Monetary policy and endogenous financial crises

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July 2, 2021

Very Preliminary

Abstract

Excessive economic booms sometimes lead to financial crises and severe recessions. We study whether central banks should systematically curb such booms to prevent crises, *i.e.* "lean against the wind" (LAW), even though this may come at the cost of inefficient fluctuations in output. We tackle this question within a textbook New Keynesian model augmented with endogenous capital accumulation and micro–founded financial crises. The model is solved globally to allow the economy to depart persistently from its steady state and feature boom–driven crises. We consider several LAW interest rate rules, under which the central bank responds more forcefully to economic booms than under the standard Taylor rule (STR) or —*a fortiori*— strict inflation targeting (SIT). Our main results are threefold. First, monetary policy affects the probability of a crisis both in the short run and in the medium run (*i.e.* over multiple years), through its effects on savings and capital accumulation. Second, the welfare gain of LAW arising from financial stability exceeds the cost due to aggregate demand externalities and nominal rigidities. Welfare is thus higher under LAW than under STR or SIT. Third, while rule–based leaning helps to avert crises, leaning discretionarily and late in a boom is conducive to financial instability.

Keywords: Financial crises, monetary policy, lean against the wind.

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[¶]The views expressed in this paper are our own and should not be interpreted as reflecting the views of the Bank for International Settlements, the Deutsche Bundesbank, or the Eurosystem. We thank F. Alvarez, C. Borio, P. Disyatat, E. Mendoza, P. Rungcharoentkitkul, E. Zakrajšek and seminar participants at the Deutsche Bundesbank and the Bank for International Settlements for helpful discussions. Galí acknowledges financial support from the European Research Council under the European Union's Horizon 2020 research and innovation program, Advanced Grant (882332–HEMPEF), from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in RD (CEX2019–000915–S), and from the Generalitat de Catalunya, through CERCA and SGR Programme (2017–SGR-1393).

1 Introduction

The last fifteen years have shown that central banks can stave off financial crises but also inadvertently foster the macro–imbalances that typically bring them about. In response to the Covid–19 shock, for example, central banks swiftly lowered interest rates and acted as lenders–of–last–resort to the financial sector. These moves likely prevented a financial collapse that would otherwise have exacerbated the damage to the economy. At the same time, empirical evidence shows that, by keeping their policy rates too low for too long, central banks may entice the financial sector to search for yield and take excessive risk. And indeed, loose monetary policy is sometimes regarded as one of the causes of the 2007–8 financial crisis.[1]

The effects of monetary policy on financial stability are complex, and likely depend on the nature of the shocks that hit the economy and the overall macro–economic context. To study these effects, we develop a New Keynesian (NK) model that speaks to the conduct of conventional monetary policy when financial markets are fragile and financial crises have multiple causes —ranging from adverse exogenous shocks to endogenous macro–economic imbalances.

Our model departs from the textbook —three–equation— NK model in four important ways. First, it features endogenous capital accumulation, so that the economy may deviate from its steady state and generate protracted booms and imbalances. Second, in our model, firms are subject to idiosyncratic technology shocks, in addition to aggregate ones. This heterogeneity gives rise to a loan market where low productivity firms lend their capital to high productivity firms. Third, we assume financial frictions that make this loan market fragile. One friction is that lenders may not be able to seize the capital stock of a defaulting borrower, thus allowing firms to borrow capital and abscond with it. This moral hazard problem induces lenders to limit the amount of capital that can be borrowed. Another friction is that idiosyncratic technologies are private information. These frictions imply that the loan rate must be above a minimum threshold to entice the least productive firms —whose opportunity cost of absconding is the lowest— to lend their capital rather than borrow and abscond. In turn, when the marginal return on capital is too low, not even high productivity firms can afford paying the minimum loan rate, and the loan market collapses. This is what we call a financial crisis. A crisis is characterized by capital mis–allocation and brings along a severe recession. In our model, the typical crisis breaks out toward the end of a protracted economic boom, when there is excess capital in the economy and the marginal productivity of capital is low. The fourth departure from the textbook model is that we solve our model globally, which allows us to capture the non–linearities embedded in the endogenous booms and busts of the loan market.

[1] Taylor (2011) refers to the period 2003-2005 as the "Great Deviation", which he characterizes as one when monetary policy became less rule–based, less predictable, and loose. Empirical evidence include *e.g.*, Maddaloni and Peydró (2011), Jiménez, Ongena, Peydró, and Saurina (2014). See also Rajan (2011) as well as CGFS (2018) for a recent survey.

We study the conduct of monetary policy in terms of whether and how hard central banks should curb such booms and busts, *i.e.* "lean against the wind" (LAW), compared to a standard Taylor rule (STR).[2] We also compare the performance of the economy under LAW and under a strict inflation targeting (SIT). In our model, LAW is akin to providing the household with an insurance against future aggregate shocks. Such insurance helps them smooth consumption, reduces their need for accumulating savings during booms, and ultimately prevents excess capital accumulation and financial crises. As these effects go through agents' expectations, they require that the central bank commit itself to leaning systematically, and only materialize themselves over multiple years.

Our main results are threefold. First, monetary policy affects the probability of a crisis not only in the short run (*i.e.* at business cycle frequency), through its usual effects on output and inflation, but also in the medium run, through its effects on savings and capital accumulation. Second, the welfare gain of LAW from addressing financial externalities (*i.e.* financial instability) may exceed the cost due to aggregate demand externalities and nominal rigidities (*i.e.* inefficient fluctuations in output). Welfare is thus higher under some LAW rules than under STR or SIT. Third, while rule–based leaning helps to avert crises, leaning discretionarily and late in a boom is conductive to financial instability.

The paper bridges two main strands of the literature. The first is on monetary policy and financial stability. The papers the closest to ours are Woodford (2012) and Gourio, Kashyap, and Sim (2018).[3] Like them, we introduce endogenous crises in a standard NK framework. The main difference is that they use reduced forms to determine how macro–financial variables (*e.g.* credit gap, credit growth, leverage) affect the likelihood of a crisis, whereas in our case financial crises —including their probability and size— are micro–founded and derived from first principles. This has important consequences in terms of the prescriptions of the model. One is that, in our model, monetary policy also influences the size of the recessions that follow crises, and therefore the welfare cost of the latter —a key element to determine *whether* central banks should lean. Another is that, even though crises can be seen as credit booms "gone wrong", as documented in Schularick and Taylor (2012), not all booms are equally "bad" and conducive to crises in our model (see also Gorton and Ordoñez (2019)) —a key element to determine *when and how hard* to lean. More generally, our findings do not depend on any postulated reduced functional form for the probability of a crisis. Ours can therefore be seen as a canonical NK model with endogenous financial crises. The second strand of the literature relates to quantitative macro–financial models with micro–founded endogenous financial crises (Boissay, Collard, and Smets (2016), Gertler, Kiyotaki, and Prestipino (2019)). As in Boissay, Collard, and Smets (2016), crises take the form of a collapse of a wholesale

[2] Our main focus will be on symmetric interest rate rules whereby central banks lean against both head– and tail–winds.

[3] See Smets (2014) for a review of the literature as well as Filardo and Rungcharoenkitkul (2016), Svensson (2017), Ajello, Laubach, López-Salido, and Nakata (2019), Bernanke and Gertler (2000) and Galí (2014).

funding market. One important difference is that we add nominal price rigidities, which allows us to study the link between monetary policy and financial stability.

Though in a more indirect way, our paper is also connected to recent works on how changes in monetary policy rules affect economic outcomes in the medium term (*e.g.* Borio, Disyatat, and Rungcharoenkitkul (2019), Beaudry and Meh (2021)) as well as to works on the link between firms' financing constraints and factor mis–allocation. In particular, the notion that financial crises impair capital re–allocation chimes well with the narrative of the 2007–8 financial crisis in the US and the literature that shows that a great deal of the recession that followed this crisis can be explained by a lack of creative destruction (Foster, Grim, and Haltiwanger (2016), Argente, Lee, and Moreira (2018)) —notably due to new entrants' inability to borrow (Campello, Graham, and Harvey (2010)).

The paper proceeds as follows. Section 2 presents our theoretical framework, with a focus on the micro–foundation of endogenous financial crises. Section 3 describes and parses the channels through which monetary policy affects financial stability. Sections 4 and 5 present the typical dynamics leading to financial crises and the effects of LAW on financial stability and welfare. In Section 6, we contrast the effects of leaning systematically versus discretionarily, as well as the effects of leaning asymmetrically against tail– or headwinds. A last section concludes.

2 Model

The economy is populated with a representative household, a sector of monopolistically competitive retailers, a sector of competitive firms, and a central bank. Our model is a version of the textbook NK model (Galí (2015)) with firms, retailers, nominal rigidities à la Rotemberg, and capital accumulation. The production sector, which is subject to idiosyncratic technology shocks, is the only non standard part of the model.

2.1 Agents

2.1.1 Representative household

The representative household is infinitely lived. Each period the household supplies N_t work hours, consumes C_t units of final goods, and invests their savings into a one–period riskless nominal bond, B_{t+1}, as well as in intermediate good firms' equity K_{t+1}, in order to maximize their expected inter–temporal lifetime utility:[4]

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}}{1-\sigma}-\vartheta\frac{N_t^{1+\nu}}{1+\nu}\right)\right]$$

subject to the sequence of budget constraints

$$P_tC_t+B_{t+1}+P_tK_{t+1}\leq P_t\omega_tN_t+(1+i_{t-1})B_t+P_t(1+r_t^k)K_t+\mathscr{X}_t$$

[4]In what follows, $\mathbb{E}_t(\cdot)$ denotes the expectation operator over the aggregate shocks conditional on the information set available at the end of period t.

for $t = 0, 1, 2...$, where $C_t \equiv \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j\right)^{\frac{\epsilon}{\epsilon-1}}$ is a standard Dixit–Stiglitz consumption index of differentiated goods with $\epsilon > 0$ a measure of their substitutability, $P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon} \mathrm{d}j\right)^{\frac{1}{1-\epsilon}}$ is the unit price of the consumption basket, ω_t is the real wage, i_{t-1} is the nominal interest rate on the bonds purchased at $t-1$ (determined in $t-1$), r_t^k is the real return on firm equity, and $\mathscr{X}_t$ are other sources of income that are of a lump–sum nature.[5] The optimality conditions describing the household's behaviour are given by (together with the transversality condition):

$$\beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (1 + r_{t+1}) \right] = Z_t \tag{1}$$

$$\beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(1 + r_{t+1}^k\right) \right] = 1 \tag{2}$$

$$\vartheta N_t^{\nu} C_t^{\sigma} = \omega_t \tag{3}$$

where $1 + r_{t+1} \equiv (1 + i_t)/(1 + \pi_{t+1})$ is the real gross rate of return on bonds, with $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$ the inflation rate, and Z_t is a demand shock à la Smets and Wouters (2007) that follows an exogenous AR(1) process $\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \varepsilon_t^z$, with $\rho_z \in [0, 1)$. The innovation ε_t^z is realized at the beginning of period t.

2.1.2 Central bank

In our baseline economy, the central bank sets its policy rate according to the standard (quarterly) Taylor rule (STR):

$$1 + i_t = \frac{1}{\beta}(1 + \pi_t)^{1.5} \left(\frac{Y_t}{Y} \right)^{0.125} \tag{STR}$$

where Y_t is aggregate output in period t and Y is the average aggregate output in the stochastic steady state (see Taylor (1993)).[6]

2.1.3 Retailers

A continuum of retailers purchase intermediate goods from firms (described in next section) at price p_t, differentiate them, and resell them in a monopolistically competitive environment subject to nominal price rigidities. Retailers live infinitely and are indexed by $j \in [0, 1]$. Each retailer j sells $Y_t(j)$ units of a differentiated final good j and sets their price $P_t(j)$ subject to Rotemberg–style adjustment costs $\frac{\varrho}{2} P_t Y_t \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 \geq 0$, where $Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j\right)^{\frac{\epsilon}{\epsilon-1}}$ is aggregate output —in which the adjustment costs are expressed. Retailers' objective is to choose their price $P_t(j)$ and quantity $Y_t(j)$ so as to maximize their expected stream of future profits:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \Lambda_{0,t} \left(\frac{P_t(j)}{P_t} Y_t(j) - \frac{p_t}{P_t} Y_t(j) - \frac{\varrho}{2} Y_t \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 \right) \right]$$

[5] These lump–sum transfers include retailers' profits and rebated menu costs.

[6] The average inflation rate in the steady state is implicitly set to $\overline{\pi} = 0$.

subject to the sequence of demand schedules

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t \tag{4}$$

where $\Lambda_{t,t+k} \equiv \beta^k \left(C_{t+k}/C_t\right)^{-\sigma}$ is the stochastic discount factor. In the symmetric equilibrium, the first–order optimality condition yields:

$$(1+\pi_t)\pi_t = \mathbb{E}_t\left(\Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}(1+\pi_{t+1})\pi_{t+1}\right) - \frac{\epsilon-1}{\varrho}\left(1-\frac{\epsilon}{\epsilon-1}\frac{1}{\mathscr{M}_t}\right) \tag{5}$$

where

$$\mathscr{M}_t \equiv \frac{P_t}{p_t} \tag{6}$$

is retailers' average markup.

2.1.4 Firms

Firms are at the core of our model. They produce the intermediate goods that retailers differentiate, are perfectly competitive, and live one period only. Firms operating in any given period t are born at the end of period $t-1$, and experience idiosyncratic productivity shocks at the beginning of period t. More precisely, a firm hit by shock q (for short "firm q") has access to a technology represented by the production function

$$y_t(q) = A_t(qK_t(q))^{\alpha} N_t(q)^{1-\alpha} \tag{7}$$

where $K_t(q)$ and $N_t(q)$ denote the capital stock and labor input, A_t is an aggregate technology shock that evolves exogenously over time according to an AR(1) process $\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_t^a$, and q is distributed over interval $[0,1]$ with cumulative distribution $\mu(q)$. To fix ideas, it is useful to think of $qK_t(q)$ as the amount of *effective* capital: insofar as firm q can only use a fraction q of its capital stock productively, its effective capital stock is $qK_t(q)$. Capital goods take the form of a Dixit–Stiglitz bundle of final goods identical to that defining the composite consumption good, and depreciate at rate δ.

When they are born at the end of period $t-1$, firms are identical, purchase the same amount of capital goods K_t at price P_{t-1}, and finance these purchases by issuing equity.[7] The productivity shocks at the aggregate (A_t) and at the firm (q) level are realized at the beginning of period t. Upon observing these shocks, firms may re–scale their capital stock, by purchasing or selling capital goods — depending on their respective qs. To fund any gap between their desired capital, $K_t(q)$, and their initial capital stock, K_t, they can use a loan market. Thus, $K_t(q) - K_t$ represents the net borrowing (resp. lending) by firm q if $K_t(q) > K_t$ (resp. $K_t(q) < K_t$). The real interest rate on the resulting loan market is denoted r_t^{ℓ}, and expressed in terms of the final good bundle.[8] Loans are

[7] Part of that capital stock is purchased from exiting firms (an amount $(1-\delta)K_{t-1}$), with the rest consisting of purchases of final goods produced in period $t-1$ (i.e. new investment).

