

# Unbundling Labor

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The views herein are those of the authors and not the Federal Reserve System

# This paper

- Has technological change made jobs more or less similar?
- What are the implications for wage inequality?
- When does such technological change arise?

1970 - Cafe



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2020 - Starbucks



# This paper

## 1. Data

**A.** Heterogeneity in skill requirements across occupations

↓ Low skill jobs , ↑ High skill jobs

**B.** Inequality in wages within occupations

↓ Low skill jobs , ↑ High skill jobs

## 2. Competitive theory

- Extend Rosen (1983), Heckman Scheinkman (1987)
- Technological change consistent with **A.** causes **B.**
- Nests three standard frameworks that are silent on links b/w **A.** and **B.**
- Endogenize **A.** as appropriate technology choice (Caselli Coleman, 2006)

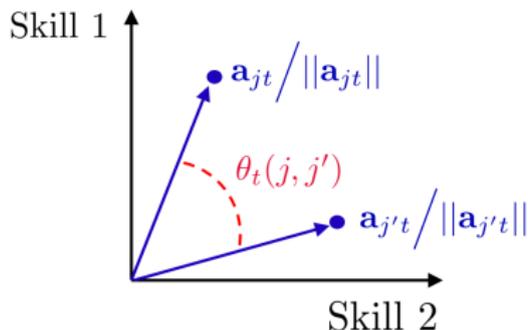
# Fact A. - Technology

*High skill jobs have become more different*

*Low skill jobs have become more similar*

## Approach

1. O\*NET data on 250+ skills and  $J$  occupations. Split: 2003-09, 2010-18
2. Reduce to  $4 \times J$  matrix of skills  $\mathbf{A}_t = [\mathbf{a}_{1t}, \dots, \mathbf{a}_{Jt}]$  (Lise Postel-Vinay, 2020)
3. Distance between occupations (Gathmann Schönberg, 2010)

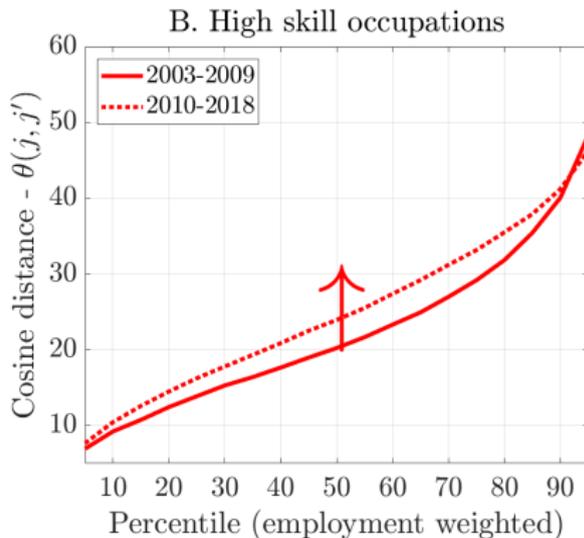
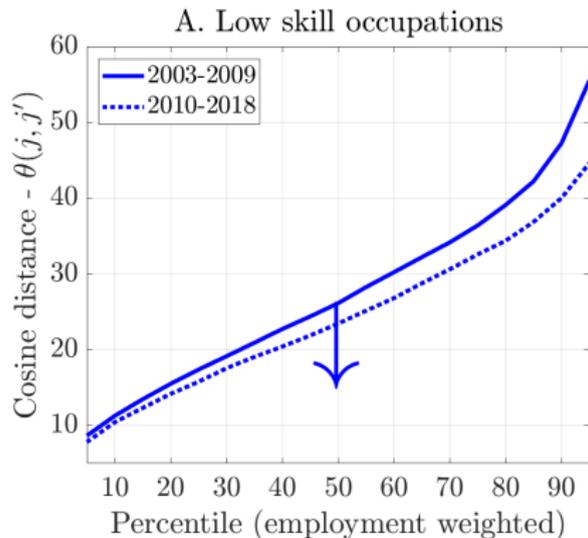


4. Compare the distribution of these distances  $\theta(j, j')$  across periods

# Fact A. - Technology

*High skill jobs have become more different*

*Low skill jobs have become more similar*



- E.g. median distance between low skill occupations down  $\approx 5$  degrees

## Fact B. - Wages

Wages in *high skill* jobs have become *more different*

Wages in *low skill* jobs have become *more similar*

### Approach

- Log annual earnings from the CPS -  $\log y_{it}$
- Residuals after controlling for observables -  $e_{it}$

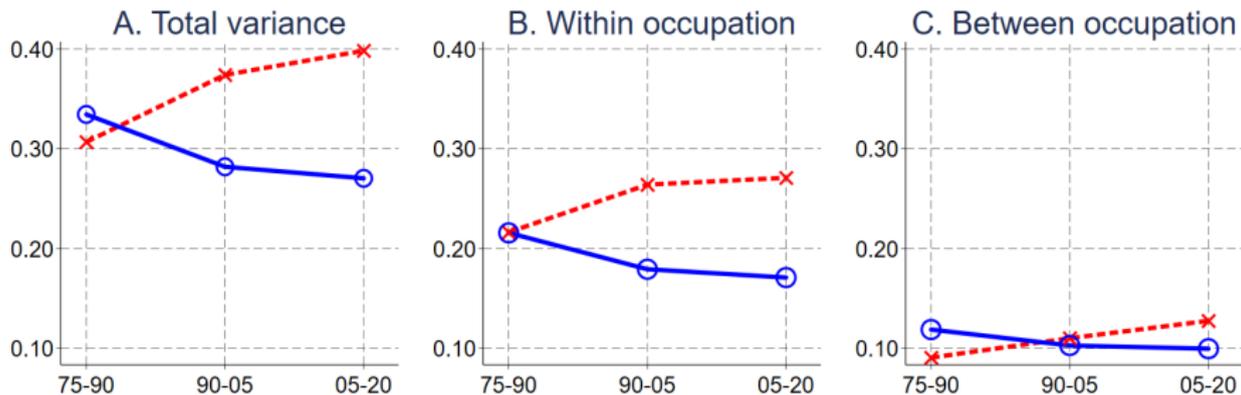
$Year_t, NAICS1_{it}, Ed_{it}, Race_{it}, Sex_{it}, FirmSize_{it}, Exp_{it}, Exp_{it}^2, Hours_{it}$

- Estimate in 15 year windows. Separately for low and high skill occupations.
- Decompose  $var(e_{it})$  into within- and between-occupation components

## Fact B. - Wages

Wages in *high skill* jobs have become *more different*

Wages in *low skill* jobs have become *more similar*



Variance of residuals. Red = High wage occupations, Blue = Low wage occupations

Robust across  $\{\text{All, Male, Female}\} \times \{\text{Fix occupations in 1980, 2010}\}$

► Details

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# Model

- General equilibrium environment

- Individual skills  $(x(i), y(i))$
- Two occupations  $j \in \{1, 2\}$ , with different skill intensities

- Competitive equilibrium wages

$$w_j(i) = \lambda_{jX}x(i) + \lambda_{jY}y(i) \quad \rightarrow \quad \text{var}\left(\log w_j(i) \middle| j\right)$$

- Within occupation inequality determined by two forces

1. Distribution of skills conditional on selection
2. Gradient of within-occupation skill prices  $\{\lambda_{jX}, \lambda_{jY}\}$

Mandelbrot (1962) - Paretian Distributions and Income Maximization

# Model

- General equilibrium environment
  - Individual skills  $(x(i), y(i))$
  - Two occupations  $j \in \{1, 2\}$ , with different skill intensities

## Mandelbrot (1962) - Paretian Distributions and Income Maximization

Suppose that  $w_j(i)$ , which is the rental price which the **occupation**  $j$  is ready to pay for the use of a man's abilities, can be written as a linear form for  $K$  independent **factors**  $x_k(i)$ , each of which is randomly distributed in the population, and “measures” one of several “abilities”. Then one can write:

$$w_j(i) = \sum_{k=1}^K \lambda_{jk} x_k(i)$$

... (The fact that the same commodity may have different prices with respect to different buyers  $w_j(i)$ , is a result of the **impossibility of renting the different factors to different employers**; we intend to discuss this question elsewhere)

# Environment

- Workers  $i \in [0, 1]$  endowed with two skills  $k \in \{x, y\}$

$$(x(i), y(i)) \sim H(x, y)$$

- Final good

$$U(C_1, C_2)$$

- Task / Occupation  $j$  technology:

$$C_j = F_j(X_j, Y_j) = \left[ \alpha_j X_j^\sigma + (1 - \alpha_j) Y_j^\sigma \right]^{\frac{1}{\sigma}}, \quad \sigma < 1$$

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$$X_j = \int x(i) \phi_j(i) di, \quad Y_j = \int y(i) \phi_j(i) di, \quad \phi_j(i) \in \{0, 1\}$$

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$$X_j = \int x(i) \phi_j(i) di, \quad Y_j = \int y(i) \phi_j(i) di, \quad \phi_j(i) \in \{0, 1\}$$

BUNDLED - Worker  $i$  must allocate  $(x(i), y(i))$  to the same task  $j$

Rosen (1983), Heckman Scheinkman (1987)

# Efficient allocation - Relaxed problem

$$\max_{\phi_{1x}(i) \in \{0,1\}, \phi_{1y}(i) \in \{0,1\}} U\left(F_1(X_1, Y_1), F_2(X_2, Y_2)\right)$$

subject to

Let  $\lambda_{jX}$  be the shadow price of  $X_j$

$$X_1 = \int \phi_{1x}(i) x(i) di \quad \longrightarrow \quad \lambda_{1X} = U_1 F_{1X}$$

$$X_2 = \int [1 - \phi_{1x}(i)] x(i) di \quad \longrightarrow \quad \lambda_{2X} = U_2 F_{2X}$$

$$Y_1 = \int \phi_{1y}(i) y(i) di \quad \longrightarrow \quad \lambda_{1Y} = U_1 F_{1Y}$$

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# Efficient allocation

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and person-by-person bundling constraints

$$\phi_{1x}(i) = \phi_{1y}(i) \quad \text{for all } i \in [0, 1]$$

## Feasible allocations

- Given  $X_1$  what is *minimum* and *maximum*  $Y_1$  bundled along with it?

BUNDLING CONSTRAINT:  $Y_1 \in [\underline{B}(X_1), \overline{B}(X_1)]$

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- Given  $X_1$  what is *minimum* and *maximum*  $Y_1$  bundled along with it?

$$\text{BUNDLING CONSTRAINT: } Y_1 \in [\underline{B}(X_1), \overline{B}(X_1)]$$

- Construct  $X_1$  using workers with highest  $x(i)/y(i)$  first

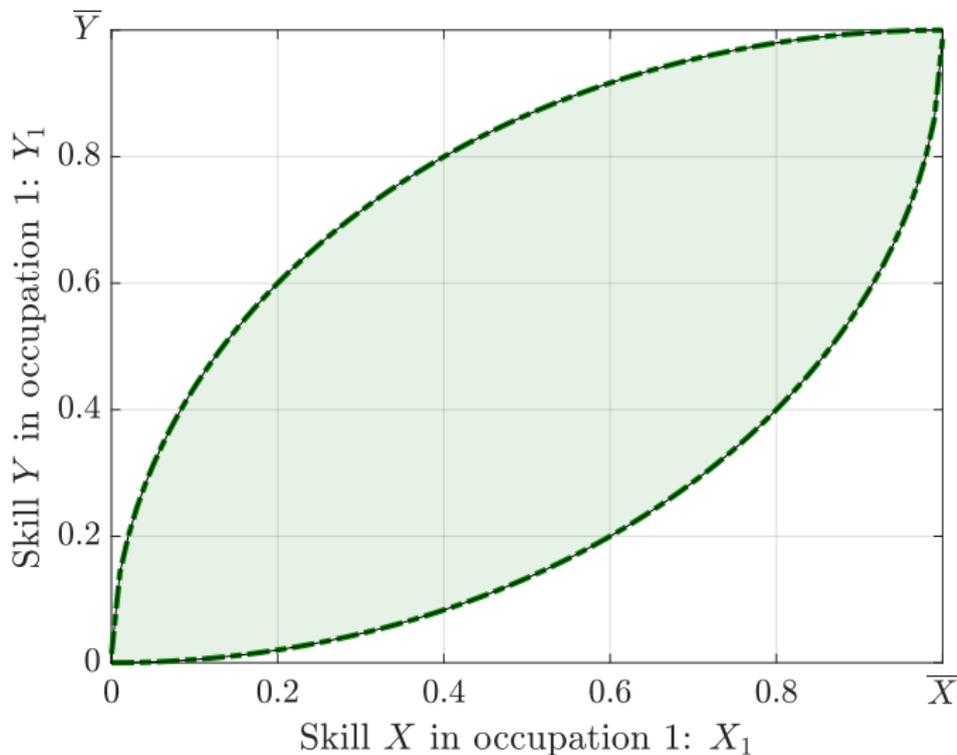
$$X_1 = \int_0^{i^*} x(i) di \quad , \quad \underline{B}(X_1) = \int_0^{i^*} y(i) di$$

**Result** - If the skill distribution  $H$  has no mass points, then

1.  $\underline{B}$  is strictly increasing, strictly *convex*
2.  $\overline{B}$  is strictly increasing, strictly *concave*
3. Continuously differentiable, with derivative  $\underline{B}'(X_1) = \frac{y(i^*)}{x(i^*)}$

# Feasible allocations

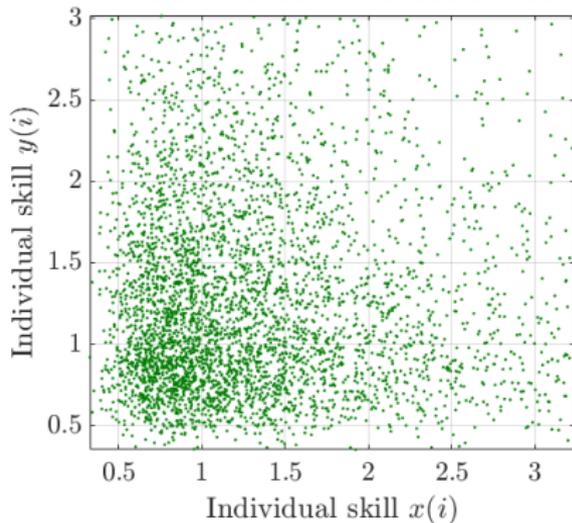
Feasible allocations must satisfy aggregate *bundling constraint*  $Y_1 \in [\underline{B}(X_1), \overline{B}(X_1)]$ .  
Determined by distribution of skill endowments only. Example:  $x(i) \sim \text{Fréchet}(\theta)$ .



