Debt as Safe Asset

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Motivation

- How much government debt can the market absorb?
- Is there a debt valuation puzzle?

  (valuation too large? risk premium too low?)

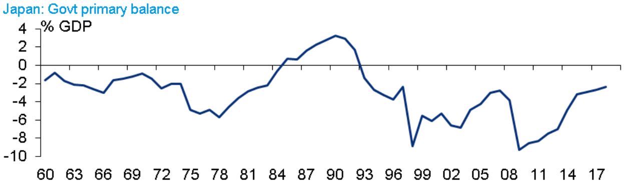
  (Jiang, Lustig, van Nieuwerburgh, Xiaolan 2019,2020)
- This paper: safe asset nature of government debt
  - model of government debt as a *countercyclical safe asset* (negative β)
  - safe asset nature matters *qualitatively and quantitatively* for debt valuation

Valuing Government Debt – Standard Approach

- Think of representative agent holding all gov. debt
  - his cash flow is the primary surplus
  - using standard asset pricing:

$$\mathcal{B}/\mathcal{P} = \mathbb{E}\left[\mathrm{PV}_r(\text{primary surplus})\right]$$

- ... but e.g. for Japan primary surplus was negative for 50 out of 60 years
- Can surpluses be negative forever? Yes, if $r < g$ (e.g. due to safe asset nature)



Valuing Government Debt when $r < g$

- PV formula when $r < g$ (Brunnermeier, Merkel, Sannikov 2020)

$$\mathcal{B}/\mathcal{P} = \underbrace{\mathbb{E}\left[\mathrm{PV}_r(\text{primary surplus})\right]}_{=-\infty \text{ if surplus } <0} \quad + \quad \underbrace{\text{bubble}}_{=\infty \text{ if surplus } <0}$$

- Alternative approach: value agents' actual portfolio strategies, then aggregate
- Discounts at higher effective rate $r^{**} > g$

$$\mathcal{B}/\mathcal{P} = \underbrace{\mathbb{E}\left[\mathrm{PV}_{r^{**}}(\text{primary surplus})\right]}_{>-\infty} + \underbrace{\mathbb{E}\left[\mathrm{PV}_{r^{**}}(\text{service flow})\right]}_{<\infty}$$

Note: Discount rate r^{**} = "representative agent" discount rate $\neq m$ (Reis 2020)

Outline

1. Model
   - Setup
   - Debt Valuation – Two Perspectives
2. Countercyclical Safe Asset and Valuation Puzzles
   - Calibrated Model Solution
   - Debt Valuation Puzzles
3. Safe Assets and the Stock Market

Outline

Model Setup

- Continuous time, infinite horizon, one consumption good
- Continuum of agents
  - operate capital subject to idiosyncratic risk, AK production technology
  - can trade capital and government bonds
  - quantitative model: add diversified equity claims
- Government
  - exogenous spending
  - taxes output
  - issues (nominal) bonds
- Financial friction: incomplete markets:
  - agents cannot trade idiosyncratic risk
  - quantitative model: must retain skin in the game
- Aggregate risk: fluctuations in volatility of idiosyncratic shocks (and capital productivity)

Model Setup – Some Formal Details

- Preferences ($i \in [0,1]$ agent index): (quantitative model: Epstein-Zin preferences)

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t} \log c_t^i dt\right]$$

- Each agent manages capital k_t^i:
  - output flow (net of reinvestment): $(a_t - \iota_t^i) k_t^i dt$
  - output tax by government: $\tau_t a_t k_t^i dt$
  - capital evolution:

$$\frac{dk_t^i}{k_t^i} = \underbrace{\left(\Phi\left(\iota_t^i\right) - \delta\right) dt}_{\text{investment and depreciation}} + \underbrace{d\Delta_t^{k,i}}_{\text{trading}} + \underbrace{\tilde{\sigma}_t d\tilde{Z}_t^i}_{\text{idio. shocks}}$$

- Government:
  - budget constraint

$$\overbrace{i_t \mathcal{B}_t}^{\text{interest payments}} = \overbrace{\mathcal{P}_t \underbrace{\left(\tau_t a_t - \mathfrak{g}\right)}_{=:s_t} K_t}^{\text{prim. surpluses}} + \overbrace{\mu_t^{\mathcal{B}} \mathcal{B}_t}^{\text{bond issuance}}$$

- $\tilde{\sigma}_t$, a_t exogenous processes

Debt Valuation – Two Perspectives

1. **"Buy and Hold" Perspective:**

$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \lim_{T\to\infty}\left(\mathbb{E}\left[\int_0^T \xi_t^i s_t K_t dt\right] + \mathbb{E}\left[\xi_T^i \frac{\mathcal{B}_T}{\mathcal{P}_T}\right]\right)$$

