Production, Amenities and Search Frictions with Two-Sided Heterogeneity

T. Lamadon, J. Lise, C. Meghir and J.M. Robin

NBER - Macro Perspective - July 2021
sildes at http://pres.lamadon.com/llmr/

Goal of the paper

How much wage inequality is driven by:

1. worker and firm heterogeneity,
2. production complementarities,
3. disutility of work and
4. mobility frictions?

To what extent does wage inequality relate to inequality in worker value?

What are potential sources of allocative inefficiences and how are they affected by labor policies (minimum wage, non-linear taxes, ...)?

What we do

We extend the sequential auction model of Postel-Vinay and Robin (2002) with:

- worker-firm interactions in production
- worker-firm interactions in disutility of labor
- preference shocks associated with mobility (observed and priced)

We prove nonparametric identification of main model primitives

- proof delivers a transparent nonparametric estimation strategy
- estimation does not require to numerically solve the equilibrium

We apply the procedure to Swedish data (preliminary)

- surplus is nonmonotonic in firm productivity
- surplus peaks at different firms for different workers
- wages are increasing in worker productivity types

Literature

Shimer and Smith (2000); Postel-Vinay and Robin (2002); Eeckhout and Kircher (2011); Bagger et al. (2014); Lise, Meghir and Robin (2016); Hagedorn, Law and Manovskii (2017); Bagger and Lentz (2019); Jarosch (2021); Lindenlaub and Postel-Vinay (2021);

Sorkin (2018); Taber and Veljin (2020); Lise and Postel-Vinay (2020); Caplin et al. (2021)

Abowd, Kramarz and Margolis (1999); Card, Heining and Kline (2013); Lamadon, Mogstad and Setzler (2021); Di Addario, Kline, Saggio, and Sølvsten (2021)

Bonhomme, Lamadon and Manresa (2019); Lentz, Piyapromdee and Robin (2021)

This talk

1. Overview of the model
   - focus on new parts
   - jump to resulting equations

2. Identification and estimation
   - key structural equations that map to data
   - preliminary estimation results on Swedish data

Agents, production and transfers

- Unit measure of workers indexed by permanent x
  - workers are either employed or unemployed

- Mass of CRS firms indexed by permanent y
  - firms have a fixed number of production units
  - production units are either matched or vacant

- Producing matches
  - firm collects output $f(x, y)$
  - worker suffers disutility of work $c(x, y)$
  - wage transfer w, valued at $u(w)$

- When not matched
  - unemployed workers get $b(x)$
  - vacancies have flow of 0

Search frictions

- The familiar parts
  - search is random
  - separation is exogeneous at rate $\delta(x, y)$
  - both unemployed (λ) and employed $(\kappa\lambda)$ search for jobs
  - jobs become vacant when the worker leaves
  - firms make present value offers to the worker
  - poaching and incumbent firms compete à la Bertrand

- The new part
  - at matching, the worker draws a preference shock ξ
  - distributed according to a logit with parameter ρ
  - ξ is associated with moving
  - ξ is common knowledge (worker, incumbent, new firm)
  - ξ is consumed immediately after accepting the offer (before the ability to make any transfer)

[A] Within-period timing

1. worker's limited commitment binds
2. production is collected and wages are paid
3. separation and search happen
4. workers and vacancies meet
5. preference shocks for moving are realized
6. moving decisions are made and preference shocks are consumed
7. move to next period (no side payments)

Long-term contract & surplus

- Optimal contract with one-sided limited commitment
  - similar to Postel-Vinay and Robin (2002) sequential contracting
  - wage never decreases on the job
  - incumbent employer matches outside offers (including ξ)

- Let $\Pi_1(x, y, W)$ be the largest profit while promising value W
  - decreasing, stricly concave, differentiable and $\frac{\partial \Pi_1(x,y,W)}{\partial W} = \frac{-1}{u'(w)}$
  - lower bound in eqm is value of a vacancy $\Pi_0(y)$
  - upper bound is at value of unemployment $W = W_0(x)$
  - it implicitely defines maximum worker surplus $S(x, y)$

$$\Pi_1(x, y, W_0(x) + S(x, y)) = \Pi_0(y)$$

Offers to unemployed workers

Worker x meets vacancy y

- draws mobility preference shock ξ
- firm chooses offer W
- worker compares $W_0(x)$ to $W + \xi$

- if $S(x, y) < 0$
  - worker's limited commitment next period restricts $W \geq W_0(x)$
  - firm cannot make a profit and doesn't make an offer
  - similarly, if $S(x, y) + \xi < 0$, no offer is made
- if $S(x, y) \geq 0$ and $\xi \geq 0$
  - firm offers lowest value $W = W_0(x)$
  - firm gets maximum profit, worker gets $W_0(x) + \xi$
- if $S(x, y) \geq 0$ and $0 > \xi \geq -S(x, y)$
  - firm offers $W = W_0(x) - \xi$
  - firm compensates the worker for low ξ

Outside offers & Bertrand competition

Worker x with value W at firm y meets vacancy y'

- draws mobility preference shock ξ
- if $S(x, y') < 0$ no offers is made by poacher
- incumbent offers W' and poacher offers W''
- the worker compares W' to $W'' + \xi$

- if $S(x, y) \geq S(x, y') + \xi$
  - poacher can't overbid incumbent, worker stays
  - incumbent might need to match offer
  - $W' = \max\{W, S(x, y') + W_0(x) + \xi\}$

- if $S(x, y) < S(x, y') + \xi$
  - poacher can outbid incumbent, worker moves
  - poacher might need compensate worker for move
  - $W'' = \max\{S(x, y) - \xi, 0\} + W_0(x)$

Key properties

1. Mobility is based on surpluses and ξ, not W
2. J2J transitions happen even when $S(x,y') < S(x,y)$
3. Unemployed workers can start with $W > W_0(x)$
4. Wages are Markov conditional on types

Properties 1, 2 & 4 allow applying Bonhomme, Lamadon and Manresa (2019) to recover unobserved types in a first step.

