Efficiency Gains from Wealth Taxation: A Theoretical Analysis

Guvenen, Kambourov, Kuruscu, Ocampo

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Taxing Capital

- **Question:** What is the optimal combination of capital income (flow) and wealth (stock) taxes in the presence of rate of return heterogeneity?
- **Our earlier work:** Quantitative analysis of optimal capital income vs. wealth tax
  - Rich OLG model with bells and whistles
  - Find: Large efficiency and welfare gains from wealth tax
  - Robust to several extensions
- **This paper:** Theoretical analysis of optimal combination of capital income and wealth taxes
  - A plain-vanilla infinite-horizon entrepreneur-worker model
  - Establish conditions for:

    (i) efficiency gains (ii) welfare gains (by agent+overall) (iii) optimal taxes

Return Heterogeneity: A Simple Example

- One-period model.
- Government taxes to finance $G = \$50$.
- Two brothers, Fredo and Mike, each with $1000 of wealth.
- **Key heterogeneity**: investment/entrepreneurial ability.
  - (Fredo) Low ability: earns $r_f = 0\%$ net return.
  - (Mike) High ability: earns $r_m = 20\%$ net return.

Capital Income vs. Wealth Tax

	Capital income tax		Wealth tax **(on book value!)**	
	$a_{i,\text{after-tax}} = a_i + (1-\tau_k) r_i a_i$		$a_{i,\text{after-tax}} = (1-\tau_a) a_i + r_i a_i$	
	Fredo ($r_f = 0\%$)	Mike ($r_m = 20\%$)	Fredo ($r_f = 0\%$)	Mike ($r_m = 20\%$)
Wealth	\$1000	\$1000	**\$1000**	**\$1000**
Before-tax Income	0	**\$200**	0	\$200
	$\tau_k = 25\% \; (= \frac{50}{\mathbf{200}})$		$\tau_a = 2.5\% \; (= \frac{50}{\mathbf{2000}})$	
Tax liability	0	\$50 $(= 20\tau_k)$	\$25 $(= 1000\tau_a)$	\$25 $(= 1000\tau_a)$
After-tax return	0%	$15\% \; (= \frac{200-50}{1000})$	$-2.5\% (= \frac{0-25}{1000})$	$17.5\% \; (= \frac{200-25}{1000})$
After-tax wealth ratio	$1.15 \; (= {}^{1150}/_{1000})$		$1.20 \; (\approx {}^{1175}/_{975})$	

- Replacing τ_k with τ_a $\rightarrow$ reallocates capital to more productive agents (**Use it or lose it**) + increases dispersion in after-tax returns & wealth.
- Market value reflects future earnings, taxing it weakens **use it or lose it** effect.

Preview of results

1. **Efficiency Gains:** A marginal increase in the wealth tax increases TFP **iff** entrepreneurial productivity is positively auto-correlated.

2. **Welfare Gain by Type:** With a marginal shift from capital income to wealth tax
   - Workers gain
   - High-productivity entrepreneurs "typically" gain
   - Low-productivity entrepreneurs "typically" lose

3. **Optimal Taxes:** Utilitarian welfare maximizing taxes depend on the elasticity of output with respect to capital (α)
   - If α is sufficiently high $\longrightarrow \tau_a^* > 0$ & $\tau_k^* < 0$
   - If α is sufficiently low $\longrightarrow \tau_a^* < 0$ & $\tau_k^* > 0$
   - If α is in between $\longrightarrow \tau_a^* > 0$ & $\tau_k^* > 0$.

Preview of Results

Extensions

- Corporate sector with no borrowing constraint
- Rents: Return $\neq$ marginal productivity
- Entrepreneurial effort in production
- Perpetual-youth model with stationary wealth distribution

Theoretical Model

- Two groups of infinitely-lived agents:
  - homogenous workers (size L)
  - heterogenous entrepreneurs (size 2)
- Workers' and entrepreneurs' preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t) \qquad \text{where } \beta < 1.$$

- Workers:
  - supply labor inelastically + consume wage income (hand-to-mouth).

