Sufficient Statistics for Nonlinear Tax Systems with Preference Heterogeneity

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Motivation

Taxation of savings, wealth, bequests, capital

- At the forefront of current policy discussions. Useful for redistribution?
- Key issue: co-variation in earning ability and preference for saving
- Challenging to measure empirically, and to accommodate generally

Atkinson & Stiglitz (1976) result

- Optimally: no differential commodity taxes with homogeneous preferences
- Intuition: should redistribute through an income tax, not a champagne tax
- Implication: savings and capital should go untaxed (consume now vs later)

This paper

What is the optimal nonlinear tax system with preference heterogeneity?

Setting

Standard 2-good model bridging capital and commodity taxation.
(Atkinson & Stiglitz '76, Saez '02, Golosov, Troshkin, Tsyvinski, Weinzierl '13)

Results

1. Optimal allocation can be implemented with (simple!) smooth tax systems
2. General sufficient statistics characterization of optimal nonlinear tax system
   - Derive key statistic for preference heterogeneity, empirically measurable.
   - Leverages active empirical literature studying causal income effects.
3. Application to saving and capital taxation in the US economy.
   - Calculates key sufficient statistic using multiple methods: recent evidence from administrative data, and new nationally representative survey.
   - Results suggest progressive optimal tax on savings.

The sufficient statistics approach

Characterize conditions of tax system that must hold at the optimum.

- Formulas written in terms of parameters that are empirically estimable. (See Kleven 2020 for review.)
- Spans variety of structural models giving rise to same sufficient statistics.
  - Preference heterogeneity, effort-based returns, income shifting...
- Complements structural approaches: intuition for forces governing policy, provides FOCs for fixed-point procedures. (Saez '01)

Model setup

Model setup

Agents

- Heterogeneous ability, preferences, indexed by type $\theta \in \mathbb{R}$.
- Preferences: $U(c, s, z; \theta)$ [regularity assumptions]
- Numeraire consumption c. Labor earnings z.
- Commodity s, with marginal rate of transformation p.
  - Examples: electricity, education, housing ...
  - Today: savings (Saez 2002, Golosov et al. 2013), $p = \frac{1}{1+r}$

Policymaker

- Maximizes weighted sum of utilities subject to resource constraint,

$$\max \int_{\Theta} \left\{ \alpha(\theta)\, U\Big(c(\theta), s(\theta), z(\theta); \theta\Big) \right\} dF(\theta)$$

$$\text{s.t.} \int_{\Theta} \Big\{ z(\theta) - c(\theta) - ps(\theta) \Big\} dF(\theta) \geq R$$

Using taxes to implement the optimal allocation

Mechanism design

- Characterize optimal allocation: $\{c(\theta), s(\theta), z(\theta)\}_{\theta \in \Theta}$ subject to individual incentive compatibility constraints:

$$U(c(\theta), s(\theta), z(\theta); \theta) \geq U(c(\theta'), s(\theta'), z(\theta'); \theta) \quad \forall \theta, \theta'$$

An intermediate result: implementing with a bivariate tax

- Prop. 1: Under regularity assumptions, *a smooth optimal incentive-compatible allocation can be implemented by smooth tax function $\mathcal{T}(s,z)$.*
- This is a relaxed problem: with smooth $\mathcal{T}(s,z)$, θ can choose bundles not chosen by other types.
- **Now:** characterize features of $\mathcal{T}(s,z)$ using sufficient statistics.

