Fiscal Policy at the Zero Lower Bound without Rational Expectations*

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Abstract

We address the question of how sensitive is the power of fiscal policy in the ZLB to the assumption of rational expectations. We do so through the lens of a standard NK model in which people are level-k thinkers. Our analysis *weakens* the case for using government spending to stabilize the economy when the ZLB binds. The less sophisticated people are, the smaller is the size of the government-spending multiplier. Our analysis *strengthens* the case for using tax policy to stabilize output when the ZLB is binding. The power of tax policy to stabilize the economy during the ZLB period is essentially undiminished when agents do not have rational expectations. Finally, we show that the way in which tax policy is communicated is critical to its effectiveness.

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1 Introduction

The *Zero Lower Bound* (ZLB) on interest rate poses a significant constraint on conventional monetary policy.[1] So, it is important to consider the role of fiscal policy in stabilizing the aggregate economy when the ZLB is binding. In fact, a large literature emphasizes that fiscal policy is particularly useful when the ZLB binds. First, the government-spending multiplier is significantly higher than under normal circumstances, see, e.g., Christiano et al. (2011), Eggertsson (2011), and Woodford (2011).[2] Second, appropriately designed tax policy can mimic the effects of conventional monetary policy on aggregate demand, see Feldstein (2003) and Correia et al. (2013).

Much of the modern literature on alternatives to conventional monetary policy assumes that people have rational expectations. A critical issue is the robustness of the findings in that literature to the rational expectations assumption. For example, it is well known that when the ZLB is binding, forward guidance is extremely powerful in standard New Keynesian (NK) models.[3] But the power of that policy is considerably diminished under reasonable deviations from rational expectations (see the literature summary below). This observation leads to the natural question: how sensitive is the power of fiscal policy in the ZLB to the assumption of rational expectations? We argue that the efficacy of government spending is quite sensitive to that assumption. Generally, the less sophisticated people are, the smaller is the multiplier. In contrast, tax policy at the ZLB is much less sensitive to deviations from rational expectations. Indeed, in our benchmark analysis, tax policy continues to be able to support the flexible-price allocation even when agents are boundedly rational and the ZLB is binding.

We reach these conclusions using a simple representative-agent NK model with sticky wages and no capital. As in Correia et al. (2013), we assume that at time zero there is an unanticipated shock to people's discount-factor that lasts for T periods. The subjective discount rate falls below zero, driving the nominal interest rate to the ZLB.

In our benchmark model, wages are fully rigid and the price level is constant. We depart from rational expectations by assuming that people form beliefs about future endogenous variables via *level-k thinking*. Individuals understand the structure of the economy, but are limited in their ability to predict the behavior of other economic agents and, as a result, the time path for the endogenous variables in the economy (e.g., aggregate

[1]We understand that interest rates can be negative. But there is some effective lower bound on interest rates. To facilitate comparisons with the literature we work with the ZLB, with the understanding that our key results would obtain when the effective lower interest rate is binding.

[2]See also the analyses in Werning (2011) and Farhi and Werning (2016).

[3]See Eggertsson and Woodford (2003) and Werning (2011) for analyses of the power of forward guidance in standard NK models.

output). Starting from an initial belief for the least sophisticated agents, individuals update their expectations about changes in the future based on a finite reasoning process about other people's behavior, involving k iterations. We are interested in how the power of fiscal policy depends on agents' level of cognitive sophistication as indexed by k.

In Section 2, we use the benchmark model to evaluate the effects of an increase in government spending and the effects of time-varying consumption taxes, when the ZLB is binding. Consistent with earlier work by Woodford and Xie (2019), and Farhi et al. (2020), we establish that the size of the government-spending multiplier depends on agents' level of cognitive sophistication, with lower sophistication implying a lower value for government-spending multipliers (Proposition 1).[4] The intuition is as follows. Despite their cognitive limitations, individuals understand that higher government spending implies an increase in taxes. Other things equal, with sticky wages, this negative wealth effect manifests itself as a decrease in consumer demand. However, higher government spending implies an increase in the demand for labor and higher labor income. The latter effect implies an increase in consumer demand. As shown by Woodford (2011), under rational expectations, in a simple NK model the two effects on consumer demand exactly cancel each other out. So, aggregate demand rises by the same amount as the increase in government spending. Since output is demand determined, the government spending multiplier is exactly one.

The less sophisticated people are, the less they take into account the positive general-equilibrium effects of higher spending. So, the negative wealth effect associated with higher taxes receives relatively more weight in people's decisions, leading to a larger drop in consumer demand. The net effect is that lower levels of cognitive sophistication result in lower values for the government spending multiplier.

We then turn to an analysis of tax policy at the ZLB. Correia et al. (2013) show that tax policy is a powerful tool for stabilizing the economy when the ZLB binds and people have rational expectations. Following these authors, we consider a policy of lowering an *ad-valorem* tax on consumption as soon as the ZLB binds and then slowly raising that tax to its pre-shock level over time. This policy has the effect of putting consumption "on sale" while the ZLB binds. We show that there always exists a time path for consumption taxes that completely stabilizes the economy at its pre-shock level, i.e., it supports the flexible-price allocation. In general, this policy depends on people's level of sophistication, k. However, suppose that, as in Farhi and Werning (2019), the least sophisticated people

[4]As discussed in the related literature section below, Angeletos and Lian (2018) obtain a similar result stemming from the assumption that people do not share common information about future government actions.

($k = 1$) think that the economy will remain at its pre-shock level. Then, the path for consumption taxes that supports the flexible-price allocation is the same regardless of how cognitively sophisticated people are.

Critically, the flexible price allocation is the same as the pre-shock steady state of the economy. So under the tax policy that supports this allocation, peoples' initial beliefs are self-confirming, i.e., they do not make any expectational errors. In this sense, the efficacy of this policy doesn't exploit peoples' lack of sophistication. Taken together these results, summarized in Proposition 2, show that tax policy for stabilizing the economy when the ZLB binds is powerful *and* robust to how sophisticated people are.

The basic intuition for why tax policy is robustly powerful is as follows. Suppose that government announces a time-path for current and future tax rates. People incorporate those rates into their personal consumption-savings decision and will substitute consumption to dates when the tax rate is lower. This basic force is operative irregardless of any GE considerations, i.e., people do not need to calculate the GE effects of the announced tax rate to adjust their personal consumption decision to the tax rates. So the policy succeeds in boosting consumption demand when the ZLB binds, even if people are very unsophisticated.

Angeletos and Sastry (2020) emphasize that, with bounded rationality, the way in which policy is communicated matters. Above we assumed that the government announces a sequence of consumption tax rates that will apply during the ZLB. Suppose instead that the government announces a rule according to which tax rates are set as a function of the output gap. We show that this form of communication leads to a substantial deterioration in the efficacy of tax rate policy.

To make this argument, we proceed as follows. Consider a rule for setting tax rates in the ZLB and calculate the corresponding sequence of tax rates that would obtain under rational expectations. We compute the equilibria in the level-k economy under the announced policy rule and the sequence of corresponding announced tax rates. We compare the decline in output under these two communication policies. It turns out that, for any k, the decline in output is larger when policy is communicated as a tax rule rather than a sequence of tax rates.

The intuition for this result is as follows. When policy is communicated as a rule, individuals must forecast the future level of output in order to predict what tax rates will be. When individuals are limited in their ability to compute general-equilibrium effects, they will also be limited in their ability to forecast future tax rates. This limitation translates into a lower efficacy of tax policy in stimulating demand.

When applied to games of strategic substitutes, standard models of level-k thinking produce a peculiar oscillatory behavior in which the equilibrium lies below its limiting point for odd levels of cognitive sophistication and above that limiting point for even levels of cognitive sophistication. Angeletos and Sastry (2020) argue that this is a "bug" of the standard level-k thinking approach, which is not present in other similar models of bounded rationality. As it turns out, under a rules-based policy for taxes, the associated strategic behavior in our economy features strategic substitution and the attendant oscillatory behavior.

In light of this feature, we consider a generalization of standard level-k thinking which is based on the cognitive hierarchy model introduced by Camerer et al. (2004). In this generalization, level-k people assume that other people are distributed between all lower levels of cognitive ability. This model nests standard level-k thinking as a special case. Our key results for the generalized model are as follows. First, for the parameterization that we consider, the generalized level-k model does not feature the oscillatory feature criticized by Angeletos and Sastry (2020). Second, we show that all of the major qualitative results for spending and tax policy in the standard level-k model continue to hold in the generalized model in appendix B.

A natural question is whether our results are robust to alternatives ways of modeling bounded rationality. In appendix B, we redo our analysis of the benchmark model using other two alternatives to the level-k thinking approach. The first alternative is that people have *reflective expectations* as developed in García-Schmidt and Woodford (2019). The second alternative is that people display *shallow reasoning* as developed in Angeletos and Sastry (2020). We show that Propositions 1 and 2 continue to hold for both cases. We infer that our key results do not depend sensitively on the benchmark model assumption that people form expectations via level-k reasoning.

Recall that in our benchmark model we assume that the price level is constant. This assumption does not hold in more general versions of the NK models. In those models, the impact of government spending in the ZLB on inflation and the real interest rate plays an important role in magnifying the size of the government spending multiplier. When the ZLB is binding, increases in government spending lead to upwards pressure on prices, which lowers the real interest rate and boosts the demand for consumption. To the extent that people do not understand these equilibrium effects, the size of the government-spending multiplier should be lower, as shown by Angeletos and Lian (2018) and Farhi et al. (2020). It is not obvious how a variable price level affects the efficacy of tax policy under bounded rationality.

To study these issues we redo our analysis in a framework where prices and wages are not constant. Specifically, in section 3, we assume that nominal wages are set subject to Calvo-style frictions as in Erceg et al. (2000).[5] Since wages are not constant, neither is the price level. We show numerically that the key results of Proposition 1 continue to hold. Turning to tax policy we suppose that the government can impose time varying tax rates on consumption and labor income. With this proviso, we show that the analog to Proposition 2 holds for the extended model. Tax policy can be used to support the flexible price allocation at the ZLB. As in the benchmark economy, the policy that supports the flexible price allocation does not depend on k if the least sophisticated agents expect that the economy will remain at steady-state level. Finally we show through a series of numerical examples that our results regarding the advantage of communicating policy via targets rather than rules continues to hold.

Taken together, our results *weaken* the case for using government spending to stabilize the economy when the ZLB binds. At the same time, our results *strengthen* the case for using tax policy to stabilize output when the ZLB is binding. The power of tax policy to stabilize the economy during the ZLB period is essentially undiminished when agents do not have rational expectations.

Related literature This paper belongs to a growing literature studying the implications of deviations from rational expectations for the effectiveness of macroeconomic policy. The form of bounded rationality that we consider is based on level-k thinking models which were originally studied by Nagel (1995) and Stahl and Wilson (1995). Farhi and Werning (2019) use this approach to study how deviations from rational expectations impacts the efficacy of forward guidance. García-Schmidt and Woodford (2019) develop a closely related form of deviation from rational expectations, which they refer to as reflective expectations. They apply this form of expectations to study the impact of forward guidance and interest rate pegs on economic activity. Under both level-k thinking and reflective expectations, individuals have a limited ability to understand the general-equilibrium consequences of policy.[6] García-Schmidt and Woodford (2019) and Farhi and Werning (2019) show that this effect limits the power of forward guidance and mitigates some anomalous implications of this policy under rational expectations.[7] Iovino and

[5]Appendix D redoes the analysis of section 3 under the assumption that nominal prices, rather than nominal wages, are subject to Calvo-style frictions.