Theoretical Model

- Entrepreneurs' technology:

$$y = (zk)^{\alpha} n^{1-\alpha}$$

  - $z \in \{z_l, z_h\}$, where $z_h > z_l \geq 0$ with a transition matrix

$$\mathbb{P} = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix} \text{ with } 0 < p < 1.$$

  - Autocorrelation is critical: $\rho = 2p - 1 > 0 \longleftrightarrow p > 1/2$.

- Aggregate output:

$$Y = \int (zk)^{\alpha} n^{1-\alpha}$$

- Government finances exogenous expenditure G with τ_k and τ_a
  - τ_a on beginning-of-period book value wealth

Financial Markets & Entrepreneurs' Production Problem

Financial markets:

- Collateral constraint ($\lambda \geq 1$): $k \leq \lambda a$, where a is entrepreneur's wealth.
- Bonds are in zero net supply.

Entrepreneurs' Production Decision:

$$\Pi^{\star}(z,a) = \max_{\mathbf{k} \leq \lambda \mathbf{a}, n} (zk)^{\alpha} n^{1-\alpha} - rk - wn.$$

Solution: $$\Pi^{\star}(z,a) = \underbrace{\pi^{\star}(z)}_{\text{Excess return above } r} \times a$$

$$\pi^{*}(z) = \begin{cases} (MPK(z) - r)\lambda & \text{if } MPK(z) > r \\ 0 & \text{otherwise.} \end{cases}$$

- $(\lambda - 1)a$: amount of external funds used by type-z if $MPK(z) > r$.

Entrepreneur's Consumption-Saving Problem

$$V(a,z) = \max_{c,a'} \log(c) + \beta \sum_{z'} \mathbb{P}(z' \mid z) V(a', z')$$

$$\text{s.t. } c + a' = \underbrace{(1-\tau_a)\,a + (1-\tau_k)\,(r + \pi^\star(z))\,a}_{\text{After-tax wealth}}.$$

- Letting $R_i \equiv (1-\tau_a) + (1-\tau_k)(r + \pi^\star(z_i)) \quad$ for $i \in \{l, h\}$,

  the savings decision (CRS + Log Utility):

$$a' = \beta R_i a \quad \longrightarrow \text{linearity allows aggregation}$$

Evolution of Aggregates

$$A_h' = \underbrace{p\beta R_h A_h}_{\text{stayers' savings}} + \underbrace{(1-p)\,\beta R_l A_l}_{\text{switchers' savings}}$$

A_h: High type wealth

$$A_l' = \underbrace{p\beta R_l A_l}_{\text{stayers' savings}} + \underbrace{(1-p)\,\beta R_h A_h}_{\text{switchers' savings}}$$

A_l: Low type wealth

Equilibrium and Steady State

Three different equilibria can arise depending on parameter values:

1. **"Interesting"** if $\lambda < \lambda^{\star} < 2$:
   - $(\lambda - 1) A_h < A_l$: low-type entrepreneurs bid down interest rate: $r = \text{MPK}(z_l)$.
   - **Unique steady state** with:
     - return heterogeneity, misallocation of capital, wealth tax $\neq$ capital income tax.
   - **Empirically relevant:** $R_h > R_l$ and $\frac{\text{Debt}}{\text{GDP}} \gg 1.3$ when $\lambda = \lambda^{\star}$.

   Debt-GDP
2. **"Uninteresting"** if $\lambda \geq 2$:
   - Unique steady state with:
     - no return heterogeneity ($R_l = R_h$), no misallocation of capital $(K_h = A_h + A_l)$, wealth tax $\equiv$ capital income tax.
3. **Unstable** if $\lambda^{*} < \lambda < 2$: No steady state.

