'You Will:' A Macroeconomic Analysis of Digital Advertising

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Abstract

An information-based model is developed where traditional and digital advertising finance the provision of free media goods and affect price competition. The economy is not efficient. Media goods are under provided. Additionally, there is excessive advertising when ads cannot be perfectly directed toward potential buyers. The tax-cum-subsidy policy that overcomes these inefficiencies in an informationally-constrained economy is characterized. The model is calibrated to the U.S. economy. Digital advertising increases welfare significantly and is disproportionately financed by better-off consumers. The welfare gain from the optimal tax-cum-subsidy policy is much smaller than the one realized by the introduction of digital advertising.

Date: July 2021, revised

Keywords: advertising, consumer welfare, directed and undirected advertising, free media goods, GDP measurement, leisure, price competition, public policy.

JEL Nos: E1, L1, O3.

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1 Opening

1.1 The Question

Free media goods are everywhere. Think about Facebook, Google, Google Maps, Pandora, Twitter, Wikipedia, YouTube, and apps for dating, dieting, exercising, playing guitar, meditation, inter alia. Often these products are financed through advertising or the sale of marketing information for advertising purposes. Digital advertising has two benefits. First, digital advertising costs less than traditional advertising. Second, it can be targeted better to consumers who might actually purchase the product. As a result it might spur competition among firms resulting in lower prices. As with traditional advertising, digital advertising provides free goods. It does so in spades. Since media goods are not sold, they do not directly show up in the national income accounts. Additionally, advertising expenditure is deducted off of firms' profits and consequently does not show up as a final expenditure in the national income accounts in the same way as physical investment spending does. Also, even if GDP is adjusted for such things, GDP and welfare are not the same thing; think about the welfare benefit of vaccines versus their cost.

To address this question a modernized variant of Butters's (1977) information-based advertising model is used. New theoretical results are presented. Quantitative analysis of the prototype model is undertaken to illustrate its real world potential. Numerical analysis is also used to explore properties of the model that can't be analyzed analytically. Since the prototype model's structure is simple, and the facts drawn upon to illuminate the framework are limited, this is somewhat an exercise in theory ahead of measurement.

Significant hot rodding has to be done to bring Butter's (1977) framework up to speed for the task at hand. First, the framework is modified to allow for both digital and traditional advertising. Both types of advertising permit firms to convey information about products and prices to consumers, as in Butters (1977). Firms choose how much digital and traditional advertising to do. This decision depends on the relative cost effectiveness of these two information delivery mechanisms. Second, advertising is associated with the provision of free media goods. To incorporate the free provision of media goods, and distinct from Butters (1977), a fully-fledged consumer sector is added. Consumers choose which varieties of goods to consume, based on the advertised prices they receive, and how much leisure to enjoy. Free media goods are taken to complement leisure in utility, in the sense of Edgeworth and Pareto. Third, consumers differ by their income, while in Butters (1977) they are all the same. Unlike Butters (1977), the maximum prices that consumers are willing to pay are endogenously determined as a function of the economic environment. These prices change as the economy evolves. A competitive equilibrium with digital and traditional advertising is characterized. As in Butters (1977), a distribution of prices emerges for a given product. This distribution differs from Butter (1977) due both to differences in consumers' incomes and the endogeneity of choke prices.

The resulting competitive equilibrium is not efficient, unlike Butters (1977), for two reasons. To start off with, free media goods are underprovided. Additionally, both digital and traditional adverts are sent to individuals who can't afford to buy the good at the advertised price. This wastes resources. The second-best tax-cum-subsidy policy that overcomes these inefficiencies in an informationally-constrained world is fully characterized. A version of the model is also considered where advertising can be directed toward only those customers who may buy the product. This is another advantage of digital advertising. Once again, the free media goods distributed with advertising are underprovided.

The developed model is calibrated using data on price markups, the ratio of advertising expenses to consumption expenditure, the ratio of spending on digital relative to traditional advertising, the click-through rate for digital advertising, the college premium, and the time spent on leisure by non-college- and college-educated individuals. This is something Butters (1977) could not have done at the time of his research. The welfare gain from the introduction of digital advertising is computed. In the baseline setting the difference between digital and traditional advertising lies in the former's cost advantage and the amount of free media goods delivered. A hybrid model is presented later where digital advertising is directed and traditional advertising undirected. In both settings the provision of free media goods boosts consumer welfare significantly. It also leads to more leisure, since media goods and leisure are complements in utility. The increase in leisure is more pronounced for the non-college educated vis à vis the college educated. The gain in utility from the rise in leisure is largely offset by a decline in regular consumption because people earn less now. The analysis suggests that affluent consumers may finance a disproportionately large share of the cost of media goods because they purchase goods at higher prices. Yet, the move toward digital advertising may benefit affluent consumers more because it stimulates price competition at the higher price end of the goods market relative to the lower end.

1.2 The Rise of Digital Advertising

Advertising has been around for eons. Babylonian merchants employed barkers who advertised their wares by shouting out. The Romans used signage outside of stores to sell wares; a bush signified a wine shop. Painted notices on the walls of bathhouses in Pompeii told of upcoming exhibitions. Marshall (1920, p. 271) noted that "A single prominent position in a great thoroughfare promotes the sale of many various things." After the arrival of the printing press came newspapers and then magazines. Benjamin Franklin published advertisements in his newspaper, the *Pennsylvania Gazette*. He is credited with publishing in 1741 the first magazine ad in the United States in the short-lived *The General Magazine and Historical Chronicle, for all the British Plantations in America.*

Advertising became an industry in the 19th century. N.W. Ayer & Son was founded in

Figure 1: A 1919 toothpaste ad in the *Saturday Evening Post* magazine for S.S. White Dental Manufacturing Co. *Source*: Ad*Access, Duke Digital Repository.

Philadelphia in 1869. It sold complete advertising campaigns for businesses. It is credited with slogans such as "A diamond is forever" used by De Beers. A typical early 20th century magazine ad is displayed in Figure 1. Direct mail advertising started in 1872 with Aaron Montgomery Ward who launched a one page catalog, which was quickly followed by the Sear's Catalog.

Things changed rapidly in the 20th century with the advent of new technologies. Radio advertising started in the 1920s. In 1922 the first paid radio ad ran in New York City to promote the sale of apartments. It cost $50 for 50 minutes of airtime. The first paid television ad was for Bulova watches. It was broadcast in 1941 before a baseball game between the Brooklyn Dodgers and Philadelphia Phillies. Television advertising expanded with the introduction of cable tv in the 1950s. MTV introduced music videos that were really just commercials for music artists. Additionally, channels were started that were devoted to advertising, such as HSN and QVC.

The information age began in the 1970s. A descendent of direct mail advertising is email marketing. This started in 1978 with an ad sent by Digital Equipment Corporation via the Arpanet to 400 DEC computer users. It didn't really take off until the 1990s when many people started to use the internet through outlets such as Microsoft's Hotmail that offered free email starting in 1996. Last, online advertising started in the 1990s. The first clickable ad was on *Hotwired.com* in 1994, then the online version of *Wired* magazine–see Figure 2. It was part of AT&T's "You will" campaign that prognosticated about the future in the information age. The ad enjoyed a click-through rate of 44 percent and cost AT&T $30,000 for three months.

The composition of advertising spending changed as new vehicles for delivering ads cropped up, as Figure 3 shows. Ads in newspapers and magazines declined with the arrival of TV.

Figure 2: The first clickable ad, part of AT&T's "You Will" campaign. *Source*: *The Atlantic*, 2017.

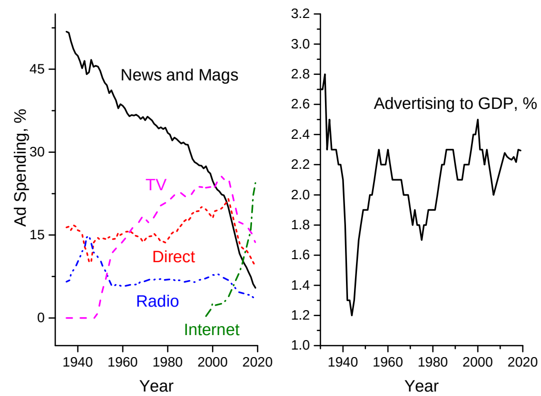

Figure 3: Advertising in the United States, 1935-2019. Advertising has consistently amounted to approximately 2 percent of GDP. Its composition has seen dramatic changes; however, as new mediums for communicating emerged. *Sources*: Douglas Galbi and AdAge.

Digital advertising rose with the advent of the information age. It's interesting to note that advertising's share of GDP has remained roughly constant in the postwar period at around 2 percent.

Online advertising is dominated by two giants, Facebook and Google. Google was founded in 1998 and Facebook in 2004. The ad revenue earned by these two companies (and Amazon) is shown in Figure 4 (right panel). Google's ad revenue shot up from around $70 million in 2001 to $135 billion in 2019. Likewise, Facebook's ascent is equally dramatic, rising from roughly $2 to $70 billion between 2010 and 2019. The first search engine was Archie, created in 1990. Alan Emtage, its creator, developed an indexing technique that allowed Archie to catalogue "freely available or Public Domain documents, images, sounds and services on the network." Yahoo! Search was the first popular search engine, arriving in 1995. The next decade saw the rise of Google Search, which yielded better search results using an iterative algorithm that ranked web pages on the number of websites that linked to them and the ranking of these websites.

The first social media website is generally attributed to Six Degrees, founded in 1997. The name was based on the idea that people are linked to each other by six, or fewer, social connections. People could create profiles and "friend" each other. It had around 3.5 million users at its pinnacle. Things took off with the creation of MySpace in 2003. Between 2005 and 2008 it was the largest social media site in the world with over 100 million users per month.

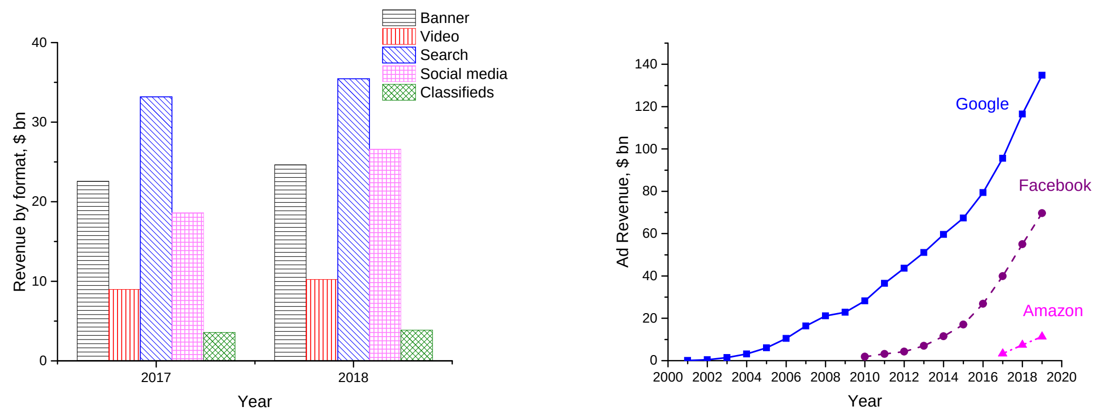

Figure 4: Right panel, ad revenue earned by Amazon, Facebook, and Google, 2001-2019. Left panel, distribution of U.S. online advertising revenue by format, 2017 and 2018. *Source: statista.*

After 2008 Facebook dominated the social media world. Facebook had 2.5 billion monthly users in 2019.

A breakdown of online advertising revenue by format is also displayed in Figure 4 (left panel). Online search is the dominant vehicle for digital advertising, followed by social media. Google inserts online ads into its products, such as Google Search, using a pay-per-click pricing model. The search advertising cost per click was \$0.69 in 2019. Google Search handled 5.4 billion search requests per day in 2019. Moving up from the third to the second position displayed by Google Search's results leads to a 31 percent increase in traffic. Advertisers pay for location. Apparently, only 0.78 percent of Google users make it to the second page of search results. The return on various mediums of advertising is presented in Figure 5, right panel. Digital search has the highest return in terms of sales per dollar spent on advertising. The left panel illustrates that spending by advertisers closely tracks the amount of time that consumers spend on the mediums.

A lot of digital content is provided for free via advertising. Think about the free goods just from Google: Chrome, Google Search, Google Maps, Gmail, Google Drive, YouTube, etc. Figure 6 shows the number of apps available in Google Play Store. In 2019 this was a whopping 2.8 million. Interestingly, consumers spend little for these products. Less than 14 percent of Google users spent more than \$10 per digital media in the Google Play Store, as the left panel illustrates.

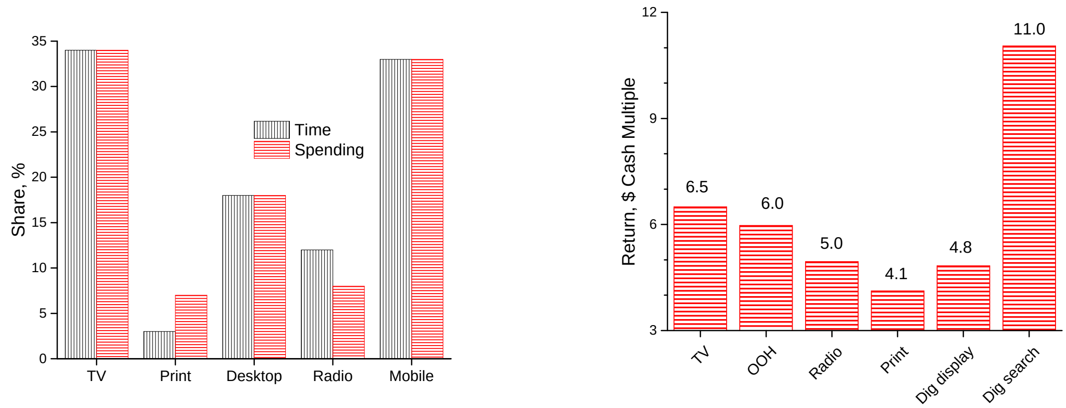

Figure 5: Right panel, the return per dollar of advertising by medium in the United States for 2017, measured as a cash multiple, 2001-2019. Left panel, U.S. advertising spending vs time spent by consumers by medium, 2018. *Source: statista.*

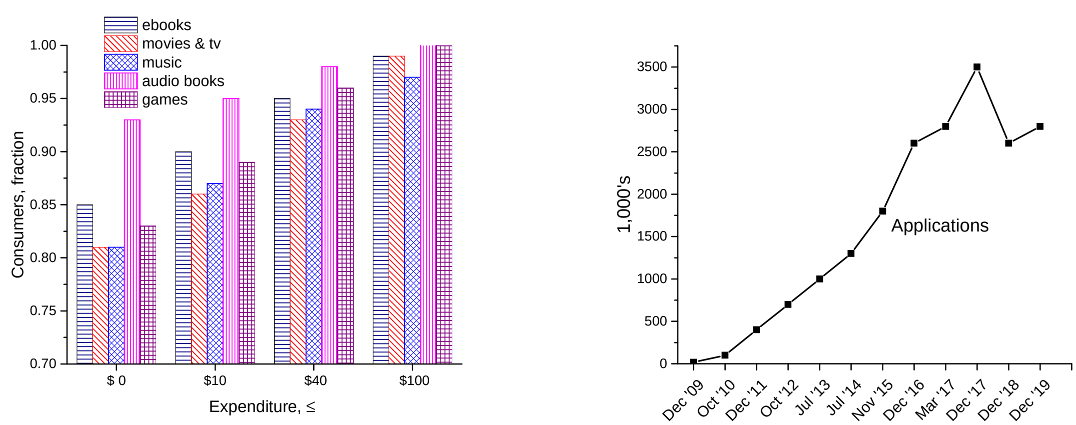

Figure 6: Right panel, applications in the Google Play Store, 2009-2019. Left panel, money spent by U.S. consumers on Google digital media products in 2017, presented in cumulative distribution form. *Source: statista.*

2 A Brief Review of the Advertising Literature

Advertising has been part and parcel of economic life for a long period of time, as Figure 3 suggests. Until the second part of the twentieth century, however, economists paid little attention to advertising. The economic analysis of advertising can be traced back to insightful work by Marshall (1920). This subject area has flourished since then.[1]

At a time when competitive equilibrium and full information were the fundamentals of economic thinking, economists struggled with the question of why consumers would respond to advertising. Two views emerged. The first one holds that advertising is persuasive, altering consumers' tastes and creating brand loyalty. Not surprisingly, according to the persuasive view, advertising has no real value to consumers. It can have important anti-competitive effects, resulting in increased economic concentration. Marshall (1920, p. 304 and 306) noted that "much of the modern expenditure on advertising is not constructive, but combative," and that "advertisements which are mainly combative generally involve social waste."

The second view holds that advertising is informative. According to this perspective, markets are characterized by imperfect consumer information that leads to inefficiencies. Here, rather than being a problem, advertising emerges as a remedy offered by the market. Clearly, according to the informative view, advertising promotes competition. Marshall (1920, p. 305) also thought that advertising could be constructive by "the assistance, which they afford to customers by enabling them to satisfy their wants without inordinate fatigue or loss of time, would be appropriate, even if the business were not in strong rivalry with others." He noted that "exceptionally constructive are all those measures needed for explaining to people generally the claims of some new thing, which is capable of supplying a great but latent want."

