Efficiency Gains from Wealth Taxation: A Theoretical Analysis

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Question: What is the optimal combination of capital income (flow) and wealth (stock) taxes in the presence of rate of return heterogeneity?

Our earlier work: Quantitative analysis of optimal capital income vs. wealth tax
- Rich OLG model with bells and whistles
- Find: Large efficiency and welfare gains from wealth tax
- Robust to several extensions

This paper: Theoretical analysis of optimal combination of capital income and wealth taxes
- A plain-vanilla infinite-horizon entrepreneur-worker model
- Establish conditions for:
  (i) efficiency gains  (ii) welfare gains (by agent+overall)  (iii) optimal taxes
Return Heterogeneity: A Simple Example

- One-period model.
- Government taxes to finance \( G = $50 \).
- Two brothers, Fredo and Mike, each with $1000 of wealth.
- **Key heterogeneity**: investment/entrepreneurial ability.
  - (Fredo) Low ability: earns \( r_f = 0\% \) net return.
  - (Mike) High ability: earns \( r_m = 20\% \) net return.
## Capital Income vs. Wealth Tax

**Capital income tax**

\[
a_{i, \text{after-tax}} = a_i + (1 - \tau_k) r_i a_i
\]

<table>
<thead>
<tr>
<th>Wealth</th>
<th>$1000</th>
<th>$1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before-tax Income</td>
<td>0</td>
<td>$200</td>
</tr>
<tr>
<td>(\tau_k) = 25% \left( \frac{50}{200} \right)</td>
<td>(\tau_k) = 25% \left( \frac{50}{200} \right)</td>
<td></td>
</tr>
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**Wealth tax (on book value!)**

\[
a_{i, \text{after-tax}} = (1 - \tau_a) a_i + r_i a_i
\]

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<td>(\tau_a) = 2.5% \left( \frac{50}{2000} \right)</td>
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<td></td>
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</tbody>
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<table>
<thead>
<tr>
<th>Tax liability</th>
<th>0</th>
<th>$50 \left( = 20\tau_k \right)</th>
<th>$25 \left( = 1000\tau_a \right)</th>
<th>$25 \left( = 1000\tau_a \right)</th>
</tr>
</thead>
<tbody>
<tr>
<td>After-tax return</td>
<td>0%</td>
<td>15% \left( = \frac{200-50}{1000} \right)</td>
<td>-2.5% \left( = \frac{0-25}{1000} \right)</td>
<td>17.5% \left( = \frac{200-25}{1000} \right)</td>
</tr>
<tr>
<td>After-tax wealth ratio</td>
<td>1.15 \left( = \frac{1150}{1000} \right)</td>
<td>1.20 \left( \approx \frac{1175}{975} \right)</td>
<td></td>
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</tbody>
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- Replacing \(\tau_k\) with \(\tau_a\) → reallocates capital to more productive agents (**Use it or lose it**) + increases dispersion in after-tax returns & wealth.
- Market value reflects future earnings, taxing it weakens **use it or lose it** effect.
1. **Efficiency Gains:** A marginal increase in the wealth tax increases TFP *iff* entrepreneurial productivity is positively auto-correlated.

2. **Welfare Gain by Type:** With a marginal shift from capital income to wealth tax
   - Workers gain
   - High-productivity entrepreneurs “typically” gain
   - Low-productivity entrepreneurs “typically” lose

3. **Optimal Taxes:** Utilitarian welfare maximizing taxes depend on the elasticity of output with respect to capital ($\alpha$)
   - If $\alpha$ is sufficiently high $\rightarrow \tau_a^* > 0 \& \tau_k^* < 0$
   - If $\alpha$ is sufficiently low $\rightarrow \tau_a^* < 0 \& \tau_k^* > 0$
   - If $\alpha$ is in between $\rightarrow \tau_a^* > 0 \& \tau_k^* > 0$. 
Preview of Results

Extensions

- Corporate sector with no borrowing constraint
- Rents: Return $\neq$ marginal productivity
- Entrepreneurial effort in production
- Perpetual-youth model with stationary wealth distribution
Theoretical Model

- Two groups of infinitely-lived agents:
  - homogenous workers (size $L$)
  - heterogenous entrepreneurs (size 2)

- Workers’ and entrepreneurs’ preferences:
  \[ E_0 \sum_{t=0}^{\infty} \beta^t \log (c_t) \quad \text{where } \beta < 1. \]