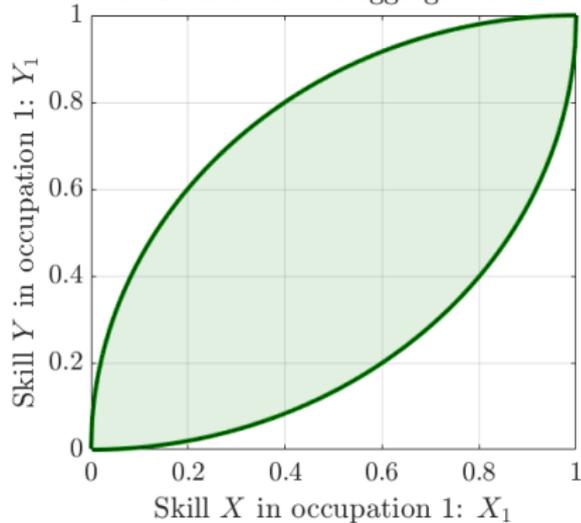
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A. Distribution of individual skills



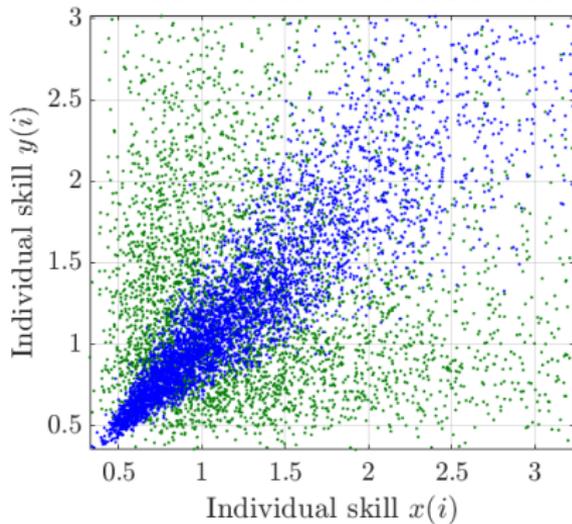
B. Constraints on aggregate skills



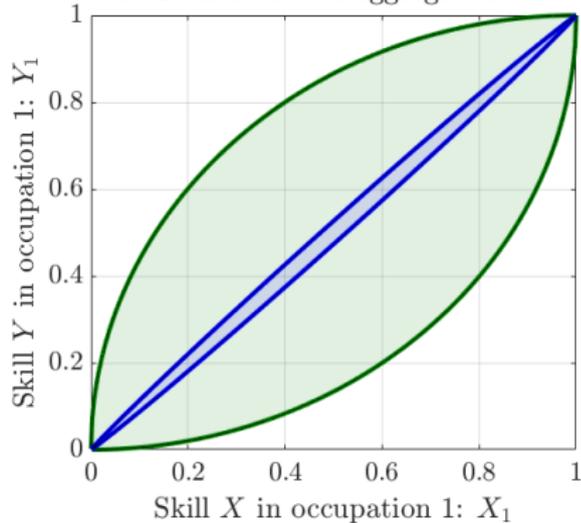
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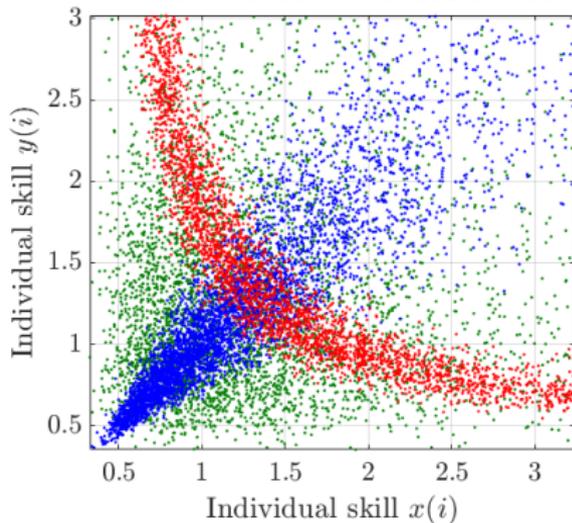
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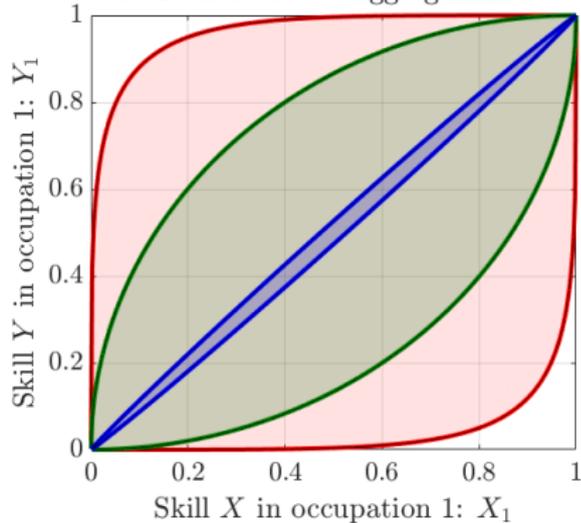
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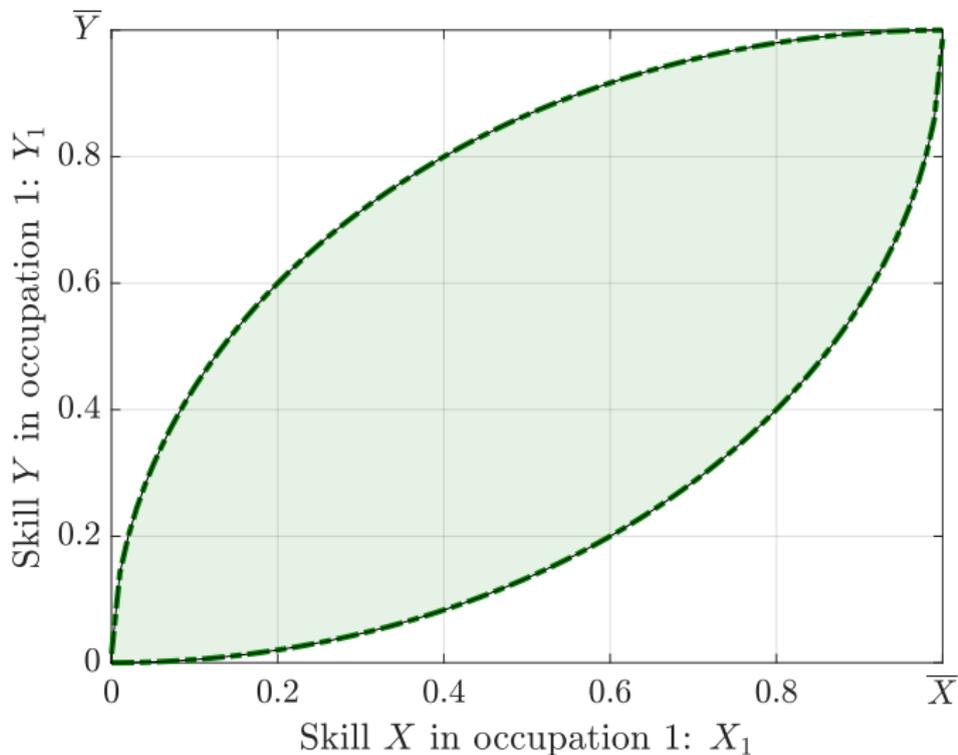


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# Feasible allocations

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## Efficient allocation

$$\max_{X_1, Y_1} U\left(F_1(X_1, Y_1), F_2(\bar{X} - X_1, \bar{Y} - Y_1)\right)$$

subject to

$$\underbrace{Y_1 \geq \underline{B}(X_1)}_{\text{Multiplier: } \underline{\mu}}$$

$$\underbrace{Y_1 \leq \bar{B}(X_1)}_{\text{Multiplier: } \bar{\mu}}$$

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First order conditions

$$X_1 : \quad \lambda_{1X} = \lambda_{2X} + \underline{\mu} \underline{B}'(X_1)$$

$$Y_1 : \quad \lambda_{1Y} = \lambda_{2Y} - \underline{\mu}$$

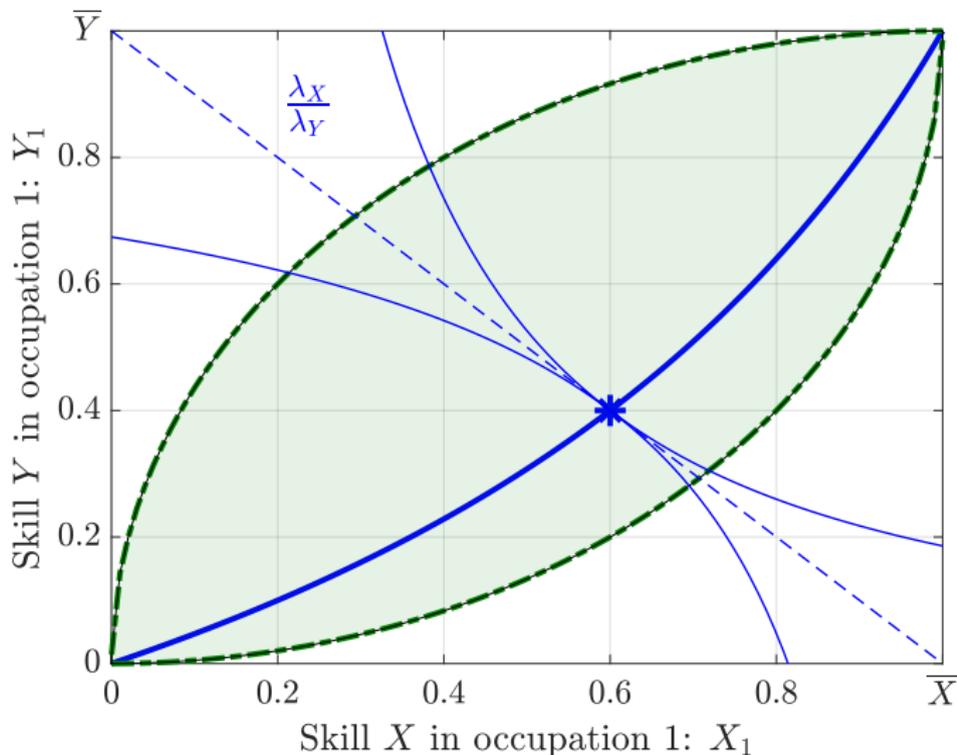
Results

1. Same allocation as ‘full’ problem, 2. Decentralization 
3. Analytical comp. statics for  $\underline{\mu}$  under Fréchet + Cobb-Douglas

# Unbundled allocation

'Contract curve' equates marginal rates of technical substitution:  $F_{1X}/F_{1Y} = F_{2X}/F_{2Y}$ .

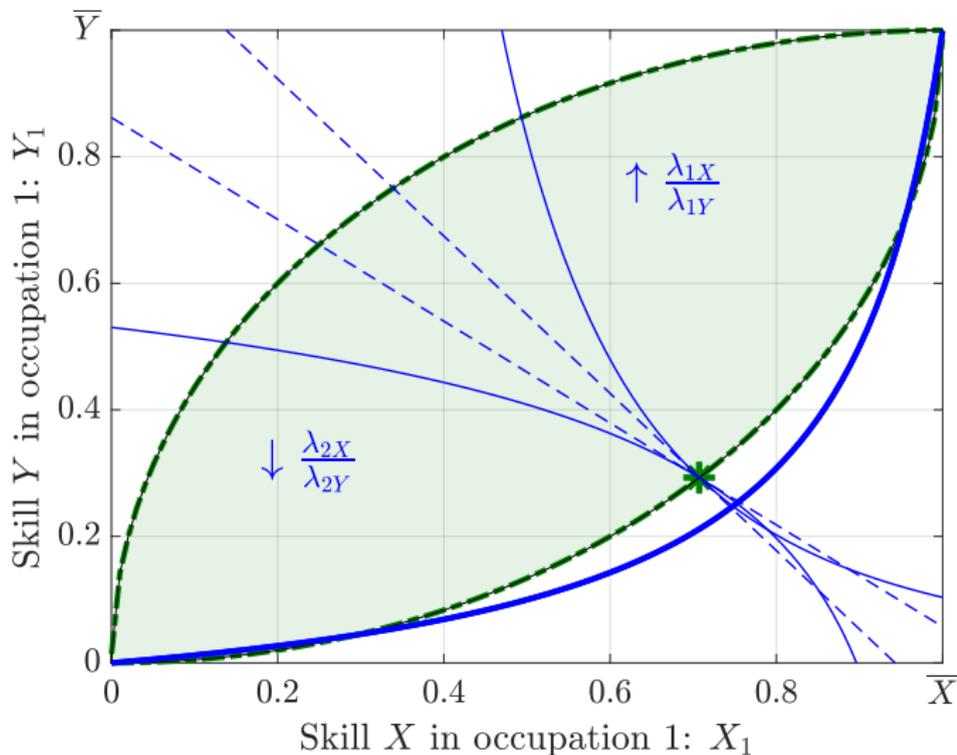
Unbundled allocation (\*) equates  $U_1/U_2$  to marginal rate of transformation  $F_{2k}/F_{1k}$ .



# Bundled allocation

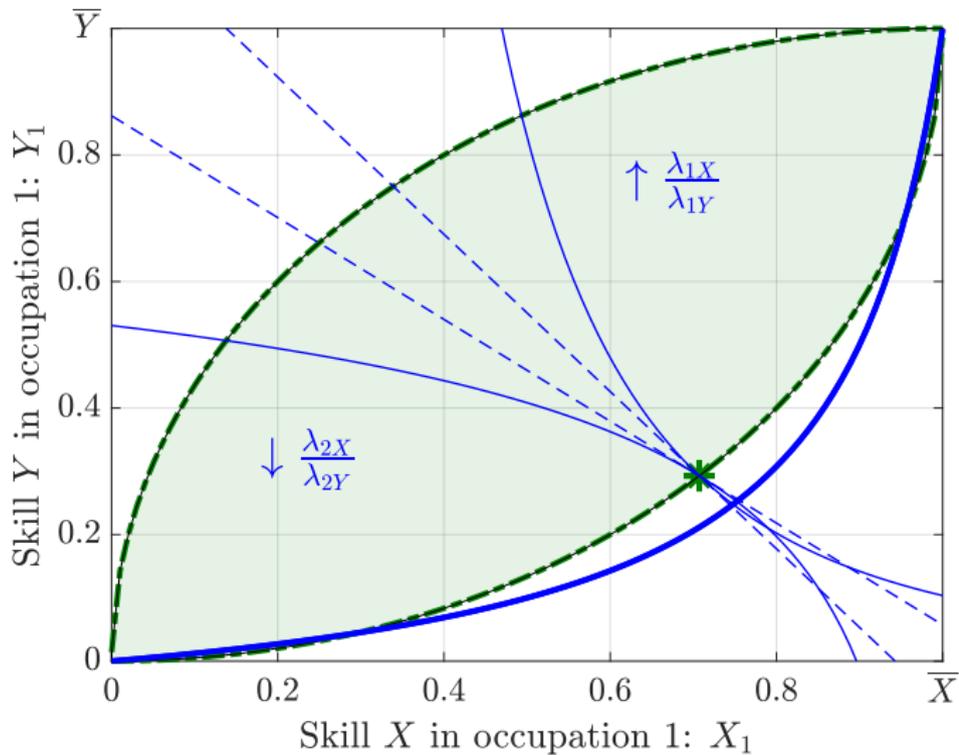
Bundling constraint binds. Cannot 'break open' workers to get at underlying skill content.

$$U_1 \left[ F_{1X} + \underline{B}'(X_1)F_{1Y} \right] = U_2 \left[ F_{2X} + \underline{B}'(X_1)F_{2Y} \right] \quad , \quad Y_1 = \underline{B}(X_1)$$