- several quantities of the model are directly recovered (e.g. sorting)
- doesn't require solving the model

Given the types, Property 1 permits a revealed preference approach to get $S(x,y)$, related to Sorkin (2019) but allowing for sorting

- straightforward estimation via MLE
- recovers $S(x,y)$ without imposing all the pieces of the model

Stationary equilibrium

- Primitives
  - utility function $u(\cdot)$, dicount rate r
  - production $f(x, y)$
  - disutility of work $c(x, y)$ and flow in unemployment $b(x)$
  - exogenous separation $\delta(x, y)$
  - meeting probabilities κ and λ
  - size of preference shock ρ

- Stationary equilibrium outcomes
  - mass of matches $H(x, y)$, vacancies $V(y)$ and unemployed $U(x)$
  - values $\Pi_1(x, y, W)$, $W_0(x)$, $\Pi_0(y)$ and implied wages and mobilities

- Equilibrium conditions
  - firms offer optimal contract subject to worker participation and incentives
  - distributions are implied by mobility decisions
  - search market clears

[A] Worker value functions

Value of being unemployed

$$rW_0(x) = (1+r)b(x) + \lambda \int \phi(x,y) \int_{-S(x,y)}^{\infty} \max\{\xi, 0\}\, g(\xi) d\xi V(y) dy.$$

Value of being employed at given wage

$$\begin{aligned} rW_1(x,y,w) &= (1+r)\left(u(w) - c(x,y)\right) + \delta(x,y)\left(W_0(x) - W_1(x,y,w)\right) \\ &+ \kappa\lambda(1-\delta(x,y)) \int \phi(x,y') \Big[\\ &\int_{W_1(x,y,w)-\Pi_0(x)-S(x,y')}^{S(x,y)-S(x,y')} \left(W_0(x) + S(x,y') + \xi - W_1(x,y,w)\right) g(\xi) d\xi \\ &+ \int_{S(x,y)-S(x,y')}^{\infty} \left(\max\{S(x,y)-\xi, 0\} + \xi + W_0(x) - W_1(x,y,w)\right) g(\xi) d\xi \Big] v(y')\, dy' \end{aligned}$$

[A] Firm value functions

Value of vacancy,

- define $J(x, y, S) = \Pi_1(x, y, S + W_0(x)) - \Pi_0(y)$

$$r\Pi_0(y) = \lambda \int \phi(x,y) \int_{-S(x,y)}^{\infty} J(x,y,-\xi)g(\xi)d\xi U(x)dx$$
$$+ \kappa\lambda \iint \phi(x,y) \int_{S(x,y')-S(x,y)} J\left(x,y,S(x,y')-\xi\right)(1-\delta(x,y))H(x,y')\, g(\xi)d\xi dx dy'$$

Value of profits

$$(r+\delta)\Pi_1(x,y,W) =$$
$$\max_w \,(1+r)(f(x,y)-w) + \delta(x,y)\Pi_0(y)$$
$$+ \kappa\lambda(1-\delta(x,y)) \int \phi(x,y') \int_{W-\Pi_0(x)-S(x,y')}^{\infty}$$
$$\Big(\max\{\Pi_1(x,y,W_0(x)+S(x,y')+\xi), \Pi_0(y)\} - \Pi_1(x,y,W)\Big)\, g(\xi)d\xi V(y')dy$$
$$s.t.\ W \leq W_1(x,y,w).$$

Model specification & data

- Model specification for estimation
  - discrete types: 10 firm and 6 worker
  - $u(w) = \log(w)$
  - Logit preference shock, truncated at 0 for unemployed
  - discount rate r set at 5% annually

- Matched employer-employee data from Sweden
  - yearly employment spells with firm and worker identifier
  - quartlerly frequency for transitions
  - full-year employment for wages
  - 2000 to 2003
  - 1.4M workers, 70k firms

Overview of identification

1. Recover type-specific distributions, transition rates and wages
   - use BLM (2019) on fixed-T matched employer-employee data

2. Get surplus $\rho S(x,y)$, vacancies $\lambda V(y)$ and κ
   - link mobility from the model to transition rates in step 1

3. Separate preference parameter ρ from surplus $S(x,y)$
   - express wage equation in terms of $\rho S(x,y)$ and ρ and link to wage growth

4. Get $\Pi_0(y)$ and $f(x,y)$
   - use optimal contract to express $\Pi_1(x,y,W)$
   - evaluate at $W = W_0(x) + S(x,y)$ to get $f(x,y)$

