Micro Risks and Pareto Improving Policies with Low Interest Rates*

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Abstract

We provide sufficient conditions for the feasibility of a Pareto-improving fiscal policy when the risk-free interest rate on government bonds is below the growth rate ($r < g$). We do so in the class of incomplete markets models pioneered by Bewley-Huggett-Aiyagari, but we allow for an arbitrary amount of ex ante heterogeneity in terms of preferences and income risk. We consider both the case of dynamic inefficiency as well as the more plausible case of dynamic efficiency. The key condition is that seigniorage revenue raised by government bonds exceeds the increase in the interest rate times the initial capital stock. The Pareto improving fiscal policies weakly expand every agent's budget set at every point in time. The policies improve risk sharing and potentially guide the economy to a more efficient level of capital. In simulations, we find that the government must rely on moderate levels of debt issuance along the transition to the new steady state.

1 Introduction

In this paper we provide sufficient conditions for a Pareto improvement when the risk-free interest rate on government bonds is below the growth rate ($r < g$). We do so in the class of incomplete markets models pioneered by Bewley-Huggett-Aiyagari, but we allow for an arbitrary amount of ex ante heterogeneity in terms of preferences and income risk. We consider both the case of dynamic inefficiency as well as the more plausible case of dynamic efficiency. The paper augments the classic dynamic inefficiency condition of Samuelson (1958) and Diamond (1965) by allowing for rich heterogeneity along dimensions other than age, including idiosyncratic income risk,

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and considering scenarios in which the economy is dynamically efficient. By looking at Pareto improvements rather than maximizing a utilitarian social welfare function, the paper also complements the important work of Aiyagari and McGrattan (1998) and Dávila, Hong, Krusell and Ríos-Rull (2012). It is the absence of cross-agent utility comparisons, or even within-person but cross-time tradeoffs in our implementation, that distinguishes our Pareto metric from the more common utilitarian welfare measures.[1]

We analyze fiscal policies where the instruments available to the government are public debt, linear taxes/subsidies, and lump-sum transfers. We study policies that leave all after-tax factor prices (and pure profits, if there are any) weakly greater at all dates, and where there are no lump-sum taxes. By weakly expanding the budget set of all agents at all dates, these policies necessarily generate a Pareto improvement. We study when these *Pareto-improving fiscal policies* are feasible, and consider versions that lead to both crowding out and crowding in of capital.

We establish sufficient conditions for the existence of these feasible Pareto-improving policies that involve only the knowledge of the aggregate savings schedule (that is, total private savings as a function of interest rates and government transfers) and technology (including the size of a mark-up, if any). In particular, the short and long run elasticities of aggregate household savings play a crucial role in determining feasibility. We emphasize that the relevant elasticities are the ones governing the *aggregate* savings schedule (and technology). One advantage of this "macro" approach is that conditional on these aggregate elasticities, the nature and extent of the underlying heterogeneity across individuals is not relevant for assessing feasibility.[2]

We conduct the analysis in an environment that builds closely on the canonical model of Aiyagari (1994), where precautionary savings motives reduce the equilibrium level of interest rates. The main restriction we impose on preferences is zero wealth effect on labor supply, as in the well known "GHH" preferences of Greenwood, Hercowitz and Huffman (1988), and omit aggregate risk considerations.[3]

To understand the role of the aggregate savings schedule, consider starting from a laissez-faire equilibrium (zero debt, zero taxes, zero transfers) in which $r < g$, and consider a government that issues some amount of debt ΔB. All else equal, this will crowd out capital and increase the risk-free rate, assuming that aggregate savings is increasing in r. Factor prices respond to the policy, with increases in the return to wealth and decreases in the wage. Our focus on fiscal policies that

[1]Our focus on Pareto-improving policies rather than policies that maximize a utilitarian metric has an antecedent in Werning (2007), who explores Pareto-efficient tax policies in a Mirrelesian environment.

[2]There are of course disadvantages to this approach. For example, our approach rules out the use of lump sum taxes (even if available) and as a result, the policy cannot exploit the link between private borrowing constraints and government liquidity identified by Woodford (1990) and Aiyagari and McGrattan (1998). Those policies however would require information on the underlying heterogeneities, frictions and intertemporal trade-offs of agents in addition to knowledge about the aggregate savings behavior.

[3]We also omit idiosyncratic return risk, as in the model of Angeletos (2007).

keep all factor prices weakly greater calls for the government to offset the decline in wages. The government has two approaches: it can subsidize wages to keep the after tax wage from falling, or it can subsidize the rental rate of capital to forestall the decline in capital. Which approach is fiscally cheaper depends on the dynamic efficiency of the economy. Taking the case in which the economy is dynamically inefficient, the crowding out of capital increases income available for consumption, as in Diamond (1965), making this the more efficient approach. The government subsidizes wages to maintain equivalence between the ex post after tax wage and that of the original laissez-faire equilibrium. We show that the fiscal cost of this policy is bounded above by $\Delta r \times K_0$, where Δr is the change in the net interest rate and K_0 is the initial capital stock (in efficiency units with exogenous labor-augmenting productivity growth).[4] This is the case in the new steady state, and we show how a similar condition holds for the transition path. In the new steady state, revenues are the seigniorage obtained from bond issuance, which in the case of exogenous technological growth at rate g, is $(g - r)\Delta B > 0$. If this exceeds $\Delta r \times K_0$, then a Pareto improvement is feasible, as r has increased and after tax wages remain constant,[5] making all agents weakly better off and those with positive assets strictly better off.

If we allow for product market mark-ups of sufficient size such that the economy is dynamically efficient, a more efficient fiscal policy is one that subsidizes the rental rate of capital and avoids crowding out. The first fiscal policy we consider for this case subsidizes the rental rate just enough to keep capital at the initial laissez faire level. This plan guarantees that output, as well as untaxed wages and profits, are also unchanged. The sufficient condition for the feasibility of a Pareto improving policy discussed above continues to suffice: the costs of this plan can again be bounded by $\Delta r \times K_0$, and the revenue remains $(g - r)\Delta B$. While the trade off remains identical in terms of the elements, the absence of crowding out makes it more difficult to achieve a Pareto improvement. This follows from the fact that the additional debt needs to be completely absorbed as new wealth (rather than replacing capital) by private households. All else equal, a given amount of debt therefore demands a larger increase in the interest rate ($\Delta r \uparrow$), raising the cost of the policy.

Given that product market mark-ups depress capital in the laissez faire equilibrium, we also consider the feasibility of Pareto improving fiscal policies that crowd in capital towards a more efficient level. For this case, we construct fiscal policies that subsidize capital and tax labor and profits, such that after-tax wages and profits remain at the laissez faire levels. These are feasible Pareto-improving policies if the tax and seignorage revenue can cover the cost of the capital subsidy. The sufficient condition here incorporates the additional net revenue for the government,

[4] More precisely, the condition is $\Delta r \times (K_0 - \underline{a})$, where $\underline{a} \leq 0$ is the loosest private household borrowing constraint. To keep the notation streamlined in the introduction, we omit this term.

[5] Contrast this with the utilitarian metric of Dávila et al. (2012), which requires that a change in *relative* factor prices improved the lot of the poorest households relative to the richest.

which is $\Delta Y - (r + \delta)\Delta K$. We find that this net revenue tends to be larger in the long-run than in the short-run because of a lower savings elasticity to interest rates in the short-run; that is, r overshoots along the transition. This property implies that along the transition debt issuance allows the government to finance the Pareto improvement. Rather than the traditional crowding out, the use of debt to engineer the transition to a higher level of capital makes debt and capital investment *complements* rather than substitutes.

In all cases we analyze, the Pareto improving fiscal policies weakly expand every agent's budget set at every point in time. This avoids the trade off between young and old in the classic over-lapping generations settings. It also avoids the trade off between the poor and rich that is the focus of the utilitarian metric common in the Bewley-Huggett-Aiyagari literature. The latter can be motivated by an ex ante "behind the veil of ignorance" rationale, or, given ex ante homogeneity, by a "renewal" argument based on the fact that all agents eventually transit through all states (see Aiyagari and McGrattan (1998) for a related discussion). Given that income and wealth differences persist across generations (Chetty, Hendren, Kline and Saez (2014)), that some agents have limited access to financial markets (Braxton, Herkenhoff and Phillips (2020)), and agents may value inter-temporal tradeoffs differently (Krusell and Smith (1998)), working through budget sets is a robust exercise. That said, the Pareto criterion is a high threshold, and as such, it should not be viewed as a necessary condition for policy intervention. But certainly, the availability of a Pareto improvement provides a sufficient condition for a government response. It is in this spirit we provide sufficient conditions for such a scenario.

As in any model in which seigniorage (or a liquidity premium) plays an important fiscal role, the willingness of private households to hold additional government bonds without a large increase in the interest rate is key. The empirical literature on whether and to what extent government borrowing increases the interest rate is challenged by identification concerns and has produced numbers with no clear consensus.[6] Theory provides some guidance, which we discuss below, but one must keep in mind that in heterogeneous agent models aggregate elasticities have a complex relationship with individual preference parameters and idiosyncratic shocks. We therefore use simulations to assess the magnitude of the aggregate savings elasticity and the feasibility of Pareto-improving fiscal policies.

We present simulation results with Epstein and Zin (1989) preferences, and calibrate the dynamically efficient economy using the income process of Krueger, Mitman and Perri (2016) and the historical data on $r - g$ in the U.S. We find scope for Pareto improving policies for a wide range of debt policies and for policies with and without capital crowding in. Our baseline experiment considers a Pareto improving fiscal policy plan that keeps capital as in the laissez faire economy

[6] See, for example, the survey papers by Bernheim (1987a) and Seater (1993) that examine the empirical evidence and seem to draw opposing conclusions on Ricardian equivalence.

and slowly increases debt to the average in the data. Welfare gains arise because fiscal policy improves risk sharing as households receive positive transfers, and their wealth increases due to a higher return. Welfare gains are larger early in the transition reflecting the larger government transfers financed by debt issuance.

The second fiscal policy plan we consider consists of the same debt path as the baseline, but with capital increasing towards the Golden Rule where the marginal product of capital equals the rate of depreciation. We find that this fiscal plan is also Pareto improving and generates even larger welfare gains to all households because here policy not only helps with risk sharing but also with efficient supply expansions. Debt is an essential part of these fiscal policies, as it provides the seigniorage revenue that is used for transfers to households and subsidies for capital expansions. We do find, however, that seigniorage revenue from bonds has limits and features a Laffer Curve: more debt increases interest rates and therefore the relative cost for servicing the debt. However, in our calibration, the upper bound on debt for Pareto improving fiscal policies is about twice the level of output.

This paper is part of a fast-growing recent literature exploring fiscal policy in environments with persistently low risk-free interest rates. Mehrotra and Sergeyev (2020) use a sample of advance economies to document that $r - g$ is often negative and develop a model to study its implications for debt sustainability.[7] Blanchard (2019)'s presidential address to the American Economics Association gave a major stimulus to the question of debt sustainability under low interest rates. Other recent papers are Bassetto and Cui (2018), Reis (2020), Brunnermeier, Merkel and Sannikov (2020), and Ball and Mankiw (2021). Our paper incorporates features of this previous work, such as borrowing constraints and the potential role of markups in opening a wedge between the interest rate and the marginal product of capital. However, our focus is on designing Pareto improving fiscal policies in the presence of individual heterogeneity and incomplete markets, as in the Bewely-Huggett-Aiyagari tradition, and the role played by $r < g$.

Our work also contributes to the literature studying the effects of fiscal policies in models with heterogeneous agents. Heathcote (2005) shows the failure of Ricardian equivalence in this class of models, as temporary tax cuts financed with public debt tend to increase consumption and output as they give households that are at the borrowing constraint extra resources that are spent. Heathcote, Storesletten and Violante (2017) study optimal labor tax progressivity in this environment and illustrate sharply the trade off between insurance and incentives motives in response to the tax system. Dyrda and Pedroni (2020) study the optimal tax system in a quantitative version of the idiosyncratic risk model. Also recently, Bhandari, Evans, Golosov and Sargent (2020) explore optimal fiscal and monetary policy within the context of the heterogeneous agent

[7] See also Mauro and Zhou (2021) and Jordà, Knoll, Kuvshinov, Schularick and Taylor (2019).

model with nominal rigidities and aggregate shocks.[8] All of these papers focus on a utilitarian welfare criteria, and do not analyze the implications of $r < g$.

The paper proceeds as follows: Section 2 lays out the environment; Section 3 provides the sufficient conditions for a Pareto improving fiscal policy; Section 3.4 discusses the aggregate interest-elasticity of household savings; Section 4 provides numerical examples; and Section 5 concludes.

2 Environment

The model hews closely to canonical environment of Aiyagari (1994). However, in many ways, our environment is more general. We allow for permanent differences in the income process or preferences across households. The framework also allows for product market mark-ups, driving a wedge between the marginal product of capital and the return on risk-free bonds. However, we do impose one assumption on preferences; namely, there is no wealth effect on labor supply. This greatly simplifies tracing the impact of a change in interest rates on labor supply. In particular, the wage is a sufficient statistic for pinning down aggregate labor, regardless of other factor prices.[9]

We suppress exogenous growth in the text, but show how the model extends in the usual straight-forward way (given homothetic preferences) to growth in Appendix C. As a rule of thumb, the key condition $r < 0$ for an interest rate r is replaced with the corresponding $r < g$ where g denotes the constant exogenous growth rate of labor-augmenting productivity.

2.1 Households

There are a measure-one continuum of households. At time t, each household, indexed by $i \in [0, 1]$, draws an idiosyncratic labor productivity $z_t^i \geq 0$. We do not impose that each household faces the same stochastic process for idiosyncratic risk. That is, some households may face a permanently lower level of productivity or additional risk. We impose a cross-sectional independence restriction below that rules out aggregate productivity risk.

If the household provides $n_t^i \geq 0$ units of labor, it receives $w_t z_t^i n_t^i$ in labor earnings, w_t is the equilibrium wage rate per efficiency unit of labor. Without loss of generality, we assume labor taxes are paid by the firm.

A household may also receive profit income. We model this as payment to entrepreneurial

[8] See also Le Grand and Ragot (2020). Other recent papers that have studied the implications of transfers and government debt in heterogeneous agent models with price rigidities are Oh and Reis (2012) and Hagedorn, Manovskii and Mitman (2019).

[9] However, as we will see below, the Frisch elasticity of labor supply is not important for the analysis, as the policies that we explore maintain a constant after-tax wage.

talent, which, like labor productivity, is an endowment that may follow a stochastic process. Let π_t^i denote household i's return to entrepreneurial talent. Define aggregate household profit income as $\Pi_t = \int \pi_t^i di$, and household $i's$ share as $\theta_t^i \equiv \pi_t^i/\Pi_t$. Household i faces a potentially stochastic process for θ_t^i that determines its share of aggregate profits, with the restriction that $\theta_t^i \in [0,1]$ and $\int \theta_t^i di = 1$ for all t.