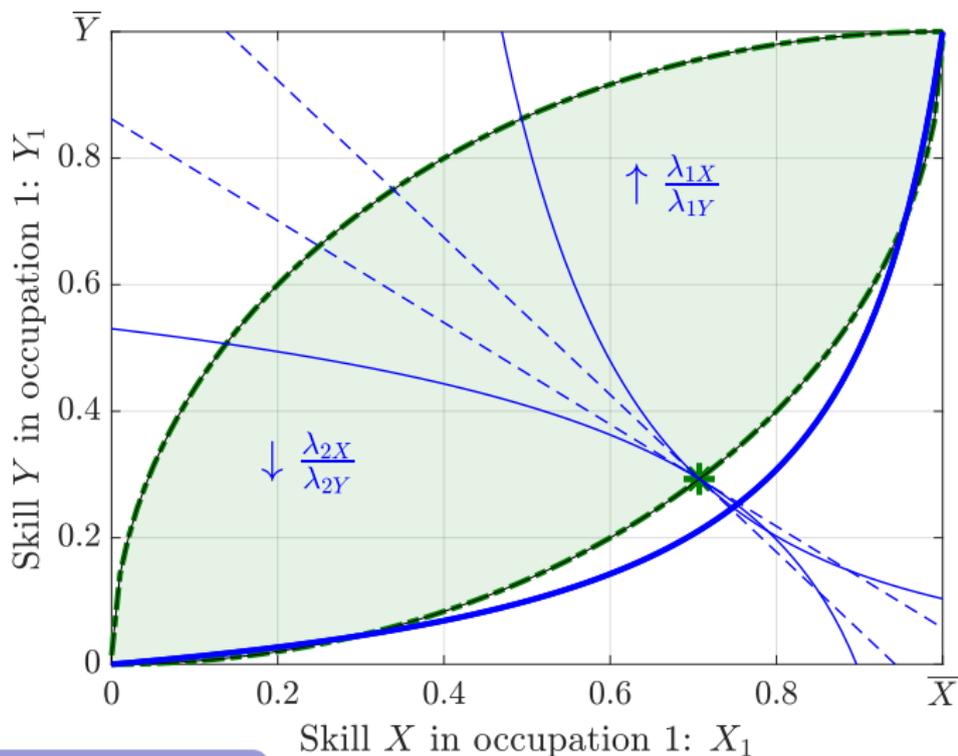
# Wages

$$w_1(i) = \lambda_{1X}x(i) + \lambda_{1Y}y(i)$$



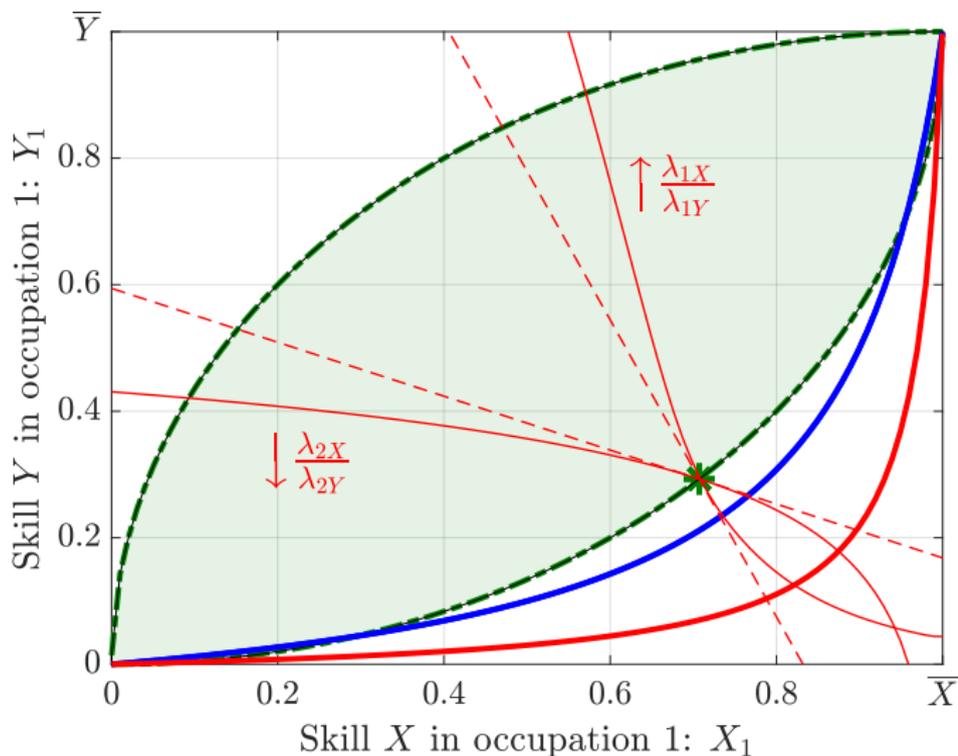
# Wages

$$\log w_1(i) = \log \lambda_{1Y} + \log y(i) + \log \left( 1 + \uparrow \left( \frac{\lambda_{1X}}{\lambda_{1Y}} \right) \left( \frac{x(i)}{y(i)} \right) \right)$$



# Wages

$$\log w_1(i) = \log \lambda_{1Y} + \log y(i) + \log \left( 1 + \uparrow \left( \frac{\lambda_{1X}}{\lambda_{1Y}} \right) \left( \frac{x(i)}{y(i)} \right) \right)$$



# Symmetric Frechet example

## 1. Skills

$$x(i) \sim \text{Frechet}(\theta) \quad , \quad y(i) \sim \text{Frechet}(\theta) \quad , \quad \text{Tail: } 1/\theta \quad , \quad \theta > 1$$

## 2. Technology

$$F_1 = \left[ \alpha X_1^\sigma + (1-\alpha) Y_1^\sigma \right]^{1/\sigma} \quad , \quad F_2 = \left[ (1-\alpha) (1-X_1)^\sigma + \alpha (1-Y_1)^\sigma \right]^{1/\sigma}$$

- Bundling constraint

$$\underline{B}(X_1) = 1 - \left( 1 - X_1^{\frac{\theta}{\theta-1}} \right)^{\frac{\theta-1}{\theta}} \quad , \quad \lim_{\theta \rightarrow \infty} \underline{B}(X_1) = X_1 \quad , \quad \lim_{\theta \searrow 1} \underline{B}(X_1) = 0$$

- If  $\alpha < \alpha^*$  then *unbundled equilibrium*

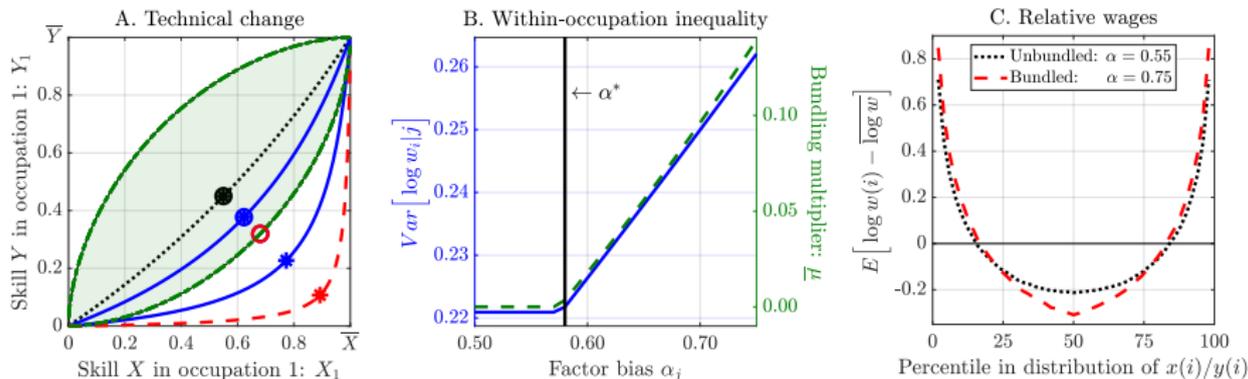
$$\uparrow \alpha^* = \frac{\uparrow \psi^{1-\sigma}}{1 + \uparrow \psi^{1-\sigma}} \quad , \quad \uparrow \psi = \frac{1}{2^{1-\uparrow 1/\theta} - 1} \in \left[ \frac{1}{2}, 1 \right]$$

1. More dispersion of skills  $\uparrow (1/\theta)$ , increase  $\alpha^* \rightarrow$  Unbundled

2. More complementary skills  $\downarrow \sigma$ , increase  $\alpha^* \rightarrow$  Unbundled

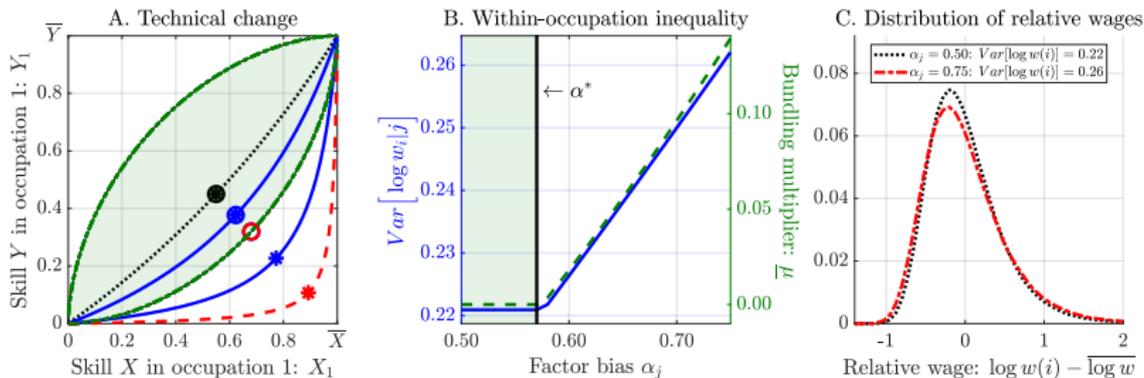
# Skill bias and inequality

Varying  $\alpha \in \{0.50, \dots, 0.75\}$ . As occupations become more different, bundling constraint binds and *primary* skill prices increase relative to *secondary* skill prices.



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# General symmetric case

## Definition - Symmetric economy

- Weight  $\alpha$  on primary skill,  $\bar{X} = \bar{Y}$ , no other restrictions on  $H(x, y)$

## Proposition 2

For a *symmetric economy*,  $\exists$  a unique factor intensity  $\alpha^*$  such that:

- (i) The equilibrium is unbundled if and only if  $\alpha \leq \alpha^*$ .
- (ii) If the unbundled, then  $X'(\alpha) > 0$ , and  $\mu(\alpha) = 0$ .
- (iii) If the bundled, then  $X(\alpha) = X(\alpha^*)$  and  $\mu(\alpha) > 0$  with  $\mu'(\alpha) > 0$ .

## Proposition 3

For each occupation  $j$  there is a unique factor intensity  $\alpha_j^{**} \geq \alpha^*$ , that depends on moments of  $H$ , on the such that  $\uparrow \alpha$  increases the variance of log wages in occupation  $j$  if and only if  $\alpha > \alpha_j^{**}$

## General symmetric case

- Amount of  $X$  in occupation 1

$$X(\alpha) = \frac{\alpha^{\frac{1}{1-\sigma}}}{(1-\alpha)^{\frac{1}{1-\sigma}} + \alpha^{\frac{1}{1-\sigma}}} \bar{X}.$$

- Cut-off

$$\bar{X} - X(\alpha^*) = \underline{B}(X(\alpha^*))$$

- Variance of log wages -  $\hat{w}_j(i) = \zeta_{jX}\hat{x}(i) + \zeta_{jY}\hat{y}(i)$ , within- $j$  deviations

$$\text{Var}_j[\hat{w}] = \text{Var}_j[\hat{y}] + \zeta_{jX}^2 \text{Var}_j[\hat{x} - \hat{y}] + 2\zeta_{jX} \text{Cov}_j[\hat{y}, \hat{x} - \hat{y}]$$

$$\zeta_{jX} = \frac{\lambda_{jX} \bar{x}_j}{\lambda_{jX} \bar{x}_j + \lambda_{jY} \bar{y}_j}$$

- Cut-off - In symmetric economy  $RHS$  depends on distribution of skills

$$\left( \frac{\alpha_1^{**}}{1 - \alpha_1^{**}} \right) / \left( \frac{\alpha^*}{1 - \alpha^*} \right) = \underbrace{\left( \frac{\text{Var}_1[\hat{y}] - \text{Cov}_1[\hat{x}, \hat{y}]}{\text{Var}_1[\hat{x}] - \text{Cov}_1[\hat{x}, \hat{y}]} \right)}_{\text{If this is } < 1, \text{ then } \alpha^{**} = \alpha^*} \left( \frac{\bar{y}_1}{\bar{x}_1} \right)$$

# Low skill occupations in the US: 1970 vs 2020

↑ *Skill bias* → *Bundled / Sorted equilibrium* → ↑ *Inequality*



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↓ *Skill bias* → *Unbundled / Unsorted equilibrium* → ↓ *Inequality*



## Three special cases

*Katz-Murphy* , *Roy* , *Lindenlaub*  
 $\underbrace{\hspace{10em}}_{\theta \rightarrow 1}$     $\underbrace{\hspace{2em}}_{\alpha_j \rightarrow 1}$     $\underbrace{\hspace{10em}}_{J \rightarrow \infty}$

# Three special cases

$$\underbrace{\text{Katz-Murphy}}_{\theta \rightarrow 1}, \underbrace{\text{Roy}}_{\alpha_j \rightarrow 1}, \underbrace{\text{Lindenlaub}}_{J \rightarrow \infty}$$

## 1. Katz-Murphy

$$F_1 = \left[ \alpha_{1L} L^\sigma + \alpha_{1H} H^\sigma \right]^{\frac{1}{\sigma}}, \quad \mathbf{x}(i) \in \left\{ (l(i), 0), (0, h(i)) \right\}$$

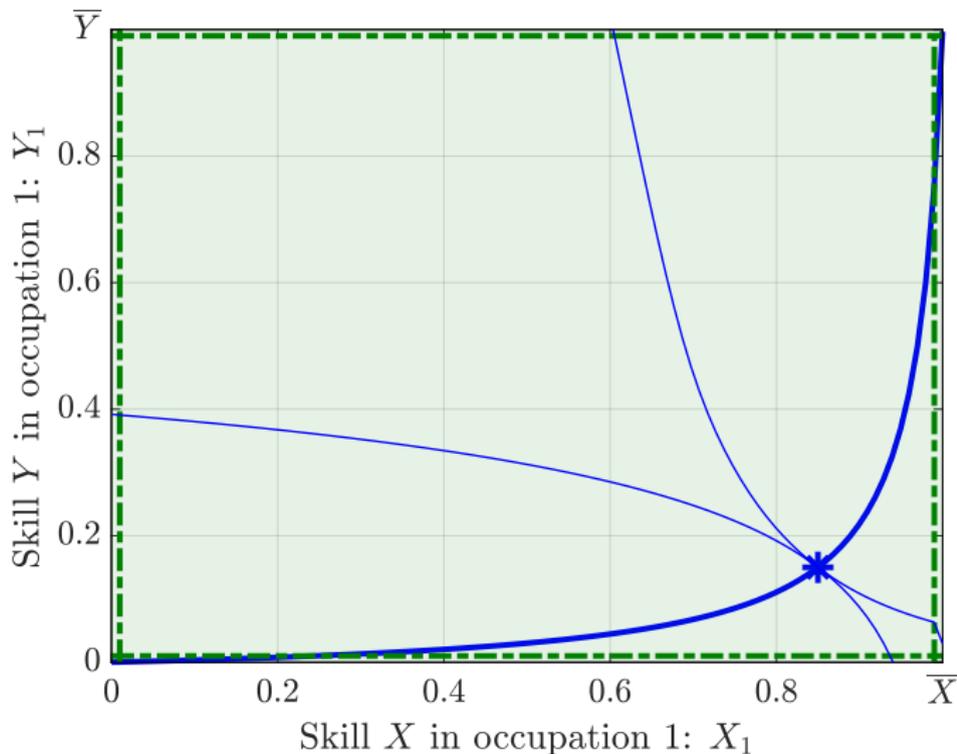
- ‘Complete’ skill supply  $\Rightarrow$  Always unbundled
- Law of one price holds for each skill

$$w(i) = \lambda_L l(i) + \lambda_H h(i)$$

$$\text{var}(\log w(i) \mid 1) = \text{var}(\log w(i))$$

# 1. Katz-Murphy

Entire set feasible. Equilibrium always unbundled, regardless of technology. Workers not sorted. All workers indifferent. No rents due to comparative advantage.  $w_j(i) = \lambda_X x(i)$



