordered in terms of decreasing  $x(i)/y(i)$ . Then:

LEMMA 1. The feasible set for the planner's problem is

$$\mathcal{B} := \left\{ (X_1, Y_1) \in \mathcal{E} : Y_1 \in [\underline{B}(X_1), \bar{B}(X_1)] \text{ for each } X_1 \in [0, \bar{X}] \right\} \quad (10)$$

where the bounds  $\underline{B}(X)$  and  $\bar{B}(X)$  are given by

$$\underline{B}(X) := \int_0^{i_{min}^*(X)} y(i) di, \quad \text{and} \quad \bar{B}(X) := \int_{i_{max}^*(X)}^1 y(i) di \quad (11)$$

and the cutoffs  $i_{min}^*(X)$  and  $i_{max}^*(X)$  satisfy

$$X = \int_0^{i_{min}^*(X)} x(i) di = \int_{i_{max}^*(X)}^1 x(i) di \quad (12)$$

The feasible set  $\mathcal{B}$  is constructed as follows. We first ask, for a given amount of skill  $X$  in occupation 1,  $X_1 \in [0, \bar{X}]$ , what is the *minimum* amount of skill  $Y$  bundled with  $X_1$ ? Call this amount  $\underline{B}(X_1)$ . Since workers are ordered in terms of decreasing  $x(i)/y(i)$  this minimum amount of  $Y$  is obtained by starting with  $i = 0$  and adding workers with higher  $i$  only until we have reached the required amount  $X_1$ . Let  $i_{min}^*(X_1)$  denote the cutoff worker implied by this assignment.<sup>6</sup> Likewise, for the same  $X_1$  we can ask what is the *maximum* amount of skill  $Y$  that can be allocated to occupation 1. Call this amount  $\bar{B}(X_1)$ . This maximum amount

<sup>6</sup>If the distribution  $H(x, y)$  has mass points, the identity of the cutoff worker may be indeterminate but if so all candidates for the cutoff worker have the same  $(x, y)$  type.

of  $Y$  is obtained by starting with  $i = 1$  and adding workers with lower  $i$  only until we have again reached  $X_1$ . Let  $i_{max}^*(X_1)$  denote the cutoff worker implied by this assignment.

In short we require

$$Y_1 \in [\underline{B}(X_1), \overline{B}(X_1)]$$

Put differently, if, for a given  $X_1$ , the amount  $Y_1$  is *not* in  $[\underline{B}(X_1), \overline{B}(X_1)]$  then  $Y_1$  must be violating *some* of the individual-level bundling constraints. In what follows in a slight abuse of terminology we now use the term *aggregate bundling constraint* to refer to the constraint  $(X_1, Y_1) \in \mathcal{B}$  with the understanding that this aggregate constraint derives from the underlying bundling of skills  $x(i), y(i)$  in individual workers.

**The feasible set  $\mathcal{B}$  is convex.** The key properties of the feasible set  $\mathcal{B}$  derive from the underlying distribution of skill types  $H(x, y)$ . In particular:

LEMMA 2.

- (i) The set  $\mathcal{B}$  is convex. The boundaries  $\underline{B}(X)$  and  $\overline{B}(X)$  are continuous increasing functions from  $\underline{B}(0) = \overline{B}(0) = 0$  to  $\underline{B}(X) = \overline{B}(X) = Y$ .
- (ii) If  $H(x, y)$  has no mass points, the lower boundary  $\underline{B}(X)$  is strictly convex and the upper boundary  $\overline{B}(X)$  is strictly concave with derivatives

$$\underline{B}'(X) = \frac{y(i_{min}^*(X))}{x(i_{min}^*(X))}, \quad \text{and} \quad \overline{B}'(X) = \frac{y(i_{max}^*(X))}{x(i_{max}^*(X))} \quad (13)$$

Geometrically, the set  $\mathcal{B}$  is a *convex lens* in the Edgeworth box  $\mathcal{E}$ , as in [Rosen \(1983\)](#) and [Heckman and Scheinkman \(1987\)](#). The basic shape of the lens is intuitive. First, if no skill  $X$  is assigned to occupation 1 then no skill  $Y$  is assigned to occupation 1 either. If more of skill  $X$  is assigned to occupation 1, there cannot be less of skill  $Y$  assigned to occupation 1, so the lower boundary  $\underline{B}(X)$  is increasing. Likewise the more of skill  $X$  that is assigned to occupation 1, the more of skill  $Y$  may also be assigned to occupation 1 too, so the upper boundary  $\overline{B}(X)$  is also increasing. In the limit, if all of skill  $X$  is assigned to occupation 1 then all of skill  $Y$  must also be assigned to occupation 1.

The properties in part (i) hold for any distribution of skill types  $H(x, y)$ . The additional curvature properties in part (ii) follow when we impose more structure on  $H(x, y)$ . If  $H(x, y)$  is smooth, containing no mass points,<sup>7</sup> we can differentiate the expression for  $\underline{B}(X)$  in equations (11) and (12) with respect to  $X$  to obtain, for example,

$$\underline{B}'(X) = y(i_{min}^*(X)) \frac{di_{min}^*}{dX}, \quad \text{and} \quad 1 = x(i_{min}^*(X)) \frac{di_{min}^*}{dX}$$

so that we find that the gradient of the boundary at  $X$  is determined by the relative endowment of the cutoff worker at that point

$$\underline{B}'(X) = \frac{y(i_{min}^*(X))}{x(i_{min}^*(X))} > 0 \quad (14)$$

<sup>7</sup>If  $H(x, y)$  is discrete the boundaries of  $\mathcal{B}$  are given by piecewise line segments, as in [Rosen \(1983\)](#).

Intuitively, there are diminishing returns to selecting workers in order of their comparative advantage, giving rise to the convexity of  $\underline{B}(X)$ . If we start by selecting workers with high comparative advantage in occupation 1, with high  $x(i)$  and/or low  $y(i)$ , then initially the gradient  $\underline{B}'(X)$  is low, but as we progress to adding workers with low comparative advantage in occupation 1, with low  $x(i)$  and/or high  $y(i)$ , the gradient  $\underline{B}'(X)$  becomes steeper.

**Example.** Let  $H_X(x)$  and  $H_Y(y)$  denote the marginal distributions of  $x$  and  $y$  and write the joint distribution  $H(x, y) = \mathcal{C}(H_X(x), H_Y(y))$  where the copula  $\mathcal{C}$  controls the dependence between  $x$  and  $y$ . For illustrative purposes, suppose the marginal distributions are Fréchet,  $H_X(x) = \exp(-x^{-\theta})$  and  $H_Y(y) = \exp(-y^{-\theta})$  with common shape parameter  $\theta$  and the copula  $\mathcal{C}$  is Gaussian with linear correlation  $\rho$ , as in Figure 3. Panel A shows a scatterplot of the individual skills  $x(i), y(i)$  drawn with positively correlated skills (blue) and negatively correlated skills (red). Panel B shows the implied feasible sets  $\mathcal{B}$  constraining the aggregate skills  $X, Y$ . When individual skills are very positively correlated, there is a strong pattern of *absolute advantage* but a weak pattern of *comparative advantage*. Individuals tend to have either  $x(i), y(i)$  both high or both low and hence for a given amount of  $X$  assigned to occupation 1 it does not much matter how workers are selected — the amount of  $Y$  bundled with that  $X$  is similar, so the bounds  $\underline{B}(X), \overline{B}(X)$  are close together. In the limit where all workers have the same  $x(i)/y(i)$  ratio, the bounds  $\underline{B}(X), \overline{B}(X)$  collapse to a single line. But when individual skills are very negatively correlated, there is instead a strong pattern of comparative advantage. Individuals who have high  $x(i)$  tend to have low  $y(i)$  while individuals with low  $x(i)$  tend to have high  $y(i)$ . Hence for a given amount of  $X$  assigned to occupation 1 it now matters greatly how workers are selected — the amount of  $Y$  bundled with that  $X$  will be low if high  $x(i)/y(i)$  workers are selected but will be high if low  $x(i)/y(i)$  workers are selected, so the bounds  $\underline{B}(X), \overline{B}(X)$  are instead far apart.

With the feasible set  $\mathcal{B}$  in hand, we can now restate the planning problem using the aggregate bundling constraint. To simplify the exposition, throughout we focus on the case of a smooth distribution of skill types  $H(x, y)$  so that the boundaries of  $\mathcal{B}$  are differentiable.

<p>INDIRECT PLANNING PROBLEM. Choose aggregate skills <math>X_j, Y_j</math> for <math>j = 1, 2</math> to maximize</p> $U(F_1(X_1, Y_1), F_2(X_2, Y_2)) \tag{15}$ <p>subject to</p> $X_1 + X_2 \leq \overline{X}, \quad Y_1 + Y_2 \leq \overline{Y} \tag{16}$ <p>and the aggregate bundling constraint</p> $\underline{B}(X_1) \leq Y_1 \leq \overline{B}(X_1) \tag{17}$
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Since  $\underline{B}(X_1)$  is strictly increasing and strictly convex while  $\overline{B}(X_1)$  is strictly increasing

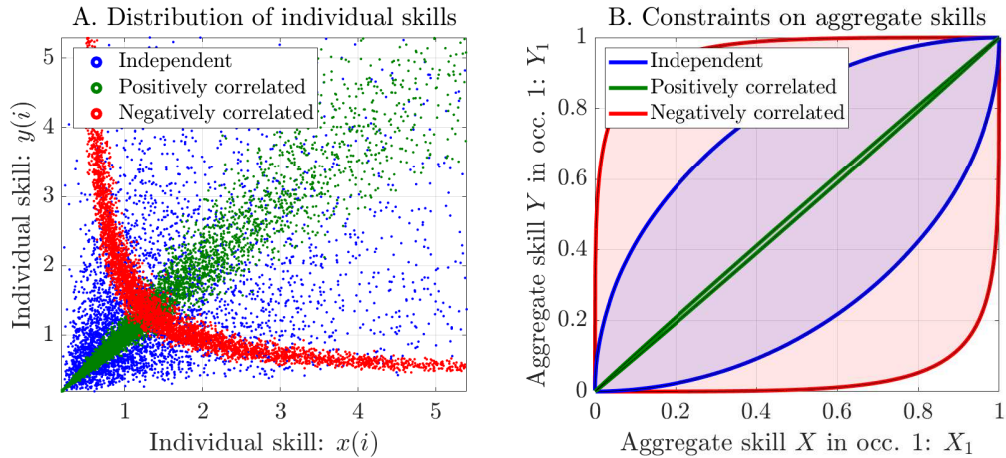


Figure 3: Distribution of individual skills and aggregate bundling constraints

Notes: Joint distribution  $H(x, y) = \mathcal{C}(H_X(x), H_Y(y))$  where the marginal distributions are Fréchet with shape parameter  $\theta = 1.3$  and the copula is Gaussian with linear correlation  $\rho$ . Panel A shows draws of individual skills  $x(i), y(i)$ . Panel B shows the implied feasible sets  $\mathcal{B}$  constraining the aggregate skills  $X, Y$ .