Theoretical sufficient statistics results

Road map for theoretical results

1. A sufficient statistic for preference heterogeneity
2. Characterizing the optimal tax $\mathcal{T}(s, z)$
3. Implications for "simple" tax systems

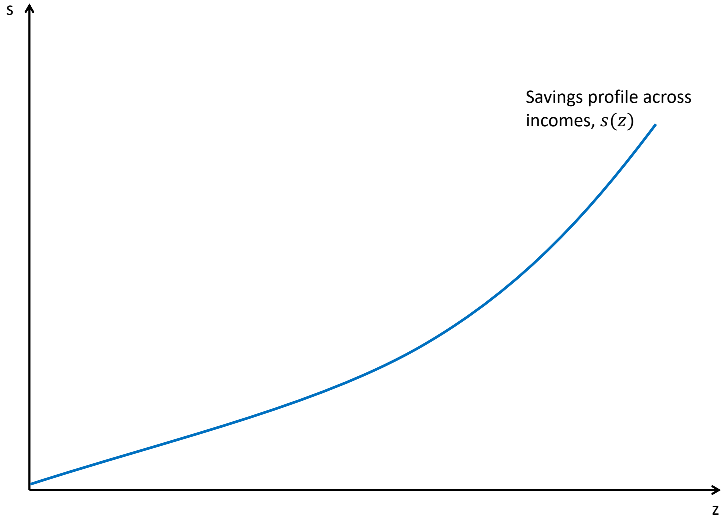
Sufficient statistics for optimal $\mathcal{T}(s, z)$

Familiar sufficient statistics.

- $\zeta_z^c(z)$: compensated elasticity of taxable income
- $\zeta_{s|z}^c(z)$: compensated savings elasticity (fixing z)
- $\hat{g}(z)$: social marginal welfare weights augmented with income effects
- $h_z(z)$: income density

Plus a sufficient statistic for local slope of *preference heterogeneity.*

Decomposing the cross-sectional profile $s(z)$



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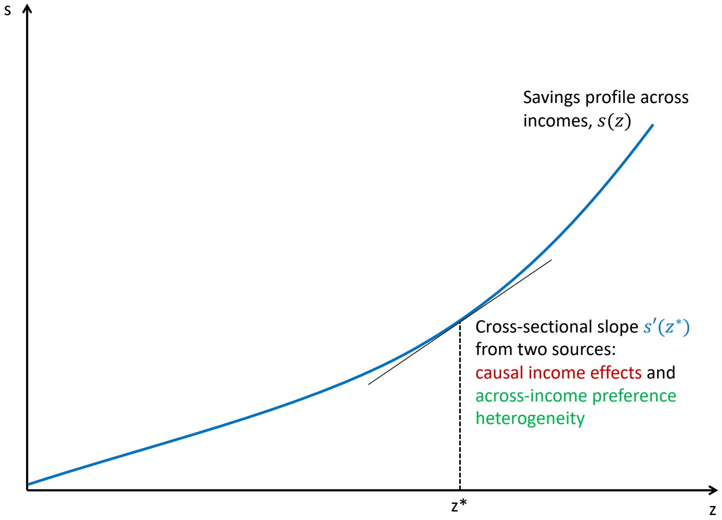

A sufficient statistic for preference heterogeneity

- $s(z;\theta) :=$ type θ's preferred choice of s given earnings z.
- Define $\vartheta(z) = \theta$ s.t. $z(\theta) = z$.
- Cross-sectional slope = causal income effect + preference heterogeneity

$$\underbrace{\left.\frac{ds(\tilde{z};\vartheta(\tilde{z}))}{d\tilde{z}}\right|_{\tilde{z}=z}}_{s'(z)} = \underbrace{\left.\frac{\partial s(\tilde{z};\vartheta(z))}{\partial\tilde{z}}\right|_{\tilde{z}=z}}_{s'_{inc}(z)} + \underbrace{\left.\frac{\partial s(z;\vartheta(\tilde{z}))}{\partial\tilde{z}}\right|_{\tilde{z}=z}}_{s'_{pref}(z)}$$

- $s'_{pref}(z)$ is the key sufficient statistic for preference heterogeneity
  - Intuition: when $s'(z)$ driven by $s'_{pref}(z)$, $s(z;\theta)$ acts like ability tag.
- Under Atkinson-Stiglitz assumptions, $s'_{inc}(z) = s'(z) \Rightarrow s'_{pref}(z) = 0$.