   Unstable Eq'm

Equilibrium and Steady State

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Equilibrium Values

Lemma: Aggregate output is

$$Y = (ZK)^{\alpha} L^{1-\alpha} \qquad (Z^{\alpha} \text{ is measured TFP})$$

where

$$Z = s_h z_\lambda + (1 - s_h) z_l : \qquad Z = \text{Wealth-weighted productivity}$$

Key variables:

- $s_h = \frac{A_h}{K}$: wealth share of high-productivity entrepreneurs.
- $z_\lambda \equiv z_h + (\lambda - 1)(z_h - z_l)$: effective productivity of high-type entrepreneurs.

Use it or lose it effect increases efficiency if $s_h \uparrow (\longrightarrow Z \uparrow)$

Steady State: 2 equations 2 unknowns

Using the law of motion for A_l and A_h we obtain two steady state equations:

Steady State K

$$(1-\tau_k)\overbrace{\alpha Z^{\alpha}\left(K/L\right)^{\alpha-1}}^{\text{MPK}}-\tau_a=\frac{1}{\beta}-1.$$

Steady State Z (depends on only τ_a!)

graph How τ_k disappears

$$\left(1-\rho\beta\left(1-\tau_a\right)\right)Z^2-\frac{z_l+z_\lambda}{2}\left(1+\rho-2\rho\beta\left(1-\tau_a\right)\right)Z+z_l z_\lambda\rho\left(1-\beta\left(1-\tau_a\right)\right)=0.$$

Proposition:

Graph $\bar{\tau}_a$ graph

For all $\tau_a < \bar{\tau}_a$ ($\longleftrightarrow \lambda < \lambda^*$), a marginal increase in τ_a **increases steady state Z** **iff** entrepreneurial productivity is autocorrelated, $\rho > 0$ ($p > 1/2$)

Corollary:

1. Wealth concentration: $s_h \uparrow$ ($Z \uparrow = s_h z_\lambda + (1 - s_h) z_l$)

2. Dispersion of after-tax returns rises with τ_a:

G.E.

$$\frac{dR_l}{d\tau_a} = (\text{use-it-lose-it} < 0) \quad + (\text{G.E. effect} < 0) \qquad < \mathbf{0}$$

$$\frac{dR_h}{d\tau_a} = (\text{use-it-lose-it} > 0) \quad + (\text{G.E. effect} < 0) \qquad > \mathbf{0}$$

Government Budget and Aggregate Variables

$$G = \tau_k \alpha Y + \tau_a K.$$

Assumption: G is a constant fraction $\theta\alpha$ of aggregate output: $G = \theta\alpha Y$.

- In what follows, τ_k adjusts in the background when $\tau_a \uparrow$

Lemma: For all $\tau_a < \overline{\tau}_a$, a marginal increase in $\boldsymbol{\tau_a}$

- **Increases** capital (K), output (Y), wage (w), h-type wealth (A_h), and G **iff** $\rho > 0$
  - Higher $\alpha \longrightarrow$ Larger response of K, Y, w
  - $A_l = (1 - s_h) K \downarrow$ iff $\alpha z_\lambda < Z$ and $\rho > 0$.

Welfare gains (across steady states)

$\text{CE}_{1,i}$ measure ($i \in \{w, l, h\}$)**:**

- (a, i) in **B**enchmark economy v.s.
  (a, i) in **C**ounterfactual economy with higher τ_a (lower τ_k)
- Welfare gains (**C**$\succ$**B**) if

$$\frac{\log\left(1+\text{CE}_{1,i}\right)}{1-\beta} = V^{\text{C}}(a,i) - V^{\text{B}}(a,i) > 0$$

  independent of a because $V(a,i) = m_i + \frac{1}{1-\beta}\log(a)$ $i \in \{i, h\}$.

- Utilitarian welfare CE_1 depends on population shares n_i's:

$$\log\left(1+\text{CE}_1\right) = \sum_i n_i \log\left(1+\text{CE}_1(a,i)\right)$$

- CE_1 does not account for changes in distribution of wealth.
  - Alternative measure CE_2 takes into account changes in wealth levels.