In the approach taken here advertising is informative. The foundation of the informative view of advertising was laid by Ozga (1960) and Stigler (1961). They saw price dispersion as a reflection of consumer ignorance and advertising as a valuable source of information for consumers that results in a reduction in price dispersion. Telser (1964) significantly advanced the theoretical and empirical foundations for the informative view, concluding that advertising is a sign of competition and is an important source of information for the consumers. Following these lines, Butters (1977) offered the first equilibrium analysis of advertising in a multi-firm model. He showed that advertising in equilibrium is efficient. Stegeman (1991) extended Butters's (1977) work with the assumption that consumers' valuations of products are heterogeneous. He demonstrated that informative advertising is then inefficient.[2]

[1] Bagwell (2007) provides a detailed survey of the literature, so only a capsule summary is given here.

[2] Digital advertising was not around at the time of Stegeman's (1991) paper. Like Butters (1977), he did not have a fully fleshed-out consumer sector, which wasn't needed for their analyses. The latter is important for the current inquiry for two reasons. First, consumer behavior changes as the economy evolves due to technological progress in advertising and, second, tastes need to be specified for the welfare analysis. Additionally, Stegeman did not present the optimal tax-cum-subsidy policy that renders the advertising economy efficient. Last, he

Extending Butters's (1977) model to an economy where firms have differing levels of productivity, Dinlersoz and Yorukoglu (2008) studied how improvements in advertising technology affect industry equilibrium. In related work, Dinlersoz and Yorukoglu (2012) analyzed how advertising technology affects firm dynamics. They showed that entry, exit, and volatility in firm size and value, increase as advertising technology improves. The equilibria in both models are efficient. Along the same lines, Gourio and Rudanko (2014) studied the role in firm and industry dynamics that the customer acquisition process has through marketing.

Incorporating advertising into macroeconomic frameworks is relatively new. Using a macroeconomic model, Hall (2014) argued that the cyclical behavior of advertising provides revealing information about the behavior of macroeconomic wedges over the cycle. In more recent work, Perla (2019) builds a model where consumers learn about firms slowly through a network of connections between consumers and firms that endogenously evolves through the life cycle of an industry. The implications of advertising for firm dynamics and economic growth through its interaction with R&D investment at the firm level are analyzed by Cavenaile and Roldan (2019). They provide empirical evidence supporting substitution between advertising and R&D using exogenous changes in the tax treatment of R&D expenditures across U.S. states. Rachel (2019) argues that the rise of leisure goods provided by advertising has an adverse impact on welfare. As leisure rises the amount of labor going into R&D declines. This stifles growth. Both Cavenaile and Roldan (2019) and Rachel (2019) take a branding approach to advertising, as opposed to the information-based one here. Advertising benefits firms by either influencing consumer tastes or as a direct input into production. It operates by allowing a firm to get a leg up on a competitors, so it has the counterproductive element to it in the flavor of Marshall (1920). In neither paper is there digital-specific technological progress in advertising or heterogenous consumer types. Last, Dinlersoz, Goldschlag, Yorukoglu, and Zolas (2021) incorporate the interaction between advertising and trademarks in a macroeconomic model to study the impact of trademarking on product quality, the reallocation of resources across firms, and welfare.

3 Setup

The analysis starts with the case of undirected advertising and then turns to directed advertising.[3] The modifications required to the setup to analyze directed advertising are minimal. To

did not take the model to data; calibration was in its infancy at the time of his research.

[3] The setup is static. Adding dynamics would greatly complicate things–see for instance Dinlersoz, Goldschlag, Yorukoglu, and Zolas (2021). Clean theoretical results would be difficult to obtain. The static setup is probably not much of a drawback for the question at hand, however, since issues such as the entry of new products and growth are abstracted from. For new products the buildup of information over time might be important. The rise of digital advertising has been very rapid, implying that any transitional dynamics would need to operate at a fast clip. Also, the depreciation rate on advertising is high, somewhere between 30 and 50 percent, so treating it as a flow rather than a stock is not a great violation for the question entered here.

this end, consider an economy with three types of goods; namely, generic consumption goods, media leisure goods, and leisure. At most a unit measure of varieties of regular consumption goods can be produced. There is free entry into the production of each variety of regular goods, $i \in [0, 1]$, subject to incurring a fixed cost of $\mathfrak{r}$. To sell its product a regular goods producer must advertise to potential customers, which is costly. Advertising can be done in two ways. The first way is through traditional advertising. The second way is via modern online advertising. A potential customer receives ads for a variety in a random manner. A producer of regular good i is free to set the price, p_i, that it wants. This can differ across variety-i producers because consumers will vary in the advertised prices that they randomly received in their information sets.

Ads are delivered via media goods, which are provided to consumers for free. There are $\mathfrak{m}$ media goods available. Media goods have a click-through rate that represents the number of ads that the good will deliver. The supply of media goods, $\mathfrak{m}$, is determined by the amount of advertising that firms want to do. The cost of providing these goods is absorbed as an advertising expense.

Turn now to the consumer/worker. Regular good-i must be consumed in the discrete quantity $c_i \in \{0, 1\}$. An individual might not consume the full spectrum of regular goods because either they didn't receive an ad for a good or because they couldn't afford them at the advertised price. Media leisure good-j is consumed in the discrete quantity $m_j \in \{0, 1\}$. Since media leisure goods are free, the consumer will enjoy the full spectrum of what is currently available. There is a unit mass of people. Each person is indexed by a talent level $\tau \in \{\underline{\tau}, \overline{\tau}\}$, where $\underline{\tau} = 1 < \overline{\tau}$. The fractions of the population with $\underline{\tau}$ and $\overline{\tau}$ are denoted by $\mathfrak{t}$ and $1 - \mathfrak{t}$. A person with ability level $\tau = \underline{\tau}$ is unskilled and earns the wage rate 1. A skilled person, $\tau = \overline{\tau}$, earns the wage $\overline{\tau}$, but must incur a fixed education cost, $\mathfrak{e}$, in terms of time. In other words, the wage rate for unskilled labor is the numeraire, which implies that all goods prices are measured in terms of unskilled labor. An individual has one unit of time that they can split between working in the market, h, leisure, l, and education, $\mathfrak{e}$. As will be seen, the education cost operates to make the skilled work more than the unskilled, since the former must recover their investment in human capital.

Preferences are given by

$$\theta \ln\left(\int_0^v c_i di\right) + \frac{(1-\theta)}{\rho} \ln[\kappa l^\rho + (1-\kappa)(\int_0^{\mathfrak{m}} m_j dj)^\rho], \text{ with } \rho < 0, \tag{1}$$

where v and $\mathfrak{m}$ demarcate the set of available regular and media goods. These preferences are well defined even when particular varieties of consumption goods are not consumed. Media goods can be mixed with leisure to generate utility; i.e., they are leisure goods. For example, you must spent time to enjoy an online game. The fact that regular consumption goods are aggregated linearly is not a undue restriction. Consumption within a variety is indivisible and hence there is no intensive margin of consumption. The consumer decides whether or not to consume an extra variety, which is a continuous decision. Total consumption for a person moves smoothly as in the standard macroeconomic model.

The assumption that $\rho < 0$ implies that leisure, l, and leisure goods, the m_j's, are Edgeworth-Pareto complements in utility–in other words, the cross partial in utility is positive. The idea is that more leisure goods increase the marginal utility of leisure. Therefore, you will want more leisure at the margin. The notion of leisure complementing goods is in Greenwood and Vandenbrouke (2008) and Kopecky (2011). Kopecky (2011) suggests the decline in the price of leisure goods encouraged the elderly to spend a larger fraction of their life in retirement. Aguiar et al (2017) use this notion to argue that part of the recent decline in hours worked by young males is due to the advent of recreational computing. Kopytov et al (2021) find that declining recreation good prices can account for much of the increase in leisure in both the United States and across the world, due to their complementarity with leisure.

Last, the individual's budget constraint is given by

$$\int_0^v p_i c_i di = \tau h(\tau) \equiv \begin{cases} \overline{\tau}(1 - l - \mathfrak{e}), & \text{skilled;} \\ 1 - l, & \text{unskilled,} \end{cases} \tag{2}$$

where, with some abuse of notation, in this context p_i represents the minimum price for good i that the consumer/worker has in his information set and $h(\tau)$ is the hours worked by a type-τ person.

4 Regular Goods Firms

Firms can freely enter into the production of any variety of regular goods subject to a fixed cost of $\mathfrak{r}$ (in units of unskilled labor). Suppose that there are v active varieties of goods with n firms producing each variety for a total of vn firms in the economy. The quantities v and n will be determined in equilibrium by the fact that firms must earn zero profits. Any variety of regular goods can be produced by a firm according to the constant-returns-to-scale production

$$o = h/\gamma,$$

where o is the output of the good and h is the amount of labor employed. The unit cost of producing a good is γ.

The constant returns to scale assumption is standard in macroeconomics. Unlike the standard model in macroeconomics, however, the number of firms producing a variety will be determinate, due to the information friction. In fact, with increasing returns there could be many firms producing the same variety. On this, even in the current setup average production costs decline with sales due to the presence of the fixed cost. High volume firms will supply consumers at low prices while low volume ones will sell at higher prices. The fact that firms can sell at different prices is discussed below. The simple production structure adopted permits new theoretical results to be obtained, which is important when crossing uncharted territory.

To sell its product at time price p, a firm must reach out to customers, which involves advertising.[4] Ads are delivered through media goods, which can be distributed through either a traditional or digital vehicle. Let a_t and a_d represent the number of traditional and digital ads that are sent out by the firm. To generate a_t traditional ads a firm must provide t media goods that each has a click through rate of $\zeta < 1$; i.e.,

$$a_t = \zeta t. \tag{3}$$

The cost (measured in terms of unskilled labor) for traditional advertising is

$$A(a_t) = \phi a_t^\alpha = \phi(\zeta t)^\alpha, \text{ with } \alpha > 1. \tag{4}$$

Likewise, digital ads are distributed via digital goods provision. A digital good has a click-through rate of $\psi < 1$. So, d digital goods will deliver a flow of ads, a_d, according to

$$a_d = \psi d. \tag{5}$$

The cost of producing a_d digital ads is

$$A(qa_d) = \phi(q\psi d)^\alpha, \tag{6}$$

where q is a technology factor reflecting the cost advantage of digital advertising. As q declines, digital advertising becomes more efficient relative to traditional advertising. Additionally, if digital advertising can directed to consumers who possibly will buy the product, then this offers a potential advantage over traditional advertising–directed advertising is turned to in Section 12. These cost functions imply that there is decreasing returns in advertising. This is a feature of the data, according to Bagwell (2007). When the click-through rate is low a lot of free content will have to be provided to achieve a given impact from advertising. As will be shown in Section 10, the click-through rate of digital advertising is lower than traditional advertising, implying that digital advertising will provide more free media goods per advertising message noticed by consumers.

Because consumers will differ in the ads that they have in their information sets, firms do not have to charge the same price. This information friction allows firms to charge a price higher than its marginal production cost, γ. Let $\underline{p}$ represent the lowest profitable price in equilibrium and likewise $\overline{p}$ denote the maximum profitable one. Now, a firm is free to charge

[4]This is a key distinction between a Butters-style advertising model and a directed search model. In a directed search model there is no friction associated with providing consumers information about posted prices. Additionally, unlike a directed search model, in an advertising model a firm can supply all customers who want to buy its product. In a directed search model customers queue up to buy a product from a firm with limited selling capacity. Thus, a customer can only expect to purchase the good with some probability. So, the friction here concerns the prices that consumers have in their information sets and not whether a consumer will be able to buy a good from the firm at the posted price.

any price p such that $\underline{p} \leq p \leq \overline{p}$. The higher the price, the less likely the firm will make a sale. The set of viable equilibrium prices, $\mathcal{P}$, is characterized later in Proposition 2. Since there is free entry into a variety, it must transpire that a firm will earn the same profit at any price, $p \in \mathcal{P}$.

4.1 Advertising

Let $S(p) = \Pr(\text{SALE}|p)$ be the probability that an ad at price p will generate a sale for the firm. This probability is exogenous for a firm and is unpacked later. The firm chooses its advertising strategy to maximize its profits at price p. So, its advertisements solve

$$\Pi(p; q) \equiv \max_{a_t, a_d}\{(p-\gamma)(a_t + a_d)S(p) - A(a_t) - A(qa_d)\}. \tag{7}$$

Here the term $p - \gamma$ represents the firm's unit profits (excluding advertising costs) while $(a_t + a_d)S(p)$ is the firm's total sales. Its advertising costs are $A(a_t) + A(qa_d)$. The first-order conditions for a_t and a_d are

$$(p-\gamma)S(p) = \phi\alpha a_t^{\alpha-1} \text{ and } (p-\gamma)S(p) = \phi\alpha q^{\alpha} a_d^{\alpha-1}. \tag{8}$$

The common lefthand side of these expressions is the expected profit (or marginal benefit) from sending out an extra ad. The righthand sides represent the marginal costs of an extra traditional or digital ad. Clearly, the marginal cost of digital advertising increases with its cost factor, q.

Proposition 1 *(Advertising) All firms do the same amount of traditional, a_t, and digital advertising, a_d, even when charging different prices for their products, $p \in \mathcal{P}$. Digital advertising decreases with its cost factor, q.*

Proof. See the Appendix. ■

To understand the logic underlying the proposition, note that in equilibrium a firm is free to pick any price it desires. So, expected unit profits, $(p-\gamma)S(p)$, must be constant across equilibrium prices. Suppose not. Then firms with higher values for $(p-\gamma)S(p)$ would make more than firms with lower values because the former could always do the same amount of advertising as the latter.

If the marginal benefit is constant across prices, then from (8) so must be the marginal costs. This implies that a_t and a_d are invariant across prices, p. Finally, from the above first-order conditions for a_t and a_d, it is immediate that

$$a_t = [\frac{(p-\gamma)S(p)}{\alpha\phi}]^{1/(\alpha-1)} \text{ and } a_d = [\frac{(p-\gamma)S(p)}{q^{\alpha}\alpha\phi}]^{1/(\alpha-1)} = q^{\alpha/(1-\alpha)}a_t, \tag{9}$$

or equivalently

$$a_t = [\frac{(\underline{p}-\gamma)S(\underline{p})}{\alpha\phi}]^{1/(\alpha-1)} \text{ and } a_d = [\frac{(\underline{p}-\gamma)S(\underline{p})}{q^{\alpha}\alpha\phi}]^{1/(\alpha-1)}, \tag{10}$$

where the second line follows from the proposition. If there are n firms producing each variety, then the total number of adverts per variety, a, is

$$a = n(a_t + a_d) = n(1 + q^{\alpha/(1-\alpha)})[\frac{(\underline{p} - \gamma)S(\underline{p})}{\alpha\phi}]^{1/(\alpha-1)}. \tag{11}$$

5 Pricing

5.1 Advertised Price Distribution

Consumers receive ads randomly, without any targeting by firms–targeting is discussed in Section 12. Assume that there is a much larger mass of consumers vis à vis firms and that no consumer receives more than one ad from the same firm. Let a represent the number of ads for a variety per consumer in the economy. The number of ads, i, that a consumer receives will be distributed according to a Poisson distribution $e^{-a}a^i/i!$.[5] Now, let $P(p) = \Pr(\text{PRICE} \leq p)$ be the fraction of ads for a variety that have a price less than or equal to p. The function $P(p)$ is characterized later in Proposition 3.

Suppose a firm sends an ad to a consumer offering to sell the good at price p. The odds of a consumer with i *other* ads having no price lower than p are $[1 - P(p)]^i$. Even when the firm's price p is the lowest one in the consumer's information set, the person may not buy the good because it is too expensive. Let $I(p;\tau) = 1$ denote the situation when a type-τ consumer buys the firm's good at price p and $I(p;\tau) = 0$ when not. It then follows that the probability of an ad with price p to a consumer will generate a sale, given that the consumer may have received $i = 0, 1, 2, \cdots$ other ads, is given by[6]

$$\begin{aligned} S(p) = \Pr(\text{SALE}|p) &= e^{-a}\sum_{i=0}^{\infty}\frac{a^i}{i!}[1 - P(p)]^i[\mathfrak{t}I(p;\underline{\tau}) + (1-\mathfrak{t})I(p;\overline{\tau})] \\ &= e^{-aP(p)}[\mathfrak{t}I(p;\underline{\tau}) + (1-\mathfrak{t})I(p;\overline{\tau})]. \end{aligned}$$

[5] To see this, imagine an economy with a discrete number of consumers, $\mathfrak{c}$, who are flooded with $a\mathfrak{c}$ ads per variety. The probability that a consumer will receive i ads is distributed according to the binomial distribution

$$\binom{a\mathfrak{c}}{i}(\frac{1}{\mathfrak{c}})^i(1 - \frac{1}{\mathfrak{c}})^{a\mathfrak{c}-i},$$

where $1/\mathfrak{c}$ is the chance that a consumer gets an ad (success) and $1-1/\mathfrak{c}$ are the odds that they won't (failure). Out of a set of $a\mathfrak{c}$ ads there are $\binom{a\mathfrak{c}}{i}$ ways each event could happen. Finally,

$$\lim_{\mathfrak{c}\to\infty}\binom{a\mathfrak{c}}{i}(\frac{1}{\mathfrak{c}})^i(1 - \frac{1}{\mathfrak{c}})^{a\mathfrak{c}-i} = e^{-a}a^i/i!.$$

[6] To go from the first to the second line, set $s = \sum_{i=0}^{\infty}(a^i/i!)x^i = [1 + ax + (ax)^2/2! + (ax)^3/3! + \cdots]$, which implies that $ds/dx = [a + a^2(ax) + a^3(ax) + \cdots] = as$. Therefore, $(1/s)ds/dx = a$ so that $s = e^{ax}$. Now, let $x = 1 - P(p)$ to get $\sum_{i=0}^{\infty}(a^i/i!)[1 - P(p)]^i = e^{a[1-P(p)]}$, from which the desired result follows.