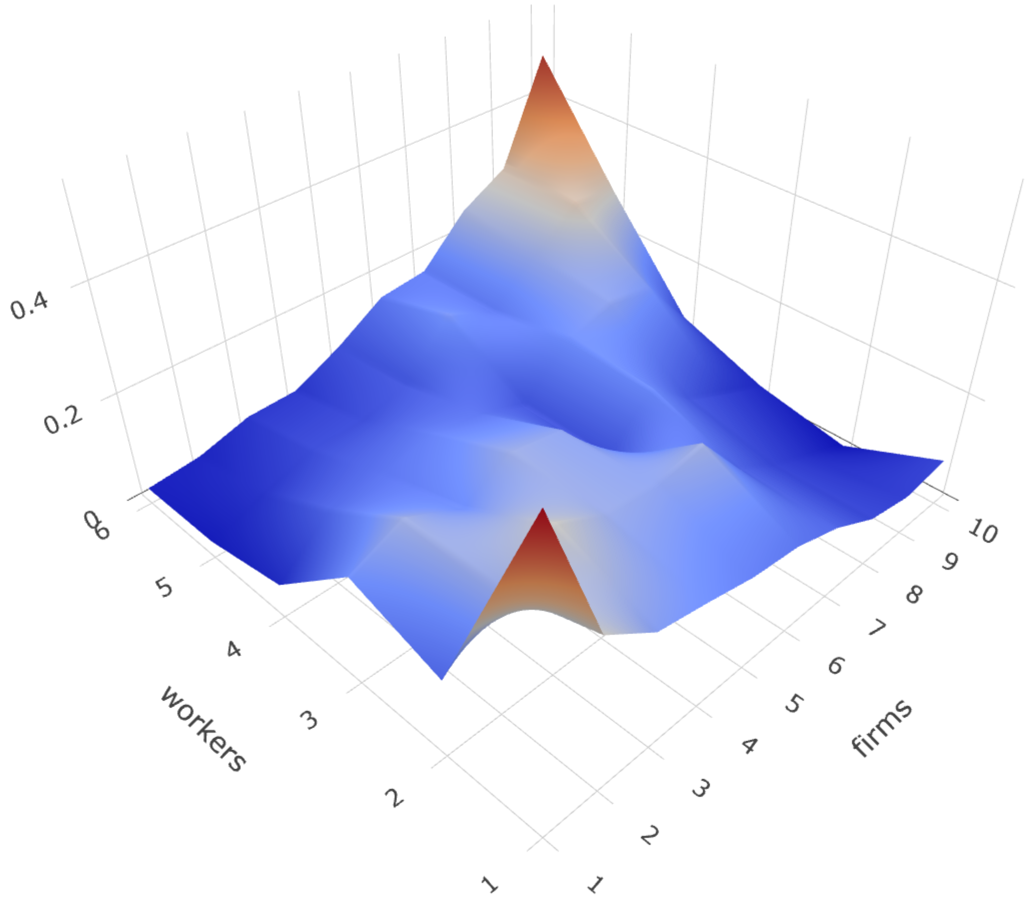
Step 1: Type-specific quantities

- Apply BLM (2019) result and estimation approach
  - show that Markov assumptions are satisfied
  - for estimation, cluster firms based on wage distributions
  - estimate type-specific distributions, transition rates and wages
  - in plots, we order workers and firms by mean wage

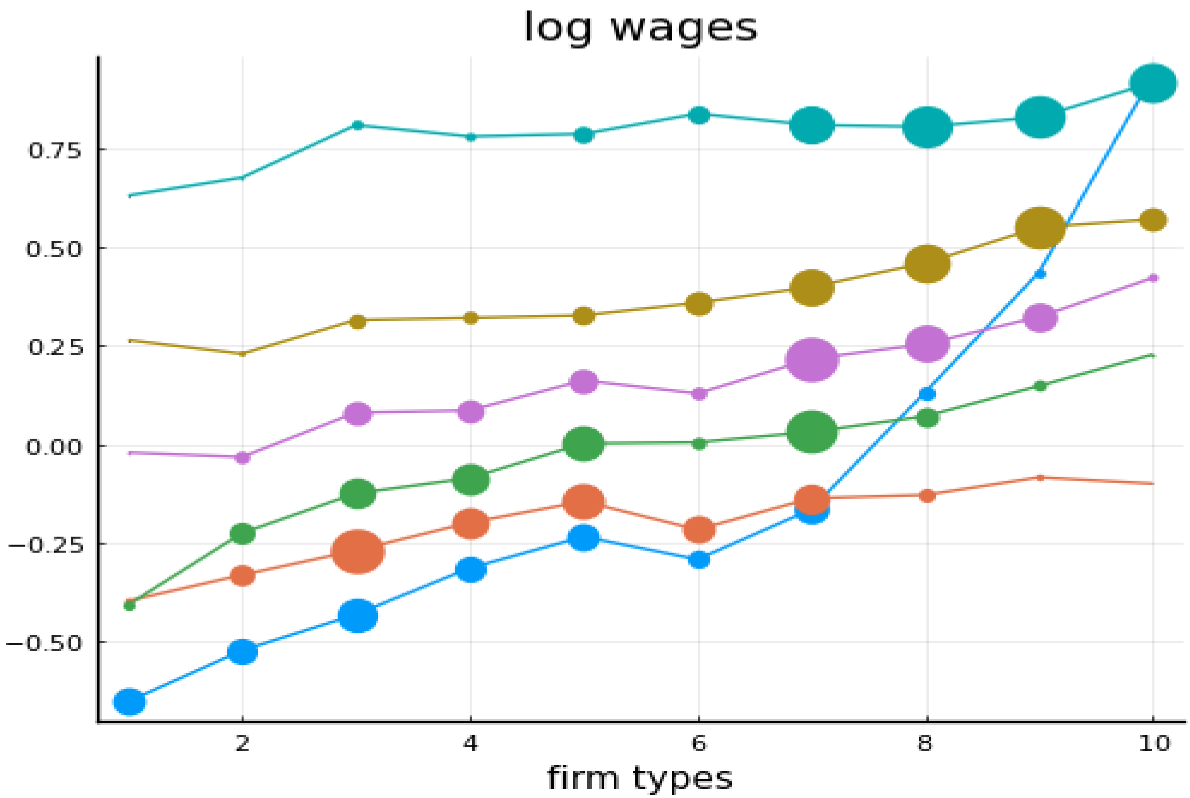
- We recover a mix of primitives and endogeneous objects
  - $U(x)$, $Pr[U2E|x]$, $Pr[y|x, U2E]$
  - $H(x, y)$, $Pr[J2J|x, y]$, $Pr[y'|x, y, J2J]$, $\delta(x, y)$
  - $Pr[w_{t+1}|w_t, x, y, EE]$, $Pr[w_{t+1}|x, y, J2J, y']$, $Pr[w_{t+1}|x, U2E, y]$