[6]Similar ideas are captured by the *calculation equilibrium* and *internal rationality* approach to bounded rationality discussed in Evans and Ramey (1992) and Adam and Marcet (2011), respectively.

[7]Similar results are derived in Woodford (2018) in a model in which individuals can only make contingent plans up to a finite number of future periods, i.e., they have *limited foresight*, Gabaix (2020) in a model in which individuals are inattentive to the interest rate, in Angeletos and Lian (2018) in a model with informational frictions and imperfect common knowledge, and in Wiederholt (2015) in a model with sticky expectations.

Sergeyev (2018) apply level-k thinking and reflective expectations to analyze the effects of quantitative easing.

Angeletos and Lian (2018) study a rational-expectations environment in which people do not have common knowledge about the relevant news and structure of the economy. They show that the absense of common knowledge dampens the general-equilibrium effects of news. In an NK model, this effect reduces the size of the government-spending multiplier. The idea that common knowledge attenuates general-equilibrium effects was initially developed in a more abstract formulation in Angeletos and Lian (2017). We obtain a similar result when people have complete information about the shocks, but are limited in their cognitive ability. While the mechanism is different, this limitation attenuates the general-equilibrium effects of those shocks as in Angeletos and Lian (2018).

The consequences of bounded rationality for the size of fiscal multipliers has been analyzed by Woodford and Xie (2019) and Farhi et al. (2020). Following the approach developed by Woodford (2018), Woodford and Xie (2019) assume that individuals can only plan for a finite number of periods but are fully rational within the planning horizon. They show that this behavioral bias may limit the size of the government-spending multiplier at the ZLB, because the stimulus effect of future government spending is zero if it occurs after the relevant planning horizon. Instead, we work with a model in which individuals have an infinite planning horizon but have a limited capacity to understand the GE effects of different policies.

Our analysis is closest to Farhi et al. (2020), who also assume that individuals are level-k thinkers. Their main focus is on the *fiscal multiplier puzzle* discussed in Farhi and Werning (2016). They begin by noting that in standard representative-agent NK economies, government-spending multipliers grow explosively as government spending is back-loaded. At the heart of this result is the fact that back-loading spending generates more inflation, which lowers the real rate when the nominal interest rate is constrained to the ZLB, thus boosting aggregate demand. Farhi et al. (2020) then examine the fiscal multiplier puzzles in models with heterogeneous agents and incomplete markets.

An important distinction between our paper and the literature just cited, is that we study how deviations from rational expectations affect the efficacy of tax policy versus government spending when the ZLB is binding. In addition, we analyze how communication affects the power of tax policy at the ZLB.

Because our model features a representative agent and Ricardian Equivalence, there is no role for countercyclical fiscal transfers as a tool for stabilization policy. McKay and Reis (2016, Forthcoming) and Kekre (2021) study the role of tax and transfer programs that affect inequality in stimulating demand in heterogeneous-agent incomplete markets economies with rational expectations. Recently, Woodford and Xie (2020) show that uniform fiscal transfers can be a powerful stabilization tool in a model in which Ricardian Equivalence fails due to bounded rationality.[8]

As mentioned above, Angeletos and Sastry (2020) analyze the implications of policy communication when agents have a particular form of bounded rationality. They address the question of whether policy communication should focus on instruments (interest rates) or on targets (unemployment). They show that the answer to this question depends on the relative importance of partial versus general-equilibrium effects of a given policy. Their substantive application is forward guidance, while we focus on tax policy. In addition, we look at rules versus instrument settings, rather than their main focus of instruments versus targets.

There is a large empirical literature estimating the causal effect of different policies on consumer expectations and decisions. Recently, D'Acunto et al. (2020) estimate the impact of forward guidance and consumption tax policies on household inflation expectations and spending. They show that while forward guidance policies had little effect on household expectations and behavior, consumption tax policies like the ones we describe effectively raise both inflation expectations and household spending. These empirical results support our theoretical finding that tax policy can still be a powerful stabilization tool, even if people are not as sophisticated as in the rational expectations paradigm.

The paper is organized as follows. Section 2 describes our benchmark NK model with level-k thinking. Section 2.1 analyzes the effects of government spending and the implications of bounded rationality for the government spending multiplier in the benchmark model. Section 2.2 presents our results on consumption tax policy in the benchmark model. Section 3 considers the extended model with time varying wages and prices. Finally, section 4 contains concluding remarks. The proofs for all propositions are contained in the appendix.

2 A benchmark model

In this section, we describe our benchmark model. Sections 2.1 and 2.2 analyze the effect of government spending and tax policy, respectively.

Consider a simple NK economy with fully rigid wages. Without loss of generality, we normalize nominal wages to one, $W_t = 1$. The representative household preferences over

[8] Wolf (2021) also considers a general model in which Ricardian Equivalence fails and shows that aggregate allocations that are implementable with interest rate policy can be equivalently implemented with uniform cash transfers.

sequences of consumption, C_t, and labor, N_t, are given by:

$$\sum_{s\geq 0} \beta^s \xi_{t+s} \left[u\left(C_{t+s}\right) - v\left(N_{t+s}\right) \right], \tag{2.1}$$

where $u(C) = C^{1-\sigma^{-1}} / \left(1-\sigma^{-1}\right)$ and $v(N) = N_{t+s}^{1+\varphi} / (1+\varphi)$. As in Correia et al. (2013), we assume that the steady state subjective discount factor $\beta \in (0,1)$ is perturbed by a *discount-factor shock:*

$$\xi_t = e^{-\chi(T-t)}, \tag{2.2}$$

for $t = 0, 1, ..., T$ and $\xi_t = 1$ for $t \geq T$. This assumption implies that the household's subjective discount rate between periods t and $t+1$ is

$$\log \frac{\xi_t}{\beta \xi_{t+1}} = \rho - \chi, \quad t \leq T-1,$$

where $\rho \equiv \log \beta^{-1}$. We assume that the shock satisfies $\chi > \rho$, so that the subjective discount rate is negative for $t \leq T-1$.

For simplicity, we assume that the production function is linear in labor, $Y_t = N_t$. The goods market clearing condition is

$$C_t + G_t = Y_t, \tag{2.3}$$

where G_t denotes government spending. In our baseline specification, we assume that government spending is zero, $G_t = 0$.

In this simple economy, the first-best (flexible-price) allocation is $Y_t^{fb} = C_t^{fb} = N_t^{fb} = 1$. Note that the discount-rate shock does not affect aggregate consumption or production in this allocation. However, implementing this allocation requires a negative real-interest rate. So that allocation cannot be achieved using only conventional monetary policy.

Firms Firms are perfectly competitive and maximize profits. An interior solution for the firms problem requires that $W_t = P_t$. Because wages are fully rigid, there is no inflation:

$$\frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} = 1. \tag{2.4}$$

Monetary and fiscal policies The monetary authority controls the nominal interest rate, R_t. During $t \leq T-1$ the nominal interest rate is at the *ZLB*,

$$R_t = 1, \tag{2.5}$$

and then goes back to its pre-shock level: $R_t = \beta^{-1}$ for $t = T, T+1, ...$

The fiscal authority sets government spending G_t, consumption taxes τ_t^c, and lump-sum taxes T_t. The government's intertemporal budget constraint is given by:

$$\sum_{s\geq 0} Q_{t,t+s} G_{t+s} + R_{t-1} B_t = \sum_{s\geq 0} Q_{t,t+s} \left[\tau_{t+s}^c C_{t+s} + T_{t+s}\right]. \tag{2.6}$$

Here $Q_{t,t+s}$ is the discount factor between t and $t+s$,

$$Q_{t,t+s} \equiv \prod_{m=t}^{t+s-1} R_m^{-1}$$

for $s \geq 1$, $Q_{t,t} \equiv 1$, and $C_t = Y_t - G_t$ is equal to households' consumption.

Households and expectations The household has perfect foresight with respect to exogenous variables so that it correctly anticipates the path for the discount rate shock, ξ_t. For now, we assume that the government announces sequences of nominal interest rates, R_t, government spending, G_t, and consumption taxes, τ_t^c. The fact that the household correctly anticipates the path for these policy variables is consistent with the idea that they see and understand policy announcements. However, the household is limited in its ability to fully predict the equilibrium changes that occur as a result of these policies. We denote by Y_t^e and T_t^e the household's beliefs about the time t values of output and lump-sum taxes, respectively. There is no uncertainty in this economy so these beliefs do not correspond to expectations over possible realizations of Y_t and T_t. They are what households think those variables will be with probability one.

Our goal is to transparently highlight the consequences of failures in predicting the general-equilibrium implications of fiscal policies for their effectiveness. We isolate this particular form of bounded rational behavior from other potential sources of non-rational expectations. So, we assume that given their beliefs for output, the household's expectations for lump-sum taxes are consistent with the government's inter-temporal budget. Formally, we assume that household beliefs for T_t^e satisfy:

$$\sum_{s\geq 0} Q_{t,t+s} T_{t+s}^e = \sum_{s\geq 0} Q_{t,t+s} \left[G_{t+s} - \tau_{t+s}^c \left(Y_{t+s}^e - G_{t+s}\right)\right] + R_{t-1} B_t. \tag{2.7}$$

The previous expression implies that Ricardian equivalence holds in our model.[9]

[9] Iovino and Sergeyev (2018) analyze the impact of central bank balance sheet policy on the economy. They do so assuming that people are level-k thinkers who do not fully understand the inter-temporal nature of the government's budget constraint. So in their model economy Ricardian equivalence does not hold.

The household enters period t with financial assets B_t which earn the interest rate R_{t-1}. As in Farhi and Werning (2019), we assume that the household knows its contemporaneous income Y_t and taxes T_t.[10] When solving its dynamic consumption-savings problem, the household maximizes its perceived utility which is evaluated based on today's consumption, C_t, and on its plans for future consumption $\widetilde{C}_{t+s}$ for $s = 1, 2, ...$ To the extent that the household makes mistakes in predicting its future disposable income, actual consumption will deviate from planned consumption.

The household solves the problem:

$$\max_{\widetilde{C}_{t+s}} \sum_{s\geq 0} \beta^s \xi_{t+s} \left[\frac{\widetilde{C}_{t+s}^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \frac{(N^e_{t+s})^{1+\varphi}}{1+\varphi} \right], \quad \text{subject to}$$

$$\sum_{s\geq 0} Q_{t,t+s} \left(1+\tau^c_{t+s}\right) \widetilde{C}_{t+s} = \sum_{s\geq 0} Q_{t,t+s} \left[Y^e_{t+s} - T^e_{t+s}\right] + R_{t-1} B_t.$$

Since wages are rigid, equilibrium output and labor are demand determined. The solution to the household's problem implies that C_t satisfies

$$C_t = \frac{Y_t - T_t + \sum_{s\geq 1} Q_{t,t+s} \left[Y^e_{t+s} - T^e_{t+s}\right] + R_{t-1} B_t}{\left(1+\tau^c_t\right) \left[1 + \sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} \frac{1+\tau^c_{t+s}}{1+\tau^c_t}\right]^{1-\sigma}\right]}.$$

Replacing the present value of lump-sum taxes using equation (2.7), we obtain:

$$C_t = \frac{(Y_t - G_t) + \sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau^c_{t+s}}{1+\tau^c_t} \left[Y^e_{t+s} - G_{t+s}\right]}{1 + \sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} \frac{1+\tau^c_{t+s}}{1+\tau^c_t}\right]^{1-\sigma}}. \tag{2.8}$$

Temporary and rational-expectations equilibria We start by defining a *temporary equilibrium*. Because this general equilibrium concept does not impose any restrictions on agents expectations, it serves as a good starting point for our analysis. Formally, for given beliefs $\{Y^e_t\}$, a temporary equilibrium is a sequence of allocations which satisfy private optimality for households and firms and the budget constraint of the government. In addition markets clear. Using equation (2.8) and imposing market clearing $Y_t = C_t + G_t$, the temporary equilibrium output is given by

$$Y_t = \mathcal{Y}_t\left(\{Y^e_{t+s}\}_{s\geq 1}\right) = G_t + \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau^c_{t+s}}{1+\tau^c_t} \left[Y^e_{t+s} - G_{t+s}\right]}{\sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} \frac{1+\tau^c_{t+s}}{1+\tau^c_t}\right]^{1-\sigma}}. \tag{2.9}$$

[10] Our results go through if we assume that the household does not see contemporaneous Y_t and C_t.