[8] In effect, a loan is akin to an outright sale of capital goods on credit, and the loan rate is net of capital depreciation.

paid back at the end of period t. To recap, firms thus have access to two external financial markets: an inter–period equity market at the end of period $t-1$, and an intra–period loan market at the beginning of period t.

Once shocks have been realized and its capital stock re–scaled, firm q hires labor $N_t(q)$ and starts the production of intermediate goods. At the end of period t, it sells its production $y_t(q)$ to retailers at price p_t (which is taken as given by both sides), sells its un–depreciated capital $(1-\delta)K_t(q)$ at price P_t, pays the workers at real wage rate ω_t, reimburses (or is reimbursed) its loans, and distributes dividends

$$\frac{1}{\mathscr{M}_t} y_t(q) - \omega_t N_t(q) + (1-\delta)K_t(q) - (1+r_t^\ell)(K_t(q) - K_t)$$

to its shareholders. Dividing the above expression by K_t, subtracting 1, and re–arranging the terms yields (by definition) firm q's net real return on equity

$$r_t^k(q) \equiv \frac{1}{\mathscr{M}_t}\frac{y_t(q)}{K_t} - \omega_t \frac{N_t(q)}{K_t} - (r_t^\ell + \delta)\frac{K_t(q) - K_t}{K_t} - \delta \tag{8}$$

Firm q's objective is to maximize $r_t^k(q)$ by choosing $K_t(q)$ and $N_t(q)$ optimally. We conjecture that in equilibrium all firms with productivity $q \geq q_t^*$ will choose to produce (*i.e.* $N_t(q) > 0$), while those with $q < q_t^*$ will not produce (*i.e.* $N_t(q) = 0$), where q_t^* will be determined endogenously.

Consider first the choice of $N_t(q)$ given $K_t(q)$. If firm q chooses $N_t(q) = 0$, its return on equity will be

$$r_t^k(q) = -(r_t^\ell + \delta)\frac{K_t(q) - K_t}{K_t} - \delta \tag{9}$$

Then, as long as $r_t^\ell > -\delta$, the firm finds it optimal to lend all its capital, in which case $K_t(q) = 0$ and its return on equity is $r_t^k(q) = r_t^\ell$. If $r_t^\ell = -\delta$, the return from the loan is the same as if the firm keeps its capital idle and sells it at the end of the period. In this case the firm is indifferent between the two options, and its return on equity is $r_t^k(q) = -\delta$.[9] Accordingly, the return on equity for firms with productivity $q < q_t^*$ is $r_t^k(q) = r_t^\ell$.

If instead the firm chooses to produce, *i.e.* $N_t(q) > 0$, then its optimal labor hiring will satisfy

$$\omega_t = \frac{1-\alpha}{\mathscr{M}_t}\frac{y_t(q)}{N_t(q)} \Leftrightarrow A_t^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\mathscr{M}_t \omega_t}\right)^{\frac{1-\alpha}{\alpha}} = \frac{y_t(q)}{qK_t(q)} \equiv \Phi_t \tag{10}$$

where Φ_t is firm q's productivity per unit of effective capital. Since technology has constant returns to scale, Φ_t is the same across all productive firms and independent of q.

To determine which firms produce and which ones don't, it is convenient to rewrite firm q's return on equity $r_t^k(q)$ in (8) —for all $q \in [0,1]$— in terms of the average return on equity r_t^k (see

[9]It should be clear that r_t^ℓ cannot be strictly below $-\delta$ in equilibrium insofar as, in this case, a firm always prefers to keep its capital idle rather than lend it out. Hence, we ignore this case.

details in Appendix 8.1):

$$r_t^k(q) + \delta = \frac{q}{\overline{q}_t} \frac{K_t(q)}{K_t}(r_t^k + \delta) - (r_t^\ell + \delta)\frac{K_t(q) - K_t}{K_t} \tag{11}$$

where $r_t^k \equiv \int_0^1 r_t^k(q)\mathrm{d}\mu(q)$ and

$$\overline{q}_t \equiv \int_{q_t^*}^1 q\frac{K_t(q)}{K_t}\mathrm{d}\mu(q) \tag{12}$$

is the weighted average of idiosyncratic productivity shocks across productive firms, using the re–scaling factors $K_t(q)/K_t$, as weights (with the latter equal to zero for inactive firms). Expression (11) emphasizes that firm q's equity return relative to the sector's average depends not only on its relative productivity (term $q/\overline{q}_t$), but also on its ability to leverage and scale up (term $K_t(q)/K_t$). Since $\partial r_t^k(q)/\partial K_t(q) > 0 \Leftrightarrow q(r_t^k + \delta) > \overline{q}_t(r_t^\ell + \delta)$, firm q will produce if and only if:

$$q \geq \overline{q}_t \left(\frac{r_t^\ell + \delta}{r_t^k + \delta}\right) \equiv q_t^* \tag{13}$$

in which case it will leverage itself up as much as possible. Otherwise, if $q < q_t^*$ the firm will either lend out its capital (if $r_t^\ell > -\delta$), or will be indifferent between lending it and keeping it idle (if $r_t^\ell = -\delta$).

Before turning to the description of the loan market equilibrium, two remarks are in order. First, note that $r_t^k(q) \geq r_t^\ell$ since any firm can always choose to stay idle, lend its capital and get a return r_t^ℓ. Accordingly, it must be the case that $r_t^k \geq r_t^\ell$ and, hence, $\overline{q}_t \geq q_t^*$. Second, for what follows, it is useful to express aggregate intermediate good output y_t as a function of the sector's average idiosyncratic productivity $\overline{q}_t$, and to express the sector's overall return on equity r_t^k as a function of y_t and $\mathscr{M}_t$. Using the definitions of Φ_t and $\overline{q}_t$ in (10) and (12), one gets

$$y_t \equiv \int_0^1 y_t(q)\mathrm{d}\mu(q) = \int_{q_t^*}^1 qK_t(q)\Phi_t\mathrm{d}\mu(q) = \Phi_t\overline{q}_tK_t \tag{14}$$

where $\overline{q}_tK_t$ is the sector's effective capital stock. Further, integrating (8) over $[0, 1]$ yields (see details in Appendix 8.1)

$$r_t^k = \frac{\alpha}{\mathscr{M}_t}\frac{y_t}{K_t} - \delta \tag{15}$$

2.2 Equilibrium in a Frictionless Loan Market

A useful benchmark is given by the case of a frictionless loan market, where firms' productivity shocks can be observed by all potential lenders, and where loan contracts are fully enforceable, with no constraints on the amounts that a firm can borrow. In that case all firms with productivity $q > q_t^*$ will seek to borrow an infinite amount of capital. Thus, for a competitive equilibrium to exist the borrowing rate r_t^ℓ ought to rise until firms with the highest productivity (*i.e.* with $q = 1$)

break even with the marginal unit of capital borrowed, crowding out all other firms. From (11) this requires

$$\frac{\partial r_t^k(1)}{\partial K_t(1)} = 0 \Leftrightarrow \frac{r_t^k + \delta}{\overline{q}_t} = r_t^\ell + \delta$$

and therefore $q_t^* = \overline{q}_t = 1$, implying that the entire production of the intermediate good is carried out by the firms with $q = 1$. Firms with $q \in [0, 1)$ lend their capital stock to firms with $q = 1$, and have a return on equity of $r_t^k(q) = r_t^\ell$. Thus,

$$r_t^k(q) = r_t^\ell = r_t^k = \frac{\alpha}{\mathscr{M}_t} \frac{y_t}{K_t} - \delta$$

for all $q \in [0, 1]$, *i.e.* the loan market perfectly hedges firms against idiosyncratic technology shocks. Moreover, since $r_t^\ell > -\delta$ no firm keeps its capital idle, and the entire capital stock of the economy is used efficiently. Total output of the intermediate good satisfies $y_t = \Phi_t K_t$, which we can rewrite using the definition of Φ_t in (10) and rearranging terms as

$$\mathscr{M}_t = \left(\frac{1-\alpha}{\omega_t}\right) \left(\frac{y_t}{K_t}\right)^{-\frac{\alpha}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} \tag{16}$$

This equation can be interpreted as determining the average markup as a function of the real wage, aggregate technology, the aggregate capital stock and total output of the intermediate good, with the latter being ultimately determined by aggregate demand. This relation and the remaining equilibrium conditions for aggregate variables result in a system that corresponds to that of the standard NK model with a representative intermediate good firm and endogenous capital accumulation.

2.3 Equilibrium in a Loan Market with Financial Frictions

Next we consider the case of financial frictions arising from asymmetric information, when lenders cannot observe a firm's productivity q and hence cannot assess its incentives to stay idle and abscond. As we show next, these frictions imply an upper bound on the leverage ratio of any individual firm.

Suppose that a firm with productivity $q < q_t^*$ were to borrow as much as allowed and abscond, keeping its entire capital idle, reselling it at the end of the period, and defaulting on its loan.[10] The implied return for that firm would be $P_t(1 - \delta)K_t(q)$. That firm will not abscond as long as this return is smaller than the return $P_t(1 + r_t^\ell)K_t$ from lending out all its capital in the loan market (which is the best alternative for firms with $q < q_t^*$). A firm's leverage ratio must therefore satisfy the incentive compatibility constraint:

$$\frac{K_t(q) - K_t}{K_t} \leq \frac{r_t^\ell + \delta}{1 - \delta} \equiv \overline{\phi}_t \tag{17}$$

for all q. If the above condition is satisfied, all firms will refrain from borrowing and absconding. Those with productivity $q < q_t^*$ will prefer to lend all their capital at the loan market rate r_t^ℓ, while

[10] We assume that keeping the capital idle is a requirement for absconding.

those with productivity $q \geq q_t^*$ will obtain a higher return by expanding their capital, leveraging up as much as possible (*i.e.* choosing $K_t(q) = (1+\overline{\phi}_t)K_t$) and producing and selling the intermediate good. The incentive–compatibility constraint (17) implies that the maximum leverage ratio is increasing in r_t^ℓ: the higher is the interest rate in the loan market, the higher is the opportunity cost of borrowing and absconding, and hence the higher is the leverage ratio consistent with the incentive not to default.

We are now in the position to express the productivity threshold q_t^* as a function of r_t^ℓ. Using (12) and (17), we obtain

$$\overline{q}_t = (1+\overline{\phi}_t)\int_{q_t^*}^1 q\mathrm{d}\mu(q) = \frac{1+r_t^\ell}{1-\delta}\int_{q_t^*}^1 q\mathrm{d}\mu(q)$$

Further, defining

$$h(q_t^*) \equiv \frac{q_t^*}{\int_{q_t^*}^1 q\mathrm{d}\mu(q)}$$

we can substitute $\overline{q}_t$ in (13) to obtain

$$h(q_t^*) = \left(\frac{1+r_t^\ell}{1-\delta}\right)\left(\frac{r_t^\ell+\delta}{r_t^k+\delta}\right) \tag{18}$$

Given that $h'(q_t^*) > 0$ and $h(0) = 0$, equation (18) implies a positive relationship between q_t^* and r_t^ℓ given r_t^k, with $q_t^* = 0$ when $r_t^\ell = -\delta$ and $q_t^* = 1$ when $r_t^\ell = r_t^k(1)$ (as in this case not even the firms with $q = 1$ gain from leveraging up). Intuitively, a higher loan rate increases the number of firms that prefer to lend capital rather than produce, and is therefore associated associated with a higher q_t^*. Relation (18) also implies a negative relation between q_t^* and r_t^k given r_t^ℓ, reflecting the fact that an increase in the overall return on capital increases the number of firms that want to borrow, expand their capital and produce, instead of lending their capital in the loan market.

Consider next the equilibrium in the loan market, conditional on a given r_t^k. The supply of loans (normalized by the aggregate stock of capital K_t) is given by $L_t^S(r_t^\ell; r_t^k) = \mu(q_t^*)$ when $r_t^\ell > -\delta$ and by the interval $[0, \mu(0)]$ when $r_t^\ell = -\delta$, since in the latter case firms with $q = 0$ are indifferent between lending out their capital or keeping it idle. The blue line in Panel (a) of Figure 1 represents that loan supply schedule. For $r_t^\ell \in (-\delta, r_t^k(1)]$ there is a positive relation between r_t^ℓ and the (normalized) supply $\mu(q_t^*)$.

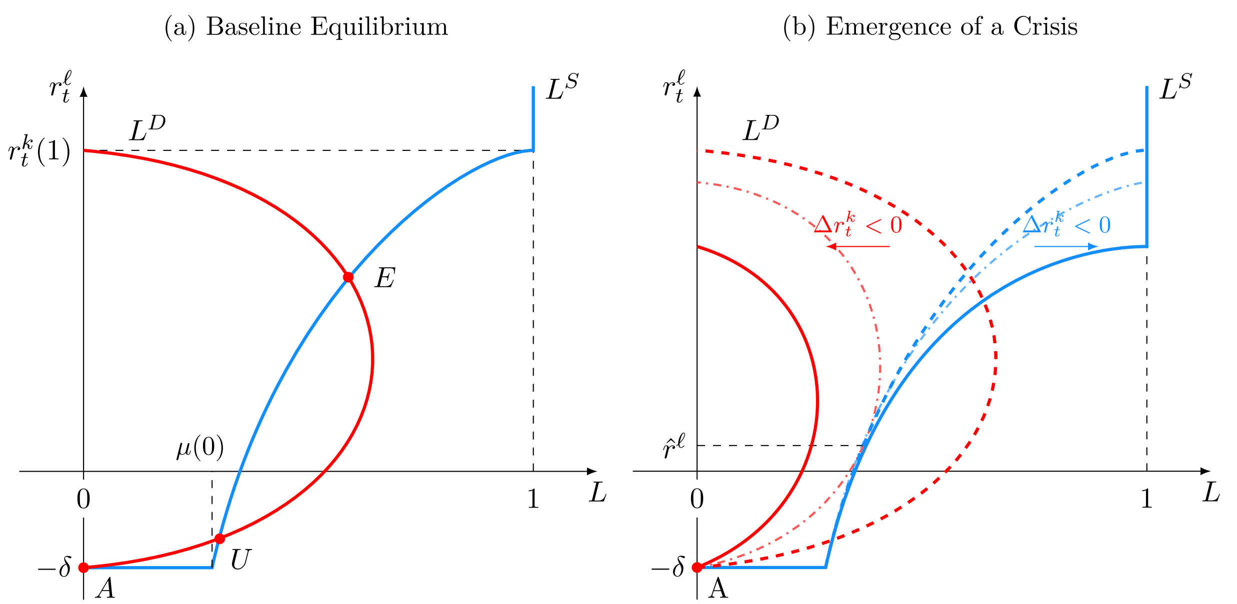
On the other hand, the (normalized) loan demand is given by $L_t^D(r_t^\ell; r_t^k) = (1-\mu(q_t^*))\overline{\phi}_t = (1-\mu(q_t^*))\frac{r_t^\ell+\delta}{1-\delta}$. An increase in r_t^ℓ affects the demand for loans through two channels with opposite effects on its extensive and intensive margins. On the extensive margin, aggregate demand goes down: a higher r_t^ℓ raises firms' opportunity cost of producing (as opposed to lending their capital in the loan market), which lowers $1-\mu(q_t^*)$, the measure of firms that are active producers and borrow to expand their capital. On the intensive margin, aggregate demand goes up: a higher

r_t^ℓ relaxes the incentive compatibility constraint, allowing for an increase in the leverage ratio $\overline{\phi}_t = \frac{r_t^\ell+\delta}{1-\delta}$ of each active firm. The demand for loans from active firms is zero when $r_t^\ell = -\delta$ (as the incentive–compatibility constraint implies $\overline{\phi}_t = 0$ in that case) and when $r_t^\ell = r_t^k(1)$ (since not even the most productive firms find it profitable to borrow at that rate). For $r_t^\ell \in (-\delta, r_t^k(1))$, the demand for loans is strictly positive and its relationship with r_t^ℓ is non–monotonic. The red line in Panel (a) of Figure 1 represents that backward bending loan demand schedule under the assumption that there is a range of interest rates values for which that demand exceeds the loan supply. Formally this corresponds to the case when

$$\max_{r_t^\ell \in [-\delta, r_t^k(1)]} L_t^D(r_t^\ell; r_t^k) - L_t^S(r_t^\ell; r_t^k) > 0 \tag{19}$$

In this case, there exist three possible loan market equilibria, denoted by A, U, and E in the figure. We focus our discussion here on equilibria A and E which, unlike U, are stable under tatônnement.