Step 1: Match distribution

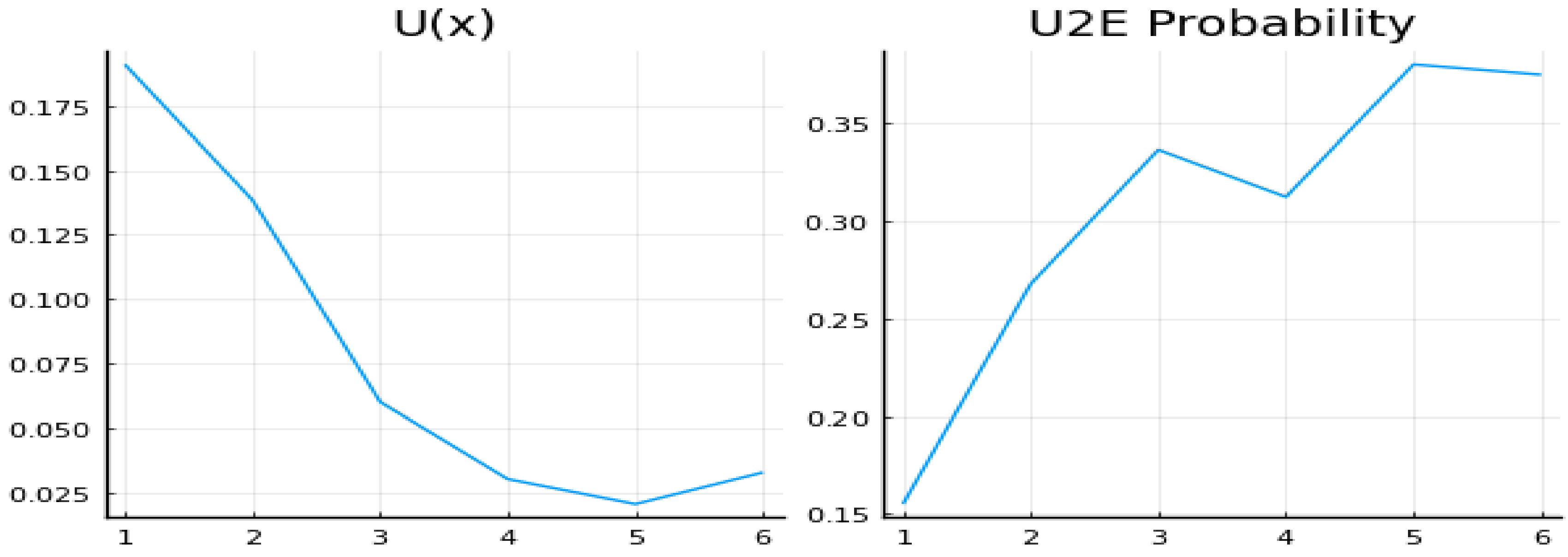
We plot $H(x, y)$ divided by product of marginals