CE_2 Details

Main Result 2: Welfare gains by type

Proposition:

For all $\tau_a < \overline{\tau}_a$, a marginally higher $\boldsymbol{\tau_a}$ changes welfare as follows **iff** $\rho > 0$

- Workers: Higher $\text{CE}_{1,w} > 0$
- High-type entrepreneurs: Higher $\text{CE}_{1,h} > 0$ iff $R_h - R_l < \kappa_R(\beta, \rho)$ κ_R
  - Taking wealth accumulation into account: $\text{CE}_{2,h} > 0$ always.
- Low-type entrepreneurs: Lower $\text{CE}_{1,l} < 0$
  - Taking wealth accumulation into account: $\text{CE}_{2,l} < 0$ if $\alpha z_\lambda < Z$.
- Lower average welfare of entrepreneurs: $\text{CE}_{1,\mathbf{E}} < 0$.

Optimal Taxation

Government chooses (τ_a, τ_k) to maximize the utilitarian social welfare CE_1 (or CE_2)

Key trade-off:

1. Higher wages (depends on α) v.s.
2. Lower (LOG) average return (higher return dispersion + negative GE effect)
   - & changes in $\{A_l, A_h\}$ if CE_2 is the objective.

Main Result 3: Optimal Taxes

Proposition: There exists a unique optimal tax combination $(\tau_a^\star, \tau_k^\star)$ that maximizes CE_1. An interior optimum $(\tau_a^\star < \bar{\tau}_a)$ is the solution to:

$$n_w \underbrace{\xi_w}_{Z\text{-Elasticity of Wages}(=\alpha/(1-\alpha))} + \frac{1-n_w}{1-\beta} \underbrace{\left(\frac{\xi_{R_l} + \xi_{R_h}}{2}\right)}_{\text{Av. } Z\text{-Elasticity of Returns}<0} = 0$$

where $\xi_x \equiv \frac{d \log x}{d \log Z}$ is the elasticity of variable x with respect to Z. Furthermore,

$\tau_a^\star \in \left[1 - \frac{1}{\beta}, 0\right)$	and $\tau_k^\star > \theta$	if $\alpha < \underline{\alpha}$
$\tau_a^\star \in \left[0, \frac{\theta(1-\beta)}{\beta(1-\theta)}\right]$	and $\tau_k^\star \in [0, \theta]$	if $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$
$\tau_a^\star > \frac{\theta(1-\beta)}{\beta(1-\theta)}$	and $\tau_k^\star < 0$	if $\alpha > \bar{\alpha}$

Extensions

- Corporate sector with no borrowing constraint
  - If $z_l < z_C < z_h$, then low-productivity agents invest in the corporate sector.
- Rents: Return $\neq$ marginal productivity.
  - Introduce zero-sum return wedges so that $R_h <> R_l$.
  - Efficiency gains from $\tau_a \uparrow$ if $\rho > 0$ **and** $R_h > R_l$.
  - Efficiency gains from $\tau_a \uparrow$ if $\rho < 0$ **and** $R_h < R_l$.
- Entrepreneurial effort in production:
  - With GHH preferences, aggregate entrepreneurial effort increases with wealth tax.
- Perpetual youth with permanent types:
  - We can solve the stationary distribution of agents.
  - $\text{CE}_{2,h} > \text{CE}_{1,h} > 0$ always.

Conclusions

Increasing τ_a:

- Reallocates capital: less productive $\rightarrow$ more productive agents.

  This reallocation increases
  - TFP, output, and wages;
  - dispersion in returns and wealth **iff $\rho > 0$**.

- Workers gain
- Entrepreneurs: High-productivity gain*, low-productivity lose*.

Optimal tax combination: depends on elasticity of output with respect to capital.

Full draft coming soon!

Thanks!

Extra

Unstable equilibrium

Back to Eq.

1. Can there be a steady state with $(\lambda - 1) A_h > A_l$? **NO.** In that case $R_h = R_l$,

$$\frac{A_h'}{A_l'} = \frac{pA_h + (1-p) A_l}{(1-p) A_h + pA_l} = \frac{A_h}{A_l},$$

   which implies that $A_h = A_l$. But then $(\lambda - 1) A_h > A_l$ is violated since $\lambda < 2$.