Three prices play a central role in the analysis; namely, the minimum price in the economy, $\underline{p}$, the maximum price at which the unskilled will buy, $p(\underline{\tau})$, and the maximum price at which the skilled will purchase, $\overline{p}$. The minimum price is determined by technological considerations while the maximum prices also depend upon the outcome of the consumer problems for the unskilled and skilled, an important distinction from Butters (1977). The determination of $\underline{p}$ and $\overline{p}$ is discussed now with the specification of $p(\underline{\tau})$ following shortly after. Consider a firm that chooses to charge the minimum price, $\underline{p}$. All the ads that this firm sends out will result in purchases by consumers, implying $S(\underline{p}) = 1$. Since there is free entry into the production of any variety, this firm will earn zero profits. Hence,

$$\Pi(\underline{p}; q) - \mathfrak{r} = 0.$$

Solving this equation gives

$$\underline{p} = [\frac{\mathfrak{r}}{\Upsilon(q)}]^{(\alpha-1)/\alpha} + \gamma, \tag{12}$$

where

$$\begin{aligned}\Upsilon(q) &\equiv (1 + q^{\alpha/(1-\alpha)})\phi^{1/(1-\alpha)}(\alpha^{1/(1-\alpha)} - \alpha^{\alpha/(1-\alpha)}) \\ &= (1 + q^{\alpha/(1-\alpha)})(\frac{1}{\phi})^{1/(\alpha-1)}(\frac{1}{\alpha})^{\alpha/(\alpha-1)}(\alpha - 1) > 0.\end{aligned}$$

(See the proof of Proposition 1 in the Appendix for guidance.) The minimum price, $\underline{p}$, is determined solely by technological factors. As a consequence so is the amount of traditional, a_t, and digital advertising, a_d, that each firm does, a fact that follows from (10) in conjunction with $S(\underline{p}) = 1$.

Since a firm is free to pick any price it must be the case that

$$\Pi(p'; q) = \Pi(p''; q), \text{ for any } p' \text{ and } p'' \in \mathcal{P}.$$

Proposition 1 states that all firms do the same amount of advertising. Therefore,

$$(p' - \gamma)S(p') = (p'' - \gamma)S(p''), \tag{13}$$

or equivalently

$$\begin{aligned}&(p' - \gamma)e^{-aP(p')}[\mathfrak{t}I(p'; \underline{\tau}) + (1 - \mathfrak{t})I(p'; \overline{\tau})] \\ &\qquad\qquad = (p'' - \gamma)e^{-aP(p'')}[\mathfrak{t}I(p''; \underline{\tau}) + (1 - \mathfrak{t})I(p''; \overline{\tau})].\end{aligned}$$

Turn to the firm that charges the highest price, $\overline{p}$. Only skilled consumers ($\tau = \overline{\tau}$) who have no other ads will buy the firm's product. Therefore, $S(\overline{p}) = e^{-a}(1 - \mathfrak{t})$, because $P(\overline{p}) = 1$ (i.e., all ads have a price lower than $\overline{p}$). Therefore, evaluating the above expression at $p' = \overline{p}$ and $p'' = \underline{p}$ gives

$$e^{-a}(1 - \mathfrak{t})(\overline{p} - \gamma) = \underline{p} - \gamma,$$

so that the maximum price at which a skilled person buys a good is

$$\overline{p} = \frac{\underline{p} - \gamma}{e^{-a}(1 - \mathfrak{t})} + \gamma = \frac{[\mathfrak{r}/\Upsilon(q)]^{(\alpha-1)/\alpha}}{e^{-a}(1 - \mathfrak{t})} + \gamma. \tag{14}$$

Next, focus on the highest price that unskilled consumers can afford, $p(\underline{\tau})$. At any higher price there will be a discrete drop off in potential customers from 1 down to $1 - \mathfrak{t}$. To recover profits there must be a discrete jump up in the lowest price above $p(\underline{\tau})$, denoted by $p_{\uparrow}(\underline{\tau})$. Since there are no prices in between $p(\underline{\tau})$ and $p_{\uparrow}(\underline{\tau})$ it transpires that $P(p(\underline{\tau})) = P(p_{\uparrow}(\underline{\tau}))$, which is formalized later in Proposition 3. The prices at the left and righthand sides of the jump must have equal profits, so that $(1 - \mathfrak{t})[p_{\uparrow}(\underline{\tau}) - \gamma] = p(\underline{\tau}) - \gamma$, which yields

$$p_{\uparrow}(\underline{\tau}) = \frac{p(\underline{\tau}) - \gamma}{1 - \mathfrak{t}} + \gamma. \tag{15}$$

Now, there must be firms charging every price, p, in the set $\mathcal{P} = [\underline{p}, p(\underline{\tau})] \cup [p_{\uparrow}(\underline{\tau}), \overline{p}]$. To understand why, suppose to the contrary that there is a hole in one of the intervals. Firms at the lower edge of the hole could increase profits by raising their price slightly, because this will not affect the number of customers they have. The proposition below describes the situation.

Proposition 2 *(Pricing) For any variety of regular goods there are firms charging every price, p, in the set $\mathcal{P} = [\underline{p}, p(\underline{\tau})] \cup [p_{\uparrow}(\underline{\tau}), \overline{p}]$. Take the aggregate amount of advertising per variety, a, as given. Then, both $\underline{p}$ and $\overline{p}$ are increasing in the entry cost, $\mathfrak{r}$, the marginal cost of production, γ, and the cost of digital advertising, q. Last, the maximum price, $\overline{p}$, is decreasing in the fraction of individuals, $1 - \mathfrak{t}$, who are skilled.*

Proof. See the Appendix. ■

It's probably obvious that an increase in the cost of doing business, as given by $\mathfrak{r}$, γ, and q, will lead to the pricing set $\mathcal{P}$ =$[\underline{p}, p(\underline{\tau})] \cup [p_{\uparrow}(\underline{\tau}), \overline{p}]$ shifting rightward, because given the free-entry assumption firms must recover their costs. When there are more skilled consumers, $1 - \mathfrak{t}$, it becomes more profitable to charge the maximum price, $\overline{p}$, since the odds of an ad landing on a skilled person increase. But, again, perfect competition will drive the maximum price down so that firms earn zero profits.

Direct attention now to characterizing the distribution of prices in the set $\mathcal{P} = [\underline{p}, p(\underline{\tau})] \cup [p_{\uparrow}(\underline{\tau}), \overline{p}]$. Using the fact that $S(\underline{p}) = 1$ in equation (13) gives

$$(p - \gamma)e^{-aP(p)}[\mathfrak{t}I(p; \underline{\tau}) + (1 - \mathfrak{t})I(p; \overline{\tau})] = \underline{p} - \gamma, \text{ for } p \in \mathcal{P}. \tag{16}$$

Since this equation must hold for all p in the pricing set, $\mathcal{P}$, it traces out the function $P(p)$.

Proposition 3 *(Advertised Price Distribution) The cumulative distribution for prices, $P(p)$,is given by*

$$P(p) = \Pr(\text{PRICE} \leq p) = \begin{cases} \ln\{(p - \gamma)/(\underline{p} - \gamma)\}/a, & \textit{for } p \in [\underline{p}, p(\underline{\tau})]; \\ \ln\{[p(\underline{\tau}) - \gamma]/(\underline{p} - \gamma)\}/a, & \textit{for } p \in [p(\underline{\tau}), p_{\uparrow}(\underline{\tau})]; \\ \ln\{(1 - \mathfrak{t})(p - \gamma)/(\underline{p} - \gamma)\}/a, & \textit{for } p \in [p_{\uparrow}(\underline{\tau}), \overline{p}]. \end{cases} \tag{17}$$

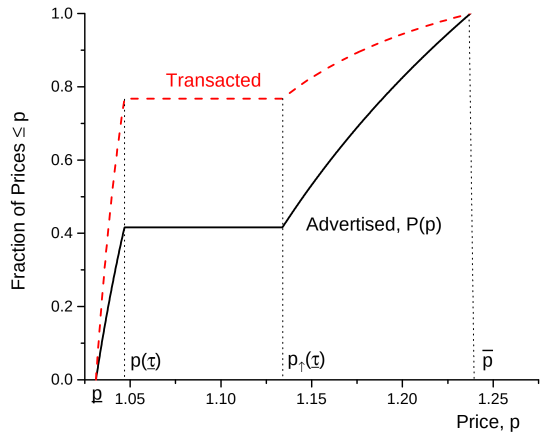

Figure 7: The cumulative distribution functions for both advertised prices, $P(p)$, and transacted prices that obtain in calibrated equilibrium for 2018–Section 10 discusses the model's calibration. It is not profitable for a firm to price in the open interval $\left(p(\underline{\tau}), p_{\uparrow}(\underline{\tau})\right)$. The distribution function for advertised prices stochastically dominates the one for transacted prices, because consumers buy at the lowest price in their information set.

The associated density function reads

$$P_1(p) = \begin{cases} 1/[a(p-\gamma)] > 0, \textit{for } p \in [\underline{p}, p(\underline{\tau})]; \\ 0, \qquad\qquad\qquad \textit{for } p \in [p(\underline{\tau}), p_{\uparrow}(\underline{\tau})]; \\ 1/[a(p-\gamma)] > 0, \textit{for } p \in [p_{\uparrow}(\underline{\tau}), \overline{p}]. \end{cases} \tag{18}$$

Proof. See the Appendix. ■

Figure 7 illustrates the cumulative distribution for prices. For subsequent use note that $P_1(p)$ represents the fraction of ads offering to sell a variety at price p.

5.2 Number of Varieties

How many varieties, v, will be produced? Since there is free entry into the production of any variety of consumption goods all possible varieties will be sold. If this wasn't the case, a producer could move into a variety where no one else is producing and earn supra-normal profits because of the lack of competition in advertised prices. Individuals will not consume all varieties, though. People won't receive ads for some varieties and even when they do get ads some varieties may be too expensive for unskilled consumers.

Proposition 4 *(Number of Varieties) All consumption goods in the feasible set* $[0, 1]$ *will be produced; i.e.,* $v = 1$.

Proof. See the Appendix. ■

5.3 Maximum Price the Unskilled will Pay, $p(\underline{\tau})$

What is the maximum price, $p(\underline{\tau})$, at which an unskilled person will buy a good? To begin with, since $S(\underline{p}) = 1$, equation (13) also implies

$$S(p) = \frac{(\underline{p} - \gamma)}{(p - \gamma)}, \text{ for } p \in \mathcal{P}. \tag{19}$$

Let $B(p) = \Pr(\text{BUY})$ represent that the probability that a consumer will buy at price p. This is not quite the same as the probability that a firm will make a sale at price p, $S(p)$, because the latter averages over both types of consumers. The two probabilities are related as follows:

$$B(p) \equiv \Pr(\text{BUY}) = \begin{cases} S(p), & \text{for } p \in [\underline{p}, p(\underline{\tau})]; \\ S(p)/(1 - \mathfrak{t}), & \text{for } p \in [p_{\uparrow}(\underline{\tau}), \overline{p}]. \end{cases}$$

For given variety, the odds of a purchase at price p by a consumer are $P_1(p)B(p)$. Since there is a unit mass of varieties, a type-τ person's budget constraint can be written as

$$a \int_{\underline{p}}^{p(\tau)} p P_1(p) B(p) dp = \tau h(\tau), \tag{20}$$

where $h(\tau)$ is hours worked and $p(\tau)$ denotes the time price of the most expensive good the person will buy; i.e., $p(\tau) = p(\underline{\tau})$, for $\tau = \underline{\tau}$, and $p(\tau) = \overline{p}$, for $\tau = \overline{\tau}$. Equation (20) pins down $p(\underline{\tau})$. To see this, set $\tau = \underline{\tau}$ in (20) and perform the required integration, while using (18) and (19), to obtain

$$a(\underline{p} - \gamma)\{\ln[\frac{p(\underline{\tau}) - \gamma}{\underline{p} - \gamma}] - \frac{\gamma}{p(\underline{\tau}) - \gamma} + \frac{\gamma}{\underline{p} - \gamma}\}/a = 1 - l(\underline{\tau}). \tag{21}$$

This equation determines $p(\underline{\tau})$.

6 Supply of Free Media Goods

By reference to (3) and (5), it is immediate that the quantity of media goods provided, $\mathfrak{m}$, is given by

$$\mathfrak{m} = n(\frac{a_t}{\zeta} + \frac{a_d}{\psi}) = n(\frac{\underline{p} - \gamma}{\alpha\phi})^{1/(\alpha - 1)}[\frac{1}{\zeta} + \frac{q^{\alpha/(1-\alpha)}}{\psi}]. \tag{22}$$

Again, to obtain the requisite impact from advertising, the lower the click-through rate, the higher will be the amount of free media good required to deliver the specified information.

7 The Consumer/Worker Problem

A consumer/worker's optimization problem is to maximize (1) subject to (2) by the choice of $\{c_i\}_i^v$ and l, for $\tau \in \{\underline{\tau}, \overline{\tau}\}$. Focus on a generic type-τ worker and index the regular goods from the lowest to the highest priced so that p_i is increasing in i. Let $c(\tau)$ signify the most expensive generic good consumed by the individual, which has the price $p(\tau)$. This also represents the person's overall consumption of generic goods because $c(\tau) = \int_0^{c(\tau)} c_i di$, as $c_i = 1$ for $i \in [0, c(\tau)]$. Now, from the budget constraint (2) it's clear that $c(\tau)$ can be written as a function of a person's productivity, τ, and hours worked, $h(\tau)$. So, write[7]

$$c(\tau) = C(h(\tau), \tau),$$

with

$$C_1(h(\tau), \tau) = \tau/p(\tau) > 0 \text{ and } C_2(h(\tau), \tau) = h(\tau)/p(\tau) > 0, \tag{23}$$

Using this fact, the consumer/worker's maximization problem can be reformulated as

$$W(\tau) = \max_{l(\tau)}\{\theta \ln[C(1 - l(\tau) - \mathfrak{e}; \tau, \mathfrak{e})] + \frac{(1-\theta)}{\rho} \ln[\kappa l(\tau)^{\rho} + (1-\kappa)\mathfrak{m}^{\rho}]\}. \tag{24}$$

The generic first-order condition for the leisure of a type-τ person, or $l(\tau)$, is:

$$\underbrace{\frac{\theta}{c(\tau)} \frac{\tau}{p(\tau)}}_{\text{Marginal Cost of Leisure}} = \underbrace{(1-\theta)\frac{\kappa l(\tau)^{\rho-1}}{\kappa l(\tau)^{\rho} + (1-\kappa)\mathfrak{m}^{\rho}}}_{\text{Marginal Benefit of Leisure}}, \text{ for } \tau \in \{\underline{\tau}, \overline{\tau}\}. \tag{25}$$

The righthand side of this equation is the marginal benefit from an extra unit of leisure. It is increasing in the quantity of media leisure goods, $\mathfrak{m}$, since $\rho < 0$. The lefthand side is the marginal cost of leisure. An extra unit of leisure leads to a drop in income for a type-τ person. This causes a drop in regular consumption, $c(\tau)$, of $\tau/p(\tau)$, where $p(\tau)$ is the price of the last regular good consumed. This is multiplied by the marginal utility of regular goods, $\theta/c(\tau)$.

The upshot of this first-order condition is given by the proposition below.

Proposition 5 *(Consumption/Leisure) An individual's consumption and leisure satisfy the following properties:*

1. *Leisure, $l(\tau)$, is increasing in the number of media leisure goods, $\mathfrak{m}$;*

2. *Regular consumption, $c(\tau)$, is decreasing in the number of media leisure goods, $\mathfrak{m}$, and is increasing in the level of skill, τ;*

3. *Work effort, $h(\tau)$, rises with the cost of an education, $\mathfrak{e}$.*

Proof. See the Appendix. ■

The first point follows from the fact that an increase in the number of media goods, $\mathfrak{m}$, raises the marginal benefit of leisure, $l(\tau)$, because the two goods are complements in the utility

[7]Note that $\int_0^v p_i c_i di = \int_0^{c(\tau)} p_i di = \tau h(\tau)$. To compute $C_1(h(\tau), \tau)$, take the total differential of the above equation while using Leibniz's rule to get $p_{c(\tau)} c_{c(\tau)} dc(\tau) = \tau dh(\tau)$ so that $dc(\tau)/dh(\tau) \equiv C_1(h(\tau), \tau) = \tau/p(\tau)$. A similar calculation gives the formula for $C_2(h(\tau), \tau)$.

function (i.e., $\rho < 0$). Next, the rise in leisure, $l(\tau)$, is connected with a drop in work effort, $h(\tau)$, that reduces regular consumption, $c(\tau)$. An increase in τ decreases the marginal cost of regular consumption in terms of forgone leisure. Hence, regular consumption rises. The third result transpires because an increase in $\mathfrak{e}$ raises the marginal cost of leisure for any given level of hours worked, $h(\tau)$, implying that regular consumption, $c(\tau)$, will be lower. This property is important because it implies that if an education is costly enough, then the skilled will work more than the unskilled. This allows the framework to explain the recent rise in the unskilled's leisure relative to the skilled's.