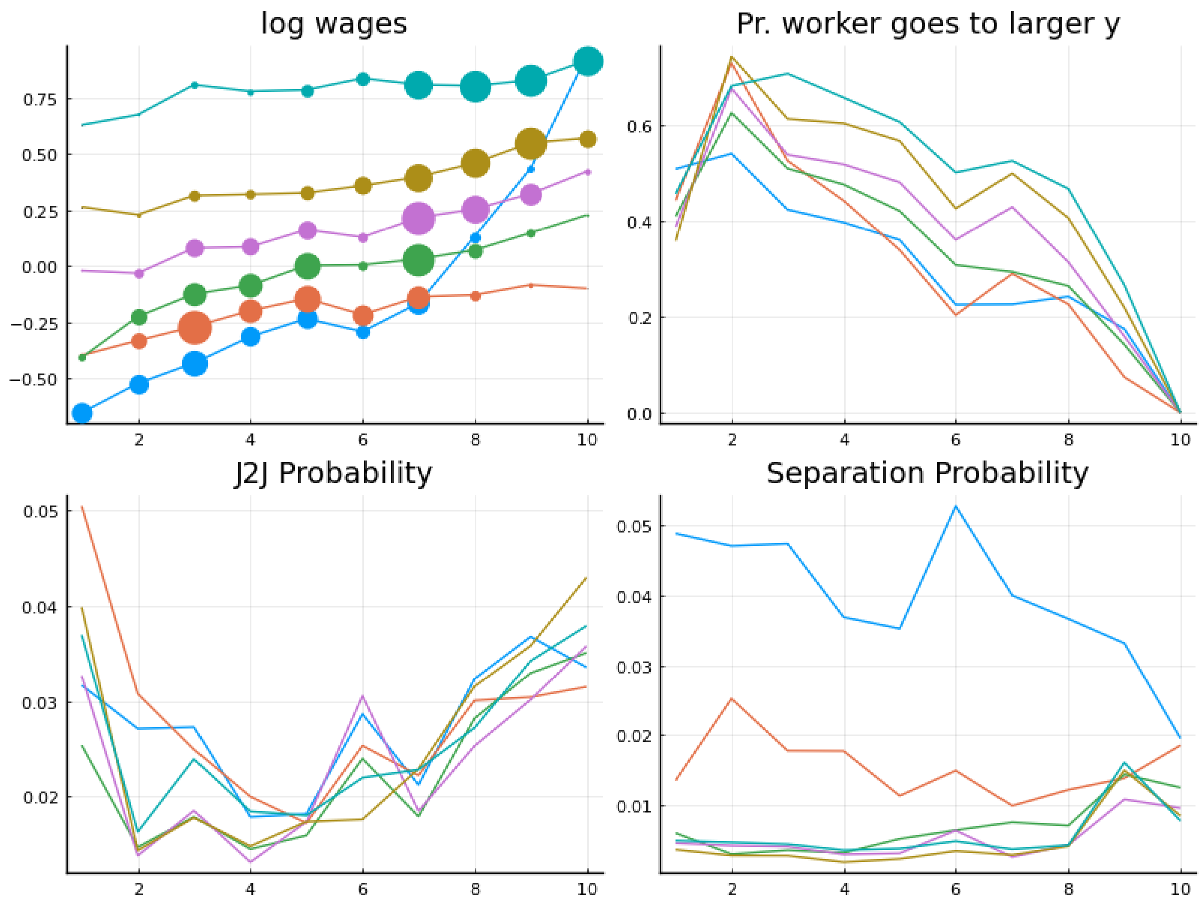
Step 1: Wages



Step 1: Unemployment quantities



Step 1: Employment quantities



Step 2: Surplus and vacancies

- Model implies
  - probability that worker x in a firm y moves to y'

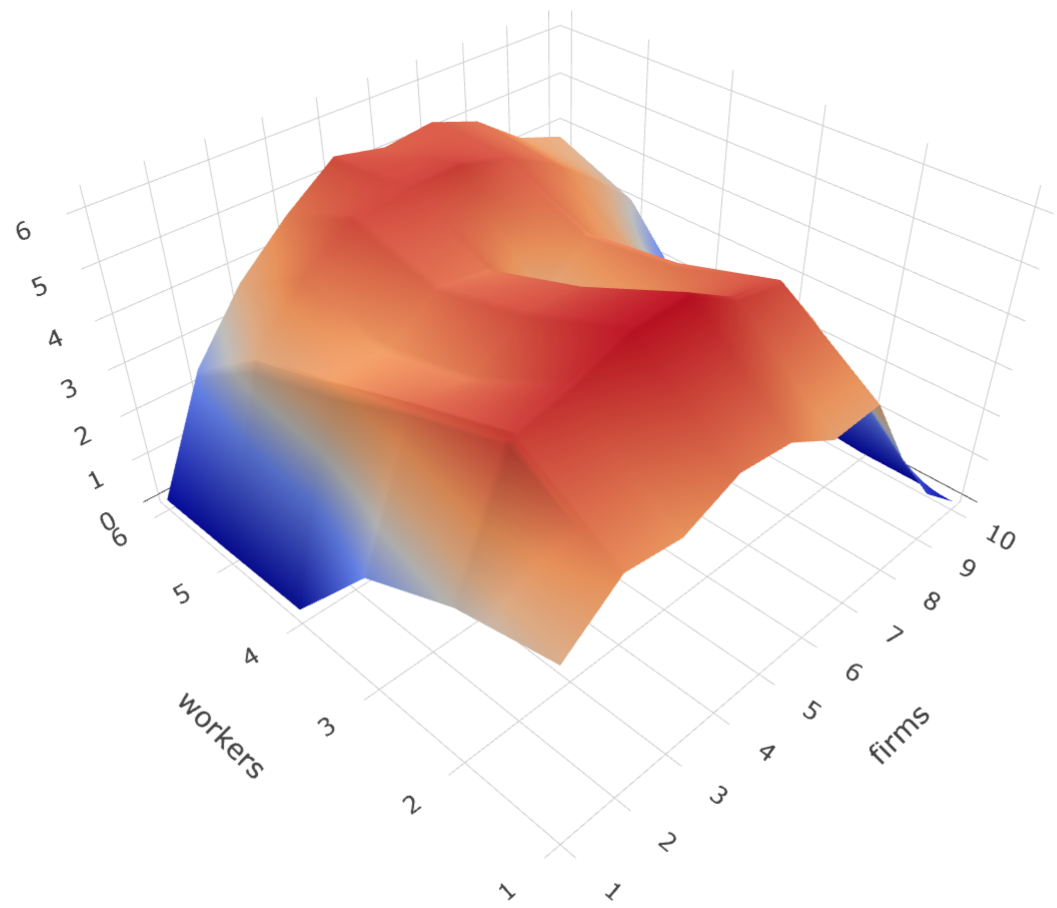
$$Pr[y', J2J|x, y] = (1 - \delta(x, y))\kappa\lambda\mathbf{1}[S(x, y') \geq 0] \frac{e^{\rho S(x,y')}}{e^{\rho S(x,y')} + e^{\rho S(x,y)}} V(y')$$

  - with a similar equation for the unemployed
  - note that this only involves types and surpluses, not wages

