Minimum Wages and Welfare

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The views expressed herein are those of the authors and not those of the Census or the Federal Reserve System.

What is the optimal (Federal) minimum wage?

We develop and quantify a general equilibrium macro model

- Firm heterogeneity, strategic interactions, worker heterogeneity

1. Accommodates different sides of the minimum wage debate
   - Firms have labor market power: $w_i = \mu_i mrpl_i$
   - Labor misallocation worsens if the minimum wage is pushed too high
2. Replicates key elasticities in the micro data that discipline these channels
   - How much firms raise wages in response to competitors Derenoncourt et al, 2021; Staiger et al, 2010
   - Small firms shrink and larger firms grow after a minimum wage increase Dustmann et al, 2021

Results

- What is the (utilitarian) optimal Federal minimum wage?
  - $14.41 $\pm$ $1.50, depending on preference parameters
  - Non-college workers want +$1.30. Short-run (putty-clay capital + exit): –$2.00
- What are the welfare gains?
  - Small! Equivalent to a 1.17 percent TFP increase. Competitive economy: 15 percent increase.
- How are welfare gains distributed?
  - Significant losses for college workers, due to lower business and capital income
  - Replace population weights with Pareto weights that reflect U.S. economy: $7.39

THEORY

Environment

Heterogeneous households $k \in \{1, \ldots, K\}$

- Measure π_k, share η_k of capital. Sends workers to a continuum of labor markets $j \in [0, 1]$
- Market j has a fixed number of firms $i \in \{1, 2, \ldots, M_j\}$, $M_j \sim G(M)$
- Disutility of supplying workers $\{n_{ijkt}\}$ across markets / firms

Firms

- Firm i has idiosyncratic productivity z_{ij}, DRS production: $y_{ijt} = z_{ij} \sum_k \xi_k \left(k_{ijkt}^{1-\gamma} n_{ijkt}^{\gamma}\right)^{\alpha}$
- Hire workers n_{ijkt}, rent capital $k_{ijt} = \sum_k k_{ijkt}$ to produce identical final good

Markets

- Local, Cournot competition for labor
- National, Walrasian markets for output and capital

Household problem - Without minimum wage

Endowed with $\eta_k K_0$, takes as given $\{w_{ijkt}, R_t, \Pi_t \}$ and chooses $\{n_{ijkt}, C_{kt}, K_{kt+1}\}$

$$\mathcal{U}_{k0} = \max_{\{n_{ijkt}, C_{kt}, K_{kt+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_{kt}/\pi_k\right)^{1-\sigma}}{1-\sigma} - \frac{1}{\tilde{\varphi}_k^{1/\varphi}} \frac{N_{kt}^{1+1/\varphi}}{1+1/\varphi} \right] \quad , \quad \beta \in (0,1)$$

where

$$N_{kt} := \left[\int_0^1 n_{jkt}^{\frac{\theta+1}{\theta}} \, dj \right]^{\frac{\theta}{\theta+1}} \quad , \quad \theta > 0$$

$$n_{jkt} := \left[\sum_{i=1}^{M_j} n_{ijkt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}} \quad , \quad \eta > \theta$$

subject to the budget constraint

$$C_{kt} + \left[K_{kt+1} - (1-\delta) K_{kt} \right] = \int_0^1 \sum_{i=1}^{M_j} w_{ijkt} n_{ijkt} \, dj + R_t K_{kt} + \eta_k \Pi_t$$

Firm problem - Cournot competition

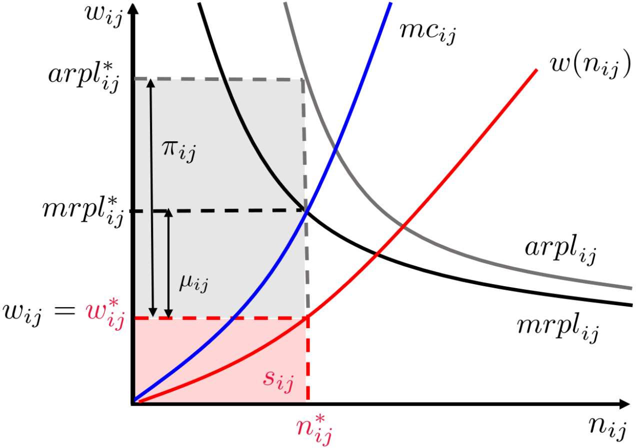
$$\max_{k_{ijk}, n_{ijk}} \pi_{ijk} = \underbrace{\widehat{z}_{ijk} {n_{ijk}}^{\widehat{\alpha}}}_{z_{ij}\xi_k \left(k_{ijk}^{1-\gamma} n_{ijk}^{\gamma}\right)^{\alpha} - R_t k_{ijk}} - w_{ijk} n_{ijk}$$

subject to

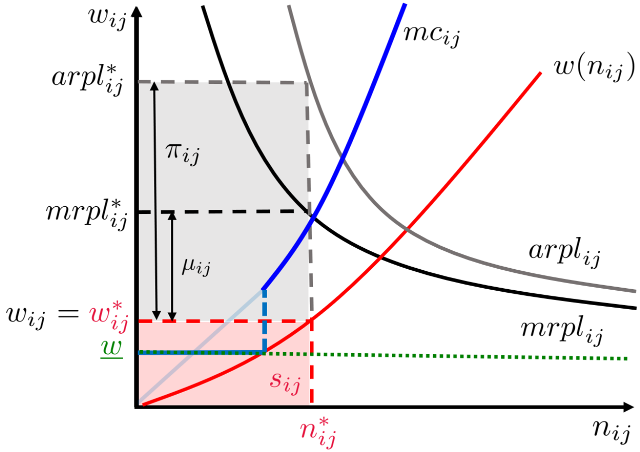
$$\underbrace{w_{ijk} = \left(\frac{n_{ijk}}{n_{jk}}\right)^{\frac{1}{\eta}} \left(\frac{n_{jk}}{N_k}\right)^{\frac{1}{\theta}} W_k}_{\text{Labor supply to the firm}} \quad , \quad \underbrace{N_k = \widetilde{\varphi}_k {W_k}^{\varphi} \left(\frac{C_k}{\pi_k}\right)^{-\varphi\sigma}}_{\text{Total labor supply}}$$

$$n_{jk} = \left[{n_{ijk}}^{\frac{\eta+1}{\eta}} + \sum_{m \neq i} {n^*_{mjk}}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