Last, the overall consumption of generic goods by a type-τ person, $c(\tau)$ for $\tau \in \{\underline{\tau}, \overline{\tau}\}$, is given by

$$c(\tau) = a \int_{\underline{p}}^{p(\tau)} B(p)P_1(p)dp \text{ [cf. (20)].}$$

where again $P_1(p)B(p)$ represents the odds of a purchase at price p. Evaluating the integral at $\tau = \underline{\tau}$ gives consumption for an unskilled person,

$$\begin{aligned} c(\underline{\tau}) &= a \int_{\underline{p}}^{p(\underline{\tau})} \frac{\underline{p} - \gamma}{a(p-\gamma)^2} dp \text{ [using (18) and (19)]} \\ &= 1 - S(p(\underline{\tau})). \end{aligned} \tag{26}$$

The expression has an intuitive interpretation since $1 - S(p(\underline{\tau}))$ represents the odds for each variety of getting at least one advertised price less than or equal to $p(\underline{\tau})$. Alternatively, when $\tau = \overline{\tau}$ the formula yields a skilled person's consumption,

$$c(\overline{\tau}) = a[\int_{\underline{p}}^{p(\underline{\tau})} \frac{\underline{p} - \gamma}{(p-\gamma)^2} dp + \frac{1}{1-\mathfrak{t}} \int_{p_{\uparrow}(\underline{\tau})}^{\overline{p}} \frac{\underline{p} - \gamma}{(p-\gamma)^2} dp]/a = 1 - e^{-a}. \tag{27}$$

Here, $1 - e^{-a} = 1 - S(\overline{p})$ is the probability of receiving at least one ad per variety.

8 Equilibrium

In equilibrium the labor market must clear. The labor-market-clearing condition reads

$$\begin{aligned} \gamma[\mathfrak{t}c(\underline{\tau}) + (1-\mathfrak{t})c(\overline{\tau})] + n[A(a_t) + A(qa_d) + \mathfrak{r}] \\ = \mathfrak{t}[1 - l(\underline{\tau})] + (1-\mathfrak{t})\overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}]. \end{aligned} \tag{28}$$

The lefthand side is the demand for labor. The first term, $\gamma[\mathfrak{t}c(\underline{\tau}) + (1-\mathfrak{t})c(\overline{\tau})]$, is the demand for labor originating from the consumption of regular goods. The second term represents the labor used in advertising and absorbed by the fixed costs associated with entry for the n regular firms, $n[A(a_t) + A(qa_d) + \mathfrak{r}]$. The righthand side is the supply of labor from unskilled and skilled workers. This condition can be thought of as tying down the number of entrants, n, into a variety of regular goods.

It's now time to take stock of things.

Definition of an Equilibrium *An equilibrium for the economy is defined by a solution for advertising, a_t, a_d, and a, overall consumption, $c(\underline{\tau})$ and $c(\overline{\tau})$, the quantity of media goods consumed, $\mathfrak{m}$, labor supply, $l(\underline{\tau})$ and $l(\overline{\tau})$, the number of firms producing a variety, n, and the prices of regular goods, $\underline{p}$, $\overline{p}$, $p(\underline{\tau})$,* and *$p_{\uparrow}(\underline{\tau})$, such that:*

1. *Advertising is done in accordance with (10) and (11), which determine a_t, a_d, and a, where $S(\underline{p}) = 1$. These solutions depend on the values for n and $\underline{p}$.*
2. *The minimum and maximum time prices for regular goods, $\underline{p}$ and $\overline{p}$, are regulated by (12) and (14), taking as given a.*
3. *The highest time price paid by an unskilled person, $p(\underline{\tau})$, is described by the pricing equation (21), assuming values for a, $l(\underline{\tau})$, and $\underline{p}$. The price for the skilled at the jump point, $p_{\uparrow}(\underline{\tau})$, is determined by (15) as a function of $p(\underline{\tau})$.*
4. *The quantity of media goods consumed, $\mathfrak{m}$, is given by (22), where the solution for $\mathfrak{m}$ is dependent on a_t, a_d, and n.*
5. *The solution to the consumer-worker's problem for $c(\tau)$ and $l(\tau)$ is governed by (25), (26), and (27) for $\tau \in \{\underline{\tau}, \overline{\tau}\}$, given $p(\tau)$ and $\mathfrak{m}$. These solutions take as given a, $\mathfrak{m}$, $\underline{p}$, $p(\underline{\tau})$, $p_{\uparrow}(\underline{\tau})$, and $p(\overline{\tau}) = \overline{p}$.*
6. *The labor market clears in accordance with (28), which gives the number of firms per variety, n, as a function of a_t, a_d, $c(\underline{\tau})$, $c(\overline{\tau})$, $l(\underline{\tau})$, and $l(\overline{\tau})$.*

9 Efficiency of the Equilibrium

The competitive equilibrium is not efficient. This transpires for two reasons why the equilibrium is not efficient. First, ads offering to sell goods at high prices are being sent to unskilled consumers that can never afford to buy them. This is a social waste of resources. Second, when engaging in advertising, firms do not take into account how the introduction of free media goods benefits the consumer. So, there is an underprovision of media goods.

The Pareto optima for the economy can be traced out by solving the following informationally-constrained planning problem, where $\xi \in [0, 1]$ is the relative planning weight that is being placed on unskilled individuals:

$$\max_{c(\underline{\tau}),c(\overline{\tau}),a_t,a_d,n,l(\underline{\tau}),l(\overline{\tau})} \Big(\xi \mathfrak{t}\theta \ln c(\underline{\tau}) + \frac{\xi \mathfrak{t}(1-\theta)}{\rho} \ln\{\kappa l(\underline{\tau})^{\rho} + (1-\kappa)[n(\frac{a_t}{\zeta} + \frac{a_d}{\psi})]^{\rho}\} \\ + (1-\mathfrak{t})\theta \ln c(\overline{\tau}) + \frac{(1-\mathfrak{t})(1-\theta)}{\rho} \ln\{\kappa l(\overline{\tau})^{\rho} + (1-\kappa)[n(\frac{a_t}{\zeta} + \frac{a_d}{\psi})]^{\rho}\} \Big), \tag{29}$$

subject to

$$1 - e^{-(a_t + a_d)n} - c(\overline{\tau}) = 0, \tag{30}$$

and

$$\mathfrak{t}[1 - l(\underline{\tau})] + (1 - \mathfrak{t})\overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}] - \mathfrak{t}\gamma c(\underline{\tau}) - (1 - \mathfrak{t})\gamma c(\overline{\tau}) - n[A(a_t) + A(qa_d) + \mathfrak{r}] = 0. \tag{31}$$

An interpretation of this problem is that the planner is giving unskilled and skilled people coupons in the amounts $c(\underline{\tau})$ and $c(\overline{\tau})$. Each coupon entitles a person to one good at the store they go to. The total amount of coupons handed out is constrained by the resource constraint (31). The advertisements give the locations of the stores that sell each variety. Without an ad the consumer will not know where to buy a variety. The odds of getting at least one ad for any particular variety are $1 - e^{-(a_t + a_d)n}$. So, equation (30) states that the consumption for the skilled is constrained by the ads they receive.

The allocations from the informationally-constrained planning problem can be supported in a competitive equilibrium using a tax-cum-subsidy scheme. The excessive amount of advertising can be corrected by levying a fine on all advertising and providing a subsidy for consumers on all goods sold. Specifically, consumers require a proportional price reduction, r, in the amount

$$r = 1 - \gamma / p(\underline{\tau}), \tag{32}$$

and all advertising should be fined at the rate

$$f = r\overline{p}(1 - \mathfrak{t})e^{-(a_t + a_d)n}. \tag{33}$$

The underprovision of media goods can be rectified by providing a subsidy per media good in the amount s, where

$$s = \mathfrak{t}\frac{(1 - \kappa)}{\kappa}(\frac{\mathfrak{m}}{l(\underline{\tau})})^{\rho - 1} + (1 - \mathfrak{t})\overline{\tau}\frac{(1 - \kappa)}{\kappa}(\frac{\mathfrak{m}}{l(\overline{\tau})})^{\rho - 1}. \tag{34}$$

This is equivalent to subsidizing traditional and digital advertising at the rates s/ζ and s/ψ. The above policy should be financed by lump-sum taxation in line with

$$\begin{aligned} &ra[\mathfrak{t}\int_{\underline{p}}^{p(\tau)} pP_1(p)B(p)dp + (1 - \mathfrak{t})\int_{\underline{p}}^{\overline{p}} pP_1(p)B(p)dp] + sn(\frac{a_t}{\zeta} + \frac{a_d}{\psi}) \\ &= \mathfrak{t}t(\underline{\tau}) + (1 - \mathfrak{t})t(\overline{\tau}) + fn(a_t + a_d), \end{aligned} \tag{35}$$

where $t(\underline{\tau})$ and $t(\overline{\tau})$ are the lump-sum taxes levied on the unskilled and skilled. The way these taxes are raised affects the economy's income distribution.

Proposition 6 *(Informationally-Constrained Efficiency) The solution to the informationally-constrained planning problem (29) can be supported as a competitive equilibrium with the tax-cum-subsidy scheme specified by (32), (33), and (34) that is financed by lump-sum taxation in accordance with (35).*

Corollary 1 *(Single agent economy) Suppose there is only one type of consumer/worker. Then only a subsidy on media goods is required.*

Proof. See Appendix 14. ∎

The intuition for the above tax-cum-subsidy scheme is this. The skilled consume more varieties than the unskilled. A certain amount of advertising is required to effect this. There is no need to do any extra advertising to support the unskilled's consumption. So, the last variety sold to an unskilled person should be priced at its marginal production cost implying that $(1-r)p(\underline{\tau}) = \gamma$, where r is the required proportional price reduction. When determining how much advertising to do firms use the price p instead of the subsidized price $(1-r)p$, where the latter reflects the value of the good to a consumer. Since $p > (1-r)p$ there is propensity toward too much advertising. This is corrected by fining advertising in general at the rate f.

Firms neglect the fact that media goods are valuable to consumers. Therefore, they under provide them. This is rectified by subsidizing media goods. The subsidy, s, is just an expenditure-weighted average of each group's marginal rate of substitution between leisure and media goods, as can be seen from (34). The marginal rates of substitution reflects how much an extra media good is worth to a person in terms of leisure. For a skilled person a unit of leisure is worth more than for an unskilled person, as reflected by $\overline{\tau}$. Last, the click-through rate specifies how efficient advertising is.

Last, the solution to the planner's problem in a world with full information can obtained undertaking the maximization in (29) while dropping the information constraint (30). This can be supported as a competitive equilibrium where all goods are sold at the marginal cost so that $p = \gamma$. A lump-sum subsidy to firms is needed to cover their fixed costs, $\mathfrak{r}$. The cost of this would have to be financed by lump-sum taxation on consumers.

10 Calibration

In order to simulate the model, values have to be assigned to the following parameters: θ, κ, ρ, $\mathfrak{m}$, γ, α, ϕ, ζ, ψ, q, $\mathfrak{r}$, $\overline{\tau}$, $\mathfrak{e}$, and $\mathfrak{t}$. Most of the parameter values are unique to this study. The strategy is to pin down parameter values by using data on markups, the advertising-to-consumption ratio, the click-through rate, the hike in the ratio of spending on digital versus traditional advertising, the college premium, and the step up in the time spent on leisure by non-college- and college-educated individuals.

Some parameter values can be set straightforwardly. The unskilled in the model are taken to be the non-college educated. They represent 65 percent of the population. In United States the income of college graduates is 1.98 times that of the non-college educated. The productivity of college graduates in model, $\overline{\tau}$, is set to match this fact. Thus, $\overline{\tau}$ is determined by the condition

$$\text{INCOME RATIO} = 1.98 = \frac{\overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}]}{1 - l(\underline{\tau})}.$$

Accordingly, $\mathfrak{t} = 0.65$ and $\overline{\tau} = 2.35$. The click-through rate on digital advertising is very low, roughly 2.5 percent. This dictates setting $\psi = 0.025$. The choice of some parameters are normalizations. On this, γ and ϕ control the units that output and advertising are measured in. Hence, $\gamma = \phi = 1$.

The rest of the parameter values are selected by targeting a set of stylized facts. The long and short of the calibration procedure is this–a detailed explanation is provided in Appendix 15. The model's calibration is divided into two parts; viz, the firm side that determines the advertising parameters and a consumer side that pins down the preference ones. These two parts are linked. On the firm side, an important parameter is the cost elasticity for advertising, α. To calibrate this parameter, a markup of 7 percent for the average transacted price over marginal production cost is chosen–this number is taken from Basu (2019).[8] As mentioned in the introduction, advertising has been roughly 2 percent of GDP for the last 100 years. This leads to the following two restrictions on the calibration exercise.

$$\begin{aligned} \text{MARKUP} &= 1.07 = E[p]/\gamma \\ &= [\frac{\int_{\underline{p}}^{p(\underline{\tau})} pB(p)P_1(p)dp + (1-\mathfrak{t})\int_{p(\underline{\tau})}^{\overline{p}} pB(p)P_1(p)dp}{\int_{\underline{p}}^{p(\underline{\tau})} B(p)P_1(p)dp + (1-\mathfrak{t})\int_{p(\underline{\tau})}^{\overline{p}} B(p)P_1(p)dp}]/\gamma, \end{aligned} \tag{36}$$

and

$$\text{A2C} = 0.02 = \frac{n[A(a_t) + A(qa_d)]}{\mathfrak{t}c(\underline{\tau}) + (1-\mathfrak{t})c(\overline{\tau})}. \tag{37}$$

The markup is calculated using the transacted price distribution, shown in Figure 7. The restrictions (36) and (37) are used to pin down a value for α, which governs the marginal cost of advertising. The backed-out value for α is similar to the one used by Dinlersoz and Yorukoglu (2012) to generate a reasonable equilibrium firm-size distribution. In fact, alternatively, if their value is used for α, then the model here would predict a markup of 7 percent. These restrictions also determine the fixed entry cost, $\mathfrak{r}$.

The ratio of spending on digital to traditional advertising rose from 0.02 percent 2003 to 0.28 in 2018. These numbers are used to calibrate the rise in the relative efficiency of digital advertising, or q, over this time period. Hence, the following condition is imposed on the calibration exercise:

$$\text{D2T} = \frac{A(qa_d)}{A(a_t)} = \begin{cases} 0.02, \text{for } 2003; \\ 0.07, \text{for } 2010; \\ 0.28, \text{for } 2018. \end{cases} \tag{38}$$

When calibrating the firm side of the model, the labor allocations from the consumer side are taken as given. Given these labor allocations, the firm-side calibration hits exactly the three data targets given by (36), (37), and (38).

[8] The size of price markups is controversial. The number used here is conservative: the larger is the price markup, the bigger will be welfare gain from digital advertising.

For the consumer side, the preference parameters θ, κ, and ρ, plus the parameter governing the cost of education, $\mathfrak{e}$, are chosen to match certain observations about leisure for the non-college and college educated for the years 2003, 2010, and 2018. Additionally, these observations are also used to infer a value for the click-through rate, ζ, on traditional advertising. This turns out to be much higher than the one for digital advertising. Thus, to obtain a given number of messages read by consumers, digital advertising supplies more free media goods than traditional advertising. (But, these media goods are less expensive to produce.) Leisure is defined as all time spent on entertainment, social activities, relaxing, active recreation, sleeping, eating, and personal care; this definition corresponds with Aguiar and Hurst (2007, Table III, measure 2). The trend in leisure is charted in Figure 8. Leisure for the non-college educate rose from 64.2 percent of time not working in 2003 to 65.4 in 2018. The increase for college graduates was from 60.7 to 61.1. In each year college graduates enjoyed less leisure than the non-college educated. Galbi (2001) has noted that, historically speaking, increases in discretionary time use are closely related to the waxing in time spent on media. So, the figure also tracks the gain in leisure since 2003 accounted for by the time consumed on media; namely, TV, radio, reading, movies, computers, and games. The model is calibrated to 2003 levels of leisures and subsequent gains in leisures linked with the increased time spent on media.

Since media goods are free their quantity is not recorded in the national income accounts. The data on time spent not working, both over time and between the non-college and college educated, is used to infer the quantity of media goods. Since media goods and leisure are Edgeworth-Pareto complements, an increase in the supply of the media goods should lead to more time spent not working, ceteris paribus; recall Proposition 5. This type of strategy was introduced in Goolsbee and Klenow (2006) and followed by Brynjolfsson and Oh (2012). Things are more complicated here, though. The advent of digital advertising also affects the prices of consumer goods, which will have an impact on leisure as well.