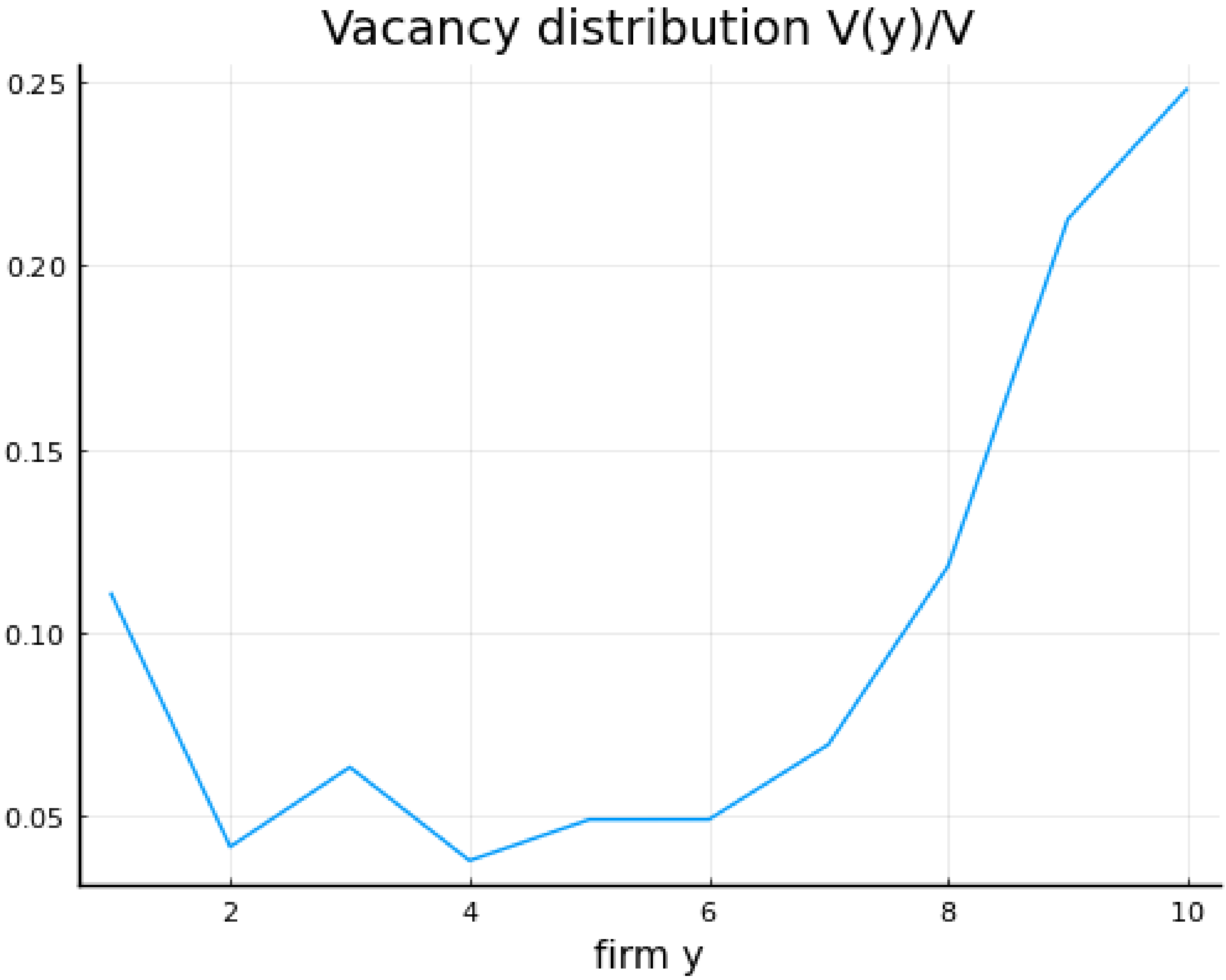
- It identifies $\rho S(x, y)$, $\lambda V(y)$ and κ
  - intuition is like in Sorkin (2018) but within x and allowing for sorting
  - inflows inform about $V(y)$ and $\rho S(x, y)$
  - outflows inform about $\rho S(x, y)$

- We estimate using the likelihood objective

Step 2: Scaled surplus $\rho S(x, y)$



Step 2: Vacancy distribution



Step 3: Separate ρ from $S(x, y)$

Model delivers a closed-form wage equation

$$\log(w_t) = c(x, y) + b(x) + \frac{1}{\rho}\mathcal{R}(x, y, \rho S_t)$$

- where $S_t \in [0, S(x, y)]$ and $u(w) = \log(w)$
- $\mathcal{R}(x, y, \cdot)$ involves only $\rho S(\cdot, \cdot)$ and other identified quantities

And the law of motion for ρS_t is known from $\rho S(\cdot, \cdot)$ since

- $\rho\xi$ is drawn from a standard $(\rho{=}1)$ Logit
- moving decisions only involve comparing $\rho S(x, y)$ and $\rho\xi$
- update rule on the job is $\rho S' = \max\{\rho S, \rho S(x, y') + \rho\xi\}$

Average wage growth on the job gives ρ (we find $\hat{\rho} = 0.73$)

Average wage net of $\mathcal{R}$ gives $c(x, y) + b(x)$

$w(x, y, S)$ is identified

[A] Expression for R

$$
\begin{aligned}
(1+r)\mathcal{R}(x,y,\rho S) &= (r+\delta(x,y))\rho S \\
&+ \lambda \int \phi(x,y) \int_{-\rho S(x,y)}^{\infty} \max\{\xi, 0\}\, g_o(\xi) d\xi V(y) dy \\
&- \kappa\lambda \int \phi(x,y') \Big[\int_{\rho S - \rho S(x,y')}^{\rho S(x,y) - \rho S(x,y')} (\rho S(x,y') + \xi - \rho S))\, g_o(\xi) d\xi \\
&- \int_{\rho S(x,y) - \rho S(x,y')}^{\infty} (\max\{\rho S(x,y) - \xi, 0\} + \xi - \rho S)\, g_o(\xi) d\xi \Big] V(y')\, dy'
\end{aligned}
$$