Figure 1: Loan market equilibrium



Note: This figure illustrates the aggregate loan supply (blue) and incentive–compatible aggregate demand (red) curves (normalized by K_t). In Panel (a), equilibrium U is unstable, whereas A and E are stable. In Panel (b), the dashed and dash–dotted lines are associated with values of r_t^k above or equal to $\hat{r}^k$, and E and A as stable equilibria. The solid lines are associated with a value of r_t^k strictly below $\hat{r}^k$ and A as unique stable equilibrium. The threshold loan rate $\hat{r}^\ell$ corresponds to the equilibrium loan rate when the supply and demand curves (dash–dotted lines) are tangent. This threshold is constant over time, and can be seen as the minimum incentive–compatible loan rate that firms require in order to lend. The figure assumes $\mu(0) > 0$. When $\mu(0) = 0$, the blue curve shifts leftward until U coincides with A, and E becomes the only stable equilibrium.

Consider equilibrium A (for "autarky"), at which $r_t^\ell = -\delta$. In that case, only firms with $q = 0$ are willing to lend out capital, with any quantity in the interval $[0, \mu(0)]$ being consistent with optimal firm behavior. However, the incentive compatibility constraint in that case prevents firms with $q > 0$ from borrowing any positive amount ($\overline{\phi}_t = 0$). As a result all firms keep their (similar) initial capital stock unchanged and produce with that amount of capital, *i.e.* $K_t(q) = K_t$ for all q. In that case $\overline{q}_t \equiv \int_0^1 q\mathrm{d}\mu(q) \equiv \overline{q}^A < 1$, where $\overline{q}^A$ is the unconditional mean of idiosyncratic productivity shocks across all firms. The total supply of the intermediate good satisfies $y_t = \overline{q}^A K_t \Phi_t$ implying the markup equation:

$$\mathscr{M}_t = \left(\frac{1-\alpha}{\omega_t}\right)\left(\frac{y_t}{\overline{q}^A K_t}\right)^{-\frac{\alpha}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} \tag{20}$$

Equilibrium E on the other hand features a loan rate $r_t^\ell > -\delta$, as shown in Panel (a). For $r_t^\ell \in (-\delta, r_t^k(1))$ one gets $q_t^* \in (0,1)$, with firms $q < q_t^*$ lending out all of their initial capital to firms $q \geq q_t^*$, each of the latter borrowing an amount $\overline{\phi}_t = \frac{r_t^\ell+\delta}{1-\delta}$. Now we have $\overline{q}_t \equiv (1+\overline{\phi}_t)\int_{q_t^*}^1 q\mathrm{d}\mu(q) \equiv \overline{q}^E$, and the aggregate quantity of intermediate goods satisfies $y_t = \overline{q}^E K_t \Phi_t$, implying the markup equation:

$$\mathscr{M}_t = \left(\frac{1-\alpha}{\omega_t}\right)\left(\frac{y_t}{\overline{q}^E K_t}\right)^{-\frac{\alpha}{1-\alpha}} A_t^{\frac{1}{1-\alpha}} \tag{21}$$

It can be easily checked that $\overline{q}^A < \overline{q}^E < 1$, *i.e.* both equilibria involve an average productivity that is inefficiently low, but more so in the autarkic equilibrium A, since there is no reallocation of capital towards the more productive firms. Furthermore, note that markup equations (16), (20) and (21) are equivalent to those that would obtain in an economy with a representative firm with idiosyncratic productivity q equal to 1, $\overline{q}^A$, and $\overline{q}^E$ respectively.

Finally, consider what happens when the average equity return r_t^k goes down, *e.g.* as a result of an adverse technology shock or a sequence of shocks that increase the capital stock well above its steady state level. As implied by (18), q_t^* will be larger for any given loan rate r_t^ℓ, *i.e.* more firms will be inactive, leading to a positive (resp. negative) shift in the loan supply (resp. demand) schedule. If the decline in r_t^k is large enough, the range of loan rates for which $L_t^D(r_t^\ell; r_t^k) > L_t^S(r_t^\ell; r_t^k)$ may vanish altogether, in which case there is only one possible equilibrium, the autarkic one, A. Panel (b) of Figure 1 illustrates that possibility. Accordingly, there exists a unique and constant threshold $\hat{r}^k$ for r_t^k at which the demand and supply curves are tangent. If $r_t^k \geq \hat{r}^k$ there exist three equilibria; if $r_t^k < \hat{r}^k$ only the autarkic equilibrium A survives. Formally, the threshold $\hat{r}^k$ solves

$$\max_{r_t^\ell \in [-\delta, r_t^k(1)]} L_t^D(r_t^\ell; \hat{r}^k) - L_t^S(r_t^\ell; \hat{r}^k) = 0 \tag{22}$$

Given $\hat{r}^k$, (22) also implicitly determines the minimum loan rate $\hat{r}^\ell$ that is consistent with an non–autarkic equilibrium in the loan market. The threshold $\hat{r}^\ell$ can be interpreted as the minimum incentive–compatible loan rate that firms require in order to lend.

In what follows we assume that when multiple equilibria are present, market participants coordinate on the most efficient one, namely, equilibrium E.[11] When $r_t^k < \hat{r}^k$ that equilibrium

[11] Note that there is another candidate equilibrium with trade, represented by point U in Figure 1. But we rule this one out because it is not tatônnement–stable. An equilibrium loan rate r_t^ℓ is tatônnement–stable if, following any small perturbation to r_t^ℓ, a standard adjustment process —whereby the loan rate goes up (down) whenever there is excess loan demand (supply)— pulls r_t^ℓ back to its equilibrium value (see Mas-Colell, Whinston, and Green (1995), Chapter 17). Since firms take r_t^ℓ as given, tatônnement stability is the relevant concept of equilibrium stability.

disappears, the loan market collapses and the autarkic equilibrium A —the only one left— prevails, characterized by a lower average productivity resulting from a greater capital mis–allocation. We refer to that event as a *financial crisis* (vs. *normal times*). In the following sections we analyze the factors that may trigger a financial crisis, as well as its macroeconomic consequences.[12]

3 General equilibrium

We solve the dynamic stochastic general equilibrium globally. The procedure consists in finding the fixed point solution for agents' expectations and any set of state variables.[13] We proceed in two steps. First, for each state variable, we conjecture agents' expectations and derive the equilibrium outcome for period t. To do so, we check whether the normal times equilibrium E exists (*i.e.* if $r_t^k \geq \hat{r}^k$). If it does, we select it; otherwise, the equilibrium is A. Second, based on the equilibrium outcome, we revise our initial conjecture for the expectations, re–compute the equilibrium, and iterate until the revised expectations coincide with those last conjectured.

3.1 Monetary policy's YMCA channels

The general equilibrium outcome depends not only on whether there is a financial crisis, but also on the household's and retailers' expectations of a financial crisis in the future, which can be summarized by the one–period–ahead crisis probability (using relations (15) and the fact that $y_t = Y_t$ in equilibrium),

$$\mathbb{E}_{t-1}\left(\mathbb{1}\left\{\frac{\alpha Y_t}{\mathscr{M}_t K_t} - \delta < \hat{r}^k\right\}\right) \tag{23}$$

where $\mathbb{1}\{\cdot\}$ is a dummy variable equal to one when the inequality inside the curly braces holds (*i.e.* there is a crisis) and to zero otherwise.

Expression (23) is key to understanding the effects of monetary policy on financial stability in our model. It emphasizes that crises may emerge through a fall in aggregate output (the "Y–channel"), a rise in retailers' markup (the "M–channel"), or excess capital accumulation (the "CA–channel"). Given a (predetermined) capital stock K_t, a crisis is more likely to break out following a shock that lowers output and/or increases the markup. Such a shock needs not to be large to trigger a crisis, if the economy has accumulated a large capital stock.[14] When K_t is high, all other things equal,

[12] Of course, there are several other ways to select the equilibrium. For example, one could include a stochastic sunspot, *e.g.* assume that firms coordinate on equilibrium E (*i.e.* are "optimistic") with some constant and exogenous probability whenever this equilibrium exists. It should be clear, however, that the central element of our analysis is condition $r_t^k \geq \hat{r}^k$ for the existence of E, not the selection of E conditional on its existence. In other terms, our results do not hinge on the equilibrium selection mechanism.

[13] As relations (1), (2), and (5) show, the household and retailers form three types of expectations about period $t+1$, which relate to the returns on bonds and equity, $\beta\mathbb{E}_t\left[u'(C_{t+1})/(1+\pi_{t+1})\right]$ and $\beta\mathbb{E}_t\left[u'(C_{t+1})(1+r_{t+1}^k)\right]K_t$, and to inflation, $\beta\mathbb{E}_t\left[u'(C_{t+1})Y_{t+1}(1+\pi_{t+1})\pi_{t+1}\right]$. The state variables of period t include the capital stock, K_t, the productivity shock A_t, and the demand shock Z_t. For more details about the algorithm used to solve the model, see Appendix 8.4.

[14] This would for example be the case after an unusually long economic boom, as we show later.

firms' real return on equity —and therefore their opportunity cost of absconding— is low, the loan market is fragile, and even a small variation in Y_t or $\mathscr{M}_t$ may trigger a crisis.

One important reason why crises break out despite their implied inefficiencies is that rational agents do not fully internalize the effects of their individual choices on Y_t, $\mathscr{M}_t$ and K_t and, through these variables, on financial fragility. To see this, assume that, for whichever reason, the representative household believes that a crisis —and therefore a severe recession— is looming. To hedge against the recession and smooth consumption, the they tend to save relatively more (or dis–save less), which contributes to increasing (or slowing down the fall in) K_t, and makes the crisis more likely through the CA–channel. Boissay, Collard, and Smets (2016) refer to this externality as "the savings glut externality". In addition, retailers' price setting behaviour may also contribute to the crisis whenever their desire to avoid (privately) costly price adjustments leads them to maintain high markups (the M–channel), thus contributing to an inefficiently low level of activity (the Y-channel), with the resulting low equity return, and the possibility a financial crisis.[15]

As the above discussion suggests, the central bank may affect the probability of a crisis both in the short run and in the medium run (*i.e.* over multiple years). In the short run, it does so through the Y– and M–channels, by adjusting its policy rate in response to shocks. The more forceful its response, the more resilient the loan market. To see this, consider our baseline economy, where the central bank follows a STR and, therefore, responds only moderately to a decline in output or a rise in markups. In the case of a negative demand shock, output falls and markups rise, leading to an increase in the crisis probability via both the Y– and M–channels. In the case of a negative technology shock, output also falls but markups decline —the latter reflecting retailers' inability to pass their higher costs onto prices, leading to a rise in crisis probability via the Y–channel. Under STR, the loan market thus tends to be vulnerable to adverse shocks —especially demand shocks. To improve its resilience, one alternative for the central bank is to commit itself to cutting its policy rate by more than under STR when output falls. Following an adverse shock, the rate cut will mitigate the fall in output. It will also mitigate the rise in markups in the case of a adverse demand shock, and magnify their fall in the case of a technology shock. The upshot is that, whatever their origin (*i.e.* technology or demand shocks), recessions are less likely to trigger a financial crisis when the central bank fights them more forcefully.

In the medium term, monetary policy affects financial stability through its impact on capital accumulation. Under the STR, the central bank commits itself —to some extent— to counteracting future variations in output, which tends to slow down capital accumulation during booms. The more forceful its response, the slower capital accumulation and, all other things equal, the higher the return on equity and the more resilient the loan market in the face of adverse shocks (*i.e.* the

[15] Note that the M– and Y–channels are also related to what Blanchard and Kiyotaki (1987) referred to as aggregate demand externalities.

weaker the CA–channel). Intuitively, smoothing the business cycle helps the central bank address the savings glut externality. This is akin to providing the household with an insurance against future aggregate shocks that helps them smooth consumption and reduces their need for accumulating savings during booms. For this reason, and as we show later, monetary rules that are more aggressive toward variations in output are more effective in lowering the crisis probability.[16] Insofar as capital accumulation takes time, this channel of monetary policy only materializes itself over multiple years.

3.2 Parametrization of the model

We parameterize our model based on quarterly data (see Table 1) and under STR (our baseline). The model is a standard NK model with endogenous capital accumulation, except that firms differ in terms of their technology —their qs. Accordingly, the only non–standard parameters in the model relate to the distribution of the idiosyncratic productivity shocks $\mu(q)$. For the sake of parsimony, we assume that $\mu(q)$ takes the following simple form:

$$\mu(q) = \mu + (1 - \mu)q^{\lambda} \tag{24}$$

As parameters λ and μ govern the dispersion of firm productivity shocks, they also determine the degree of asymmetric information and, therefore, the size of financial frictions in the economy.