Empirical measurement

$$s'_{pref}(z) = s'(z) - s'_{inc}(z)$$

- Simple example with heterogeneous discount rates:

$$U(c, s, z; \theta) = \ln(c) + \delta(\theta)\ln(s) - \psi(z/\theta),$$

  - then $s'_{pref}(z) \propto \frac{d}{dz}\frac{\delta(z)}{1+\delta(z)}$
  - but δ may be difficult to measure.
- $s'(z)$ is directly observable from data.
- $s'_{inc}(z)$ can be measured using standard empirical tools. (Prop. 2)
  - $s'_{inc}(z) =$ marginal propensity to consume s (if weak separability)
  - or $\frac{\partial s}{\partial z}$ from earnings responses to exogenous shocks, e.g. income tax reforms.

Captures more than preference heterogeneity

Difference $s'(z) - s'_{inc}(z)$ captures all type-specific across-income heterogeneity, not just intrisic preferences.

Heterogeneous prices $p(s;\theta)$

- Scale effects related to s contribute to $s'_{inc}(z)$
- Premium related to type θ contributes to $s'_{pref}(z)$
- Adds to lit. on taxation with heterogeneous returns, $p(s;\theta) = \frac{1}{1+r(s;\theta)}$

Income shifting, e.g., from labor to capital gains

- Scale effects related to earnings z contribute to $s'_{inc}(z)$
- Premium related to type θ contributes to $s'_{pref}(z)$

Road map for theoretical results

1. A sufficient statistic for preference heterogeneity
2. Characterizing the optimal tax $\mathcal{T}(s,z)$
3. Implications for "simple" tax systems

Optimal savings tax rates

Prop. 3: In an optimal smooth tax system, at each bundle $(s(z), z)$, marginal savings tax rates satisfy:

$$\frac{\mathcal{T}_s'(s,z)}{1+\mathcal{T}_s'(s,z)} = s_{pref}'(z)\frac{1}{s\,\zeta_{s|z}^c(z)}\frac{1}{h_z(z)}\int_{x=z}^{\bar{z}}\left(1-\hat{g}(x)\right)h_z(x)\,dx$$

- Savings tax rate is proportional to local preference heterogeneity $s_{pref}'(z)$.
- Note Atkinson-Stiglitz corollary: $s_{pref}'(z) = 0 \Rightarrow \mathcal{T}_s'(s,z) = 0$.

Optimal earnings tax rates

Prop. 3 (cont.): In an optimal smooth tax system, at each bundle $(s(z), z)$, marginal earnings tax rates satisfy:

$$\frac{\mathcal{T}_z'(s,z)}{1-\mathcal{T}_z'(s,z)} = \frac{1}{z\,\zeta_z^c(z)}\frac{1}{h_z(z)}\int_{x=z}^{\bar{z}}\Big(1-\hat{g}(x)\Big)\,h_z(x)\,dx - s_{inc}'(z)\frac{\mathcal{T}_s'(s,z)}{1-\mathcal{T}_z'(s,z)}$$

- Equity-efficiency trade-off, extended with savings responses through $s_{inc}'(z)$.
- Under Atkinson-Stiglitz, $\mathcal{T}_s'(s,z) = 0 \Rightarrow$ last term drops out.

Road map for theoretical results

1. A sufficient statistic for preference heterogeneity

2. Characterizing the optimal tax $\mathcal{T}(s, z)$

3. Implications for "simple" tax systems

A taxonomy of simple tax systems

Focus on three common functional restrictions on general $\mathcal{T}(s,z)$

Type of tax system	$\mathcal{T}(s,z)$
SL: Separable Linear	$\tau_s s + T_z(z)$
SN: Separable Nonlinear	$T_s(s) + T_z(z)$
LED: Linear Earnings-Dependent	$\tau_s(z)s + T_z(z)$

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Select examples (more in paper)

Country	Wealth	Capital Gains	Property	Pensions	Inheritance
France	–	Other	Other	SL, SN	SN
Italy	SL, SN	SL	SL	SL	SL, SN
New Zealand	–	Other	SN	SL, LED	–
Norway	SN	SL	SL	SN	–
United States	–	LED	SL	SN	SN

Props. 10, 11: Conditions where optimal $\mathcal{T}(s,z)$ can be implemented by an SN system (very general) or by a LED system (fairly general).