## Three special cases

$$\underbrace{\text{Katz-Murphy}}_{\theta \rightarrow 1}, \underbrace{\text{Roy}}_{\alpha_j \rightarrow 1}, \underbrace{\text{Lindenlaub}}_{J \rightarrow \infty}$$

### 2. Roy model

$$F_1 = Z_1 X_1 \quad , \quad X_1 = \int x(i) \phi_1(i) di \quad , \quad x(i) = \exp\left(\beta'_X \xi(i)\right)$$

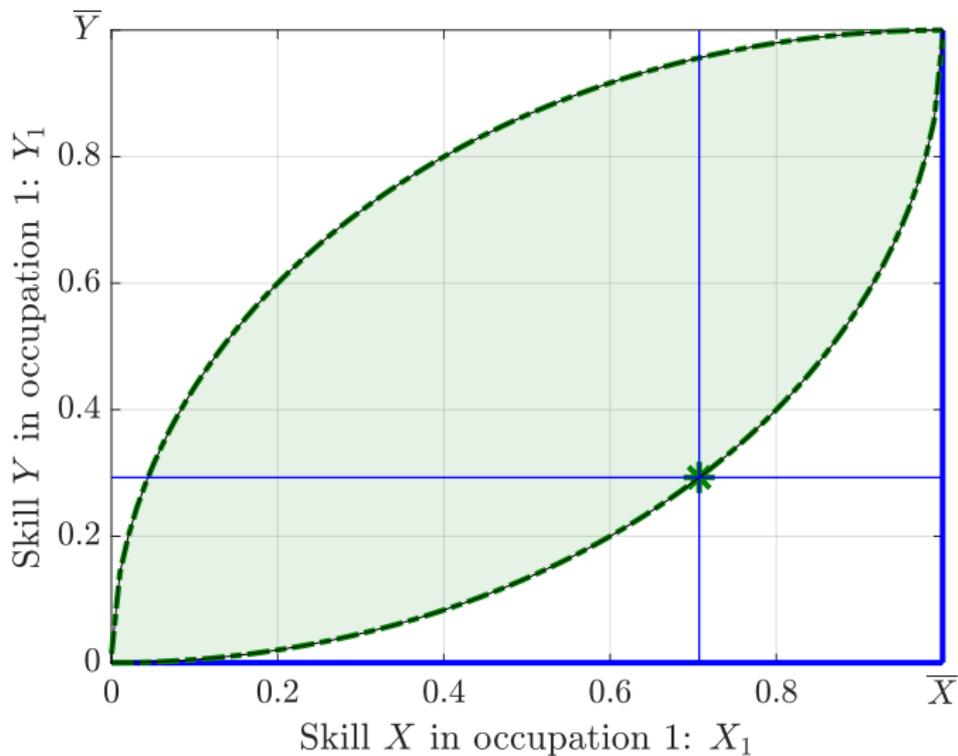
- *Extreme factor bias*  $\Rightarrow$  Always bundled
- One positive price for each 'skill' composite

$$w_1(i) = \lambda_{1X} x(i)$$

$$\text{var}\left(\log w(i) \mid 1\right) = \text{var}\left(\log x(i) \mid i < i^*\right)$$

## 2. Roy model

Equilibrium always bundled. Workers sorted by comparative advantage. Skill prices  $\lambda_{1X}/\lambda_{2Y}$  pinned down by relative skills of marginal worker,  $i^*$ .  $w_1(i) = \lambda_{1X}x(i)$



## Technology $\rightarrow$ Skill Prices $\rightarrow$ Inequality

- Roy - Returns to individual characteristics  $\xi(i)$  are exogenous:

$$\log w_1(i) = \log \lambda_{1X} + \beta'_X \xi(i)$$

Skill prices enter only through occupation fixed effect

- Our model - To a first order approximation

$$\log w_1(i) \approx \log \bar{w}_1 + \tilde{\beta}'_1 \xi(i), \quad \tilde{\beta}_1 = \tilde{\lambda}_1 \beta_X + (1 - \tilde{\lambda}_1) \beta_Y$$

Returns to individual characteristics  $\xi(i)$  are endogenous to skill prices

$$\tilde{\lambda}_1 = \frac{\lambda_{1X} \bar{x}_1}{\lambda_{1X} \bar{x}_1 + \lambda_{1Y} \bar{y}_1}$$

Changes re-weight characteristics  $\xi(i)$  via changes in skill prices  $\lambda_{1X}, \lambda_{1Y}$ . Roy model is special case where  $\lambda_{1Y} = 0$  always.

## Three special cases

$$\underbrace{\text{Katz-Murphy}}_{\theta \rightarrow 1}, \underbrace{\text{Roy}}_{\alpha_j \rightarrow 1}, \underbrace{\text{Lindenlaub}}_{J \rightarrow \infty}$$

### 3. Lindenlaub

$$\int_0^j Y(m) dm = \int_0^j X(m) dm \quad \text{for all } j \in [0, J] \quad \rightarrow \quad \underline{\mu}_j$$

- Continuum  $\alpha(j) \in [0, 1] \Rightarrow 1:1 \text{ matching} \Rightarrow$  All workers are marginal
- Continuum of skill prices

$$w_j(i) = \lambda_X(j)x(i) + \lambda_Y(j)y(i)$$

$$\text{var}(\log w(i) \mid j) = 0$$

# Three special cases

$$\underbrace{\text{Katz-Murphy}}_{\theta \rightarrow 1}, \underbrace{\text{Roy}}_{\alpha_j \rightarrow 1}, \underbrace{\text{Lindenlaub}}_{J \rightarrow \infty}$$

## 3. Lindenlaub

$$\int_0^j Y(j') dj' = \int_0^j X(j') dj' \quad \text{for all } j \in [0, J] \quad \rightarrow \quad \underline{\mu}_j$$

- Continuum  $\alpha(j) \in [0, 1] \Rightarrow 1:1 \text{ matching} \Rightarrow$  All workers are marginal
- Suppose  $\alpha(0) = 1$ , all weight on  $X$ , then

$$\lambda_X(j) = \lambda_X(0) - \int_0^j \underline{\mu}_j \times \left( \frac{y(i^*(j))}{x(i^*(j))} \right) dj$$

E.g. *technology more diverse*, constraints tighten,  $\uparrow$  gradient,  $\uparrow$  inequality

# This paper

## 1. Data

A. Heterogeneity in skill requirements across occupations

↓ Low skill jobs ,    ↑ High skill jobs

B. Inequality in wages within occupations

↓ Low skill jobs ,    ↑ High skill jobs

## 2. Theory

- Extend Rosen (1983), Heckman Scheinkman (1987)
- Technological change consistent with **A.** causes **B.**
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  - Endogenous bundling when skills  $X$  and  $Y$  are complements

**Thank you!**

# Appendix

# Endogenous technology

*Under what conditions do these changes in factor intensities emerge endogenously from an expansion in the set of available technologies?*

## 1. Production function

$$F_j = \left[ \alpha_j (a_{jX} X_j)^\sigma + (1 - \alpha_j) (a_{jY} Y_j)^\sigma \right]^{1/\sigma}, \quad \sigma < 1$$

# Endogenous technology

Under what conditions do these *changes in factor intensities* emerge endogenously from an *expansion in the set of available technologies*?

## 1. Production function

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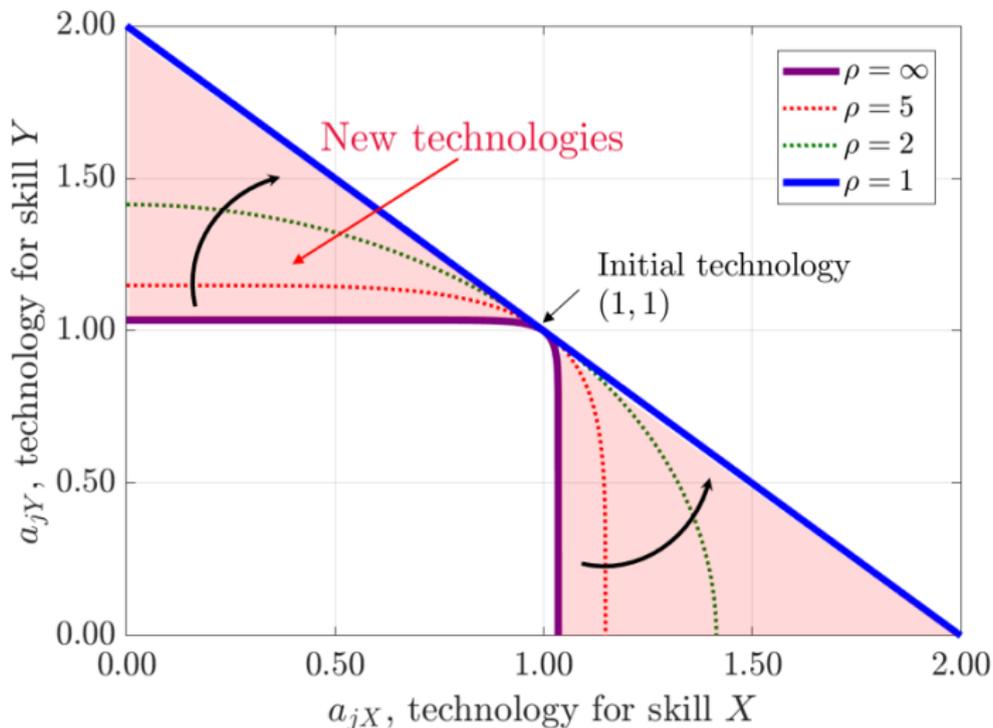
## 2. Minimize marginal cost subject to *available technologies*

$$\min_{a_{jX}, a_{jY}} \left[ \left( \frac{\lambda_{jX}}{\alpha_j^{1/\sigma} a_{jX}} \right)^{\frac{\sigma}{\sigma-1}} + \left( \frac{\lambda_{jY}}{(1-\alpha_j)^{1/\sigma} a_{jY}} \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\sigma-1}{\sigma}}$$

s.t.  $\left[ a_{jX}^\rho + a_{jY}^\rho \right]^{1/\rho} = \bar{A}_j, \quad \rho > 1$

# Available technologies

Technology frontier  $[a_{jX}^\rho + a_{jY}^\rho]^{1/\rho} = \bar{A}_j$ . As  $\rho \searrow 1$  can reach more combinations of  $a_{jX}, a_{jY}$  for given  $\bar{A}_j$ .



# Competitive equilibrium

- Skill prices determine technology adoption

$$\lambda_{jk} \quad \Longrightarrow \quad a_{jk}^*$$

Caselli-Coleman (2006)

- Adopted technology determines sorting and skill premia

$$a_{jk}^* \quad \Longrightarrow \quad \underline{\mu} \geq 0 \quad \Longrightarrow \quad \lambda_{jk}$$

Rosen (1983), Heckman Scheinkman (1987)

## Example

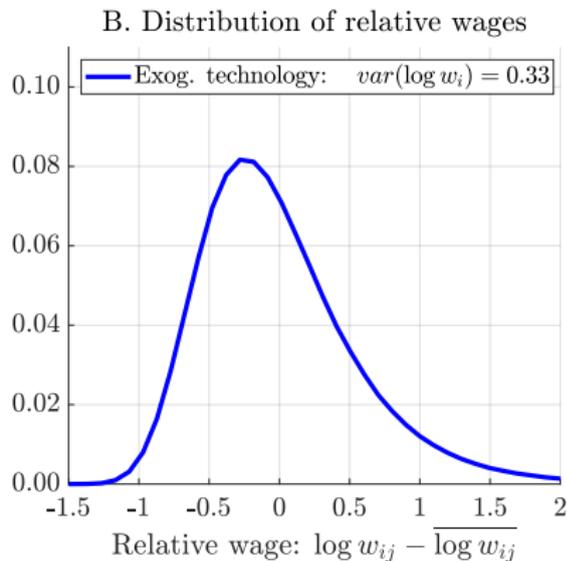
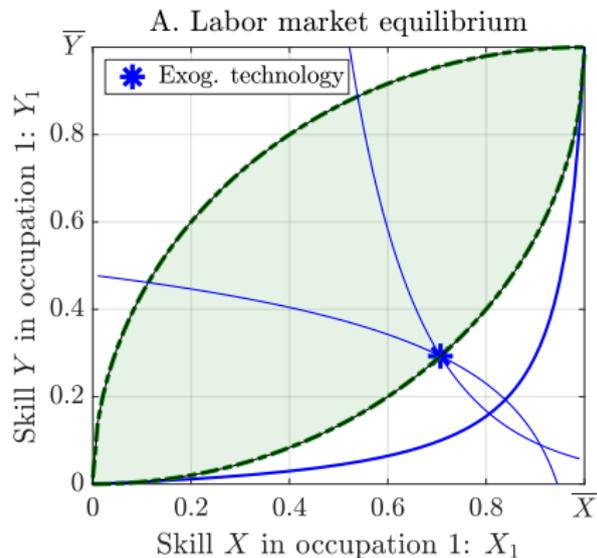
- Symmetric sectors
- Innate skill bias  $\alpha_j = 0.8$
- Short-run  $\rho = \infty \implies a_{jk} = 1$
- Long-run  $\rho = 1$ , choose technologies
- Production function CES with e.o.s.  $\sigma$
- Result

$\sigma > 0$  skills are substitutes  $\rightarrow$  *bundling*  $\sim$  High skill occupations

$\sigma < 0$  skills are complements  $\rightarrow$  *unbundling*  $\sim$  Low skill occupations

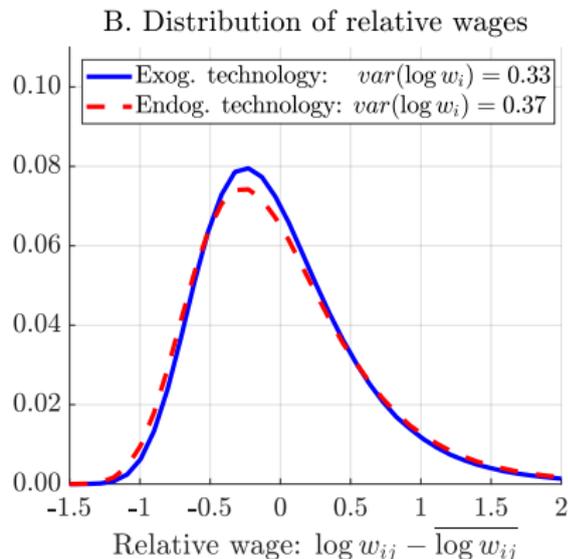
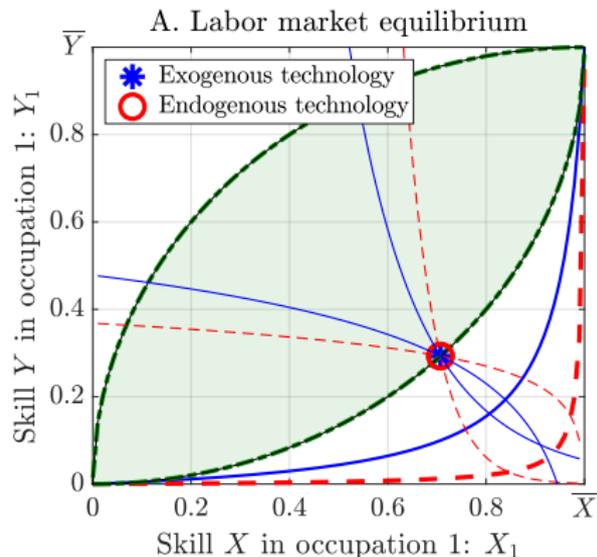
# Bundling labor: $\sigma > 0$

Skills are substitutes,  $\sigma > 0$ .