A *rational-expectations equilibrium* is a temporary equilibrium in which expectations are consistent with the equilibrium path for these variables: $Y_t^e = Y_t$. The RE equilibrium, Y_t^*, solves the fixed-point problem

$$Y_t^* = \mathcal{Y}_t\left(\{Y_{t+s}^*\}_{s\geq 1}\right).$$

Level-k equilibria We now describe the concept of a level-k equilibrium for our model economy. Let Y_t^k denote the time-t level of output in an economy for which all agents are level k. Also $Y_t^{e,k}$ denotes the household's beliefs about output.

We take as given the beliefs of level-1 people, $\left\{Y_t^{e,1}\right\}$. For example one could assume that level-1 people believe that output will stay at its pre-shock level, i.e., $Y_t^{e,1} = 1$. This assumption is consistent with the approach in Farhi and Werning (2019), and captures the intuitive idea that level-1 individuals do not take into account how the shocks and policy will affect the future state of the economy.

Given these beliefs, a level-1 equilibrium is given by

$$Y_t^1 = \mathcal{Y}_t\left(\left\{Y_{t+s}^{e,1}\right\}_{s\geq 1}\right).$$

In the standard model of level-k thinking, individuals believe that all other agents are exactly one level below them in terms of cognitive ability. So level-2 people believe the economy is entirely populated by level-1 people. Moreover, level-2 people are able to calculate the market equilibrium in an economy populated entirely by level-1 people. So, level-2 people think that the market equilibrium is given $Y_t^{e,2} = Y_t^1$. The level-2 equilibrium is therefore given by

$$Y_t^2 = \mathcal{Y}_t\left(\left\{Y_{t+s}^{e,2}\right\}_{s\geq 1}\right) = \mathcal{Y}_t\left(\left\{Y_{t+s}^{1}\right\}_{s\geq 1}\right).$$

Level-3 thinkers can work through the reasoning process of both level-1 and level-2 individuals. So they think that the market equilibrium is given $Y_t^{e,3} = Y_t^2$. The level-3 equilibrium is given by

$$Y_t^3 = \mathcal{Y}_t\left(\left\{Y_{t+s}^{2}\right\}_{s\geq 1}\right).$$

More generally, level-k people think that the market equilibrium is given by $Y_t^{e,k} = Y_t^{k-1}$ so that level-k equilibrium is

$$Y_t^k = \mathcal{Y}_t\left(\left\{Y_{t+s}^{k-1}\right\}_{s\geq 1}\right). \tag{2.10}$$

2.1 Government spending multipliers

In this section we assume that consumption taxes are kept at their steady state level $\tau_t^c = \tau^c$ for all periods and consider an increase in government spending, ΔG_t, during the ZLB periods, i.e., for $t \leq T - 1$.

Rational expectations In this model, the monetary authority pegs the real interest rate. It is widely understood that, under such a policy, there are multiple equilibria in the standard rational expectations NK model. As in Farhi and Werning (2019), we focus on rational expectations equilibria for which $Y_t^* \to 1$ as $t \to \infty$, i.e., the equilibrium converges to the pre-shock steady state. The household's Euler equation then implies that

$$C_t = C_{t+1} = C_{t+2} = \lim_{s \to \infty} C_{t+s} = 1$$

for all $t \geq T$.

During the ZLB period, the real interest rate is higher than the subjective discount rate. So consumption is lower than in the pre-shock steady state:

$$C_t = (\beta e^{\chi})^{-\sigma} C_{t+1} = ... = e^{-\sigma(T-t)(\chi-\rho)}. \tag{2.11}$$

Here, $\rho \equiv -\log(\beta)$. The rational expectation equilibrium level of output is given by

$$Y_t^* = G_t + e^{-\sigma(T-t)(\chi-\rho)}.$$

Consistent with Woodford (2011), equation (2.11) implies that consumption is not affected by government spending in the rational expectations equilibrium. So, the government-spending multiplier is exactly equal to one

$$\frac{\Delta Y_t^*}{\Delta G_t} = 1, \tag{2.12}$$

where ΔY_t denotes the difference in output relative to the output level in the equilibrium without government spending.

Note that the multiplier does not depend on the path of government spending. As it turns out this result depends on the assumption of rational expectations.[11] To show this formally we now turn to the temporary equilibrium.

[11]The multiplier would not be independent of the path of G_t in more general versions of the NK model or a neo-classical growth model with savings, flexible hours worked and/or time varying prices.

With bounded rationality Relation (2.9) implies that the temporary equilibrium is given by

$$\mathcal{Y}_t\left(\{Y^e_{t+s}\}\right) = G_t + \frac{\sum_{s\geq 1} Q_{t,t+s}\left[Y^e_{t+s} - G_{t+s}\right]}{\sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} Q^{1-\sigma}_{t,t+s}}.$$

It seems natural to assume that level-1 people believe that the economy goes back to its steady state after the shock reverts to its pre-shock value, i.e., $Y^{e,1}_t = 1$ for $t \geq T$. This assumption implies that Y_t is equal to its steady-state level for $t \geq T$. It follows that $Y^{e,k}_t = 1$ for all k and $t \geq T$. It follows that we can write the equilibrium level of output for $t \leq T-1$ as follows

$$Y_t = G_t + \Omega_t \left\{\sum_{s=1}^{T-t-1}\left[Y^e_{t+s} - G_{t+s}\right] + \frac{1}{1-\beta}\right\},$$

where $\Omega_t \equiv \left[e^{\sigma(\chi-\rho)}\left[\frac{1-e^{\sigma(\chi-\rho)(T-t-1)}}{1-e^{\sigma(\chi-\rho)}} + \frac{e^{\sigma(\chi-\rho)(T-t-1)}}{1-\beta}\right]\right]^{-1} \in (0,1]$.

Lemma 1. *In a temporary equilibrium, the government spending multiplier is given by*

$$\frac{\Delta Y_t}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1}\left[\frac{\Delta Y^e_{t+s}}{\Delta G_{t+s}} - 1\right]\frac{\Delta G_{t+s}}{\Delta G_t}. \tag{2.13}$$

Note that in a temporary equilibrium, the time t government spending multiplier depends on people's beliefs regarding future income. Recall that this dependency is not a feature of the rational expectations equilibrium for our simple model.

The intuition about how beliefs about future government spending affect the time t multiplier is as follows. First, if expectations for future incomes do not change with the policy ($\Delta Y^e_{t+s} = 0$) then the effect of future spending on current output is negative,

$$-\Omega_t \sum_{s=1}^{T-t-1}\frac{\Delta G_{t+s}}{\Delta G_t}.$$

We refer to this effect as the *partial-equilibrium effect* of government spending: higher taxes associated with higher current and future spending leads to a negative wealth effect that causes people to reduce consumption.

The *general-equilibrium effect* of government spending is given by

$$\Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta Y^e_{t+s}}{\Delta G_{t+s}} \frac{\Delta G_{t+s}}{\Delta G_t}.$$

Higher future spending leads people to believe that their future incomes will be higher. The associated positive wealth effect leads to an increase in current consumption. Other things equal this increase leads to a rise in actual current output. The fact that the government spending multiplier is exactly one under rational expectations reflects that the partial and general equilibrium effects exactly offset each other.

We now consider the level-k economy and show that the less sophisticated people are, the less they take GE effects into account. This effect leads to a lower government spending multiplier. Suppose that level-1 people believe that the aggregate output will remain at its pre-shock level, so that $\Delta Y_t^{e,1} = 0$. So the government spending multiplier in a level-1 equilibrium is

$$\frac{\Delta Y^1_t}{\Delta G_t} = 1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t}.$$

The previous formula shows that the multiplier, $\Delta Y^1_t / \Delta G_t$, is less than one because level-1 agents only take into account the partial-equilibrium effect of a change in government spending.

More generally, the government spending multiplier for a level-k economy can be computed using the iterative equation

$$\frac{\Delta Y^k_t}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y^{k-1}_{t+s}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t}.$$

Here

$$\Delta Y^{k-1}_{t+s} / \Delta G_{t+s} = \Delta Y^{e,k}_{t+s} / \Delta G_{t+s}$$

where the RHS of the previous equation denotes the household's belief about future government spending multipliers. Note that if $\Delta Y^{k-1}_{t+s} / \Delta G_{t+s} \leq 1$ for all t and s, then $\Delta Y^k_t / \Delta G_t \leq 1$ for all t. It follows that the government spending multiplier for a level-k economy is always lower than the multiplier under rational expectations.

The next proposition summarizes this result and two additional implications. First, we provide a sufficient condition under which the government spending multiplier is increasing as a function of cognitive sophistication. Second, we assume that $\Delta G_t = \gamma^t \Delta G_0$ and study how the government spending multiplier depends on the persistence parameter γ. As it turns out, for a fixed k, more persistent spending leads to lower multipliers because the negative wealth effect is stronger.

Proposition 1. *Suppose that $\Delta Y_t^{e,1}/\Delta G_t = 0$. Then, the level-k government spending multiplier is lower than one, i.e., $\Delta Y_t^k/\Delta G_t \leq 1$. Furthermore, if $1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \geq 0$ for all t, then the government spending multiplier is increasing in k:*

$$\frac{\Delta Y_t^k}{\Delta G_t} \geq \frac{\Delta Y_t^{k-1}}{\Delta G_t}.$$

Finally, suppose that $\Delta G_t = \zeta^t \Delta G_0$, then the level-k government spending multiplier is decreasing in ζ:

$$\frac{d\Delta Y_t^k/\Delta G_t}{d\zeta} < 0.$$

In sum, departing from rational expectations by introducing level-k thinking implies a decline in the size of the government spending multiplier. As discussed above, all households internalize the effects of higher taxes associated with higher government spending. However, understanding the expansionary effects of government spending requires that people compute how, in equilibrium, higher government spending leads to higher labor income. The less sophisticated people are, the less weight they give to the expansionary effect, the lower is their expected future disposable income and the lower is their current consumption. In this way, lower levels of sophistication leads to lower values of government spending multipliers.

2.2 Consumption tax policy

In this section, we discuss the efficacy of consumption tax policy when the ZLB is binding. Following Correia et al. (2013), we show that consumption tax policy can implement the flexible-price allocation allocation under rational expectations. We then evaluate the efficacy of consumption-tax policy under level-k thinking and show that there alway exists a policy that supports that allocation. Under plausible assumptions, that policy does not depend on the value of k and its success does not depend on people making systematic errors in their beliefs about economy-wide variables.