and strictly concave, *at most one* of these constraints will bind. Since we assumed occupation 1 is skill  $X$  biased, the constraint  $Y_1 \leq \overline{B}(X_1)$  will be slack. Let  $\mu \geq 0$  denote the multiplier on the remaining constraint  $\underline{B}(X_1) \leq Y_1$ . The efficient allocation consists of  $X_1, Y_1$  that satisfy the two first order conditions

$$X_1 : \quad U_1 F_{1X} = U_2 F_{2X} + \mu \underline{B}'(X_1) \quad (18)$$

$$Y_1 : \quad U_2 F_{2Y} = U_1 F_{1Y} + \mu \quad (19)$$

To interpret these conditions, observe that if the bundling constraint is slack,  $\mu = 0$ , then we recover the *unbundled allocation* we previously defined, characterized by a single shadow price for each skill, say  $\lambda_X = U_1 F_{1X} = U_2 F_{2X}$  and  $\lambda_Y = U_2 F_{2Y} = U_1 F_{1Y}$ . However, if the bundling constraint binds,  $\mu > 0$ , we have a *bundled allocation*. The bundling constraint inhibits the planner's efforts to allocate more of skill  $X$  and skill  $Y$  towards the occupations intensive in their use, creating a wedge between the value of skill  $X$  in occupation 1 relative to occupation 2 and a wedge between the value of skill  $Y$  in occupation 2 relative to occupation 1. These wedges are larger the larger is the multiplier and are amplified by the relative endowment of the marginal worker,  $\underline{B}'(X_1) = y(i^*)/x(i^*)$ . If the marginal worker has high  $y(i)$  relative to  $x(i)$ , then shifting the worker from occupation 2 to occupation 1 does little to increase  $X_1$ .

Substituting out the multiplier, the allocation  $X_1, Y_1$  satisfies

$$U_1 [F_{1X} + \underline{B}'(X_1) F_{1Y}] = U_2 [F_{2X} + \underline{B}'(X_1) F_{2Y}], \quad Y_1 = \underline{B}(X_1) \quad (20)$$

The first condition is intuitive. A small increase in  $X_1$  increases the production of occupation 1 by  $F_{1X} + \underline{B}'(X_1) F_{1Y}$ , i.e., the direct marginal product  $F_{1X}$  plus the effect of the additional  $\underline{B}'(X_1)$  units of skill  $Y$  bundled with the increase in  $X_1$ , each of which has marginal product  $F_{1Y}$ . Multiplying by  $U_1$  then gives the social marginal product of this increase in occupation



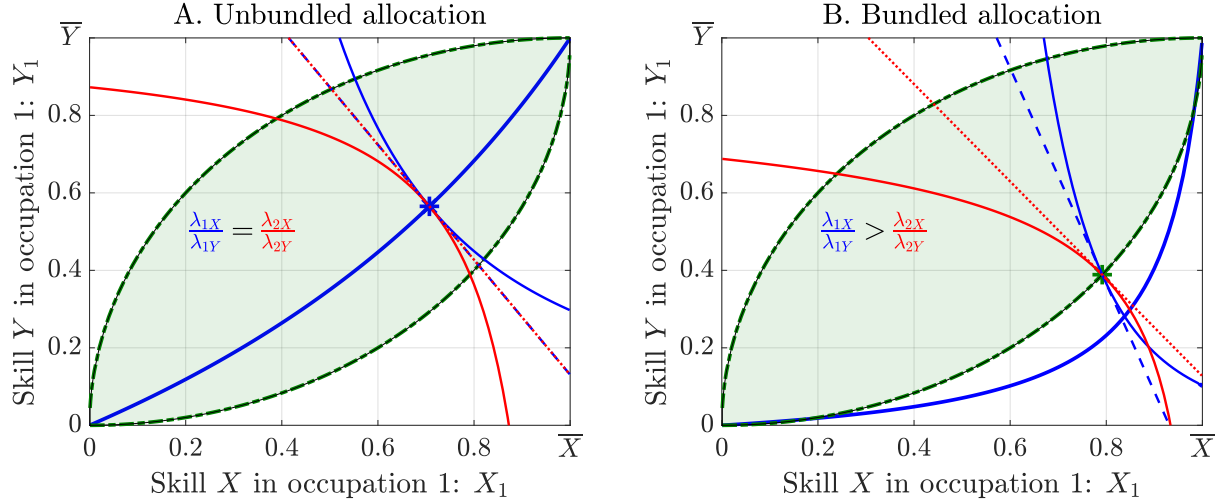


Figure 4: Unbundled and bundled allocations.

[X]

1. The opportunity cost of this increase in occupation 1 is a decrease in occupation 2. Specifically, a small increase in  $X_1$  decreases production of occupation 2 by  $F_{2X} + \underline{B}'(X_1)F_{2Y}$ , i.e., the direct marginal product  $F_{2X}$  plus the effect of the additional loss of  $\underline{B}'(X_1)$  units of skill  $Y$ , each of which has marginal product  $F_{2Y}$ . Multiplying by  $U_2$  then gives the social marginal product of this decrease in occupation 2. At the optimum, the planner cannot be made better off by such changes, giving equation (20).

We illustrate the two possibilities in Figure 4. In each panel we show the Edgeworth box  $\mathcal{E}$  and the technical contract curve, the locus of points of equalized marginal rates of technical substitution between skills  $F_{jX}/F_{jY}$ . In Panel A we show an *unbundled allocation* where the marginal rate of substitution  $U_1/U_2$  between occupations is equated to the marginal rate of transformation  $F_{2k}/F_{1k}$  at a point that is interior to  $\mathcal{B}$ . The aggregate bundling constraint is slack,  $\mu = 0$  and the allocation  $X_j, Y_j$  for  $j = 1, 2$  is found where  $U_1/U_2 = F_{2X}/F_{1X} = F_{2Y}/F_{1Y}$  given  $X_2 = \bar{X} - X_1$  and  $Y_2 = \bar{Y} - Y_1$ . In Panel B we show a *bundled allocation* where the marginal rate of substitution between occupations  $U_1/U_2$  cannot be equated to the marginal rate of transformation  $F_{1k}/F_{2k}$  at a point interior to  $\mathcal{B}$ . The aggregate bundling constraint binds,  $\mu > 0$  and the allocation  $X_j, Y_j$  is found on the boundary where  $Y_1 = \underline{B}(X_1)$  at the point where the gradient satisfies

$$\underline{B}'(X_1) = \frac{U_1 F_{1X} - U_2 F_{2X}}{U_2 F_{2Y} - U_1 F_{1Y}}$$

given  $X_2 = \bar{X} - X_1$  and  $Y_2 = \bar{Y} - Y_1$ . In solving the model we first guess  $\mu = 0$ , compute an unbundled allocation, check if that candidate solution satisfies  $Y_1 \geq \underline{B}(X_1)$ . If it does, we are done. If not we let  $\mu > 0$  and solve for an unbundled allocation.

An unbundled allocation is supported by a gradient for shadow skill prices  $\lambda_X/\lambda_Y$  common to both occupations. But in a bundled allocation there are distinct gradients  $\lambda_{jX}/\lambda_{jY}$  for

each occupation  $j = 1, 2$ . Ordinarily such a difference in valuations would suggest the planner can achieve a better allocation by moving  $X$  and  $Y$  from where they are valued less to where they are valued more. In the example in Panel B, the planner would like to reallocate  $X$  from occupation 2 to occupation 1 and reallocate  $Y$  from occupation 1 to occupation 2, but this deviation is infeasible because it violates the aggregate bundling constraint.

**Shadow price for each worker type.** The solution to the planner's problem yields a set of aggregate shadow prices  $\lambda_{jX}, \lambda_{jY}$  for each aggregate skill  $X_j, Y_j$ . These aggregate shadow prices also imply shadow prices for each worker type and hence each individual worker. The shadow price for a worker of type  $x, y$  in occupation  $j$  is

$$\Lambda_j(x, y) = \lambda_{jX} x + \lambda_{jY} y = U_j (F_{jX} x + F_{jY} y) \quad (21)$$

and the overall shadow price for worker  $x, y$  is then  $\Lambda(x, y) = \max_j \Lambda_j(x, y)$ . In an unbundled allocation, at the efficient allocation of  $X_j, Y_j$ , the worker's value  $\Lambda_j(x, y)$  is the same in both occupations  $j = 1, 2$ , and the planner is indifferent to the allocation of individual workers to occupations since any given worker has the same value everywhere. In a bundled allocation the planner sorts workers into occupations depending on where they have highest value.

Before characterizing the conditions under which the aggregate bundling constraint binds or is slack, and hence the efficient allocation is bundled or unbundled, we show that the planner's problem can be decentralized as a competitive equilibrium. In this equilibrium, the shadow price  $\Lambda(x, y)$  corresponds to a worker's *wage*.

### 3.3 Competitive equilibrium

In this section we provide a decentralization of the planner's problem above. Since the competitive equilibrium of the decentralized economy coincides with the solution to the planner's problem, the competitive equilibrium is efficient.

**Final good producers.** Let the final good be the numeraire and let  $P_j$  denote the price of output from occupation  $j$  in final good units. The final good is produced by many identical perfectly competitive firms with the technology  $U(C_1, C_2)$ . Profit maximization by the final good producers gives the standard condition  $U_1/U_2 = P_1/P_2$ .

**Occupations.** Let  $W_j(x, y)$  denote the wage of a worker with type  $x, y$  in occupation  $j$ . Output in each occupation is produced by many identical perfectly competitive firms with the technology  $F_j(X, Y)$  in terms of the skill aggregates  $X, Y$ . The problem of the representative firm in occupation  $j$  can be broken into two steps: (i) choose the cost-minimizing bundles of skills (workers) to achieve given amounts of  $X_j, Y_j$ , then (ii) choose the amounts of  $X_j, Y_j$  to maximize profits.