Table 1: Parametrization

Parameter	Target	Value
Preferences		
β	4% annual real interest rate	0.989
σ	Logarithmic utility on consumption	1.000
ν	Inverse Frish elasticity equals 2	0.500
ϑ	Steady state hours equal 1	0.757
Technology and price setting		
α	64% labor share	0.289
δ	6% annual capital depreciation rate	0.015
ϱ	Same slope of the Phillips curve as with Calvo price setting	105.000
ϵ	11% markup rate	10.000
Aggregate shocks		
ρ_a	Persistence of TFP	0.950
σ_a	Standard deviation of TFP innovation (in %)	0.700
ρ_z	Persistence in Smets and Wouters (2007)	0.220
σ_z	Standard deviation of risk–premium innovation in Smets and Wouters (2007) (in %)	0.230
Idiosyncratic productivity shocks		
λ	2pp spread in normal times	23.000
μ	The economy spends 10% of the time in a crisis	0.0176

All other things equal, the lower λ or the higher μ, the bigger the mass of low–q firms, the higher the aggregate supply of loans, the lower the equilibrium loan rate, and the higher the probability of

[16]We will discuss this point more extensively when we analyse and compare the effects of several leaning against the wind policies in Sections 5 and 6.

a crisis.[17] Parameters λ and μ also affect the spread between the average equity return r_t^k and the lending rate r_t^ℓ. We set parameters λ and μ jointly so that, in the stochastic steady state of the economy, (i) this spread is on average equal to 2 percentage points (in annualized terms) in normal times and (ii) the economy spends 10% of the time in a crisis. Existing databases on financial crises report that countries spend on average 6.6% (Laeven and Valencia (2018)) to 11.9% (Reinhart and Rogoff (2014)) of the time in a financial crisis —these figures take into account that a crisis typically lasts several quarters. We purposely set our target in the upper range of the distribution (10%) to account for the fact that crisis–fighting policies were deployed in most —if not all— historical crisis episodes, and that the observed time spent in a crisis would likely have been longer without them.[18]

All the remaining parameters are standard. The utility function is logarithmic with respect to consumption ($\sigma = 1$). The parameters of labor dis–utility are set to $\vartheta = 0.757$ and $\nu = 0.5$ so as to normalize hours to one in the deterministic steady state and to obtain an inverse Frish labor elasticity of 2 —this is in the ballpark of the estimates for industrialized countries. We set the discount factor β so that the annualized average return on equity is 4%. The elasticity of substitution between intermediate goods ϵ is set to 10 in order to generate a markup of 11% in the steady state. Given this, we set the capital elasticity parameter α to 0.289 in order to obtain a labor income share of 64% in the steady state. We assume that capital depreciates by 6% per year ($\delta = 0.015$). We set the price adjustment cost parameter to $\varrho = 105$, so that the model generates the same slope of the Phillips curve as in a Calvo pricing model with an average duration of prices of 4 quarters. The parametrization of the technology shock is also standard, with $\rho_a = 0.95$ and $\sigma_a = 0.007$. That of the demand shock is borrowed from Smets and Wouters (2007), with $\rho_z = 0.22$ and $\sigma_z = 0.0023$.

4 Typical path to a crisis

The aim of this section is to describe the typical dynamics around the beginning of financial crises, for an economy that experiences both technology and demand shocks. We proceed in two steps. First, we solve our non–linear model numerically and globally.[19] Second, starting from the stochastic steady state, we feed the model with the two shocks, simulate it over 1,000,000 periods, and identify the crises' starting dates as well as the sequences of technology and demand shocks around them. We then compute the average dynamics 20 quarters around these dates. The outcome is reported in Figure 2.

[17] Our model encompasses the standard NK model with homogeneous firms, which corresponds to the case with $\lambda = +\infty$ and $\mu = 0$, when all firms have $q = 1$.

[18] In Section 5, we show that adequate monetary policy can significantly reduce the time spent in a crisis.

[19] Our model cannot be solved linearly because of discontinuities in the decision rules. And it cannot be solved locally because crises may break out when the economy is far away from its steady state (*e.g.* when K_t is high). Details on the numerical solution method are provided in the appendix.

The first result —reminiscent of Boissay, Collard, and Smets (2016)— that emerges from the analysis is that the typical crisis is not caused by an unusually large adverse exogenous shock. Rather, it occurs on the heels of a protracted economic boom (Figure 2, panel (f)) driven by a long sequence of relatively small positive technology and demand shocks (panels (a) and (b)). Throughout the boom, the economy accumulates capital (panel (c)), which over time gradually exerts downward pressures on firms' realized return on equity (panel (k)).[20] At first, these pressures are more than compensated by the series of favorable exogenous shocks that hit the economy. In the first phase of the boom, firms' realized return on equity and, hence, their opportunity cost to keeping their capital idle is relatively high (*i.e.* above its steady state). The loan market reallocates capital effectively to the most productive firms (panels (g) and (h)), and the probability of a crisis is relatively small (panel (i)). The boom in output ends around five quarters before the crisis (panel (f)), once the sequence of favorable supply and demand shocks runs its course. Just before the crisis, productivity and demand shocks recede and output falls back toward its steady state, leaving firms with excess capital. As a result, firms' return on equity goes down and the probability of a crisis goes up (panel (i)). The crisis eventually breaks out in the face of relatively mild adverse shocks, which on impact reduce the TFP and aggregate demand exogenous components (A_t and Z_t) by around 0.5% (panels (a) and (b)). The typical crisis is characterized by a collapse of the loan market (a fall in $\overline{\phi}_t$), capital mis–allocation (a fall in $\mu(q_t^*)$), and a recession (see also Table 2).

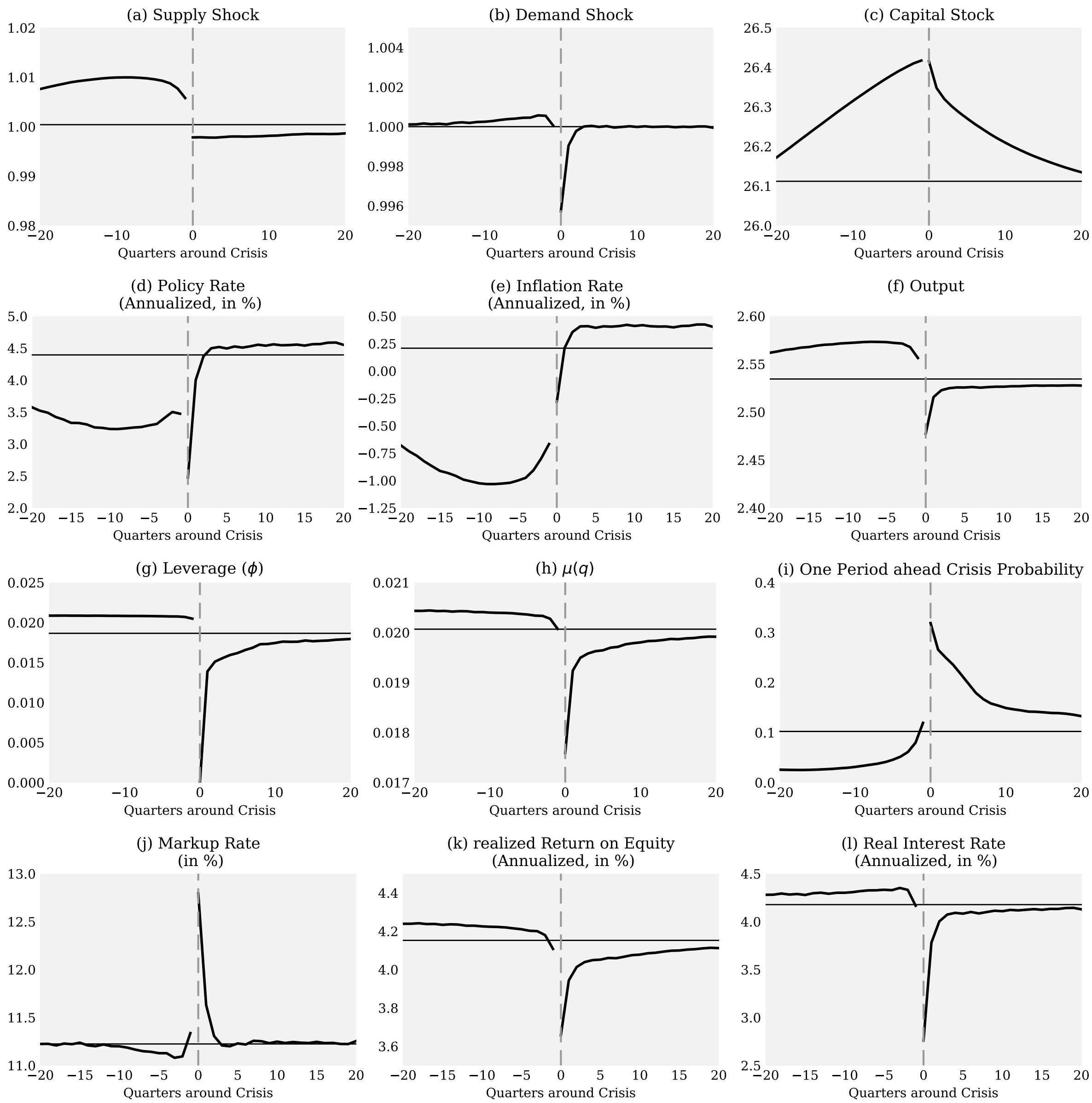
Importantly, these adverse shocks are not the root cause of the crisis but only its trigger, in the sense that an identical sequence of shocks would not have led to a crisis, had the economy (notably the capital stock) been closer to its steady state. The presence of a large capital overhang is thus a necessary condition for a financial crisis, in the absence of large adverse exogenous shocks. Figure 3 illustrates this point. The center panels show the impulse response functions of aggregate output to a negative one–standard deviation technology or demand shock, for moderate (black line), fairly high (orange line) and very high (green line) initial capital stocks.[21] The right hand panels show how the effect of the shock depends on the initial level of capital. When capital is relatively low, a negative shock has a limited effect on output on impact. The economy is initially in normal times and remains there following the shock (black dot). When the capital stock is initially so high that the economy is already in a crisis, the effect is also limited. The economy is initially in a crisis and remains there following the shock (green dot). The interesting case is for intermediate values of the initial capital stock, when the economy is on the brink of a crisis. In this case, a one–standard deviation shock suffices to trigger the crisis, which in turn induces a larger drop in output (orange

[20] Note that there is nothing systematic about the prevalence of boom–driven crises, as the typical crisis could *a priori* have been caused by large adverse shocks. This result can be explained by the asymmetric effects the household's consumption smoothing behaviour on financial stability. Following adverse shocks, the household tends to dis–save, which slows down capital accumulation and, all else equal, augments the resilience of the loan market to adverse shocks (see expression (23)). In good times, in contrast, the economy accumulates more capital, which —all else equal— hampers the resilience of the loan market to adverse shocks.

[21] For the impulse responses of the other variables of the model, see Figure 8.1 in the appendix.

dot). This experiment highlights the central role of capital accumulation in the dynamics of financial crises, and therefore the endogenous nature of the latter. Excess capital accumulation in normal times weighs on firms' realized real return on equity and opportunity cost of keeping their capital idle, impairs the economy's resilience to adverse shocks, and lays the ground for a crisis.

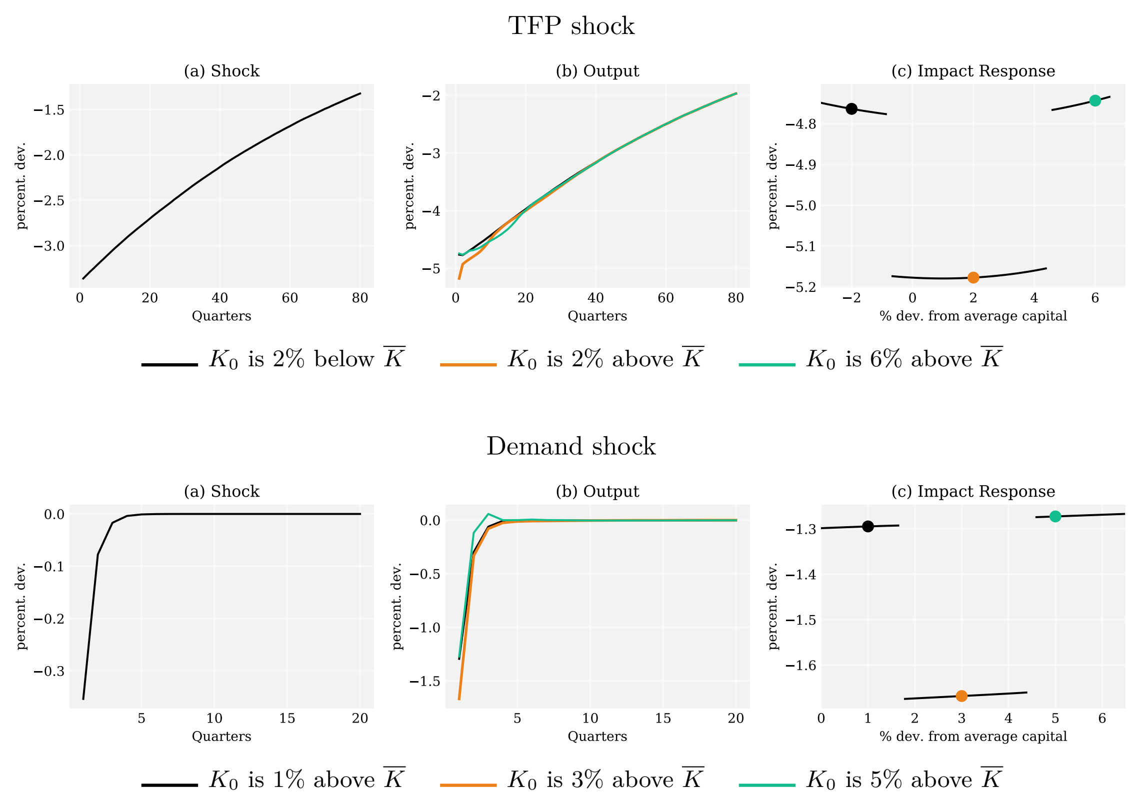
Figure 2: Typical path to crisis



Note: Simulations for the STR economy. Average dynamics of the economy around the beginning of a new crisis (in quarter 0). To filter out the potential noise due to the aftershocks of past crises, we only report averages for new crises, *i.e.* crises that follow at least 20 quarters of normal times. Panels (a) and (b) show the average dynamics of the technology and demand shocks.

Even though in our baseline (STR) economy the central bank commits to respond to *future* variations in output, which helps to slow down capital accumulation and stave off some crises, the economy still spends 10% of the time in a crisis —the CA–channel. How monetary policy stabilizes output and inflation (and markups) in response to *current* adverse shocks is also an important factor of financial (in)stability. Under STR, the central bank lets output decline in the face of both negative technology and demand shocks. This partly explains why crises occur at the end of economic booms —the Y–channel. At the same time, markups increase following negative demand shocks, and decrease following negative technology shocks (see Figure 8.1 in the appendix). These shocks therefore have opposite effects on financial stability through the M–channel in the short run (see Figure 2). Ultimately, whether a crisis breaks out depends on the strength of the YMCA channels and the relative size of the shocks.

Figure 3: Impulse response function of output depends on initial conditions



Note: Simulations for the STR economy. Left hand panel: dynamics of the technology shock following a negative one standard deviation from its average level. Center panel: generalized impulse response function (IRF) of output obtained from 100,000 draws. K_0 and $\overline{K}$ denote, respectively, the level of the capital stock at the time of the shock and the average level of the capital stock in the stochastic steady state. Note that these IRFs take into account that the shock may accelerate or delay the transition toward the steady state, depending on the initial level of capital. Right hand panel: effect of the shock on output on impact (y–axis) depending on the initial level of capital (x–axis).