- Workers:
  - supply labor inelastically + consume wage income (hand-to-mouth).
Entrepreneurs’ technology:

\[ y = (zk)^\alpha n^{1-\alpha} \]

- \( z \in \{z_l, z_h\} \), where \( z_h > z_l \geq 0 \) with a transition matrix

\[ P = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix} \text{ with } 0 < p < 1. \]

- Autocorrelation is critical: \( \rho = 2p - 1 > 0 \iff p > \frac{1}{2}. \)

Aggregate output:

\[ Y = \int (zk)^\alpha n^{1-\alpha} \]

Government finances exogenous expenditure \( G \) with \( \tau_k \) and \( \tau_a \)

- \( \tau_a \) on beginning-of-period book value wealth
Financial Markets & Entrepreneurs’ Production Problem

Financial markets:

▶ Collateral constraint ($\lambda \geq 1$): $k \leq \lambda a$, where $a$ is entrepreneur’s wealth.
▶ Bonds are in zero net supply.

Entrepreneurs’ Production Decision:

$$
\Pi^* (z, a) = \max_{k \leq \lambda a, n} (zk)^\alpha n^{1-\alpha} - rk - wn.
$$

Solution:

$$
\Pi^* (z, a) = \pi^* (z) \times a
$$

Excess return above $r$

$$
\pi^* (z) = \begin{cases} 
(MPK(z) - r) \lambda & \text{if } MPK(z) > r \\
0 & \text{otherwise.}
\end{cases}
$$

▶ $(\lambda - 1) a$: amount of external funds used by type-$z$ if $MPK(z) > r$.  

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Entrepreneur’s Consumption-Saving Problem

\[ V(a, z) = \max_{c, a'} \log(c) + \beta \sum_{z'} \mathbb{P}(z' | z) V(a', z') \]

s.t. \( c + a' = (1 - \tau_a) a + (1 - \tau_k) (r + \pi^*(z)) a \).

After-tax wealth

- Letting \( R_i \equiv (1 - \tau_a) + (1 - \tau_k) (r + \pi^*(z_i)) \) for \( i \in \{l, h\} \),
  the savings decision (CRS + Log Utility):

\[ a' = \beta R_i a \quad \rightarrow \text{linearity allows aggregation} \]
Evolution of Aggregates

\[ A'_h = p\beta R_h A_h + (1 - p) \beta R_l A_l \]

stayers' savings \quad switchers' savings

\[ A'_l = p\beta R_l A_l + (1 - p) \beta R_h A_h \]

stayers' savings \quad switchers' savings

\( A_h \): High type wealth

\( A_l \): Low type wealth
Equilibrium and Steady State

Three different equilibria can arise depending on parameter values:

1. **“Interesting”** if \( \lambda < \lambda^* < 2 \):
   - \((\lambda - 1)A_h < A_l\): low-type entrepreneurs bid down interest rate: \( r = MPK(z_l) \).
   - **Unique steady state** with:
     - return heterogeneity, misallocation of capital, wealth tax \( \neq \) capital income tax.
   - **Empirically relevant**: \( R_h > R_l \) and \( \frac{\text{Debt}}{\text{GDP}} \gg 1.3 \) when \( \lambda = \lambda^* \).

2. **“Uninteresting”** if \( \lambda \geq 2 \):
   - Unique steady state with:
     - no return heterogeneity (\( R_l = R_h \)), no misallocation of capital (\( K_h = A_h + A_l \)), wealth tax \( \equiv \) capital income tax.

3. **Unstable** if \( \lambda^* < \lambda < 2 \): No steady state.
Equilibrium and Steady State

Three different types of equilibria can arise depending on parameter values:

1. **“Interesting”** if $\lambda < \lambda^* < 2$:  
   - **The equilibrium**
     - $(\lambda - 1) A_h < A_l$: low-type entrepreneurs bid down interest rate: $r = \text{MPK}(z_l)$.
     - **Unique steady state** with:
       - return heterogeneity, misallocation of capital, wealth tax $\neq$ capital income tax.
   - **Empirically relevant**: $R_h > R_l$ and $\frac{\text{Debt}}{\text{GDP}} \gg 1.3$ when $\lambda = \lambda^*$.

2. **“Uninteresting”** if $\lambda \geq 2$:
   - Unique steady state with:
     - no return heterogeneity ($R_l = R_h$), no misallocation of capital ($K_h = A_h + A_l$), wealth tax $\equiv$ capital income tax.