Let $\text{LEISURE}_t(\tau)$ represent the leisure target for a type-τ person in year t. Then, formally speaking, the parameter values in question solve

$$\min_{\theta,\kappa,\rho,\mathfrak{e},\zeta} \sum_{\tau=\underline{\tau},\overline{\tau}} \sum_{t=03,10,18} [l_t(\tau) - \text{LEISURE}_t(\tau)]^2, \tag{39}$$

subject to (36), (37), and (38). The constraints take into account how the choice of the preference parameters interacts with the firm-side calibration. The upshot from the calibration procedure is displayed in Tables 1 and 2.

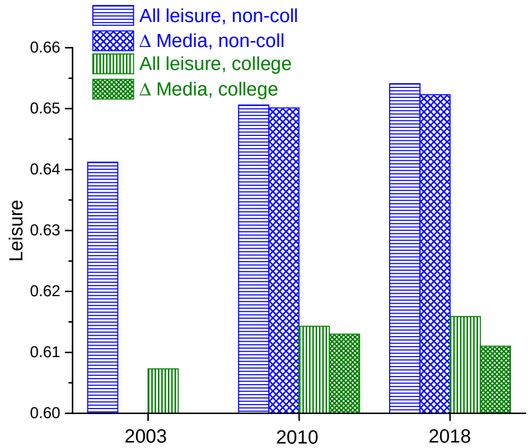

Figure 8: Leisure for the non-college and college educated. Also shown is the increase in leisure since 2003 comprised by shifts in time spent on media (Δ Media). The bars for 2003 and the cross-hatched ones for 2010 and 2018 are used in the model's calibration. *Source*: American Time Use Survey.

Calibrated Parameter Values

Parameter Values	*Description*	*Identification*
Consumers		
$\theta = 0.3499$	Consumption weight	Data, Eq (39)
$\kappa = 0.0076$	Weight on leisure, CES	Data, Eq (39)
$\rho = -4.9896$	Elasticity of substitution	Data, Eq (39)
$\mathfrak{t} = 0.65$	Low-type fraction	Data
$\overline{\tau} = 2.3506$	High-type productivity	Data
$\mathfrak{e} = 0.0953$	Cost of skill	Data, Eq (39)
Firms		
$\gamma = 1$	Marginal production cost	Normalization
$\mathfrak{r} = 0.0028$	Entry fixed cost	Data, Eqs (36) and (37)
Advertising		
$\alpha = 3.0148$	Cost elasticity	Data, Eqs (36) and (37)
$\phi = 1$	Constant	Normalization
$q_{03} = 12.0920, q_{10} = 5.9132$	Efficiency of digital adv.	Data, Eq (38)
$q_{18} = 2.3302$		
$\psi = 0.025$	Click-through rate, digital	Data
$\zeta = 0.4410$	Click-through rate, traditional	Data, Eq (39)

Table 1: The parameter values that result from the calibration procedure.

Data Targets

Description	*U.S. Data*	*Model*
Income ratio	1.98	1.98
Markup, 2018	1.07	1.07
Advertising/consumption, 1919-2019	0.022	0.022
Digital/traditional advertising		
2018	0.282	0.282
2010	0.070	0.070
2003	0.024	0.024
Leisure		
Non-college, 2018	0.6523	0.6520
College, 2018	0.6110	0.6115
Non-college, 2010	0.6501	0.6505
College, 2010	0.6130	0.6124
Non-college, 2003	0.6412	0.6411
College, 2003	0.6073	0.6074

Table 2: The data targets used in the calibration exercise and the corresponding numbers for the model. The calibration procedure hits the firm side numbers exactly while maximizing the model's fit for leisure.

11 Welfare

A person's welfare, $W(\tau)$, reads

$$W(\tau) = \theta \ln(c(\tau)) + \frac{(1-\theta)}{\rho} \ln[\kappa l(\tau)^{\rho} + (1-\kappa)\mathfrak{m}^{\rho}], \text{ for } \tau \in \{\underline{\tau}, \overline{\tau}\},$$

where $c(\tau)$, $l(\tau)$, and $\mathfrak{m}$ represent the allocations for consumption, leisure, and media goods under some particular scenario. From this it is clear that any change in welfare can be broken down into changes in $c(\tau)$, $l(\tau)$, and $\mathfrak{m}$. Now consider two different scenarios, A and B. In order to move to regime B a type-τ person living in regime A would have to be compensated by boosting his regime-A consumption by the factor

$$\text{EV}(\tau) = e^{[W_B(\tau) - W_A(\tau)]/\theta} - 1.$$

That is, $\text{EV}(\tau)$ measures a type-τ person's equivalent variation.[9]

[9] In otherwords, $\text{EV}(\tau)$ solves the equation

$$W_B(\tau) = \theta \ln[(1 + \text{EV}(\tau))c_A(\tau)] + \frac{(1-\theta)}{\rho} \ln[\kappa l_A(\tau)^{\rho} + (1-\kappa)(\mathfrak{m}_A)^{\rho}].$$

11.1 The Change in Welfare from 2003 to 2018

Between 2003 and 2018, advertising became more efficient. This had three effects. First, consumers benefited from the introduction of new media goods.[10] Second, leisure rose. Third, the reduction in hours was associated with a decline in consumption. By how much did welfare improve overall?

Table 3 shows the results. Welfare increased for the non-college and college educated by 2.5 and 2.7 percent, in terms of consumption. To attain this, the efficiency of digital advertising relative to traditional advertising rose (or q fell) at about 11 percent per year. For both groups of individuals, there is a significant increase in welfare due to the expansion of free media goods connected with digital advertising. The non-college educated realize a significant gain in welfare from their rise in leisure. This occurs because media goods and leisure are complements in utility; recall Proposition 5. The welfare gain from the increase in leisure is mostly offset by a decline in non-college educated consumption. The college educated enjoyed a smaller improvement in welfare from the rise in leisure. Their decline in consumption is negligible. The reduced work effort by the college-educated is counteracted by a reduction in prices stimulated by increased competition. This estimate of the improvement in welfare is not out of line with other work. Goolsbee and Klenow (2006) calculate that the internet was worth somewhere between 2 to 3 percent of income to the average consumer in 2005, but this could be as high as 27 percent depending on the preferred specification. Greenwood and Kopecky (2013) place the welfare gain from the introduction of personal computers at somewhere between 2 to 3 percent of GDP in 2004. Brynjolfsson and Oh (2012) find, using the Greenwood and Kopecky (2013) method, that the introduction of free media goods was worth about 5 percent of consumption in 2011.

The large boost in welfare generated by the free provision of media goods is not reflected in GDP. First, GDP is not the same as economic welfare. One might think that adding the implicit value of the free media goods to GDP would cure this problem. In particular, suppose that GDP is measured as $p_{\mathfrak{c}}c+p_{\mathfrak{m}}\mathfrak{m}$, where c is aggregate consumption, p_c is the price index for consumption, and $p_{\mathfrak{m}}$ is the implicit price index for media goods. Standard reasoning suggests measuring this implicit price by the marginal rate of substitution between leisure and media goods. This differs across the rich and the poor, so take an expenditure-weighted average. It then turns out that $p_{\mathfrak{m}} = s$, where s is given by equation (34). Doing this would increase GDP by 7.8 percent in 2003, 1.3 percent in 2010, and 0.02 percent in 2018. This seems counter intuitive because $\mathfrak{m}$ increased substantially over this time period, so how could media goods contribution to GDP decline? But, as media goods increase their implicit price falls. So, once again, GDP is not the same as welfare. For example, electricity constitutes around 2 percent of

[10] Marshall (1920, p. 307) notes "the dependence of newspapers and magazines on receipts from advertisements. They are thereby enabled to provide a larger amount of reading matter than would otherwise be possible ..."

The Increase in Welfare from 2003 to 2018				
	EV	Consumption	Media Goods	Leisure
Non-college	2.5%	-2.43%	1.85%	3.00%
College	2.7%	-0.03%	1.42%	1.22%

Table 3: The welfare gains from the expansion of free media goods arising from the advent of digital advertising. These welfare gains are decomposed into the effects that digital advertising had on regular consumption, media goods provision, and leisure.

expenditure yet Greenwood and Kopecky (2013) estimate it has a compensating variation of 92 percent with there existing no equivalent variation; i.e., it isn't possible to give a person today enough income to compensate them for living without electricity. Second, some researchers have suggested that advertising is an intangible investment and should be treated the same way as physical investment in the national income and product accounts. This boosts GDP. Advertising spending is deducted from firm's profits in the GDP accounts unlike physical investment spending.[11] Corrado, Hulten, and Sichel (2009) recommend counting (a portion of) advertising as an intangible investment in the GDP accounts–McGrattan and Prescott (2010) express a similar view. This would increase GDP by advertising's share of GDP, or around 2 percent over the last century. This adjustment would be constant over this time period and would not reflect the welfare gain from digital advertising–the ratio of advertising spending to GDP has been stable.

It's dangerous to prognosticate about the future, but suppose, solely as a *thought experiment*, that technological advance in digital advertising continues until 2040 at the same rate as between 2003 and 2018. From 2018 to 2030 the non-college educated would see their welfare climb by an additional 1.7 percent, while the college educated would enjoy a benefit of 4.1 percent. By 2040 the respective numbers would be 3.0 and 7.8 percent. The cumulative welfare gains from 2003 on are shown in Figure 9. These welfare gains can be broken down. The free provision of media goods see strong diminishing returns kick in after 2018. The extra supply of free media goods increases welfare for the non-college- and college-educated population by 0.05 and 0.04 percent for the 2018-2040 period. This is trivial compared with

[11] For those not familiar with the issue, write the national income identity as

$$\text{CONSUMPTION} + \text{INVESTMENT} + \ldots = \text{LABOR INCOME} + \text{PROFITS} + \ldots .$$

Advertising is not currently counted as a component of investment. Suppose alternatively that advertising spending is added as a component of investment spending. On the lefthand side of the accounts, INVESTMENT would then increase by ADVERTISING. On the righthand side, ADVERTISING is no longer deducted from profits so that PROFITS increase by ADVERTISING. That is, advertising expenditure is now capitalized instead of expensed. This retains a balance between the left- and righthand sides of the national income accounts balance. GDP now increases by ADVERTISING.

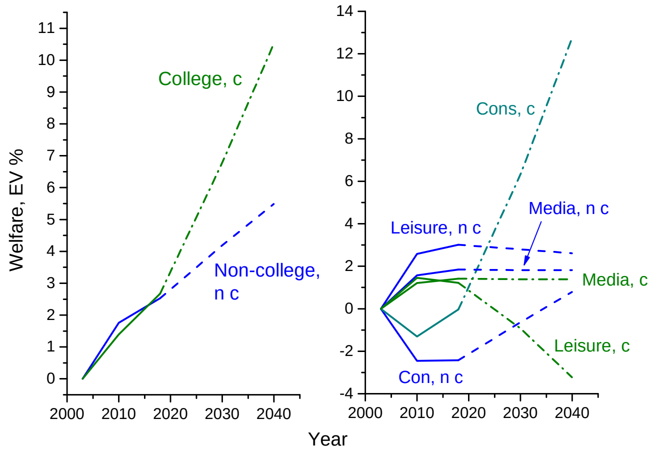

Figure 9: Cumulative welfare changes for the non-college and college educated population. The dashed and dot-dashed portions of the lines show the extrapolations from 2018 to 2040. Over this future period the college-educated gain a lot in welfare from generic consumption due to increased competition at the upper end of the price distribution. This is partially offset by a decline in welfare because of a reduction in leisure motivated by the rise in the return on working for the college educated.

the gain between 2003 and 2018. Most of the hike in welfare over this period derives from more generic consumption resulting from more intense price competition; for the two parties, the numbers are 3.2 and 12.8 percent. Interestingly, leisure drops for both parties, which contributes welfare losses of 0.4 and 4.5 percent. The more precipitous loss for the college educated occurs because they realize a significant boost in their effective real wage because of a drop in prices at the upper end of the price distribution.

11.2 Who pays?

Who is implicitly paying for the provision of free media goods? Specifically, does consumption by the upper end of the population help the lower end by stimulating the supply of free media goods? To begin with, the non-college share of advertising expenditure is given by

$$\frac{\mathfrak{t}[1 - l(\underline{\tau}) - \gamma c(\underline{\tau})] - \mathfrak{t}P(p(\underline{\tau}))n\mathfrak{r}}{n[A(a_t) + A(qa_d)]}.$$

The numerator is non-college income less the variable cost of their consumption, $\mathfrak{t}[1 - l(\underline{\tau}) - \gamma c(\underline{\tau})]$, minus their prorated share of the fixed cost of production, $\mathfrak{t}P(p(\underline{\tau}))n\mathfrak{r}$, where $\mathfrak{t}P(p(\underline{\tau}))$ is the share of advertised prices below $p(\underline{\tau})$ that are sent to the non-college educated. This represents the slice of non-college income that is absorbed in advertising costs. The denominator is aggregate advertising expenditure. By this metric the non-college educated pay 27 percent of the cost of advertising–see Table 4. Note that the non-college educated represent $100 \times \mathfrak{t} = 65$ percent of the population, so the percentage share per person is only 42 percent.

Share of Digital Advertising Costs

	Undirected		*Directed*	
	Share	Share/(Pop Share)	Share	Share/(Pop Share)
Non-college	27.04%	42%	39.50%	61%
College	72.96%	208%	60.50%	173%

Table 4: The fraction of the cost of free good provision paid for by the non-college- and college-educated populations. The last two columns refer to the directed advertising model introduced in Section 12.

The college-educated pay more than their share because they buy goods at higher prices where the markups are larger.

11.3 Public Policy

By how much would welfare improve if the second-best tax-cum-subsidy scheme proposed in Section 9 was implemented? The upshot is presented in Table 5. Moving to the informationally-constrained efficient equilibrium has a small welfare gain, worth about 0.02 percent for the non-college educated and about 0.03 percent for the college educated. These are smaller than some of the magnitudes calculated in traditional welfare analyses, such as Rees's (1963) estimate of the welfare cost of labor unions, which he found to be 0.13 percent of GDP. They are bigger than Lucas's (1987) estimate of the welfare gains from eliminating business cycles.

Implementing the informationally-constrained efficient equilibrium would require a fairly large intervention in the economy. The purchase of consumption goods would have to be subsidized at 6 percent in order to align the marginal price paid by the non-college educated to its marginal production cost. Advertising in general would face a small fine of 1.0 percent. Media goods provision would have to be subsidized at an insignificant rate to compensate for the underprovision of media goods. Last, the lump-sum taxes required to implement the program would amount to 4.71 percent of labor income for the non-college educated and 3.23 percent for the college educated. While in the rarefied confines of the model such a policy is desirable, this is unlikely to be the case in the real world especially given the small welfare gain. The advertising equilibrium modeled is surprisingly close to being efficient in an informationally-constrained economy.

The second-best informationally-constrained equilibrium is still some distance away from the first-best full-information equilibrium. To see this, the planner's problem in a full-information world is solved using the same utility weights as in the informationally-constrained problem, so as to keep things comparable. As can be seen, there is a big utility gain for

Implementing the Efficient Equilibrium

EV, Non-college	EV, College	r	f	s	$\frac{t(\underline{\tau})}{1-l(\underline{\tau})}$	$\frac{t(\overline{\tau})}{\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}]}$
Informationally Constrained						
0.02%	0.03%	5.9%	1.0%	0.00%	4.71%	3.23%
Full Information						
-6.45%	16.27%	0	0	0	9.18%	-5.45%

Table 5: The tax-cum-subsidy policy needed to make the competitive equilibrium efficient and the welfare gains from doing so.

college-educated consumers and a loss for non-educated ones. In the full-information world, college-educated consumers benefit from a large drop in prices. This effect is much smaller for non-college educated consumers and is offset by the lump-taxes needed to cover firms' fixed costs. Both parties work more, and so lose leisure, especially the non-college educated. This reinforces the message made in Table 4 that in the competitive equilibrium with advertising the college-educated are paying a disproportionate share of the cost of free media costs via the high markups on their consumption.

Lest anyone worries, clearly resources could be redirected away from the college-educated toward the non-college educated that result in everyone being better off by a move to the first-best equilibrium; i.e., the full-information planning problem could be resolved placing more weight on the non-college educated. But, there is no way of implementing the first-best equilibrium without a mechanism for costlessly getting price information to consumers.

11.4 Growth in TFP

Suppose TFP changes. Would this change the results in a material way? The answer is no. To address this question, let productivity in the production, advertising, and the cost of entry grow in a balanced fashion at the gross rate g so that $g = (1/\gamma')/(1/\gamma) = (1/\phi')/(1/\phi) = \mathfrak{r}/\mathfrak{r}' > 1$. Additionally, suppose that the number of varieties, v, also expands at this rate implying $v'/v = g$. Last, assume that there is technological progress in the household sector that augments the benefit of leisure at rate g. Then, it can be shown that, when there is no digital-specific technological progress, the economy will evolve along a balanced growth path–the details are in Appendix 16. Fernald (2014) calculates that TFP in the U.S. economy grew at roughly 0.5 percent per year. So, set $g = 1.005$; as will be seen, the exact number isn't material. Also, as before, assume that there is technological improvement in digital advertising so that $q_{03} > q_{10} > q_{18}$, with the q's calibrated in the manner described in Section 10. The upshot is presented now. The full set of results for the setting with growth in TFP are provided in Appendix 16.