Firm problem

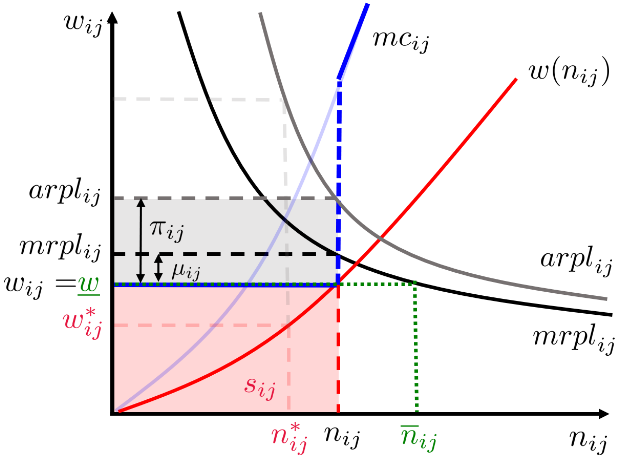


Minimum wage increase - Region I

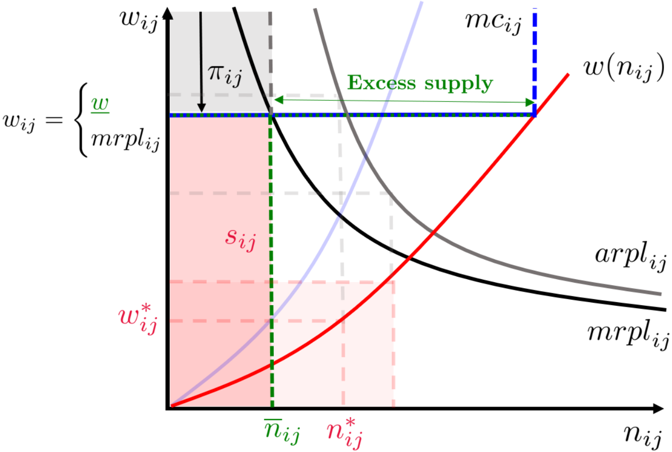


- Firm unaffected: w^*_{ij}, n^*_{ij}

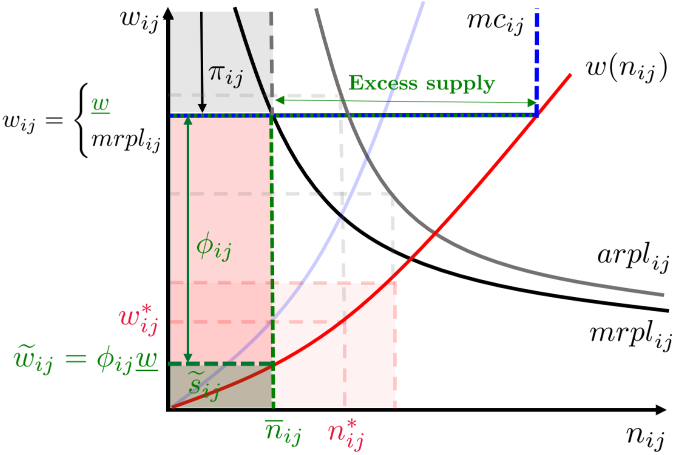
Minimum wage increase - Region II



Minimum wage increase - Region III



Minimum wage increase - Region III



- Equilibrium labor as if households receives the *shadow wage*: $\widetilde{w}_{ij} = \phi_{ij}\underline{w}$

Household problem - Without minimum wage

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$$n_{jkt} := \left[\sum_{i=1}^{M_j} n_{ijkt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}} \quad , \quad \eta > \theta$$

s.t. the b.c.

$$C_{kt} + \left[K_{kt+1} - (1-\delta) K_{kt} \right] = \int_0^1 \sum_{i=1}^{M_j} w_{ijkt} n_{ijkt} \, dj + R_t K_{kt} + \eta_k \Pi_t \qquad \left[\lambda_{kt}\right]$$

Household problem - With minimum wage

Endowed with $\eta_k K_0$, takes as given $\{w_{ijkt}, R_t, \Pi_t, \overline{n}_{ijkt}\}$ and chooses $\{n_{ijkt}, C_{kt}, K_{kt+1}\}$

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s.t. the b.c. and *labor rationing constraints*: $n_{ijkt} \leq \overline{n}_{ijkt}$

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s.t. the b.c. and *labor rationing constraints*: $n_{ijkt} \leq \overline{n}_{ijkt}$ $\quad \left[\lambda_{kt} w_{ijkt} (1 - \phi_{ijkt})\right]$

$$C_{kt} + \left[K_{kt+1} - (1-\delta) K_{kt} \right] = \int_0^1 \sum_{i=1}^{M_j} w_{ijkt} n_{ijkt} \, dj + R_t K_{kt} + \eta_k \Pi_t \qquad \left[\lambda_{kt}\right]$$

Household labor supply - With minimum wage

- Firm shadow wage

$$\widetilde{w}_{ijk} := \phi_{ijk} \; w_{ij} \quad , \quad \phi_{ijk} \in (0,1]$$

- Aggregate shadow wage

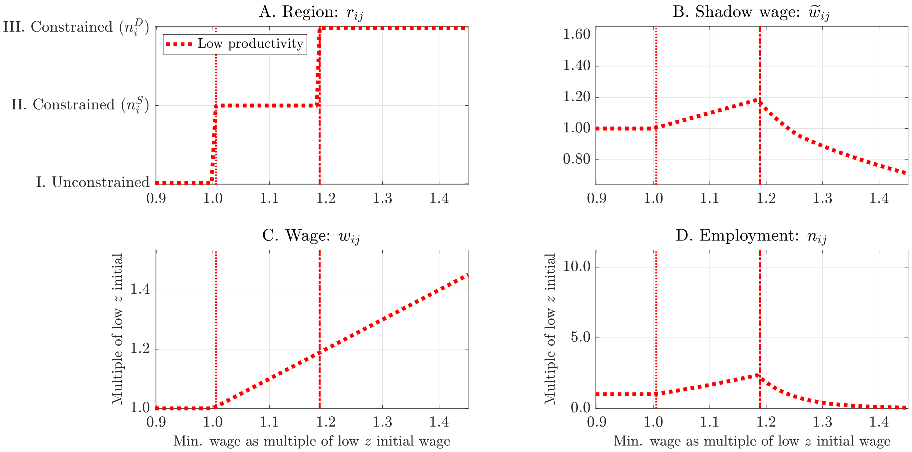
$$\widetilde{w}_{jk} = \left[\sum_{i \in j} \widetilde{w}_{ijk}^{1+\eta} \right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{W}_k = \left[\int \widetilde{w}_{jk}^{1+\theta} \, dj \right]^{\frac{1}{1+\theta}}$$

- Firms' labor supply

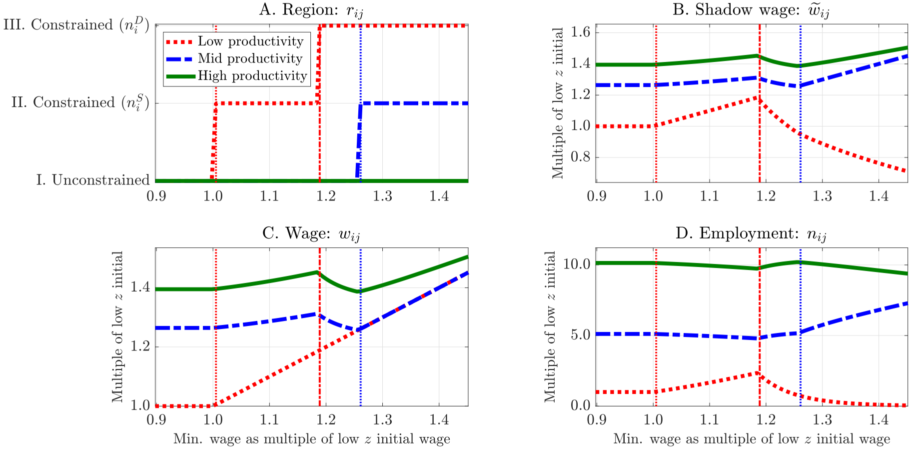
$$\underbrace{n_{ijk} = \left(\frac{\widetilde{w}_{ijk}}{\widetilde{w}_{jk}} \right)^{\eta} \left(\frac{\widetilde{w}_{jk}}{\widetilde{W}_k} \right)^{\theta} N_k}_{\text{Labor supply to the firm}} \quad , \quad \underbrace{N_k = \widetilde{\varphi}_k \widetilde{W}_k^{\varphi} \left(\frac{C_k}{\pi_k} \right)^{-\varphi\sigma}}_{\text{Total labor supply}}$$

- Result - *Strategic complementarities in shadow wages, not actual wages*

Market equilibrium



Market equilibrium



Aggregation

Given the market equilibrium, compute $\underbrace{\widetilde{\mu}_{ijk} := \widetilde{w}_{ijk} / mrpl_{ijk}}_{\text{Shadow markdown}}$, then

$$\underbrace{\widetilde{\mu}_{jk} = \left[\sum_{i\in j}\left(\frac{z_{ijk}}{z_{jk}}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}}\widetilde{\mu}_{ijk}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}}_{\text{Shadow markdown}}, \quad \underbrace{\omega_{jk} = \sum_{i\in j}\left(\frac{z_{ijk}}{z_{jk}}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}}\left(\frac{\widetilde{\mu}_{ijk}}{\widetilde{\mu}_{jk}}\right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}}}_{\text{Misallocation}}$$

then market employment, shadow wage and output satisfy:

$$\underbrace{n_{jk} = \left(\frac{\widetilde{w}_{jk}}{\widetilde{W}_k}\right)^{\theta} N_k}_{\text{Labor supply}} \quad , \quad \underbrace{\widetilde{w}_{jk} = \widetilde{\mu}_{jk}\,\alpha z_{jk} n_{jk}^{\alpha-1}}_{\text{Labor demand}} \quad , \quad \underbrace{y_{jk} = \omega_{jk}\, z_{jk} n_{jk}^{\alpha}}_{\text{Output}}$$