3. **Unstable** if $\lambda^* < \lambda < 2$: No steady state.
Equilibrium Values

**Lemma:** Aggregate output is

\[ Y = (ZK)^{\alpha} L^{1-\alpha} \quad (Z^\alpha \text{ is measured TFP}) \]

where

\[ Z = s_h z_{\lambda} + (1 - s_h) z_l \]: \( Z \) = Wealth-weighted productivity

**Key variables:**

- \( s_h = \frac{A_h}{K} \): wealth share of high-productivity entrepreneurs.
- \( z_{\lambda} \equiv z_h + (\lambda - 1) (z_h - z_l) \): effective productivity of high-type entrepreneurs.

**Use it or lose it effect** increases efficiency if \( s_h \uparrow (\rightarrow Z \uparrow) \).
Using the law of motion for $A_l$ and $A_h$ we obtain two steady state equations:

**Steady State $K$**

\[
(1 - \tau_k) \alpha Z^{\alpha} \left(\frac{K}{L}\right)^{\alpha - 1} - \tau_a = \frac{1}{\beta} - 1.
\]

**Steady State $Z$ (depends on only $\tau_a$!)**

\[
(1 - \rho \beta (1 - \tau_a)) Z^2 - \frac{Z_I + Z_L}{2} (1 + \rho - 2 \rho \beta (1 - \tau_a)) Z + Z_I Z_L \rho (1 - \beta (1 - \tau_a)) = 0.
\]
Main Result 1: Efficiency Gains from Wealth Taxation

Proposition:

For all $\tau_a < \bar{\tau}_a$ ($\lambda < \lambda^*$), a marginal increase in $\tau_a$ increases steady state $Z$ iff entrepreneurial productivity is autocorrelated, $\rho > 0$ ($\rho > \frac{1}{2}$)

Corollary:

1. Wealth concentration: $s_h \uparrow (Z \uparrow = s_h Z_\lambda + (1 - s_h) Z_l)$

2. Dispersion of after-tax returns rises with $\tau_a$:

$$\frac{dR_l}{d\tau_a} = (\text{use-it-lose-it} < 0) + (\text{G.E. effect} < 0) < 0$$

$$\frac{dR_h}{d\tau_a} = (\text{use-it-lose-it} > 0) + (\text{G.E. effect} < 0) > 0$$
Government Budget and Aggregate Variables

\[ G = \tau_k \alpha Y + \tau_a K. \]

**Assumption:** \( G \) is a constant fraction \( \theta \alpha \) of aggregate output: \( G = \theta \alpha Y. \)

- In what follows, \( \tau_k \) adjusts in the background when \( \tau_a \uparrow \)

**Lemma:** For all \( \tau_a < \bar{\tau}_a \), a marginal increase in \( \tau_a \)

- **Increases** capital (\( K \)), output (\( Y \)), wage (\( w \)), h-type wealth (\( A_h \)), and \( G \) iff \( \rho > 0 \)
  - Higher \( \alpha \) \( \rightarrow \) Larger response of \( K, Y, w \)
  - \( A_l = (1 - s_h) K \downarrow \) iff \( \alpha z_\lambda < Z \) and \( \rho > 0. \)
Welfare gains (across steady states)

**CE₁,ᵢ measure** \( i \in \{w, l, h\} \):

- \( (a, i) \) in **Benchmark** economy v.s. \( (a, i) \) in **Counterfactual** economy with higher \( τ_a \) (lower \( τ_k \))
- Welfare gains \( C \succ B \) if
  \[
  \frac{\log (1 + CE₁,ᵢ)}{1 - β} = V^C (a, i) - V^B (a, i) > 0
  \]
  independent of \( a \) because \( V (a, i) = m_i + \frac{1}{1 - β} \log (a) \ i \in \{i, h\} \).
- Utilitarian welfare CE₁ depends on population shares \( nᵢ \)’s:
  \[
  \log (1 + CE₁) = \sum_i n_i \log (1 + CE₁ (a, i))
  \]

- CE₁ does not account for changes in distribution of wealth.
  - Alternative measure CE₂ takes into account changes in wealth levels.
Main Result 2: Welfare gains by type

Proposition:
For all \( \tau_a < \bar{\tau}_a \), a marginally higher \( \tau_a \) changes welfare as follows iff \( \rho > 0 \)

- **Workers:** Higher \( CE_{1,w} > 0 \)

- **High-type entrepreneurs:** Higher \( CE_{1,h} > 0 \) iff \( R_h - R_l < \kappa_R (\beta, \rho) \)
  - Taking wealth accumulation into account: \( CE_{2,h} > 0 \) always.