Assume that government spending does not respond to the discount rate shock, so that $G_t = 0$ remains at its steady state value of zero. Consumption taxes change during the ZLB period and converge back to their pre-shock level, τ^c, once the economy exits the ZLB ($t = T$).

Rational expectations With time-varying consumption taxes, the household's Euler equation for $t \leq T-1$ can be written as as

$$Y_t = Y_{t+1}\left(\beta\frac{\xi_{t+1}}{\xi_t}R_t\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)^{-\sigma}$$

where we have set $C_t = Y_t$. This expression makes clear that the relevant relative price of consumption at time t versus time $t+1$ is given by the real interest rate times the ratio of consumption taxes, $R_t\left(1+\tau_t^c\right)/\left(1+\tau_{t+1}^c\right)$.

We write this Euler equation in log terms,

$$y_t = y_{t+1} - \sigma\left(r_t + \log\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right) - (\rho-\chi)\right), \tag{2.14}$$

where $r_t = \log R_t = 0$. Note that, for $t \geq T$, the real interest rate returns to its pre-shock level, $r_t = \rho$, and $y_t = 0$ (or $Y_t = 1$).

Suppose that, at time 0, the government announces that taxes will follow the path $\tau_t^c = \tau_t^{c,*}$, where

$$\tau_t^{c,*} = \left(1+\tau^c\right)e^{-(T-t)(\chi-\rho)} - 1 \tag{2.15}$$

for $t \leq T$. With this specification, the consumption tax falls at time 0 and then slowly converges back to its pre-shock value. Note that

$$\log\left(\frac{1+\tau_t^{c,*}}{1+\tau_{t+1}^{c,*}}\right) = \rho - \chi.$$

Under this assumption, the relative price of consumption is equal to the subjective discount rate even if the nominal interest rate is at the ZLB.

Equation (2.14) implies that under this policy $y_t = y_{t+1}$ for all t. Since $y_t \to 0$ in the limit, it follows that this tax policy implements the flexible-price allocation, i.e., $y_t = 0$ for all t. The conclusion that tax policy can effectively circumvent the ZLB and achieve the flexible-price allocation is the key result in Correia et al. (2013).[12] In our derivation, we assumed that the government has access to lump-sum taxes. Correia et al. (2013) show that even if lump-sum taxes are unavailable, consumption taxes can still be used to fully offset the ZLB restriction and support the flexible price allocation.

As emphasized by Correia et al. (2013), consumption taxes affect the relative price of

[12] In a more general setting, Correia et al. (2008) show that fiscal policy can be used to neutralize the effects of price stickiness in standard NK models.

leisure. So in general, the government must change labor income taxes to compensate for the effects of changes in consumption taxes on labor supply. But in our simple model hours worked are demand determined so that labor income taxes are equivalent to lump-sum taxes. We return to this point in section 3.

Bounded rationality Suppose that the government announces a path for consumption taxes, τ_t^c, such that taxes go back to their pre-shock level as soon as the economy exits the ZLB, i.e., $\tau_t^c = \tau^c$ for $t \geq T$. In addition, suppose that everyone expects the economy to go back to its pre-shock steady state once the ZLB is no longer binding. Then the temporary equilibrium level of output is given by:

$$Y_t = \left(\frac{1+\tau^c}{1+\tau_t^c}\right)^\sigma \frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^c}{1+\tau^c}\right)Y_{t+s}^e + 1}{(1-\beta)\sum_{s=1}^{T-t-1} e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma} + e^{(T-t)\sigma(\chi-\rho)}}. \tag{2.16}$$

Equation (2.16) highlights the effect of time-varying consumption taxes on consumption and equilibrium output. For $t = T-1$, we can write this equation as

$$Y_{T-1} = \left(\frac{1+\tau^c}{1+\tau_{T-1}^c}\right)^\sigma e^{-\sigma(\chi-\rho)}.$$

This expression makes clear that setting $\tau_{T-1}^c = (1+\tau^c)\, e^{-(\chi-\rho)} - 1$ implements $Y_{T-1} = 1$.

It follows directly from (2.16) that , for given beliefs Y_t^e, there always exists an appropriate choice of τ_t^c for which $Y_t = 1$ for all t. However, in models of belief revision like level-k thinking, beliefs are endogenous to the policy that is implemented. Proposition 2 shows that for every level of cognitive ability k, there exists an appropriately chosen path for consumption taxes which implements the flexible price allocation. The key technical aspect of the proof is that future beliefs are only a function of future consumption taxes. As agents become more sophisticated, this policy approaches the rational expectations optimal policy, $\tau_t^{c,*}$. if the expectations of unsophisticated agents are anchored at the initial steady state, then the policy that achieves full stabilization is the same regardless of k. Moreover, that policy coincides with the optimal policy under rational expectations.

Proposition 2. *Suppose that level-*1 *people believe that the economy goes back to steady state after the ZLB period, i.e.,* $Y_t^{e,1} = 1$ for $t \geq T$.

1. *For each k, there exists a policy announcement* $\left\{\tau_t^{c,k}\right\}$ *which implements the flexible-price allocation.*

2. *Suppose that* $Y_t^{e,1} = 1$ *for all* $t \geq 0$*, then the policy announcement* $\{\tau_t^{c,*}\}$ *implements the flexible-price allocation for all k.*

In the appendix, we prove the first result in the Proposition. Specifically, we show how to construct the path for consumption taxes that implements the flexible-price allocation for a given level of cognitive sophistication. In general, this policy is a function of k, which is to say that its correct design would require the government to know the people's level of cognitive sophistication.

A simple proof of the second result in the Proposition is as follows. Recall that under the tax policy $\{\tau_t^{c,*}\}$, the rational expectations equilibrium is $Y_t^* = 1$. By definition, this equilibrium is a fixed point of the temporary equilibrium relation (2.16). If level-1 individuals expect the economy to stay at steady state, then they will adjust their behavior so that it is the same equilibrium outcome, i.e., $Y_t^1 = 1$. Since level-2 individuals believe that the equilibrium is $Y_t^{e,2} = Y_t^1 = 1$, then the level-2 equilibrium ends up being exactly the same as the level-1 equilibrium. The same logic applies for any k. We conclude that under the proposed tax policy, the belief $Y_t^{e,1} = 1$ is self confirming. It immediately follows that to support the flexible price allocation, the proposed tax policy does not rely on people making mistakes. To the contrary, the tax policy leads to an equilibrium in which people's beliefs coincide with actual outcomes.

2.2.1 Rules versus targets

Proposition 2 provides a strong rationale for using tax policy to fight recessions at the ZLB. In this section, we highlight that the efficacy of the policy depends crucially on how it is communicated. We consider two communication strategies. First, tax policy is communicated and implemented as a sequence of *targets* for consumption taxes. Second, tax policy is communicated and implemented as a *rule* involving endogenous objects like the output gap. We refer to these two strategies as target-based and rule-based communication policy. The reason that communication matters in our setting is straightforward. Under target-based communication, individuals immediately know what tax rates will be in the future and incorporate those rates into their decisions. But under rule-based communication, individuals must work out the future general-equilibrium effects of the policy in order to understand what current and future tax rates will be. In a world populated by level-k thinkers this difference matters.

Assume that monetary policy is given by a Taylor rule subject to a ZLB constraint

$$R_t = \max\left\{\beta^{-1} Y_t^{\phi_y}, 1\right\} \Leftrightarrow r_t = \max\left\{\rho + \phi_y y_t, 0\right\} \tag{2.17}$$

where ϕ_y denotes the elasticity of R_t to the output gap.[13] As in the quantitative analysis of Correia et al. (2013), we assume that consumption taxes are set as:

$$\frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \min\left\{\beta^{-1} Y_t^{\phi_y}, 1\right\} \Leftrightarrow \log \frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \min\left\{\rho + \phi_y y_t, 0\right\}. \tag{2.18}$$

Under this policy, the consumption tax rate do not change when the ZLB doesn't bind. But if the ZLB does bind, then consumption tax rates do change. Given this choice of how tax rates change, people face the same relative price of consumption over time, regardless of whether the ZLB binds or not:

$$R_t \frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \beta^{-1} Y_t^{\phi_y}.$$

Critically, under this announced policy, agents must predict current and future values of output in order to forecast what future tax rates will be, a calculation that involve general equilibrium effects.

The temporary equilibrium is given by

$$\mathcal{Y}_t\left(\{Y_{t+s}^e\}\right) = \left[\beta^{\sigma} \frac{\sum_{s=1}^{\infty} Q_{t+1,t+s}^e\left(\frac{1+\tau_{t+s}^{c,e}}{1+\tau_{t+1}^{c,e}}\right) Y_{t+s}^e}{\sum_{s=1}^{\infty}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t+1,t+s}^e \frac{1+\tau_{t+s}^{c,e}}{1+\tau_{t+1}^{c,e}}\right]^{1-\sigma}}\right]^{\frac{1}{1+\sigma\pi}}. \tag{2.19}$$

where $Q_{t+1,t+s}^e\left(\frac{1+\tau_{t+s}^{c,e}}{1+\tau_{t+1}^{c,e}}\right) \equiv \beta^{s-1} \prod_{\tau=t+1}^{t+s-1} \left(Y_t^e\right)^{-\phi_y}$.

Rational expectations As before, once the economy exits the ZLB, output returns to its pre-shock steady state, so that $y_t = 0$ for $t \geq T$. For earlier dates, we can find the equilibrium using the individual's Euler equation:

$$y_t = y_{t+1} - \sigma\left(\rho + \phi_y y_t - \rho + \chi\right) \Leftrightarrow y_t = \frac{y_{t+1} - \sigma\chi}{1+\sigma\phi_y}.$$

Iterating forward, we obtain the rational-expectations level of log-output

$$y_t^* = -\frac{\chi}{\phi_y}\left[1 - \frac{1}{\left(1+\sigma\phi_y\right)^{T-t}}\right]. \tag{2.20}$$

[13] We do not include inflation in the Taylor rule because inflation is always zero for our simple economy.

As long as the policy is not infinitely reactive ($\phi_y \to \infty$), then the rules-based policy will not achieve the flexible-price allocation.

The equilibrium path for consumption taxes under this policy is:

$$\log\left(\frac{1+\tau_t^{c,r}}{1+\tau_{t+1}^{c,r}}\right) = \rho - \chi\left[1-\left(1+\sigma\phi_y\right)^{-(T-t)}\right], \tag{2.21}$$

when $r_t = 0$, and $\tau_t^{c,r} = \tau^c$ for $t \geq T$.

In order to evaluate the relative power of rules- versus targets-based policy under bounded rationality, we compute the level-k equilibrium under a rules-based policy and the policy that announces consumption tax targets which satisfy (2.21). This comparison preserves the underlying rational expectations equilibrium under each type of policy communication.

Bounded rationality We now describe the implications of bounded rationality for the efficacy of rules-based policy. As before, we assume that initial beliefs are anchored at the initial steady state, i.e., $y_t^{e,1} = 0$. We begin by describing the equilibrium for level-1 individuals. In the appendix, we show that the level-1 equilibrium log-output if policy is announced as a rule takes the form

$$y_t^1 = -\frac{\sigma\chi + \varphi_t}{1+\sigma\phi_y}, \tag{2.22}$$

where $\varphi_t \equiv \log\left[(1-\beta)\frac{1-e^{(T-t-1)(\sigma\chi-\rho)}}{1-e^{\sigma\chi-\rho}} + e^{(T-t-1)(\sigma\chi-\rho)}\right]$. When policy is communicated as the target path which satisfies (2.21), the equilibrium can be computed using (2.16). Our next proposition summarizes our main result.