For step (i), let  $L_j(x, y)$  denote the amount of workers of type  $x, y$  employed in occupation  $j$ . The cost of  $X_j, Y_j$  is then

$$\text{Cost}_j(X_j, Y_j) := \min_{L_j(x, y)} \int_0^1 W_j(x(i), y(i)) L_j(x(i), y(i)) di \quad (22)$$

subject to

$$X_j = \int_0^1 x(i) L_j(x(i), y(i)) di, \quad \text{and} \quad Y_j = \int_0^1 y(i) L_j(x(i), y(i)) di \quad (23)$$

Let  $\lambda_{jX}, \lambda_{jY}$  denote the multipliers for this problem. Since the objective and constraints are linear in  $L_j(x, y)$ , the solution is generally at a corner. The firm has labor demand  $L_j(x, y) = 1$  whenever  $W_j(x, y) < \lambda_{jX} x + \lambda_{jY} y$ . At the solution, the cost is

$$\text{Cost}_j(X_j, Y_j) = \lambda_{jX} X_j + \lambda_{jY} Y_j \quad (24)$$

For step (ii), the profit maximizing choice of  $X_j, Y_j$  is then pinned down by  $P_j F_{jX} = \lambda_{jX}$  and  $P_j F_{jY} = \lambda_{jY}$ . Since the technology has constant returns, these conditions imply zero profits.

**Workers.** Let  $w_j(i) = W_j(x(i), y(i))$  denote the wage of worker  $i$  in occupation  $j$ . All workers have linear utility over final output  $u(c) = c$  and maximize utility by choosing the occupation  $j$  that maximizes their wage income  $w(i) = \max_j w_j(i)$ , i.e., they choose occupation 1 if and only if  $w_1(i) \geq w_2(i)$ . Worker  $i$  then has consumption  $c(i) = w(i)$ .

**Equilibrium.** A *competitive equilibrium* is a price system  $P_j, W_j(x, y)$  and allocation  $X_j, Y_j$  such that (i) taking the prices as given, firms maximize profits, (ii) taking the prices as given, workers maximize utility, and (iii) markets clear.

Our first main theoretical result is then:

PROPOSITION 1.

- (i) The allocation  $X_j, Y_j$  and wages  $W_j(x, y)$  of the *competitive equilibrium* coincide with the allocation and shadow prices  $\Lambda_j(x, y)$  of the *indirect planning problem*.
- (ii) The allocation and shadow prices of the *indirect planning problem* coincide with the allocation and shadow prices of the *direct planning problem*.

The significance of this result is not that the equilibrium is efficient per se (part i), but by combining (i) and (ii) we can use the solution to the indirect planning problem to establish properties of wages in the competitive equilibrium. In particular, we can use the multiplier  $\mu$  on the aggregate bundling constraint, an object that does not appear in either the competitive equilibrium problem or the direct planning problem, to characterize wage inequality in equilibrium. Changes in the occupational technologies  $F_j(X, Y)$  or changes in endowments  $H(x, y)$  change wages both through the usual marginal conditions and through their effects on the bundling multiplier  $\mu$ .

**Equilibrium wages and sorting.** To illustrate these links, consider equilibrium wages and the sorting of workers into occupations. Firms in occupation  $j = 1, 2$  hire a worker of type  $x, y$  if and only if  $\lambda_{jX} x + \lambda_{jY} y \geq W_j(x, y)$  and workers choose occupation 1 if and only if  $W_1(x, y) \geq W_2(x, y)$ . Therefore, in equilibrium, workers in occupation  $j$  have wages

$$w_j(i) = \underbrace{W_j(x(i), y(i))}_{\text{Competitive equilibrium}} = \overbrace{\lambda_{jX} x(i) + \lambda_{jY} y(i)}^{\text{Planning problem}} = \Lambda_j(x(i), y(i)) \quad (25)$$

and the marginal worker is pinned down by

$$\underbrace{\frac{y(i^*)}{x(i^*)}}_{\text{Competitive equilibrium}} = \overbrace{\frac{\lambda_{1X} - \lambda_{2X}}{\lambda_{2Y} - \lambda_{1Y}}}^{\text{Planning problem}} = \underline{B}'(X_1) \quad (26)$$

To summarize, the multiplier on the aggregate bundling constraint  $\mu$  and the curvature of the boundary  $\underline{B}(X)$  play key roles in determining equilibrium skill premia. Shocks to technology or the distribution of skills that tighten the bundling constraint will tend to increase the premia that each skill commands in the occupation intensive in its use. As we will see next, the differentiability of the boundary  $\underline{B}(X)$  greatly simplifies the analysis of such shocks.

## 4 Technology and wage inequality

We now use our model to understand the effects of changes in technology on wage inequality within occupations. To begin with, [Section 4.1](#) explains how, in our model, increasing skill premia *between* occupations leads to increasing wage inequality *within* occupations. Analytical solutions for comparative statics that highlight these effects. [Section 4.2](#) considers symmetric changes in factor intensity when occupational technologies are mirror-images of each other. We show that as technologies become increasingly primary skill intensive, the bundling constraint tightens, increasing skill premia and increasing within-occupation inequality. In the Appendix we relax the symmetry assumption but instead assumes occupational technologies are Cobb-Douglas and skills are drawn from independent Fréchet distributions — parametric assumptions that allow for a complete analytical solution of the model.

### 4.1 From between-occupation to within-occupation inequality

**Wage inequality.** The wage of worker  $i$  in occupation  $j$  is given by  $w_j(i) = \lambda_{jX} x(i) + \lambda_{jY} y(i)$ . Consistent with [Section 2](#), our measure of wage inequality is the variance of log wages. To calculate this in our model, we first log-linearize the wage  $w_j(i)$ . Let  $\bar{w}_j :=$

$\exp(\mathbb{E}_j[\log w])$  denote the geometric mean wage in occupation  $j$ , and  $\widehat{w}_j(i) := \log w_j(i) - \log \bar{w}_j$  denote the log deviation of the wage from its geometric mean. Similarly  $\widehat{x}_j(i)$  and  $\widehat{y}_j(i)$  denote the within- $j$  log-deviations of skills from their geometric means  $\bar{x}_j$  and  $\bar{y}_j$ . Then to a first-order approximation

$$\widehat{w}_j(i) \approx \zeta_{jX} \widehat{x}_j(i) + \zeta_{jY} \widehat{y}_j(i) \quad (27)$$

where the elasticities  $\zeta_{jX}, \zeta_{jY}$  are weighted averages of the skill prices  $\lambda_{jX}, \lambda_{jY}$ , namely

$$\zeta_{jX} := \frac{\lambda_{jX} \bar{x}_j}{\lambda_{jX} \bar{x}_j + \lambda_{jY} \bar{y}_j}, \quad \zeta_{jY} := \frac{\lambda_{jY} \bar{y}_j}{\lambda_{jX} \bar{x}_j + \lambda_{jY} \bar{y}_j}, \quad (28)$$

Taking variances within occupation  $j$  and using  $\zeta_{jX} + \zeta_{jY} = 1$  gives

$$\text{Var}_j[\widehat{w}] = \text{Var}_j[\widehat{y}] + \zeta_{jX}^2 \text{Var}_j[\widehat{x} - \widehat{y}] + 2\zeta_{jX} \text{Cov}_j[\widehat{y}, \widehat{x} - \widehat{y}] \quad (29)$$

This expressions makes clear two determinants of within-occupation wage inequality: (i) within-occupation skill heterogeneity, conditional on selection, entering through the conditional moments of the distributions of  $X$  and  $Y$ , and (ii) the gradient of within-occupation skill prices, entering through the elasticity  $\zeta_{jX}$ .

**Role of skill heterogeneity.** To see the role of skill heterogeneity in isolation, consider the limit  $\zeta_{jX} \rightarrow 1$  so that only skill  $X$  matters. In this case  $\text{Var}_j[\widehat{w}] \rightarrow \text{Var}_j[\widehat{x}]$  so that wage inequality within occupation  $j$  reflects only the distribution of skill  $X$  selected into  $j$ . Likewise at the other extreme  $\zeta_{jX} \rightarrow 0$  so that only skill  $Y$  matters and  $\text{Var}_j[\widehat{w}] \rightarrow \text{Var}_j[\widehat{y}]$  so that wage inequality within occupation  $j$  reflects only the distribution of skill  $Y$  selected into  $j$ . In these extremes, implicitly, the skill price gradient is so steep that only one skill matters. For intermediate levels of  $\zeta_{jX}$  the dispersion in both skills matter.

**Role of skill price gradient.** To see the role of the skill price gradient  $\lambda_{jX}/\lambda_{jY}$  observe from (28) that the elasticity  $\zeta_{jX}$  is in one-to-one correspondence with the skill price gradient and that (29) is strictly convex in  $\zeta_{jX}$ . This gives:

**LEMMA 3.** The variance of log wages within occupation  $j$  is increasing in the skill price gradient  $\lambda_{jX}/\lambda_{jY}$  if and only if

$$\frac{\lambda_{jX}}{\lambda_{jY}} > \left( \frac{\text{Var}_j[\widehat{y}] - \text{Cov}_j[\widehat{x}, \widehat{y}]}{\text{Var}_j[\widehat{x}] - \text{Cov}_j[\widehat{x}, \widehat{y}]} \right) \left( \frac{\bar{y}_j}{\bar{x}_j} \right) \quad (30)$$

If occupation  $j$  uses skill  $X$  sufficiently intensively then an increase in the relative price of skill  $X$  increases wage inequality within occupation  $j$ . Roughly speaking, this suggests that shocks to the economy that increase the relative price of skill  $X$  will tend to increase wage inequality within occupations that are sufficiently intensive in skill  $X$ .

In the rest of this section we develop comparative statics results for special cases of our model for which we can explicitly link the endogenous variables on the left- and right-hand-sides of (30) to exogenous parameters. Our main results link the factor intensities that determine how different the occupational technologies are,  $\alpha_j$ , to equilibrium skill prices and then use (30) to link equilibrium skill prices to changes in wage inequality.

## 4.2 Symmetric changes in factor intensity

First we consider symmetric changes in the intensity with which each occupation  $j = 1, 2$  uses its primary skill, i.e., in the intensity  $\alpha_1$  with which occupation 1 uses skill  $X$  and the intensity  $1 - \alpha_2$  with which occupation 2 uses skill  $Y$ . To fix ideas, consider the following:

**SYMMETRIC ECONOMY.** The occupational technologies are *mirror images* of each other,  $\alpha := \alpha_1 = 1 - \alpha_2 \geq 1/2$  and equally productive,  $Z_1 = Z_2$ . Occupations have equal expenditure shares in final output,  $\eta = 1/2$ , and the aggregate amounts of each skill are the same,  $\bar{X} = \bar{Y}$ .

The Symmetric Economy places no restrictions on the elasticity of substitution in the occupational technologies,  $1/(1 - \sigma)$  or on the distribution of skills  $H(x, y)$ , though we continue to assume it is smooth such  $\underline{B}(X)$  is differentiable.