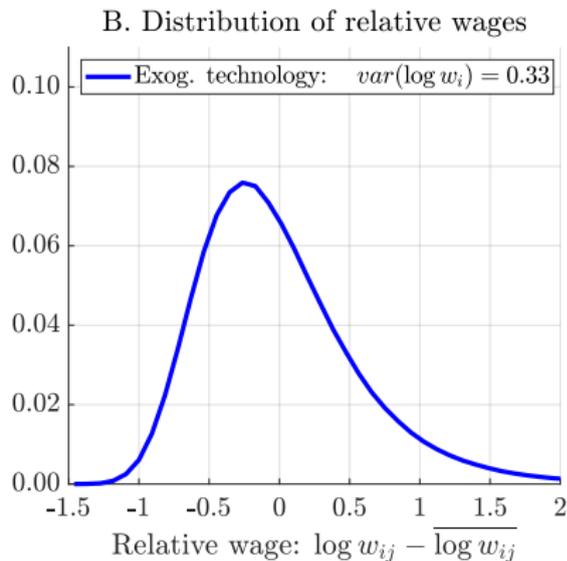
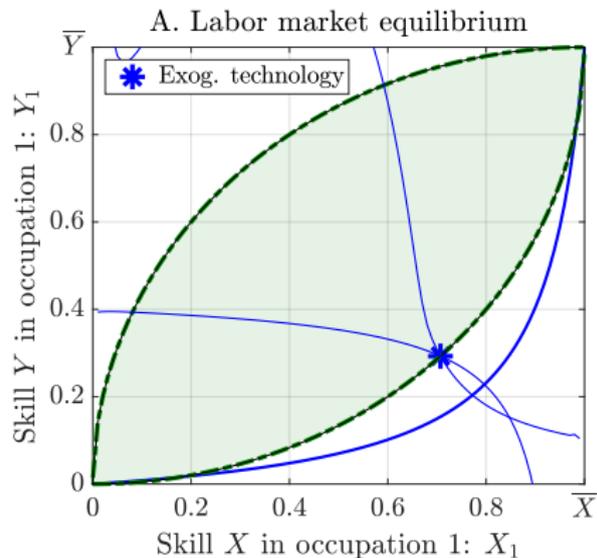
# Bundling labor: $\sigma > 0$

Skills are substitutes,  $\sigma > 0$ . Choose technology more skill biased. *Endogenously more 'Roy-like'. Bundling constraints tighter. Specialist wages increase. Increasing inequality.*



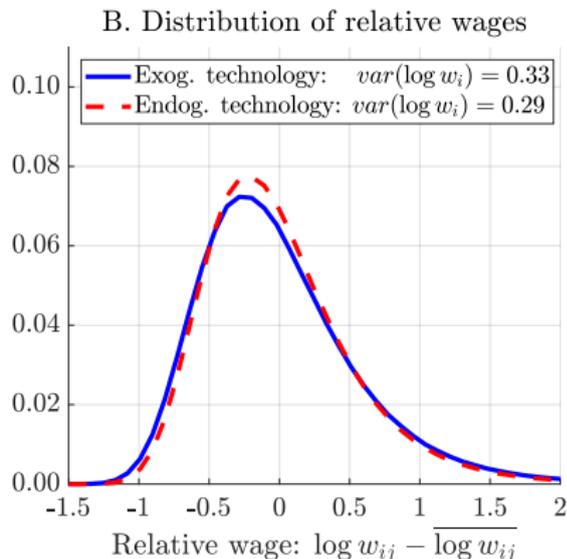
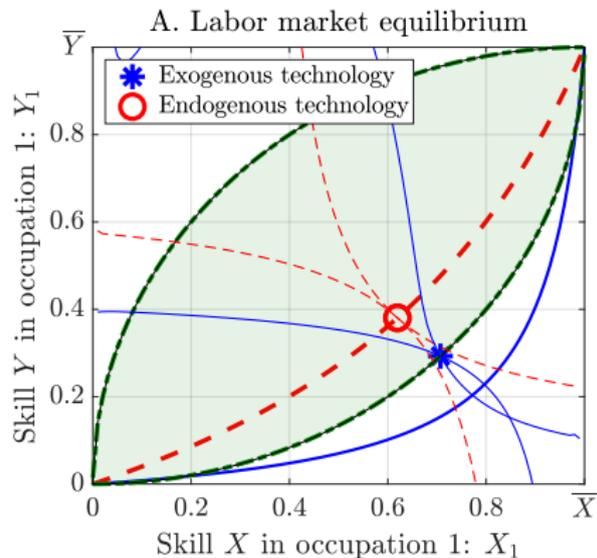
# Unbundling labor: $\sigma < 0$

Skills are complements,  $\sigma < 0$ .



# Unbundling labor: $\sigma < 0$

Skills are complements,  $\sigma < 0$ . Choose technology less skill biased. *Bundling constraints slack. Wage gains for generalists. Wage losses for specialists. Decreasing inequality.*



# This paper

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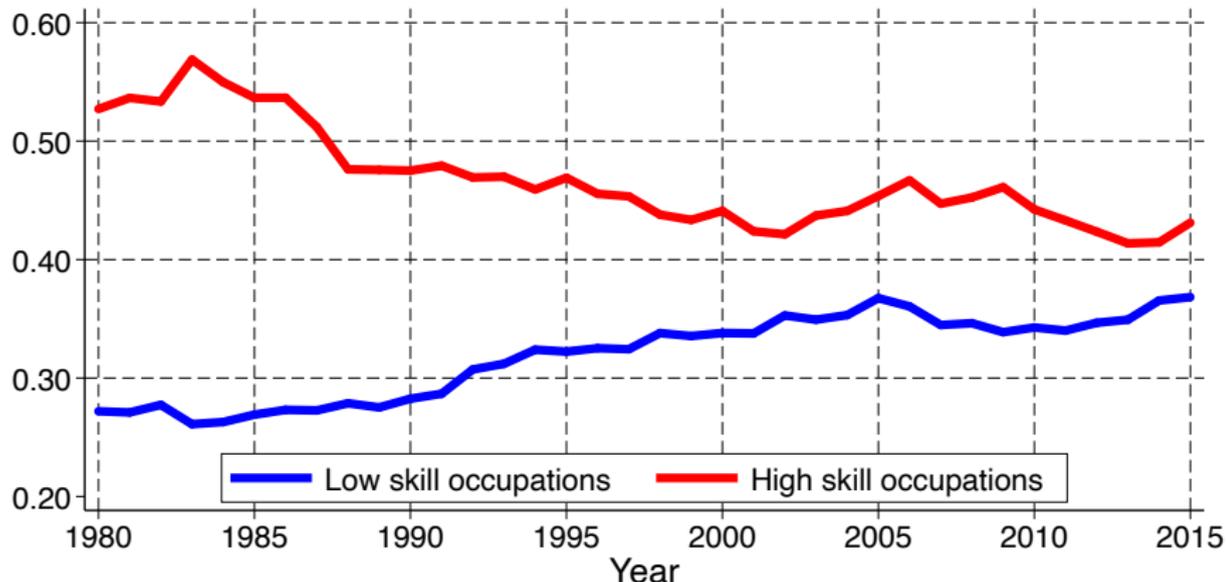
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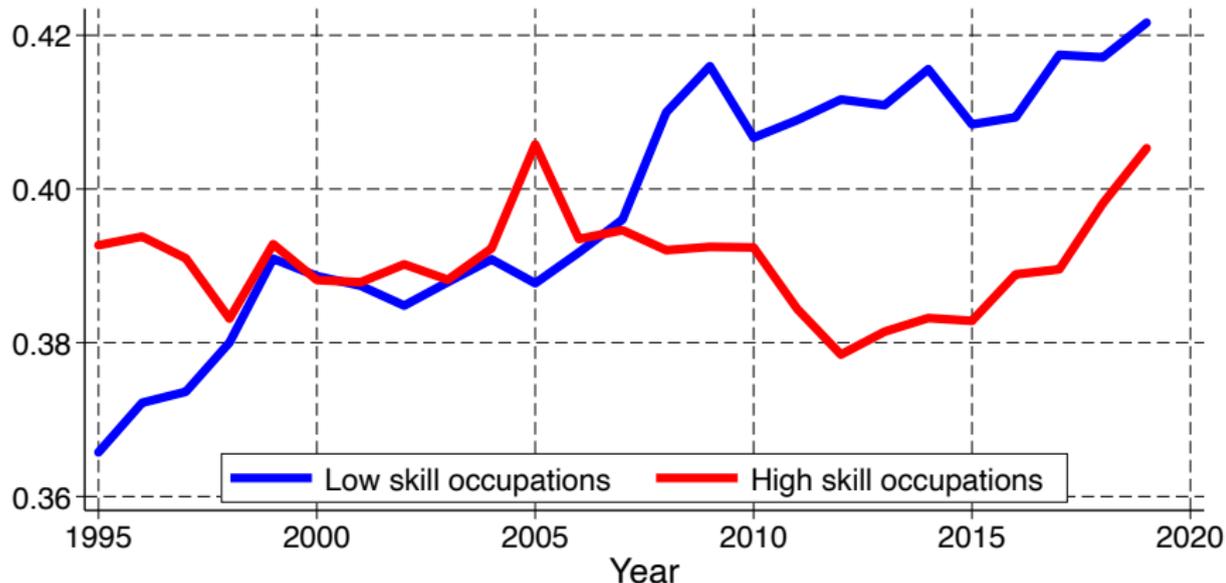
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# 1. Occupation switching



Fraction of male workers experiencing  $\{E_{March}, \dots, U_m, \dots, E_{March'}\}$   
that swap 1-digit occupations across  $\{E_{March}, E_{March'}\}$

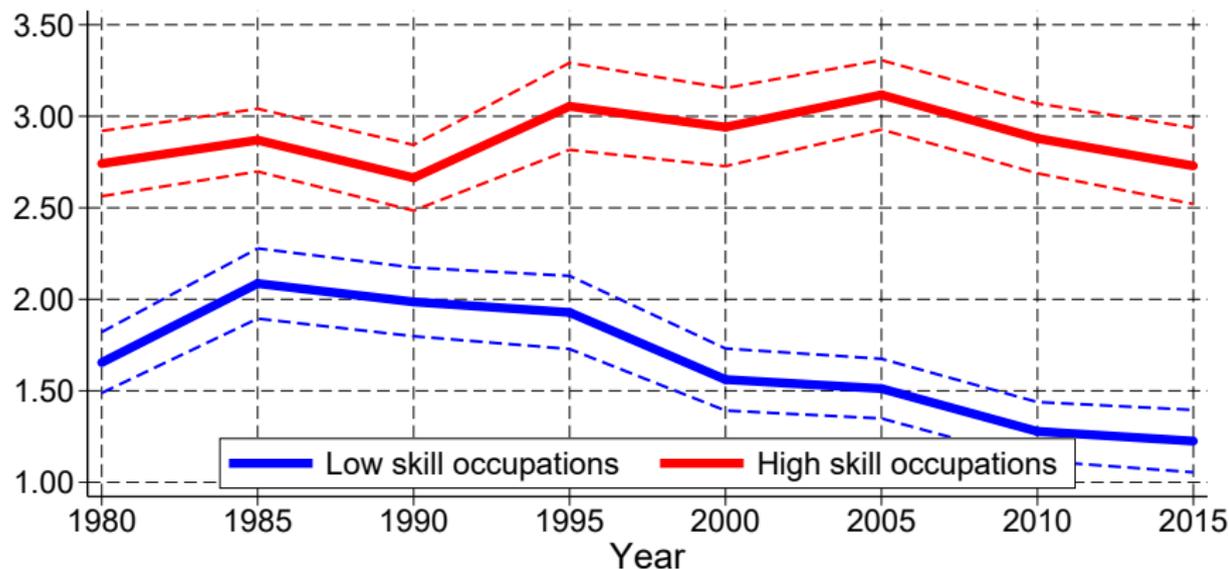
# 1. Occupation switching



Fraction of male workers experiencing  $\{E_{Month}, E_{Month+1}\}$

that swap 1-digit occupations across  $\{E_{Month}, E_{Month+1}\}$

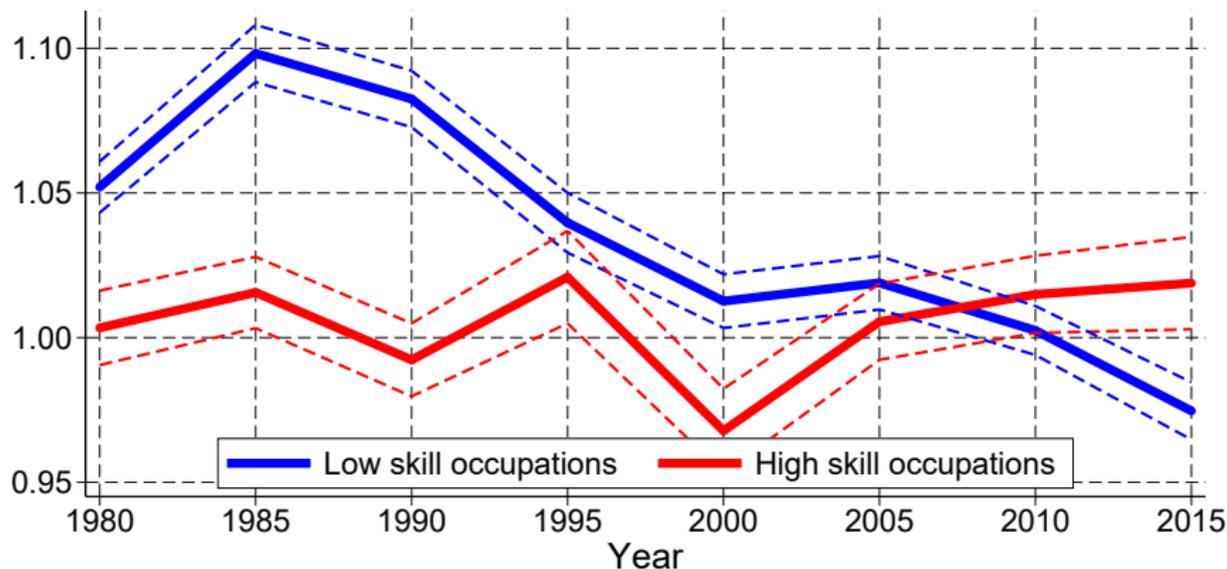
## 2. Experience premium



One extra year experience associated with 2 to 3 percent higher wage

$$\log Inc_{it} = \alpha + \beta_{Exp}^{\tau} Exp_{it} + \beta_{Exp^2}^{\tau} Exp_{it}^2 + \beta_{Hours}^{\tau} \log Hours_{it} + \beta_{Size}^{\tau} Size_{it} \dots \\ + \beta_X^{\tau} [Year_t, Race_{it}, NAICS1_{it}, Ed_{it}, Sex_{it}]$$