Aggregation

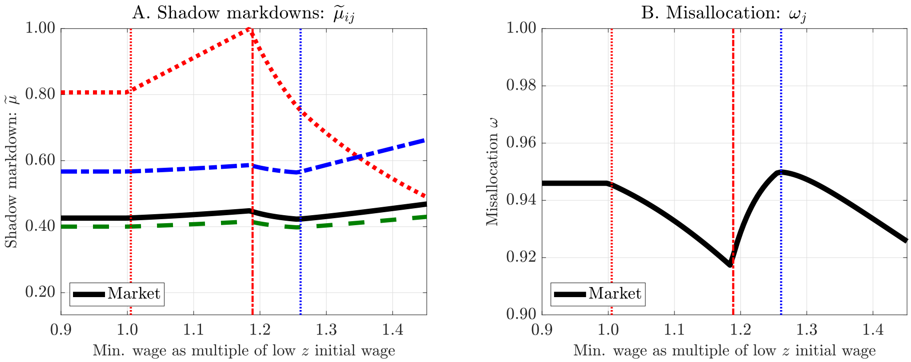
Given the market equilibrium, compute $\underbrace{\widetilde{\mu}_{ijk} := \widetilde{w}_{ijk} / mrpl_{ijk}}^{\text{Shadow markdown}}$, then

$$\underbrace{\widetilde{\mu}_{jk} = \left[\sum_{i\in j}\left(\frac{z_{ijk}}{z_{jk}}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}}\widetilde{\mu}_{ijk}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}}_{\text{Shadow markdown}} , \quad \underbrace{\omega_{jk} = \sum_{i\in j}\left(\frac{z_{ijk}}{z_{jk}}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}}\left(\frac{\widetilde{\mu}_{ijk}}{\widetilde{\mu}_{jk}}\right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}}}_{\text{Misallocation}}$$

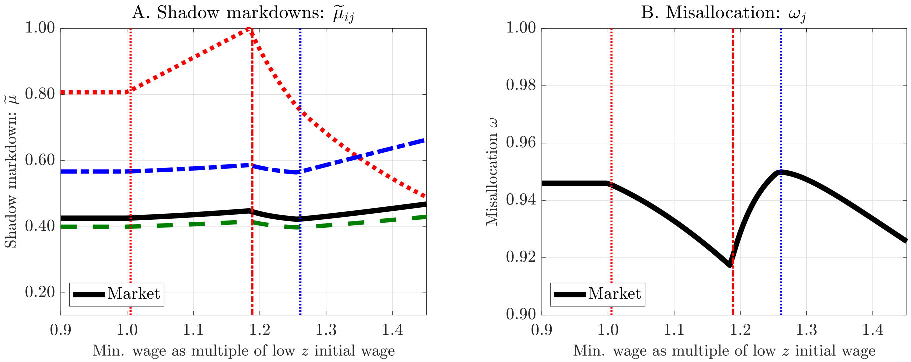
... and if we keep on aggregating ...

$$\underbrace{\mathcal{N} = \overline{\varphi}\left(\frac{\widetilde{W}}{\mathcal{N}}\right)^{\varphi}\mathcal{C}^{-\sigma\varphi}}_{\text{Labor supply}} , \quad \underbrace{\widetilde{W} = \widetilde{\boldsymbol{\mu}}\,\alpha Z\mathcal{N}^{\alpha-1}}_{\text{Labor demand}} , \quad \underbrace{Y = \boldsymbol{\omega} Z\mathcal{N}^{\alpha}}_{\text{Output}}$$

2. Market equilibrium



2. Market equilibrium



$\widetilde{\mu}$ - Replicate Derenoncourt et al (2021) $\rightarrow$ We get strategic responses of firms right

ω - Replicate Dustmann et al (2021) $\rightarrow$ We get reallocation across firms right

CALIBRATION

Calibration - From BHM (2021)

- Distribution of M_j taken from Census data. Market = NAICS3 $\times$ CZ
  - Average of 113 firms per market
- Productivity log normal $\sigma_z = 0.33$ $\rightarrow$ Payroll weighted payroll concentration 0.11
- DRS and output elasticities (α, γ) $\rightarrow$ Capital and labor share
- $(\theta, \eta) = (0.45, 6.96)$ chosen to match new evidence on s_{ij}-dependence of n_{ij} and w_{ij} responses to changes in state corporate taxes

Baseline preference parameters: $\sigma = 1.05$, $\varphi = 0.50$

Calibration - CPS, CEX, LBD

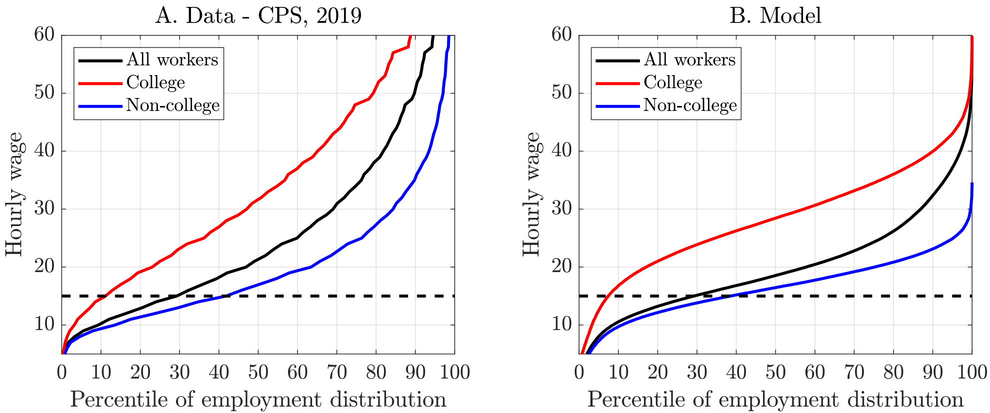
π_k	College workers are 35% of workers
ξ_k	... earn 1.8 times higher wages,
$\widetilde{\varphi}_k$	... 57% of total labor income,
η_k	... and account for 59% of consumption.
$\overline{\varphi}$	The average firm has 22.83 workers,
$\overline{Z}$	... and a $15 minimum wage would bind for 29% of workers

Non-college workers

Population share $= 0.65$, Implied pareto weight $\psi = 0.39$

▸ Details - Table of parameters and moments

Distribution of wages by type



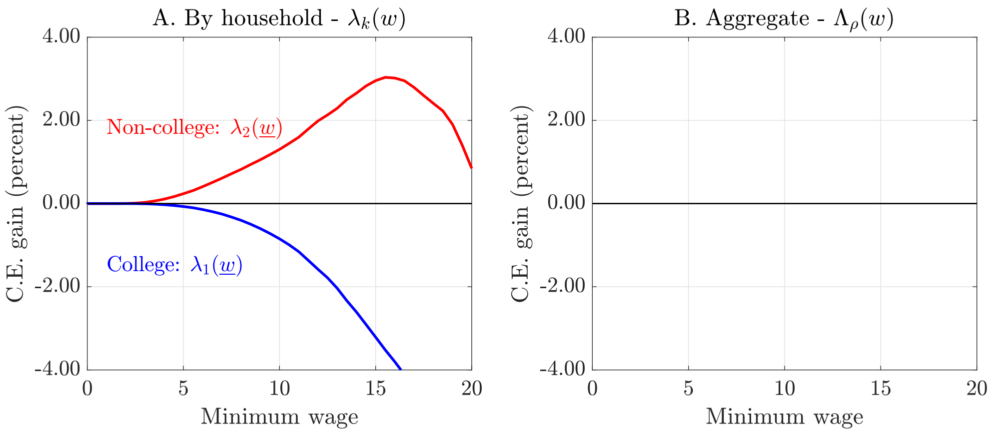
- Data uses 2019 MORG and March surveys. Wages are weekly earnings (`earnweek`) divided by usual weekly hours worked (`uhrsworkt`).