Proposition 3. *Suppose that $y_t^{e,1} = 0$ for all t. If policy is announced as a target for consumption tax rates, then $y_t^1 \geq y_t^*$ with equality only if $t = T-1$. Suppose that $\beta \geq \left(1+\sigma\phi_y\right)^{-1}$. If policy is announced as a rule, then $y_t^1 \leq y_t^*$ with equality only if $t = T-1$.*

The condition that $\beta \geq \left(1+\sigma\phi_y\right)^{-1}$ is easily satisfied in standard calibrations. For example, the calibration for the medium-scale DSGE model in Christiano et al. (2011) features $\sigma = 0.5$ and $\phi_y = 0.25$, which implies that $\left(1+\sigma\phi_y\right)^{-1} = 0.89$, which is lower than the value of β that they assume.

According to Proposition 3, when policy is communicated as a rule consumption tax policy is less powerful than when policy is communicated via targets. The intuition is as follows. Under a rules (and targets) based policy, level-1 people do not understand that future output will be lower after the discount-rate shock. Other things equal, this error implies that their consumption will be higher than it would be under rational expectations. Under a rules policy, level-1people do not think output will change. So they do not think that future consumption tax rates will change. Other things equal, this error implies that their consumption will be lower than it would be under rational expectations. If $\beta \geq \left(1+\sigma\phi_y\right)^{-1}$, then the effect of the second error dominates the effect of the first error and output is *lower* in the level-1 equilibrium than in the rational expectations equilibrium.

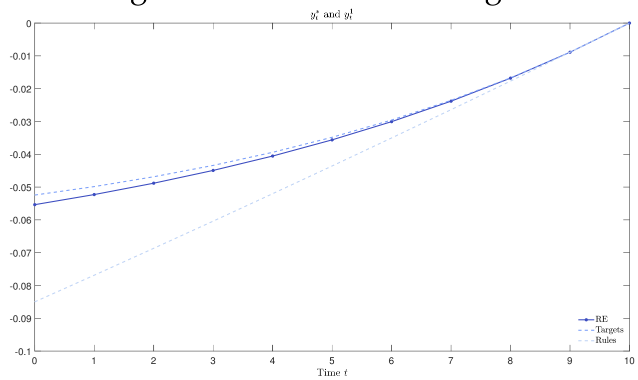
When policy is communicated as a sequence of targets, level-1 people internalize the exact path of future consumption tax rates. So, the expansionary effects of the tax rate change become operative even if people are not very sophisticated. This effect is as strong as it would be under rational expectations. But level-1 people still underestimate the decline in their future income. So, consumption and output are *higher* than under rational expectations.

Figure 2.1 illustrates the properties of the rational expectations equilibrium and the level-1 equilibria under rules- and targets-based policy. We set the discount factor β to 0.99, the intertemporal elasticity of substitution σ to 0.5 , and the coefficient on output in the Taylor rule, ϕ_y, equal to 0.25. We assume that the ZLB lasts for 10 periods, $T = 10$, and we choose the discount rate shock so that $\beta e^{\chi} = 1.01$, so that $\chi = 0.02$. Four findings emerge from Figure Figure 2.1. First, equilibrium output under target-based communication is close to the rational expectations equilibrium level of output. Second, equilibrium output under rule-based communication is much lower than the output in the rational expectations equilibrium. Third, the poor performance of rule-based communication is more pronounced the earlier we are in the ZLB episode, i.e., the longer the episode is expected to last. Finally, Figure 2.1 shows that, with targets based communication, output is higher in the level-1 equilibrium than in the rational expectations equilibrium. In that sense, the same policy achieves even stronger stabilization when people are not very sophisticated. As it turns out, this result holds for all levels of k.

Proposition 4. *Suppose that the government announces the target for tax policy τ_t^r, given by (2.21), and suppose that $y_t^{e,1} = 0$ for all t. Then, for any k, output is higher than under rational expectations, i.e., $y_t^k \geq y_t^*$. Furthermore, y_t^k converges monotonically to y_t^* as $k \to \infty$.*

This proposition shows that, with targets, the consumption-tax policy under consideration becomes more powerful the less sophisticated people are. The intuition follows from the discussion after proposition 3. As k increases, people expect an increasingly large recession after the discount rate shock. So, equilibrium consumption and output drop by more as k increases, eventually converging to the rational expectations equilibrium.

Figure 2.1: Rules versus targets



As it turns out, under rules-based communication, the level-k model under consideration exhibits a peculiar type of oscillatory behavior as a function of k. The equilibrium level of output lies below the rational expectations equilibrium level for odd levels of k but is above it when k is even. The log-linearized version of the temporary equilibrium is given by

$$y_t = -\left[\beta - \frac{1}{1+\sigma\phi_y}\right] \sum_{s=1}^{T-t-1} \beta^{s-1} y^e_{t+s} - \frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta}. \tag{2.23}$$

Since the level-1 equilibrium has lower output that the rational expectations equilibrium, level-2 people believe $y_t^{e,2} = y_t^1 < y_t^*$. This implies that the level-2 equilibrium has higher output than the rational expectations equilibrium, $y_t^2 > y_t^*$, since $\beta - \left(1+\sigma\phi_y\right)^{-1} > 0$. This oscillatory pattern emerges more generally as a function of k.

This peculiar oscillatory feature reflects a more general type of oscillatory behavior in level-k thinking models that is discussed in Angeletos and Sastry (2020). They argue that this feature is a "bug" of the standard level-k thinking approach, which is not present in other similar models of bounded rationality.

A key question is whether our key results are are robust to other models of bounded rationality which are known not to feature this bug. To address this question, we proceed as follows. First, in the main text, we redo the analysis in the previous figure for various levels of k in a generalized level-k thinking model. Second, in appendix B, we redo the analysis of this section for (i) a *generalized level-k thinking* model based on Camerer et al. (2004), (ii) the *reflective expectations* as modeled in García-Schmidt and Woodford (2019), and (iii) the *shallow reasoning* model as developed by Angeletos and Sastry (2020). All of our previous results go through for these alternative models of bounded rationality.

Generalized level-k thinking In this section, we consider the effects of rules-based policy in a generalized level-k economy for the log-linearized economy. Following Camerer et al. (2004), we assume that level-k individuals think that other people are distributed over lower levels of cognitive ability according to the distribution $f_k(j)$ for $0 \leq j \leq k-1$. The reasoning process underlying the generalized level-k model is analogous to the process in the standard level-k model. As in Farhi and Werning (2019), we assume that contemporaneous output, y_t, is observed.

To analyze this economy we must introduce the concept of a level-0 person. This type of person continues to act as they did before the discount rate shock, i.e. $y_0^0 = 0$. It is always possible to specify beliefs $\left\{y_t^{e,0}\right\}$ that support such an action.

Level-1 individuals believe that the economy is populated by level-1 people so $y_t^{e,1} = y_t^0$. Given current output y_t,

$$c_t^1(y_t) = -\left(\beta\left(1+\sigma\phi_y\right)-1\right)y_t - \left(\beta\left(1+\sigma\phi_y\right)-1\right)\sum_{s=1}^{\infty}\beta^s y_{t+s}^{e,1} - \sigma\beta\chi\frac{1-\beta^{T-t}}{1-\beta}. \tag{2.24}$$

Suppose that the economy is populated entirely by level-1 individuals. Solving 2.24 for y_t^1 yields,

$$y_t^1 = -\left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{\infty}\beta^{s-1}y_{t+s}^{e,1} - \frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta}.$$

Level-2 individuals believe that a fraction $f_2(j)$ of the population is level $j = 0, 1$ and work through the problem of level-0 and level-1 people. So they believe that y_t^2 is the solution to

$$y_t^{e,2} = \sum_{j=0}^{1} f_2(j)\, c_t^j\left(y_t^{e,2}\right).$$

More generally, level-k people believe that output is the solution to

$$y_t^{e,k} \equiv \sum_{j=0}^{k-1} f_k(j)\, c_t^j\left(y_t^{e,k}\right). \tag{2.25}$$

Since output is contemporaneously observed, people with different cognitive levels expect different consumption levels for people less sophisticated than themselves. Technically, this means that level-k people think that level-j people behave according to

$$c_t^j(y_t) = -\left(\beta\left(1+\sigma\phi_y\right)-1\right)y_t - \left(\beta\left(1+\sigma\phi_y\right)-1\right)\sum_{s=1}^{\infty}\beta^s y_{t+s}^{e,j} - \sigma\beta\chi\frac{1-\beta^{T-t}}{1-\beta}, \tag{2.26}$$

for $j \geq 1$. For technical reasons, we assume that this equation holds for level-0 people.[14]

Using conditions (2.25) and (2.26), the beliefs of level-k individuals can be written as

$$y_t^{e,k} = \sum_{j=0}^{k-1} f_k(j)\, y_t^j,$$

where

$$y_t^j = -\left(\beta - \frac{1}{1+\sigma\phi_y}\right) \sum_{s=1}^{\infty} \beta^{s-1} y_{t+s}^{e,j} - \frac{\sigma\chi}{1+\sigma\phi_y} \frac{1-\beta^{T-t}}{1-\beta}.$$

Camerer et al. (2004) assume that the distributions $f_k(\cdot)$ are consistent with the physical distribution of cognitive levels in the economy. In contrast, we maintain the representative agent assumption, so that everyone shares the same level k. We assume that agents of different cognitive levels agree on the relative proportions of lower cognitive levels. The distributions $f_k(\cdot)$ are such that for any $k_1 < k_2$ and $s, s' < k_1$

$$\frac{f_{k_1}(s)}{f_{k_1}(s')} = \frac{f_{k_2}(s)}{f_{k_2}(s')}. \tag{2.27}$$

Let $\gamma_k \equiv f_k(k-1)$ for all k. Then assumption (2.27) implies that $f_k(j) = (1-\gamma_k) f_{k-1}(j)$ for $j \leq k-2$. We can write the expectation of level-k individuals as follows:

$$y_t^{e,k} = (1-\gamma_k) \sum_{j=0}^{k-2} f_{k-1}(j)\, y_t^j + \gamma_k y_t^{k-1} = (1-\gamma_k)\, y_t^{e,k-1} + \gamma_k y_t^{k-1}. \tag{2.28}$$

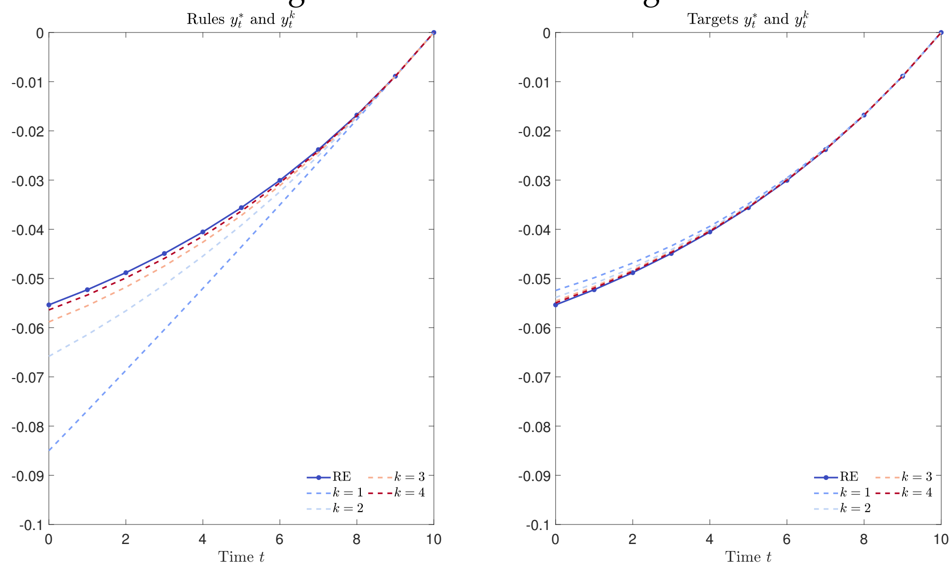
Intuitively, the beliefs of a level-k thinker are given by a weighted average of the beliefs of level $k-1$ agents and the equilibrium that would arise if everyone in the economy was a level-$(k-1)$ thinker. Standard level-k thinking corresponds to the case of $\gamma_k = 1$. By varying γ_k, we can control the intensity of updating across level-k iterations.