**Symmetric equilibrium.** The equilibrium of the symmetric economy is obviously symmetric, such that the quantities of primary skills are equal  $X_1 = Y_1$ . In a slight abuse of notation, let  $X$ , which is to be determined, denote these quantities:  $X := X_1 = Y_2$  such that  $X_2 = Y_1 = \bar{X} - X$ . Output of each occupation can then be expressed in terms of the allocation of primary skill alone, such that we can drop the  $j$  subscript

$$C = F(X) := Z [\alpha X^\sigma + (1 - \alpha)(\bar{X} - X)^\sigma]^{1/\sigma}, \quad \alpha \geq 1/2, \quad \sigma < 1 \quad (31)$$

Under symmetry marginal utilities are constant  $U_1 = U_2 = 1/2$ . The derivative  $F'(X)$  can therefore be interpreted as the *social marginal product of reallocating skills*:  $U_1 F_{1X} - U_2 F_{2X} = F'(X)/2$ . Using this, the first order conditions (18) of the indirect planning problem simplify:

$$\begin{aligned} F'(X)/2 &= \mu \underline{B}'(X) \\ F'(X)/2 &= \mu \end{aligned}$$

These conditions clearly bifurcate. An *unbundled equilibrium* of the symmetric economy has  $\mu = 0$  and the allocation of primary skill solves  $F'(X) = 0$ . There is zero social marginal product associated with reallocating skills. A *bundled equilibrium* of the symmetric economy has  $\mu = F'(X)/2 > 0$  and the allocation of primary skill solves  $\bar{X} - X = \underline{B}(X)$ . That  $F'(X) > 0$  indicates a positive social marginal product of reallocating skills at the optimum. Geometrically, the allocation is found where the downward sloping line  $\bar{X} - X$  extending from the northwest to southeast corners of the Edgeworth box  $\mathcal{E}$  intersects the boundary  $\underline{B}(X)$  of the feasible set  $\mathcal{B}$ . The corresponding premium for the primary skill is the multiplier  $\mu$ .

**Symmetric changes in factor intensity.** Consider changes in factor intensity  $\alpha \geq 1/2$ . Let  $\mu(\alpha)$  and  $X(\alpha)$  denote the multiplier and equilibrium allocation of primary skill.

PROPOSITION 2. For any *symmetric economy*, there is a unique factor intensity  $\alpha^*$  such that:

- (i) The equilibrium is unbundled if and only if  $\alpha \leq \alpha^*$ .
- (ii) If the equilibrium is unbundled, then  $X'(\alpha) > 0$ , and  $\mu(\alpha) = 0$ .
- (iii) If the equilibrium is bundled, then  $X(\alpha) = X(\alpha^*)$  and  $\mu(\alpha) > 0$  with  $\mu'(\alpha) > 0$ .

Intuitively, an increase in the factor intensity  $\alpha$  increases the demand for the primary skill. If the equilibrium is unbundled, then the allocation is interior to  $\mathcal{B}$  and the increase in demand can be accommodated by the distribution of skills. But if the equilibrium is bundled, the allocation is already on the boundary  $\underline{B}(X)$ , and an increase in  $\alpha$  only tightens the aggregate bundling constraint further, increasing the multiplier  $\mu(\alpha)$  and hence increasing the premium paid to the primary skill, without changing the allocation.

To see this, (31) delivers the marginal product of reallocating skills

$$F'(X; \alpha) = ZF(X; \alpha)^{1-\sigma} [\alpha X^{\sigma-1} - (1-\alpha)(\bar{X} - X)^{\sigma-1}] \quad (32)$$

In an unbundled equilibrium with  $\mu = 0$ , the allocation  $X(\alpha)$  solves  $F'(X; \alpha) = 0$  giving

$$X(\alpha) = \frac{\alpha^{\frac{1}{1-\sigma}}}{(1-\alpha)^{\frac{1}{1-\sigma}} + \alpha^{\frac{1}{1-\sigma}}} \bar{X}. \quad (33)$$

As the factor intensity  $\alpha$  increases from 1/2 to 1, this is strictly increasing from  $\bar{X}/2$  to  $\bar{X}$ . As  $X(\alpha)$  increases and  $\bar{X} - X(\alpha)$  decreases the economy eventually hits the boundary of the feasible set  $\mathcal{B}$ . Let  $\alpha^*$  denote the unique factor intensity solving

$$\bar{X} - X(\alpha^*) = \underline{B}(X(\alpha^*)) \quad (34)$$

In short we have  $\mu(\alpha) = 0$  and  $X(\alpha)$  given by equation (33) for all  $\alpha \leq \alpha^*$  while  $X(\alpha) = X(\alpha^*)$  for all  $\alpha > \alpha^*$  with multiplier  $\mu(\alpha) = F'(X(\alpha^*); \alpha)/2 > 0$ . In this range equation (32) shows that the multiplier is increasing in  $\alpha$ : the marginal product is increasing in  $\alpha$  for any given  $X$ , while at the boundary of  $\mathcal{B}$ ,  $X(\alpha^*)$  is constant. In other words, if the equilibrium is bundled and  $\alpha$  increases, the primary skill becomes even more valuable in its occupation.

The cutoff  $\alpha^*$  determined by equation (34) implicitly depends on the shape of the skill distribution  $H(x, y)$  via the boundary  $\underline{B}(X)$  and on the elasticity of substitution  $1/(1-\sigma)$ . Changes in the skill distribution  $H(x, y)$  shift the boundaries of  $\mathcal{B}$  and thereby change the cutoff. For example, more skill dispersion widens  $\mathcal{B}$ , increasing the cutoff  $\alpha^*$ , requiring greater asymmetry in technologies for the equilibrium to be bundled.

## Implications for wage inequality.

**PROPOSITION 3.** For each occupation  $j$  there is a unique factor intensity  $\alpha_j^{**} \geq \alpha^*$  such that an increase in  $\alpha$  increases the variance of log wages in occupation  $j$  if and only if  $\alpha > \alpha_j^{**}$ .

Intuitively, an increase in the factor intensity  $\alpha$  steepens the skill price gradient. When this effect is strong enough we know, from (30), that the variance of log wages will increase. Perhaps surprisingly, the cutoff  $\alpha_j^{**}$  is in general occupation specific. This is because we have not imposed any symmetry assumptions on the shape of the underlying joint distribution of skills  $H(x, y)$ . For example, if skill  $X$  and skill  $Y$  have different amounts of dispersion the cutoffs  $\alpha_j^{**}$  may differ across  $j$ .

To obtain this result, we first compute the skill price gradients  $\lambda_{jX}/\lambda_{jY} = F_{jX}/F_{jY}$  for  $j = 1, 2$  under the symmetric allocation. These work out to be

$$\frac{\lambda_{1X}}{\lambda_{1Y}} = \frac{\lambda_{2Y}}{\lambda_{2X}} = \begin{cases} 1 & \alpha \leq \alpha^* \\ \left(\frac{\alpha}{1-\alpha}\right) / \left(\frac{\alpha^*}{1-\alpha^*}\right) & \alpha > \alpha^* \end{cases} \quad (35)$$

If  $\alpha \leq \alpha^*$  we have an unbundled equilibrium, and the symmetric allocation  $X(\alpha)$  given by (33) is supported by a skill price gradient of 1 in both occupations. If  $\alpha > \alpha^*$  we have a bundled equilibrium, the symmetric allocation is constant at  $X(\alpha^*)$ , supported by skill price gradients that are strictly increasing in  $\alpha$ . In other words, if the equilibrium is unbundled an increase in the intensity  $\alpha$  of the primary skill is absorbed by an increasing allocation  $X(\alpha)$  to the primary skill but an unchanged relative price. But if the equilibrium is bundled an increase in the intensity  $\alpha$  of the primary skill is absorbed by an increasing relative price but an unchanged allocation  $X(\alpha^*)$ . Combining (35) with (30) we then find that the cutoff  $\alpha_1^{**}$  for increasing wage inequality in occupation  $j = 1$  is given by

$$\left(\frac{\alpha_1^{**}}{1-\alpha_1^{**}}\right) / \left(\frac{\alpha^*}{1-\alpha^*}\right) = \left(\frac{\text{Var}_1[\hat{y}] - \text{Cov}_1[\hat{x}, \hat{y}]}{\text{Var}_1[\hat{x}] - \text{Cov}_1[\hat{x}, \hat{y}]}\right) \left(\frac{\bar{y}_1}{\bar{x}_1}\right) \quad (36)$$

whenever the statistic on the RHS is larger than 1, otherwise we set  $\alpha_1^{**} = \alpha^*$ . Reversing the roles of  $X$  and  $Y$  we likewise find that the cutoff  $\alpha_2^{**}$  for increasing wage inequality in occupation  $j = 2$  is given by

$$\left(\frac{\alpha_2^{**}}{1-\alpha_2^{**}}\right) / \left(\frac{\alpha^*}{1-\alpha^*}\right) = \left(\frac{\text{Var}_2[\hat{x}] - \text{Cov}_2[\hat{x}, \hat{y}]}{\text{Var}_2[\hat{y}] - \text{Cov}_2[\hat{x}, \hat{y}]}\right) \left(\frac{\bar{x}_2}{\bar{y}_2}\right) \quad (37)$$

whenever the statistic on the RHS is larger than 1, otherwise we set  $\alpha_2^{**} = \alpha^*$ .

Importantly, in a symmetric bundled equilibrium the conditional moments on the RHS of these expressions are independent of the factor intensity  $\alpha$ . The gradient of the aggregate bundling constraint is  $\underline{B}'(X) = y(i^*)/x(i^*) = 1$  independent of  $\alpha$ . In other words, if  $\alpha > \alpha^*$  then further increases in  $\alpha$  have no additional selection effects. So we can be sure that for high enough  $\alpha$  the skill price gradient exceeds the RHS and further increases in  $\alpha$  increase wage inequality within each occupation.