At the start of period t, the household has a_t^i units of financial assets, which receive a risk-free return $(1+r_t)$ in period t. Letting $T_t \geq 0$ denote lump sum transfers from the government, which is uniform across i, the household's budget constraint is:

$$c_t^i + a_{t+1}^i \leq w_t z_t^i n_t^i + \theta_t^i \Pi_t + (1+r_t)a_t^i + T_t,$$

where c_t^i is consumption in period t.

Households are subject to a (potentially idiosyncratic) borrowing constraint $a_t^i \geq \underline{a}^i$ for all t. The fact that some households may have a tighter constraint than others captures the possibility that access to financial markets may be heterogeneous. Let $\underline{a} \equiv \inf_i \underline{a}^i$ denote the loosest borrowing constraint faced by households.[10]

The main restriction on preferences is the absence of a wealth effect on labor supply, as in the well known "GHH" preferences of Greenwood et al. (1988). In particular, let $x^i(c,n) \equiv c^i - v^i(n)$ for some convex function v^i. We write preferences recursively as $V_t^i = \phi^i(x_t, h_t^i(V_{t+1}^i))$, where V_t^i is household i's value and h^i represents a certainty equivalent operator over z_{t+1}, conditional on $z_t = z$ and the household's stochastic process for productivity. This notation nests both standard "CRRA" utility as well as the recursive utility of Kreps and Porteus (1978) and Epstein and Zin (1989). We incorporate the latter to explore the different roles of risk aversion and inter-temporal elasticity of substitution in the feasibility of a Pareto improvement.

The idiosyncratic state variables for an individual household are $s \equiv (a, z, \theta)$, and the aggregate states are the (perfect foresight) sequences for factor prices $\{r_t, w_t\}$, aggregate profits $\{\Pi_t\}$, and transfers $\{T_t\}$. The household's problem can be written in levels as follows

$$V_t^i(a,z,\theta) = \max_{a' \geq \underline{a}^i, n \in [0,\bar{n}^i], c \geq 0} \phi^i(x^i(c,n), h_t^i(V_{t+1}^i(a',z',\theta'))) \tag{1}$$

$$\text{subject to: } c + a' \leq w_t z n + \theta \Pi_t + (1+r_t)a + T_t.$$

Note that as preferences can vary across households, we can accommodate distinct labor supply elasticities, as well as hand-to-mouth households.[11] In particular, the framework nests

[10] We assume below that the borrowing constraint is always above the natural borrowing limit. See Bhandari, Evans, Golosov and Sargent (2017) and Heathcote (2005) for a discussion on the role of such ad-hoc limits in breaking Ricardian equivalence.

[11] To see the later, consider the case of an aggregator $\phi^i(x,v) = h^i(x)$ for some household i. This corresponds to a

the classic Aiyagari (1994) with inelastic labor supply.[12]

Assuming an interior labor supply decision, household i's first-order condition with respect to labor is:

$$v_i'(n_t^i) = w_t z_t^i.$$

This implies a policy function $n^*_{i,t}(z)$, where the subscript t captures the equilibrium wage at period t.

Similarly, we let $a^*_{i,t}(a, z, \theta)$ and $c^*_{i,t}(a, z, \theta)$ denote the optimal saving and consumption policy functions respectively. The aggregate stock of savings chosen in period t and carried into period $t+1$ is $A_{t+1} \equiv \int a^*_{i,t}(a_t^i, z_t^i, \theta_t^i)di$.

We now state our independence assumption. Let $\boldsymbol{z}_t \equiv \{z_t^i\}_{i\in[0,1]}$ denote the state vector for productivity across households at time t.[13] Let

$$N(w_t, \boldsymbol{z}_t) \equiv \int z_t^i n^*_{i,t}(z_t^i)di = \int v_i'^{-1}(w_t z_t^i)di.$$

We make the assumption that N is independent of $\boldsymbol{z}_t$. This is a generalization of the typical assumption that v is common across households and that z is i.i.d. across i and t. The current environment requires only that aggregate labor supply is independent of the distribution, which is weaker than assuming that households are ex ante identical.[14]

2.2 Firms

The representative firm has a constant-returns technology given by $F(k, l)$, where k is capital and l effective units of labor. Firm's hire labor and rent capital in competitive markets at rates r_t^k and w_t, respectively. Let τ_t^n and τ_t^k denote linear taxes on factor payments for labor and capital, respectively.

Firms may have market power in the product market. For simplicity, we assume that firms charge a price that is a constant mark up over marginal cost. Let $\mu \geq 1$ be the ratio of price to

household that does not value future consumption (it has a discount factor equal to 0). As a result, this household does not save and consumes its entire disposable income every period.

[12] This can be achieved by setting $v^i = 0$. In this case, the labor supply decision is not interior and the corresponding first order condition below does not hold.

[13] We use the word vector loosely, as $\{z_t^i\}$ is a continuum of random variable realizations indexed by $i \in [0, 1]$.

[14] For example, households could belong to one of J types, each with non-trivial measure. Then within a type we can assume that the law of large numbers holds, and the aggregate is simply a weighted average across types.

marginal cost. The representative firm's first-order conditions are (suppressing t in what follows):

$$F_k(k,l) = \mu(1+\tau_t^k)r_t^k$$
$$F_l(k,l) = \mu(1+\tau_t^n)w_t.$$

Firm (pre-tax) profits are given by

$$\hat{\Pi} = \left(\frac{\mu-1}{\mu}\right)F(k,l) = F(k,l) - (1+\tau_t^k)r_t^k k - (1+\tau_t^n)w_t l.$$

Profits are taxed by the government at rate τ_t^π, so after tax profits are $\Pi = (1-\tau_t^\pi)\hat{\Pi}$. We can think of the representative firm hiring a bundle of entrepreneurial talent that is in constant aggregate supply at after tax price Π.

2.3 Financial Intermediaries

We assume that the capital is owned by financial intermediaries.[15] Such intermediaries are competitive and borrow from the households at rate r_t, and, in turn, rent capital to firms at r_t^k and invest in government bonds at rate r_t^b. Capital depreciates at rate δ. Competition in the intermediary market ensures the following equilibrium condition at all t:

$$r_t = r_t^b = r_t^k - \delta.$$

Given the first equality, we drop the distinction between r and r^b in what follows. As noted in the introduction, there is also no maturity mismatch on the intermediaries' balance sheet.

2.4 Government

The government's policy consists of a sequence of taxes $\{\tau_t^n, \tau_t^k, \tau_t^\pi\}$, as well as a sequence of one-period debt issuances, $\{B_t\}$. The lump-sum transfers T_t are such that the sequential budget constraint holds at all periods:

$$T_t = \tau_t^n w_t N_t + \tau_t^k r_t^k K_t + \tau_t^\pi \hat{\Pi}_t + B_{t+1} - (1+r_t)B_t$$

[15] As usual, this is not crucial. We could have equivalently assumed that the capital is owned directly by firms, which finance capital purchases with risk-free bonds issued to households.

2.5 Market Clearing

Given r_t^k and w_t and taxes, let K_t and L_t solve the representative firm's first-order conditions. Market clearing in the financial market requires $A_t = K_t + B_t$. Market clearing in the labor market requires $L_t = N_t$, where we recall that N_t is aggregate efficiency units of labor supplied by households. Finally, goods market clearing requires $C_t \equiv \int c_{i,t}^* di = F(K_t, N_t) - K_{t+1} + (1-\delta)K_t$. By Walras law, one of the market clearing conditions is redundant given that the government and household budget constraints are satisfied.

Definition 1 (Equilibrium Definition)**.** *Given an initial distribution of household wealth and idiosyncratic shocks $\{a_0^i, z_0^i, \theta_0^i\}_{i\in[0,1]}$ and a fiscal policy $\{B_t, \tau_t^n, \tau_t^k, \tau_t^\pi\}_{t\geq 0}$ with initial debt B_0, an equilibrium is a sequence of quantities $\{A_t, K_t, N_t, L_t\}_{t\geq 0}$, prices $\{r_t, r_t^k, w_t\}_{t\geq 0}$, and transfers $\{T_t\}_{t\geq 0}$ such that A_t and N_t are consistent with household optimization given prices and transfers, K_t and L_t are consistent with firm optimization given prices and taxes, T_t is the lump sum transfer necessary to satisfy the sequential government budget constraint, $r_t^k = r_t + \delta$, and financial, labor, and good markets clear.*

We define a *stationary* equilibrium to be an equilibrium in which all sequences are constant over time.[16]

3 Pareto Improvements to a *Laissez Faire* Economy

We begin with a stationary equilibrium without a government. That is, let us consider the case where $\tau_t^j = T_t = B_t = 0$ for all t. This will be the benchmark from which we will search for Pareto improving policies and discuss the special role of $r < 0$. Let (w_0, r_0, Π_0) denote the wage, interest rate, and (un-taxed) aggregate profits in the initial stationary equilibrium of the laissez faire economy, and let $(N_0, K_0 = A_0)$ denote the associated aggregate labor supply and capital stock. Unless otherwise stated, we will assume $r_0 < 0$ in what follows.

3.1 Pareto-Improving Fiscal Policies

The thought experiment we consider is a government that, starting from this environment, unexpectedly announces a new fiscal policy. Normalize $t = 0$ to be the last period in which taxes and maturing debt are zero.

[16]In the analysis that follows, we will assume that such an stationary equilibrium exists. Note that this may require additional assumptions on the stochastic processes for labor productivity and the profit share as well as on their initial cross-sectional distribution. See Açıkgöz (2018), Light (2018), and Achdou, Han, Lasry, Lions and Moll (2021) for results on existence and uniqueness of stationary equilibria in Bewley models.

In period $t = 0$, the government announces a sequence of debt issuances, taxes, and transfers $\{B_{t+1}, \tau_t^n, \tau_t^k, \tau_t^\pi, T_t\}_{t\geq 0}$. We assume that factor taxes are zero in the initial period; that is, $\tau_0^k = \tau_0^n = \tau_0^\pi$. As K_0 is inherited from the laissez faire equilibrium, this implies w_0 and Π_0 remain unchanged in this initial period. Other than the announcement, the only action of the government in period zero is the issuance of new bonds B_1 due next period, the proceeds of which are lump-sum rebated to households $T_0 = B_1$. Subsequent to the announcement, there is perfect foresight.

The fiscal policy will potentially involve a new sequence of transfers and factor prices going forward. We *focus on policies that keep the wage stable at w_0 and aggregate after-tax profits at Π_0*. This ensures that no agent experiences a decline in labor or profit income at each t and idiosyncratic state (z_t^i, θ_t^i).

In period zero, each household re-optimizes its consumption-saving policy to incorporate a new sequence of interest rates and transfers, $\{r_t, T_t\}_{t\geq 0}$, with r_0 given, as well as the original (w_0, Π_0). Starting from the laissez faire stationary equilibrium in period 0, let $\mathcal{A}_{t+1}(\{r_\tau, T_\tau\}_{\tau\geq 0})$ denote the aggregate household saving in period t generated by the new household policy.

Asset market clearing imposes a restriction on the possible combinations of transfers, capital, debt issuances, and interest rates. We formalize this restriction in the following definition:[17]

Definition 2. *A sequence $\{r_t, T_t, B_t, K_t\}_{t=0}^{\infty}$ constitutes an* "admissible sequence" *if for all $t \geq 1$:*

$$\mathcal{A}_t(\{r_\tau, T_\tau\}_{\tau=0}^{\infty}) = B_t + K_t,$$

$\{r_0, K_0\}$ represent the initial laissez-faire stationary equilibrium outcomes, $B_0 = 0$, and there exists a $\overline{B} < \infty$ such that $B_t \leq \overline{B}$ for all $t \geq 1$.

It is useful to clarify what is and is not imposed by admissibility. It imposes household optimality over consumption savings decisions given the sequence $\{r_t, T_t\}$ and a fixed wage, w_0, as well as asset market clearing. Given that the wage is constant and with GHH preferences, labor market clearing is satisfied for an aggregate labor supply equal to N_0. Admissibility does not impose goods market clearing and the government budget constraint. By Walras Law, either one of these is sufficient to establish an allocation is achievable in equilibrium. The following result gives properties of admissible sequences that are sufficient for the feasibility of Pareto-improving fiscal policies relative to laissez faire:

Proposition 1. *If there is an admissible sequence $\{r_t, T_t, B_t, K_t\}_{t=0}^{\infty}$ such that for all $t \geq 0$:*

(i) $r_t \geq r_0$;

[17]Note that the definition imposes that the debt sequence has a finite upper bound, $\overline{B}$. This is to rule out Ponzi schemes by the government.

(ii) $T_t \geq -(r_t - r_0)\underline{a}$*;*

(iii) and

$$B_{t+1} - (1 + r_t)B_t - T_t \geq F(K_0, N_0) - F(K_t, N_0) - (r_0 + \delta)K_0 + (r_t + \delta)K_t, \tag{2}$$

with either (i) or (ii) strict for at least one $t \geq 0$, then there exists a feasible fiscal policy that implements a Pareto improvement.

This proposition delivers a set of conditions that are sufficient for the existence of a Pareto improvement. Note that household heterogeneity only matters for the admissibility of a sequence. And for this, only knowledge of the aggregate savings schedule, $\mathcal{A}$, is required.[18]

We establish the result in three steps, providing some expository remarks as we proceed. We first describe the set of tax instruments used by the government, then compute the tax revenue, and finally establish the result.

For step one, in order for the government to keep the household wage constant, it must tax/-subsidize the firm's labor input such that:

$$\frac{F_N(K_t, N_0)}{(1 + \tau_t^n)\mu} = w_0.$$

Note that, due to the GHH preferences, keeping after-tax wages constant at w_0 ensures the aggregate labor supply remains at its initial level N_0.[19] Moreover, if the issuance of debt crowds out capital ($K_t < K_0$), this involves a labor subsidy $\tau_t^n < 0$, given that $F_K(K_0, N_0) = \mu w_0$.

Similarly, the government taxes/subsidizes profits so that:

$$\Pi_t = (1 - \tau_t^\pi)\hat{\Pi}_t = (1 - \tau_t^\pi)(\mu - 1)F(K_t, N_0)/\mu = \Pi_0$$

[18] As noted, Walras Law implies an alternative representation of condition (2) in the proposition that verifies goods market clearing. Let C_t be aggregate consumption:

$$\begin{aligned} C_t &\equiv w_0 N_0 + \Pi_0 + (1 + r_t)A_t - A_{t+1} + T_t \\ &= F(K_0, N_0) - (r_0 + \delta)K_0 + (1 + r_t)A_t - A_{t+1} + T_t. \end{aligned}$$

Then, for an admissible sequence, condition (2) is equivalent to the aggregate resource constraint:

$$C_t + K_{t+1} \leq F(K_t, N_0) + (1 - \delta)K_t.$$

[19] This is the major simplification introduced by GHH. We do not need to keep track of the aggregate labor supply. And more importantly, we do not need to check that such aggregate supply is consistent with the aggregation of households' labor optimality conditions.