Figure 2.2 displays the numerical solution for this economy under rational expectations as well as the four lowest levels of cognitive sophistication. For illustrative purposes, we assume that $\gamma_k = 0.5$ so that level-k people think that half of the population is level $k-1$. In practice, we find that our qualitative results are reasonably robust to perturbations of γ_k. The left and right panels show the equilibrium for the case in which policy is communicated as a rule and as a sequence of targets, respectively.

A number of key results emerge from Figure 2.2. First, in this model economy, rule-

[14]This assumption is convenient because we suppose that people see contemporaneous aggregate output when making consumption decisions. In a continuous-time version of our economy, consumption would effectively not depend on output.

Figure 2.2: Rules versus targets



based communication does not lead to oscillatory behavior as people become more sophisticated. The reason is that expectations about income are updated more smoothly than under standard level-k thinking. Second, target-based communication does better than rules-based communication in terms of stabilizing output. For any given k, target-based communication results in a higher level of output than under rational expectations. But the opposite is true of rule-based communication. The intuition for these results follows from our discussion of the level-1 economy. Third, under rules-based communication, the level of people's sophistication is an important determinant of the size of the recession. Indeed, if people are not very sophisticated, output can be two to three percentage points lower than under rational expectations. In contrast, the level of sophistication is quantitatively less relevant under target-based communication. Finally, as was the case under standard level-k thinking, under rules-based communication, the differential impact of k on output is larger the longer the ZLB period is expected to last.

3 A model with Calvo-style wage rigidities

In this section, we extend the baseline model to allow for time-varying prices and wages. We do so by introducing Calvo-style wage rigidities as in Erceg et al. (2000) and Schmitt-Grohé and Uribe (2005). In Appendix D, we show that our results are robust to assuming Calvo-style price rigidities.

The model economy is populated by a continuum of households, a continuum of unions, goods producers and the government. Each household has a continuum of workers who have different labor skills. Output can be used for private or government consumption so that the aggregate resource constraint is still given by (2.3).

Goods producer The final good is produced by a representative firm using a Cobb-Douglas technology from a fixed stock of capital, $\overline{K}$, and a composite labor input, N_t:

$$Y_t = A\overline{K}^{\alpha} N_t^{1-\alpha}, \tag{3.1}$$

where $A > 0$ denotes total-factor productivity and $\alpha \in [0,1]$ denotes the capital share of output. We assume that capital is fixed for simplicity and to avoid complications in modeling investment decisions when agents have bounded rationality. This assumption can be rationalized for business cycle dynamic analysis if there are large capital adjustment costs (see for example Rotemberg and Woodford, 1997 and Farhi and Werning, 2019).

The composite labor input N_t is generated using a continuum of labor varieties according to the technology:

$$N_t = \left[\int_0^1 n_{u,t}^{\frac{\theta-1}{\theta}} du\right]^{\frac{\theta}{\theta-1}}, \tag{3.2}$$

where $\theta > 1$ captures the elasticity of substitution across the labor varieties. The firm, which is perfectly competitive in both the goods and the labor market, produces final output using the technology given by (3.1) and (3.2). The firm maximizes

$$P_t Y_t - \int_0^1 w_{u,t} n_{u,t} du$$

subject to (3.1) and (3.2). Here P_t denotes the price of the consumption good and $w_{u,t}$ denotes the wage of $n_{u,t}$. Cost minimization implies that

$$n_{u,t} = \left(\frac{w_{u,t}}{W_t}\right)^{-\theta} N_t, \tag{3.3}$$

where

$$W_t = \left[\int_0^1 w_{u,t}^{1-\theta} du\right]^{\frac{1}{1-\theta}}. \tag{3.4}$$

The firm's first-order condition for N_t implies

$$\frac{W_t}{P_t} = (1-\alpha) A \left(\frac{\overline{K}}{N_t}\right)^{\alpha}. \tag{3.5}$$

Households The household enters period t with financial assets B_t which earn the interest rate R_{t-1}. As in section 2, we assume that the household knows its present income Y_t and taxes T_t. When solving its dynamic consumption-savings problem, the household maximizes its perceived utility which is evaluated based on today's consumption, C_t, and on its plans for future consumption $\widetilde{C}_{t+s}$ for $s = 1, 2, ...$ Labor supply is determined by labor unions as described as below. We denote by L_t the total hours worked by the household,

$$L_t = \int_0^1 n_{u,t}.$$

With price dispersion induced by nominal rigidities, L_t is not to equal N_t.

The representative household maximizes (2.1) subject to

$$(1 + \tau^c_{t+s})\, P^e_{t+s} \widetilde{C}_{t+s} + \widetilde{B}_{t+s+1} = (1 - \tau^n_{t+s})\, W^e_{t+s} N^e_{t+s} + \Omega^e_{t+s} + R_{t+s-1} \widetilde{B}_{t+s} - T^e_{t+s},$$

where Ω^e_{t+s} denotes lump-sum profits from firms and τ^n_t denotes the time t tax rate on labor income.

The household has perfect foresight with respect to exogenous variables, including the discount rate shock, ξ_t. For now, we assume that the government announces sequences of nominal interest rates, $\{R_t\}$, government spending, $\{G_t\}$, and taxes $\{\tau^c_t, \tau^n_t\}$. Household beliefs for T^e_t satisfy:

$$\sum_{s \geq 0} Q_{t,t+s} T^e_{t+s} = \sum_{s \geq 0} Q_{t,t+s} \left[G_{t+s} - \tau^c_{t+s} P^e_{t+s} C^e_{t+s} - \tau^n_{t+s} W^e_{t+s} N^e_{t+s} \right] + R_{t-1} B_t. \tag{3.6}$$

Along with our other assumption, 3.6 implies that Ricardian equivalence holds in our model.

As shown in appendix C.1, the solution to the household's problem implies

$$C_t = \frac{\sum_{s \geq 0} Q_{t,t+s} \frac{P^e_{t+s}(1+\tau^c_{t+s})}{P_t(1+\tau_t)} \left[Y^e_{t+s} - G_{t+s} \right]}{1 + \sum_{s \geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t} \right)^{\sigma} \left[Q_{t,t+s} \frac{P^e_{t+s}(1+\tau^c_{t+s})}{P_t(1+\tau_t)} \right]^{1-\sigma}}. \tag{3.7}$$

Labor market and unions Wages are decided by unions. In the presence of sticky wages, actual employment is demand determined. Each household supplies $n_{u,t}$ units of type u labor to a union indexed by $u \in [0, 1]$. Union u faces labor demand given (3.3).

The union sets wages subject to Calvo-style frictions. At each date, $1 - \lambda$ unions are randomly selected to adjust their wage, $w_{u,t}$. For the other λ unions, $w_{u,t} = w_{u,t-1}$. Unions act on behalf of households and choose wages and labor hours to maximize the expected household's valuation of labor income.

In a symmetric equilibrium all unions that can reset their wage $w_{u,t}$ choose the same value. We denote that common new reset wage by W_t^*. In appendix C.2, we show that W_t^* satisfies

$$\frac{W_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(\frac{P_{t+s}^e}{P_t}\right)^{\theta} \left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta} N_{t+s}^e v' \left(L_{t+s}^e\right)}{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(\frac{P_{t+s}^e}{P_t}\right)^{\theta-1} \left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta} N_{t+s}^e u' \left(C_{t+s}^e\right) \frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}}. \tag{3.8}$$

The union has perfect foresight with respect to exogenous variables, but is boundedly rational with respect to endogenous variables. In particular, we assume that the union forms beliefs about future aggregate prices, P_t^e, wages, W_t^e,consumption C_t^e, the labor composite N_t^e and labor input, L_t^e using level-k thinking.

Monetary and fiscal policies Nominal interest rates during and after the ZLB period are as described in the benchmark model. The fiscal authority sets government spending G_t, consumption taxes τ_t^c, labor income taxes τ_t^n, and lump-sum taxes T_t, subject to the intertemporal budget constraint:

$$\sum_{s\geq 0} Q_{t,t+s} P_{t+s} G_{t+s} + R_{t-1} B_t = \sum_{s\geq 0} Q_{t,t+s} \left[\tau_{t+s}^c P_{t+s} C_{t+s} + \tau_{t+s}^n W_{t+s} N_{t+s} + T_{t+s}\right]. \tag{3.9}$$

Temporary Equilibrium As in Farhi and Werning (2019), we assume that peoples' beliefs regarding future nominal prices and wages are scaled by P_t/P_t^e. This assumption allows people to incorporate current and past surprise inflation into their beliefs, leaving beliefs about future inflation and real wages unchanged.

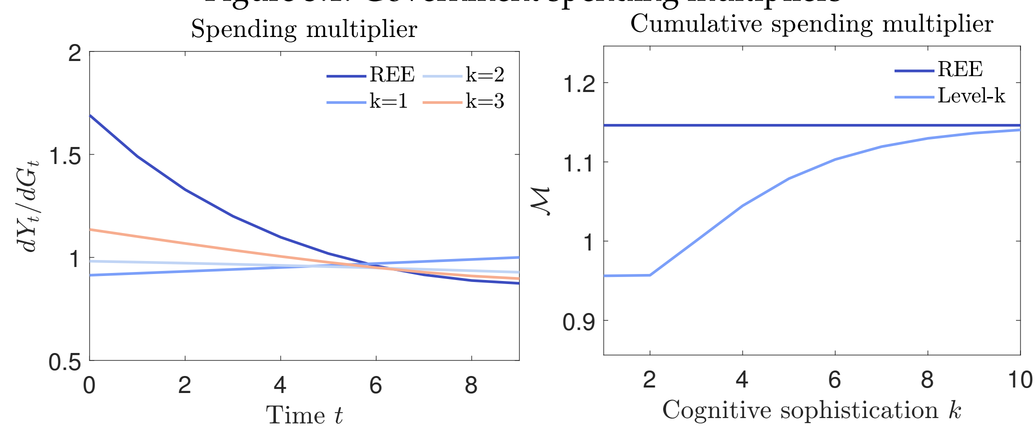
For each date t, given beliefs $\mathcal{A}_t^e = \left\{Y_t^e, C_t^e, N_t^e, L_t^e, P_t^e/P_{t-1}^e, W_t^e/P_t^e\right\}$, a temporary equilibrium is a sequence of allocations and prices $\mathcal{A}_t = \{Y_t, C_t, N_t, L_t, P_t/P_{t-1}, W_t/P_t\}$ in which households, firms and unions solve their optimization problem, and goods markets clear. In appendix C, we summarize the set of equations whose solution defines an equilibrium for this economy. In addition, we present the log-linearized system and show how to compute generalized level-k equilibria in which beliefs evolve analogously to those in equation (2.28).