VALIDATION

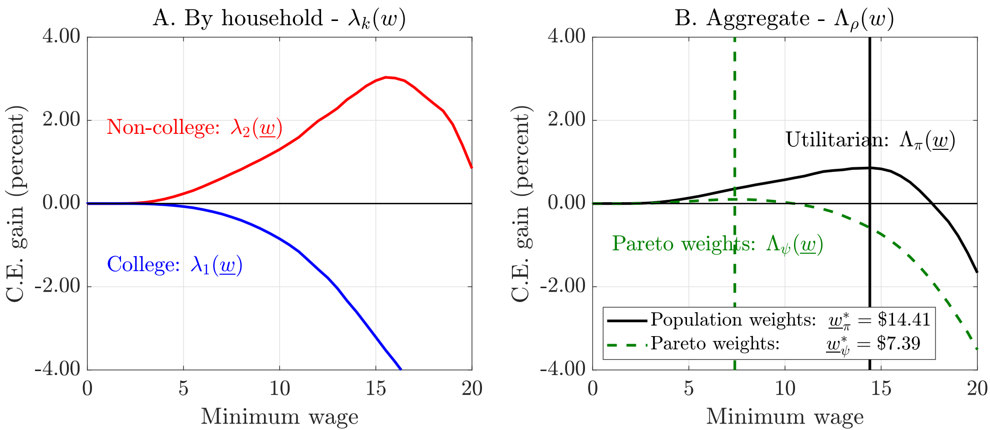
▸ Validation exercise

Optimal Minimum Wage

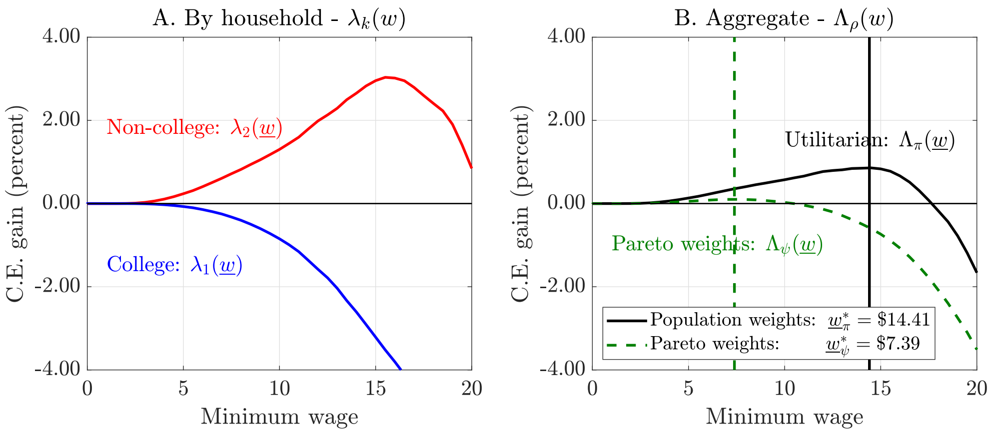
Consumption equivalent welfare gain $\Lambda_\rho(\underline{w})$



Consumption equivalent welfare gain $\Lambda_\rho(\underline{w})$



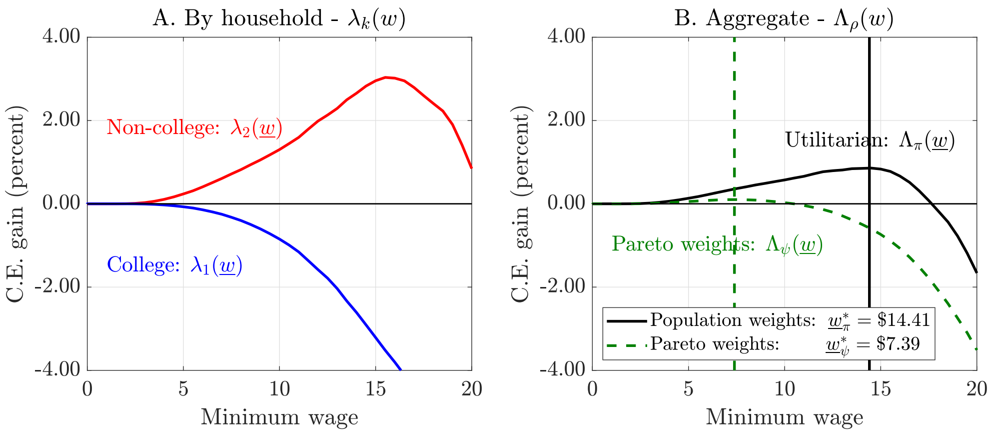
Consumption equivalent welfare gain $\Lambda_\rho(\underline{w})$



- Utilitarian weights: Same welfare increase as a 1.17 percent increase in TFP $\uparrow \overline{Z}$

▸ Details - Welfare equations: $\Lambda_\rho(\underline{w})$ is a weighted average of $\lambda_k(\underline{w})$

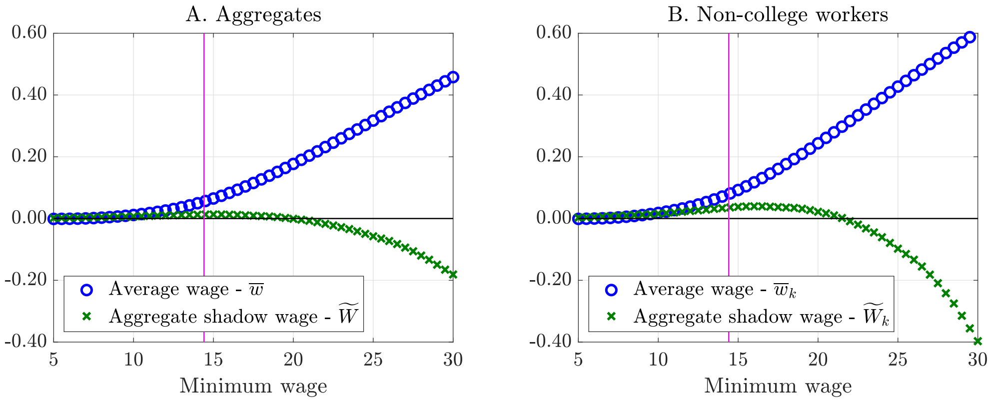
Consumption equivalent welfare gain $\Lambda_\rho(\underline{w})$



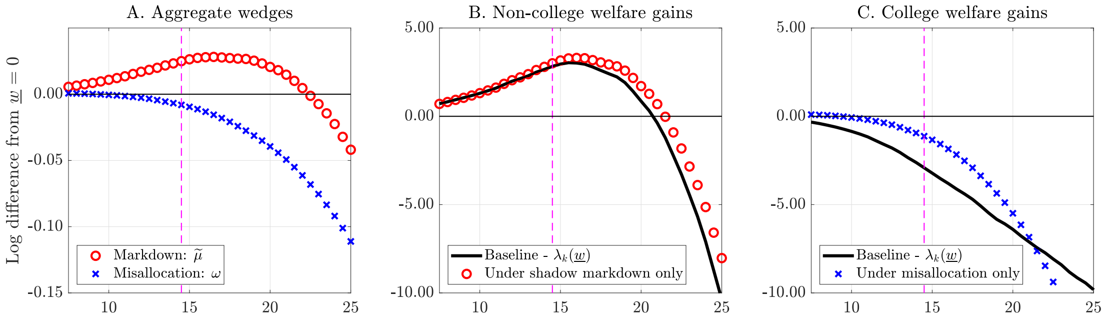
- Consistent with Heathcote Tsujiyama (2021) - U.S. policy reflects weights on wealthy

▸ Details - Welfare equations: $\Lambda_\rho(\underline{w})$ is a weighted average of $\lambda_k(\underline{w})$