Finally, the government must ensure that the representative firm's choice of capital is consistent with the risk-free interest rate:

$$F_K(K_t, N_0) = (1 + \tau_t^k)\mu r_t^k = (1 + \tau_t^k)\mu(r_t + \delta).$$

The total government revenue (before transfers) of this tax policy is given by:

$$\begin{aligned}
\text{Revenue} &= \tau_t^n w_0 N_0 + \tau_t^k r_t^k K_t + \tau_t^\pi \hat{\Pi}_t \\
&= (1 + \tau_t^n) w_0 N_0 + (1 + \tau_t^k) r_t^k K_t - (1 - \tau_t^\pi)\hat{\Pi}_t - w_0 N_0 - r_t^k K_t + \hat{\Pi}_t \\
&= \frac{F_N(K_t, N_0) N_0 + F_K(K_t, N_0) K_t}{\mu} - \Pi_0 - w_0 N_0 - r_t^k K_t + \frac{(\mu - 1) F(K_t, N_0)}{\mu} \\
&= F(K_t, N_0) - \Pi_0 - w_0 N_0 - r_t^k K_t,
\end{aligned}$$

where the third line uses: $(1 - \tau_t^\pi)\hat{\Pi}_t = \Pi_0$; the firm's first-order condition for labor and capital; and $\hat{\Pi}_t = (\mu - 1)F/\mu$. The last line follows from Euler's theorem. We can then use $r^k = r + \delta$ and $\Pi_0 = F(K_0, N_0) - r_0^k K_0 - w_0 N_0$ to obtain:

$$\text{Revenue} = F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0. \tag{3}$$

A convenient feature of this result is that the costs of a policy are pinned down by the aggregate capital stock and the interest rate. No additional information is needed, despite the potentially complicated nature of policies necessary to keep all factor prices and profits weakly increasing.[20]

Note that if $K_t < K_0$, equation (3) implies that revenue is necessarily negative. To see this, strict concavity of F implies $F(K_0, N_0) - F(K_t, N_0) > F_K(K_0, N_0)(K_0 - K_t)$. From the firm's first-order condition in the laissez faire equilibrium, we have $F_K(K_0, N_0) = \mu(r_0 + \delta) \geq r_0 + \delta$, and hence (as $K_t < K_0$), $F(K_0, N_0) - F(K_t, N_0) > (r_0 + \delta)(K_0 - K_t)$. This implies that the value in (3) is strictly less than $(r_0 - r_t)K_t \leq 0$.

Equation (2) follows from equation (3) and the government's budget constraint. The right-hand side is the negative of equation (3). Bringing that to the other side, we have that bond issuances plus tax revenues minus lump-sum transfers must be non-negative. The inequality reflects that we allow the government to dispose of any surplus.

Finally, we verify that the new equilibrium is a Pareto improvement. By construction, household wage and profit income remain the same as in laissez faire in every t and idiosyncratic state.

[20] Part of this tractability rests on the representative firm assumption. This allows us to track how the marginal product of labor changes in response to a policy using just the knowledge of aggregates. If there were a distribution of firms with heterogeneous capital-labor ratios, we would need to track the entire distribution's response to policy in order to compute the labor subsidy necessary to keep wages constant.

The fact that the return to financial wealth weakly increases makes every saver at time t better off. However, those with negative positions (debt) are worse off. The fact that $T_t \geq -(r_t - r_0)\underline{a}$ ensures that lump-sum transfers are large enough to make debtors weakly better off, and strictly if $a_t^i > \underline{a}$. From the household's perspective, resources are weakly greater at every t and at every idiosyncratic state, and strictly greater for at least one household as there exists a t such that $r_t > r_0$ or $T_t > 0$. This establishes that the fiscal policy results in a Pareto improvement and concludes the proof of the proposition.

The following result provides a simpler sufficient condition for equation (2) to hold:

Claim 1. *If $F(K_t, N_0) - F(K_0, N_0) \geq r_t^k (K_t - K_0)$ and transfers are minimized $T_t = -(r_t - r_0)\underline{a}$, then a sufficient condition for (2) to hold is:*

$$B_{t+1} - (1 + r_t)B_t \geq (r_t - r_0)(K_0 - \underline{a}), \tag{4}$$

for all $t \geq 1$.

Proof. The premise and equation (3) implies that we have a lower bound on tax revenue:

$$\begin{aligned} \text{Revenue} &= F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 \\ &\geq (r_t + \delta)(K_t - K_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 \\ &= -(r_t - r_0)K_0. \end{aligned}$$

Substituting into (2) and setting $T_t = -(r_t - r_0)\underline{a}$ yields equation (4). □

The condition $F(K_t, N_0) - F(K_0, N_0) \geq r_t^k (K_t - K_0)$ holds immediately in two useful benchmarks. One is zero crowding out of capital, so that $K_t = K_0$. The second is when $F_K(K_t, N_0) = r_t^k$, that is, capital is undistorted relative to the risk free rate faced by households, and the inequality holds by concavity of F in K.

To provide intuition for Proposition 1 and Claim 1, and to more fully characterize the nature of the policies, we consider two alternative environments in turn. The first environment assumes the economy is *competitive* (so that $\mu = 1$). In the second environment, firms charge a markup, which may be large enough so that the economy is *dynamically efficient*. We will consider Pareto-improving policies tailored to each of these cases.

3.2 The Competitive Benchmark

Consider first the case where firms are competitive, $\mu = 1$, and profits are zero. The fact that $F_K = (r_0 + \delta) < \delta$ implies that the economy is dynamically inefficient. Suppose the government issues additional bonds. At a given level of capital, in order to induce households to hold more assets, the equilibrium interest rate must increase $r_t \geq r_0$, assuming that aggregate savings is increasing in r.[21] The higher r induces additional savings and crowds out capital. This is beneficial at the margin given the dynamic inefficiency. This implies $\tau_t^n < 0$, i.e., labor is subsidized, and represents the primary fiscal cost of the policy.

Suppose the policy sets $\tau^k = 0$, so that capital is undistorted relative to the risk-free rate. We consider a broader set of policies after providing some intuition for this case. From Claim 1, the cost of the labor subsidy is bounded by $(r_t^k - r_0^k)K_0 = (r_t - r_0)K_0$, which is the change in payments to capital. If this is large, then the share of revenue paid by firms to labor falls significantly and the subsidy to labor must be large ($\tau^n << 0$). Another interpretation is obtained by letting $w = \phi(r)$ denote the factor price frontier in the competitive laissez faire equilibrium (so that w and r correspond to the associated factor marginal products). We have $dw/dr = -K$, and so the change in wages is approximately $\Delta w \approx -K_0 \Delta r$. This is the amount the government must make up through subsidy.

Let ss denote the new stationary equilibrium. In the limit as $t \longrightarrow \infty$, the feasibility condition becomes:

$$-r_{ss}B_{ss} \geq (r_{ss} - r_0)(K_0 - \underline{a}). \tag{5}$$

The "seigniorage" revenue ($r < 0$) from bonds must be large enough to offset the movement along the factor price frontier as well the compensation to borrowers.

Figure 1 depicts the trade-off in the canonical capital market equilibrium diagram from Aiyagari (1994). The underlying calibration is provided in Section 4, but the qualitative features are fairly general. At each interest rate on the vertical axis r, the associated rental rate of capital is $r^k = r + \delta$. Holding labor supply $N = N_0$ constant, the downward sloping red line traces out a capital demand equation from the firm's first-order condition $F_K(K, N_0) = r^k$. This is the relevant capital demand curve for the case of $\tau_t^k = 0$.

Similarly, at each candidate r, A denotes the aggregate steady-state saving of household when the wage is fixed at w_0. These two curves intersect at the laisssez faire equilibrium interest rate r_0. Note that in this parameterization, $r_0 < 0$, which is the case of interest. The quantities reflected on the horizontal axis are normalized by $Y_0 = F(K_0, N_0)$.

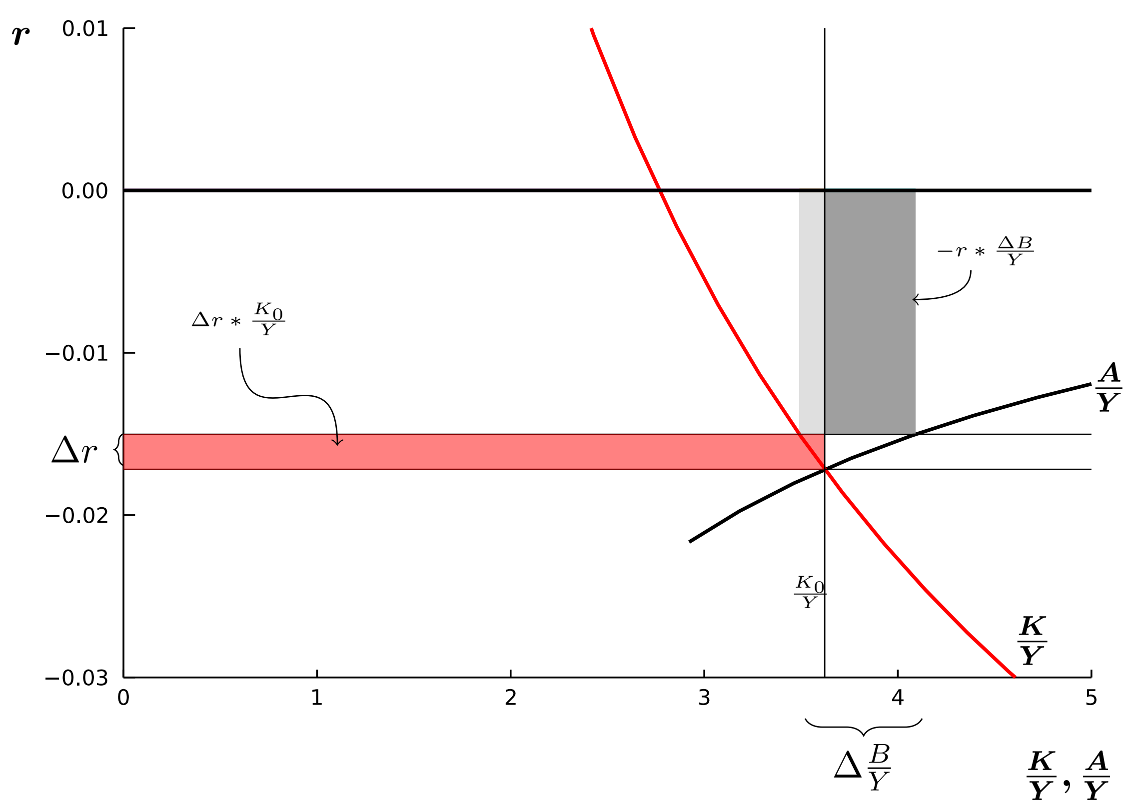
The distance between the capital demand equation and the household saving line is the amount of government bonds as a share of laissez faire output.[22]

[21] More precisely, the interest rate must increase at some point along the path and converge to $\lim_{t\to\infty} r_t > r_0$. To rule out alternative cases which trade-off lower interest rates along the transition against higher long-run rates, we include $r_t \geq r_0$ as a condition for all t in Proposition 1.

[22] At each point on the household saving line, there is an associated lump-sum transfer that satisfies the government's budget constraint. At each r and implied $B = S - K$, the upward sloping line solves the household's problem for the associated transfer.

In this example, $\underline{a} = 0$. The width of the gray rectangle is $\Delta B/Y_0 = \frac{A_{ss}-K_{ss}}{Y_0}$ and its height is the equilibrium interest rate at the new equilibrium, hence its area is $-r_{ss}\Delta B/Y_0$. The red rectangle has height $r_{ss} - r_0$, where r_0 is the interest rate in the laissez faire equilibrium. Its width is K_0/Y_0, where K_0 is the capital stock in the laissez faire equilibrium. The area of this rectangle is $(r_{ss} - r_0) * K_0/Y_0$. From equation (5), if the area of the gray rectangle exceeds that of the red, then a Pareto improvement is feasible.

Figure 1: Fiscal Tradeoff in the Steady State



Note: This figure is a graphical depiction of the fiscal tradeoff from equation (5). All elements are normalized by the laissez faire stationary equilibrium output $Y = Y_0$. The downward sloping line K/Y represents firm demand from capital ($r = F_K - \delta$) and the upward sloping line A/Y depicts aggregate household saving associated with the interest rate r and the laissez faire wage as well as the transfers generated by any fiscal surplus. The intersection is the initial laissez faire stationary equilibrium. Fiscal costs are represented by $\Delta r * K_0/Y$, the area shaded in red, and seigniorage revenue by $-r * \Delta B/Y$, the area shaded in gray. The part of seigniorage generated by displacing capital is to the left of the vertical line K_0/Y and is shaded a lighter color. The remaining seigniorage represents additional household saving.

The inequality (5) is likely to be satisfied if consumers are willing to hold new debt without a sharp increase in the interest rate. That is, if the response of savings to r is large. The intuition is that the return to capital cannot increase significantly for a small issuance of B, as the increase in the return to capital is the amount of subsidy necessary to keep the household's labor earnings constant.

The elasticity of the interest rate to government debt is a primary concern when discussing the "crowding out" of capital. Here it is playing an additional role. Namely, even if capital were inelastic, this elasticity matters because it speaks to the fiscal impact of debt issuance on the subsidy to wages.

On the capital demand side, from the firm's first order condition we have $\partial K/\partial r = -1/F_{kk}$. If F_{kk} is small in magnitude, then capital demand is very elastic and crowding out of capital will be significant. This actually makes the feasibility condition easier to achieve. The intuition is the same as in Diamond (1965). As $r < 0$ and $F_k = r + \delta$, the return to capital net of depreciation is negative. As aggregate consumption in the stationary equilibrium equals $F(K, N) - \delta K$, a reduction in capital actually increases the resources available for consumption. This part of calculation will change below when we consider a dynamically efficient equilibrium.

Note that there may be a broader set of fiscal policies that are feasible. Namely, those that increase the crowding out of capital to make "room" for additional debt issuance at a given interest rate. This can be implemented by a positive τ^k that depresses K_t given $r^k = r + \delta$. While the additional seigniorage and revenue from $\tau^k > 0$ relax the fiscal constraints (and may bring the economy closer to the golden rule), the lower capital stock requires a larger wage subsidy. There is thus a limit to feasibility of this additional crowding out.

However, it is straightforward to show that if the economy is dynamically inefficient, the government can always generate additional resources by crowding out capital to the golden rule level K^* (that is, the value that solves $F_K(K^*, N_0) = \delta$):

Claim 2. *Consider the case where the laissez-faire stationary economy is dynamically inefficient (that is, $r_0 < 0$). Then the sequence $\{r_t, T_t, B_t, K_t\}$ with $r_t = r_0$, $T_t = 0$, $K_t = K^*$ and $B_t = K_0 - K^* > 0$ for all $t \geq 1$ and $T_0 = 0$ is admissible. In addition, such sequence satisfies the conditions (i), (ii) and (iii) of Proposition 1. And in particular, the inequality (2) is strict for all $t \geq 0$; that is, the government raises strictly positive revenue in all periods without affecting any household's utility.*

Proof. That the sequence is admissible follows directly from the fact that $\{r_t, T_t\}$ remain as in the stationary laissez-faire equilibrium, and thus $A_t = K_0$ for all $t \geq 0$; $B_t + K_t = A_t = K_0$; and B_t is bounded.