   - valuation of strategy that buys and holds a fixed fraction of outstanding debt

Notation: ξ_t^i: agent i's SDF, $d\xi_t^i/\xi_t^i = -r_t^f dt - \varsigma_t dZ_t - \tilde{\varsigma}_t^i d\tilde{Z}_t^i$, $\tilde{\sigma}_t^c$: idiosyncratic consumption volatility

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2. **Dynamic Trading Perspective**

$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \mathbb{E}\left[\int_0^\infty \underbrace{\left(\int \xi_t^i \eta_t^i di\right)}_{=\xi_t^{**}} s_t K_t dt\right] + \mathbb{E}\left[\int_0^\infty \underbrace{\left(\int \xi_t^i \eta_t^i di\right)}_{=\xi_t^{**}} (\tilde{\sigma}_t^c)^2 \frac{\mathcal{B}_t}{\mathcal{P}_t} dt\right]$$

   - valuation of *actual cash flows from individual bond portfolios*, including trading cash flows (aggregated over all agents i to obtain total value of debt)

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Expected Bond Return and Discount Rates (with convenience yields)

Bond valuation formula

$$\text{value of bond stock} = \mathbb{E}[PV_{\text{discount rate}}(\text{surpluses})] + \mathbb{E}[PV_{\text{discount rate}}(\text{service flow})]$$

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$$\text{expected bond return} = \underbrace{\underbrace{\rho + \mu^c - \overbrace{\left((\sigma^c)^2 + (\tilde{\sigma}^c)^2\right)}^{\text{precautionary savings}}}_{\text{risk-free rate } r^f} + \text{risk premium}}_{\text{discount rate}} - \underbrace{\text{convenience yield}}_{\text{service flow}}$$

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- **Dynamic Trading Perspective** (ξ^{**})

$$\text{expected bond return} = \underbrace{\underbrace{\rho + \mu^c - (\sigma^c)^2}_{\text{"risk-free rate" } r^{f**}} + \text{risk premium}}_{\text{discount rate}} - \underbrace{\left((\tilde{\sigma}^c)^2 + \text{convenience yield}\right)}_{\text{service flow}}$$

Note: r^{f**} is risk-free rate in "representative agent" economy that leads to same allocation

Outline

Quantitative Model Fit (Selected Moments)

moment		model	data
symbol	description		(post 1985)
$\sigma(Y)$	output volatility	0.019	0.010
$\sigma(C)$	consumption volatility	0.010	0.007
$\sigma(S/Y)$	surplus volatility	0.004	0.011
$\mathbb{E}[q^K K/Y]$	average capital-output ratio	3.206	≈ 3
$\mathbb{E}[q^B K/Y]$	average debt-output ratio	0.672	0.714
$\mathbb{E}[dr^E - dr^{\mathcal{B}}]$	average equity premium	5.6%	$\approx 6.4\%$
$\frac{\mathbb{E}[dr^E - dr^{\mathcal{B}}]}{\sigma\left(dr^E - dr^{\mathcal{B}}\right)}$	equity sharpe ratio	0.436	≈ 0.5

Notes: $\sigma(x)$ denotes the standard deviation of x and $\rho(x, y)$ denotes the correlation of x and y, both at a quarterly frequency. Inputs x and y are HP-filtered with smoothing parameter 1600. For $x, y \in \{Y, C\}$, we take logarithms before filtering. $\mathbb{E}[x]$ denotes expectations over the ergodic model distribution, inputs x are not HP-filtered. Y: (aggregate) output, C: consumption, S: primary surplus, q^K, q^B, dr^B, dr^E are defined as in Section 2.

Debt Valuation Puzzles

- Properties of US primary surpluses
  - average surplus ≈ 0
  - surpluses procyclical (> 0 in booms, < 0 in recessions)

- Two valuation puzzles from standard perspective of $\mathcal{B}/\mathcal{P} = \mathbb{E}[\mathrm{PV}(\text{surpluses})]$
  (Jiang, Lustig, van Nieuwerburgh, Xiaolan 2019, 2020)

  1. "Public Debt Valuation Puzzle"
     - empirically $\mathbb{E}[\mathrm{PV}(\text{surpluses})] < 0$, yet $\mathcal{B}/\mathcal{P} > 0$