Not surprisingly when growth in TFP is allowed, the improvement in welfare for both the poor and rich is much larger. Table 6 shows the results after technological progress in the household sector is factored out. The welfare gain for both types of consumers is large,

THE INCREASE IN WELFARE WITH GROWTH IN TFP

	EV	Consumption	Media Goods	Leisure
Non-college	10.5%	5.62%	1.85%	3.00%
College	10.9%	8.22%	1.42%	1.22%

Table 6: The welfare gains when there is growth in TFP.

10.5 and 10.9 percent. Most of the dividend in welfare comes from consumption growth. Interestingly, the welfare gains accruing from the increase in leisure and media goods are virtually identical to the results obtained for the baseline model.

11.5 Annoying Ads

Ads provide important facts for consumers; viz, prices, product specifications, and information about new goods. They can also be annoying. How much is an open question. In a randomized experiment of its 35 million customers the music streaming service Pandora found that as they increased the number of ads per hour less people tuned in and more people signed up for the $4.99 per month ad-free version. An extra ad per hour led to a 2 percent drop in listeners and a 0.14 percent increase in paid subscribers. The increased revenue from the paid subscription service, however, did not make up for the loss in ad revenue. Only 30 percent of viewers for the video-on-demand service Hulu purchase the $11.99-per-month no-commercials version versus the $5.99 ad-supported plan–undirected TV and movie ads are probably the most disruptive form of advertising. In 2018 Hulu earned $1.5 billion in ad revenue. Last, the opt-out rate on marketing emails is low, somewhere between 0.2 and 0.5 percent. Industry is endeavouring to find the sweet spot between the amount of advertising and fee for service. Consumers love free goods and services. The large networks built by Facebook, Google, and other tech giants allow for the rapid diffusion of the information contained in advertising. It's a profitable business model for these tech companies to use free goods as a vehicle to distribute advertising.

How ads enter consumers' preferences is an open question. Suppose that they just detract from the enjoyment of media goods. Specifically, assume that they reduce the enjoyment of a media good by the gross factor ξ. This is effectively a renormalization of the constant term κ on media goods.[12] Therefore, nothing changes in the above analysis and the welfare gain from media goods can be thought of as purging the nuisance of ads. Alternatively, perhaps consumers hate ads in their own right. In particular, subtract the disutility term $H(na)$ from preferences. All of the positive analysis done here still goes through unaltered. The welfare

[12] Write preferences as $\theta \ln(\int_0^v c_i di) + (1-\theta)\ln[\kappa l^\rho + (1-\kappa)(\xi \mathfrak{m})^\rho/\beta]/\rho$. If β is a free parameter, then setting $\beta = \xi^\rho$ shows that this is really a renormalization of the current setup. Without further information the two scenarios are observationally equivalent.

analysis will change though. It would be difficult to parameterize the function H without a lot of additional information. Additionally, the evidence suggests that consumers aren't willing to pay much for ad-free content. Could companies pay people (a negative price) to view ads? It might be hard to get the more affluent to view the ads for a small negative price. Becker and Murphy (1993) suggest that consumers could sell their "attention" to advertisers, but then just ignore the ads; therefore, there is a moral hazard problem with negative pricing.

12 Directed Advertising

Advertisers now collect vast amounts of information on consumers. Suppose instead that advertising can be directed only toward those consumers who will potentially buy the product, but that anyone can use the free media goods used to disseminate the ads. In such a setting there is no point sending an ad with a very high price to a consumer who can't afford to purchase the good at this price. So, directed advertising is more efficient than undirected advertising. It also is probably less annoying. To operationalize this idea the economy is split into two mutually exclusive spheres of economic activity, one for each consumer type. A firm can decide which group of consumers to sell to and at what price. These two spheres are only linked via the free-entry condition and the provision of free media goods. A capsule summary of the revised setup is now presented.

First, the number of firms per variety in each sphere is different, denoted by n_τ, for $\tau \in \{\underline{\tau}, \overline{\tau}\}$. Within each realm firms solve an advertising problem of the form (7). Since firms' profits must be the same across groups and prices, all firms in the economy will do the same amount of traditional and digital advertising, a_t and a_d. Denote the total amount of adverts within any variety for a group by $a_\tau = n_\tau(a_t + a_d)$, for $\tau \in \{\underline{\tau}, \overline{\tau}\}$.

Second, each group of consumers faces their own advertised price distribution, $P_\tau(p)$ for $\tau \in \{\underline{\tau}, \overline{\tau}\}$. This occurs because they are targeted separately. As before, let the maximum prices for each group be represented by $p(\underline{\tau})$ and $\overline{p}$. For the non-college and college educated these prices respectively solve

$$[p(\underline{\tau}) - \gamma]e^{-a_{\underline{\tau}}/\mathfrak{t}} = \underline{p} - \gamma \text{ and } (\overline{p} - \gamma)e^{-a_{\overline{\tau}}/(1-\mathfrak{t})} = \underline{p} - \gamma.$$

The minimum price, $\underline{p}$, is the same as in the equilibrium with undirected advertising because, as was mentioned, this price depends only on technological considerations. The two advertised price distributions are

$$P_\tau(p) = \Pr(\text{PRICE} \leq p) = \begin{cases} \ln\{(p-\gamma)/(\underline{p}-\gamma)\}\mathbf{t}/a_{\underline{\tau}}, & \text{for } \tau = \underline{\tau}; \\ \ln\{p-\gamma]/(\underline{p}-\gamma)\}(1-\mathbf{t})/a_{\overline{\tau}}, & \text{for } \tau = \overline{\tau}. \end{cases}$$

Neither price distribution exhibits a flat portion associated with a jump in prices.

Third, there are separate resource constraints for each of the two spheres:

$$\gamma c(\underline{\tau}) + n_{\underline{\tau}}[A(a_t) + A(qa_d) + \mathfrak{r}]/\mathbf{t} = 1 - l(\underline{\tau}),$$

Move Toward Directed Advertising				
	EV	Consumption	Media Goods	Leisure
Non-college	-0.03%	0.05%	-0.03%	-0.05%
College	5.05%	9.90%	-0.02%	-4.40%

Table 7: The welfare gains from a move toward directed advertising.

and

$$\gamma c(\overline{\tau}) + n_{\overline{\tau}}[A(a_t) + A(qa_d) + \mathfrak{r}]/(1 - \mathfrak{t}) = \overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}].$$

Last, the consumption of media goods, $\mathfrak{m}$, for both groups of individuals is given by

$$\mathfrak{m} = (n_{\underline{\tau}} + n_{\overline{\tau}})(a_t/\zeta + a_d/\psi).$$

How would a move from a world where advertising is undirected to a world where it is directed affect welfare? To conduct this experiment, the parameter values from the benchmark economy are retained to keep things comparable. The results are somewhat surprising–see Table 7. Welfare drops ever so slightly by 0.03 percent for the non-college educated but moves up for the college educated by 5.05 percent. Media goods consumption falls insignificantly for both groups because now there is marginally less advertising overall. This leads to a loss in welfare, ceterus paribus. The non-college educated reduce their leisure, because leisure and media goods are Edgeworth-Pareto complements in utility. The college educated realize a large gain in welfare from increased consumption because price competition is stimulated at the upper end of the price distribution. For the college educated the average price that they pay for goods drops by 4.6 percent. The maximum price paid by the college educated falls by 11.8 percent. As can be seen from the first-order condition (25) for the college educated, this amounts to an increase in the college-educated real wage, $\overline{\tau}/\overline{p}$, that stimulates work effort and discourages leisure. Figure 10 shows the shift in the transacted price distribution for the college educated. There is virtually no impact on the transacted price distribution the non-college educated. Last, note that the share of directed advertising paid for by the college educated drops–see Table 4.

12.1 Public Policy

The directed advertising economy is virtually efficient. A move to the informationally-constrained efficient equilibrium leads to infinitesimal welfare gains of 0.003 and 0.001 percent for the non-college and college educated–see Table 8.

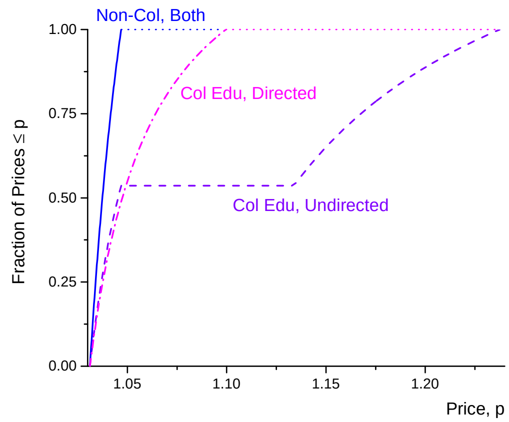

Figure 10: The cumulative distribution functions for transacted prices under both directed and undirected advertising. The college-educated purchase from a much better price distribution when advertising is directed. For the non-college educated the two prices distributions are virtually identical.

SECOND-BEST EFFICIENCY WITH DIRECTED ADVERTISING

EV, Non-college	EV, College	r	f	s	$\frac{t(\underline{\tau})}{1-l(\underline{\tau})}$	$\frac{t(\overline{\tau})}{1-l(\overline{\tau})-\mathfrak{e}}$
0.003%	0.001%	0%	0%	0.008%	0.10%	0.13%

Table 8: The tax-cum-subsidy policy needed to make the competitive equilibrium with directed advertising (informationally-constrained) efficient and the welfare gain from doing so.

12.2 A Hybrid Model

It's hard to know how much advertising is directed versus undirected. Here theory is outpacing measurement. Given this void, suppose that all digital advertising is directed while all traditional advertising is undirected. To model the extent of direction in ads, imagine an economy with both types of consumers living in two separate spheres: directed and undirected. The relative size of the population living in the directed sphere is targeted so that it grows over time in accordance with the observed diffusion of digital advertising in United States. Since the efficiency gain accruing from digital advertising derives from the fact that it can be directed, the cost advantage of digital advertising is eliminated; i.e., $q_t = 1$ for all t. The 2.5 percent click-through rate for digital advertising is retained.

The hybrid model is calibrated so that the average across spheres in 2018 for the price markup, the advertising-to-consumption ratio, and the relative earnings of the college-educated match the data. Additionally, the average leisure time across spheres for workers in 2003, 2010, and 2018 is also targeted. In the analogue to data matching problem (39) the average price markup, the advertising-to-consumption ratio, and the relative earnings of the college educated are added as targets. The estimated parameters are now $\alpha, \mathfrak{r}, \theta, \kappa, \rho, \overline{\tau}$, $\mathfrak{e}$, ζ, and $\mathfrak{s}_t$ (for $t = 2003, 2010, 2018$) where $\mathfrak{s}_t$ is the relative size of the digital sector. The results for the hybrid model are remarkably similar to the baseline model–see Appendix 17. Non-college-educated workers enjoy a welfare gain between 2003 and 2018 of 2.0 percent while for the college educated the number is 2.8. Notice that the skilled fare better relative to the unskilled with the move toward digital advertising in the hybrid model.

13 Closing

An information-based model is developed where firms must advertise to sell goods. There are two modes of advertising; namely, traditional and digital. Advertising is executed via the provision of free media goods. In the baseline version of the model, digital advertising costs less than traditional advertising. It also delivers more free media goods per message received by consumers. These media goods complement leisure in utility. Since there is randomness in the ads that consumers receive, firms set different prices for the exact same product. Hence, an equilibrium distribution of prices emerges. The advertising equilibrium is not efficient. First, free media goods are underprovided. Second, some advertising is wasteful in the sense that ads are sent to consumers who can't afford to purchase the good at the posted price. A second-best tax-cum-subsidy policy that overcomes these inefficiencies is developed. Part of this policy involves subsidizing media goods provision and taxing advertising.

The developed model is matched up with some stylized facts from the U.S. data; in particular, the average price markup, the ratio of advertising expenses to consumption expenditure, the click-through rate for digital advertising, the growth in the ratio of spending on digital advertising relative to traditional advertising, the college premium, and the rise in the time spent on leisure that was connected with media for both non-college- and college-educated people. Interestingly, the framework is consistent with the recent decrease in hours worked for the non-college educated relative to the college educated. The provision of free media goods via advertising is connected with a large increase in welfare. GDP is not a good measure of welfare when new goods are introduced into an economy. Adding an imputed value for the new media goods to GDP may not accurately reflect the gain in welfare. Additionally, counting advertising as component of investment in the GDP accounts may not capture the benefit of the digital advertising revolution. College-educated consumers pay a disproportionately large share of the cost of these media goods because they purchase products at higher prices. They may benefit from the introduction of digital advertising, however, due to the expansion of price competition at the upper end of the goods market relative to the lower end. The tax-cum-subsidy policy that overcomes these inefficiencies associated with advertising has a small impact on welfare, which is swamped by the welfare gain from the free provision of media goods.

The competitive equilibrium with undirected advertising is compared with one where advertising is directed toward consumers that might actually buy the product. There is a slightly smaller supply of free media goods in the world with directed advertising because there is less advertising. This (negligibly) hurts those consumers who wouldn't have bought high-priced products in the economy with undirected advertising. It benefits those consumers who bought high-priced goods in the economy with undirected advertising because now there is more price competition, which results in increased consumption. A hybrid model is entertained where all digital advertising is directed and traditional advertising is undirected. After recalibrating, the hybrid model delivers similar results to the baseline one. Compared with the baseline model the skilled fare relatively better than the unskilled with the shift toward directed advertising.

An interesting extension of the model would be see if it can mimic observed price distributions. If it can, then an advertising model is a viable alternative to search models. Doing this would probably involve introducing further heterogeneity in consumers, both in tastes and incomes. Additionally, heterogeneity in firms' productivities might be required. All this involves breaking new theoretical ground, gathering further facts on advertising and prices, and pushing the quantitative analysis forward. These are challenges, but the return could be high.

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14 Appendix: Proofs

14.1 Competitive Equilibrium with Undirected Advertising

Proof of Proposition 1 (Advertising). Plugging these solutions for a_t and a_d, given by the first line of (9), into the objective function (7) gives

$$\Pi(p;q) = [(p-\gamma)S(p)]^{\alpha/(\alpha-1)}\Upsilon(q),$$

where

$$\begin{aligned}\Upsilon(q) &\equiv (1+q^{\alpha/(1-\alpha)})\phi^{1/(1-\alpha)}(\alpha^{1/(1-\alpha)} - \alpha^{\alpha/(1-\alpha)}) \\ &= (1+q^{\alpha/(1-\alpha)})(\frac{1}{\phi})^{1/(\alpha-1)}(\frac{1}{\alpha})^{\alpha/(\alpha-1)}(\alpha-1) > 0.\end{aligned}$$

Now, consider two firms charging two different prices, p' and p'', in the set $\mathcal{P}$. It must transpire that $\Pi(p';q) = \Pi(p'';q)$, which can only be true if $[(p'-\gamma)S(p')]^{\alpha/(\alpha-1)} = [(p''-\gamma)S(p'')]^{\alpha/(\alpha-1)}$. But then from (9), the solutions for a_t and a_d must be the same. ■

Proof of Proposition 2 (Pricing). It's trivial to see from (12) and (14) that $\underline{p}$ and $\overline{p}$ are increasing in $\mathfrak{r}, \gamma$, and q–note that $\Upsilon(q)$ is decreasing in q. Last, $\overline{p}$ falls with $(1-\mathfrak{t})$, as is immediate from (14). ■

Proof of Proposition 3 (Price Distribution). An exponentiation of equation (16) implies that

$$P(p) = \frac{1}{a}\ln\{\frac{(p-\tau)[\mathfrak{t}I(p;\underline{\tau}) + (1-\mathfrak{t})I(p;\overline{\tau})]}{\underline{p}-\gamma}\}.$$

The result follows by noting that $\mathfrak{t}I(p;\underline{\tau}) + (1-\mathfrak{t})I(p;\overline{\tau}) = 1$, when $p \in [\underline{p}, p(\underline{\tau})]$, and $\mathfrak{t}I(p;\underline{\tau}) + (1-\mathfrak{t})I(p;\overline{\tau}) = 1-\mathfrak{t}$, when $p \in [p_{\uparrow}(\underline{\tau}), \overline{p}]$. Last, since there are no firms that price in the range $[p(\underline{\tau}), p_{\uparrow}(\underline{\tau})]$ the distribution function is flat over this interval. ■

Proof of Proposition 4 (Number of Varieties). Suppose that some consumption good i is not produced. A producer could enter the variety, charging the maximum price, $\overline{p}$, while advertising in the amounts a_t *and* a_d*.* All high-type consumers receiving an ad would buy this good. The resulting level of supra-normal profits is

$$\begin{aligned}&(\overline{p}-\gamma)(a_t+a_d)(1-\mathfrak{t}) - A(a_t) - A(qa_d) - \mathfrak{r} > \\ &\qquad\qquad (\overline{p}-\gamma)(a_t+a_d)e^{-a}(1-\mathfrak{t}) - A(a_t) - A(qa_d) - \mathfrak{r} = 0.\end{aligned}$$

On the righthand side of the above equation $e^{-a} = S(\overline{p})$ is the odds that a firm selling another variety at price $\overline{p}$ will make a sale. These positive profits violate the zero-profit condition. ■

Proof of Proposition 5 (Consumption/Leisure). To conserve on notation let $c = c(\tau)$,

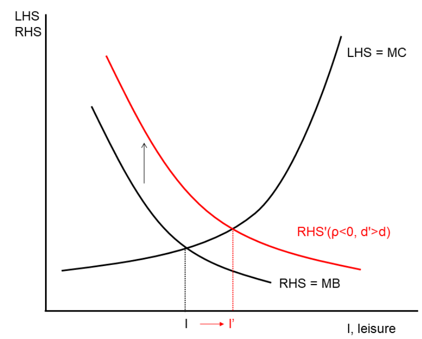

Figure 11: Leisure, l, is determined at the point where the marginal benefit and marginal cost curves intersect. The diagram shows what happens when the number of leisure goods increase from d to d'.