Conditions (i) and (ii) in Proposition 1 are satisfied. From the fiscal resource condition (2), we have for $t = 0$ the government raises $B \equiv K_0 - K^*$ in additional resources by issuing bonds. As K_0 is fixed, there are no fiscal costs in the initial period. For $t > 0$ the fiscal resource condition (2) is:

$$
\begin{aligned}
&B - (1 + r_0)B + F(K^*, N_0) - F(K_0, N_0) - (r_0 + \delta)K^* + (r_0 + \delta)K_0 \\
&= F(K^*, N_0) - F(K_0, N_0) - \delta K^* - r_0(K^* + B) + \delta K_0 + r_0 K_0 \\
&= F(K^*, N_0) - \delta K^* - (F(K_0, N_0) - \delta K_0),
\end{aligned}
$$

where we use $K^* + B = K_0$ for the last equality. By definition of the golden rule capital stock, K^* maximizes $F(K, N) - \delta K$. Hence, the policy generates strictly positive resources for all $t \geq 0$. Such policy does not affect any household's utility as all prices and incomes remain unchanged. □

In terms of Figure 1, distorting capital at a given r increases the width of the gray rectangle without increase the cost. The claim states the government can do this at the initial r_0, generating zero net costs. The fact that this generates a surplus suggests that there is scope to raise welfare by providing a public good, as long as the public good does not alter the household's saving or labor-supply choices. The result extends the classic Samuelson-Diamond result to the Aiyagari framework. As we will see, dynamic inefficiency is not a necessary condition for Pareto-improving fiscal policies once we re-introduce mark-ups. However, the next result states that it is necessary for the competitive case, at least for the class of policies considered by Proposition 1.

Given the incompleteness of markets, a natural question is whether a Pareto-improving fiscal policy is feasible even if $r_0 > 0$. In the competitive benchmark, we can state that there are no admissible sequences that satisfy Proposition 1 if the interest rate is positive in the steady state:

Claim 3. *Consider $\mu = 1$. If $r_0 > 0$, then there is no admissible sequence that satisfies the conditions of Proposition 1.*

Proof. Condition (iii) in Proposition 1 requires

$$B_{t+1} - (1 + r_t)B_t + F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 - T_t \geq 0.$$

If $\mu = 1$, we have $F_K(K_0, N_0) = (r_0 + \delta)$. Concavity of F implies

$$F(K_0, N_0) - F(K_t, N_0) \geq (r_0 + \delta)(K_0 - K_t).$$

Thus a necessary condition for condition (iii) is:

$$B_{t+1} - (1 + r_t)B_t + (r_0 - r_t)K_t - T_t \geq 0.$$

Re-arranging:

$$B_{t+1} - B_t \geq r_t B_t + (r_t - r_0)K_t + T_t.$$

If $r_t > r_0$ or $T_t > 0$ for some t, the right-hand side is strictly positive at t even if $B_t = 0$, implying a strictly positive increase in debt. After this period, the right-hand side is always strictly positive as $r_t \geq r_0 > 0$ and $T_t \geq 0$, generating an explosive path of debt, violating the upper bound on debt required for an admissible sequence. □

3.3 Mark-ups

We now turn to the case in which mark-ups drive a wedge between the marginal product of capital and the return on bonds. The presence of a mark-up per se is not important, only the extent to which the economy my be dynamically inefficient. In particular, Claim 2 holds even if $\mu > 1$.

To provide some intuition of how a mark-up affects the analysis, consider the same exercise as in the previous sub-section. Specifically, starting from a laissez faire stationary equilibrium, the government embarks on a new sequence of fiscal policies that is a component of an admissible sequence. Again, we start with the case of $\tau^k = 0$, so that $F_K = \mu(r_t + \delta)$. The complication relative to the competitive case is that with a mark-up, a Pareto improvement requires not only weak increases in factor prices but also a weak increase in profits. That is, $\tau_t^\pi \leq 0$ so that $(1-\tau_t^\pi)\hat{\Pi}_t = \Pi_0$.

Using the fact that $\hat{\Pi}_t = \Pi_0/(1-\tau_t^\pi)$ and $\tau^k = 0$, equation (3) implies:

$$\tau_t^n w_0 N_0 + \frac{\tau_t^\pi}{1-\tau_t^\pi}\Pi_0 = F(K_t, N_0) - F(K_0, N_0) - r_t^k K_t + r_0^k K_0,$$

Concavity implies $F(K_0, N_0) \leq F(K_t, N_0) + F_K(K_t, N_0)(K_0 - K_t)$. Using the fact that $F_K(K_t, N_0) = \mu r_t^k$, we have:

$$\tau_t^n w_0 N_0 + \frac{\tau_t^\pi}{1-\tau_t^\pi}\Pi_0 \geq (\mu - 1) r_t^k (K_t - K_0) - (r_t - r_0) K_0.$$

The first term on the right-hand side is due to the mark-up. As $K_t < K_0$ it adds to the fiscal burden. The fall in K leads to a fall in profits, which must be offset by subsidies in order to ensure a Pareto improvement. Here, it makes the improvement harder to achieve if capital is elastic.

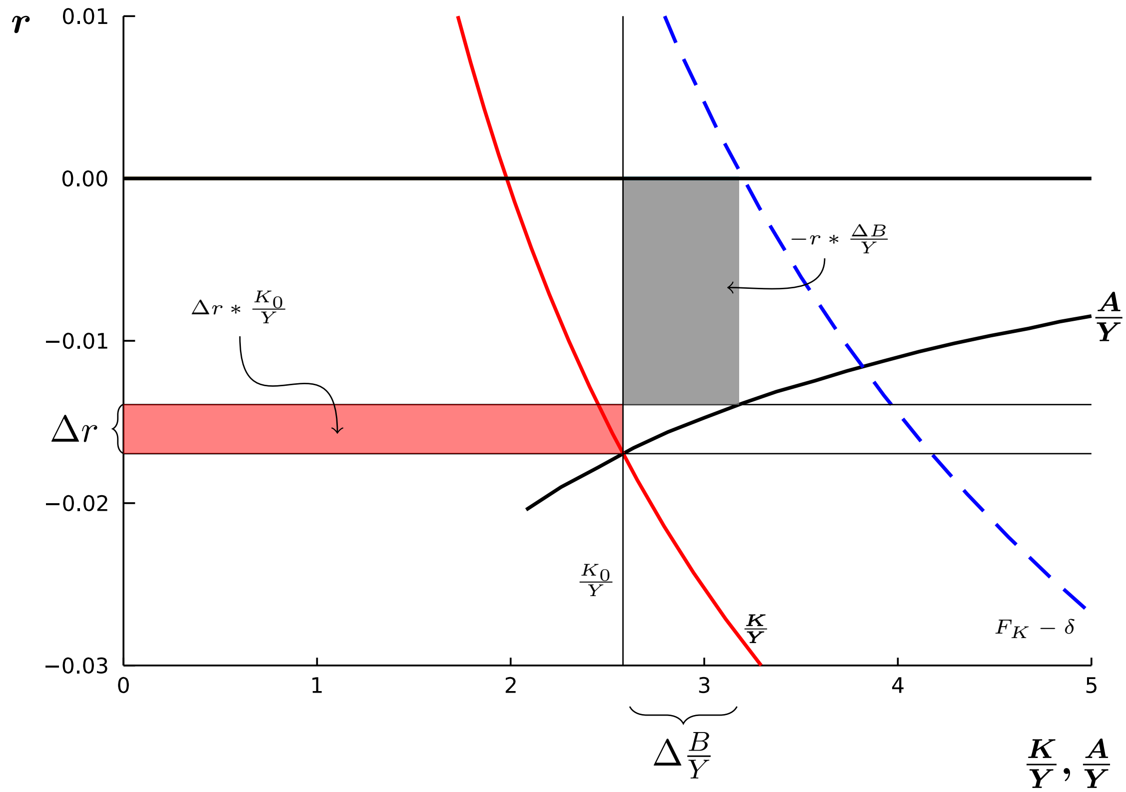
The counter-part to condition (4) is now

$$B_{t+1} - (1 + r_t)B_t \geq -(\mu - 1) r_t^k (K_t - K_0) + (r_t - r_0)(K_0 - \underline{a}).$$

For the steady-state comparison of Figure 1, we can think of losing part of the gray rectangle over the ΔK part of the horizontal axis if $\mu > 1$. The mark-up implies some of the gray area is used to compensate the decline in profits.

As in the competitive case, dynamic inefficiency implies a return to crowding out capital. Thus, more revenue can be generated by setting $\tau^k > 0$. In particular, Claim 2 required only dynamic inefficiency, not $\mu = 1$.

Figure 2: Net Resource Cost with Markups



Note: This figure reproduces the asset demand and supply curves from Figure 1 for the case with a mark-up. The dashed downward sloping line represents the marginal product of capital minus depreciation. The capital demand curve K/Y is distorted relative to this benchmark by the markup. In this example, policy holds capital at the initial laissez faire capital sock.

Now suppose μ is large enough that $\mu(r + \delta) = F_K > \delta$, despite the fact that $r < 0$. This is the case depicted in Figure 2. Relative to Figure 1, Figure 2 draws a distinction between capital demanded by firms at each r and the marginal product of capital. That is, demand is the solid downward sloping schedule defined by the firm's first-order condition, $r = F_K(K, N_0)/\mu - \delta$, while the dashed line is $r = F_K - \delta$. At the depicted laissez faire equilibrium, $F_K - \delta > 0$, and hence the economy is dynamically efficient. Crowding out of capital is costly, as resources available for consumption decline with a decrease in K.

In the case of dynamic efficiency, a potentially cheaper path to a Pareto improvement utilizes fiscal policies that avoid crowding out capital. Specifically, suppose the government *subsidizes* the return from renting capital. That is, let r_0^k be the rental rate in the laissez faire equilibrium, with $r_0 = r_0^k - \delta$. Let r_t be the net interest rate on government bonds in the new equilibrium at time t. Let $\tau_t^k < 0$ be a subsidy to capital such that firms pay $r_0^k = (1 + \tau_t^k)(r_t + \delta)$, and households receive $r_t \geq r_0$. As firms are paying the same after-tax rental rate, then $K_t = K_0$, and hence profits, wages, and total output remain unchanged.

From equation (3), the cost of this policy is $-(r_t - r_0)K_0$. The amount of seigniorage need is thus given by equation (4). However, relative to the previous cases in which capital was crowded out, r_t will need to be greater for a given quantity of debt. In terms of Figure 2, the trade off is now the horizontal region $\Delta r * K_0$ relative to the gray region $r * \Delta A = r * \Delta B$, and ΔK no longer plays a role in the calculus.

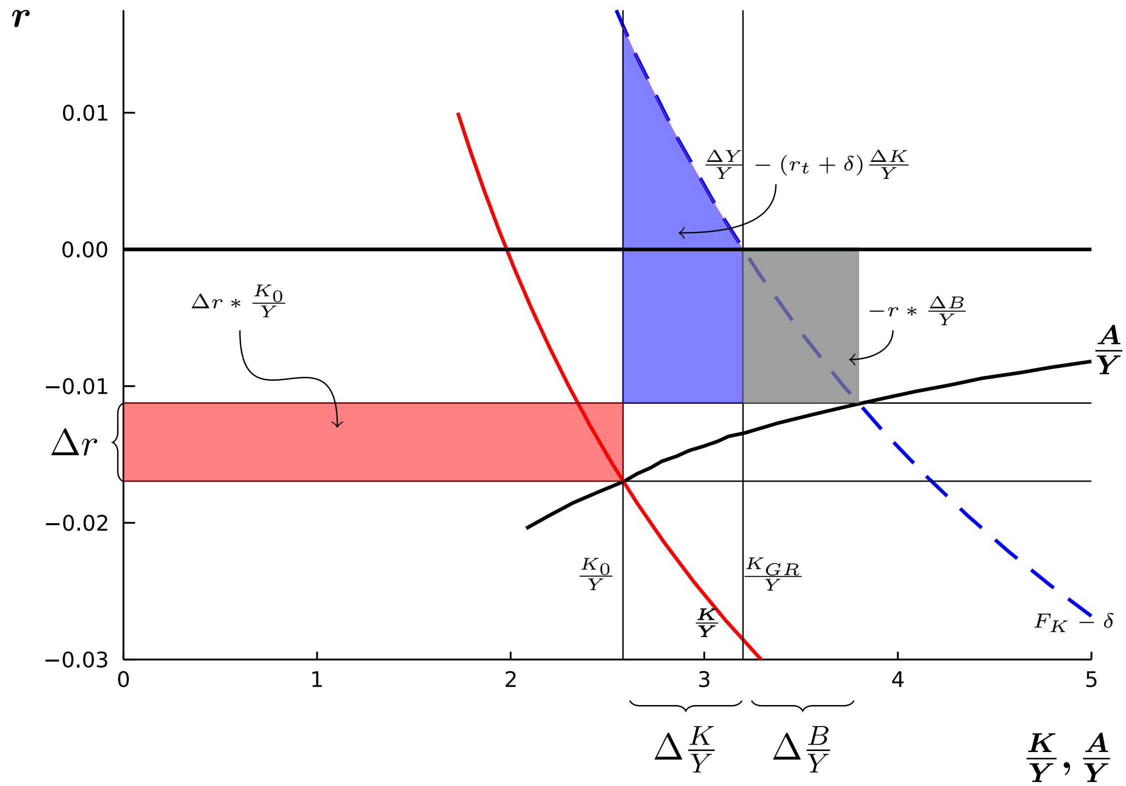
Given that the mark-up depresses the level of capital in the laissez faire equilibrium relative to the first-best, there may be scope for "crowding in" of capital. In particular, rather than using government debt to increase the supply of safe assets, the government may also find it beneficial to induce additional investment. It can do so by subsidizing capital to the point that $(1+\tau^k)(r_t+\delta) < r_0 + \delta$. The additional capital raises pre-tax profits and wages. This begs the question of whether the government can tax these latter factors, keeping the after-tax return constant, and generate sufficient revenue to subsidize capital.

In particular, suppose we take the extreme of zero revenue from seigniorage, so $B_t = 0$ for all t. Instead, the government implements policy to increase the capital stock, $K_t > K_0$. For simplicity of exposition, let $\underline{a} = 0$ and $T_t = 0$, so that any excess revenue is discarded.

For this crowding-in policy to be an equilibrium, households must be willing to hold additional wealth. That is, it must be part of an admissible sequence $\{r_t, T_t = 0, B_t = 0, K_t\}_{t \geq 1}$. Recall that admissible sequences are generated by the household's problem, and reflect the mapping from sequences $\{r_t, T_t\}$ to aggregate wealth, A_t, given (w_0, Π_0). The experiment sets $A_t = K_t = A_0 + \Delta K_t$ for $t \geq 1$, where $\Delta K_t \equiv K_t - K_0$.