Conditions for optimal *simple* taxes on savings

Prop. 4:

- Optimal *Separable Linear* tax system, $\mathcal{T}(s,z) = \tau_s s + T_z(z)$:

$$\frac{\tau_s}{1+\tau_s} = \frac{1}{\bar{\zeta}^c_{s|z}\bar{s}} \int_z \left(s'_{pref}(z) \int_z^{\bar{z}} (1-\hat{g}(x)) dH_z(x) \right) dz.$$

- Special cases:
  1. $s'_{pref}(z) \equiv 0 \Rightarrow \tau_s = 0$ (Atkinson Stiglitz '76).
  2. $s'_{pref}(z) \equiv s'(z) \Rightarrow$ generalized "many person Ramsey rule" (Diamond '75)

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  2. $s'_{pref}(z) \equiv s'(z) \;\Rightarrow$ generalized "many person Ramsey rule" (Diamond '75)
- Or: what condition ensures tax is Pareto efficient among SL systems?

$$\frac{\tau_s}{1+\tau_s} = \frac{1}{\int_z \zeta^c_{s|z}(z) s(z)\, dH_z(z)} \int_z s'_{pref}(z)\, \zeta^c_z(z)\, z\, \frac{T'_z(z) + s'_{inc}(z)\tau_s}{1-T'_z(z)} h_z(z) dz.$$

Conditions for optimal *simple* taxes on savings

What condition must tax on s satisfy to be Pareto efficient among simple systems?
Prop. 4:

- *Separable Nonlinear* tax system, $\mathcal{T}(s,z) = T_s(s) + T_z(z)$:

$$\frac{T_s'(s(z))}{1+T_s'(s(z))} = s'_{pref}(z) \frac{\zeta_z^c(z)z}{\zeta_{s|z}^c(z)s(z)} \frac{T_z'(z) + s'_{inc}(z) T_s'(s(z))}{1-T_z'(z)}$$

- *Linear Earnings-Dependent* tax system, $\mathcal{T}(s,z) = \tau_s(z)\, s + T_z(z)$:

$$\frac{\tau_s(z)}{1+\tau_s(z)} = s'_{pref}(z) \frac{\zeta_z^c(z)z}{\zeta_{s|z}^c(z)s(z)} \frac{T_z'(z) + \tau_s'(z)\, s(z) + s'_{inc}(z)\tau_s(z)}{1-T_z'(z) - \tau_s'(z)\, s(z)}$$

Primary message: $s'_{pref}(z)$ is the key statistic for characterizing optimal tax on s in all of these different systems.

Extension 1: multidimensional heterogeneity

Prop. 5: generalizes Prop. 4
Same measurable statistics are still key to quantifying optimal simple taxes.

- SL, LED: take conditional expectations at each earnings level.[Formula]
- SN: take conditional expectations at each level of *savings*.
- Numerically, we find multidimensionality has modest effects on optimal simple tax rates.

Extension 2: when government wants agents to save more

Prop. 6 Suppose policymaker values savings more than individual.
(Spans present focus, or Farhi Werning (2010) misalignment about bequests.)

$$U\left(c,s,z;\theta\right)=u\left(c;\theta\right)-k\left(z;\theta\right)+\beta v\left(s;\theta\right)$$

- Gov't maximizes $\int_{\Theta}\left[U(c,s,z;\theta)+\nu v(s;\theta)\right]dF(\theta)$
  - e.g., $\nu=1-\beta$
- Generates separable corrective term.

$$\frac{\mathcal{T}_s'\left(s(z),z\right)}{1+\mathcal{T}_s'\left(s\left(z\right),z\right)}=$$
$$s_{pref}'\left(z\right)\frac{1}{\zeta_{s|z}^{c}(z)}\frac{1}{s(z)h_z(z)}\int_{x\geq z}\left(1-\hat{g}(x)\right)dH_z(x)-\underbrace{\frac{\nu(z)}{\beta(z)}g(z)}_{\text{corrective term}}.$$

- $s_{pref}'(z)$ still key statistic for redistributive motive.
- If correction stronger at low z $\rightarrow$ subsidize low savings, more progressive.