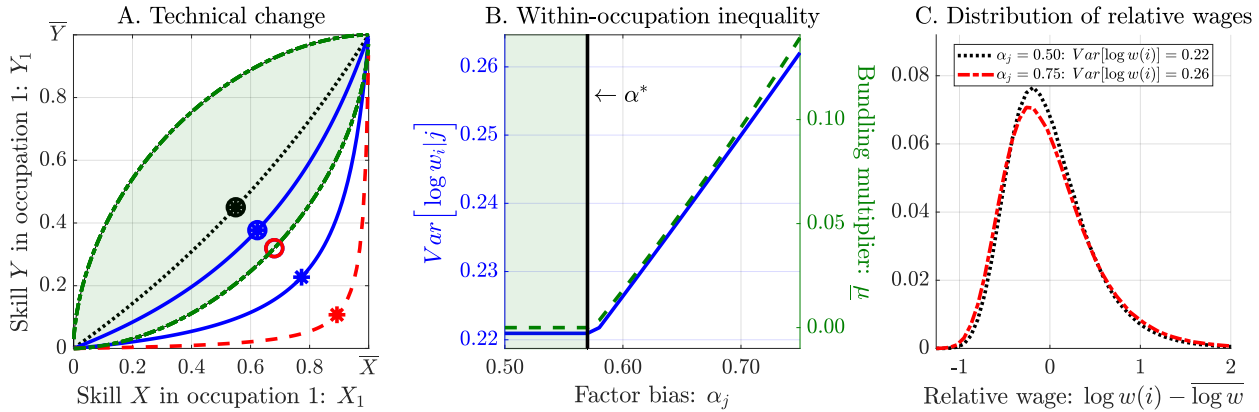


Figure 5: Increasing factor intensity  $\alpha$  and within-occupation wage inequality

**Numerical example.** We illustrate these effects in Figure 5. We suppose that skills  $H(x, y)$  are independent Fréchet and consider a sequence of symmetric economies indexed by  $\alpha$  from  $\alpha = 1/2$  to 1. Panel A shows the feasible set  $\mathcal{B}$  and the contract curves implied by the symmetric technology as  $\alpha$  increases. At  $\alpha = 1/2$  the allocation of primary skill  $X(\alpha)$  is inside the feasible set  $\mathcal{B}$  and the equilibrium is unbundled with  $\mu(\alpha) = 0$ . As  $\alpha$  increases the allocation  $X(\alpha)$  increases until we hit the boundary  $\underline{B}(X)$  of the feasible set. This implicitly determines the cutoff  $\alpha^*$ . As  $\alpha$  continues to increase, the allocation stays constant at  $X(\alpha^*)$  while the multiplier on the aggregate bundling constraint  $\mu(\alpha)$  increases. Panel B shows the implications for within-occupation wage inequality. As  $\alpha$  increases, the dispersion in log wages increases.

Once the aggregate bundling constraint binds,  $\mu(\alpha) > 0$ , increases in  $\alpha$  lead to a demand for the primary skill that cannot be met. Bringing in more of the primary skill would bring with it more of the secondary skill bundled with it. In this scenario with symmetrically increasing demand for primary skill in both occupations this is infeasible and the multiplier  $\mu(\alpha)$  rises accordingly. Put differently, in a bundled equilibrium there is a relative shortage of the primary skill and a relative excess of the secondary skill relative to the underlying demand from the technology. Once the bundling constraint binds, higher  $\alpha$  only amplifies this effect, increasing the premium paid to the primary skill which is in short supply. Wage inequality increases because wages increase for the workers that are strongly selected into their occupation (those with a relatively high endowment of the primary skill), but decrease for those who were only marginally selected into their occupation.

The same effects work in reverse. Consider a symmetric decrease in factor intensity  $\alpha$  starting from  $\alpha > \alpha^*$ . Initially the equilibrium remains bundled, the allocation of workers is unchanged but the multiplier  $\mu(\alpha)$  falls and reduces the premium for the primary skill. Wage inequality decreases because wages increase for the workers that are only marginally selected into their occupation (and who are at the bottom of the wage distribution) but decrease for those who were strongly selected into their occupation.

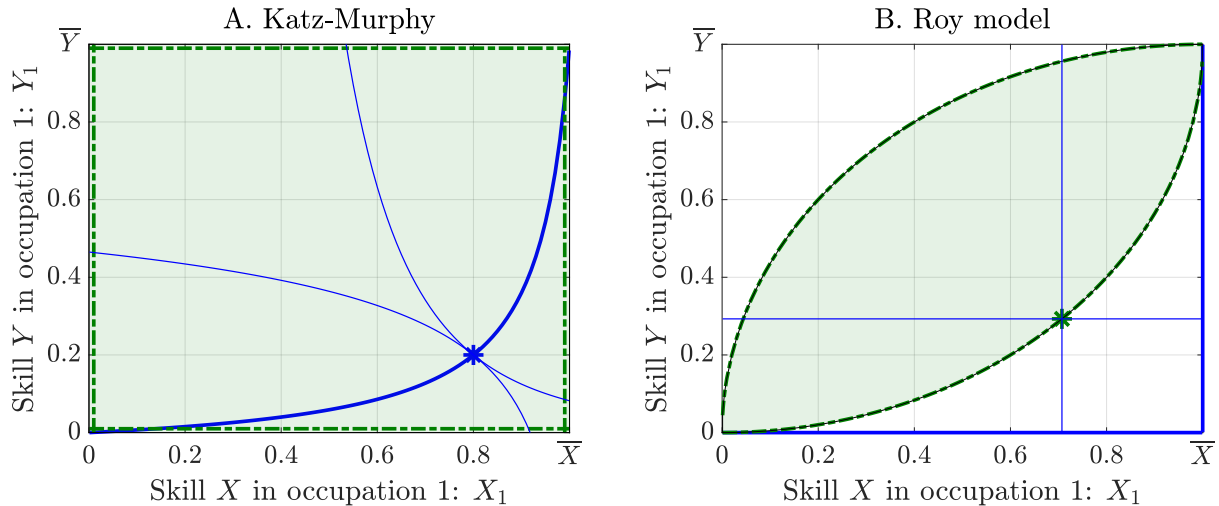


Figure 6: Special cases of bundled and unbundled equilibria

These symmetric examples are intuitive but raise many questions. What happens if only one occupation is becoming more intensive in its primary skill? What if both occupations are becoming more intensive in the same skill (for example occupation 1 is becoming more intensive in its primary skill but occupation 2 is becoming less intensive in its primary skill). In the next section we consider an extended example that allows us to relax the symmetry assumption and answer these questions.

## 5 Alternative competitive models

To help see the value of our framework, we show that three alternative models of skill and technology heterogeneity in competitive markets are, by contrast, *silent* on the relationship between changes in the differential skill intensity of technologies and changes in within-occupation wage inequality. Two of these alternatives — the Katz-Murphy model and the Roy model — are nested as special cases of our benchmark framework, while the third, a one-to-one matching model, can be obtained as a simple extension.

### 5.1 Katz-Murphy model

The Katz-Murphy model is silent on the relationship between technology and within-occupation inequality. To see this, consider the following interpretation of our model. We rename the skill aggregates *low skill*  $L$  and *high skill*  $H$  and write the occupational technologies

$$C_j = Z_j \left[ \alpha_j L_j^\sigma + (1 - \alpha_j) H_j^\sigma \right]^{1/\sigma}, \quad L_j = \int_{i \rightarrow j} l(i) di, \quad H_j = \int_{i \rightarrow j} h(i) di$$

Workers are either low-skill workers with type  $(l, h) = (l(i), 0)$  or high-skill workers with type  $(l, h) = (0, h(i))$ . In this setting, the bundling of skills in workers is irrelevant. Individuals

with one skill do not come bundled with another. This makes the entire Edgeworth box  $\mathcal{E} = [0, \bar{L}] \times [0, \bar{H}]$  feasible,  $\mathcal{B} = \mathcal{E}$ . In a sense, the orthogonal low-skill workers and high-skill workers are like ‘Arrow securities’ and the existence of a complete set of Arrow securities makes it possible to span the whole space, i.e., any point in  $\mathcal{E}$  can be obtained by choosing the appropriate combination of low-skill and high-skill types. Panel (a) of [Figure 6](#) illustrates.

Because the equilibrium is always unbundled, the price of each skill is equalized across occupations,  $\lambda_{1L} = \lambda_{2L}$  and  $\lambda_{1H} = \lambda_{2H}$ . Individual workers receive the same wage in both occupations,  $w(i) = \lambda_L l(i) + \lambda_H h(i)$ . In particular, regardless of occupation, low-skill workers get wage  $\lambda_L l(i)$  while high-skill workers get wage  $\lambda_H h(i)$ . Hence shocks to technology (or preferences) shift wages proportionately through the multiplicative skill prices  $\lambda_L, \lambda_H$  alone. The within-occupation variance of log wages remains constant, e.g., for low-skill workers  $\text{Var}(\log w(i) | j) = \text{Var}(\log l(i))$ , independent of changes in technology.

In this sense, the Katz-Murphy model is silent on the relationship between changes in the skill-intensity of technologies and within-occupation wage inequality.

## 5.2 Roy model

The Roy model is a popular framework for theoretical and empirical work on the allocation of heterogeneous workers across occupations and the implication of such allocations for wages. Where the Katz-Murphy model features a single dimension of skill but multiple inputs in production, the Roy model features multiple dimensions of skill but a single input in production, leading to a meaningful occupational choice problem for workers.

**Basic Roy model.** The basic Roy model can be obtained as a special case of our framework by taking the factor intensities  $\alpha_1 \rightarrow 1$  and  $\alpha_2 \rightarrow 0$  so that each occupation uses a single factor,  $C_1 = Z_1 X_1$  and  $C_2 = Z_2 Y_2$ . While the distribution of skills in the Katz-Murphy model is such that the equilibrium is always unbundled, regardless of the technology, the technology in the Roy model is such that the equilibrium is bundled, almost regardless of the distribution of skills.<sup>8</sup> Panel (b) of [Figure 6](#) illustrates. Given the technologies, it would seem natural to assign all of skill  $X$  to occupation 1 and all of skill  $Y$  to occupation 2. But this would put the allocation outside  $\mathcal{B}$ , violating the bundling constraints. Instead the equilibrium is found on the boundary of  $\mathcal{B}$  as shown.

In this equilibrium there are only two positive skill prices,  $\lambda_{1X}$  and  $\lambda_{2Y}$ . The amounts  $X_2 = \bar{X} - X_1$  and  $Y_1 = \bar{Y} - Y_2$  are unused and have skill prices  $\lambda_{2X} = \lambda_{1Y} = 0$ . Although perhaps unusual to analyze the Roy model in this way, our framework provides a new interpretation of wages in the Roy model in terms of the bundling multiplier

$$\lambda_{1X} = \mu \underline{B}'(X_1), \quad \text{and} \quad \lambda_{2Y} = \mu$$

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<sup>8</sup> Except in the degenerate case where workers have only one skill in positive amount, which puts us back in the Katz-Murphy framework where  $\mathcal{B} = \mathcal{E}$ .