Associated with the admissible sequence are taxes such that $F_K(K_t, N_0) = \mu(1 + \tau_t^k)(r_t + \delta)$, $F_N(K_t, N_0) = (1 + \tau_t^n)w_0$, and $(1 - \tau_t^\pi)F(K_t, N_0) = F(K_0, N_0)$, with $K_t = K_0 + \Delta K$. From the proof of

Figure 3: Net Resource Cost with Crowding In



Note: This figure reproduces the asset demand and supply curves from Figure 2 for the case in which policy *crowds in* capital. The dashed downward sloping line represents the marginal product of capital minus depreciation. The capital demand curve K/Y is distorted relative to this benchmark by the markup. In this example, policy crowds in capital to the “golden rule” level K_{GR}. The gain from crowding in of capital is the area between $F_K - \delta$ and the new steady state r.

Proposition 1, tax revenues are:

$$F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0.$$

Given that $B_t = T_t = 0$ in this experiment, feasibility requires this expression must be weakly greater than zero. A positive mark-up implies that $F_K(K_0, N_0) > (r_0 + \delta)K_0$, so for small ΔK, resources are increasing in K_t *for a given interest rate*. For a small increase $\Delta K \approx 0$, revenues are approximately

$$F_K(K_0, N_0)\Delta K_t - (r_0 + \delta)\Delta K_t - \Delta r_t K_0,$$

where $\Delta r_t \equiv r_t - r_0$ and we set second-order terms to zero.[23] The representative firm's first order condition in the laissez faire equilibrium implies $F_K(K_0, N_0) = \mu(r_0 + \delta)$. Thus, to a first order and assuming $\Delta r_t > 0$, a sufficient condition for revenue to be weakly positive is

$$\xi_0 \geq \frac{1}{\mu - 1}. \tag{6}$$

where $\xi_0 \equiv \frac{\Delta K_t}{\Delta r_t}\left(\frac{r_0+\delta}{K_0}\right)$.

The right-hand side of (6) is the inverse mark-up, and the left-hand side is the elasticity of aggregate wealth to an increase in the gross return to capital, using the fact that $K_t = A_t$ in this experiment.[24] This condition is easier to satisfy if the distortion of capital due to market power is large and if the elasticity of wealth to the interest rate is large. For such a combination, to a first order crowding in of capital is a feasible Pareto improvement.

Crowding in of capital allows the economy to (eventually) produce more efficiently, but foregoes the revenue generated by issuing bonds along the path. In the initial period, issuing bonds generates more resources than devoting extra saving to capital, but in the steady state, the policy of crowding in capital generates greater fiscal resources.

Of course, policy does not have to be only debt or only crowding-in of capital. In fact, there is a theoretical case for a combination of both. As noted, equation (6) is easier to satisfy the greater the elasticity of private saving to the interest rate. In many settings, the long-run elasticity is greater than short-run elasticity, a property Paul Samuelson linked to the LeChatelier principle original developed in chemistry. This property holds in the quantitative Aiyagari model explored in Section 4. In the current context of inducing greater investment, this potentially implies an

[23] Note that we impose that Δr_t is of the same order as ΔK_t, which implicitly assumes a continuous mapping from the sequence of interest rates to the sequence of aggregate wealth.

[24] Keep in mind that aggregate wealth at time t is a function of the entire sequence of interest rates. In this sense, we use the term "elasticity" for the ratio $(\Delta K_t/\Delta r_t) * ((r_0 + \delta)/K_0)$ loosely.

over-shooting of the interest rate along the transition. A higher interest rate raises the fiscal cost of capital subsidies. If the short-run elasticity is too small to satisfy (6), the government can issue debt along the transition to finance the associated crowding-in fiscal policy. In the new stationary equilibrium, the additional resources can then be used to service (if $r_\infty > 0$) or pay down the debt. Thus, along the transition, debt and additional capital become *complements* rather than substitutes in engineering a Pareto improvement. We give an example of such a hybrid policy in Section 4.

A graphical depiction of the steady state after such a hybrid policy is given in Figure 3. The novelty is the area between the curve labelled $F_K - \delta$ and the new long-run interest. In particular, recall that fiscal revenue before transfers in the new steady state is given by

$$F(K_\infty, N_0) - F(K_0, N_0) - (r_\infty + \delta)K_\infty + (r_0 + \delta)K_0$$
$$= \int_{K_0}^{K_\infty} (F_K - \delta - r_\infty)dK - \Delta r * K_0.$$

In Figure 3, the shaded area under the marginal product curve and above the new steady-state r represents the first term, while the rectangle $\Delta r * K_0$ represents the final term. The additional shaded rectangle labelled $-r\Delta B$ represents seigniorage from debt issuance. Again, this depiction refers to the stationary equilibrium and ignores the transition. Nevertheless, it graphically highlights the fiscal resources gained by crowding in capital when $F_K - \delta > r$ due to a markup distortion.

3.4 The Elasticity of Aggregate Savings

The robust conclusion from the above is that a Pareto improvement is facilitated by a very elastic aggregate savings function with respect to the interest rate. Feasibility turns on this key statistic. Unfortunately, there is little clear cut empirical or theoretical guidance on the magnitude of this elasticity.

Testing the sensitivity of interest rates to changes in government debt or deficits was an active area of empirical research in the 1980s and 1990s.[25] Perhaps surprisingly, there are a number of empirical studies that conclude the Ricardian equivalence benchmark of no change in the interest rate is a reasonable description of the data. Nevertheless, there are other empirical estimates that conclude otherwise, and our reading of this literature is that there is no clear consensus.

In the Bewley-Huggett-Aiyagari literature there are a few theoretical results. For example, for the case of CRRA utility, Benhabib, Bisin and Zhu (2015) show that as $a \to \infty$, the household saving function's sensitivity to the risk-free interest rate is increasing in the IES. A similar result

[25] See the surveys and associated references of Barth, Iden and Russek (1984); Bernheim (1987b); Barro (1989); Elmendorf and Gregory Mankiw (1999); Gale and Orszag (2003); and Engen and Hubbard (2005).

is proved by Achdou et al. (2021). Thus the derivative with respect to r is governed by the intertemporal elasticity of substitution (IES), with a larger IES indicates a more elastic response, at least for the very wealthy.

At the other end of the asset domain, Achdou et al. (2021) shows that, for those at the lowest income realization and approaching the borrowing constraint, the sensitivity of savings to r also depends positively on the IES.

These results pertain to individual savings behavior at the extremes of the asset distribution. To explore the elasticity of *aggregate* saving to the interest rate in a stationary equilibrium, and how this elasticity varies with preference parameters, we turn to the a calibrated version of the model in the next section. The quantitative model can also speak to whether the sufficient conditions for a Pareto improving policy are satisfied in a plausible calibration. This is the topic of the Section 4. Before turning to simulations, we conclude the analytical discussion with an instructive benchmark – the representative agent neoclassical growth model.

3.5 The Complete Markets Benchmark

The complete-markets, representative agent benchmark is a useful environment to shed light on three key facets of the above analysis. The first is the role of the elasticity of aggregate savings. In the complete markets case, the long-run elasticity is infinite, and transition dynamics form the household side are pinned down by the consumer's Euler equation. The second facet is the role of debt along the transition. This is not independent of the first, as the finite short-run elasticity of savings implies the interest rate overshoots its long-run level, and the government can use debt along the transition to smooth the costs associated with the temporarily high interest rate. The third facet is that the mark up on its own, independent of risk sharing considerations or $r < 0$, opens the door to Pareto-improving fiscal policy, despite the fact that after-tax profits remain bounded below by the laissez faire equilibrium.

Note that the representative agent benchmark is nested in our notation. The technology side is the same as in the neoclassical growth model, but augmented with a constant mark-up. Given the Ricardian structure of the neoclassical model, we can set transfers to zero without loss of generality. Thus, the household budget constraint and national income accounting imply "admissible sequences" satisfy the same resource conditions:

$$\begin{aligned} C_t &= F(K_t, N_0) + (1-\delta)K_t - K_{t+1} \\ &= w_0 N_0 + \Pi_0 + (1+r_t)A_t - A_{t+1}, \end{aligned}$$

where $A_t = K_t + B_t$. Household saving behavior is characterized by the representative agent's

Euler equation

$$u_c(C_t, N_0) = \beta(1 + r_t)u_c(C_{t+1}, N_0),$$

where we assume the case of time-separable preferences. This condition is the restriction imposed by complete markets. Thus, admissible sequences are those that satisfy the resource condition, the Euler equation, and the upper bound on government debt $\overline{B}$.

To make the analysis as transparent as possible, we consider a very simple policy. At time 0, starting from the laissez faire steady state, the government induces a small, permanent increase in the capital stock, $K_t = K_1 > K_0$. We also assume the government does not issue government bonds, $B_1 = 0$, implying $A_1 = K_1 > K_0 = A_0$.For this to be consistent with household optimization, we require $r_1 > 1/\beta - 1 = r_0$. We assume for $t \geq 2$, the economy is in steady state. Thus, for $t \neq 1$, $r_t = 1/\beta - 1 = r_0$.

Working backwards from $t = 2$, the fact that $r_2 = 1/\beta - 1$ implies that $C_1 = C_2$ due to the Euler equation. The household's budget constraint implies:

$$C_1 = w_0 N_0 + \Pi_0 + (1 + r_1)A_1 - A_2$$
$$C_2 = w_0 N_0 + \Pi_0 + r_0 A_2,$$

where the second line uses the fact we are in steady state for $t \geq 2$. These expressions plus the fact that $C_1 = C_2$ and $r_1 > r_0$ implies that $A_2 > A_1$. As $K_1 = K_2$, we have $B_2 = A_2 - K_1 = A_2 - A_0 > 0$. Thus, the government issues debt in period $t = 1$, and then rolls it over indefinitely.

To see the role of debt from the household's perspective, note that the interest rate is falling between periods $t = 1$ and $t = 2$. Consumption smoothing induces the household to save some of the temporarily high capital income, and the government accommodates this by issuing debt. Note that the government is issuing debt at r_0, taking advantage of the infinite elasticity of savings in the steady state.

From the household's budget constraints, plus $A_2 = B_2 + A_1 = B_2 + K_1$, we can solve out

$$B_2 = \left(\frac{r_1 - r_0}{1 + r_0}\right) K_1.$$

Thus, the larger is r_1, the more the government issues debt.

The size of r_1 depends on the amount of investment being induced, $K_1 - K_0$, as well as the inter-temporal elasticity of substitution. Let $r_1 + \delta = \boldsymbol{R}(K_1)$ define the mapping from K_1 to $r_1 + \delta$, conditional on preferences.[26] The short-run response of the interest rate to the government

[26] $\boldsymbol{R}$ is defined by the Euler equation and budget constraint in periods 0. In particular, $C_0 = w_0 N_0 + \Pi_0 + (1 + r_0)K_0 - K_1$. $C_1 = C_2 = w_0 N_0 + \Pi_0 + r_0(K_1 + B_2))$, where B_2 is defined in the text as a function of r_1 and K_1. Then $r_1 = \boldsymbol{R}(K_1)$ solves

policy is given by $\boldsymbol{R}'(K)$. Let $\xi_0^{CM} \equiv \boldsymbol{R}(K_0)/(\boldsymbol{R}'(K_0)K_0)$ represent the (short-run) elasticity of the aggregate asset supply in this complete markets case.

We can now state:

Claim 4. *A Pareto improvement is feasible if*

$$\xi_0^{CM} \geq r_0 \frac{1}{\mu - 1}$$

Proof. □

This condition contains many of the characteristics of the incomplete markets case. In particular, a large mark-up creates room for a Pareto improvement. In addition, the smaller the (short-run) increase in interest rate, the more easily the condition is satisfied. However, the markup term is now scaled by r_0, which reflects the advantage offered the government by an infinite long-run elasticity. That is, the ability to issue debt in the transition at r_0 is unique to the complete markets example, and does not hold in the incomplete markets case.

4 Simulations

In this section we present simulation results for various policy experiments. Our benchmark focuses on the dynamically efficient economy with mark-ups. In this setting, we explore policies that leave capital unchanged as well as those that crowd in additional capital. For contrast, we also briefly present a competitive economy that is dynamically inefficient.

The simulated economies allow us to assess the scope for Pareto-improving fiscal policy in a calibrated quantitative model as well as compute transition dynamics and welfare implications. The quantitative experiments will also underscore how government debt is used in implementing Pareto-improving policies.

4.1 Parameter Settings

The utility function we consider for households is of the Epstein-Zin form

$$V_{it} = \{(1-\beta)x_{it}^{1-\xi} + \beta(\mathbb{E}_z V_{it+1}^{1-\gamma})^{\frac{1-\xi}{1-\gamma}}\}^{\frac{1}{1-\xi}}$$

$u_c(C_0, N_0) = \beta(1 + r_1)u_c(C_2, N_0)$.

where β is the discount factor, $1/\xi$ is the elasticity of intertemporal substitution, γ is the risk aversion coefficient, and x is the composite of consumption and labor

$$x_{it} = c_{it} - n_{it}^{1/\nu}.$$

The parameter ν controls the Frisch elasticity of the labor supply. We set some of the preference parameters to conventional values in the literature and others as part of the calibration. The elasticities of substitution and of labor supply are set to the common parameters values of 1 and 0.2, respectively. The discount factor and coefficient of risk aversion are set as part of the calibration exercise described below. We set the borrowing constraint to zero for all households.

An important part of the parametrization is the stochastic structure for idiosyncratic shocks. We adopt the structure and estimates from Krueger et al. (2016) that use micro data on after tax labor earnings from the PSID. Idiosyncratic productivity shocks z_{it} contain a persistent and transitory component and their process is as follows

$$\log z_{it} = \tilde{z}_{it} + \varepsilon_{it}$$
$$\tilde{z}_{it} = \rho^z \tilde{z}_{it-1} + \eta_{it}$$

with persistence ρ^z and innovations of the persistent and transitory shocks (ε, η), with associated variances given by $(\sigma_\varepsilon^2, \sigma_\eta^2)$. We set the three parameters controlling this process $(\rho^z, \sigma_\varepsilon^2, \sigma_\eta^2)$ to .9695, .0384, and .0522 respectively to reflect the estimated earnings risk in Krueger et al. (2016) for employed individuals. We discretize this process into 10 points based on Tauchen (1986).

We take a parsimonious approach to allocating profits to households. In particular, we assume a distinct class of entrepreneurs that are endowed with managerial talent and consume profit distributions in a hand-to-mouth manner. While stark, this approach offers a number of advantages. First, it approximates that a significant share of entrepreneurial rents accrue to a small share of the population. Second, under this assumption, they do not affect factor prices, and so we can solve the economy without taking a stand on the idiosyncratic details of the entrepreneurial class. Finally, and related to the previous point, the analysis is invariant to the extent to which profits are offset by fixed costs versus representing pure rents.