To get a sense of which type of shock is most conducive to a crisis, we solve and simulate our model separately for two "counterfactual" economies that experience either technology or demand shocks —not both. Figure 8.2 in the appendix, which reports the typical paths to crises in these two economies, shows that technology–driven crises (solid line) tend to follow large booms whereas demand–driven ones (dashed line) do not (panel (c)). This difference is due to the nature of the shocks and the capacity of the loan market to re–allocate a larger capital stock. Following persistent positive technology shocks, the average return on equity goes up and firms have less incentives to keep capital idle. Being more resilient, the loan market can reallocate a larger stock of capital, implying that more capital accumulation is "required" to lead to a crisis. In contrast, demand–driven crises are mostly caused by adverse shocks. One reason is that favorable demand shocks are not persistent enough to generate booms. Since the latter are not accompanied with productivity gains, even a mild boom can lead to a crisis. Another reason is that, as already noted, under STR adverse demand shocks induce both a decline in output and a rise in markups, which makes the economy particularly vulnerable in the short run. The last two rows in Table 2 report the statistics on crises in the two counterfactual economies. We find that demand–driven crises occur with a 1.19% probability and are almost twice as frequent as technology–driven ones (0.72%). However, the latter last much longer (almost eight quarters) than the former (one quarter) and, therefore, also tend to be deeper (with output falling by 5.4%). This analysis thus suggests that the bulk of "crisis time" originates from technology shocks, reflecting the persistence of the latter (see Table 1).

Table 2: Crisis statistics and origins

	% Crisis time	Length	% Nb crises	Output loss
Baseline model	[10.00]	1.86	5.48	−2.73
Model with TFP shocks only	5.53	7.67	0.72	−5.39
Model with demand shocks only	1.25	1.05	1.19	−2.65

Note: The first row reports statistics of the stochastic steady state ergodic distribution in our baseline model with both technology and demand shocks. The second and third rows report the same statistics, when we shut down the demand or technology shocks, *i.e.* in a model with only one type of shock. We use the latter two "counterfactuals" to get a rough sense of the frequency, size and length of the crises that are due to technology or demand shocks in our baseline model. "% Crisis time" is the percentage of the time that the economy spends in a crisis. By construction, it is equal to 10% in the baseline economy, which features technology and demand shocks (square brackets). "Length" is the average duration of a crisis (in quarters). "% Nb crises" is the number of new (distinct) crisis episodes per quarter (in %), *i.e.* the ratio % Crisis time/Length. The output loss is the percentage fall in output from one quarter before the crisis till the trough of the crisis.

The dynamics of the policy rate further informs us about which type of shock prevails during booms. To see this, consider again Figure 8.2. In the economy with technology shocks only, the policy rate is always below its steady state during booms (panel (d)), as the central bank fights the deflationary pressures induced by the productivity gains. In the economy with demand shocks only, in contrast, the policy rate is always (slightly) above its steady state during the boom, as the central banks seeks to address the inflationary pressures induced by excess aggregate demand.

5 Leaning against the wind

The aim of this section is to study whether the central bank should lean against the wind, *i.e.* set its interest rate rule to stabilize not only output and inflation —as under STR, but also to avert financial crises. To study this question, we consider the following “augmented” interest rate rule,

$$1+i_t = \underbrace{\frac{1}{\beta}(1+\pi_t)^{1.5}\left(\frac{Y_t}{Y}\right)^{0.125}}_{\text{STR}} \times \underbrace{\left(\frac{Y_t}{Y}\right)^{\alpha_y}\left(\frac{1+\overline{\phi}_t}{1+\overline{\phi}}\right)^{\alpha_\phi}}_{\text{LAW component}} \tag{LAW}$$

where $\overline{\phi}$ is the average borrowing limit in the stochastic steady state, and $\alpha_y \geq 0$ and $\alpha_\phi \geq 0$. When $\alpha_y = \alpha_\phi = 0$, this rule corresponds to the STR.

Table 3: Interest rate rules

	α_y	α_ϕ
STR	0.000	0.000
LAW–ϕ	0.000	0.389
LAW–$y^{(\mathrm{L})}$	0.343	0.000
LAW–$y^{(\mathrm{M})}$	0.448	0.000
LAW–$y^{(\mathrm{H})}$	0.600	0.000
SIT	$\pi_t = 0\ \forall t$	

Note: The parameters of the LAW–ϕ and LAW–$y^{(\mathrm{L})}$ rules are set so that the economy spends 1.91% of the time in a crisis, which corresponds to the time that a SIT economy spends in a crisis. The parameters of the LAW–$y^{(\mathrm{M})}$ and LAW–$y^{(\mathrm{H})}$ rules are set so that the economy arbitrarily spends 1% and 0.5% of the time in a crisis (see Table 4).

We motivate the above specification by our finding that the typical crisis follows a boom with output and corporate leverage above their steady state level.[22] When $\alpha_y > 0$ or $\alpha_\phi > 0$, the central bank responds more strongly to output or to corporate leverage, as a way to rein in the booms that precede crises. In what follows, we experiment with several values of α_y and α_ϕ, and refer to these rules as LAW rules (see Table 3). In addition, we consider a SIT rule. This benchmark is useful because, under SIT, retailers' markup is constant, the real economy reaches the same outcome as if prices were fully flexible, demand shocks do not have real effects, and all demand–driven crises are eliminated.

5.1 Leaning in the absence of financial frictions

SIT is the optimal monetary policy for an economy with a frictionless loan market and whose only distortion is the presence of sticky prices. TBC

[22] We also considered cases where the central bank responds to the departure of the capital stock from its steady state but —the capital stock being sluggish— such rules do not perform as well in terms of their ability to avert crises.

5.2 Leaning in the presence of financial frictions

The goal of this section is to study the net welfare gain from LAW policies. As a first step, we assess the effects of these policies on financial stability. To do this, we simulate the economy under the rules described in Table 3, and compute statistics on the frequency and severity of crises. Two results stand out (see Table 4). First, under SIT, the central bank reduces the time spent in a crisis down to 1.91% (from 10%) but crises last longer (more than four quarters) and are deeper than under STR (5.8% against 2.73% output loss). Second, under LAW, the central bank can reduce the overall time spent in a crisis further below 1.91%, while keeping the duration or severity of crises essentially unchanged relative to STR. These findings lead us to conclude that LAW is more effective than both STR and SIT to shield the economy against financial crises.

Table 4: Monetary policy rules and financial crises

	Crisis statistics				YMCA channels			
	% Crisis time	Length	% Nb crises	Output loss	$\sigma(Y_t)$	$\sigma(\mathscr{M}_t)$	$\sigma(K_{t-1})$	$\rho(Y_t, \mathscr{M}_t)$
STR	[10]	1.86	5.48	-2.73	4.36	1.07	4.39	-0.06
SIT	1.91	4.47	0.43	-5.84	4.49	0.00	4.90	0.00
LAW–ϕ	[1.91]	1.72	1.10	-1.54	4.14	1.05	4.14	0.16
LAW–$y^{(\mathrm{L})}$	[1.91]	1.80	1.06	-2.23	3.59	0.94	3.27	0.79
LAW–$y^{(\mathrm{M})}$	[1.00]	1.86	0.54	-2.22	3.40	1.06	2.99	0.87
LAW–$y^{(\mathrm{H})}$	[0.50]	1.78	0.28	-2.27	3.17	1.23	2.63	0.93

Note: This table reports statistics of the stochastic steady state ergodic distribution under the STR, SIT, LAW–ϕ, LAW–$y^{(\mathrm{L})}$, LAW–$y^{(\mathrm{M})}$ or LAW–$y^{(\mathrm{H})}$ (see Table 3). "% Crisis time" is the percentage of the time that the economy spends in a crisis. By construction, and for the purpose of comparison across experiments, it is normalized to 10% under STR, to 1.91% under LAW–ϕ and LAW–$y^{(\mathrm{L})}$ (this corresponds to the time the SIT economy spends in a crisis), to 1% in the LAW–$y^{(\mathrm{M})}$ economy, and to 0.5% in the LAW–$y^{(\mathrm{H})}$ economy (square brackets). "Length" is the average duration of a crisis (in quarters). "% Nb crises" is the number of new (distinct) crisis episodes per quarter (in %), *i.e.* the ratio % Crisis time/Length. The output loss is the percentage fall in output from one quarter before the crisis till the trough of the crisis.

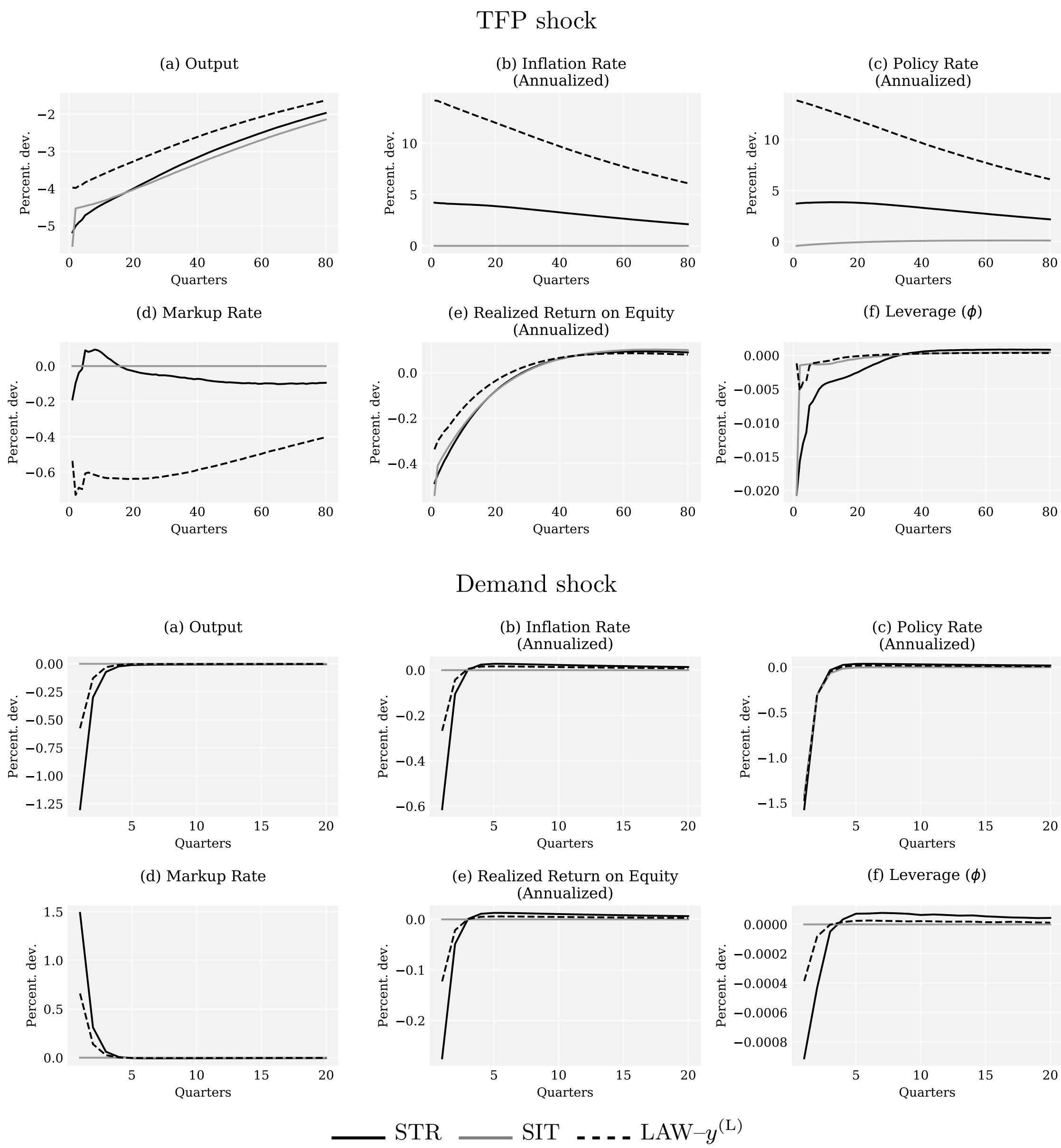
To build intuition, it is useful to examine these results through the lens of the YMCA channels described in Section 3, the impulse response functions under the three policy rules in Figure 4, which speaks to how the central bank's response to shocks affects the economy in the short run, and the counterfactual analysis in Figure 5. The latter compares how the STR, SIT, LAW–ϕ and LAW–$y^{(\mathrm{L})}$ economies evolve when they experience the same sequences of shocks as those that lead to a crisis in the STR economy.[23] This experiment, in which the economies only differ in terms of the central bank's interest rate rules, allows us to compare the strength of the CA–channel under the different moneta policy rules.

We start with the comparison of SIT and STR. The reason why there are fewer crises under SIT than under STR is twofold. First, SIT insulates the economy from the effects of demand–driven shocks. As a result, it directly eliminates all the demand–driven crises as well as those due to a mix

[23] The average sequence of shocks is represented in Figure 2, panels (a) and (b). The counterfactuals for LAW–$y^{(\mathrm{M})}$ and LAW–$y^{(\mathrm{H})}$ are similar as for LAW–$y^{(\mathrm{L})}$ and reported in Figure 8.3 in the appendix.

of demand and technology shocks. Second, by shutting down the M–channel, SIT also prevents the rise in markups that typically precedes technology–driven crises. The counterfactual experiment in Figure 5 indeed shows that, for the same underlying sequence of shocks, markups are slightly lower during booms under SIT than under STR (panel (b), grey versus black lines), which enhances the resilience of the loan market. The finding that crises are deeper and longer under SIT than under STR can be explained by the fact that SIT does not stabilize output as much as STR in the face of negative technology shocks (Y–channel); see Figure 4 (panel (a)).