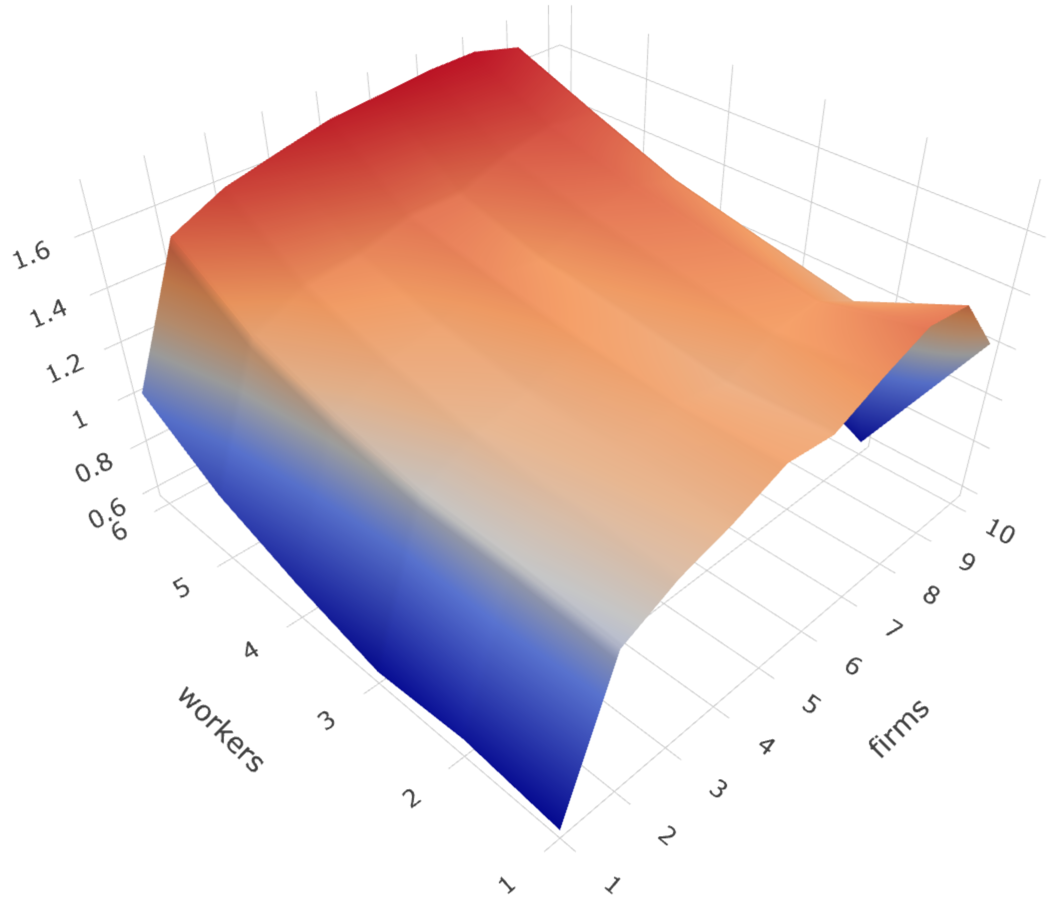
Step 4: $\Pi_0(y)$ and $f(x,y)$

- Get $\Pi_0(y)$ from firm surplus
  - define firm surplus $J(x,y,S) = \Pi_1(x,y,S+W_0(x)) - \Pi_0(y)$
  - we know that $J(x,y,S(x,y)) = 0$ and $\frac{\partial J(x,y,S)}{\partial S} = \frac{-1}{u'(w(x,y,S))}$
  - integrating over S identifies $J(x,y,S)$
  - equation for $\Pi_0(y)$ only involves $J(x,y,S)$