Empirical application

Calibrating a model of savings taxes in the U.S.

Model interpretation

- 2 representative periods: work-life, and retirement
- z : labor income during work-life (annualized)
- s : retirement savings (annualized)
- $p = \frac{1}{(1+r)^N}$: price of retirement savings, returns compounded N years
- τ_s, $T_s(s)$, $\tau_s(z)$: remap model to report these as functions of gross retirement savings, measured in 2nd period dollars. [Details]

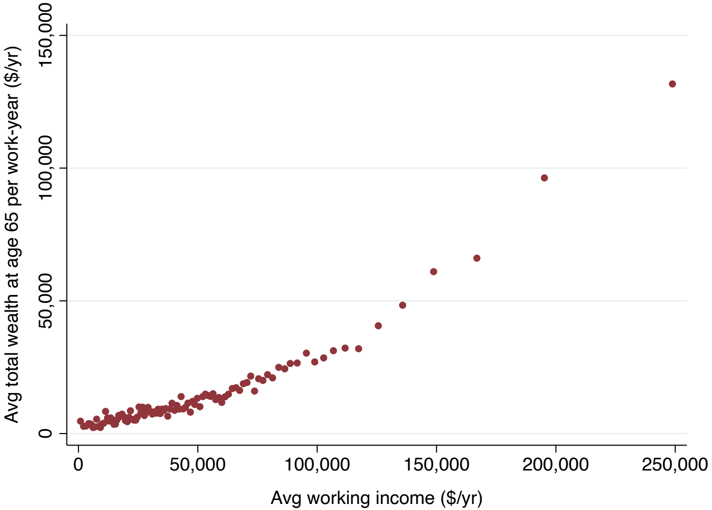
Elasticities

- Compensated earnings elasticity $\zeta_z^c = 0.33$ (Chetty, 2012)
- Compensated savings elasticity $\zeta_{s|z}^c = 1$ (Jakobsen et al, 2020)

Calibration output

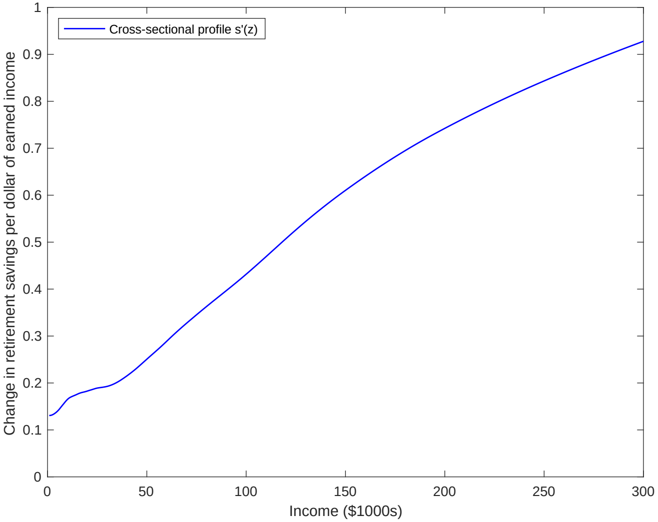
- Compute Pareto-efficiency formulas using observed earnings, savings and income distributions.
- Tests for Pareto efficiency, and approximates optimal simple tax reform. (Not exact: statistics may be endogenous).