2. Can there be a steady state with $(\lambda - 1) A_h < A_l$? If the answer is yes, then we are already focusing on that SS and that SS implies that $\lambda < \lambda^*$.

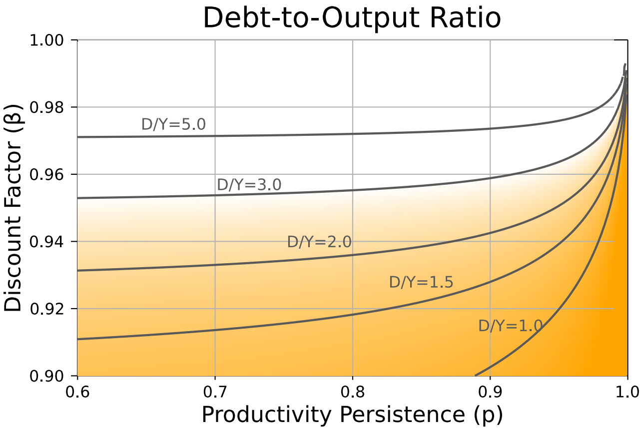
3. If $(\lambda - 1) A_h > A_l$ in the transition, then $A_h > A_l$ since $\lambda < 2$ and

$$\frac{A_h'}{A_l'} = \frac{pA_h + (1-p) A_l}{(1-p) A_h + pA_l} < \frac{A_h}{A_l}.$$

   Then at some point, we will have $(\lambda - 1) A_h < A_l$ and we will be in the heterogenous-return case. If this converges to a a steady state, it is the one with $\lambda < \lambda^*$.

Is λ^* too restrictive?

Back to Eq.

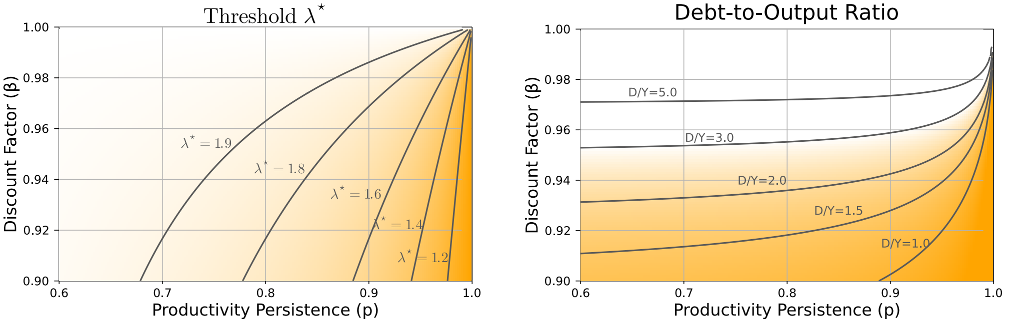


Debt-to-output ratio when $\lambda = \lambda^\star$ computed as $^{(\lambda^\star - 1)A_h}/_{Y}$.

Is λ^* too restrictive?

Back to Eq.

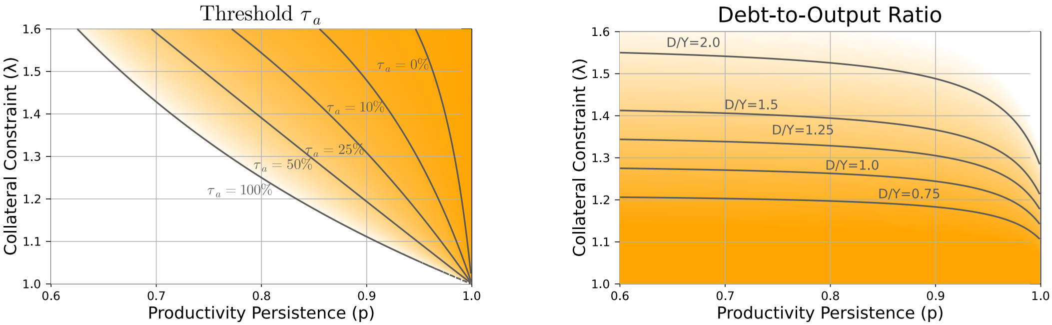
Figure 1: Conditions for Steady State with Heterogeneous Returns



$z_l = 0$, $z_h = 2$, $\tau_k = 25\%$, and $\alpha = 0.4$.