Calibration As in section 2, we assume that the elasticity of intertemporal substitution is $\sigma = 0.5$, $\beta = 0.99$, $\chi = 0.02,$ and $G/Y = 0.2$. Consistent with the evidence in Chetty et al. (2011) we set the Frisch elasticity is $\varphi^{-1} = 0.75$. We normalize $\overline{K} = 1$ and set the capital share, α, to 0.33. In addition, we set total factor productivity, A, so that steady state output is equal to one. Following Correia et al. (2013), we assume that the elasticity of substitution across labor types θ is equal to 3, and the Calvo parameter λ is 0.85. We set the steady-state tax rates τ^c and τ^n equal to 0.05 and 0.28, respectively. Finally, as in section 2, we assume that level-1 beliefs are anchored at the initial steady state.

3.1 Government spending multipliers

In this section we briefly illustrate the analog to Proposition 1 for the case in which tax rates are constant and government spending rises by ΔG during the ZLB period.

Figure 3.1: Government spending multipliers



Panel A of Figure 3.1 displays the government spending multiplier, $\Delta Y_t/\Delta G_t$, computed under the assumption of rational expectations and for various levels of k. Under rational expectations, this multiplier is initially close to 1.5. Consistent with results in the NK literature, the large size of this multiplier reflects the fact that government spending induces inflation, which lowers the real interest rate during the ZLB period. Because of inter-temporal substitution effects, this fall induces households to raise their demand for consumption which raises output. Other things equal, perfectly rational agents understand that these inter-temporal substitution effects increase current and future output. In a virtuous cycle, the rise in future income raises peoples' permanent income which raises current spending and inflation. The latter effect lowers the real interest rate which further strengthens the inter-temporal substitution effect. The net effect is a sequence of large multipliers, exceeding one in value.[15]

[15]Note that in our example the multiplier for low k can exceed the rational expectation multiplier for large t. This feature reflects the fact that, with $\alpha > 0$, the marginal productivity of labor is decreasing less over time after a rise in government spending.

To assess the impact of level-k thinking, it is useful to define the cumulative spending multiplier as[16]

$$\mathcal{M} \equiv \frac{\sum_t \Delta Y_t}{\sum_t \Delta G_t} = \sum_t \frac{\Delta G_t}{\sum_t \Delta G_t} \frac{\Delta Y_t}{\Delta G_t}.$$

Panel B of Figure 3.1 shows that the cumulative multiplier increases with k. The intuition is as follows. The lower the cognitive level of individuals, the less they understand the general equilibrium effects of spending on total GDP and inflation. So lower level-k people predict a relatively small rise in their income and in inflation in response to a rise in government spending. The result is that the lower is k,the smaller is the rise in consumption induced by government spending. Indeed level-1 and level-2 people cut their spending because the tax effects of a rise in government spending outweigh the income effects. Finally note that consistent with the discussion in Farhi et al. (2020), the less "sticky" wages are (the smaller is λ) the larger is the reduction in the multiplier associated with a given level of k.

Taken together the results in this section reinforce the message from the benchmark model: bounded rationality weakens the case for the efficacy of government spending as a tool for stabilizing output in the face of a shock that causes the ZLB to bind.

3.2 Consumption tax policy

In this section, we consider the efficacy of tax policy in the extended version of our benchmark economy. Our key result is that Proposition (2) continues to hold so that tax policy can be used to support the flexible-price allocation even when prices and wages are not fully rigid.

Under rational expectations, the requisite tax policy sets consumption taxes according to

$$\tau_t^{c,*} = (1 + \tau^c)\, e^{-(T-t)(\chi - \rho)} - 1.$$

Recall that in the benchmark economy wages are fully rigid. Employment is entirely determined entirely by the demand for labor. In the extended model, consumption taxes, $\tau_t^{c,*}$, induce distortions in labor supply which affect the equilibrium because wages aren't perfectly rigid. In order to support the flexible-price allocation, the government must adjust labor taxes to undo these distortions:

$$\frac{1 - \tau_t^{n,*}}{1 + \tau_t^{c,*}} = \frac{1 - \tau^n}{1 + \tau^c}.$$

[16]Since the cumulative multiplier can be decomposed into a weighted sum of the time t multipliers, the results in Proposition 1 for the benchmark model also hold for the cumulative multiplier.

Under this policy, the tax wedge on labor supply is constant over time. Critically, the government announces its policy for $\tau_t^{c,*}$ and $\tau_t^{n,*}$ as a sequences of tax rate *targets*.

We now state the analog to Proposition (2) for the extended model.

Proposition 5. *Suppose that level-*1 *people believe that the economy goes back to steady state after the ZLB period, i.e.,* $\mathcal{A}_t^{e,1} = \mathcal{A} \equiv \{Y, C, N, L, 1, W/P\}$ *for* $t \geq T$*. Consider the log-linearized version of the model economy. Then,*

1. *For each k, there exists a policy* $\left\{\tau_t^{c,k}, \tau_t^{n,k}\right\}$ *which implements the flexible-price allocation.*
2. *Suppose that* $\mathcal{A}_t^{e,1} = \mathcal{A}$ *for all* $t \geq 0$*, then the policy* $\{\tau_t^{c,*}, \tau_t^{n,*}\}$ *implements the flexible-price allocation for all k.*

Here $\mathcal{A}_t^{e,k}$ denote the beliefs of level-k people. This proposition generalizes Proposition 2 to the extended model and demonstrates that tax policy is still very powerful even under bounded rationality in the presence of time-varying wages and prices.

3.2.1 Rules versus targets

In this section, we revisit the effectiveness of rules-based communication in the extended model. As in Correia et al. (2013), we assume that the interest rate is given by a Taylor rule subject to a ZLB constraint,

$$R_t = \max\left\{\beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} Y_t^{\phi_y}, 1\right\}. \tag{3.10}$$

Here ϕ_π is the coefficient on realized inflation and ϕ_y is the elasticity of the interest rate with respect to the output gap. The rule for consumption taxes and labor-income taxes is

$$\frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \min\left\{\beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} Y_t^{\phi_y}, 1\right\}, \tag{3.11}$$

and

$$\frac{1-\tau_t^n}{1+\tau_t^c} = \frac{1-\tau^n}{1+\tau^c}. \tag{3.12}$$

Critically, the government announces tax policy in form of the *rules*, (3.10)-(3.12).

Proposition 3 follows trivially for the extended model with $k = 1$ because everyone expects inflation to be zero and output to remain at its steady-state level. However, it is not possible, in general, to prove the analog proposition for $k > 1$. However we can show numerically that the basic results in that Proposition continue to hold. We follow Christiano et al. (2011) and set $\phi_\pi = 1.5$ and $\phi_y = 0.25$.

Figure 3.2 displays our results under rational expectations and level-k thinking. The (1,1) element of Figure 3.2 displays the shock to the subjective discount factor χ_t. The (1,2), (1,3) and (2,1) elements show the log deviation of output (y_t), consumption (c_t), and labor (n_t), from their steady state levels, respectively. Finally, the (2,2) and (2,3) elements show inflation, π_t, and the after-tax real interest rate, $r_t - \pi_{t+1} - \Delta\hat{\tau}^c_{t+1}$.

Recall that in the flexible price allocation, all quantities remain at that their pre-shock steady-state values. The solid blue lines depict the equilibrium under the rules-based monetary and fiscal policies (3.10)-(3.12). Correia et al. (2013) show that, under rational expectations, the proposed fiscal policy has a powerful stabilizing influence on the economy. For example, in our model economy, if tax rates are kept constant, the maximal drop in output exceeds seven percent. Under the proposed fiscal policy, the maximal decline in output would be roughly two percent (see Figure 3.2).

With level-k thinking, rules-based fiscal policy is much less powerful than under rational expectations. For example, when $k = 1$, the maximal decline in output is slightly over five percent. As k rises, the efficacy of rules-based fiscal policy rises as people are better able to understand the evolution of future tax rates. As k goes to infinity, the response of the model economy converges to the rational-expectations equilibrium.

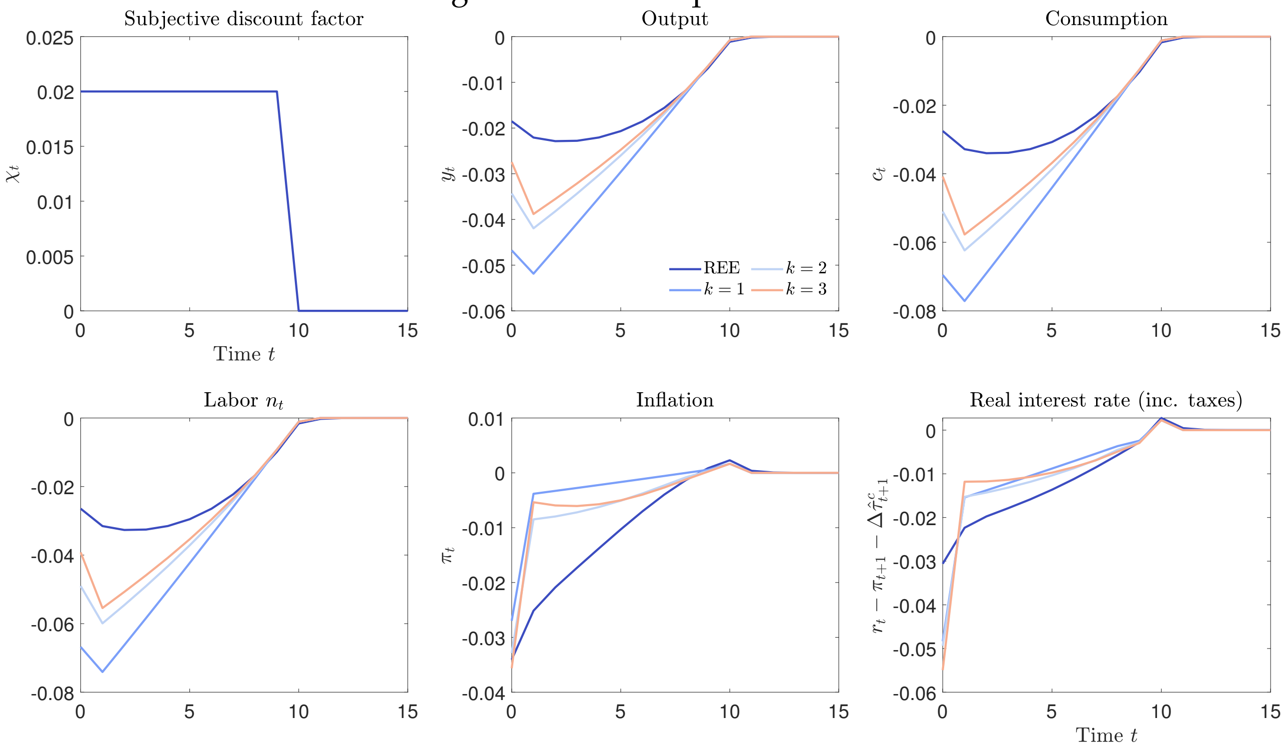
Taken together the results in this subsection reinforce the message from the benchmark model. When agents are level-k thinkers, target-based communication is more effective than rules-based communication in terms of its ability to stabilize economic activity when the ZLB is binding.