The technology specification is Cobb-Douglas, $F(K, N) = K^\alpha N^{1-\alpha}$. We use standard values for the coefficient α and for the depreciation rate of capital δ. The values are $\alpha = 0.3$ and $\delta = 0.1$. The mark-up parameter μ is set to 1.4, which is within the estimates in Basu (2019).[27]

We calibrate the discount factor and the coefficient of relative risk aversion by targeting a

[27] As noted above, some of this mark-up may represent fixed costs. The aggregate mark-up may also reflect smaller mark-ups at different stages of production in a vertical supply chain, as in Ball and Mankiw (2021). In fact, 1.4 is close to the number they use in their numerical exercises.

capital-output ratio of 2.5, based on Aiyagari and McGrattan (1998) and Krueger et al. (2016), and an interest rate of -1.4%, which is the difference between the average one year treasury rates and average nominal GDP growth in the United States since 1962.[28] While our focus is on Pareto-improving policies relative to a laissez faire benchmark, the empirical moments are generated from an economy with government debt. Hence, we simulate a stationary economy with a debt-to-output ratio of 60%, which is the average value in the US since 1966, and choose preference parameters to match moments from this economy to the data. The resulting values are $\{\beta = 0.993, \theta = 5.5\}$.

We treat the economy with debt as being generated from a Pareto-improving fiscal policy starting from the laissez faire benchmark. That is, in simulating the economy with government debt during moment matching, we assume that tax policy is such that after-tax wages and capital are the same as if the economy had zero debt and zero taxes. We refer to this set of policies as our *baseline fiscal policy*.

Table 1: Baseline and Laissez Faire Economies

	Data	Baseline Policy	Laissez Faire
Aggregates			
Public Debt (% output)	60	60	0
Interest Rates(%)	-1.4	-1.4	-1.7
Capital (rel. output)	2.5	2.6	2.6
Wealth Distribution			
Q1 Wealth Share	-1	1	1
Q2 Wealth Share	1	4	4
Q3 Wealth Share	4	11	10
Q4 Wealth Share	13	24	23
Q5 Wealth Share	83	61	63

Table 1 presents some moments in the steady states of our economy with baseline fiscal policy and the laissez faire economy. The level of public debt, interest rates, and capital in the economy with the baseline fiscal policy matches the data moments by construction.[29] The table shows that an increase in debt to output of 60% increases interest rates by 0.3%. We also present some moments on the wealth distribution in the steady states, namely the wealth share of each asset quintile, and compare it with data as reported in Krueger et al. (2016). Our model economies generate skewed distributions of wealth, with a most of the wealth being held by the top quintile of the distribution, although not quite as skewed as the data. Also in our model economies, a

[28]This estimate is consistent with the ones in Blanchard (2019) and Mehrotra and Sergeyev (2020).

[29]The economy is dynamically efficient also by construction. To see this, $F_K = \alpha Y/K = 0.3/2.5 = 0.12$, which is greater than the depreciation rate of 0.10.

small fraction of agents are at their borrowing constraint at any period, about 2%.

4.2 Transitions under Pareto-Improving Fiscal Policy

We now describe the transition as the government implements its Pareto-improving fiscal policy. The economy starts in the laissez-faire steady state and transitions to the steady state with fiscal policy. We perform two policy experiments, one in which capital is held constant and one in which capital eventually reaches the Golden Rule level. Both of these experiment are undertaken in our benchmark dynamically efficient economy. We postpone discussion of an economy without mark-ups to the final section.

The two experiments confirm that Pareto-improving fiscal policies are feasible in a calibrated model. In particular, the calibrated aggregate savings schedule is sufficiently elastic to allow the government to increase both debt and capital without resorting to lump sum taxation.

4.2.1 Baseline Fiscal Policy

We start with a policy plan that transitions from a laissez faire stationary equilibrium to a new steady state with the baseline fiscal policy. In particular, the government takes the economy from zero debt to a level of 60% of output, while keeping after-tax wages and profits constant. Our posited path of public debt is depicted at the top left panel of Figure 4; debt increases monotonically until it reaches its steady state level of 60% of output. Also be construction, capital is held fixed at the laissez faire level, as depicted in the top middle panel of Figure 4. Given the policy of constant capital and wages, output and consumption (reported in the lower middle panel) are not changing. This will be different in the subsequent experiment with capital crowding in.

Given this path of debt and capital, we solve for the equilibrium interest rates path r_t and associated government transfers T_t. The computational algorithm and other details are reported in Appendix A.

The top right panel of Figure 4 plots the path for government transfers and seigniorage revenue from debt issuance $B_{t+1} - (1 + r_t)B_t$, both relative to output. Transfers are larger on impact, about 5% of output, remain positive throughout the transition, and settle to a small positive level in the steady state, of about 0.1% of output. The difference between transfers and seigniorage revenue is equal to the tax revenues, which is negative due to the capital subsidies.

The bottom left panel in Figure 4 plots the path for the interest rate. Interest rates rise with public debt to induce households to hold a greater stock of aggregate wealth. Note that interest rates overshoot during the transition, which is the Le Chatelier principle at work; namely, the short-run elasticity of assets to interest rates is lower than its long run level. The sharp spike in interest rates makes the policy fiscally expensive, as exposited in Section 3. However, the cost is more than offset by the funds raised directly by debt issuance as seen by the elevated transfers early in the transition.

The bottom right panel plots the dispersion of household consumption relative to the laissez-faire dispersion. Consumption dispersion decreases by about 10% upon the introduction of the fiscal policy plan, as households with low assets and low productivity benefit from government transfers to support their consumption. As transfers fall over time, consumption dispersion increases, but remains about 2% below the one in the laissez-faire economy. The smaller long-run consumption dispersion reflects that households on average hold a greater stock of precautionary savings given the elevated interest rate.

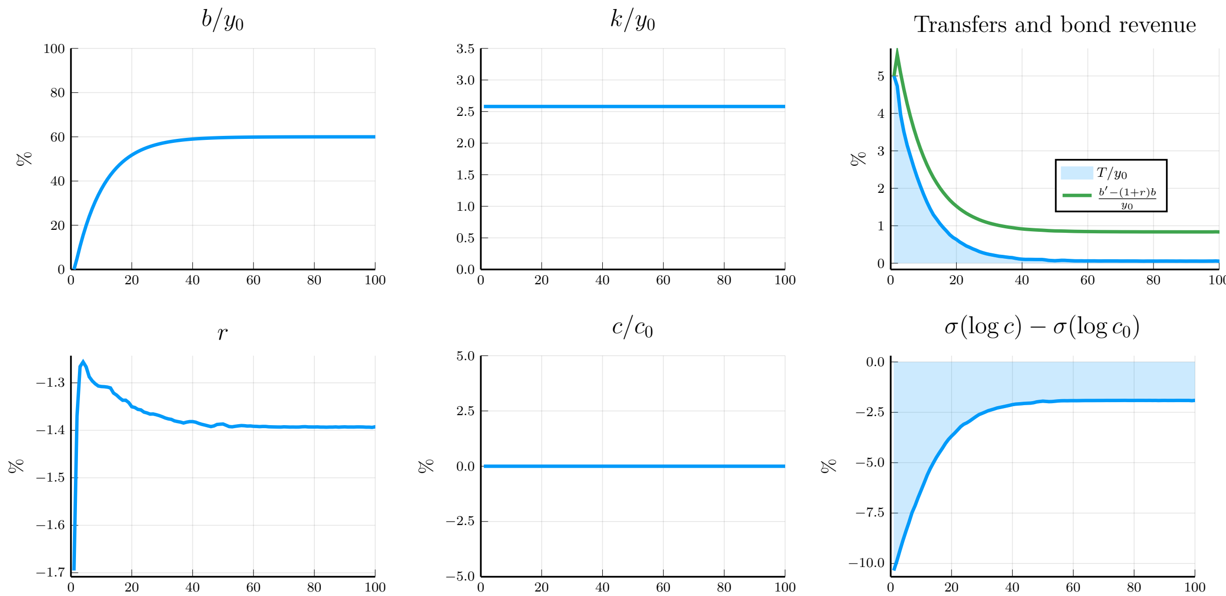


Figure 4: Baseline Fiscal Policy Transition

The transition paths of positive transfers and higher interest rates imply that our baseline fiscal policy is Pareto improving. We now evaluate the magnitude of the welfare gains. Table 2 Column 1 reports welfare for various households upon the announcement of the policy. Welfare is measured in consumption equivalence units relative to the laissez faire economy. Across the distribution of households for assets and productivity (a, z), the economy with fiscal policy delivers higher welfare for every household. The table reports five measures welfare gain: the mean gain, the minimum gain, and the mean gains for the bottom ten percent of the asset distribution, the 40-60th percentiles of the asset distribution, and the top 10 percent of the asset distribution. The mean welfare gains are computed by integrating over idiosyncratic states, conditional on belonging to the respective asset bin, weighted by the invariant distribution of the laissez faire economy.

The mean welfare gain is 2.62% and the minimum gain is 2.09%. Looking across the wealth distribution, welfare gains are greatest for the poorest households. While all households receive the same transfer, the poorer households benefit relatively more in percentage terms. However, gains are not monotonic in wealth. The top decile of asset holders experience a greater welfare gain than those in the middle of the asset distribution. This reflects the fact that the benefits of a higher interest rate are increasing in wealth. At some point in the distribution, this effect dominates the uniform transfer, generating a non-monotonicity in percentage welfare gain as a function of initial wealth.

We can also compute welfare gains comparing the new steady state to the laissez faire steady state, ignoring the transition. Welfare gains are more modest in the new steady state relative to the gains enjoyed at $t = 0$ due to the declining path of transfers. Nevertheless, all households are better off in the new steady state, with an average welfare gain of 1.8%.

Table 2: Changes in Welfare

	Policy Baseline	Policy Crowding-In
Welfare Gains at Announcement (%)		
Overall Mean	2.62	5.16
Overall Minimum	2.09	4.48
Poor ($\leq$ 10 pct)	3.57	5.19
Middle Wealth (40-60 pct)	2.30	4.95
Rich (>90 pct)	2.82	7.12

The preceding established that a Pareto-improving fiscal policy targeting a long-run debt-to-output level of 60% is feasible. This result reflects that a debt level of this magnitude can be absorbed by households with only a modest increase in interest rates. This raises the question of whether even higher levels of debt are feasible while still ensuring all agents are weakly better off.

To answer this question, we revisit the logic of Figure 3. In particular, long-run seigniorage is given by $-rB$, while the costs are captured by $\Delta r * K_0$. In Figure 5 we plot these two components for stationary equilibria with different levels of debt to output. At each debt level, tax policy is set to deliver laissez faire taxes and profits.

Up until debt levels of roughly twice the level of output, seigniorage exceeds fiscal costs, implying positive lump sum transfers to households. Beyond this level of debt, the increase in interest rate makes weakly positive transfers infeasible. Note that these two curves intersect while seigniorage is still increasing in debt, and eventually r becomes close enough to zero that seigniorage begins to decline in debt. The peak of this Laffer curve occurs at debt levels roughly four times output. Feasible Pareto-improving levels of debt, however, are much lower than this peak.

While Figure 5 establishes only that the policy is feasible in the new steady state, the analysis of transition dynamics in the baseline case above suggest that feasibility in the steady state is the critical metric. Along the transition, the government is a net issuer of bonds. As long as the revenue from the net issuances dominates any overshooting of the interest rate, feasibility rests on long-run considerations.

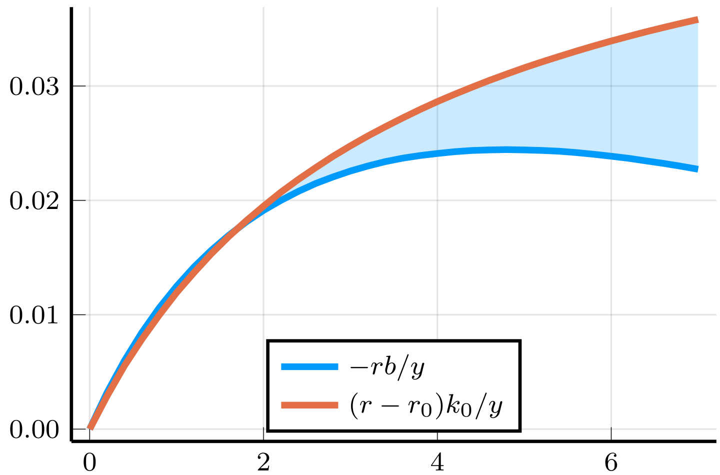


Figure 5: Steady State Seigniorage and Tax Revenue across Debt

4.3 The Role of Preferences

The analysis of Section 3 revealed that a key consideration in the feasibility of Pareto-improving policies is the elasticity of the aggregate savings function with respect to the interest rate. The short-run and long-run elasticities, in turn, depend on household preferences and the extent of exposure to idiosyncratic risk in a non-trivial manner. To assess this sensitivity quantitatively, we perform our baseline policy experiment in environments with alternative household preferences. In particular, we vary the inter-temporal elasticity of substitution, the coefficient of risk aversion, and the discount factor, and trace out the path of interest rates and transfers in response to an increase in government debt. We present the case for alternative IES, and defer the sensitivity to risk aversion and β to the appendix. In all experiments, the increase in government debt is 60 percent of initial aggregate income, keeping in mind that the initial capital stock (and hence income) will vary across experiments.

In Figure 6 we plot the path of interest rates (panel (a)) and transfers (panel (b)) for different values of the IES; namely 0.5, the benchmark 1.0, and 1.5, holding constant risk aversion at the benchmark 5.5. The pattern confirms the conjecture that a higher IES requires a smaller increase in interest rates to absorb the government debt, both in the short run and long run.

However, note that the initial interest rate is very different, as well. In particular, a low elasticity implies a very low level of interest rates throughout the transition, which also matters for feasibility. In Panel (b), we plot the associated transfers. It is the *more* elastic preferences that eventually require negative transfers (albeit very small, on the order of 10^{-4} of initial output). While the small increase in interest rates requires a small subsidy to capital, the fact that rates are close to zero implies lower seigniorage revenue, and the latter slightly dominates the former.[30]

Appendix Figures A.2 and A.3 contain the same simulated time series for alternative coefficients of relative risk aversion (CRRA) and time discount factors β. Intuitively, a higher CRRA implies a lower laissez faire interest rate and as well as a less elastic aggregate savings function. Nevertheless, the former dominates, implying feasibility for a CRRA of 10.0, but not for a CRRA of 2.0. Similarly, a lower discount factor (more impatience), implies a higher initial interest rate, leaving less fiscal space for Pareto-improving policies.

4.3.1 Fiscal Policy with Crowding In of Capital

We now consider a fiscal policy plan that engineers an increase in capital that reaches the Golden Rule level in the new steady state. In particular, capital relative to output increases from 2.5 to 3.0.[31] We assume that the government also pursues the same path of debt issuance as in the previous experiment. Capital subsidies are set to ensure firms rent the targeted level of capital given the prevailing interest rate, and labor and profit taxes are set such that after-tax wages and profits remain constant. We find that transfers are positive throughout, and hence the fiscal plan is a feasible Pareto improvement.

Figure 7 plots the variables of interest during this transition. The layout of the figure is the same as Figure 4. Along the path, we normalize quantities by the initial laissez faire income, keeping in mind that contemporaneous income is increasing with capital.