Debt-to-output ratio when $\lambda = \lambda^\star$ computed as $(\lambda^\star - 1)A_h/Y$

Figure 2: Conditions for Steady State with Heterogeneous Returns



$z_l = 0$, $z_h = 2$, $\tau_k = 25\%$, and $\alpha = 0.4$.

Debt-to-output ratio with $\tau_a = 0$ (benchmark) computed as $(\lambda^\star - 1) A_h / Y$

Steady State: 2 equations 2 unknowns

Back to ss | Back to Eff.

SteadyState K:

$$(1-\tau_a)+(1-\tau_k)\overbrace{\alpha Z^{\alpha}\left({}^{K}/_{L}\right)^{\alpha-1}}^{\text{Marginal Product } K}=\frac{1}{\beta}$$

Steady State R:

$$R_i=(1-\tau_a)+(1-\tau_k)\overbrace{\alpha\left({}^{ZK}/_{L}\right)^{\alpha-1}}^{\text{Marginal Product } ZK} z_i \qquad \text{Equilibrium R}$$

$$R_i=(1-\tau_a)+(1-\tau_k)\,\alpha Z^{\alpha}\left({}^{K}/_{L}\right)^{\alpha-1}\frac{z_i}{Z} \qquad \text{Change to MPK}$$

$$R_i=(1-\tau_a)+\left(\frac{1}{\beta}-(1-\tau_a)\right)\frac{z_i}{Z} \qquad \text{Steady State}$$

Key: Steady state K adjusts to maintain constant (after-tax) MPK:

$$(1-\tau_k)\,\text{MPK}=\frac{1}{\beta}-(1-\tau_a)$$

As in NGM τ_k affects level of K but not long run (after-tax) MPK $\left({}^{1}/_{\beta}-1+\tau_a\right)$.

Welfare gains (with changes in wealth)

Back to CE_1

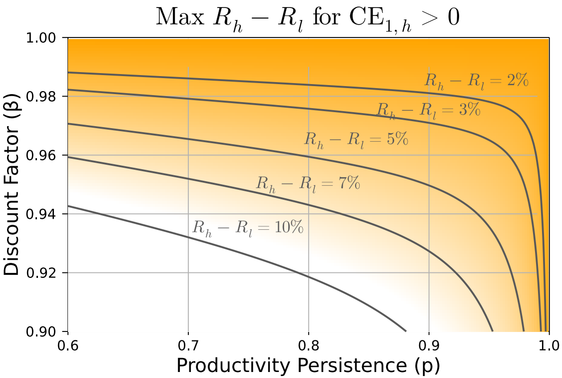
$\mathrm{CE}_{2,i}$ measure $(i \in \{w, l, h\})$:

- Evaluate welfare gain at average wealth levels for each economy.
- (A_i^{B}, i) in the **B**enchmark economy v.s. (A_i^{C}, i) in the **C**ounterfactual economy.
- Welfare gains (**C**$\succ$**B**) if

$$\frac{\log\left(1 + \mathrm{CE}_{2,i}\right)}{1-\beta} = V^{\mathrm{C}}\left(A_i^{\mathrm{C}}, i\right) - V^{\mathrm{B}}\left(A_i^{\mathrm{B}}, i\right) > 0 \qquad i \in \{w, l, h\}$$