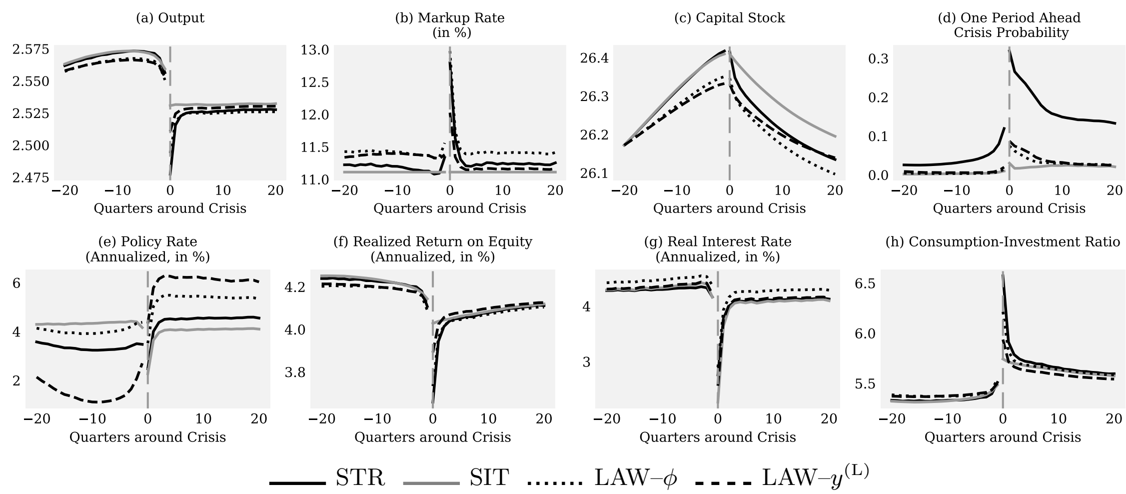
Figure 4: Impulse response functions around steady state



Note: Generalized impulse response functions following a negative technology or demand shock under STR, SIT and LAW–$y^{(\mathrm{L})}$, around the average of the ergodic distribution in the stochastic steady state.

Next, we compare LAW with STR. LAW stabilizes the loan market mainly through the Y– and CA–channels. Table 4 indeed shows that output and capital are less volatile ($\sigma(Y_t)$ and $\sigma(K_{t-1})$ are lower) under LAW, which reduces the probability that the return on capital falls below the crisis threshold. These lower volatilities are both due to the central bank's stronger stabilization of output in the face of adverse (demand and technology) shocks (see Figure 4) and to a slower capital accumulation during booms (see Figure 5, panel (c)). As the central bank commits itself to reining booms more aggressively under LAW than under STR, the household indeed expects lower equity returns during expansions, and increases their equity investment by less, shifting resources from investment toward consumption (panel (h)).

Figure 5: Counterfactuals



Notes: For STR: typical path to crisis as in Figure 2. For SIT, LAW–ϕ, and LAW–$y^{(L)}$: counterfactual average dynamics, when the economy starts with the same capital stock in quarter $t = -20$ and is fed with the same technology and demand shocks as the STR economy.

Table 4 further shows that LAW may foster financial stability compared with SIT. While both rules may insulate the economy from demand–driven and mixed crises, LAW can be more effective in preventing the technology–driven ones. The reason is twofold. First, LAW discourages capital accumulation (see Figure 5, panel (c)). Second, compared with SIT, LAW stabilizes better firms' equity returns in the face of adverse technology shocks (see Figure 4, panel (e)), because the central bank does not allow output to decline (*i.e.* $\sigma(Y_t)$ is smaller). In that case, it lets the markup fall (*i.e.* $\sigma(\mathscr{M}_t)$ is higher), but this entails a positive correlation between output and markup ($\rho(Y_t, \mathscr{M}_t) > 0$), which further contributes to stabilising the loan market (see expression (23)). The upshot is that the central bank averts more crises when it leans aggressively enough than when it follows SIT.[24]

Note that, unlike conventional wisdom, LAW does not necessarily require the central bank to

[24]Under LAW–$y^{(M)}$ and LAW–$y^{(H)}$, notably, the central bank commits itself to stabilizing aggregate demand more aggressively. Following an adverse technology shock, firms must meet resilient aggregate demand despite their productivity loss. As a result their production costs and prices go up, and retailers' markup falls.

set its policy rate higher than under STR during economic booms (Figure 5, panel (e)). Indeed, recall that most booms are driven by a long sequence of positive technology shocks. As a result, during booms, LAW entails stronger deflationary pressures than STR (see Figure 4, panel (b)), which in turn calls for larger policy rate cuts. Moreover, under LAW, the central bank's stronger commitment to raising the policy rate (and hence contracting demand) if output increases tends to anchor the economy around an equilibrium where persistent productivity gains only entail a moderate rise in permanent income and, therefore, in aggregate demand. As a result, for a given shift in aggregate supply, equilibrium output increases by less under LAW than under STR, and deflationary pressures are stronger (see Figure 4, panel (a)).

Finally, we turn to the net welfare gains of SIT and LAW over STR (see Table 5). As expected, the net gain from SIT is positive. Compared to STR, SIT raises welfare by 0.05% (last column). This is because SIT not only eliminates inefficient fluctuations due to nominal rigidities in the face of technology and demand shocks, but also reduces the probability of financial crises compared to STR (see Table 4). However, welfare can be further improved by leaning against the wind. In particular, the net gain under LAW–$y^{(\mathrm{H})}$ is 0.06%. This suggests that the welfare loss due to inefficient variations in output and inflation in response to technology shocks under LAW are more than offset by the gains in terms of financial stability. The latter can be explained by less frequent and milder crises, which contributes to smoothing consumption over time: $\sigma(C_t)$ is smaller under LAW–$y^{(\mathrm{H})}$ than under STR or SIT. The upshot is that LAW improves welfare over both STR and SIT.

Table 5: Net welfare gain

	$\sigma(C_t)$	$\sigma(h_t)$	PCE
STR	3.84	1.49	–
SIT	4.02	0.72	0.0560
LAW–ϕ	3.69	1.29	0.0273
LAW–$y^{(\mathrm{L})}$	3.31	0.83	0.0535
LAW–$y^{(\mathrm{M})}$	3.17	0.83	0.0611
LAW–$y^{(\mathrm{H})}$	3.00	0.88	0.0641

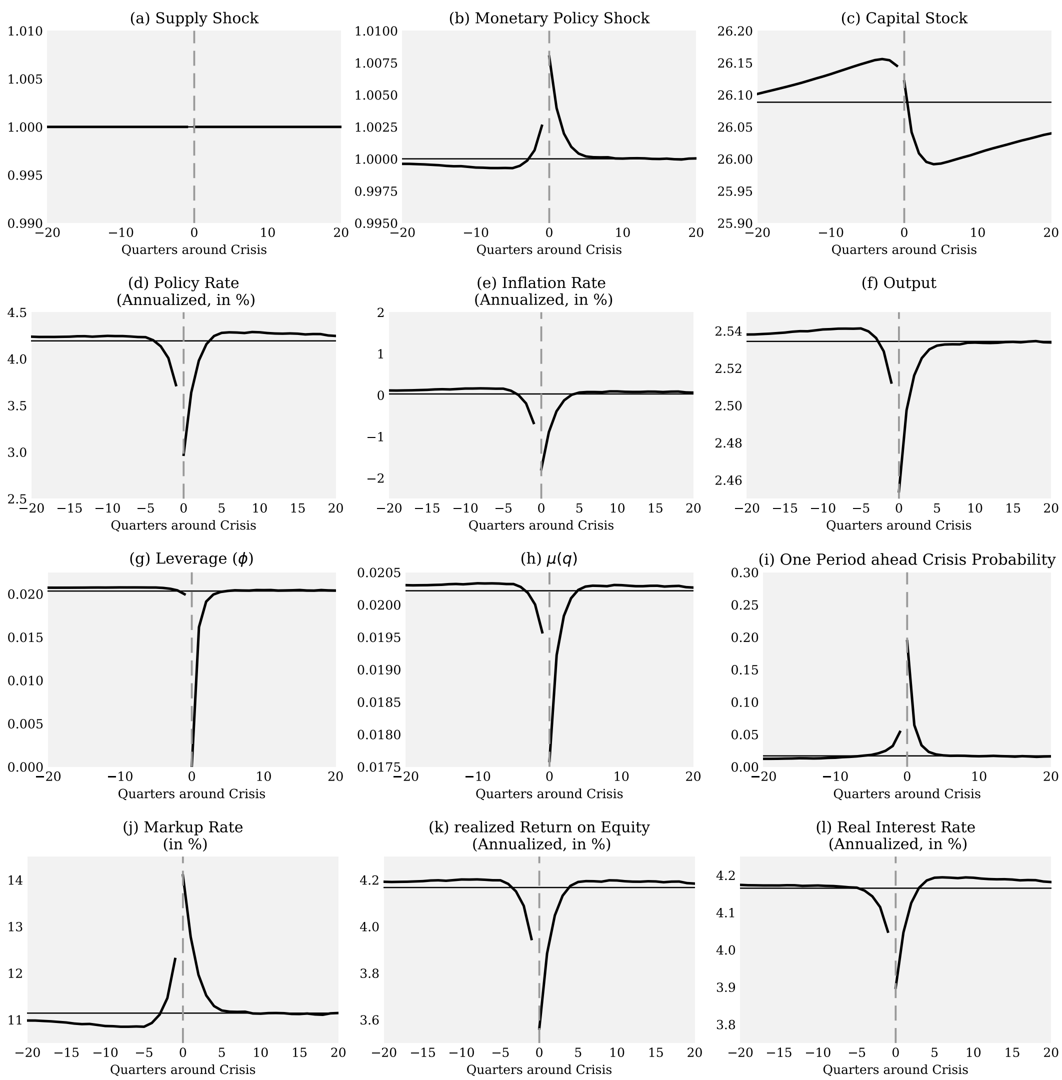
Note: This table reports statistics of the stochastic steady state ergodic distribution under the STR, SIT, LAW–ϕ, LAW–$y^{(\mathrm{L})}$, LAW–$y^{(\mathrm{M})}$ or LAW–$y^{(\mathrm{H})}$ (see Table 3), as well as the permanent consumption equivalent increase (PCE, in %) that the household should be given to live in the STR economy (which spends 10% of the time in a crisis) rather than in economies that spend less time in a crisis (see Table 4); see Lucas (1987). The PCE does not take into account the cost of lower consumption in the transition from an economy to another. Welfare is computed by truncating the welfare criterion after 5,000 periods. Expectations are computed relying on Monte Carlo techniques (1000 draws).

6 Discussion

6.1 Monetary surprises and discretionary leaning against the wind

The aim of this section is to assess the effects of monetary surprises —as opposed to rules— on financial stability. To do so, we consider a STR economy that experiences monetary shocks only, taking the form of random deviations from the STR.[25]

Figure 6: Typical path to a crisis due to monetary policy shocks



Note: Simulations for the STR economy. Average dynamics of the economy around the beginning of a new crisis (in quarter 0). To filter out the potential noise due to the aftershocks of past crises, we only report averages for new crises, *i.e.* crises that follow at least 20 quarters of normal times. Panel (b) shows the average dynamics of the monetary policy surprises.

[25]The process of these shocks is standard (see Galí (2015)). More specifically, we consider the monetary policy rule $1+i_t=\frac{1}{\beta}(1+\pi_t)^{1.5}\left(\frac{Y_t}{Y}\right)^{0.125}\varsigma_t$ where ς_t follows an exogenous AR(1) process $\ln(\varsigma_t)=\rho_\varsigma\ln(\varsigma_{t-1})+\epsilon_t^\varsigma$, with $\rho_\varsigma=0.5$ and $\sigma_\varsigma=0.0025$. In the context of our model, monetary policy shocks are isomorphic to demand shocks.

Figure 6 shows the path to the subset of crises due to the monetary policy shocks in this new environment. We find that crises occur after a long period of unexpected monetary easing, where the policy rate remains below the one prescribed by STR for almost five years (panel (b)). Persistently lower rates fuel the boom that precedes the typical crisis. The crisis is triggered by three consecutive, unwarranted (according to the rule), and abrupt interest rate hikes toward the end of the boom, at a time when the crisis probability is very high (third row, right hand panel). This finding is consistent with the recent empirical evidence in Schularick, Ter Steege, and Ward (2021) that unanticipated "last minute" interest rate hikes at the end of a boom are more likely to trigger a crisis than to avert it (see also Taylor (2012)). Schularick, Ter Steege, and Ward (2021) refer to such policy as "discretionary" leaning against the wind.

Overall, our analysis highlights an important difference between discretionary and rule–based leaning against the wind, *i.e.* that leaning discretionarily and late in the boom is conducive to financial crises. To understand this, consider a booming economy and the central bank's policy options. If the central bank unexpectedly and abruptly increases its policy rate, it may trigger a sudden fall in aggregate demand and firms' return on equity, and ultimately a crisis. If instead it unexpectedly reduces its policy rate, it may boost aggregate demand further and avert the crisis in the short term. But this will only kick the can down the road. Following the rise in aggregate demand and equity returns, the household will keep on accumulating capital, making the financial sector even more fragile in the medium term. None of these discretionary policies helps to avert the crisis. Rather, our analysis prescribes that the central bank address financial externalities by committing itself to curbing the boom should it persist and propping up the economy should a recession breaks out. In effect, systematically leaning against the wind amounts to providing the household with an insurance against future aggregate shocks. By helping smooth consumption, such insurance inhibits the household's saving behaviour, slows down —and prevents excess— capital accumulation over the medium term, and shields the economy against crises.

6.2 Asymmetric leaning

The aim of this section is to disentangle the effects of LAW into those due to leaning against headwinds (LAHW) and those due to leaning against tail–winds (LATW). We consider also a specific case of LAHW, whereby the central bank only departs from the STR (*i.e.* is more accommodative) during crises. We refer to the latter case as "cleaning" (CLEAN). Our goal is to determine whether LAHW and CLEAN only help to mitigate the effects of adverse shocks and crises, or whether they also contribute to curbing booms ahead of a crisis through expectations —and therefore also help to stave off crises. Our contention is that the household may not need to accumulate so much precautionary savings if they expect the central bank to LAHW or CLEAN. If so, these asymmetric rules could be "enough" to prevent crises, and the central bank would only need to commit itself to responding to negative shocks or crises, without hindering economic booms inappropriately. TBC

7 Conclusion

We have developed a version of the NK model with capital accumulation and heterogeneous firms and inter–firm lending. In the absence of financial frictions the equilibrium outcome collapses to that of the standard model with a representative firm. With financial frictions, however, there is an upper bound on the leverage ratio of any individual firm resulting from an incentive–compatibility constraint, which prevents capital from being fully reallocated to the most efficient firms. When the average return on capital is low, possibly due to a capital overhang at the end of a long boom, the loan market collapses, triggering a financial crisis and a fall in activity due to capital mis–allocation.

Our analysis of varied monetary policies through the lens of that model points to the advantages of systematic leaning–against–the–wind policies, whereby the central bank commits itself to raising the interest rate in response to an investment boom and a loosening of credit constraints.