For example, if skills become less dispersed (or more correlated), the feasible set  $\mathcal{B}$  shrinks and the bundling multiplier tightens, increasing both skill prices and increasing wages across the board. In other words, this way of analyzing the Roy model links the dispersion in skills to the *level* of wages, not just the relative wage.<sup>9</sup>

Although the equilibrium in the Katz-Murphy model is always unbundled while the equilibrium in the Roy model is almost always bundled, the two models are similarly limited in their ability to speak to within-occupation wage inequality. The common thread is that in both models wages depend on a *single factor*. In the Roy model wages are  $w_1(i) = \lambda_{1X} x(i)$  for workers who choose occupation 1 and  $w_2(i) = \lambda_{2Y} y(i)$  for workers who choose occupation 2. The role of technology here is limited to *selection*, i.e., the workers in occupation 1 do not represent the full distribution of  $x(i)$  but rather that set of workers who choose to specialize in occupation 1. Apart from these selection effects, the variance of log wages is again independent of skill prices and hence independent of technology.

**Generalized Roy model.** In practice, applied researchers typically work with a version of the Roy model that allows for richer heterogeneity. The *generalized Roy model* can be expressed as follows, consistent with, for example, [Firpo et al. \(2011\)](#) and [Roys and Taber \(2020\)](#). Let  $\boldsymbol{\xi}(i)$  denote a vector of worker characteristics and suppose the efficiency of a worker at a given task is an *occupation-specific* function of these characteristics, say

$$x(i) = \exp(\boldsymbol{\xi}(i)' \boldsymbol{\beta}_X), \quad y(i) = \exp(\boldsymbol{\xi}(i)' \boldsymbol{\beta}_Y) \quad (38)$$

This leads to equilibrium log wages of the form

$$\begin{aligned} \log w_1(i) &= \log \lambda_{1X} + \boldsymbol{\xi}(i)' \boldsymbol{\beta}_X, \\ \log w_2(i) &= \log \lambda_{2Y} + \boldsymbol{\xi}(i)' \boldsymbol{\beta}_Y, \end{aligned}$$

Notice that the only priced objects are the efficiency units in each occupation, which have skill prices  $\lambda_{1X}$  and  $\lambda_{2Y}$  respectively. These skill prices affect the intercept but not the slope coefficients on individual worker characteristics. The coefficients  $\boldsymbol{\beta}_{1X}, \boldsymbol{\beta}_{2Y}$  are simply exogenous ‘technological’ parameters of the problem. By contrast, using the log-linear approximation in (27), we see that in our model equilibrium log wages have the form

$$\begin{aligned} \log w_1(i) &= \log \bar{w}_1 + \boldsymbol{\xi}(i)' \tilde{\boldsymbol{\beta}}_1, & \tilde{\boldsymbol{\beta}}_1 &= \tilde{\lambda}_{1X} \boldsymbol{\beta}_X + \tilde{\lambda}_{1Y} \boldsymbol{\beta}_Y \\ \log w_2(i) &= \log \bar{w}_2 + \boldsymbol{\xi}(i)' \tilde{\boldsymbol{\beta}}_2, & \tilde{\boldsymbol{\beta}}_2 &= \tilde{\lambda}_{2X} \boldsymbol{\beta}_X + \tilde{\lambda}_{2Y} \boldsymbol{\beta}_Y \end{aligned}$$

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<sup>9</sup> As usual in the Roy model, the relative wage is pinned by the relative endowments of the marginal worker. In our framework this condition can be written in terms of the gradient of the boundary of the aggregate bundling constraint, specifically

$$\frac{y(i^*)}{x(i^*)} = \frac{\lambda_{1X}}{\lambda_{2Y}} = \frac{P_1 Z_1}{P_2 Z_2} = \frac{U_1 F_{1X}}{U_2 F_{2Y}} = \underline{B}'(X_1)$$

where  $\tilde{\lambda}_{jk}$  denotes the skill price elasticities from (28) with  $\sum_k \tilde{\lambda}_{jk} = 1$  for each  $j$  and where the intercepts  $\log \bar{w}_j$  are weighted averages depending on the elasticities  $\tilde{\lambda}_{jk}$  and moments of the distribution of efficiency units in occupation  $j$  (see the Appendix for details). The generalized Roy model is nested as the special where  $\tilde{\lambda}_{1X} = \tilde{\lambda}_{2Y} = 1$ . But outside that special case, the characteristics  $\xi(i)$  are now genuinely priced in equilibrium. In particular, the equilibrium price of a characteristic in  $\xi(i)$  is given by the relevant entry in the skill-price weighted average coefficient  $\tilde{\beta}_j = \sum_k \tilde{\lambda}_{jk} \beta_{jk}$ . Shocks to technology (or preferences) that change the equilibrium skill prices will not just change the intercept, as in the generalized Roy model, but now also endogenously reweight the contributions of  $\xi(i)$  to wages, thereby changing wage inequality.

### 5.3 One-to-one assignment.

Unlike the previous two frameworks, there are alternative competitive models in the matching literature in which both production and skills are heterogeneous in two dimensions. A prominent example is Lindenlaub (2017), which extends to two dimensions the one-to-one matching model with single-dimensional types of Becker (1973).<sup>10</sup> The key difference relative to our framework is precisely that assignment is *one-to-one*, not the *many-to-one* assignment we emphasize. Our framework is intended to capture the idea that there is vastly more heterogeneity in people than there is heterogeneity in jobs.

**Many occupations.** To make contact with one-to-one assignment models, consider the following version of our setup. Suppose there is a continuum of occupations  $j \in [0, 1]$ . Each occupation  $j$  is associated with a factor intensity  $\alpha(j) \in [0, 1]$ . We assume these factor intensities are continuously distributed with full support on  $[0, 1]$  and without loss of generality order them most skill  $X$  intensive,  $\alpha(0) = 1$ , to least skill  $X$  intensive,  $\alpha(1) = 0$ . Within each occupation  $j$  is a large number of identical competitive firms. With equal measures of workers and occupations, each worker is allocated to a unique occupation, and paid a wage  $w(i, j) = \lambda_X(j)x(i) + \lambda_Y(j)y(i)$  evaluated at the assignment,  $i^*(j)$  say.

In this setting, one could consider a change in the gradient of  $\alpha(j)$  across occupations, which, as in our benchmark model, would leave the allocation unchanged while skill prices respond. But since each occupation is assigned a unique type of worker, there is no notion of within-occupation inequality. All workers in a given occupation are identical. Economically, in a one-to-one matching model all workers are *marginal*, there are none of the *inframarginal rents* that determine within-occupation inequality in our benchmark model.

To see how skill prices are determined in this setting, consider the sorting condition for the planner. For any occupation  $j$ , it must be the case that the allocation of workers to occupations  $j' < j$  was feasible given the distribution of skills in the economy. Using this we

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<sup>10</sup> Lindenlaub and Postel-Vinay (2019) consider this setup in a static, frictional environment with directed search. Lise and Postel-Vinay (2020) consider a dynamic job ladder version of the model with random search.

express the sorting condition which must hold for an allocation of aggregate skills  $X(j), Y(j)$  in terms of a sequence of aggregate bundling constraints of the form

$$\int_0^j Y(j') dj' = \underline{B}_j \left( \int_0^j X(j') dj' \right), \quad \text{for all } j \in [0, 1]$$

That is, the cumulative skill  $Y$  allocated to occupations  $[0, j]$  must be consistent with the cumulative skill  $X$  allocated to occupations  $[0, j]$  for all  $j \in [0, 1]$ .

As in the Roy model, specifying the planning problem in terms of the aggregate bundling constraints gives additional insights into the determinants of skill prices. Here the equilibrium condition between two adjacent occupations can be expressed as a derivative

$$\underbrace{\lambda_{2X} - \lambda_{1X} = -\mu \left( \frac{y(i^*)}{x(i^*)} \right)}_{\text{Benchmark model}}, \quad \underbrace{\frac{d\lambda_X(j)}{dj} = -\mu_j \left( \frac{y(i^*(j))}{x(i^*(j))} \right)}_{\text{One-to-one model}}$$

Integrating from the most skill  $X$  intensive occupation,  $j = 0$ , up to any  $j$  delivers an intuitive expression for skill prices

$$\lambda_X(j) = \lambda_X(0) - \int_0^j \mu_{j'} \left( \frac{y(i^*(j'))}{x(i^*(j'))} \right) dj', \quad \lambda_X(0) = \left. \frac{\partial F_0(x, y)}{\partial x} \right|_{(x, y) = (x(0), y(0))}$$

The price of skill  $X$  in occupation  $j$  is equal to its price in the most skill  $X$  intensive occupation,  $j = 0$ , minus an adjustment that reflects the tightness of the sequence of bundling constraints in less skill  $X$  intensive occupations. The price of skill  $X$  in occupation  $j = 0$  is pinned down by the marginal productivity of worker  $i = 0$  who has the highest  $x(i)/y(i)$  ratio. For example, if skills became less diverse, these constraints would tighten, leading to a steeper profile of skill prices across occupations. Less diverse skills would lead to lower economy-wide wage inequality, while steeper gradients of skill prices would lead to greater economy-wide wage inequality.

To summarize, this section delivers two results. First, leading alternative competitive models with two-dimensional firm and worker heterogeneity are silent on the relationship between relative skill intensity and within occupation-inequality. Second, framing these problems in terms of the aggregate bundling constraint delivers a simple characterization of skill prices and wages and clean comparative statics with respect to technology and skills.

## 6 Endogenous Technology

In this section we allow firms to *choose* an appropriate technology. Following [Caselli and Coleman \(2006\)](#), we allow firms to choose factor intensities from a menu of possible technologies. Firms choose technologies that depend on the skill prices they face. But in equilibrium the skill prices the firms face also depend on the technologies they choose. As the technology choices of firms respond to changing economic conditions, the equilibrium can switch from bundled to unbundled thereby compressing skill prices across and within occupations.