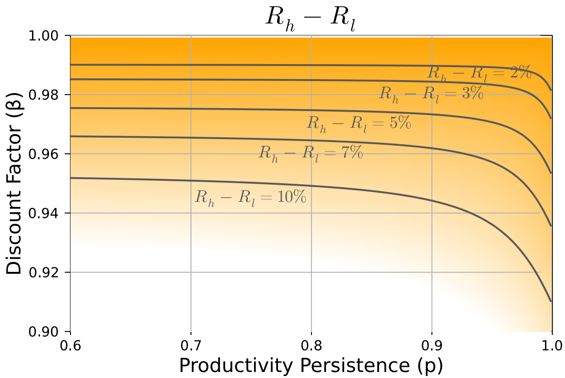
Return Dispersion for Welfare Gains of High-Type Entrepreneurs

Back to CE_1



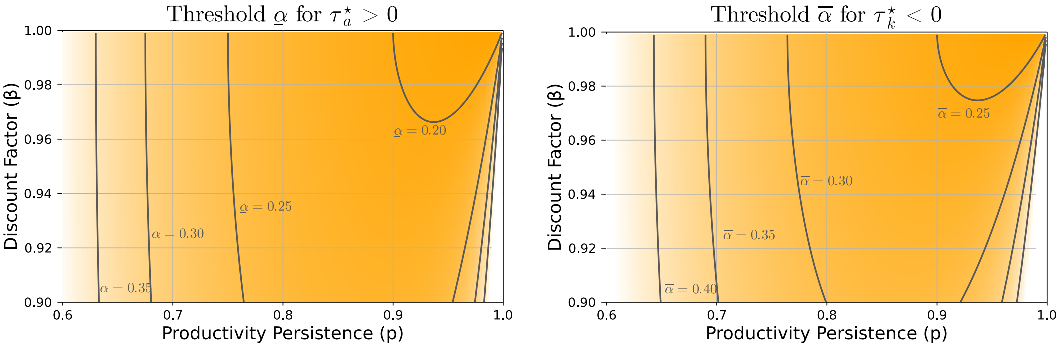
Return dispersion $R_h - R_l$:

Back to α-thresholds



α-thresholds for Optimal Wealth Taxes

Back to opt. tax

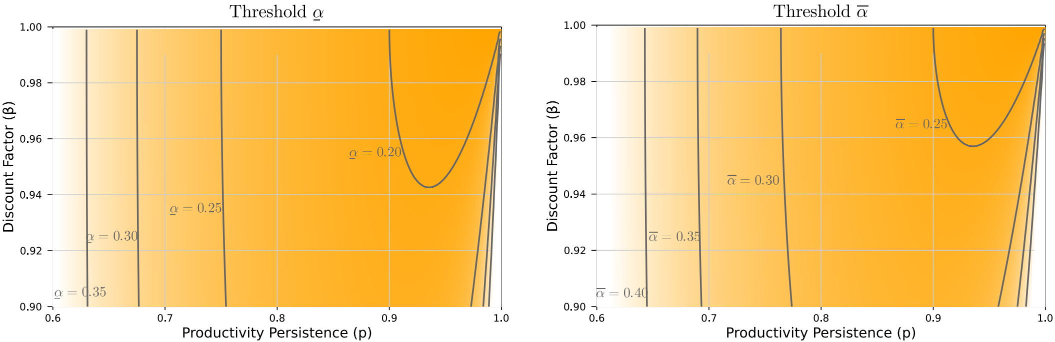


$z_l = 0$, $z_h = 2$, $\lambda = 1.3$, *and* $\theta = 25\%$.