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8 Appendix

8.1 Derivation of expressions (11) and (15)

Using (10), we can rewrite firm q's return on equity in (8) as

$$
\begin{aligned}
r_t^k(q) &= \frac{\alpha}{\mathscr{M}_t}\frac{y_t(q)}{K_t} - (r_t^\ell + \delta)\frac{K_t(q) - K_t}{K_t} - \delta && (25)\\
&= \frac{qK_t(q)}{K_t}\frac{\alpha}{\mathscr{M}_t}\frac{y_t(q)}{qK_t(q)} - (r_t^\ell + \delta)\frac{K_t(q) - K_t}{K_t} - \delta \\
&= q\frac{K_t(q)}{K_t}\frac{\alpha}{\mathscr{M}_t}\Phi_t - (r_t^\ell + \delta)\frac{K_t(q) - K_t}{K_t} - \delta && (26)
\end{aligned}
$$

for all firms with $q \in [0, 1]$. The expression for $r_t^k(q)$ in terms of the average return on equity r_t^k is obtained by integrating (26) over $[0, 1]$:

$$
r_t^k \equiv \int_0^1 r_t^k(q)\mathrm{d}\mu(q) = \int_{q_t^*}^1 q\frac{K_t(q)}{K_t}\mathrm{d}\mu(q)\frac{\alpha}{\mathscr{M}_t}\Phi_t - (r_t^\ell + \delta)\int_0^1 \frac{K_t(q) - K_t}{K_t}\mathrm{d}\mu(q) - \delta
$$

which, using (12) and the identity $K_t \equiv \int_0^1 K_t(q)\mathrm{d}\mu(q)$, can be re–written as

$$
\frac{\alpha}{\mathscr{M}_t}\Phi_t = \frac{r_t^k + \delta}{\bar{q}_t}
$$

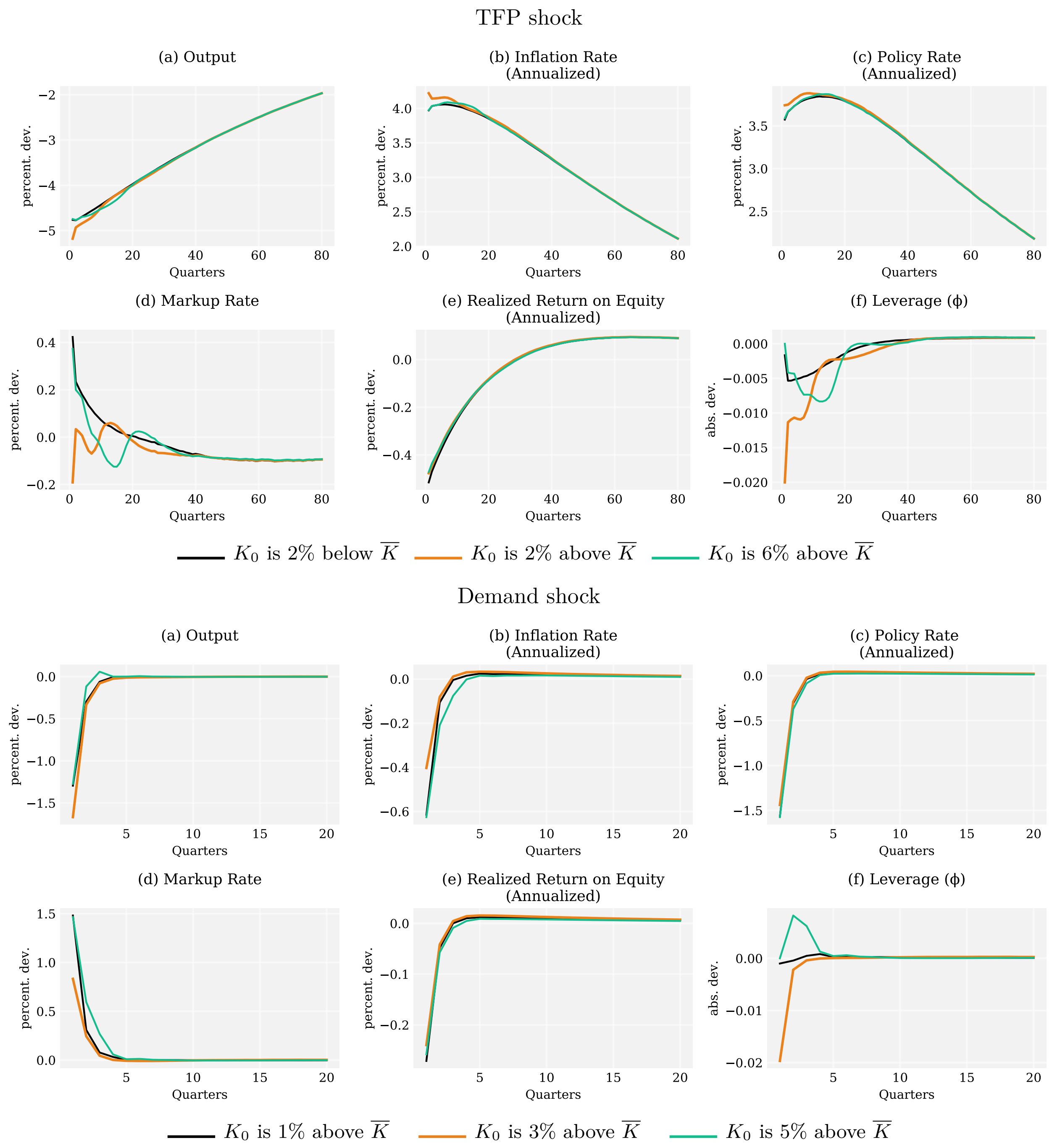
Substituting the above term in (26) then yields $r_t^k(q)$ in terms of r_t^k:

$$
r_t^k(q) + \delta = \frac{q}{\bar{q}_t}\frac{K_t(q)}{K_t}(r_t^k + \delta) - (r_t^\ell + \delta)\frac{K_t(q) - K_t}{K_t} \tag{11}
$$

Finally, expression (15) is obtained by integrating (25) over $[0, 1]$ and using the identities $r_t^k \equiv \int_0^1 r_t^k(q)\mathrm{d}\mu(q)$, $K_t \equiv \int_0^1 K_t(q)d\mu(q)$, and $y_t \equiv \int_0^1 y_t(q)\mathrm{d}\mu(q)$.

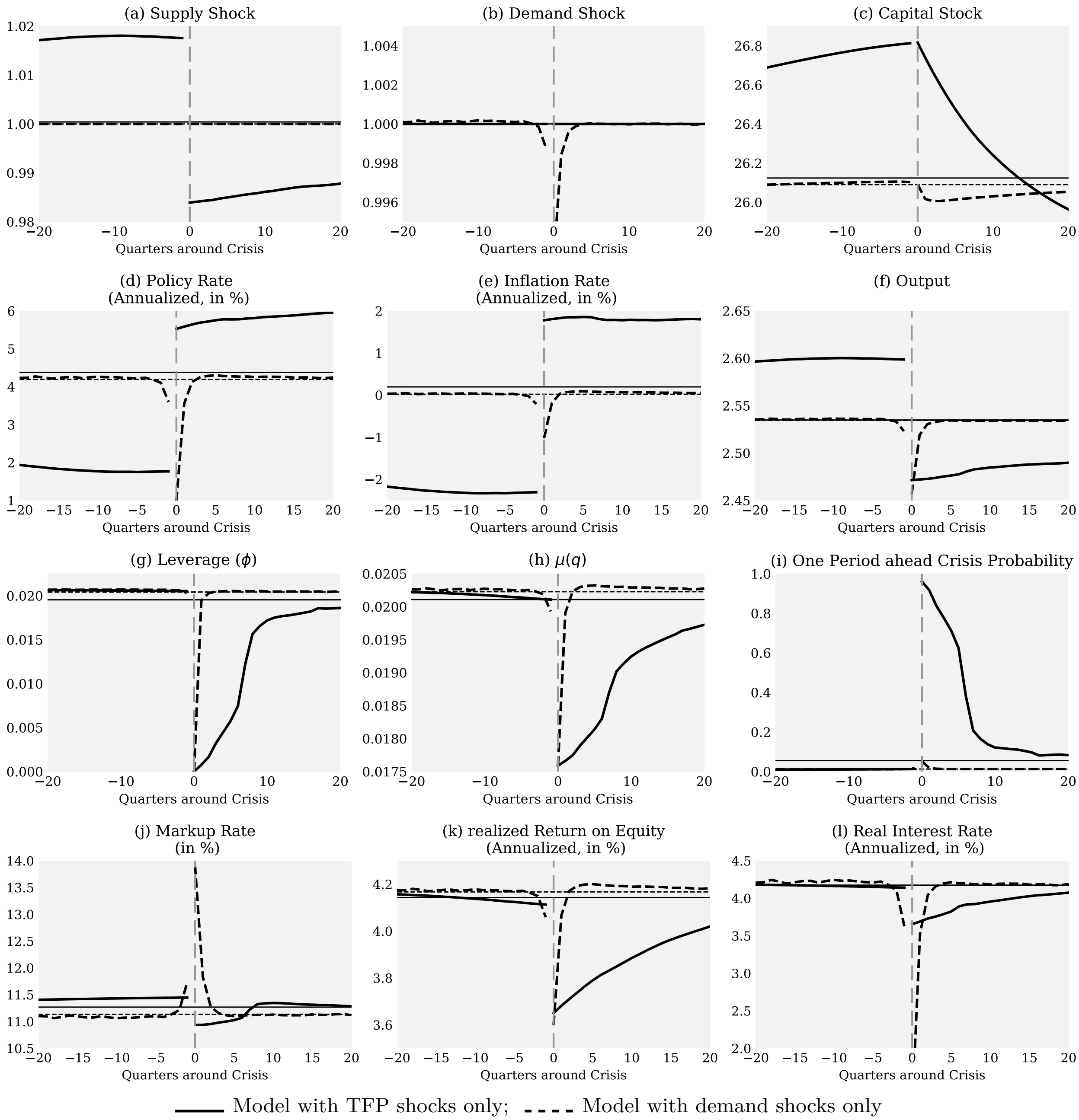
8.2 Additional figures

Figure 8.1: Impulse response functions depend on initial conditions



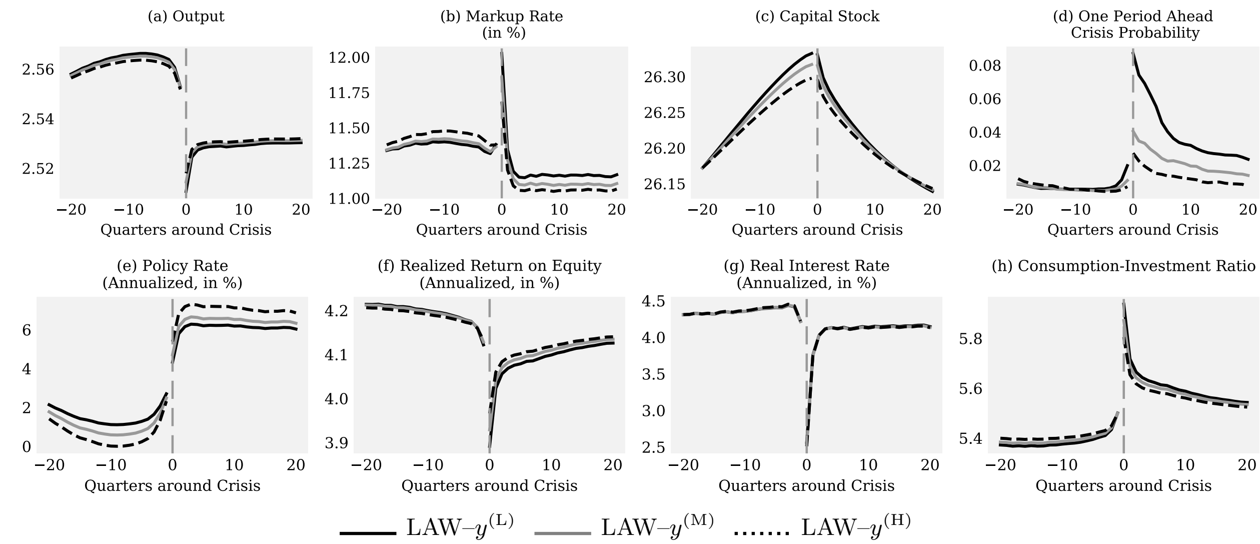
Note: Simulations for the STR economy. Left hand panel: dynamics of the technology shock following a negative one standard deviation from its average level. Center panel: generalized impulse response function (IRF) of output obtained from 100,000 draws. K_0 and $\overline{K}$ denote, respectively, the level of the capital stock at the time of the shock and the average level of the capital stock in the stochastic steady state. Note that these IRFs take into account that the shock may accelerate or delay the transition toward the steady state, depending on the initial level of capital. Right hand panel: effect of the shock on output on impact (y–axis) depending on the initial level of capital (x–axis).

Figure 8.2: Typical path to crisis — Technology versus demand shocks



Note: Simulations for the STR economy. Average dynamics of the economy around the beginning of a new crisis (in quarter 0). To filter out the potential noise due to the aftershocks of past crises, we only report averages for new crises, *i.e.* crises that follow at least 20 quarters of normal times. The model is solved and simulated with either the technology or the demand shocks. The processes for these shocks are the same as in the baseline calibration. Panels (a) and (b) show the average dynamics of the technology and demand shocks. The horizontal lines correspond to the averages of the ergodic distributions.

Figure 8.3: Counterfactuals — Comparison of LAW rules



Notes: Counterfactual average dynamics, when the economy starts with the same capital stock in quarter $t = -20$ and is fed with the same technology and demand shocks as the STR economy.