Wages

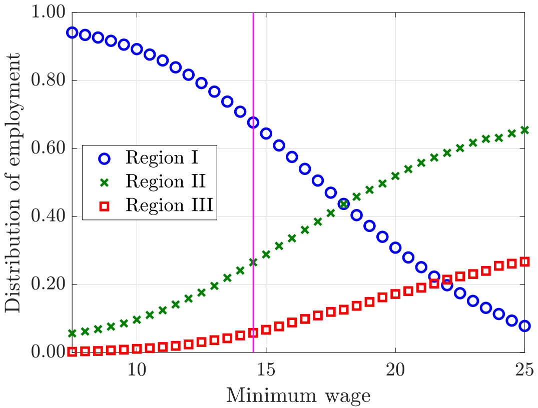

A. Aggregates
0.60
0.40
0.20
0.00
-0.20
-0.40
5
10
15
20
25
30
Minimum wage
Average wage - $\overline{w}$
Aggregate shadow wage - $\widetilde{W}$
B. Non-college workers
0.60
0.40
0.20
0.00
-0.20
-0.40
5
10
15
20
25
30
Minimum wage
Average wage - $\overline{w}_k$
Aggregate shadow wage - $\widetilde{W}_k$

Optimal minimum wage - Wedges



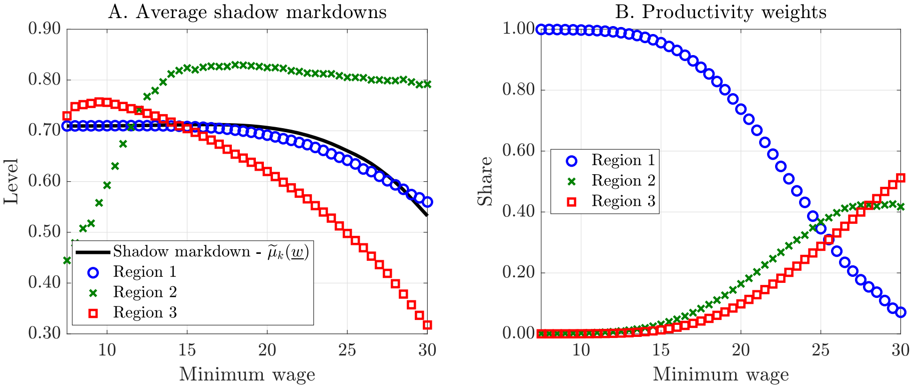
- Short-lived gains from resolving some misallocation
- Non-college worker shadow markdown deteriorates rapidly in Region III
- Misallocation reduces output and capital demand: $\downarrow K$, $\downarrow \Pi$

Optimal minimum wage - Non-college employment across regions



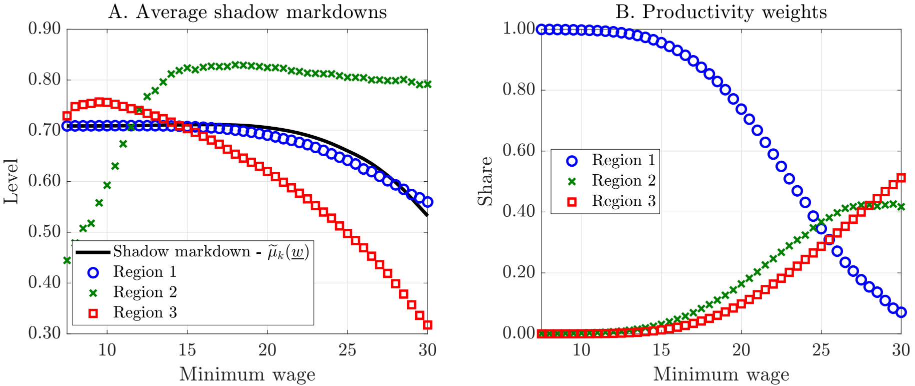
1. Even with high minimum wages, the majority of firms, employment, pay is unconstrained
2. Despite this unconstrained firms' share of resources is declining

Optimal minimum wage - What determines $\widetilde{\mu}$? - Non-college welfare



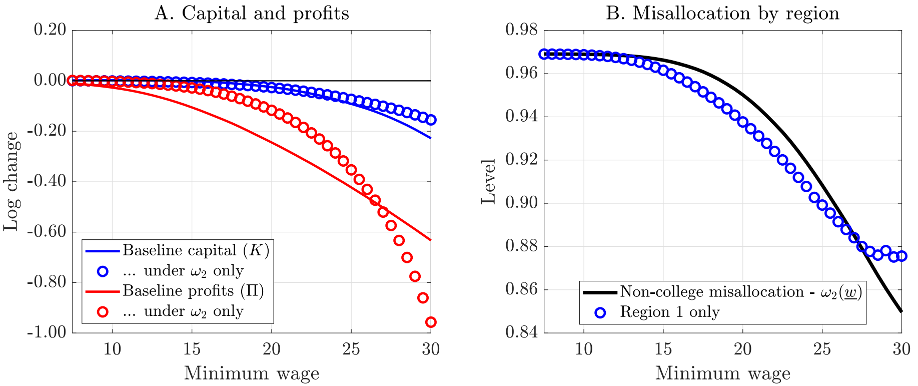
$$\widetilde{\mu}_{jk} = \underbrace{\left[\sum_{i \in j} \left(\frac{z_{ijk}}{z_{jk}} \right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \widetilde{\mu}_{ijk} \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}}_{\text{Market shadow markdown}}$$

Optimal minimum wage - What determines $\widetilde{\mu}$? - Non-college welfare



- Region I - Give firms more market power, $\downarrow \mu_i$
- Region III - Tighten rationing constraints, $\downarrow \widetilde{\mu}_i$

Optimal minimum wage - Role of $\widetilde{\omega}$ - College welfare



- Misallocation reduces K and profits Π. Hits non-college C
- Dominated by misallocation in Region I. Reallocation: Wholefoods $\rightarrow$ Supermarket

Optimal minimum wage - Alternative assumptions

		A. Baseline	B. Alternative wealth distributions	
		Match CEX (1)	Hand-to-mouth (2)	Equal per capita (3)
1. Targets in red, Outcomes in blue				
Initial non-college consumption share (%)	c_2 / C	**40.9**	36.0	60.2
Non-college share of capital and profit income (%)	η_2	9.60	**0**	**65.2**
2. Comparison of population share and Pareto weights				
Non-college population share (%)	$\pi_2/(\pi_1+\pi_2)$	65.2	65.2	65.2
Implied non-college Pareto weight (%)	ψ_2	39.8	34.2	59.0
3. Optimal minimum wage under utilitarian welfare weights				
Optimal minimum wage	$\underline{w}^*$	\$14.41	\$14.78	\$7.82
Aggregate welfare gain	$\Lambda_\pi(\underline{w}^*)$	0.86%	1.30%	0.13%
4. Alternative welfare weights				
Implied Pareto weights	$\underline{w}^*$	\$7.39	\$7.22	\$7.60
Only care about non-college workers	$\underline{w}_2^*$	\$15.71	\$15.92	\$8.95

Conclusion

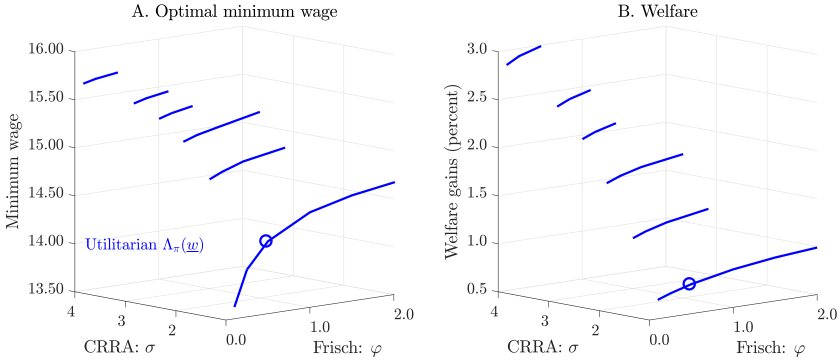
1. General equilibrium macro model of oligopsony under minimum wages
   - Characterize a simple solution to micro model of Nash equilibrium using shadow wages
   - Show how this aggregates
2. Rationalizes recent empirical studies on minimum wages
3. Decompose optimal minimum wage
   - 'Federal' minimum wage of $13.50-15.50 but limited welfare effects
   - Advocates' 'monopsony' arguments undone by steeply worsening misallocation