Input: cross-sectional savings profile $s(z)$



Source: DINA micro-files for the US (Piketty, Saez, Zucman, 2018)

Slope of cross-sectional savings profile $s'(z)$



Estimating the causal income effect $s'_{inc}(z)$

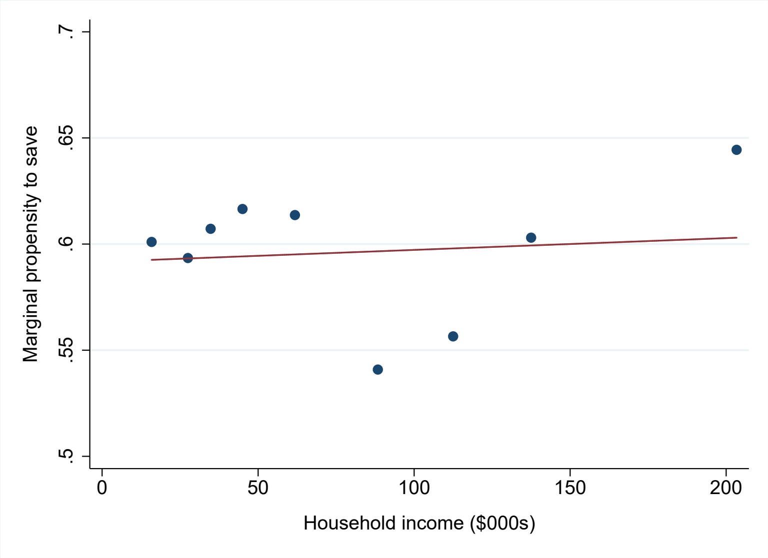
Active area of research. We draw from 2 sources:

1. Fagereng et al. (2020) uses lottery prizes linked with admin data in Norway
   - Estimates 1-year causal MPC of net-of-tax windfall income is 0.52.
   - Estimates a 5-year causal MPC of 0.9, stable across incomes.
   - Imposing that $1 - MPC$ is saved $\Rightarrow$ $s'_{inc}(z) = (1 + r)0.1(1 - T'(z))$
2. New representative survey of US adults.
   - Fielded to 1,703 adults through nationally representative AmeriSpeak panel:

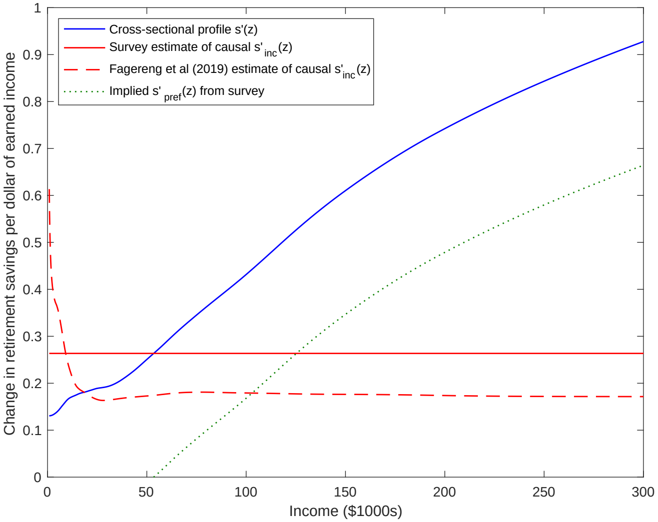
   Imagine you received a raise such that your income was $1000 higher than expected in each of the next 5 years. How much more would you save each year?

   - Asks directly about *savings* response to *earned* income. (Caveats: hypothetical, short-run.)
   - Average short-run MPS = 0.6, consistent with Fagereng et al.

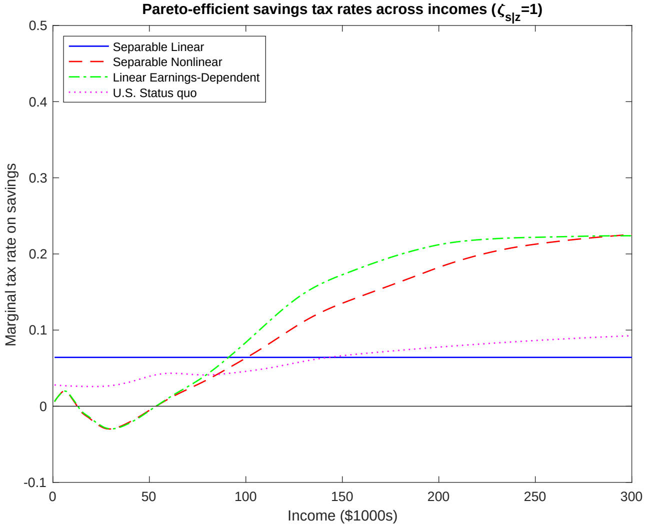
Survey: short-run marginal propensity to save