- **Low-type entrepreneurs:** Lower \( CE_{1,l} < 0 \)
  - Taking wealth accumulation into account: \( CE_{2,l} < 0 \) if \( \alpha z \lambda < Z \).

- **Lower average welfare of entrepreneurs:** \( CE_{1,E} < 0 \).
Government chooses \((\tau_a, \tau_k)\) to maximize the utilitarian social welfare \(CE_1\) (or \(CE_2\)).

**Key trade-off:**

1. Higher wages (depends on \(\alpha\)) v.s.

2. Lower (LOG) average return (higher return dispersion + negative GE effect)

   & changes in \(\{A_l, A_h\}\) if \(CE_2\) is the objective.
Main Result 3: Optimal Taxes

**Proposition:** There exists a unique optimal tax combination \((\tau_a^*, \tau_k^*)\) that maximizes \(CE_1\). An interior optimum \((\tau_a^* < \bar{\tau}_a)\) is the solution to:

\[
n_w \underbrace{\xi_w} + \frac{1 - n_w}{1 - \beta} \underbrace{\left( \frac{\xi_{R_l} + \xi_{R_h}}{2} \right)}_{\text{Av. Z-Elasticity of Returns} < 0} = 0
\]

where \(\xi_x \equiv \frac{d \log x}{d \log Z}\) is the elasticity of variable \(x\) with respect to \(Z\). Furthermore,

\[
\begin{align*}
\tau_a^* &\in \left[ 1 - \frac{1}{\beta}, 0 \right] \quad \text{and} \quad \tau_k^* > \theta \quad \text{if} \quad \alpha < \bar{\alpha} \\
\tau_a^* &\in \left[ 0, \frac{\theta (1 - \beta)}{\beta (1 - \theta)} \right] \quad \text{and} \quad \tau_k^* \in [0, \theta] \quad \text{if} \quad \alpha \leq \alpha \leq \bar{\alpha} \\
\tau_a^* &> \frac{\theta (1 - \beta)}{\beta (1 - \theta)} \quad \text{and} \quad \tau_k^* < 0 \quad \text{if} \quad \alpha > \bar{\alpha}
\end{align*}
\]
Extensions

- **Corporate sector** with no borrowing constraint
  - If \( z_l < z_c < z_h \), then low-productivity agents invest in the corporate sector.

- **Rents**: Return \( \neq \) marginal productivity.
  - Introduce zero-sum return wedges so that \( R_h < \neq R_l \).
  - Efficiency gains from \( \tau_a \uparrow \) if \( \rho > 0 \) and \( R_h > R_l \).
  - Efficiency gains from \( \tau_a \uparrow \) if \( \rho < 0 \) and \( R_h < R_l \).

- **Entrepreneurial effort** in production:
  - With GHH preferences, aggregate entrepreneurial effort increases with wealth tax.

- **Perpetual youth with permanent types**:
  - We can solve the stationary distribution of agents.
  - \( CE_{2,h} > CE_{1,h} > 0 \) always.
Conclusions

Increasing $\tau_a$:

- **Reallocates capital**: less productive $\rightarrow$ more productive agents.
  
  This reallocation increases
  
  - TFP, output, and wages;
  - dispersion in returns and wealth **iff** $\rho > 0$.

- Workers gain

- Entrepreneurs: High-productivity gain*, low-productivity lose*.

**Optimal tax combination**: depends on elasticity of output with respect to capital.

Full draft coming soon!
Thanks!
Extra
1. Can there be a steady state with \((\lambda - 1) A_h > A_l\)? **NO.** In that case \(R_h = R_l\),

\[
\frac{A'_h}{A'_l} = \frac{pA_h + (1 - p) A_l}{(1 - p) A_h + p A_l} = \frac{A_h}{A_l},
\]

which implies that \(A_h = A_l\). But then \((\lambda - 1) A_h > A_l\) is violated since \(\lambda < 2\).

2. Can there be a steady state with \((\lambda - 1) A_h < A_l\)? If the answer is yes, then we are already focusing on that SS and that SS implies that \(\lambda < \lambda^*\).