$*$ In the paper
   - Sensitivity to $\varphi, \sigma \rightarrow \underline{w}^* \in (\$13.50, \$15.50)$
   - Comparison to homogeneous household economy $\rightarrow$ 50% $\widetilde{\mu}$ and ω gains at optimum, $\underline{w}^* = \$8.33$
   - More concentrated markets respond more positively ... then more negatively
   - Wage inequality, concentration, labor share all behave monotonically ... contra hump-shaped $\Lambda(\underline{w})$
   - Long run vs. Short-run (putty-clay capital, exit) $\rightarrow \underline{w}^*_{SR} = \$12.30$

APPENDIX I

HETEROGENEITY AND ALTERNATIVE PREFERENCES

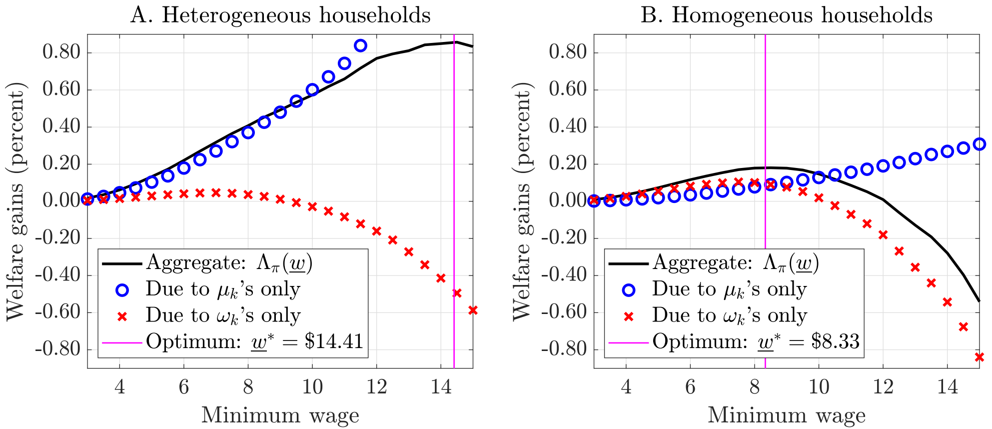
Optimal minimum wage - Alternative preferences (σ, φ)



$\uparrow \sigma$ More weight on $\lambda_k(\underline{w})$ of low C_k/π_k households $\rightarrow$ higher $\uparrow \underline{w}^*$

$\uparrow \varphi$ Increase in W due to narrower markdown $\uparrow \tilde{\mu}$ really increases N $\rightarrow$ higher $\uparrow \underline{w}^*$

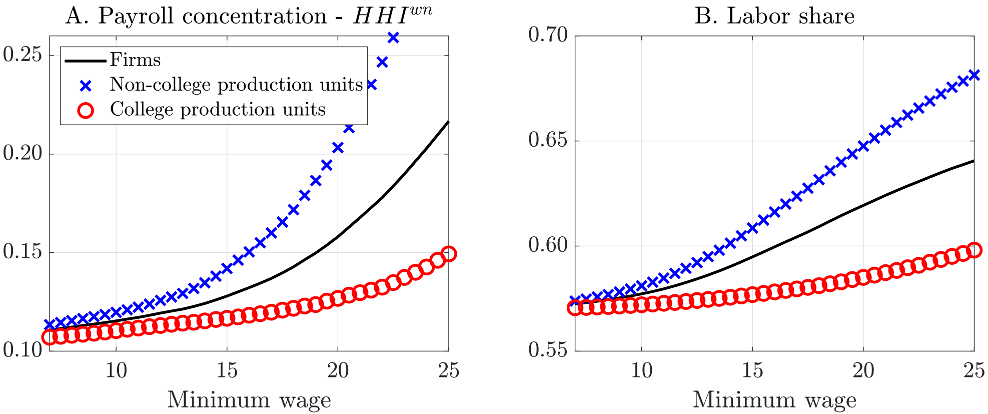
Optimal minimum wage - Role of heterogeneity



- Homogeneous: $\xi_k = \pi_k = \widetilde{\varphi}_k = 1$.

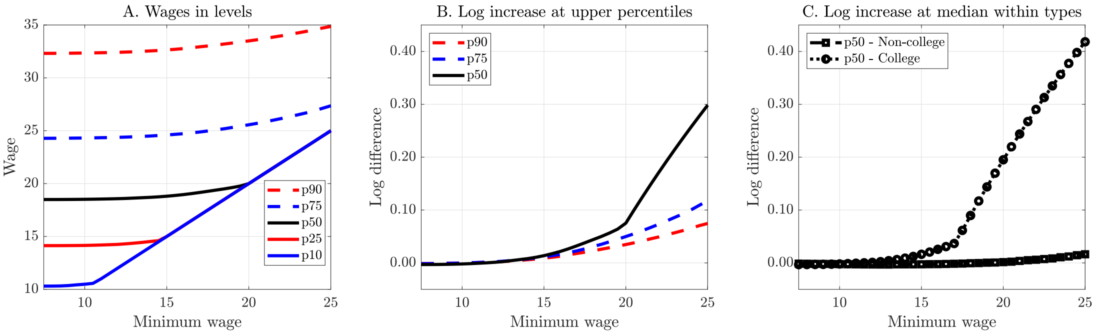
Wages and Inequality

Other statistics - Concentration and labor share



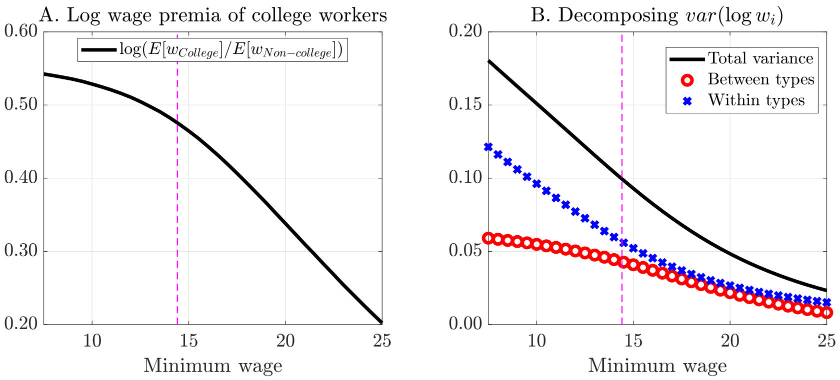
- Across economies in steady-state: $\uparrow HHI$, $\downarrow LS$
- In response to higher $\underline{w}$: $\uparrow HHI$, $\uparrow LS$, ... with welfare up, then down

Other statistics - Spillovers



- Positive spillovers up the wage distribution
- Poses issues for empirical strategies that use higher percentiles as controls

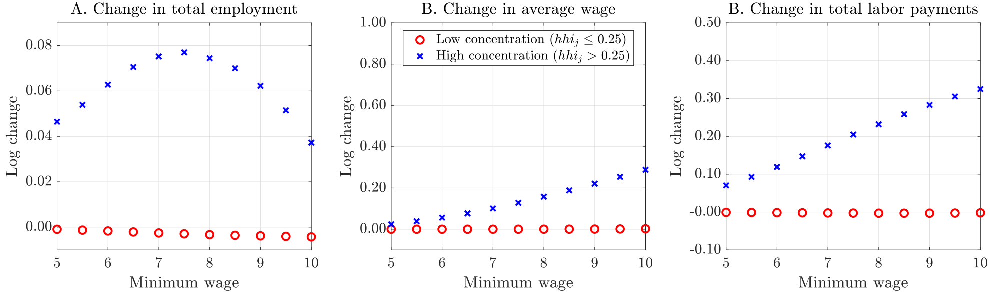
Other statistics - Wage inequality



- College premium narrows by 15 percent. Wage inequality falls by more than a third.
- Most of the reduction is within types