Calibration input: $s'(z) - s'_{inc}(z) = s'_{pref}(z)$



Savings taxes across incomes



Conclusion

This paper: optimal nonlinear tax systems with preference heterogeneity

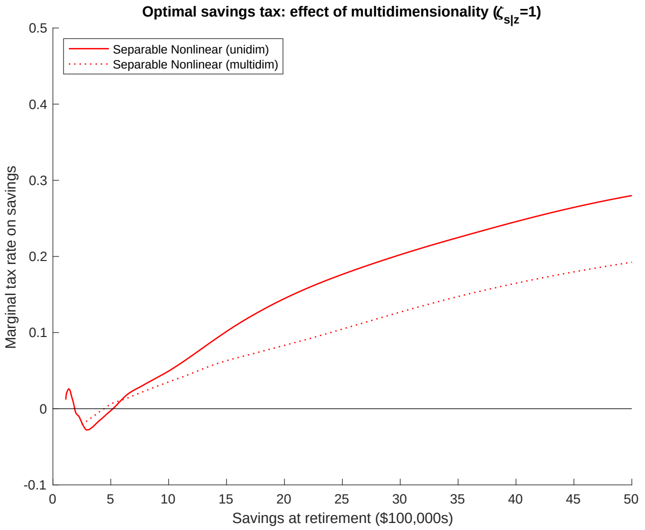
1. Optimal allocation can be implemented with (simple) smooth tax systems
2. General sufficient statistic characterization of optimal nonlinear tax systems
3. Application to savings and capital taxation in the US economy

Take-away: difference between cross-sectional profile and causal income effects is key statistic for optimal tax systems.

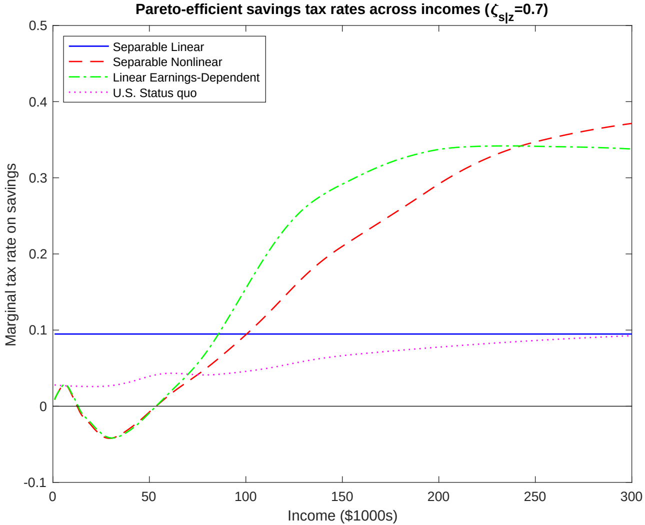
- Driven by intrisic preference heterogeneity and other type-specific factors
- Can complement structural approaches when underlying ability and preferences are difficult to measure.
- Unifies many existing "violations" of Atkinson Stiglitz in a single framework.

Thank you!

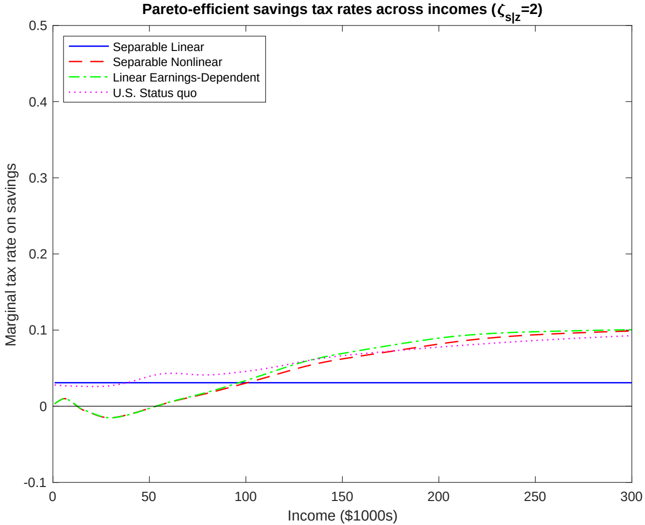
Multidimensionality: optimal Separable Nonlinear tax