3. If \((\lambda - 1) A_h > A_l\) in the transition, then \(A_h > A_l\) since \(\lambda < 2\) and

\[
\frac{A'_h}{A'_l} = \frac{pA_h + (1 - p) A_l}{(1 - p) A_h + p A_l} < \frac{A_h}{A_l}.
\]

Then at some point, we will have \((\lambda - 1) A_h < A_l\) and we will be in the heterogenous-return case. If this converges to a steady state, it is the one with \(\lambda < \lambda^*\).
Is $\lambda^*$ too restrictive?

Debt-to-output ratio when $\lambda = \lambda^*$ computed as $(\lambda^* - 1)A_h/Y$. 
Is $\lambda^*$ too restrictive?

**Figure 1**: Conditions for Steady State with Heterogeneous Returns

$z_l = 0, z_h = 2, \tau_k = 25\%, \text{ and } \alpha = 0.4.$

*Debt-to-output ratio when $\lambda = \lambda^*$ computed as $(\lambda^* - 1)A_h/Y$*
Figure 2: Conditions for Steady State with Heterogeneous Returns

\[ z_l = 0, z_h = 2, \tau_k = 25\%, \text{ and } \alpha = 0.4. \]

Debt-to-output ratio with \( \tau_a = 0 \) (benchmark) computed as \( (\lambda^* - 1)A_h / Y \).
Steady State: 2 equations 2 unknowns

**Steady State K:**

\[
(1 - \tau_a) + (1 - \tau_k) \alpha Z^\alpha \left(\frac{K}{L}\right)^{\alpha-1} = \frac{1}{\beta}
\]

**Steady State R:**

\[
R_i = (1 - \tau_a) + (1 - \tau_k) \alpha \left(\frac{ZK}{L}\right)^{\alpha-1} z_i
\]

Equilibrium R

\[
R_i = (1 - \tau_a) + (1 - \tau_k) \alpha \left(\frac{K}{L}\right)^{\alpha-1} \frac{z_i}{Z}
\]

Change to MPK

\[
R_i = (1 - \tau_a) + \left(\frac{1}{\beta} - (1 - \tau_a)\right) \frac{z_i}{Z}
\]

Steady State

**Key:** Steady state K adjusts to maintain constant (after-tax) MPK:

\[
(1 - \tau_k) \text{ MPK} = \frac{1}{\beta} - (1 - \tau_a)
\]

As in NGM \(\tau_k\) affects level of K but not long run (after-tax) MPK \(\left(\frac{1}{\beta} - 1 + \tau_a\right)\).
Welfare gains (with changes in wealth)

**CE}_{2,i} measure (i \in \{w, l, h\}):**

- Evaluate welfare gain at average wealth levels for each economy.
- \((A^B_i, i)\) in the Benchmark economy v.s. \((A^C_i, i)\) in the Counterfactual economy.
- Welfare gains \(C \succ B\) if
  \[
  \frac{\log (1 + CE_{2,i})}{1 - \beta} = V^C(A^C_i, i) - V^B(A^B_i, i) > 0 \quad i \in \{w, l, h\}
  \]
Return Dispersion for Welfare Gains of High-Type Entrepreneurs

$$\text{Max } R_h - R_i \text{ for } CE_{1,h} > 0$$

Diagram showing the relationship between Discount Factor ($\beta$), Productivity Persistence ($\rho$), and the difference in returns ($R_h - R_i$) with specific values for different thresholds.
Return dispersion $R_h - R_l$:
$\alpha$-thresholds for Optimal Wealth Taxes

$z_l = 0, z_h = 2, \lambda = 1.3, \text{ and } \theta = 25\%.$
$\alpha$-thresholds for Optimal Wealth Taxes

$z_l = 0.5, z_h = 1.5, \lambda = 1.2, and \theta = 25\%.$
Optimal Wealth Taxes and Welfare Gain

Optimal $\tau_a$

$z_l = 0, z_h = 2, \theta = 25\%, and \lambda = 1.3.$
Optimal Wealth Taxes

Optimal Taxes and $\alpha$ Thresholds
Existence and Uniqueness of Steady State (when $p > 0.5$)
What happens to $Z$ if $\tau_a \uparrow$?

$$\frac{dh(x)}{d\tau_a} = (2p - 1) (x - z_l) (x - z_\lambda) < 0 \iff p > 0.5 \text{ and } z_l < x < z_\lambda$$