8.3 List of the equations of baseline model

$$A.1.\quad Z_t = \mathbb{E}_t\left\{\Lambda_{t,t+1}(1+r_{t+1})\right\}$$

$$A.2.\quad 1 = \mathbb{E}_t\left\{\Lambda_{t,t+1}(1+r^k_{t+1})\right\}$$

$$A.3.\quad \omega_t = \vartheta N_t^{\nu} C_t^{\sigma}$$

$$B.1.\quad Y_t = A_t\left(\overline{q}_t K_t\right)^{\alpha} N_t^{1-\alpha}$$

$$B.2.\quad \omega_t = (1-\alpha)\frac{Y_t}{N_t \mathscr{M}_t}$$

$$B.3.\quad r^k_t + \delta = \alpha\frac{Y_t}{\mathscr{M}_t K_t}$$

$$B.4.\quad (1+\pi_t)\pi_t = \mathbb{E}_t\left(\Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}(1+\pi_{t+1})\pi_{t+1}\right) - \frac{\epsilon-1}{\varrho}\left(1-\frac{\epsilon}{\epsilon-1}\cdot\frac{1}{\mathscr{M}_t}\right)$$

$$C.1.\quad \overline{\phi}_t \equiv \frac{q^*_t}{\overline{q}_t}\frac{r^k_t+\delta}{1-\delta}$$

$$C.2.\quad q^*_t \equiv \overline{q}_t\frac{r^\ell_t+\delta}{r^k_t+\delta}$$

$$D.1.\quad 1+i_t = \frac{1}{\beta}(1+\pi_t)^{\alpha_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\alpha_y}$$

$$E.1.\quad Y_t = C_t + K_{t+1} - (1-\delta)K_t$$

$$E.2.\quad \begin{cases} (1-\mu\left(q^*_t\right))\left(1+\overline{\phi}_t\right) = 1 \text{ , if } \max_{q^*_t}(1-\mu(q^*_t))\left(1+\dfrac{q^*_t}{\overline{q}_t}\dfrac{r^k_t+\delta}{1-\delta}\right) \geq 1 \\ q^*_t = 0, \text{ otherwise} \end{cases}$$

$$F.1.\quad \Lambda_{t,t+1} \equiv \beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$$

$$F.2.\quad 1+r_t = \frac{1+i_{t-1}}{1+\pi_t}$$

$$F.3.\quad \overline{q}_t = (1+\overline{\phi}_t)\int_{q^*_t}^{1} q\,\mathrm{d}\mu(q)$$

8.4 Solution Method

The model is solved by approximating expectations using a collocation technique (see Christiano and Fisher (2000)). We first discretize the distribution of the shocks using the approach proposed by Tauchen (1986). This leads to a Markov chain representation of the shock, z_t, with $z_t \in \{z_1, \ldots, z_{n_z}\}$ and transition matrix $\Pi = (\varpi_{ij})_{i,j=1}^{n_z}$ where $\varpi_{ij} = \mathbb{P}(z_{t+1} = z_j | z_t = z_i)$. In what follows, we use $n_z = 9$. We look for an approximate representation of each expectatio-n in the model, as a function of the endogenous state variables. Exploiting the smoothness of expectations spares us from looking for the endogenous thresholds (*e.g.* for K_t) above which the economy switches from normal times to crisis. We approximate the following three expectations:

$$\mathscr{E}_{c,t} = \beta \mathbb{E}_t \left[\frac{u'(C_{t+1})}{1 + \pi_{t+1}} \right] \tag{27}$$

$$\mathscr{E}_{k,t} = \beta \mathbb{E}_t \left[u'(C_{t+1})(1 + r_{t+1}^k) \right] K_t \tag{28}$$

$$\mathscr{E}_{\pi,t} = \beta \mathbb{E}_t \left[u'(C_{t+1}) Y_{t+1} (1 + \pi_{t+1}) \pi_{t+1} \right] \tag{29}$$

Each of this expectation is approximated by a smooth function of the form

$$\Psi_x(K; z = z_i) \equiv \sum_{j=0}^{p} \theta_j^x(z_i) T_j(\nu(K_t))$$

where z_i denotes a particular level of the shock in the grid. $T_j(\cdot)$ is a Chebychev polynomial of order j, $\theta_j^x(z_i)$ is the loading coefficient of this polynomial when approximating the expectation function x in state z_i. $\nu(K_t)$ is a function that maps the level of physical capital into the interval (-1,1).[26]

8.4.1 Algorithm

The algorithm proceeds as follows.

1. Choose a domain $[K_m, K_s]$ of approximation for K_t and a stopping criterion $\tau > 0$. The domain is chosen such that K_m and K_s are located 30% away from the deterministic steady state of the model (located in the normal regime). We chose $\tau = 1e^{-6}$.

2. Choose an order of approximation p, compute the $p+1$ roots of the Chebychev polynomial of order $p+1$ as
$$\zeta_\ell = \cos\left(\frac{(2\ell - 1)\pi}{2(p+1)}\right) \text{ for } \ell = 1, \ldots, p+1$$
and formulate an initial guess for $\theta^x(z_i)$ for $c = \{c, k, \pi\}$ and $i = 1, \ldots, n_z$. We find that $p = 4$ is sufficient to obtain an accurate approximation of the expectations.

[26]More precisely, for any, $K \in [K_m; K_s]$, we have $\nu(K) = 2\frac{K - K_m}{K_s - K_m} - 1$.

3. Compute K_ℓ, $\ell = 1, \dots, p+1$ as

$$K_\ell = (\zeta_\ell + 1)\frac{K_s - K_m}{2} + K_m$$

for $\ell = 1, \dots, p+1$.

4. Using a candidate solution $\theta^x(z_i) = \{\theta^x_j(z_i); j = 1 \dots p+1\}$, compute approximate expectations $\Psi_c(K; z = z_i)$, $\Psi_k(K; z = z_i)$ and $\Psi_\pi(K; z = z_i)$ for each level of K_ℓ, $\ell = 1, \dots, p+1$ and the over quantities of the model using the definition of the general equilibrium of the economy (see below). In particular, compute $K_{t+1}(K_\ell; z_i)$ for each $\ell = 1, \dots, p+1$ and $i = 1 \dots n_z$.

5. Using the values $K_{t+1}(K_\ell; z_i)$ and the candidate approximation, solve the general equilibrium to obtain next period quantities and prices entering the expectations (27)–(29). Note that these quantities depend on $K(\ell)$, z_i and z'_s the next period level of the shock.

6. Evaluate the expectations as

$$\widetilde{\mathscr{E}}_{c,t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[\frac{u'(C'(K_\ell, z_i, z'_s))}{1 + \pi'(K_\ell, z_i, z'_s)} \right] \tag{30}$$

$$\widetilde{\mathscr{E}}_{k,t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[u'(C(K_\ell, z_i, z'_s))(1 + r^{k'}(K_\ell, z_i, z'_s))K'(K_\ell, z_i) \right] \tag{31}$$

$$\widetilde{\mathscr{E}}_{\pi,t} = \beta \sum_{s=1}^{n_z} \varpi_{i,s} \left[u'(C(K_\ell, z_i, z'_s))Y'(K_\ell, z_i, z'_s)(1 + \pi'(K_\ell, z_i, z'_s))\pi'(K_\ell, z_i, z'_s) \right] \tag{32}$$

7. Project each $\widetilde{\mathscr{E}}_{x,t}$ on the Chebychev polynomial $T_j(\cdot)$ to obtain a new candidate $\widetilde{\theta}^x(z_i)$ vector of approximation coefficients. If $\max_x \|\widetilde{\theta}^x(z_i) - \theta^x(z_i)\| < \tau$ then a solution was found, otherwise update the candidate solution as

$$\xi\widetilde{\theta}^x(z_i) + (1 - \xi)\theta^x(z_i)$$

where $\xi \in (0, 1]$ can be interpreted as a learning rate, and go back to step 4.

8.4.2 Computing the General Equilibrium

In this section, we explain how the general equilibrium is solved in the case of a technology shock. The procedure is the same for a demand shock and for both shocks. Given a candidate solution $\theta^x(z)$ $x \in \{c, k, \pi\}$, we present the solution for a given level of the capital stock K, a particular realization of the shock z, and given guesses on $\mathscr{E}_c$, $\mathscr{E}_k$ and $\mathscr{E}_\pi$. For convenience, and to save on notation, we drop the time index.

We start by formulating a guess on the monetary policy rate i. Given the guess and $\mathscr{E}_c$, we immediately get

$$C = ((1 + i)\mathscr{E}_c)^{-\frac{1}{\sigma}}$$

Likewise, knowing $\mathscr{E}_k$ we get the next period capital stock

$$K' = \frac{\mathscr{E}_k}{C^{-\sigma}}$$

which leads to the investment level

$$X = K' - (1-\delta)K$$

and hence the level of output (given that retailers' profits and menu costs are rebated to the household)

$$Y = C + X$$

We now first need to establish the regime in which the economy is for a given pair (K, z). We obtain this information by deriving the maximum of the function

$$\Gamma(q) \equiv (1 - \mu(q))(1 + \phi)$$

with respect to q (where the borrowing limit ϕ is an endogenous object that also depends on q) under the conjecture that equilibrium E exists. If this maximum is attained for $q \in [0,1]$ and lies above one, then the economy admits E (normal times) as equilibrium. If not, then our conjecture was incorrect and equilibrium A prevails (*i.e.* there is a crisis). We start by constructing this function. In the labor market equilibrium, we get

$$\mathscr{M} = \frac{1-\alpha}{\vartheta} C^{-\sigma} \frac{Y}{N^{1+\nu}}$$

which using

$$N = \left(\frac{Y}{z(\overline{q}K)^{\alpha}}\right)^{\frac{1}{1-\alpha}}$$

yields

$$\mathscr{M} = \frac{1-\alpha}{\vartheta} C^{-\sigma} z^{\frac{1+\nu}{1-\alpha}} Y^{-\frac{\alpha+\nu}{1-\alpha}} (\overline{q}K)^{\frac{\alpha(1+\nu)}{1-\alpha}}$$

Substituting the above expression in

$$\overline{\phi} = \frac{q^*}{\overline{q}} \frac{r^k + \delta}{1-\delta} = \frac{q^*}{\overline{q}} \frac{1}{1-\delta} \frac{\alpha Y}{\mathscr{M} K}$$

we obtain

$$\overline{\phi} = q^* \frac{\alpha\vartheta}{(1-\alpha)(1-\delta)} \frac{(Y/z)^{\frac{1+\nu}{1-\alpha}}}{C^{-\sigma}} (\overline{q}K)^{-\frac{1+\alpha\nu}{1-\alpha}}$$

In normal times, we know that $(1+\overline{\phi})(1-\mu(q^*)) = 1$ and therefore that

$$\overline{q} = (1+\overline{\phi}) \int_{q^*}^{1} q \mathrm{d}\mu(q) = \frac{1}{1-\mu(q^*)} \int_{q^*}^{1} q \mathrm{d}\mu(q) \frac{\lambda}{\lambda+1} \frac{1-{q^*}^{\lambda+1}}{1-{q^*}^{\lambda}}$$

which given the functional form $\mu(q) = \mu + (1-\mu)q^{\lambda}$ implies

$$\overline{q} = \frac{\lambda}{\lambda+1} \frac{1-{q^*}^{\lambda+1}}{1-{q^*}^{\lambda}}$$

Using the above expression, we obtain

$$\overline{\phi} = \Omega(q^*; K, z) \equiv \frac{\alpha\vartheta}{(1-\alpha)(1-\delta)} \frac{(Y/z)^{\frac{1+\nu}{1-\alpha}}}{C^{-\sigma}} K^{-\frac{1+\alpha\nu}{1-\alpha}} q^* \left(\frac{\lambda}{\lambda+1} \frac{1-{q^*}^{\lambda+1}}{1-{q^*}^{\lambda}}\right)^{-\frac{1+\alpha\nu}{1-\alpha}}$$

and

$$\Gamma(q; K, z) = (1-\mu)\left(1-q^{\lambda}\right)(1+\Omega(q; K, z))$$

Let us define $\hat{q}(K,z) = argmax_q \, \Gamma(q; K, z)$. $\hat{q}(K,z)$ is obtained by a Golden section algorithm over the range [0,1]. The value of $\Gamma(\hat{q}(K,z); K, z)$ then determines whether the equilibrium is E or A:

- If $\Gamma(\hat{q}(K,z); K, z) > 1$, then there exists a value q^* for $q^* \in [\hat{q}(K,z); 1]$ such that $\Gamma(q^*; K, z) = 1$ and hence that solves $(1-\mu(q^*))(1+\overline{\phi}) = 1$. In this case, the equilibrium is E. The value q^* is then simply obtained by a bisection algorithm over the range $[\hat{q}(K,z); 1]$:

$$\overline{\phi} = \Omega(q^*)$$
$$\overline{q}^E = \frac{\lambda}{\lambda+1} \frac{1-{q^*}^{\lambda+1}}{1-{q^*}^{\lambda}}$$

- Otherwise, the equilibrium is A and $q^* = 0$, which implies

$$\overline{\phi} = 0$$
$$\overline{q}^A = \frac{\lambda}{\lambda+1}$$

We are then in a position to compute the remaining variables of the model

$$N = \left(\frac{Y}{z(\overline{q}K)^{\alpha}}\right)^{\frac{1}{1-\alpha}}$$
$$\mathscr{M} = \frac{1-\alpha}{\vartheta} C^{-\sigma} \frac{Y}{N^{1+\nu}}$$
$$r^k = \alpha \frac{Y}{\mathscr{M}K} - \delta$$
$$r^{\ell} = \frac{q^*}{\overline{q}}(r^k + \delta) - \delta$$

Finally the inflation rate solves

$$\pi(1+\pi) = \Psi \equiv \frac{\mathscr{E}_{\pi}}{C^{-\sigma}Y} - \frac{\epsilon-1}{\varrho}\left(1 - \frac{\epsilon}{\epsilon-1}\frac{1}{\mathscr{M}}\right)$$

which admits

$$\pi = \frac{\sqrt{1+4\Psi}-1}{2}$$

as solution[27] Note that this equilibrium allocation is obtained given a particular guess for the monetary policy rate, which may or may not satisfy the restrictions imposed by the monetary policy rule. This problem then defines a fixed point between i and the Taylor rule. This aspect of the problem is addressed relying on a Gauss-Newton algorithm.

[27] The other (negative) solution is not economically meaningful.

8.4.3 Accuracy

In order to asses the accuracy of the approach, we compute the relative errors an agent would makes it they used the approximate solution. In particular, we compute the quantities

$$\mathscr{R}_c(K,z) = \frac{C_t - \left(\beta \mathbb{E}_t\left[C_{t+1}^{-\sigma}(1+r_{t+1})\right]\right)^{-1/\sigma}}{C_t}$$

$$\mathscr{R}_k(K,z) = \frac{K_{t+1} - \beta \mathbb{E}_t\left[C_{t+1}^{-\sigma}(1+r_{t+1}^k)\right]/C_t^{-\sigma}}{K_{t+1}}$$

$$\mathscr{R}_\pi(K,z) = (1+\pi_t)\pi_t - \beta \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{Y_{t+1}}{Y_t}(1+\pi_{t+1})\pi_{t+1}\right) + \frac{\epsilon-1}{\varrho}\left(1-\frac{\epsilon}{\epsilon-1}\cdot\frac{1}{\mathscr{M}_t}\right)$$

$\mathscr{R}_c(K,z)$ is the relative error in terms of consumption an agent would make by using the approximate expectation rather than the "true" rational expectation. $\mathscr{R}_k(K,z)$ is the relative error the agent would make in terms of capital decision. Finally, $\mathscr{R}_\pi(K,z)$ corresponds to the inflation error. All these errors are evaluated for values for the capital stock that lie outside of the grid that was used to compute the solution. We used 1,000 values uniformly distributed between K_m and K_s. Table 8.1 reports the average quadratic ($e_x^2 = 100 \times \|\mathscr{R}_x(K,z)\|^2$) and the maximum absolute error expressed in percentages ($e_x^\infty = 100 \times \max_{K,z}|\mathscr{R}_x(K,z)|$) in our baseline economy (STR).

Table 8.1: Accuracy

	Technology shock			Demand shock		
	$\mathscr{R}_c(K,z)$	$\mathscr{R}_k(K,z)$	$\mathscr{R}_\pi(K,z)$	$\mathscr{R}_c(K,z)$	$\mathscr{R}_k(K,z)$	$\mathscr{R}_\pi(K,z)$
e^2	0.0010	0.0018	0.0080	0.0015	0.0030	0.0112
e^∞	0.0262	0.0346	0.0623	0.0268	0.0368	0.0570

Note: All accuracy measures are expressed in percentage.

We find that, in the case of the model featuring technology shocks, an agent using the approximate expectation rather than the "true" solution in their consumption saving decision would make a maximal relative error of 0.026% (quadratic error of 0.001%) in terms of consumption. The error would amount to 0.035% (quadratic error of 0.0018%) when deciding the capital stock. Similar accuracy is obtained in the case of a demand shock.