Savings taxes across incomes: lower savings elasticity



Savings taxes across incomes: higher savings elasticity



Regularity assumptions

Regularity assumptions on utility

- $U(.)$ is twice continuously differentiable
- Increasing and weakly concave in c and s
- Decreasing and strictly concave in z
- U'_c and U'_s are bounded.

Regularity assumptions for $\mathcal{T}(s,z)$ to implement optimal allocation

Under the optimal incentive-compatible allocation,

- $c(\theta)$, $s(\theta)$, $z(\theta)$ are smooth and strictly increasing functions of θ,
- Any type θ strictly prefers its allocation over any other,
- Defining MRS's $\mathcal{S}(c,s,z;\theta) := \frac{U'_s(c,s,z;\theta)}{U'_c(c,s,z;\theta)}$ and $\mathcal{Z}(c,s,z;\theta) := \frac{U'_z(c,s,z;\theta)}{U'_c(c,s,z;\theta)}$, the extended Spence-Mirrlees condition $\mathcal{Z}'_\theta + \frac{s'(\theta)}{z'(\theta)}\mathcal{S}'_\theta \geq 0$ holds for all θ.

[Back]

Extension 1: multidimensional heterogeneity formulas

Prop. 5: generalizes Prop. 4: $s'_{inc}(s,z)$ is still the key statistic for simple tax systems.

- SL, LED: take conditional expectations at each earnings level, e.g.,

$$\frac{\tau_s}{1+\tau_s}\int_z\left\{\mathbb{E}\left[s\zeta^c_{s|z}(s,z)\middle|z\right]\right\}dH_z(z)=$$
$$\int_z\left\{\mathbb{E}\left[\left(1-\hat{g}(s,z)\right)s\middle|z\right]-\mathbb{E}\left[\frac{T'_z(z)+s'_{inc}(s,z)\tau_s}{1-T'_z(z)}z\zeta^c_z(s,z)\,s'_{inc}(s,z)\middle|z\right]\right\}dH_z(z)$$

[Back]

Remapping $\mathcal{T}(s, z)$ to a tax on gross savings

- In model, $c = z - \frac{1}{1+r}s - \mathcal{T}(s, z)$.
  - Taxes all levied at once, in units of c (in "period 1 dollars").
  - But tax is a function of real net-of-tax savings s (in "period 2 dollars").
- Can re-express our formulas as period-2 tax on gross savings, in two steps.

1. **Express savings tax as function of *gross* savings, in period 1 dollars.**
   - Write tax separably: $\mathcal{T}(s, z) = T_z(z) + T_s(s, z)$.
   - Define gross-of-tax savings $s_g(s) := s + (1 + r)T_s(s, z)$ (monotonic).
   - Define $T_s^g(s_g, z) = T_s(s(s_g), z)$.
   - Prop 12: optimal $\frac{\partial T_s^g(s_g,z)}{\partial s_g}$ formulas are identical to $\frac{\partial T_s(s,z)}{\partial s}$, provided s_g replaces s everywhere (including elasticities).
2. **Express savings tax in "period 2 dollars."**
   - Re-express $T_s(s, z)$ (or T_s^g) in period 2 dollars: $T_2(s, z) := T_s(s, z)(1 + r)$.
   - Then marginal savings tax rates are $\frac{\partial T_2(s,z)}{\partial s} = (1 + r)\frac{\partial T_s(s,z)}{\partial s}$.

[Back]