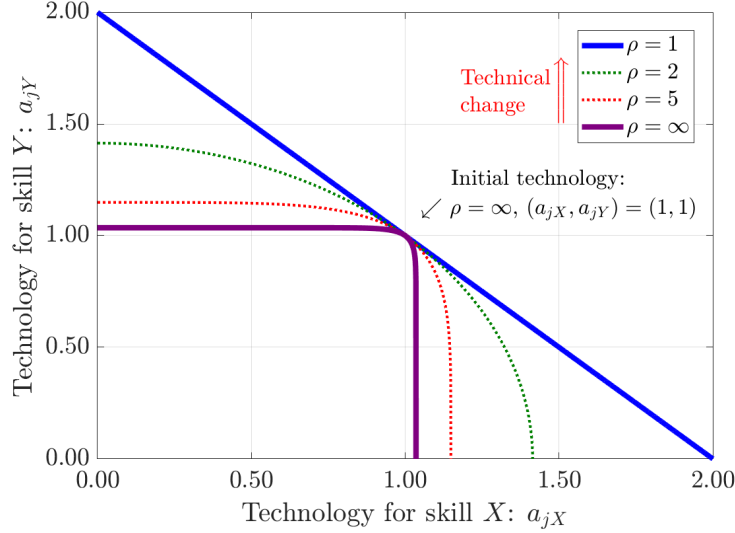


Figure 7: Changing the technology frontier

**Production function.** Let the production function for occupation  $j$  be

$$Y_j = Z_j [\alpha_j (a_{jX} X_j)^\sigma + (1 - \alpha_j) (a_{jY} Y_j)^\sigma]^{1/\sigma}, \quad 0 < \alpha_j < 1, \quad \sigma < 1 \quad (39)$$

for some endogenous factor-augmenting coefficients  $a_{jk}$  to be determined. The underlying parameters  $\alpha_j$  measure the innate factor bias of each occupation  $j$ . By changing the coefficients  $a_{jk}$  a firm can modify the effective factor intensities of its technology. The marginal cost of a firm with coefficients  $a_{jk}$  facing skill prices  $\lambda_{jk}$  is

$$\frac{1}{Z_j} \left[ \left( \frac{\lambda_{jX}}{\alpha_j^{1/\sigma} a_{jX}} \right)^{\frac{\sigma}{\sigma-1}} + \left( \frac{\lambda_{jY}}{(1 - \alpha_j)^{1/\sigma} a_{jY}} \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\sigma-1}{\sigma}} \quad (40)$$

**Menu of available technologies.** Firms choose their coefficients  $a_{jk}$  to minimize marginal cost (40) subject to a CES *technology frontier*

$$[a_{jX}^\rho + a_{jY}^\rho]^{1/\rho} \leq \bar{A}_j, \quad \rho > 1 \quad (41)$$

where  $\bar{A}_j > 0$  is an exogenous coefficient that sets the overall level of technology and where the curvature parameter  $\rho > 1$  governs the degree of substitutability between the technologies that are appropriate for skill  $X$  and the technologies that are appropriate for skill  $Y$ . When  $\rho \rightarrow 1$  these technologies are perfect substitutes, so the technology frontier is linear in  $a_{jX}$  and  $a_{jY}$ . When  $\rho \rightarrow \infty$  these technologies cannot be substituted at all.

Our comparative static exercise is to consider an increase in the set of available technologies via a decrease in  $\rho$ . This is pictured in Figure 7. As  $\rho$  declines, the set of available technologies expands, giving firms a richer set of production functions to operate.

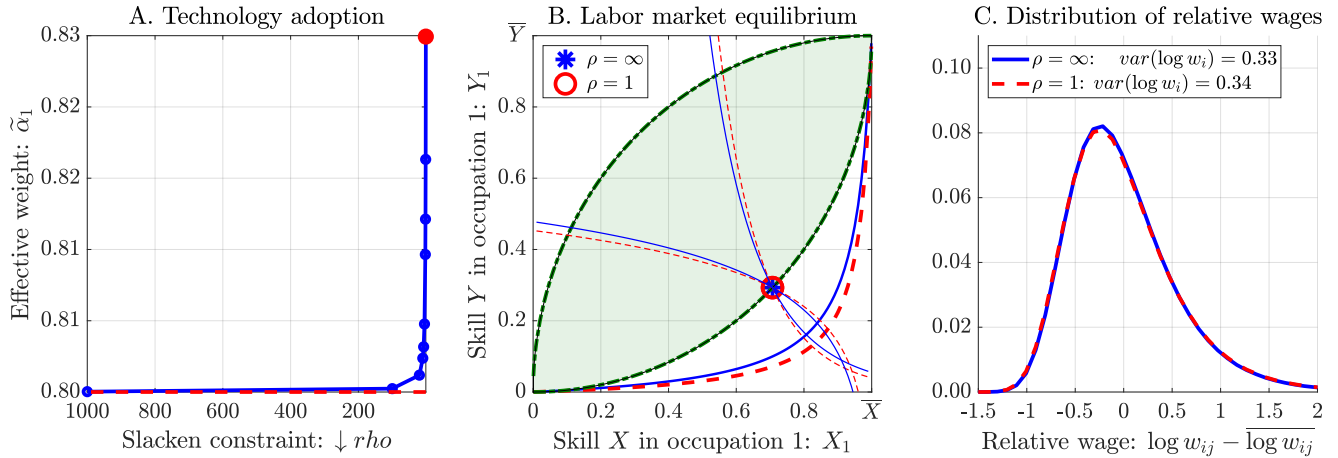


Figure 8: High skill occupations — Bundling labor when skills are substitutes  $\sigma > 0$

**Chosen technologies.** Minimizing marginal cost (40) subject to the technology frontier (41) gives the key condition

$$\frac{a_{jX}}{a_{jY}} = \left( \frac{\lambda_{jX}}{\lambda_{jY}} \right)^{\frac{\rho\sigma}{\sigma-\rho(1-\sigma)}} \left( \frac{1-\alpha_j}{\alpha_j} \right)^{\frac{\rho}{\sigma-\rho(1-\sigma)}} \quad (42)$$

which pins down the relative coefficients  $a_{jX}/a_{jY}$  in terms of the relative skill prices  $\lambda_{jX}/\lambda_{jY}$  that firms take as given. The levels of  $a_{jX}, a_{jY}$  are then determined by plugging this ratio back into the technology frontier.

**General equilibrium with endogenous technologies.** In our benchmark model, we solved for equilibrium skill prices  $\lambda_{jk}$  and allocations of skills  $X_j, Y_j$  taking the technologies  $a_{jk}$  as given. We now need to solve a larger fixed point problem. We look for equilibrium skill prices  $\lambda_{jk}$ , allocations of skills  $X_j, Y_j$ , and technologies  $a_{jk}$  that are consistent both with the occupational choices of workers, the technology adoption choices of firms, and market clearing. Intuitively, we fix candidate skill prices  $\lambda_{jk}$ , determine the technology choices of firms  $a_{jk}$  from (41)-(42), and then use these choices of technology to solve for an equilibrium as in our benchmark model. We then check to see if the resulting skill prices are the same as the ones we started with.

**Bundling and unbundling labor.** The comparative statics of the model with respect to  $\rho$  with endogenous technologies depend crucially on whether the skills  $X$  and  $Y$  in the underlying production function (39) are substitutes or complements, i.e., on whether  $\sigma > 0$  or  $\sigma < 0$ . Figures 8 and 9 describe these two cases.<sup>11</sup>

<sup>11</sup> In both cases, the initial technologies have  $\alpha_{1X} = \alpha_{2Y} = 0.80$ , skills distributed independently according to a Fréchet with  $\theta = 2$ . For the case of substitutes (complements) the elasticity of substitution of the production function is  $\sigma = 0.10$  ( $\sigma = -0.70$ ). The new equilibrium corresponds to technology adoption under a technology frontier with curvature parameter  $\rho$ .



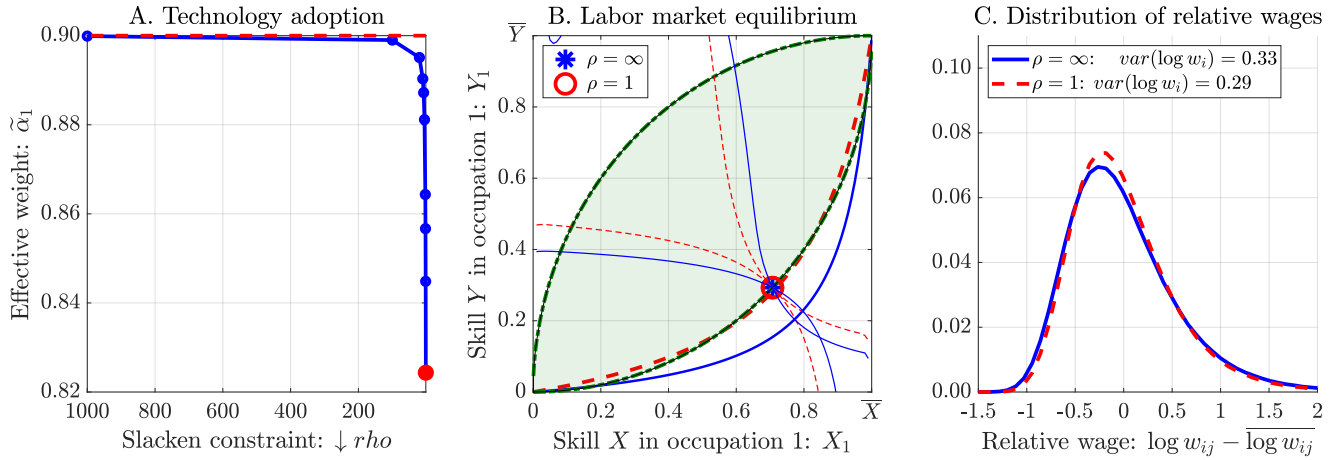


Figure 9: Low skill occupations — Unbundling labor when skills are complements  $\sigma < 0$

When skills are substitutes,  $\sigma > 0$ , expanding the set of available technologies leads firms to choose technologies that are *more factor biased*, that is, firms choose technologies that are endogenously more ‘Roy-like’. If the underlying equilibrium absent technology choice is already bundled, the endogenous technology choice tightens the bundling constraints further. If the underlying equilibrium absent technology choice is unbundled, the endogenous technology choice can switch the equilibrium from being unbundled to bundled. This leads to relatively *polarized* wage gains, in particular, to large wage gains for specialists, workers with relatively high endowments in one skill or the other. The key intuition here is that if skills are good substitutes,  $\sigma > 0$ , then firms find it relatively easy to substitute away from ‘weak links’ — i.e., away from skills that are expensive relative to their marginal product. In other words, firms invest in making skills that are already highly productive relative to their skill prices even more productive. This reinforces any pre-existing factor bias in the technologies, making the equilibrium technologies more factor-biased than the underlying exogenous technologies, and making it more likely that the resulting equilibrium is bundled with relatively large skill premia between occupations.

But when skills are complements,  $\sigma < 0$ , expanding the set of available technologies leads firms to choose technologies that are *less factor biased*, so endogenously less ‘Roy-like’. If the underlying equilibrium absent technology choice is bundled, the endogenous technology choice can switch the equilibrium from being bundled to unbundled. This leads to relatively large wage gains for generalists, workers with relatively similar endowments in both skills, and to decreasing wage inequality. The key intuition here is that if skills are not good substitutes,  $\sigma < 0$ , firms can not substitute away from skills that are ‘weak links’ and so address this problem by making weak links less weak. In other words, firms invest in ‘bringing up to scratch’ skills that would otherwise be holding the firm back. This mitigates any pre-existing factor bias in the technologies, making the equilibrium technologies less factor-biased than the underlying exogenous technologies, and making it more likely that the resulting equilibrium

is unbundled with relatively small skill premia between occupations.