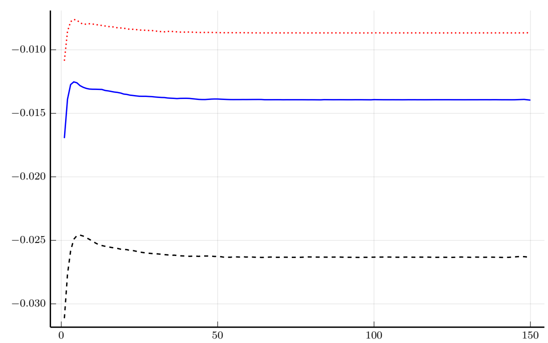
The first two panels of the figure's top row represent the posited path of debt and capital. The top right panel illustrates that government transfers are positive throughout the transition. These transfers fall in the middle of the transition and increase towards the end of the transition. Transfers increase towards the end because interest rates are declining and capital is increasing, easing the fiscal burden.

As in the baseline policy experiment, seigniorage revenue from borrowing falls during the transition but settles at a lower level, due to the higher interest rates. As seen in the bottom left panel of the figure, interest rates rise more with a fiscal policy that crowds in capital because

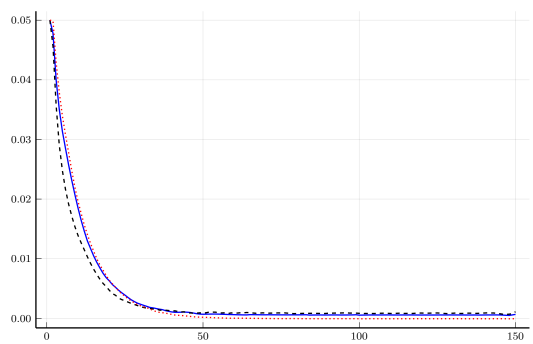
[30] The fact that the laissez faire interest rate varies with the IES while holding risk aversion constant stems from the fact that precautionary savings depends on more than the extent of risk aversion. Kimball and Weil (1992) show that with Kreps-Porteus preferences, the strength of the precautionary savings motive is determined by attitudes toward both risk and inter-temporal substitution.

[31] Recall that $F_K = \alpha Y/K$, and $\delta = 0.10$, and hence given $\alpha = 0.3$ the Golden Rule is achieved at $K/Y = 3.0$.

Figure 6: Alternative Inter-temporal Elasticities



(a) Path of r_t



(b) Path of T_t

Note: This figure displays the path of interest rates (Panel a) and transfers as a fraction of laissez faire output (Panel b) associated with the baseline fiscal policy under alternative preference parameterizations. In both panels, the solid line is the benchmark IES=1.0; the dotted red line is IES=1.5; and the dashed black line is IES=0.5.

households need to be induced to hold the additional capital as well as debt.

The bottom middle panel shows that aggregate consumption falls early in transition, as the economy increases investment in new capital, and settles above the laissez faire level in the new steady state with higher capital. The dispersion of household consumption, however, remains uniformly below the level in the laissez-faire economy throughout the transition. As seen in the bottom right panel, the standard deviation drops about 9%, and increases to about 4% lower. Thus, fiscal policy improves risk sharing because of larger transfers as well as a larger stock of household wealth, which earns a higher rate of return.

The second column of Table 2 reports the welfare gains for this experiment. Welfare increases for all households both upon the fiscal policy announcement and also in the new steady state. The mean welfare gain is 5.16% and the minimum gain is 4.48%. In this case, fiscal policies benefit the rich households more than poor households, with gains upon impact of 7.12% and 5.19%, respectively. Nevertheless, as before, households in the middle of the wealth distribution gain

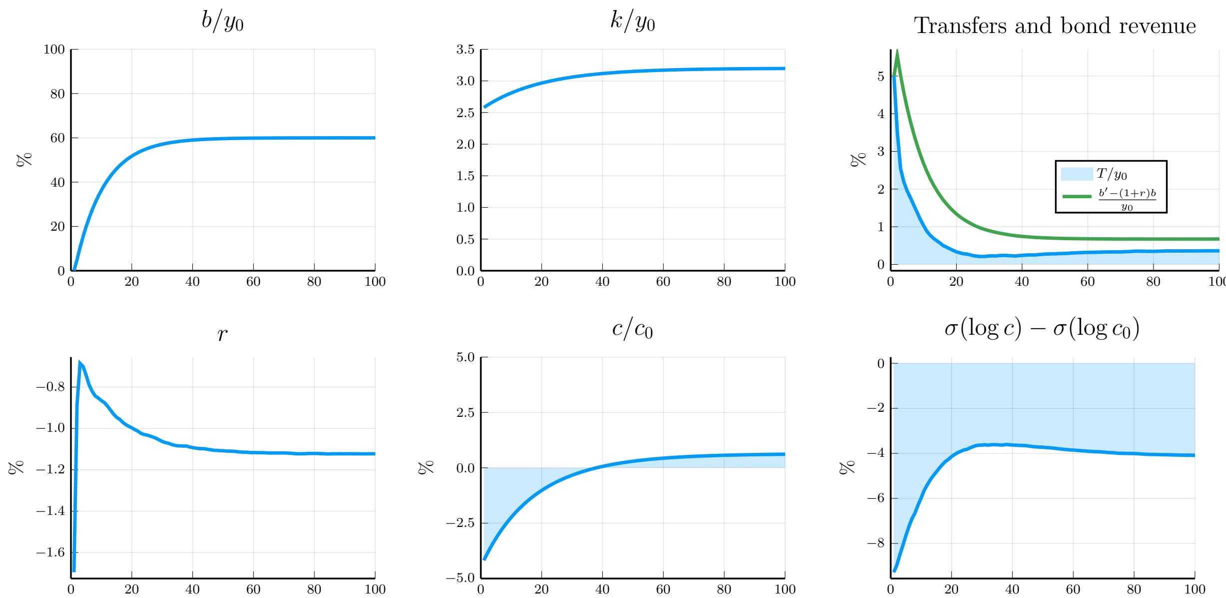


Figure 7: Fiscal Policy Transition with Crowding In

the least in percentage terms. The gains in this policy experiment are much larger than for the baseline policy, because they not only reflect better risk sharing but also a higher level of capital and consumption in the long run.

The crowding-in experiment assumed that the government issued debt in the same manner as it did in our baseline policy. In the analysis of Section 3, we argued that debt issuance may be useful along the transition to a higher capital stock, if the short-run elasticity of household savings is significantly lower than the long-run elasticity. This configuration made debt issuance a complement to capital accumulation. We can explore this property in greater depth using the quantitative model.

Specifically, we study an alternative fiscal policy that implements the same path of capital as in our crowding-in experiment, but with zero debt issuance. The transition dynamics for this case are presented in Figure 8.

The top right panel of the figure shows that without public debt, the government needs to lump-sum tax households early in the transition. The large increase in the interest rate necessary to induce households to hold more wealth (the bottom left panel) implies large fiscal costs from capital rental subsidies. In the transition with debt, the government could use debt issuance to smooth this burden. Without debt, the government must lump sum tax, which implies some households may be strictly worse off along the transition. These losses are also reflected by the higher standard deviation of consumption early in the transition, which is plotted in the lower right panel.

This experiment suggests that public debt is an important tool in Pareto-improving capital expansions. In this sense, government debt and capital expansions can be complements rather than the traditional substitutes, providing a contrast with Diamond (1965).

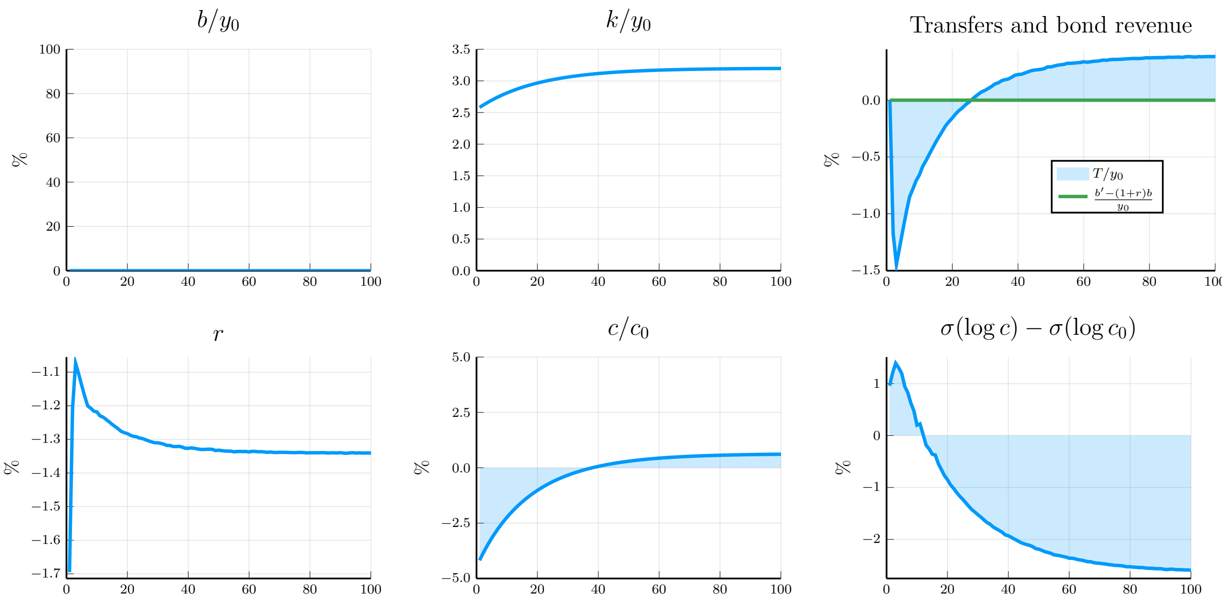


Figure 8: Fiscal Policy Transition with Crowding In and No Debt

4.4 Pareto Improvements in Dynamic Inefficient Economies

We now discuss the effects of fiscal policies in the competitive economy that is dynamically inefficient. The fiscal policy we analyze here consists of a time path for debt, labor subsidies, and the resulting transfers for households. Recall that we design labor subsidies to guarantee that the wage households receive with fiscal policy equals that in the laissez-faire economy. The path for government debt is identical to the one considered in the baseline fiscal policy experiment, while capital adjusts freely with interest rates. Finally, the parameters of this economy are the same as those we calibrated for the baseline policy, with the exception that mark-ups are zero.

We find that this fiscal policy plan in the competitive economy is also Pareto improving. As in the baseline economy, fiscal policy in this case improves risk sharing because it delivers positive transfers and higher returns on households wealth. An additional force in the dynamically inefficient economy is that government debt is also useful because it crowds out unproductive capital.

As shown in Appendix B, the transition dynamics share many features of the baseline economy: the standard deviation of consumption falls about 8% on impact and settles at about 1% below the laissez faire, transfers are higher early in the transition than later in the transition, and interest rates overshoot early in the transition. Capital to output in the laissez faire economy is larger in the absence of mark-ups. Moreover, capital falls with fiscal policy because the increase in interest rates crowds out capital. The decline in investment early in the transition boosts aggregate consumption, which settles at a higher level than in the laissez faire economy because lower capital increases consumption in the dynamically inefficient economy. The fiscal policy plan here also gives rise to significant welfare gains of 3.0 % on average upon the announcement of the policy.

5 Conclusion

We provided sufficient conditions for the feasibility of Pareto-improving fiscal policies when the risk-free interest rate on government bonds is below the growth rate ($r < g$) in the Bewley-Huggett-Aiyagari model. The key condition is that seigniorage revenue raised by government bonds exceeds the increase in the interest rate times the initial capital stock. As long as the aggregate household savings schedule is sufficiently elastic, such Pareto-improving policies are feasible. In this sense, we have shown that feasibility of a Pareto improvement depends on an aggregate elasticity, not on the finer details of idiosyncratic heterogeneity that underpin this elasticity. In calibrated examples using U.S. data on household heterogeneity and historical data on interest rates and growth rates, we find scope for Pareto improving policies for a wide range of debt and tax policies.

The government uses seigniorage debt revenue to provide transfers to households and to subsidize factor prices. These policies are welfare improving for all households because they improve risk sharing and can give rise to beneficial supply expansions. We find scope for Pareto improving fiscal policies with and without capital crowding-in, and in both dynamically efficient and inefficient economies. We find that debt is a useful tool, specially for fiscal policies that expand capital.

Many governments around the world are rapidly expanding their public debt in the context of low interest rates. Our analysis points to a force that increases the benefits of such expansions. The analysis provided simple conditions for fiscal feasibility, complementing the typical dynamic inefficiency condition of Samuelson (1958) and Diamond (1965). We have tried for analytical clarity in an extension of the canonical Bewley environment, rather than a full fledged quantitative model for policy design in the current context. In particular, we have abstracted from aggregate risk. Integrating the possibility that interest rates rise in response to aggregate shocks would certainly increase the fiscal costs of higher debt. The benefits from increasing debt to improve risk sharing and for supply expansions would then have to be balanced against the costs of having to tax future generations to pay for the debt if interest rates rise.

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Appendix A Computational Algorithm

This appendix describes the computational algorithm we use in solving the model. Our procedure consists of two steps. We first compute the initial and final stationary equilibria. The initial one is the laissez faire equilibrium and the final one has fiscal policy. We then compute the transition of this economy with shooting algorithms. We describe the algorithm for the dynamically efficient economy with markups.

A.1 Stationary Equilibrium

Initial The laissez faire initial stationary equilibrium is the standard Bewley-Hugget-Aiyagari model. We compute it with a value function iteration over a savings grid, and solve for the equilibrium wages and interest rates that clear markets. The objects we record are prices $\{w_0, r_0\}$, aggregate capital, labor, profits, and the limiting distribution of households over idiosyncratic state $\{a, z\}$, namely $\{K_0, N_0, \Pi_0, \Lambda_0(a, z)\}$. We denote the objects in the initial stationary equilibrium by 0.

Final The final stationary equilibrium is indexed by H.

1. We set a target levels for capital K_H and debt B_H.
2. Choose fiscal policies $\{\tau_H^n, \tau_H^\pi\}$ to keep wages and profits as in the initial equilibrium using

$$\begin{aligned} F_L(K_H, N_0) &= \mu(1 + \tau_H^n) w_0 \\ \Pi_0 &= (1 - \tau_H^\pi)(\mu - 1) F(K_H, N_0)/\mu \end{aligned}$$

3. Guess an equilibrium interest rate r_H.
4. We recover the initial implied τ_H^k from

$$F_K(K_H, N_0) = \mu(1 + \tau_H^k)(r_H + \delta)$$

5. Use the government budget is a constraint to recover transfers T_H given our settings and the guess for interest rates

$$T_H = F(K_H, N_0) - F(K_0, N_0) - (r_H + \delta)K_H + (r_0 + \delta)K_0 + r_H B_H$$

6. Solve households problem

$$V_H(a, z; r_H) = \max_{a' \geq \underline{a}, c, n} \phi(x(c, n), EV_H(a', z'; r_H))$$
$$\text{subject to: } c + a' \leq w_0 z n + (1 + r_H)a + T_H$$

   - Gives value and savings policy functions $V_H(a, z; r_H)$, $a'_H(a, z; r_H)$ and labor supply without wealth effects $n_H(z)$, and a household distribution $\Lambda_H(a, z; r_H)$

7. Use the asset market clearing condition and firm's optimal capital condition to obtain a new guess on interest rates $\tilde{r}_H$

$$
\begin{aligned}
B_H + \tilde{K}_H &= \int a'_H(a, z; r_H)\pi(z', z) d\Lambda_H(a, z; r_H) \\
F_K(\tilde{K}_H, N_0) &= \mu(1 + \tau^k_H)(\tilde{r}_H + \delta)
\end{aligned}
$$

8. We go back to step 3, and repeat the procedure until $\tilde{K}_H$ is close to the target capital level.

A.2 Transition

At time 0, the government announces a sequence of fiscal policies that implement a sequence of capital and debt $\{K_t, B_t\}_{t=0}^H$. We will assume that at period H the economy is in the final stationary equilibrium.