### 3. Hours premium



(= 1): wage independent of hours, ( $\geq 1$ ): wage increasing in hours

$$\log Inc_{it} = \alpha + \beta_{Exp}^{\tau} Exp_{it} + \beta_{Exp^2}^{\tau} Exp_{it}^2 + \beta_{Hours}^{\tau} \log Hours_{it} + \beta_{Size}^{\tau} Size_{it} \dots \\ + \beta_X^{\tau} [Year_t, Race_{it}, NAICS1_{it}, Ed_{it}, Sex_{it}]$$

## Interpreting other facts

1. Increasing *occupation switching* in low skill jobs
2. Declining *experience premium* in low skill jobs
3. Declining *overtime premium / part-time penalty* in low skill jobs

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1. Increasing *occupation switching* in low skill jobs
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2. Declining *experience premium* in low skill jobs
  - Add learning by doing in the direction of occupation skill bias  
Cavounidis Lang (JPE, 2020)
  - Experience premium  $\leftrightarrow$  Inframarginal rents
  - Unbundling labor reduces gradient of primary / secondary skill prices
  - Reduces observed experience premium
3. Declining *overtime premium* / *part-time penalty* in low skill jobs

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  - Unbundling labor reduces gradient of primary / secondary skill prices
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3. Declining *overtime premium / part-time penalty* in low skill jobs
  - Requires more work to extend the model
  - Unbundling labor  $\leftrightarrow$  Workers are more 'substitutable'

# Conclusions

- Deviations from law of one price for skills if either
  - (i) technologies sufficiently factor biased, or
  - (ii) weak pattern of comparative advantage in skills
- Can generate opposite trends in *within-occupation wage inequality* from technology adoption consistent with the data
- If skills *substitutes*, technology adoption *tightens bundling constraints*
  - ↑ returns to comparative advantage, ↑ sorting
  - ↑ within-occupation wage inequality
  - Consistent with experience of *high skill occupations*
- If skills *complements*, technology adoption *can cause unbundling*
  - ↓ returns to comparative advantage, ↓ sorting
  - ↓ within-occupation wage inequality
  - Consistent with experience of *low skill occupations*



## Fact A. - Technology

- Input is a  $J \times K$  normalized matrix of skill measures  $\mathbf{A}$  from O\*NET

1. Apply principal components with  $K^* \ll K$

$$\mathbf{A}_{[J \times K]} = \widehat{\mathbf{A}}_{[J \times K^*]} \widehat{\mathbf{P}}_{[K^* \times K]} + \mathbf{U}_{[J \times K]}$$

2. To *name* skills, rotate principal components s.t. satisfy  $K^*$  orthogonality conditions

$$\mathbf{A}_{[J \times K]} = \left( \widehat{\mathbf{A}}_{[J \times K^*]} \Psi \right) \left( \Psi^{-1} \widehat{\mathbf{P}}_{[K^* \times K]} \right) + \mathbf{U}_{[J \times K]} \rightarrow \mathbf{A}^* = \widehat{\mathbf{A}} \Psi$$

$\implies$  Final skill 1, places a weight of 1 on  $k = 1$ , and zero on  $k \in \{2, \dots, K^*\}$

3. Use as  $K^*$  ‘anchoring’ skills those used by Acemoglu Autor (2011)

- **Non-routine cognitive: Analytical** - “Analyzing data / information”
- **Non-routine cognitive: Interpersonal** - “Maintaining relationships”
- **Routine cognitive** - “Importance of repeating the same tasks”
- **Routine manual** - “Controlling machines and processes”

# Comparative statics

1. *Symmetric change in factor bias* -  $\alpha$
2. *Task-biased change* -  $Z_1$  
3. *Skill-biased change* -  $\psi_A$  
4. *Task-skill-biased change* -  $\zeta_{1A}$  

$$U(Y_1, Y_2) = \left[ \eta Y_1^{\frac{\phi-1}{\phi}} + (1-\eta) Y_2^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \quad \phi > 1$$

$$Y_1 = Z_1 \left[ \zeta_{1A} \psi_A \alpha X_1^\sigma + (1-\alpha) Y_1^\sigma \right]^{\frac{1}{\sigma}}$$

$$Y_2 = \left[ \psi_A (1-\alpha) X_2^\sigma + \alpha Y_2^\sigma \right]^{\frac{1}{\sigma}}$$

# Within-occupation skill prices and inequality

## 1. Wages

$$w_1(i) = \lambda_{1X} x(i) + \lambda_{1Y} y(i)$$

## 2. Sorting

- Occupation 1 chosen by individuals with high  $\uparrow x(i)/y(i)$

## 3. Inequality

- Increases as gradient of skill prices steepens  $\uparrow \lambda_{1X}/\lambda_{1Y}$
- Decreases as gradient of skill prices flattens  $\downarrow \lambda_{1X}/\lambda_{1Y}$

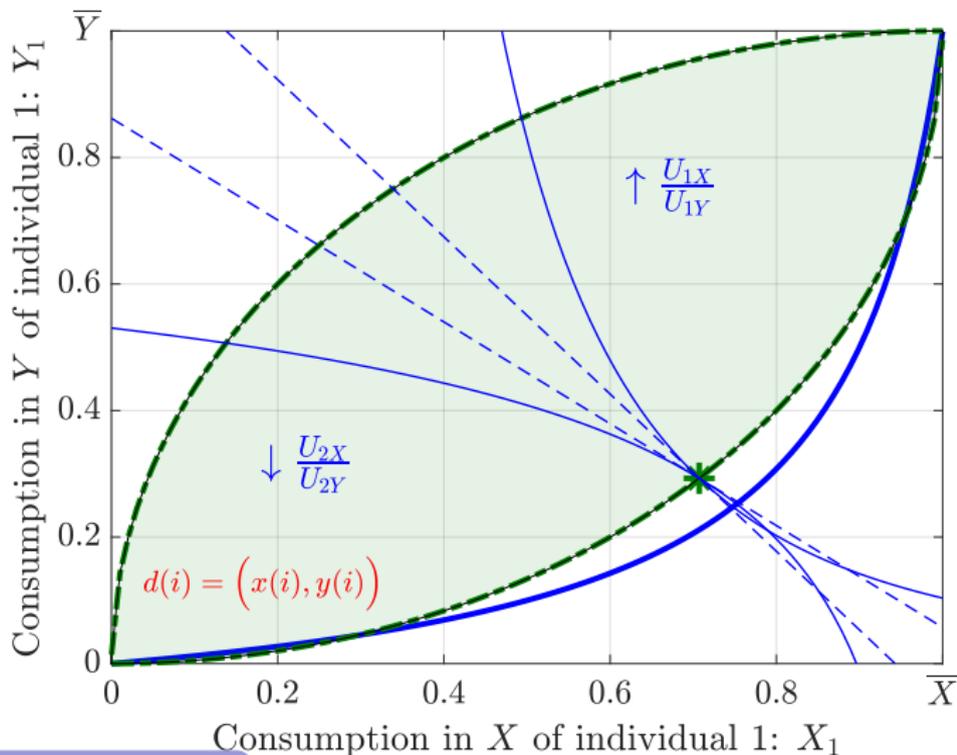
## In the paper

- Closed form example under  $(x(i), y(i)) = (e^{\alpha(1-i)}, e^{\alpha i})$
- Log-linear approximation to compute conditional variance
- Decomposes  $\text{var}(\log w(i)|j)$  into (i) Endowments, (ii) Prices

# Incomplete markets allocation

Bundling constraint binds. Cannot 'break open' assets to get at underlying arrow securities

$$U_{1A} + \underline{C}'(C_{1A})U_{1B} = U_{2A} + \underline{C}'(C_{1A})U_{2B}$$



# Link to Bais, Hombert, Weill (2020)

- **Setup** - Two agents  $j \in \{1, 2\}$  consume in two states  $k \in \{A, B\}$
- **Preferences** - Expected utility of consumption

$$F_j(C_{jA}, C_{jB}) = \pi_A \alpha_j \frac{C_{jA}^{1-\gamma}}{1-\gamma} + \pi_B (1 - \alpha_j) \frac{C_{jB}^{1-\gamma}}{1-\gamma} \quad , \quad \alpha_1 > \frac{1}{2} > \alpha_2$$

- **Trees** - Physical assets indexed  $i \in [0, 1]$  have payoffs

$$d(i) = (d_A(i), d_B(i)) \quad , \quad d_A(i)/d_B(i) \text{ decreasing in } i$$

- **Budget constraints** - Period-0 and Period 1, State- $k$

$$\int Q(i) \phi_j(i) di + q_{AA} a_{jA} + q_{BA} a_{jB} \leq \phi_j^0 \int Q(i) di$$

$$C_{jk} = \int \phi_j(i) d_k(i) di + a_{jk}$$

- **Incentive compatibility** - Only short arrow securities up to  $(1 - \delta)$  of tree payoffs

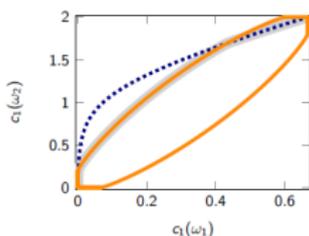
$$C_{jk} \geq \delta \int \phi_j(i) d_k(i) di \quad , \quad k \in \{A, B\} \quad \text{Slack if } \delta = 0. \text{ No shorts if } \delta = 1$$

- **Feasibility** - What IC  $(C_{1A}, C_{2A})$  can be supported by a set of trees?

$$C_{1A} = \delta \int_0^{k^*} d_A(i) di \rightarrow k^*(C_{1A}) \rightarrow \underline{C}_{1B}(C_{1A}) \geq \delta \int_0^{k^*(C_{1A})} d_B(i) di$$

# Link to Bais, Hombert, Weill (2020)

the model in an edgeworth box



a graphical analysis of the incentive feasible set (IF set)

- area inside the orange curve: IF set with many trees and  $\delta < 1$
- dotted-blue curve: Pareto set without IC constraints
- highlighted-grey curve: Pareto set with IC constraints

- Here w/out IC, trees redundant. Trade in Arrow securities.  $Q(i) = \sum_k q_k d_k(i)$ .
- If IC binds, ratios of marginal utilities not equated:  $\lambda_{1X}/\lambda_{1Y} > \lambda_{2X}/\lambda_{2Y}$
- The price of tree  $i$  depends on which agent  $j$  holds it

$$Q_1(i) = q_A d_A(i) + (q_B - \delta \mu_{1B}) d_B(i), \quad Q_2(i) = (q_A - \delta \mu_{1A}) d_A(i) + q_B d_B(i)$$

- In equilibrium  $\lambda_{1X} > \lambda_{2X}$  and  $\lambda_{1Y} < \lambda_{2Y}$ , which implies  $\lambda_{1X} > \lambda_{1Y}$
- **Result** - Securities with more extreme pay-offs (specialists) are more expensive
- **Result** - Price of tree encodes constraint, lower than replicating arrow securities

# Competitive equilibrium

$$\Pi_1 = \max_{X_1, Y_1} P_1 F_1(X_1, Y_1) - \text{Cost}_1(X_1, Y_1)$$

$$\text{Cost}_1(X_1, Y_1) = \min_{\tilde{\phi}_1(i)} \int \tilde{\phi}_1(i) w_1(i) di$$

subject to

$$X_1 = \int \tilde{\phi}_1(i) x(i) di \quad [\tilde{\lambda}_{1X}] \quad \longrightarrow \quad \tilde{\lambda}_{1X} = P_1 F_{1X} \quad (MC_{1X} = MRPL_{1X})$$

$$Y_1 = \int \tilde{\phi}_1(i) y(i) di \quad [\tilde{\lambda}_{1Y}] \quad \longrightarrow \quad \tilde{\lambda}_{1Y} = P_1 F_{1Y} \quad (MC_{1Y} = MRPL_{1Y})$$

Labor demand for each type

$$\tilde{\phi}_1(i) = \begin{cases} 1 & , \text{ if } \tilde{\lambda}_{1X} x(i) + \tilde{\lambda}_{1Y} y(i) > w_1(i) \\ 0 & , \text{ if } \tilde{\lambda}_{1X} x(i) + \tilde{\lambda}_{1Y} y(i) < w_1(i) \\ \in (0, 1) & , \text{ if } \tilde{\lambda}_{1X} x(i) + \tilde{\lambda}_{1Y} y(i) = w_1(i) \end{cases}$$

# Competitive equilibrium

- Prices per efficiency unit of skill

$$\begin{aligned}w_j(l_A, l_B) &= \omega_{jA}l_A + \omega_{jB}l_B \\ \omega_{jk} &= P_j F_{jk} = U_j F_{jk}\end{aligned}$$

- Worker  $(l_A, l_B)$  chooses occupation  $j = 1$  only if

$$w_1(l_A, l_B) > w_2(l_A, l_B)$$

- Cutoff worker indifferent

$$\underbrace{\frac{\lambda_{1X} - \lambda_{2X}}{\lambda_{2Y} - \lambda_{1Y}}}_{\text{Benefit of } j = 1} = \underbrace{\left(\frac{l_B}{l_A}\right)^*}_{\text{Relative skill in } j = 2} = \underline{B}'(X_1)$$

Under  $\{\omega_{jk} = U_j F_{jk}\}$ , this is the same condition as in the planner's problem

# Competitive equilibrium

- *Bundled equilibrium*: Sorting premia are increasing in  $\underline{\mu}$

$$\lambda_{1X} - \lambda_{2X} = \underline{\mu} \underline{B}'(X_1)$$

$$\lambda_{2Y} - \lambda_{1Y} = \underline{\mu}$$

- Inframarginal workers earn rents due to comparative advantage, determined by sorting premia.
- Additional source of within-occupation wage inequality
- *Unbundled equilibrium*: Sorting premia are zero, indeterminate sorting

$$\lambda_{1X} - \lambda_{2X} = 0$$

$$\lambda_{2Y} - \lambda_{1Y} = 0$$

- All workers are marginal. No rents due to comparative advantage.