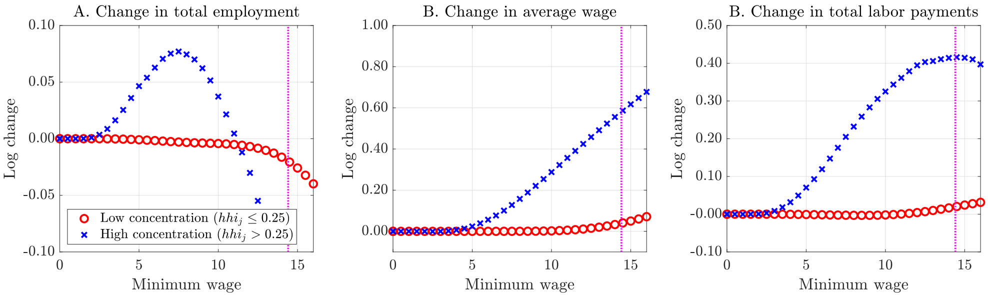
Concentration and Minimum Wage Effects

Concentration and the minimum wage - Non-college workers



Azar, Marinescu, Taska, Von Wachter (2019) - *“While increases in the minimum wage are found to significantly decrease employment of workers in low concentration markets, minimum wage-induced employment changes become less negative as labor concentration increases, and are even estimated to be positive in the most highly concentrated markets”*

Concentration and the minimum wage - Non-college workers

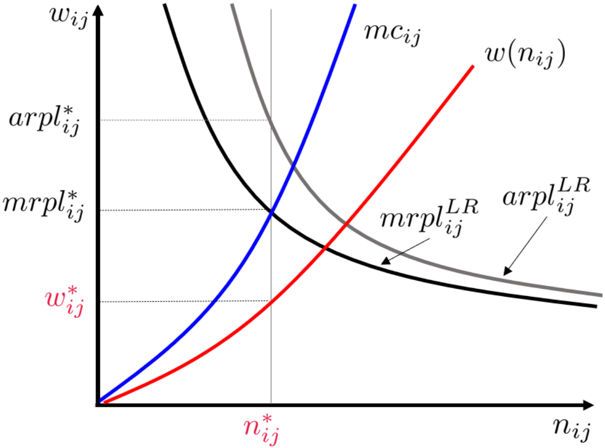


Holds between \$5 and \$10, which is the empirical range studied, ...

but then flips in welfare relevant range.

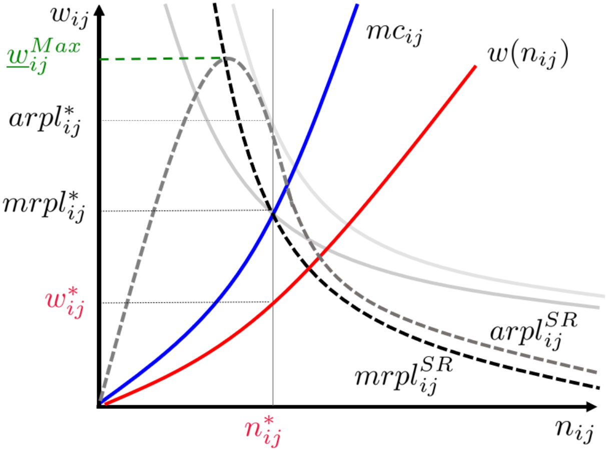
SHORT VS. LONG RUN OPTIMAL MINIMUM WAGE

Long-run flexible capital - $k_{ij} = \sum_k k_{ijk}$



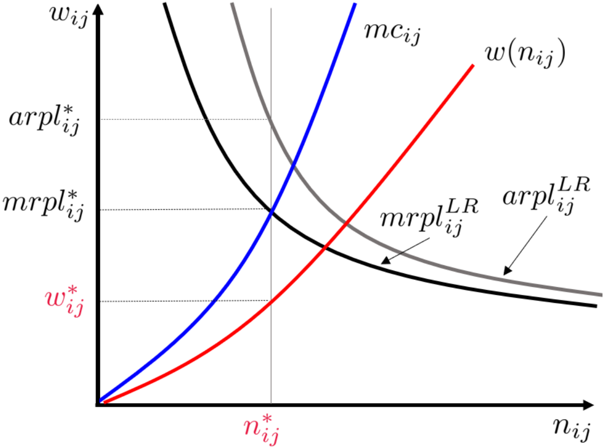
- Putty-like capital assigned to any worker

Short-run fixed capital - $\overline{k}_{ijk}$

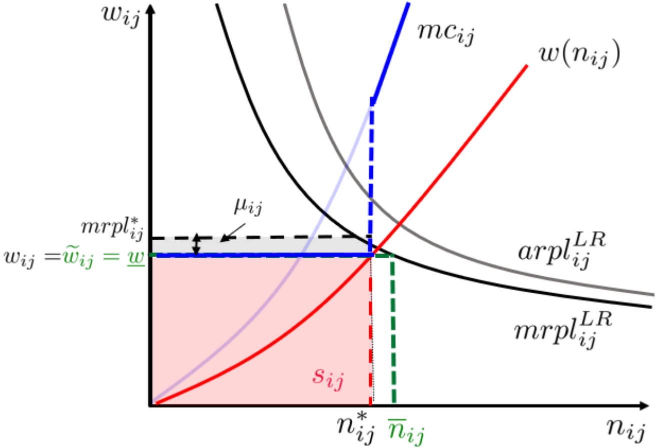


- Steeper labor demand curve, and maximum minimum wage

Long-run flexible capital - With minimum wage

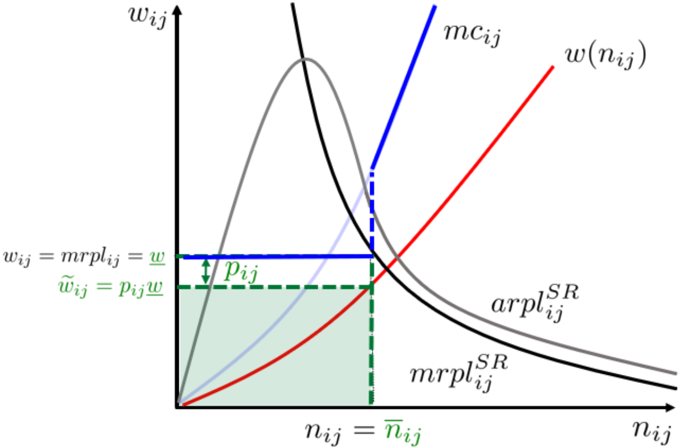


Long-run flexible capital - With minimum wage



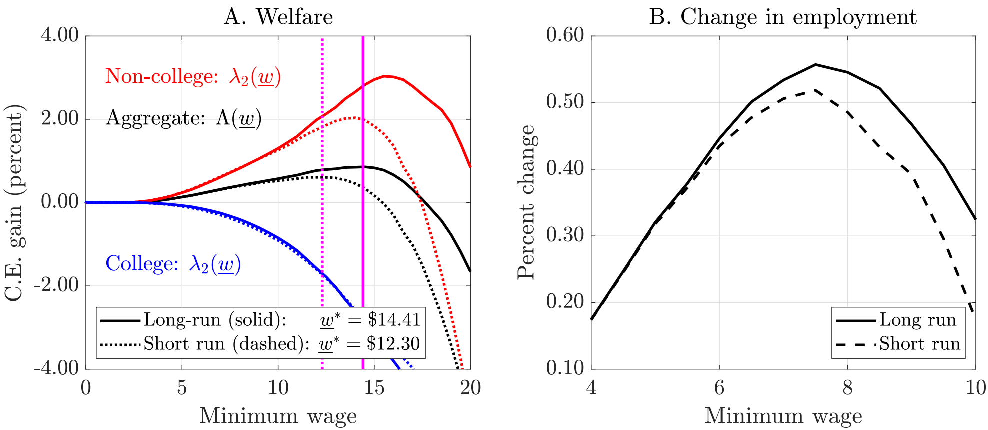
- Region II: Increases in minimum wage increase employment