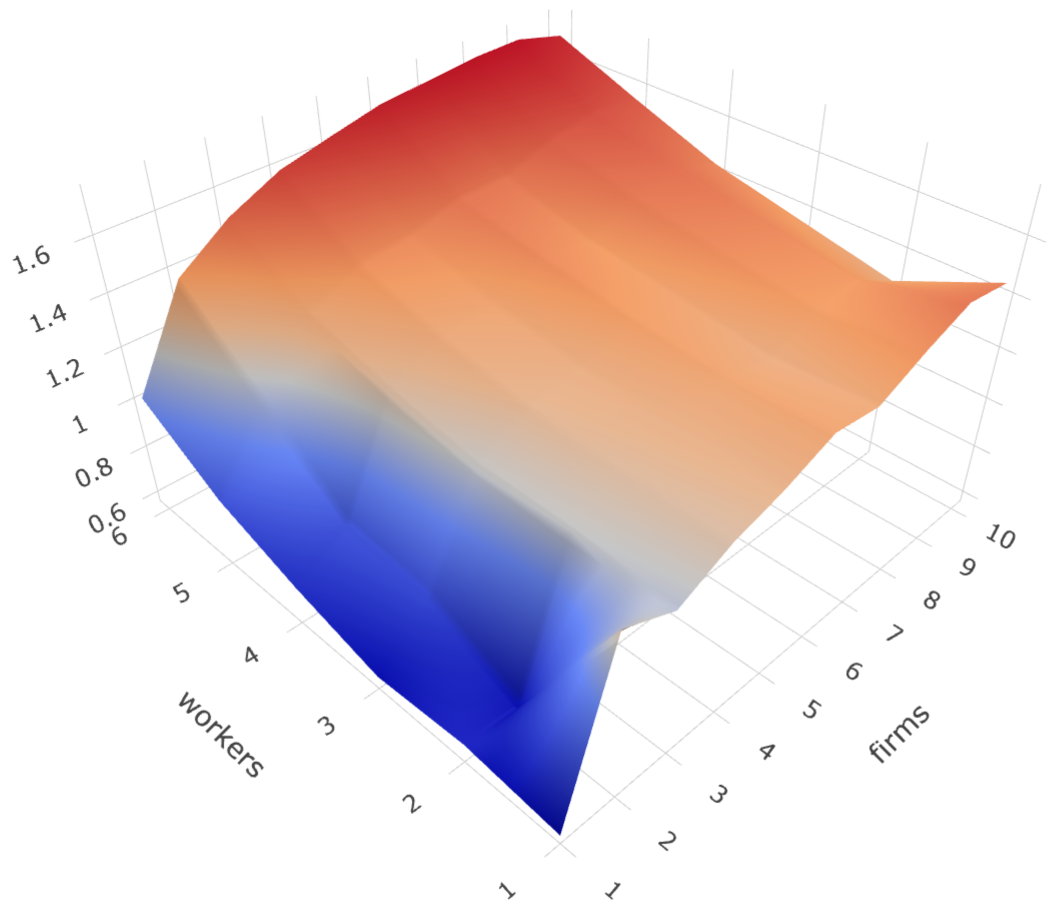
- Get $f(x,y)$ from the firm equation at reservation value

$$
\begin{aligned}
r\Pi_1(x,y,S(x,y)+W_0(x)) &= f(x,y) - w(x,y,S(x,y)) + 0 \\
&= r\Pi_0(y)
\end{aligned}
$$

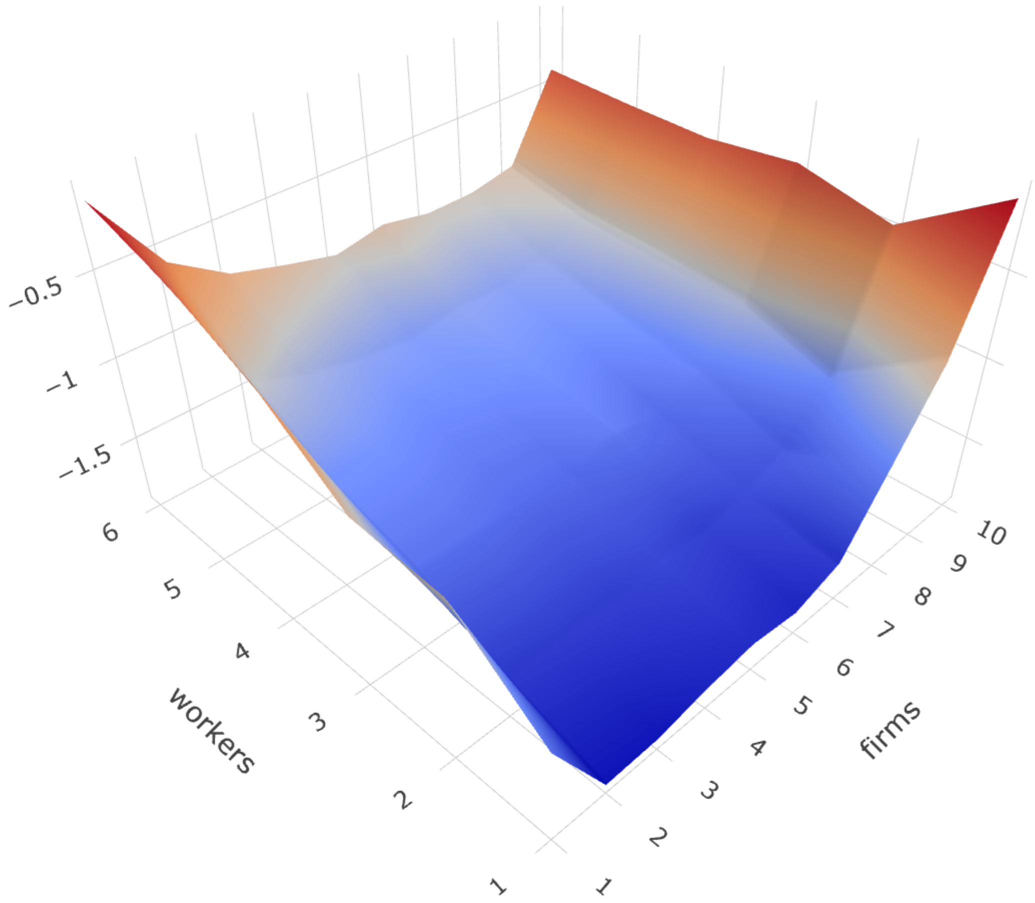
Step 4: Production function



Step 4: Production function reordered



Disutility of labor $c(x, y) + b(x)$



Conclusion & roadmap

- What we did
  - a two-sided search model of the labor market with OTJ search
  - introduce a preference shock in sequential contracting
  - model delivers structural equations with clear empirical counterparts
  - nonparametric identification and estimation for main model primitives
  - estimate on Swedish data

- Where we are going
  - evaluate the model
  - decompose wage dispersion
  - quantify misallocation
  - quantify effect of minimum wage, progressive taxation, etc.

[A] Event study