# Generalized Roy model

- Individual-occupation specific output

$$y_j(i) = \exp\left(\alpha_{j1}\xi_1(i) + \alpha_{j2}\xi_2(i)\right) \quad , \quad Y_j = \int \phi_j(i)y_j(i) di$$

- The *only* priced objects are  $y_1(i)$ ,  $y_2(i)$  with prices  $w_1, w_2$

$$\log w_j(i) = \log w_j + \alpha_{jA}x(i) + \alpha_{jB}y(i)$$

- In our case

$$\log w_j(i) \approx \log \bar{w}_j + \tilde{\omega}_{jA}\hat{l}_A(i) + \tilde{\omega}_{jB}\hat{l}_B(i)$$

1. Technology affects wages directly through the technology coefficients
2. Within occupation inequality effects are silo-ed:
  - Suppose that technology changes in occupation 2
  - All changes in the economy are encoded in the *occupation skill price*  $w_j$ , i.e. the occupation fixed effect
  - No change in incumbent *within occupation inequality* in occupation 1

# Wage inequality - Closed form example

- Skills for individuals  $i \in [0, 1]$

$$(x(i), y(i)) = (\gamma e^{\alpha(1-i)}, \gamma e^{\alpha i}) \rightarrow y(i)/x(i) = e^{\alpha(2i-1)}$$

- Approximate log wage around mean log skills conditional on selection  $i^*$

$$\log w(i, j) = \log [\lambda_{1X} e^{\log x(i)} + \lambda_{1Y} e^{\log y(i)}]$$

- Within occupation inequality

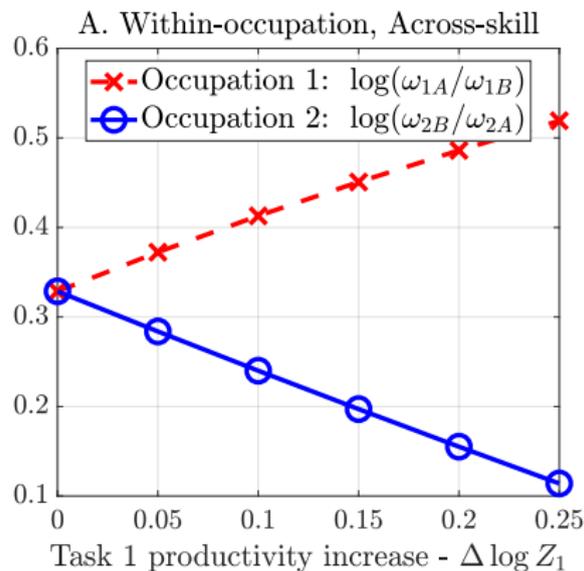
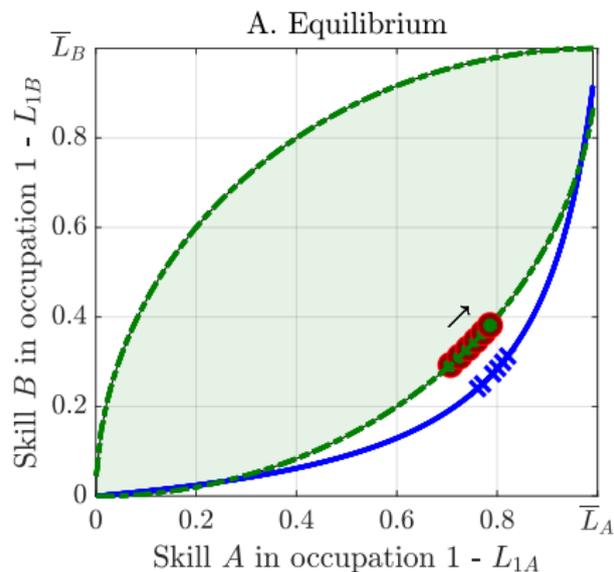
$$\text{var}(\log(w(i)) \mid j^*(i) = 1) = \underbrace{\left( \frac{\left( \frac{\lambda_{1X}}{\lambda_{1Y}} \right) e^{\alpha(1-i^*)} - 1}{\left( \frac{\lambda_{1X}}{\lambda_{1Y}} \right) e^{\alpha(1-i^*)} + 1} \right)}_{\text{Bundling}} \underbrace{\alpha^2 \frac{i^{*2}}{12}}_{\text{Roy}}$$

1. Roy As  $\lambda_{1X}/\lambda_{1Y} \rightarrow \infty$ , bundling terms goes to zero
2. Bundling With finite  $\lambda_{1X}/\lambda_{1Y}$ , inequality increasing in ratio

## 2. Task-Biased Change

Exogenous  $\uparrow Z_1$ , with  $\phi > 1$ :  $\uparrow Y_1, \downarrow Y_2$ .

Marginal worker has more Skill  $B$ , pushes up  $\lambda_{1X}/\lambda_{1Y}$ . Opposite for task 2.

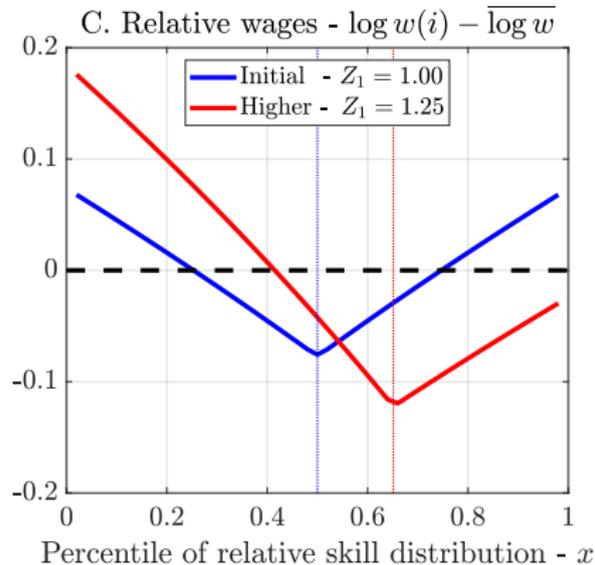
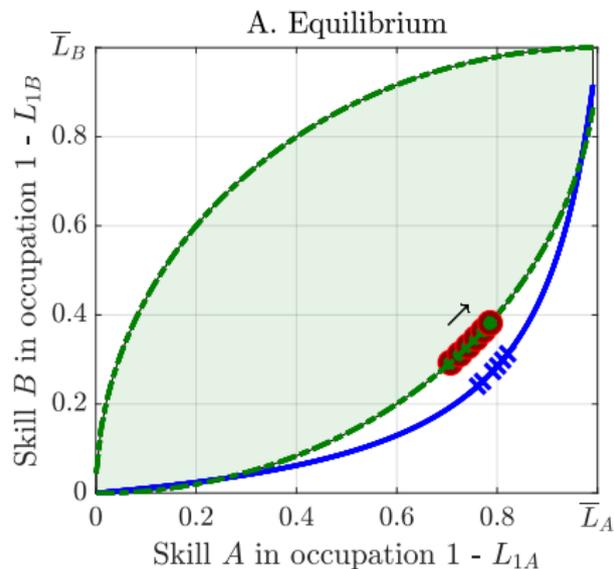


Other parameters:  $\alpha_{1A} = \alpha_{2B} = 0.80$ ,  $\sigma = 0.20$ ,  $\theta = 2$ ,  $\bar{L}_1 = \bar{L}_2 = 1$ ,  $Z_2 = 1$ .

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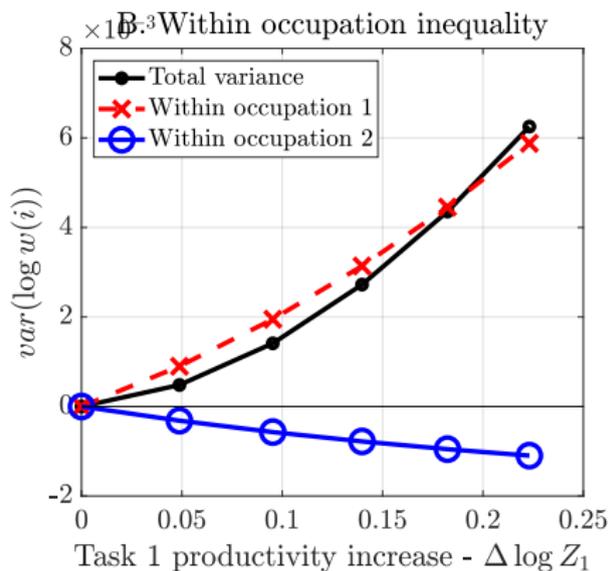
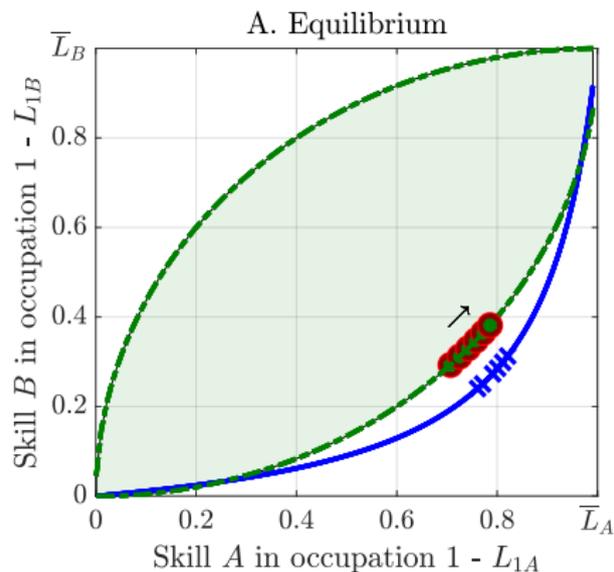


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Exogenous  $\uparrow Z_1$ , with  $\phi > 1$ :  $\uparrow Y_1, \downarrow Y_2$ .

Marginal worker has more Skill  $B$ , pushes up  $\lambda_{1X}/\lambda_{1Y}$ . Opposite for task 2.

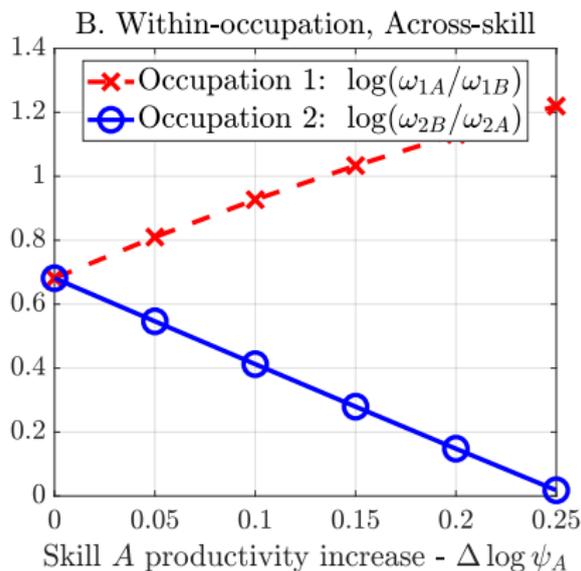
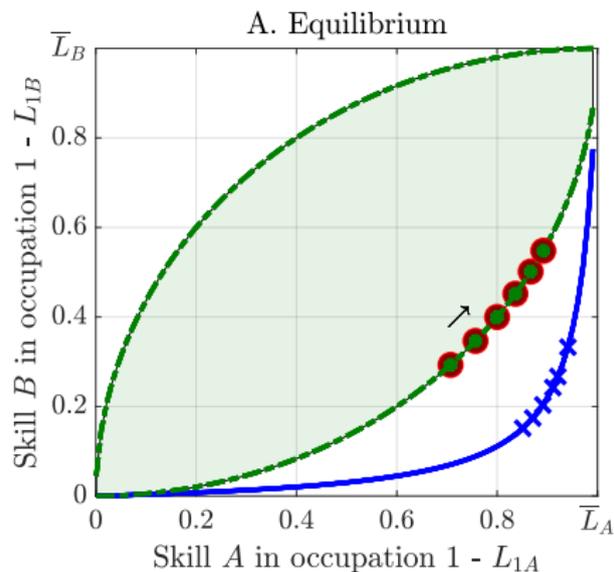


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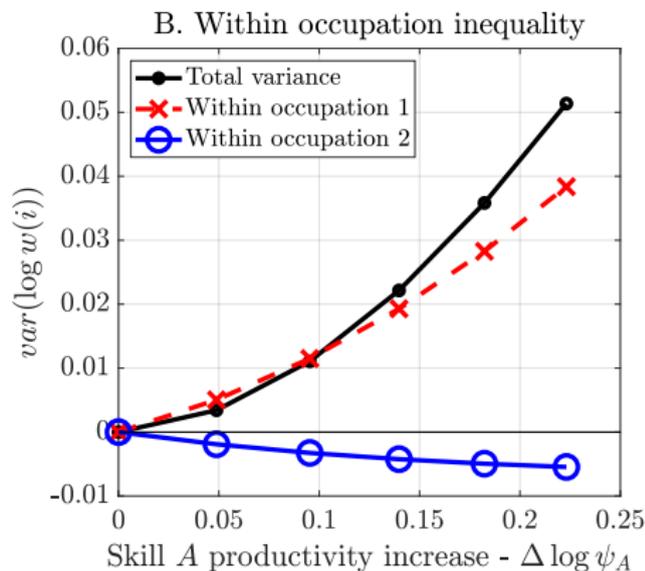
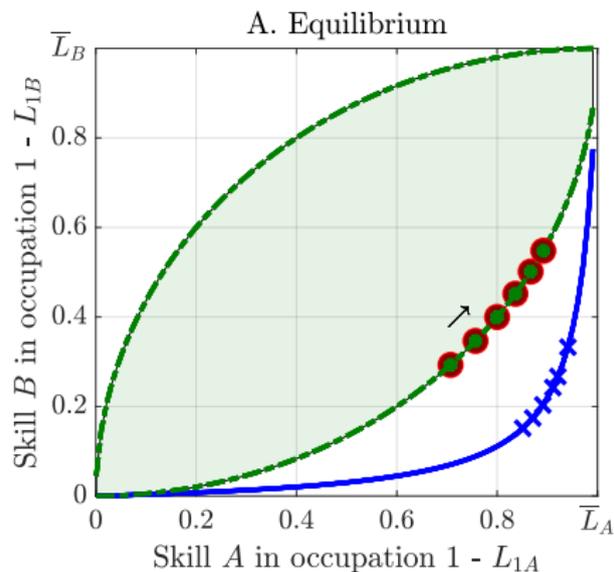


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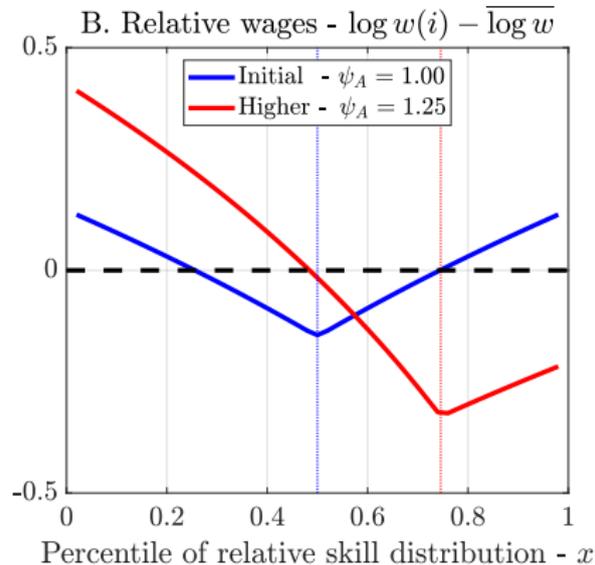
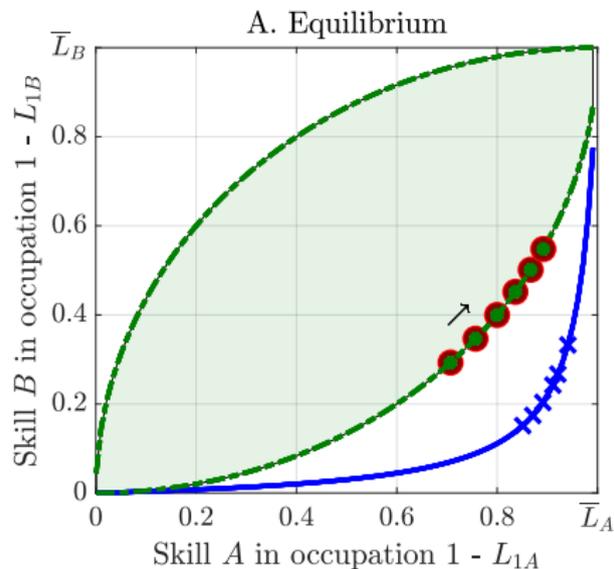