Alt. Parameters | $R_h - R_l$ | Opt. Tax and Welfare Gains

α-thresholds for Optimal Wealth Taxes

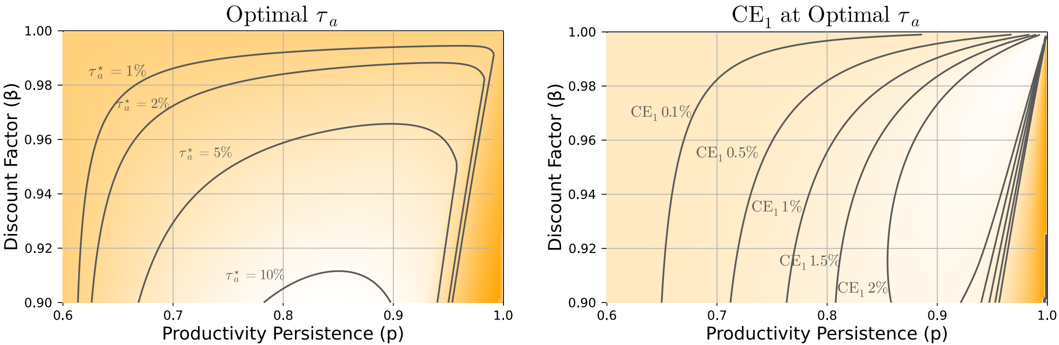
Back to opt. tax



$z_l = 0.5$, $z_h = 1.5$, $\lambda = 1.2$, and $\theta = 25\%$.

Optimal Wealth Taxes and Welfare Gain

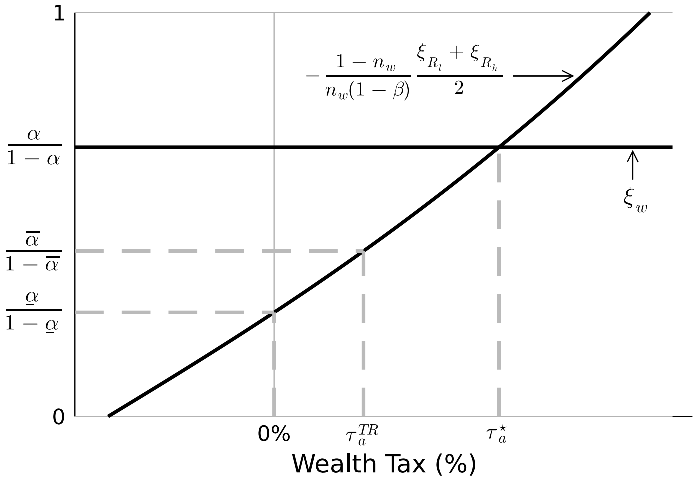
α-thresholds
Back to opt. tax



$z_l = 0$, $z_h = 2$, $\theta = 25\%$, *and* $\lambda = 1.3$.

Optimal Wealth Taxes

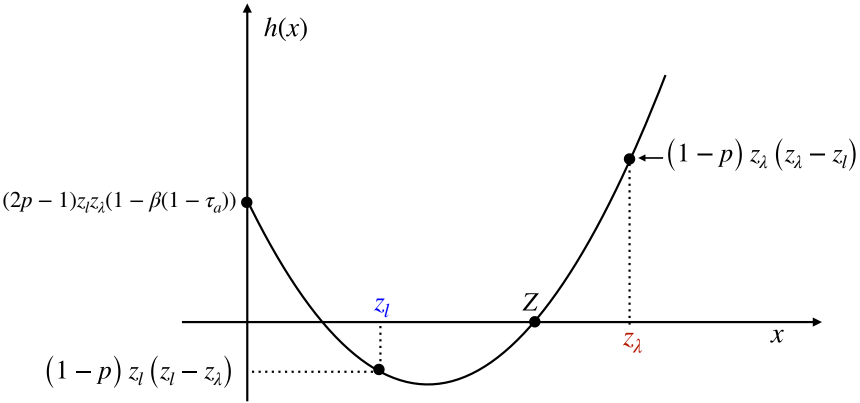
Back to opt. tax



Optimal Taxes and α Thresholds

Existence and Uniqueness of Steady State (when $p > 0.5$)

Back to ss



What happens to Z if $\tau_a \uparrow$?

Back to eff. gain

$$\frac{dh(x)}{d\tau_a} = (2p-1)(x-z_l)(x-z_\lambda) < 0 \text{ iff } p > 0.5 \text{ and } z_l < x < z_\lambda$$