**Changes in the technology frontier.** Consider an exogenous decrease in the curvature parameter  $\rho$  of the technology frontier. As  $\rho \searrow 1$  the technology frontier becomes more linear, the coefficients  $a_{jX}, a_{jY}$  become more substitutable. Put differently, more combinations of  $a_{jX}, a_{jY}$  are feasible for a given technology level  $\bar{A}_j$ .

The effects of this decrease in  $\rho$  again depend on whether the skills  $X$  and  $Y$  in the underlying production function are substitutes or complements. If skills are substitutes,  $\sigma > 0$ , a decrease in  $\rho$  allows firms to choose technologies that are increasingly more factor-biased and eventually shifts the equilibrium from unbundled to bundled driving up skill premia between occupations. If skills are complements,  $\sigma < 0$ , a decrease in  $\rho$  leads firms to choose technologies that are less factor-biased and eventually shifts the equilibrium from bundled to unbundled driving down skill premia between occupations. As the  $\rho$  decreases the bundled equilibrium obtains and the premia earned by skill  $X$  in occupation 1 relative to occupation 2 disappears.

A key prediction of the model, then, is that as technologies within a given occupation become ‘less similar’ in the sense that highly productive skills within a sector experience faster factor-augmenting productivity growth than less productive skills, then we should see increasing within-occupation wage inequality as the greatest wage gains accrue to workers with relatively specialized skill endowments. But as technologies within a given occupation become ‘more similar’ then we should see decreasing within-occupation wage inequality as the greatest wage gains accrue to workers with generalist skill endowments.

## 7 Understanding trends in experience premia

In this section we show that simple extensions of the baseline model of Section 3 can rationalize additional new empirical time-series facts that we present:

1. **Experience premia.** The experience premia associated with longer labor market experience has decreased in magnitude in low skill occupations, but has remained relatively constant in high skill occupations (Figure 10).
2. **Hours premia.** The wage premia (penalty) associated with working longer (shorter) hours has decreased in magnitude in low skill occupations, but has remained relatively constant in high skill occupations (Figure 11).
3. **Occupational mobility.** The rate at which workers in low skill occupations has increased, but has decreased for workers in high skill occupations (Figure 12).

*To be completed.*

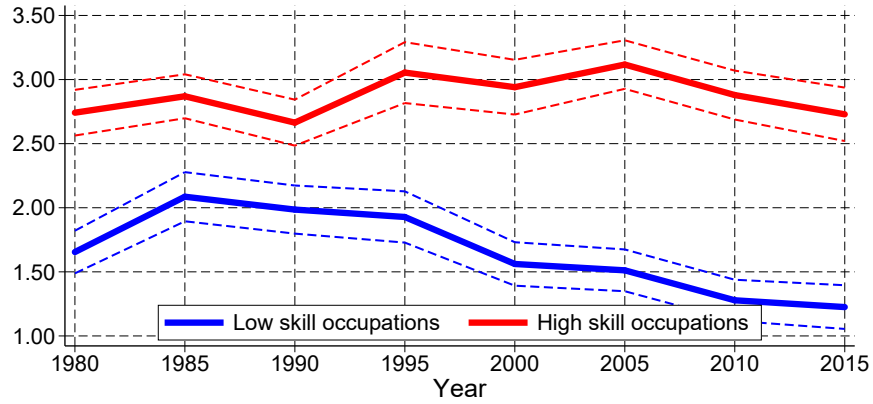


Figure 10: Experience premium

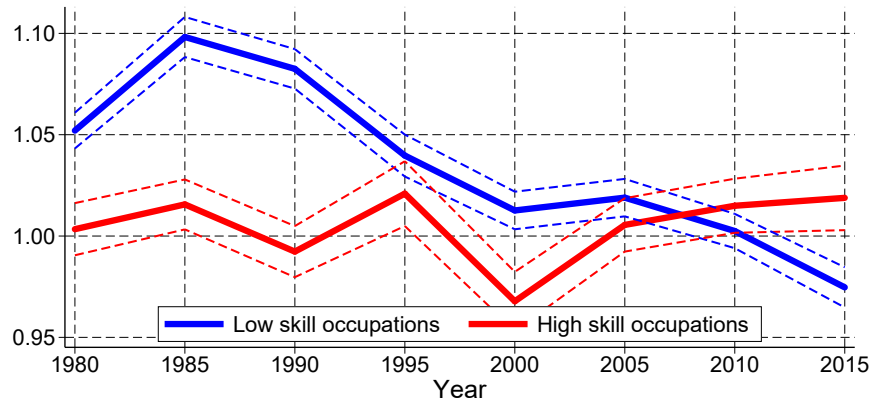


Figure 11: Hours premium

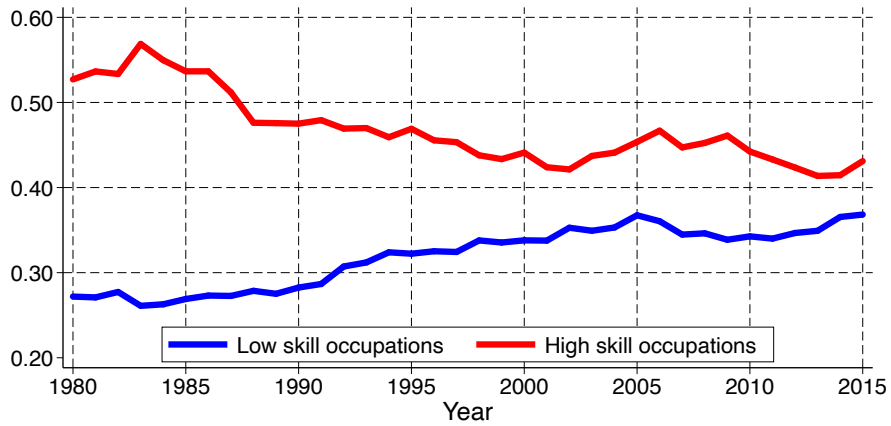


Figure 12: Workers in low (high) skill occ. now switch jobs more (less) after unemployment  
 Notes Fraction of **male** workers experiencing  $\{E_{March}, \dots, U_m, \dots, E_{March'}\}$  that swap **1-digit** occupations across  $\{E_{March}, E_{March'}\}$ .

## 8 Conclusion

We study the effects of technology adoption on wage inequality in a general equilibrium model with heterogeneous workers endowed with multiple skill characteristics. In our model, skill characteristics are indivisibly *bundled* in individual workers, they must be supplied to a single occupation. This gives rise to the possibility that a given skill characteristic is priced differently in different occupations. Our model makes clear predictions about the effects of changing occupation-specific skill prices on both between- and within-occupation wage inequality. Our model nests the standard Roy model as a special case, but makes a richer set of predictions about the effects of changing skill prices on within-occupation wage inequality and moreover provides a clear general equilibrium framework that links those changing skill prices to underlying technology adoption decisions of firms and labor market conditions. Consistent with recent trends in the US labor market, we find that changing technology can increase wage-inequality within some occupations while at the same time decreasing wage-inequality in other occupations.

When skills are complements in production, technology adoption can cause an initially bundled equilibrium to *unbundle*. As technologies that were closely associated with specific occupations become more similar, workers that were initially strongly sorted across occupations become less strongly sorted. Workers with relatively high endowments of one skill once had a preferred sector, earning rents due to their comparative advantage. But after technology adoption these same workers are essentially indifferent with regards to their occupational and within-occupation inequality flattens. We argue that this is consistent with technology adoption and the recent evolution of wages and conditions in low-skill occupations in the US.

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# APPENDIX

## A Additional figures and tables

## A.1 Occupational mobility

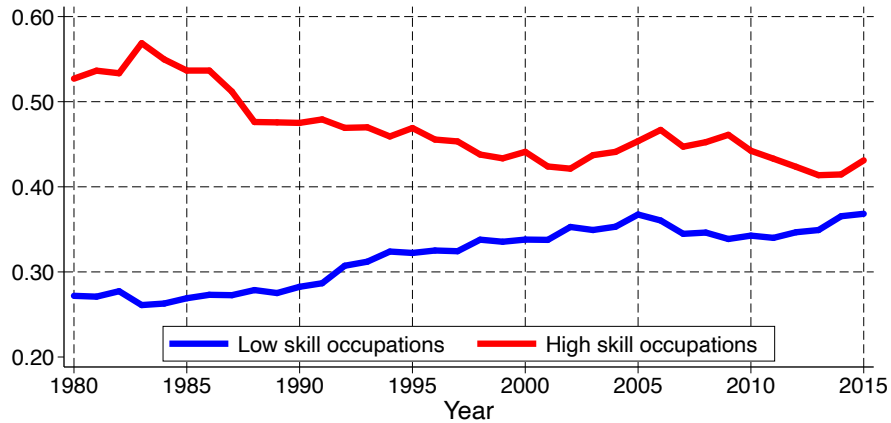


Figure A1: Workers in low (high) skill occupations now switch jobs more (less) after unemployment

Notes Fraction of **male** workers experiencing  $\{E_{March}, \dots, U_m, \dots, E_{March'}\}$  that swap **1-digit** occupations across  $\{E_{March}, E_{March'}\}$ .

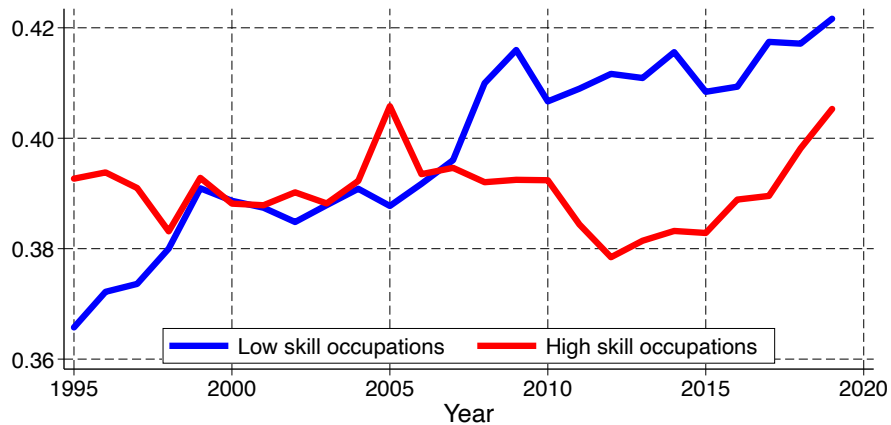


Figure A2: Hours premium

Notes Fraction of **male** workers experiencing  $\{E_{Month}, E_{Month+1}\}$  that swap **1-digit** occupations across  $\{E_{Month}, E_{Month+1}\}$