1. Choose fiscal policies $\{\tau^n_t, \tau^\pi_t\}$ to keep wages and profits as in the initial equilibrium using

$$
\begin{aligned}
F_L(K_t, N_0) &= \mu(1 + \tau^n_t) w_0 \\
\Pi_0 &= (1 - \tau^\pi_t)(\mu - 1) F(K_t, N_0)/\mu
\end{aligned}
$$

2. Guess sequence of interest rates $\{r_t\}_{t=0}^H$

   - Recover the capital taxes given our target capital sequence and interest rate guess

$$
F_K(K_t, N_0) = \mu(1 + \tau^k_t)(r_t + \delta)
$$

   - Recover transfers from government budget constraint using sequence of debt

$$
T_t = F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 + B_{t+1} - (1 + r_t)B_t
$$

3. Solve households problem backwards

   - Start with period $H - 1$ problem. Note that we have the value at period H from the stationary equilibrium

$$
\begin{aligned}
V_t(a, z) = &\max_{a' \geq \underline{a}, c, n} \phi(x(c, n), h(V_{t+1}(a', z'))) \\
&\text{subject to: } c + a' \leq w_0 z n + (1 + r_t)a + T_t
\end{aligned}
$$

   - Store saving decision rules: $a'_t(a, z)$.

4. Iterate forwards, update interest rates: The resulting aggregate savings from Step 2 will not be equal to the targets.

   - Start with initial distribution $\Lambda_0(a, z)$ and apply the decision rules from above.

- Use the asset market clearing condition to obtain the resulting capital sequence $\tilde{K}_{t+1}$

$$B_{t+1} + \tilde{K}_{t+1} = \int a'_t(a, z)\pi(z', z)d\Lambda_t(a, z)$$

- Use firm's optimal capital condition to obtain a candidate new sequence of interest rates $\tilde{r}_t$

$$F_K(\tilde{K}_t, N_0) = \mu(1 + \tau_t^k)(\tilde{r}_t + \delta)$$

- Update the sequence of interest rates such that $r_t^{new} = \lambda r_t^{old} + (1 - \lambda)\tilde{r}_t$, for attenuation parameter $\lambda = 0.5$. Go back to step 2.

Check whether implementing these target levels of $\{K_t, B_t\}_{t=0}^{H}$ result in a Pareto improvement $\{r_t \geq 0, T_t \geq 0\}_{t=0}^{H}$. If not, we modify the target levels of capital and debt.

Appendix B Dynamic Inefficient Transition

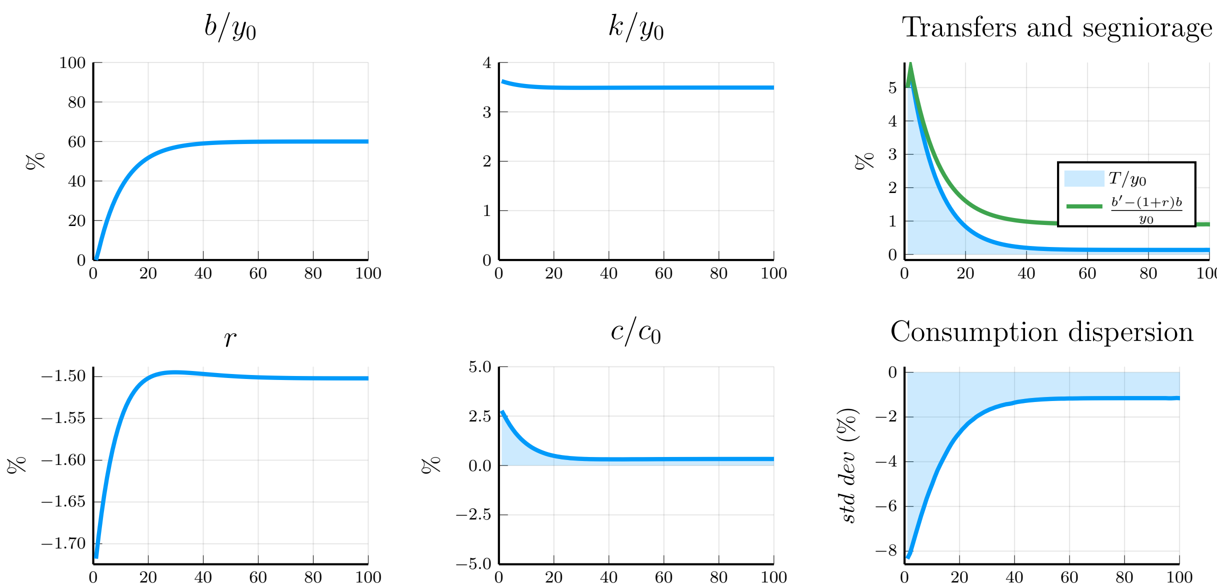


Figure A.1: Fiscal Policy Transition in Dynamic Inefficient Economy

Appendix C The Growth Economy

In this appendix, we show how the key expressions of Section 2 are modified by the presence of exogenous labor-augmenting technological growth. The derivations are standard and are included for completeness.

Assume technology is given by

$$Y_t = F(K_t, (1+g)^t L_t),$$

where $g \geq 0$ is the constant rate of growth of labor-augmenting technology. Letting a tilde denote variables divided by $(1+g)^t$, constant returns implies:

$$\tilde{Y}_t \equiv (1+g)^{-t} Y_t = F(\tilde{K}_t, L_t).$$

The representative firm's first-order conditions are (dropping t subscripts):

$$F_k(\tilde{K}, L) = \mu(1+\tau^k) r^k$$
$$F_l(\tilde{K}, L) = \mu(1+\tau^n) \tilde{w}.$$

We also have $\tilde{\Pi} = (1-\tau^\pi)(\mu - 1)F(\tilde{K}, L)/\mu$.

Given the absence of a wealth effect on labor supply, we assume that the disutility of working grows at rate g as well (dropping i and t indicators):

$$x(c, n) = c - (1+g)^t v(n),$$

giving us

$$\tilde{x}(\tilde{c}, n) \equiv (1+g)^{-t} x(c, n) = \tilde{c} - v(n).$$

We also assume that the borrowing constraint is scaled by $(1+g)^t$.

We can write the household's problem as:

$$\begin{aligned} V_t(a, z, \theta) = \max_{a', n, c} \phi(x(c, n), h(V_{t+1}(a', z', \theta'))) \\ \text{s.t. } c + a' \leq w_t z n + \theta \Pi_t + (1+r_t)a + T_t \\ a' \geq (1+g)^{t+1} \underline{a}, \end{aligned}$$

where we have altered the last constraint to account for growth and h is a certainty equivalent operator. The constraint set can be rewritten as

$$\begin{aligned} \tilde{c} + (1+g)\tilde{a}' \leq \tilde{w}_t z n + \theta \tilde{\Pi}_t + (1+r_t)\tilde{a} + \tilde{T}_t \\ \tilde{a}' \geq \underline{a}. \end{aligned}$$

Thus, if (c, n, a') is feasible at time t then $(\tilde{c}, n, \tilde{a}')$ satisfies the normalized constraint set, and vice versa. Assuming ϕ is constant-returns in x and h is homogeneous of degree 1, if $V_t(a, z, \theta)$ satisfies the consumer's Bellman equation, then $\tilde{V}_t(\tilde{a}, z, \theta) \equiv (1+g)^{-t} V_t(a, z, \theta)$ satisfies

$$\tilde{V}_t(\tilde{a}, z, \theta) = \max_{\tilde{c}, n, \tilde{a}'} \phi(\tilde{x}(\tilde{c}, n), (1+g)h(\tilde{V}_{t+1}(\tilde{a}', z', \theta'))),$$

subject to the normalized constraint set, and vice versa.[32]

[32]For the simulations, we use $\phi(x, h) = ((1-\beta)x^{1-\xi} + \beta h^{1-\xi})^{1/(1-\xi)}$. In this case, we can define $\tilde{\beta} \equiv \beta(1+g)^{1-\xi}$, and write $\tilde{\phi}(\tilde{x}, h) = ((1-\tilde{\beta})\chi \tilde{x}^{1-\xi} + \tilde{\beta} h^{1-\xi})^{1/(1-\xi)}$, where $\chi \equiv (1-\beta)/(1-\tilde{\beta})$. This is well defined as long as $\tilde{\beta} \leq 1$. Growth

Note that for an interior optimum for n, the first-order condition can be expressed:

$$v'(n) = z\tilde{w}.$$

Hence, labor supply is constant as long as $\tilde{w}$ remains constant.

The government's budget constraint can be rewritten in normalized form:

$$\tilde{T}_t = \tau_t^n \tilde{w}_t N_t + \tau_t^k r_t^k \tilde{K}_t + \tau_t^\pi \tilde{\Pi}_t/(1-\tau_t^\pi) + (1+g)\tilde{B}_{t+1} - (1+r_t)\tilde{B}_t.$$

Let $\tilde{X}_t \equiv \tau_t^n \tilde{w}_0 N_0 + \tau_t^k r_t^k \tilde{K}_t + \tau_t^\pi \tilde{\Pi}_0/(1-\tau_t^\pi)$ denote normalized tax revenue before transfers when keeping after tax normalized wages and profits constant. Following the same steps as the proof of Proposition 1, we have

$$\tilde{X}_t = F(\tilde{K}_t, N_0) - F(\tilde{K}_0, N_0) - (r_t + \delta)\tilde{K}_t + (r_0 + \delta)\tilde{K}_0.$$

Condition (iii) of Proposition 1 (equation (2)) becomes:

$$(1+g)\tilde{B}_{t+1} - (1+r_t)\tilde{B}_t - \tilde{T}_t \geq F(\tilde{K}_0, N_0) - F(\tilde{K}_t, N_0) - (r_0 + \delta)\tilde{K}_0 + (r_t + \delta)\tilde{K}_t.$$

Condition (ii) becomes $\tilde{T}_t \geq -(r_t - r_0)\underline{\tilde{a}}$, and condition (i) remains unchanged. Note that in a steady state (that is, relevant aggregates grow at rate g), Condition (iii) becomes

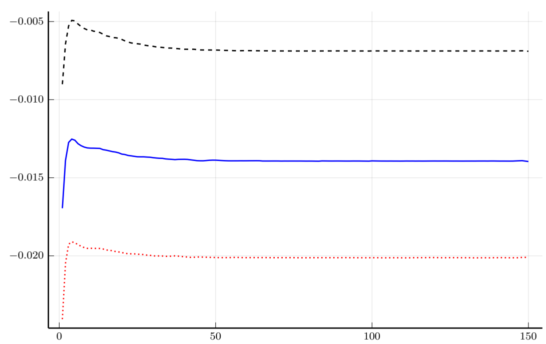
$$(g - r_{ss})\tilde{B}_{ss} - \tilde{T}_{ss} \geq F(\tilde{K}_0, N_0) - F(\tilde{K}_{ss}, N_0) - (r_0 + \delta)\tilde{K}_0 + (r_t + \delta)\tilde{K}_{ss}.$$

Hence, debt increases government revenues in the steady state as long as $g > r_{ss}$. Expressions in Claims 1 and 2 are adjusted in a similar fashion to obtain normalized equivalents.

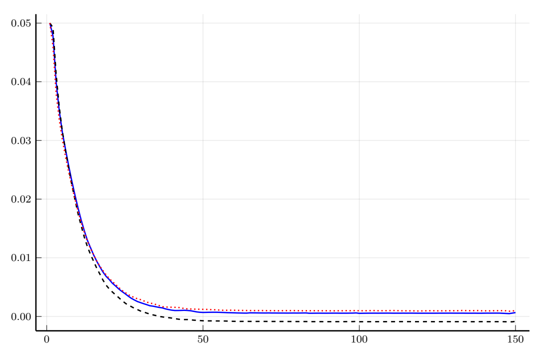
Appendix D Additional Comparative Statics with respect to Preference Parameters

can be accommodated by rescaling the discount factor, as expected with homogeneous preferences.

Figure A.2: Alternative Relative Risk Aversion Coefficients



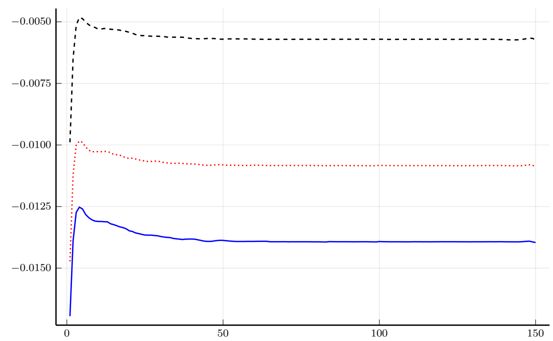
(a) Path of r_t



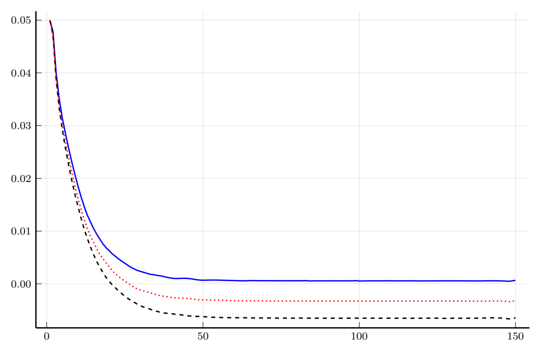
(b) Path of T_t

Note: This figure displays the path of interest rates (Panel a) and transfers as a fraction of laissez faire output (Panel b) associated with the baseline fiscal policy under alternative preference parameterizations for the coefficient of relative risk aversion (CRRA). In both panels, the solid line is the benchmark CRRA=5.5; the dotted red line is CRRA=10.0; and the dashed black line is CRRA=2.0.

Figure A.3: Alternative Discount Factors



(a) Path of r_t



(b) Path of T_t

Note: This figure displays the path of interest rates (Panel a) and transfers as a fraction of laissez faire output (Panel b) associated with the baseline fiscal policy under alternative preference parameterizations for time discount factors. In both panels, the solid line is the benchmark β-0.993; the dotted red line is β=0.98; and the dashed black line is β=0.97.