$l = l(\tau)$, and $p_c = p(\tau)$. Focus on the first-order condition (25), which can be rewritten as[13]

$$\underbrace{\frac{\theta}{c}\frac{\tau}{p_c}}_{\text{MC}} = \underbrace{(1-\theta)\frac{\kappa}{\kappa l + (1-\kappa)\mathfrak{m}^{\rho} l^{1-\rho}}}_{\text{MB}}. \tag{40}$$

The above first-order condition can be represented diagrammatically, as shown in Figure 11. The lefthand side of (40) represents the marginal cost of leisure (MC). This is increasing in l, so the marginal cost curve is upward sloping. On this, note that both c and p_c are decreasing in l by (23)–recall that prices are ordered from the lowest to the highest. The righthand side is the marginal benefit of leisure (MB). The righthand side is decreasing in l so the marginal benefit curve is downward sloping.

1. To demonstrate the proposition's first point that leisure, l, will increase with the number of media goods, $\mathfrak{m}$, note that

$$\frac{d\text{MB}}{d\mathfrak{m}} = -\rho(1-\theta)\frac{\kappa(1-\kappa)l^{1-\rho}}{[\kappa l + (1-\kappa)\mathfrak{m}^{\rho} l^{1-\rho}]^2}\mathfrak{m}^{\rho-1} > 0, \text{ as } \rho < 0. \tag{41}$$

   The marginal cost curve will stay in position, because it is not a function of $\mathfrak{m}$.

2. If leisure, l, increases with the free provision of media goods, then work effort, $h(\tau)$, and income, $\tau h(\tau)$, must fall. This leads to a drop in regular consumption, c. To show that regular consumption, c, is increasing in the level of skill, τ, convert the first-order

[13] The righthand side is the marginal utility of leisure, l. Note that the marginal utility of media goods, $\mathfrak{m}$, has the symmetric form

$$(1-\alpha)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}}{\kappa l^{\rho} + (1-\kappa)\mathfrak{m}^{\rho}} > 0.$$

Taking the derivative of this with respect to l also gives the cross partial given in (41). That is, if leisure is an Edgeworth-Pareto complement with media goods, then media goods are an Edgeworth-Pareto complement with leisure.

condition (40) for l into one for c by using the budget constraint (2). For a skilled person this will read

$$\frac{\theta}{c}\frac{\overline{\tau}}{p_c} = (1-\theta)\frac{\kappa}{\kappa[1-\mathfrak{e}-(1/\overline{\tau})\int_0^c p_i di] + (1-\kappa)\mathfrak{m}^{\rho}[1-\mathfrak{e}-(1/\overline{\tau})\int_0^c p_i di]^{1-\rho}}. \quad (42)$$

(For the unskilled person just set $\overline{\tau}=1$ and $\mathfrak{e}=0$.) The lefthand side is the marginal benefit of regular consumption, c, while the righthand side is its marginal cost. The marginal cost curve rises in c while the marginal benefit curve declines in c. Here an increase in τ decreases the marginal cost of consumption, while it raises the marginal benefit. Hence, c will increase.

3. Last, to establish that work effort for the skilled, $h(\overline{\tau}) = 1-l-\mathfrak{e}$, is increasing in the cost of education, $\mathfrak{e}$, return to equation (40). Note that the marginal cost of leisure rises with $\mathfrak{e}$ because $c = C(h(\overline{\tau}),\overline{\tau})$ will be smaller at any given level of l by (23) because $h(\overline{\tau}) = 1-l(\overline{\tau})-\mathfrak{e}$. The righthand side is unaffected by $\mathfrak{e}$.

■

14.2 Efficiency of the Undirected Advertising Equilibrium

To conserve on notation, let the subscript 1 denote an allocation for the unskilled person and 2 the skilled one. The informationally-constrained planning problem (29) then rewrites as

$$\max_{c_1,c_2,a_t,a_d,n,l_1,l_2} \Big(\xi\mathfrak{t}\theta\ln c_1 + \frac{\xi\mathfrak{t}(1-\theta)}{\rho}\ln[\kappa l_1^{\rho} + (1-\kappa)[n(\frac{a_t}{\zeta}+\frac{a_d}{\psi})]^{\rho}] + (1-\mathfrak{t})\theta\ln c_2 + \frac{(1-\mathfrak{t})(1-\theta)}{\rho}\ln[\kappa l_2^{\rho} + (1-\kappa)[\underbrace{n(\frac{a_t}{\zeta}+\frac{a_d}{\psi})}_{=\mathfrak{m}}]^{\rho}]\Big),$$

subject to

$$(1-\mathfrak{t})(1-e^{-(a_t+a_d)n}) - (1-\mathfrak{t})c_2 = 0,$$

and

$$\mathfrak{t}(1-l_1) + (1-\mathfrak{t})\overline{\tau}(1-l_2-\mathfrak{e}) - \mathfrak{t}\gamma c_1 - (1-\mathfrak{t})\gamma c_2 - n[A(a_t)+A(qa_d)+\mathfrak{r}] = 0.$$

Attach the Lagrange multiplier ω to the first constraint and the one λ to the second.

The first-order conditions are:

$$\xi\theta\frac{1}{c_1} = \lambda\gamma, \quad (43)$$

$$\theta\frac{1}{c_2} = \omega + \lambda\gamma = \lambda(\omega/\lambda+\gamma), \quad (44)$$

$$\xi\mathfrak{t}(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}n/\zeta}{\kappa l_1^{\rho} + (1-\kappa)\mathfrak{m}^{\rho}} + (1-\mathfrak{t})(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}n/\zeta}{\kappa l_2^{\rho} + (1-\kappa)\mathfrak{m}^{\rho}} + (1-\mathfrak{t})\omega n e^{-(a_t+a_d)n} = \lambda n A_1(a_t),$$

(45)

$$\xi\mathfrak{t}(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}n/\psi}{\kappa l_1^\rho+(1-\kappa)\mathfrak{m}^\rho}+(1-\mathfrak{t})(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}n/\psi}{\kappa l_2^\rho+(1-\kappa)\mathfrak{m}^\rho} + (1-\mathfrak{t})\omega n e^{-(a_t+a_d)n} = \lambda n q A_1(qa_d),$$

(46)

$$\xi\mathfrak{t}(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}(a_t/\zeta+a_d/\psi)}{\kappa l_1^\rho+(1-\kappa)\mathfrak{m}^\rho}+(1-\mathfrak{t})(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}(a_t/\zeta+a_d/\psi)}{\kappa l_2^\rho+(1-\kappa)\mathfrak{m}^\rho} + (1-\mathfrak{t})\omega(a_t+a_d)e^{-(a_t+a_d)n} = \lambda[A(a_t)+A(qa_d)+\mathfrak{r}],$$

(47)

$$\xi(1-\theta)\frac{\kappa l_1^{\rho-1}}{\kappa l_1^\rho+(1-\kappa)\mathfrak{m}^\rho}=\lambda, \tag{48}$$

and

$$(1-\theta)\frac{\kappa l_2^{\rho-1}}{\kappa l_2^\rho+(1-\kappa)\mathfrak{m}^\rho}=\lambda\overline{\tau}. \tag{49}$$

Following Negishi (1960), the question asked is whether or not there is a competitive equilibrium with the set of taxes and subsidies specified by (32), (33), (34), and (35) that shares the planning problem's allocations for $c_1, c_2, a_t, a_d, n, l_1,$ and l_2. If so, then the competitive equilibrium with the proposed tax-cum-subsidy scheme is (informationally-constrained) Pareto optimal.

Before proceeding to proving that the competitive equilibrium with the proposed tax-cum-subsidy scheme is Pareto optimal, motivated by (46), conjecture that the subsidy on each media goods, s, is

$$s=[\xi\mathfrak{t}(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}}{\kappa l_1^\rho+(1-\kappa)\mathfrak{m}^\rho}+(1-\mathfrak{t})(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}}{\kappa l_2^\rho+(1-\kappa)\mathfrak{m}^\rho}]/\lambda. \tag{50}$$

Using (48) and (49) it can be seen that the righthand side of this expression collapses so that

$$\begin{aligned} s &= \frac{(1-\kappa)\mathfrak{m}^{\rho-1}}{\kappa}[\mathfrak{t}\frac{1}{l_1^{\rho-1}}+(1-\mathfrak{t})\overline{\tau}\frac{1}{l_2^{\rho-1}}] \\ &= \mathfrak{t}\frac{(1-\kappa)}{\kappa}(\frac{\mathfrak{m}}{l(\underline{\tau})})^{\rho-1}+(1-\mathfrak{t})\overline{\tau}\frac{(1-\kappa)}{\kappa}(\frac{\mathfrak{m}}{l(\overline{\tau})})^{\rho-1}. \end{aligned}$$

This subsidy per media good is equivalent to subsidizing traditional and digital advertising at the rates s/ζ and s/ψ. The proportional price reduction on generic goods, r, implies that

$$(1-r)p(\underline{\tau})=\gamma \text{ [cf (32)]}. \tag{51}$$

Proof of Proposition 6 (Informationally-Constrained Efficiency). To start with focus on the consumption/leisure allocations, while assuming that the solutions for advertising and the number of firms agree in both situations. To show that the planning problem with the specified planning weight ξ can be supported as a competitive equilibrium with the proposed subsidy-cum-tax policy, let

$$\frac{\omega}{\lambda} + \gamma = (1 - r)\overline{p}.$$

Using this together with equations (44) and (49) gives the skilled consumer's first-order condition in the competitive equilibrium. Under both regimes $c_2 = 1 - e^{-(a_t + a_d)n}$. This, along with the consumption/leisure first-order condition, implies that the solution for the skilled person's leisure, l_2, will be the same in both scenarios. Analogously, using (51) in conjunction with equations (43) and (48) gives the unskilled consumer's first-order condition. Then, the labor-market-clearing condition (28) implies the unskilled person's consumption, c_1, is the same. So, the allocations for c_1, c_2, l_1, and l_2 from the planning problem can be supported as a competitive equilibrium with the proposed subsidy-cum-tax policy.

Now turn to advertising. In the competitive equilibrium, $\underline{p} - \gamma = A_1(a_t) + f - s/\zeta$ and $\underline{p} - \gamma = A_1(qa_d) + f - s/\psi$. Rewriting equation (45) while using the formula for s and adding f to both sides yields

$$s/\zeta + (1 - \mathfrak{t})(\omega/\lambda)e^{-(a_t + a_d)n} + f = A_1(a_t) + f.$$

Using formula (33) for the fine on advertising, f, then gives

$$s/\zeta + (1 - \mathfrak{t})(\omega/\lambda + r\overline{p})e^{-(a_t + a_d)n} = A_1(a_t) + f.$$

Noting that $\omega/\lambda = (1 - r)\overline{p} - \gamma$ leads to

$$s/\zeta + (1 - \mathfrak{t})(\overline{p} - \gamma)e^{-(a_t + a_d)n} = A_1(a_t) + f$$

or

$$\underline{p} - \gamma = A_1(a_t) + f - s/\zeta, \tag{52}$$

because $(1 - \mathfrak{t})(\overline{p} - \gamma)e^{-(a_t + a_d)n} = \underline{p} - \gamma$. This is the first-order condition for a_t in a competitive equilibrium. Similarly, from equation (46) it can be seen that

$$s/\psi + (1 - \mathfrak{t})(\omega/\lambda)e^{-(a_t + a_d)n} + f = qA_1(qa_d) + f,$$

implying

$$\underline{p} - \gamma = qA_1(qa_d) + f - s/\psi. \tag{53}$$

This is the efficiency condition for digital advertising, a_d, that arises in the competitive equilibrium. Therefore, the competitive solutions for a_t and a_d, under the proposed subsidy-cum-tax policy, satisfy the planning problem.

Last, move on to the number of firms. Multiply (45) by a_t/λ and (46) by a_d/λ and then sum the resulting equations to get

$$s(a_t/\zeta + a_d/\psi)n + (1 - \mathfrak{t})(\omega/\lambda)n(a_t + a_d)e^{-(a_t + a_d)n} = na_tA_1(a_t) + nqa_dA_1(qa_d),$$

where formula (50) for s has been used. Similarly, multiply (47) by n/λ and subtract the result from the above equation to obtain

$$a_t A_1(a_t) + q a_d A_1(q a_d) = A(a_t) + A(q a_d) + \mathfrak{r}. \tag{54}$$

Finally, multiplying the efficiency conditions (52) and (53) for traditional and digital advertising by a_t and a_d, respectively, and then summing the two equations while making use of (54) gives

$$\begin{aligned}(\underline{p} - \gamma)(a_t + a_d) &= a_t A_1(a_t) + q a_d A_1(q a_d) + (a_t + a_d) f - s(a_t/\zeta + a_d/\psi) \\ &= A(a_t) + A(q a_d) + \mathfrak{r} + (a_t + a_d) f - s(a_t/\zeta + a_d/\psi).\end{aligned}$$

This is the zero-profit condition for a firm when there is both a subsidy for media goods provision and a fine on advertising. This implies that the solution for n from the planning problem will be shared by the competitive economy with the proposed subsidy-cum-tax policy. ■

Suppose now that there is only one type of consumer/worker. Without loss in generality, let this be the high type. For this specialized case just a subsidy on media goods is required in the amount

$$s = \frac{(1-\kappa)\mathfrak{m}^{\rho-1}}{\kappa}\overline{\tau}\frac{1}{l_2^{\rho-1}}.$$

Proof of Corollary 1 (Single agent economy). The proof is a straightforward modification of the previous proof. When there is only the high-type person, the first-order conditions (43) and (48) no longer appear, so disregard them. Now, set $\omega/\lambda + \gamma = \overline{p}$. Using this together with equations (44) and (49) gives the consumer's first-order condition in a competitive equilibrium. To complete things, set $f = r = \mathfrak{t} = 0$. Then parrot the remaining steps in the above proof (ignoring the ones for the unskilled person) while using the revised formula for s. ■

14.3 Recovering r, f, s, $t(\underline{\tau})$, and $t(\overline{\tau})$ from the Planning Problem

The tax-cum-subsidy scheme that renders the competitive equilibrium efficient can be recovered from the solution to the planning problem. First, the subsidy on digital advertising, s, can be calculated from (34) using the planning problem allocations for a_t, a_d, $l(\underline{\tau})$, $l(\overline{\tau})$, and n.

Second, the proportional price reduction, r, and the fine on advertising, f, are immediate from (32) and (33), if the prices $p(\underline{\tau})$ and $\overline{p}$ are known. To recover these two prices, from the competitive equilibrium it transpires that

$$1 - S(p(\underline{\tau})) = 1 - \frac{\underline{p} - \gamma}{p(\underline{\tau}) - \gamma} = c_1 \text{ [equations (19) and (26)]},$$

which implies

$$p(\underline{\tau}) = \frac{\underline{p} - \gamma}{1 - c_1} + \gamma = \frac{e^{-a}(1-\mathfrak{t})(\overline{p} - \gamma)}{1 - c_1} + \gamma,$$

where the term on the far right follows from substituting out for $\underline{p} - \gamma$ using (14). Next, from the two consumer's problems, in the competitive equilibrium with the proposed tax-cum-subsidy scheme, it transpires that

$$p(\underline{\tau}) = \Xi \overline{p},$$

where

$$\Xi \equiv (\frac{c_2}{c_1}) \frac{\kappa l(\underline{\tau}) + (1-\kappa)\mathfrak{m}^{\rho} l(\underline{\tau})^{1-\rho}}{\overline{\tau}[\kappa l(\overline{\tau}) + (1-\kappa)\mathfrak{m}^{\rho} l(\overline{\tau})^{1-\rho}]}.$$

Substituting the second formula for $p(\underline{\tau})$ into the first one for $p(\underline{\tau})$ then gives

$$\overline{p} = \frac{\gamma(1-\Delta)}{\Xi - \Delta} \text{ and } p(\underline{\tau}) = \Xi \frac{\gamma(1-\Delta)}{\Xi - \Delta},$$

where

$$\Delta \equiv \frac{e^{-a}(1-\mathfrak{t})}{1-c_1}.$$

Since a, c_1, c_2, $\mathfrak{m}$, $l(\underline{\tau})$, and $l(\overline{\tau})$, are known from the planning problem so are Ξ and Δ.