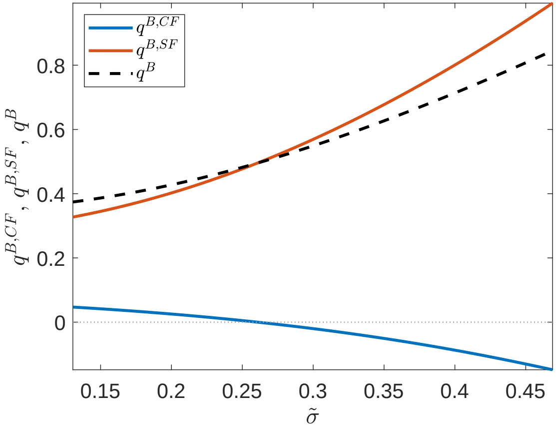
     $\rightarrow$ our model: bubble component can close the gap

  2. "Gov. Debt Risk Premium Puzzle"
     - debt should be positive-β asset, but markets do not price it that way

     $\rightarrow$ our model: can be rationalized by countercyclical bubble / service flow component

Dynamic Trading Perspective Decomposition

$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \overbrace{\mathbb{E}\left[\int_0^\infty \left(\int \xi_t^i \eta_t^i di\right) s_t K_t dt\right]}^{\mathbb{E}\text{PV(cash flow)}} + \overbrace{\mathbb{E}\left[\int_0^\infty \left(\int \xi_t^i \eta_t^i di\right) (\tilde{\sigma}_t^c)^2 \frac{\mathcal{B}_t}{\mathcal{P}_t} dt\right]}^{\mathbb{E}\text{PV(service flow)}}$$



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Why Does Safe Asset Property Survive in Presence of Stock Market?

- Diversified stock portfolio *free of idiosyncratic risk*
  $\rightarrow$ trading in stocks can also self-insure idiosyncratic risk

- But: *poor hedge against aggregate risk*, looses value in recessions (positive β)

- Why positive β? (after all, r^f goes down in recessions, lowers discount rates)
  - equity are claims to capital, marginal capital holder is insider
  - insider bears idiosyncratic risk, must be compensated
  - $\tilde{\sigma}_t \uparrow \Rightarrow$ insider premium $\mathbb{E}_t[dr_t^K] - \mathbb{E}_t[dr_t^E] \uparrow \Rightarrow$ payouts to stockholders fall

Stock Market Volatility due to Flight to Safety

- "Aggregate Intertemporal Budget Constraint":

$$\underbrace{q_t^K K_t + \mathcal{B}_t/\mathcal{P}_t}_{\text{total (net) wealth}} = \mathbb{E}_t \left[\int_t^\infty \frac{\int \xi_s^i \eta_s^i di}{\int \xi_t^i \eta_t^i di} C_s ds \right] \qquad (*)$$

- Lucas-type models: $\mathcal{B}_t/\mathcal{P}_t = 0$ (also $C_t = Y_t$ & no idiosyncratic risk)
    - then: value of equity (Lucas tree) = PV of consumption claim
    - volatility equity values require volatile rhs of $(*)$

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  - volatility equity values require volatile rhs of $(*)$
- This model: even for constant rhs of $(*)$, $q_t^K K_t$ can be volatile due to *flight to safety*:

$$\begin{aligned} \text{increase in } \tilde{\sigma}_t \quad &\rightarrow \quad \text{portfolio reallocation from capital to bonds} \\ &\rightarrow \quad q^K K \downarrow, \mathcal{B}/\mathcal{P} \uparrow \end{aligned}$$

- Does this matter quantitatively?

  Yes, excess return volatility in equivalent bondless model ($s = 0$ and no bubble): 2.9% (compared to 12.9% in ours)

Other Topics (in Paper)

1. Deficit financing by "bubble mining": Debt Laffer Curve
   - max. sustainable permanent primary deficit $\approx 2\%$ of GDP
   - aggregate risk (negative beta) quantitatively important
2. Alternative equilibria and loss of safe asset status
   - loss of safe asset status: bubble can pop – need (off-equilibrium) fiscal space to prevent
   - equilibria with bubbles on private debt: same macro dynamics but worse for government budget

Conclusion

- Safe asset $=$ good friend
  - individually: allows self-insurance through retrading
  - aggregate: appreciates in bad times (negative β)

- Asset pricing with safe assets
  - service flow term matters

    (service flow $>$ convenience yield)
  - flight to safety creates
    - countercyclical safe asset valuation
    - larger stock market volatility

- Implications for government budget (in paper)
  - government can "mine the bubble" within limits (max. 2% of GDP)
  - bubble can pop: loss of safe asset status