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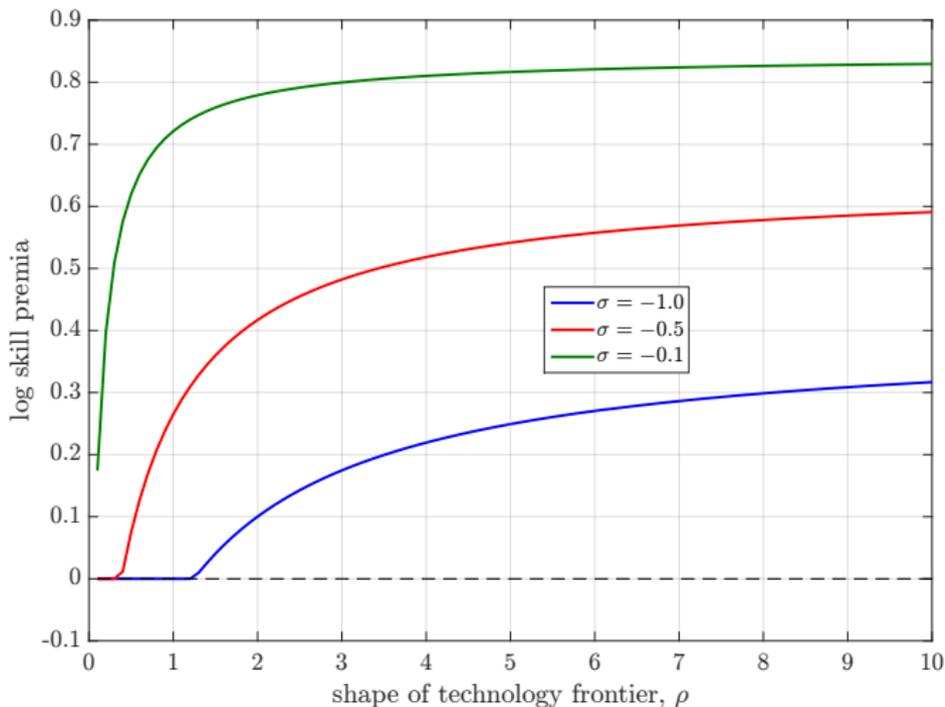
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## Unbundling Labor: $\downarrow \rho, \sigma < 0$

As  $\rho$  falls, technologies become 'more substitutable'. If  $\sigma < 0$ , firms undo existing skill bias, bundling constraints loosen, skill premia fall, wage gains for generalists.  $p_A = \lambda_{1X} - \lambda_{2X}$



# Extensions I

- Absolute vs. comparative advantage

$$(l_1, l_2) = (\psi, \psi x) \quad , \quad (\psi, x) \sim H(\psi, x)$$

+ fixed utility of being out of the labor market

- Selection on *x margin* (occupation) and on  *$\psi$  margin* (participation)
- **RESULT:** Competitive equilibrium allocation is efficient

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- **RESULT:** Competitive equilibrium allocation is efficient
- What are the effects of adding a mass of *low-productivity unspecialized workers* ( $\downarrow \psi, x \approx 1$ )?
  - (sr) wages and allocations for fixed technology
  - (lr) wages and allocations for endogenous technology

# Empirics - Details

- All data based on March CPS 'last year' questions
- Occupation, Industry - Dorn's 1990 harmonized cross-walk
  - Drop military
  - Occupation skill = Fraction of workers with high-school or less
  - Occupations sorted on occupation skill
- Use HPV (RED, 2010)
  - Earnings = Wage income +  $(2/3) \times$  Self employment income
  - Annual hours = *Weeks worked last year*  $\times$  *Usual hours worked per week*
  - Wage = Earnings / Annual hours
  - Age 25-65, Wage  $> 0.5 \times$  Federal minimum wage, Hours  $>$  One month of 8hr days
- Regression controls for residualized wage:
  - Worker education (3 levels), Industry (1 digit), Experience, Experience<sup>2</sup>  
Race, Log hours,
  - Experience = (age - max(years in school,12)) - 6

# Empirics - Regressions

## 1. Workers in low skill occupations getting paid more ‘similarly’.

- Reduced form empirical evidence from the CPS

$$\log Earnings_{i,t} = \gamma_t + \delta_{period}^{Occ} + \beta'_{period} \mathbf{X}_{i,t} + \varepsilon_{i,t}$$

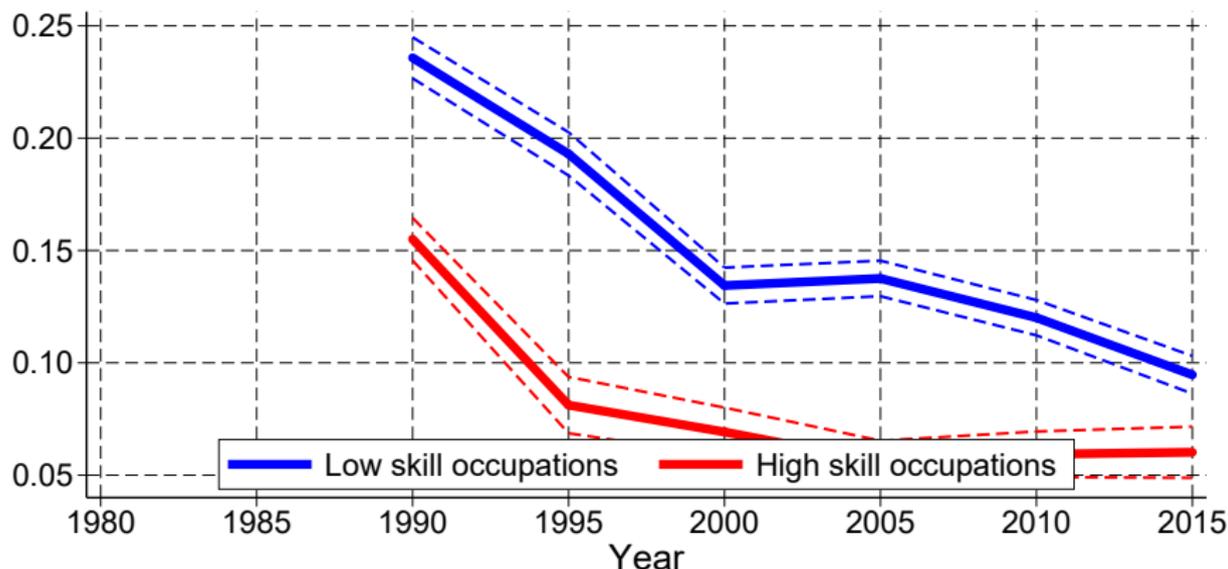
$$\mathbf{X}_{i,t} = [Year_t, NAICS1_{it}, Ed_{it}, Race_{it}, Sex_{it}, FirmSize_{it}, Exp_{it}, Exp_{it}^2, Hours_{it}]$$

- **Low skill:** Decline in  $\downarrow \hat{\beta}_{period}$  for (i) experience, (ii) hours, (iii) large firm
- **High skill:** No change

## 2. Anecdotal evidence from US labor market

- Goldin Katz (2012) vs. David Weil (2014)
- Hard to explain declining level of ‘attachment’ of working age men

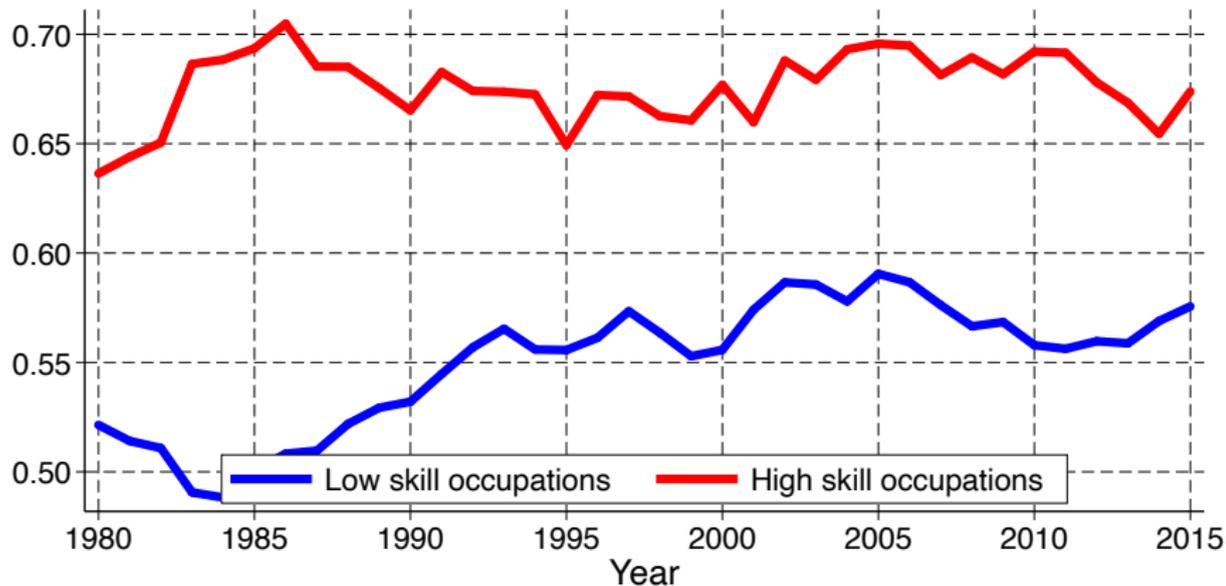
## Decreasing size premium in low skill occ



1000+ employee firms associated with a 10 to 15 percent premium

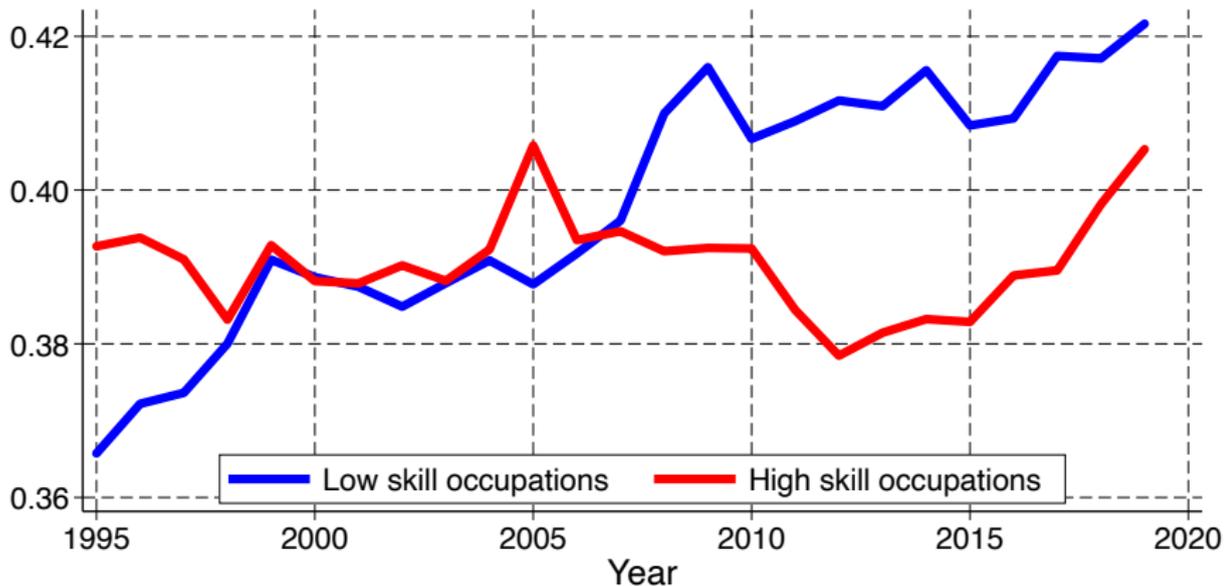
$$\log Inc_{it} = \alpha + \beta_{Hours}^T \log Hours_{it} + \beta_{Exp}^T Exp_{it} + \beta_{Exp^2}^T Exp_{it}^2 + \beta_{Size}^T Size_{it} \dots \\ + \beta_X^T [Year_t, Race_{it}, NAICS1_{it}, Ed_{it}, Sex_{it}]$$

## Increasing switching in low skill occ



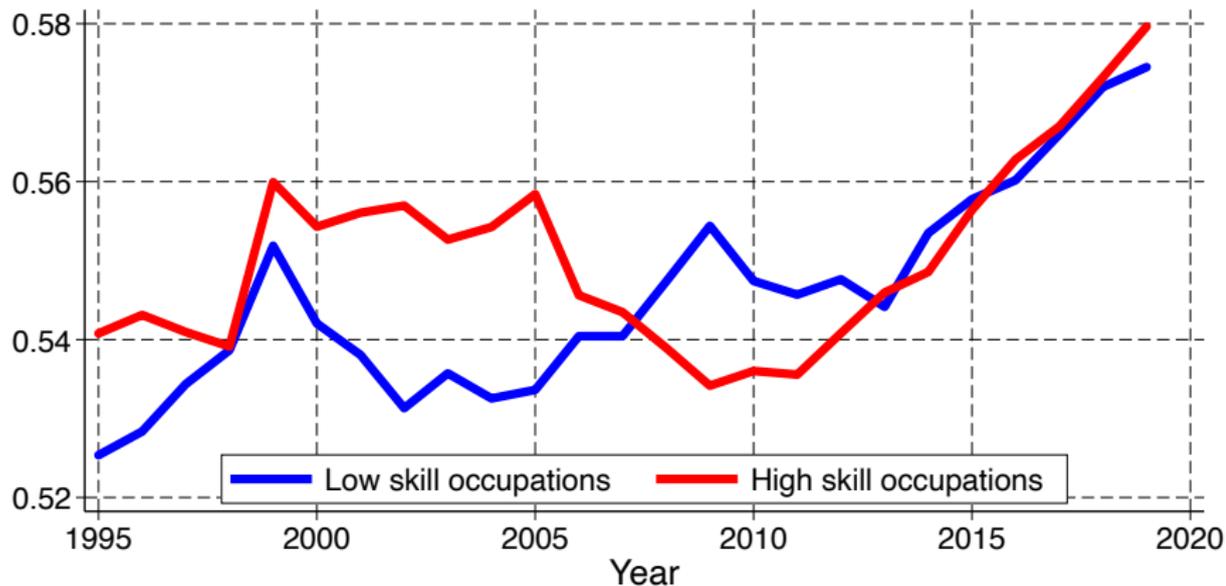
Fraction of male workers experiencing  $\{E_{March}, \dots, U_m, \dots, E_{March'}\}$   
that swap 3-digit occupations across  $\{E_{March}, E_{March'}\}$

## Increasing switching in low skill occ



Fraction of male workers experiencing  $\{E_{Month}, E_{Month+1}\}$   
that swap 1-digit occupations across  $\{E_{Month}, E_{Month+1}\}$

## Increasing switching in low skill occ



Fraction of male workers experiencing  $\{E_{Month}, E_{Month+1}\}$   
that swap 3-digit occupations across  $\{E_{Month}, E_{Month+1}\}$