Finally, by modifying (26) and (27), the lump-sum taxes levied on the unskilled and skilled, $t(\underline{\tau})$ and $t(\overline{\tau})$, read as

$$t(\underline{\tau}) = 1 - l(\underline{\tau}) - (1-r)a(\underline{p} - \gamma)\{\ln[\frac{p(\underline{\tau}) - \gamma}{\underline{p} - \gamma}] - \frac{\gamma}{p(\underline{\tau}) - \gamma} + \frac{\gamma}{\underline{p} - \gamma}\}/a$$

and

$$\begin{aligned} t(\overline{\tau}) = {} & \overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}] \\ & -(1-r)a(\underline{p} - \gamma)\{\ln[\frac{p(\underline{\tau}) - \gamma}{\underline{p} - \gamma}] - \frac{\gamma}{p(\underline{\tau}) - \gamma} + \frac{\gamma}{\underline{p} - \gamma}\}/a \\ & -\frac{(1-r)a(\underline{p} - \gamma)}{1-\mathfrak{t}}\{\ln[\frac{\overline{p} - \gamma}{p_{\uparrow}(\underline{\tau}) - \gamma}] - \frac{\gamma}{(\overline{p} - \gamma)} + \frac{\gamma}{p_{\uparrow}(\underline{\tau}) - \gamma)}\}/a, \end{aligned}$$

where

$$p_{\uparrow}(\underline{\tau}) - \gamma = \frac{p(\underline{\tau}) - \gamma}{1-\mathfrak{t}}.$$

15 Appendix: Calibration

To solve the model values for the following parameter values are needed: θ, κ, ρ, γ, $\mathfrak{r}$, α, ϕ, ζ, ψ, q, $\mathfrak{e}$, $\mathfrak{t}$, and $\overline{\tau}$. The idea is to pin down values for $\mathfrak{r}$, α, q, ζ, ψ, θ, κ, ρ, and $\mathfrak{e}$ by using data on markups, the advertising-to-consumption ratio, the click-through rate on digital advertising, the increase in the ratio of spending on digital versus traditional advertising, and the rise in time spent on leisure using media goods by non-college-educated and college-educated persons. Out of the remaining parameters, $\mathfrak{t}$ and $\overline{\tau}$ can be assigned values directly from the data. The last two parameters, γ and ϕ, are normalized to 1. As will be seen, at the calibration point the calibration procedure will determine a value for n. The steps in the procedure are as follows:

1. *Calibrating α.* Two facts are used to do this, namely the average markup, MARKUP, and advertising's share of consumption, A2C. These facts are taken to apply for the whole period in question, and therefore for the year 2018.

   (a) A formula for α. In the model all firms have the same advertising expenses, zero profits, and hence revenue net of production costs. Focus on the firms charging the lowest price, $\underline{p}$. To start with, equation (8) implies

$$(\underline{p}-\gamma)a_t = \phi\alpha a_t^{\alpha} \text{ and } (\underline{p}-\gamma)a_d = \phi\alpha q^{\alpha} a_d^{\alpha}.$$

   This gives

$$(\underline{p}-\gamma)n(a_t+a_d) = \alpha n[A(a_t)+A(qa_d)].$$

   Dividing through by total sales, $\mathfrak{t}[1-l(\underline{\tau})]+(1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}]$, then results in

$$\frac{(\underline{p}-\gamma)n(a_t+a_d)}{\mathfrak{t}[1-l(\underline{\tau})]+(1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}]} = \alpha\frac{n[A(a_t)+A(qa_d)]}{\mathfrak{t}[1-l(\underline{\tau})]+(1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}]}$$
$$= \alpha \times \text{A2Cs},$$

   where it should be noted that sales equal consumption expenditure in the model. Therefore,

$$\alpha = \frac{1}{\text{A2C}} \times \frac{(\underline{p}-\gamma)a}{\mathfrak{t}[1-l(\underline{\tau})]+(1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}]}.$$

   To use this formula, values are needed for a, $\underline{p}-\gamma$, $1-l(\underline{\tau})$, and $1-l(\overline{\tau})-\mathfrak{e}$. The latter two quantities come from the consumer side of the calibration; that is, the model's predictions for $1-l(\underline{\tau})$ and $1-l(\overline{\tau})-\mathfrak{e}$ at the 2018 calibration point, as shown in Table 2. Information on the average price markup, MARKUP, is used to solve for a and $\underline{p}-\gamma$.

   (b) Using the MARKUP to determine a and $\underline{p}-\gamma$. This will involve solving three equations in three unknowns, as discussed now. In the model the average price markup, is given by

$$\text{MARKUP} = \frac{E[p]}{\gamma} = \frac{1}{\gamma}\frac{\int_{\underline{p}}^{p(\underline{\tau})} pB(p)P_1(p)dp + (1-\mathfrak{t})\int_{p(\underline{\tau})}^{\overline{p}} pB(p)P_1(p)dp}{\int_{\underline{p}}^{p(\underline{\tau})} B(p)P_1(p)dp + (1-\mathfrak{t})\int_{p(\underline{\tau})}^{\overline{p}} B(p)P_1(p)dp}$$
$$= \frac{1}{\gamma}\frac{\mathfrak{t}[1-l(\underline{\tau})]+(1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}]}{1-\mathfrak{t}[(\underline{p}-\gamma)/(p(\underline{\tau})-\gamma)]-(1-\mathfrak{t})e^{-a}}. \quad (55)$$

   The numerator follows from (28) since this is proportional to aggregate spending. The denominator is proportional to aggregate consumption and follows from (26) and (27). Next, the labor-market-clearing condition (28) implies that

$$\gamma[\mathfrak{t}c(\underline{\tau})+(1-\mathfrak{t})c(\overline{\tau})]+a(\underline{p}-\gamma) = \mathfrak{t}[1-l(\underline{\tau})]+(1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}],$$

since $n[A(a_t) + A(qa_d) + \mathfrak{r}] = a(\underline{p} - \gamma)$ by the zero-profit condition. Solving out for $c(\underline{\tau})$ and $c(\overline{\tau})$ using (26) and (27), while noting that $B(p(\underline{\tau})) = (\underline{p} - \gamma)/[p(\underline{\tau}) - \gamma]$, then yields

$$\gamma[1 - \mathfrak{t}(\frac{\underline{p} - \gamma}{p(\underline{\tau}) - \gamma}) - (1 - \mathfrak{t})e^{-a}] + a(\underline{p} - \gamma) = \mathfrak{t}[1 - l(\underline{\tau})] + (1 - \mathfrak{t})\overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}]. \quad (56)$$

Last, $p(\underline{\tau})$ must solve

$$a(\underline{p} - \gamma)\{\ln[\frac{p(\underline{\tau}) - \gamma}{\underline{p} - \gamma}] - \frac{\gamma}{p(\underline{\tau}) - \gamma} + \frac{\gamma}{\underline{p} - \gamma}\}/a = 1 - l(\underline{\tau}). \quad (57)$$

Equations (55), (56), and (57) represent a system of three equations in three unknowns, which can be used to find a solution a, $\underline{p} - \gamma$, and $p(\underline{\tau})$ predicated upon the observed markup, MARKUP, and the labor allocations for 2018 at the calibration point reported in Table 2.

2. *Calibrating the fixed entry cost,* $\mathfrak{r}$. Since all firms earn zero profits, zero in on firms selling at the lowest price, $\underline{p}$. Their zero-profit condition gives

$$\mathfrak{r} = (\underline{p} - \gamma)^{\alpha/(\alpha-1)}(1 + q^{\alpha/(1-\alpha)})\phi^{1/(1-\alpha)}(\alpha^{1/(1-\alpha)} - \alpha^{\alpha/(1-\alpha)}) > 0,$$

where $\underline{p}$, γ, q, and α have all been previously determined.

3. *Calibrating the cost of advantage of digital advertising, q, for the years 2003, 2010, and 2018.* These can be recovered from the observed ratio of digital ad spending to traditional ad spending, D2T, for the years 2003 and 2018. For the year 2010 an interpolated value is used for D2T. To see this, from (4), (6), and (10) it is apparent that

$$\frac{A(qa_d)}{A(a_t)} = (\frac{qa_d}{a_t})^{\alpha} = (qq^{\alpha/(1-\alpha)})^{\alpha} = q^{\alpha/(1-\alpha)} = \text{D2T}.$$

Therefore,

$$q = (\text{D2T})^{(1-\alpha)/\alpha},$$

where α is known from the first step.

Calibrating the preference parameters, θ, κ, ρ, the cost of an education, $\mathfrak{e}$, and the click-through rate on traditional advertising, ζ. This is done by solving problem (39) which tries to match up the model's predictions for leisure versus 6 observations on leisure from U.S. data for non-college- and college-educated people for the years 2003, 2010, and 2018. Central to this data matching problem is the first-order condition

$$\frac{\theta}{c(\tau)}\frac{\tau}{p(\tau)} = (1 - \theta)\frac{\kappa}{\kappa l(\tau) + (1 - \kappa)\mathfrak{m}^{\rho} l(\tau)^{1-\rho}}, \text{ for } \tau \in \{\underline{\tau}, \overline{\tau}\}.$$

The quantity of digital media goods consumed, na_d/ψ, and the price of the last good consumed, $p(\tau)$, are quantities that can be recovered from the information produced in Steps 1 to 3, conditional upon values for $1 - l(\underline{\tau})$ and $1 - l(\overline{\tau}) - \mathfrak{e}$. The quantity of traditional media goods consumed, na_t/ζ, depends upon the click-through rate, ζ, for which there is no information available. So, ζ must be calibrated. The model's leisure quantities, $l_t(\underline{\tau})$ and $l_t(\overline{\tau})$, come from calibrating θ, κ, ρ, $\mathfrak{e}$, and ζ so as to match up, as close as possible, the model's predictions for leisure, $l_t(\tau)$, with the stylized facts from the data, $\text{LEISURE}_t(\tau)$ for $t = 2003, 2010$, and 2018, and $\tau = \{\underline{\tau} = \text{non-college}, \overline{\tau} = \text{college}\}$.

16 Appendix: Growth in TFP

First, it will be shown that given the assumptions in the main text the economy will follow a balanced growth path. Second, the results for the model when TFP is allowed to change are presented in Tables 9 and 10. To begin with, to incorporate technology advance in the household sector rewrite tastes as

$$\theta \ln(\int_0^v c_i di) + \frac{(1-\theta)}{\rho} \ln[\kappa(zl)^\rho + (1-\kappa)(\int_0^{\mathfrak{m}} m_j dj)^\rho],$$

where z represents a technological progress in the home sector. This is easy to justify using household production theory. Next, as assumed in the text, let

$$(1/\gamma')/(1/\gamma) = (1/\phi')/(1/\phi) = \mathfrak{r}/\mathfrak{r}' = v'/v = z'/z = g > 1.$$

Then, there will exist an balanced growth path with the following properties:

1. The price markup at the lowest price, $\underline{p} - \gamma$, grows at the rate $1/g < 1$–this is actually a decline. This fact follows from equation (12) and the assumption that $(1/\phi')/(1/\phi) = \mathfrak{r}/\mathfrak{r}' = g$. From (14) the markup at the maximum price will grow at this rate too, provided that advertising per variety is constant. By eyeballing (21) it is then apparent that $p(\underline{\tau})$ follows a similar time path–on this note that the lefthand side of (21) will now be multiplied by v, which grows at rate g so that $va(\underline{p} - \gamma)$ is constant.

2. The above implies that the price distribution shifts to the left by the factor $1/g$.

3. The first-order condition (8) for advertising shows that a_t and a_d will remain constant when both $p - \gamma$ and ϕ shift by the factor $1/g$.

4. The number of media goods, $\mathfrak{m}$, will grow at rate g. This follows from the righthand side of equation (22), which should now be multiplied by v. On this, note that $(\underline{p} - \gamma)/(\alpha\phi)$ will be constant.

5. The lefthand side of the labor-market clearing condition still holds, since aggregate consumption $c(\underline{\tau}) + (1 - \mathfrak{t})c(\overline{\tau})$ will grow at the rate g due to the expansion of varieties, while γ expands at $1/g$. Likewise, $A(a_t) + A(qa_d) + \mathfrak{r}$ grows at rate $1/g$, which will be offset by an increase in the number of firms from n to gn. Therefore, the number of firms per variety, n, remains constant.

Calibrated Parameter Values for the Model with TFP Growth

Parameter Values	*Description*	*Identification*
Consumers		
$\theta = 0.3499$	Consumption weight	Data, Eq (39)
$\kappa = 0.0076$	Weight on leisure, CES	Data, Eq (39)
$\rho = -4.9896$	Elasticity of substitution	Data, Eq (39)
$\mathfrak{t} = 0.65$	Low-type fraction	Data
$\overline{\tau} = 2.3506$	High-type productivity	Data
$\mathfrak{e} = 0.0953$	Cost of skill	Data, Eq (39)
Firms		
$\gamma/\gamma' = 1.0053$	Annual Growth in TFP	Data
$\gamma_{03} = 1, \gamma_{10} = (\gamma/\gamma')^7, \gamma_{18} = (\gamma/\gamma')^{15}$	Marginal production cost	Assumed
$\mathfrak{r}_{03} = 0.0028, \mathfrak{r}_{10} = \mathfrak{r}_{03}(\gamma/\gamma')^7, \mathfrak{r}_{18} = \mathfrak{r}_{03}(\gamma/\gamma')^{15}$	Entry fixed cost	Data, Eqs (36) and (37)
$v_{03} = 1, v_{10} = (\gamma'/\gamma)^7, v_{18} = (\gamma'/\gamma)^{15}$	Varieties	Assumed
Advertising		
$\alpha = 3.0148$	Cost elasticity	Data, Eqs (36) and (37)
$\phi_{03} = 1, \phi_{10} = (\gamma/\gamma')^7, \phi_{18} = (\gamma/\gamma')^{15}$	Constant	Assumed
$q_{03} = 12.0920, q_{10} = 5.9132$	Efficiency of digital adv.	Data, Eq (38)
$q_{18} = 2.3302$		
$\psi = 0.025$	Click-through rate, digital	Data
$\zeta = 0.4410$	Click-through rate, traditional	Data, Eq (39)

Table 9: The parameter values that result from the calibration procedure for the model with TFP growth.

6. Last, with above utility function the first-order condition for leisure is

$$\frac{\theta}{c(\tau)}\frac{\tau}{p(\tau)} = (1-\theta)\frac{\kappa z^{\rho} l(\tau)^{\rho-1}}{\kappa z^{\rho} l(\tau)^{\rho} + (1-\kappa)\mathfrak{m}^{\rho}}, \text{ for } \tau \in \{\underline{\tau}, \overline{\tau}\}.$$

Therefore, if $c(\tau)$ grows at rate g and $p(\tau)$ by the factor $1/g$, then the lefthand side will be constant. The righthand side will also be constant because $\mathfrak{m}$ and z both grow at rate g. Thus, $l(\tau)$ will be constant.

17 Appendix: Hybrid Model

The results for the hybrid model are presented in Tables 11 and 12. In the hybrid model all digital advertising is directed while all traditional advertising is undirected. See the main text for more detail.

Data Targets

Description	*U.S. Data*	*Model with TFP Growth*
Income ratio	1.98	1.98
Markup, 2018	1.07	1.07
Advertising/consumption, 1919-2019	0.022	0.022
Digital/traditional advertising		
2018	0.282	0.282
2010	0.070	0.070
2003	0.024	0.024
Leisure		
Non-college, 2018	0.6523	0.6520
College, 2018	0.6110	0.6115
Non-college, 2010	0.6501	0.6505
College, 2010	0.6130	0.6124
Non-college, 2003	0.6412	0.6411
College, 2003	0.6073	0.6074

Table 10: The data targets used in the calibration exercise and the corresponding numbers for the model with TFP growth.

Calibrated Parameter Values for the Hybrid Model

Parameter Values	*Description*	*Identification*
Consumers		
$\theta = 0.3497$	Consumption weight	Data, Eq (39)–analogue
$\kappa = 0.0073$	Weight on leisure, CES	Data, Eq (39)–analogue
$\rho = -4.9731$	Elasticity of substitution	Data, Eq (39)–analogue
$\mathfrak{t} = 0.65$	Low-type fraction	Imposed
$\overline{\tau} = 2.3185$	High-type productivity	Data, Eq (39)–analogue
$\mathfrak{e} = 0.0924$	Cost of skill	Data, Eq (39)–analogue
Firms		
$\gamma = 1$	Marginal production cost	Normalization
$\mathfrak{r} = 0.0026$	Entry fixed cost	Data, Eq (39)–analogue
Advertising		
$\alpha = 3.0074$	Cost elasticity	Data, Eq (39)–analogue
$\phi = 1$	Constant	Normalization
$q_{03} = q_{10} = q_{18} = 1.0$	Efficiency of digital sector	Imposed
$\mathfrak{s}_{03} = 0.03, \mathfrak{s}_{10} = 0.09, \mathfrak{s}_{18} = 0.29$	Size digital sector	Data, Eq (39)–analogue
$\psi = 0.025$	Click-through rate, digital	Data
$\zeta = 0.4285$	Click-through rate, traditional	Data, Eq (39)–analogue

Table 11: The parameter values that result from the calibration procedure for the hybrid model.

Data Targets

Description	*U.S. Data*	*Hybrid Model*
Income Ratio	1.98	1.98
Markup, 2018	1.07	1.07
Advertising/consumption, 1919-2019	0.022	0.022
Digital/traditional advertising		
2018	0.282	0.282
2010	0.070	0.070
2003	0.024	0.024
Leisure		
Non-college, 2018	0.6523	0.6525
College, 2018	0.6110	0.6108
Non-college, 2010	0.6501	0.6503
College, 2010	0.6130	0.6126
Non-college, 2003	0.6412	0.6408
College, 2003	0.6073	0.6080
EV		
Non-college		2.0%
College		2.8%

Table 12: The data targets used in the calibration exercise and the corresponding numbers for the hybrid model.