Short-run fixed capital - With minimum wage



- Region III: Increases in minimum wage decrease employment

Short run optimal minimum wage is lower



- Short-run optimal minimum wage is $\sim$ \$2 lower, suggests gradual increase

Conclusion

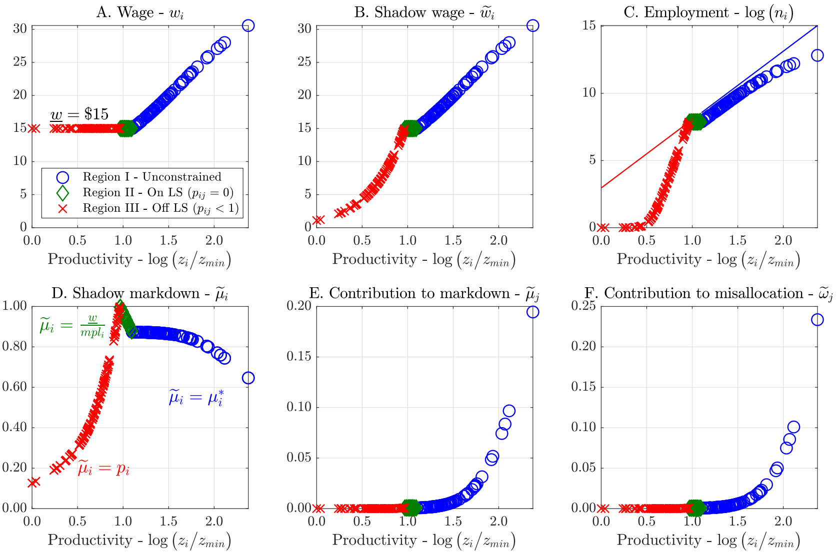
1. Model of oligopsony under minimum wages
   - Characterize a simple solution to Nash equilibrium using shadow wages
   - Partial / Market / General equilibrium

2. Rationalizes recent empirical studies on minimum wages

3. Decompose optimal minimum wage
   - 'Federal' minimum wage of $13.50-15.50 but limited welfare effects
   - Advocates' 'monopsony' arguments undone by steeply worsening misallocation
   - On-going: Upper bound of roughly $4\times$ on welfare improvements through targeting

APPENDIX II - LINKS

2. Market equilibrium



Calibration

Parameters		Value	Moment and source	Value
A. External				
Risk free rate	r	0.04		
Depreciation rate	δ	0.10		
Aggregate Frisch elasticity	φ	0.50		
Coefficient of risk aversion	σ	1.05		
Number of markets	J	5,000		
Across market substitutability	θ	0.45	Estimate from BHM (2021)	
Within market substitutability	η	6.96	Estimate from BHM (2021)	
B. Aggregate shifters				
Productivity shifter	$\widetilde{Z}$	14.33	Fraction of workers below \$15/hr wage (CPS)	0.29
Disutility shifter	$\overline{\varphi}$	1.52×10^6	Average firm size (LBD)	22.83
C. Relative shifters (College workers normalized: $\pi_1 = \xi_1 = \widetilde{\varphi}_1 = \psi_1 = 1$ and $\eta_1 = 1 - \eta_2$)				
Relative population	π_2	1.874	Population share of non-college workers (CPS)	0.65
Relative productivity	ξ_2	0.553	Relative average wage of non-college workers (CPS)	0.58
Relative disutility	$1/\widetilde{\varphi}_2^{\varphi}$	1.014	Labor income share of non-college workers (CPS)	0.57
Share of asset income	η_2	0.096	Consumption share of non-college workers (BLS, CEX)	0.41
D. Internally estimated				
Productivity dispersion	$\text{Std}[\log z_{ij}]$	0.268	Payroll weighted $\mathbb{E}[HHI^{wn}]$ (LBD)	0.11
Decreasing returns in production	α	0.957	Labor share	0.57
Labor exponent in production	γ	0.812	Capital share	0.18
Implied Pareto weight	ψ_2	0.66		

Back - Calibration

Replicate minimum wage studies

Want

- Make sure that the mechanisms important for $(\widetilde{\mu}, \omega)$ are quantitatively correct

Replications

1. Minimum wages reallocate employment across firm distribution - ω
    - Dustmann et. al. (2021) - *Reallocation Effect of Minimum Wages*
2. Firms respond to competitors wage changes - $\widetilde{\mu}$
    - Derenoncourt el. al. (2021) - *Spillover Effects from Voluntary Employer Minimum Wages*
    - Staiger et al (2010) - *Is there Monopsony in the Labor Market?*

Replicate minimum wage studies - Derenoncourt et al (2021)

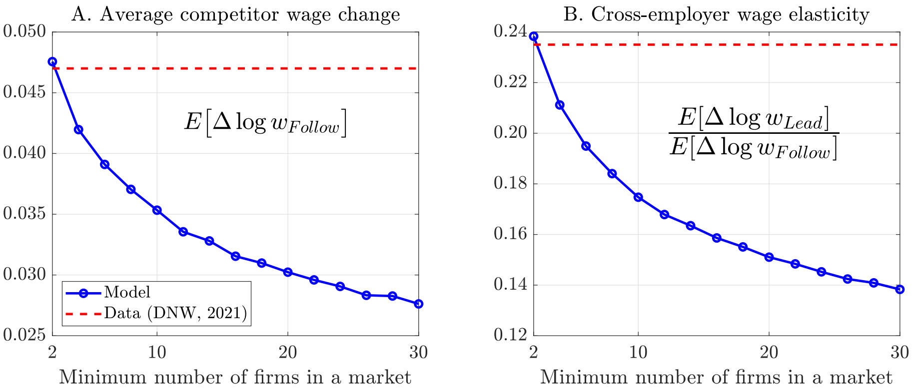
Setting

- Amazon institutes a $15 minimum wage. Increases wages by $\uparrow 20\%$
- Competitors respond with $\uparrow 4.6\%$. *Cross-employer wage elasticity* of 24 percent

Replication

- Narrow leaders' markdowns ζ percent toward competitive $\mu_{ij}^{*} = 1$
- Choose ζ to match $\uparrow 20\%$ wage increase
- Find 15 to 24 percent cross-employer wage elasticity

Replicate minimum wage studies - Derenoncourt et al (2021)



- Replicates response of firms to competitors' wage changes.
- Key point: Its less than 1

Replicate minimum wage studies - Dustmann et al (2021)

Setting

- Germany introduces $\underline{w} = €8.50$ minimum wage in January, 2015
- Regions vary by how much total wages will have to increase to get to $\underline{w}$: Gap_j

Replication

- Introduce minimum wage at $\underline{w}$ that covers 15 percent of initial workers
- Treat the economy as a single region and compute Gap
- Replicate cross-sectional regressions of $\Delta \log y$ on Gap: $\widehat{\beta}_y = \Delta \log y \,/\, Gap$

Replicate minimum wage studies - Dustmann et al (2021)

Variable y in elasticity: $\Delta \log y / Gap$	**A. Model**	**B. Data**	
		Data 1	Data 2
Employment	0.01	0.02	0.38
Firm exit (number of firms with $n_{ij} \geq 1$)	-0.83	-0.19	-0.24
Number of 'micro' firms (with $1 \leq n_{ij} \leq 2$)	-0.31	-0.27	-0.35
Average firm size	0.84	0.15	0.31
Average revenue per worker	0.51	0.31	1.04

- Overall small employment effects
- Reallocation from small unproductive firms to more productive firms

▸ Back - Validation title slide

Welfare

- Consumption equivalent welfare gains

$$\sum_k \rho_k U\left(\left(1+\Lambda_\rho\left(\underline{w}\right)\right)\frac{C_k}{\pi_k}, N_k\right) = \sum_k \rho_k U\left(\frac{C_k\left(\underline{w}\right)}{\pi_k}, N_k\left(\underline{w}\right)\right).$$

- Harmonic mean

$$\left(1+\Lambda_\rho(\underline{w})\right) = \left[\sum_k \widetilde{\rho}_k \left(1+\lambda_k(\underline{w})\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

- Two cases

Utilitarian: $\rho_k = \pi_k$, U.S.: $\rho_k = \psi_k$

Back - Optimal minimum wage