4 Conclusions

In this paper we address the question: how sensitive is the power of fiscal policy in the ZLB to the assumption of rational expectations? We do so through the lens of a standard NK model in which people are level-k thinkers, i.e, they go through k rounds of deductive reasoning about the economy.

Our analysis *weakens* the case for using government spending to stabilize the economy when the ZLB binds. The reason is that the efficacy of government spending is quite sensitive to how sophisticated people are. Using a variant of the standard NK model, we find that the less sophisticated people are, the smaller is the size of the government-spending multiplier. The basic reason is that the power of government spending depends on people's ability to compute and internalize the general-equilibrium effects of spending

Figure 3.2: Rules equilibrium



on their own incomes. The less sophisticated people are, the less they understand these general equilibrium effects, the more they cut their consumption and the more output falls during the ZLB period.

Our analysis *strengthens* the case for using tax policy to stabilize output when the ZLB is binding. Correia et al. (2013) argue that tax policy is a powerful way to stabilize the economy when the ZLB binds and people have rational expectations. We show that the power of tax policy during the ZLB period is essentially undiminished when agents do not have rational expectations. Indeed, even when people have low levels of sophistication, it is always possible to achieve the flexible-price allocation during a binding ZLB period. Suppose that the least sophisticated people think that the economy will remain at its pre-shock level. Then, the path for consumption taxes that supports the flexible-price allocation is the same regardless of how cognitively sophisticated people are. Critically, under this tax policy, peoples' initial beliefs are self-confirming, so that the efficacy of the policy doesn't exploit peoples' lack of sophistication. Taken together these results show that tax policy for stabilizing the economy when the ZLB binds is powerful *and* robust to how sophisticated people are.

The basic intuition is that, in contrast to government spending, well-communicated tax policy immediately influences people's decisions. People do not have to understand the equilibrium consequences of tax rate changes for those changes to support the flexible-price allocation.

Finally, we show that when people have limited cognitive abilities, how tax policy is communicated becomes critical to its effectiveness. Tax policy is more effective when it is communicated as a sequence of tax rates, as opposed to a rule involving equilibrium objects like the output gap. The reason is simple: when policy is communicated as a sequence of tax rates, people can immediately incorporate those rates into their decisions. When policy is communicated via a tax rule, people must deduce the implications of the rule for the variables that they care about, like consumption tax rates. In our model, unsophisticated people typically underestimate how stimulative future policy will be, so that tax policy will be less powerful at stabilizing output. We conclude that in a world where people have less than fully rational expectations, communication matters.

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A Appendix to section 2

A.1 Proof of proposition 1

We can solve for the government spending multiplier using

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t},$$

where the level-1 government spending multiplier is given by

$$\frac{\Delta Y_t^1}{\Delta G_t} = 1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t}.$$

Note that since $\Delta G_{t+s}/\Delta G_t > 0$, then $\Delta Y_t^1/\Delta G_t \leq 1$ for all t. By induction, suppose that $\Delta Y_t^{k-1}/\Delta G_t \leq 1$ for all t, then

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\underbrace{\frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}} - 1}_{\leq 0} \right] \frac{\Delta G_{t+s}}{\Delta G_t} \leq 1,$$

for all t. The first result follows.

Furthermore, if $1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \geq 0$ for all t, then $\Delta Y_t^1/\Delta G_t > 0$ for all t. Note that, with this assumption,

$$\frac{\Delta Y_t^2}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y_{t+s}^1}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \geq 1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^1}{\Delta G_t}.$$

By induction, suppose that $\Delta Y_t^k/\Delta G_t \geq \Delta Y_t^{k-1}/\Delta G_t$, then

$$\frac{\Delta Y_t^{k+1}}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y_t^k}{\Delta G_t} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \geq 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y_t^{k-1}}{\Delta G_t} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^k}{\Delta G_t}.$$

Then the second result follows.

Finally, suppose that $\Delta G_t = \gamma^t \Delta G_0$, then

$$\frac{\Delta Y_t^1}{\Delta G_t} = 1 - \Omega_t \sum_{s=1}^{T-t-1} \gamma^s = 1 - \Omega_t \gamma \frac{1 - \gamma^{T-t-1}}{1 - \gamma},$$

and note that

$$\frac{d\Delta Y_t^1/\Delta G_t}{d\gamma} = -\Omega_t \sum_{s=1}^{T-t-1} s\gamma^{s-1} \leq 0$$

with strict inequality if $t < T-1$. Furthermore, note that

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}} - 1\right]\gamma^s,$$

so,

$$\frac{d\Delta Y_t^k/\Delta G_t}{d\gamma} = \Omega_t \sum_{s=1}^{T-t-1} \left[\underbrace{\frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}} - 1}_{\leq 0}\right] s\gamma^{s-1} + \Omega_t \sum_{s=1}^{T-t-1} \frac{d\Delta Y_{t+s}^{k-1}/\Delta G_{t+s}}{d\gamma}\gamma^s.$$

This implies that if $\frac{d\Delta Y_t^{k-1}/\Delta G_t}{d\gamma} \leq 0$ for all t, then $\frac{d\Delta Y_t^k/\Delta G_t}{d\gamma} \leq 0$ for all t. The third results follows.

A.2 Proof of proposition 2

(1) As we show in the main text, for any level of cognitive sophistication, setting

$$1 + \tau_{T-1} = (1+\tau)\, e^{-(\chi-\rho)} \tag{A.1}$$

implements $Y_{T-1}^k = 1$ for all k. Note that for any t and k, the equilibrium level of output at time t is a function only of current and future consumption taxes plus beliefs about future output:

$$Y_t = \left(\frac{1+\tau^c}{1+\tau_t^c}\right)^\sigma \frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^c}{1+\tau^c}\right) Y_{t+s}^e + 1}{(1-\beta)\sum_{s=1}^{T-t-1} e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma} + e^{(T-t)\sigma(\chi-\rho)}}.$$

As a result, for any cognitive level k, $Y_{t+s}^{e,k}$ is independent of τ_t. This means that, for a fixed k, we can construct the policy as follows.

Set τ_{T-1} to the value implied by (A.1). Then, proceed recursively from that date. For each $t \leq T-2$, fix τ_{t+s} for $s \geq 1$. These imply a path for Y_{t+s}^{k-1} for $s \geq 1$. Let us choose τ_t so that

$$\left(\frac{1+\tau}{1+\tau_t}\right)^\sigma \frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}}{1+\tau}\right) Y_{t+s}^{e,k} + 1}{(1-\beta)\sum_{s=1}^{T-t-1} e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}}{1+\tau}\right]^{1-\sigma} + e^{(T-t)\sigma(\chi-\rho)}} = 1$$

or, equivalently,

$$1+\tau_t = (1+\tau)\left(\frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}}{1+\tau}\right)Y_{t+s}^{e,k}+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}}{1+\tau}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}\right)^{1/\sigma}.$$

This implies that

$$Y_t^k = 1$$

for all t.

(2) Suppose that $Y_t^{e,1} = 1$. Then,

$$Y_t^1 = \left(\frac{1+\tau^c}{1+\tau_t^{c,*}}\right)^{\sigma}\frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^{c,*}}{1+\tau^c}\right)Y_{t+s}^{e}+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^{c,*}}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}} = 1.$$

This implies that $Y_t^{e,k} = Y^{k-1} = 1$ for all k, and then $Y_t^k = 1$ for all t and k.

A.3 Rules-based equilibrium

Under a rules-based policy, the temporary equilibrium is given by

$$\mathcal{Y}_t\left(\{Y_{t+s}^e\}\right) = \frac{\sum_{s=1}^{T-t-1}Q_{t,t+s}^e\left(\frac{1+\tau_{t+s}^{c,e}}{1+\tau_t^{c,e}}\right)Y_{t+s}^e+\sum_{s=T-t}^{\infty}Q_{t,t+s}^e\left(\frac{1+\tau_{t+s}^{c,e}}{1+\tau_t^{c,e}}\right)}{\sum_{s=1}^{T-t-1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}^e\frac{1+\tau_{t+s}^{c,e}}{1+\tau_t^{c,e}}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}\sum_{s=T-t}^{\infty}\beta^{\sigma(s-(T-t))}\left[Q_{t,t+s}^e\frac{1+\tau_{t+s}^{c,e}}{1+\tau_t^{c,e}}\right]^{1-\sigma}}$$

where

$$Q_{t,t+s}^e\frac{1+\tau_{t+s}^{c,e}}{1+\tau_t^{c,e}} = \begin{cases}\beta^s\prod_{\tau=t}^{t+s-1}\left(Y_\tau^e\right)^{-\phi_y} & \text{if } s\le T-t-1\\ \beta^s\prod_{\tau=t}^{T-1}\left(Y_\tau^e\right)^{-\phi_y} & \text{if } s\ge T-t.\end{cases}$$

Assuming that $Y_t^{e,1} = 1$, implies that

$$Q_{t,t+s}^e\frac{1+\tau_{t+s}^{c,e}}{1+\tau_t^{c,e}} = \beta^s$$

for all t and s. This implies that,

$$\mathcal{Y}_t\left(\left\{Y_{t+s}^{e,1}\right\}\right) = \frac{e^{-\frac{\sigma\chi}{1+\sigma\phi_y}}}{\left[(1-\beta)\frac{1-e^{(T-t-1)(\sigma\chi-\rho)}}{1-e^{\sigma\chi-\rho}}+e^{(T-t-1)(\sigma\chi-\rho)}\right]^{\frac{1}{1+\sigma\phi_y}}},$$

or in logs:

$$y_t^1 \equiv \log Y_t^1 = -\frac{\sigma\chi + \log\left((1-\beta)\frac{1-e^{(T-t-1)(\sigma\chi-\rho)}}{1-e^{\sigma\chi-\rho}} + e^{(T-t-1)(\sigma\chi-\rho)}\right)}{1+\sigma\phi_y}.$$

A.4 Proof of proposition 3

Targets-based policy Note that, under rational expectations, the targets based policy with $\{\tau_t^{c,r}\}$ implements the same equilibrium

$$y_t^* = -\frac{\chi}{\phi_y}\left[1 - \frac{1}{(1+\sigma\phi_y)^{T-t}}\right] < 0.$$

Now, suppose that the government announces the sequence of policies $R_t = 1$ for $t \leq T-1$, $R_t = \beta^{-1}$ for $t \geq T$, and $\{\tau_t^{c,r}\}$. Then, the level-1 equilibrium is given by

$$y_t^1 = \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}} e^{y_{t+s}^{e,1}}}{\sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\},$$

where $y_t^{e,1} \equiv \log Y_t^{e,1}$. Since $y_t^{e,1} = 0 \geq y_t^*$, then

$$y_t^1 \geq \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}} e^{y_{t+s}^{*}}}{\sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} = y_t^*.$$

Rules-based policy The basic proof is constructed as follows. First, we note that if $\chi = 0$, then $y_t^* = y_t^1 = 0$. Second, we show that both y_t^* and y_t^1 are decreasing in χ. Third, y_t^* is linear in χ, while y_t^1 is concave in χ. Fourth, we show that $dy_t^1/d\chi < dy_t^*/d\chi$ as long as $\beta \geq (1+\sigma\phi_y)^{-1}$. The collection of these results finally implies that

$$y_t^1 \leq \frac{dy_t^1}{d\chi}|_{\chi=0}\cdot\chi \leq \frac{dy_t^*}{d\chi}|_{\chi=0}\cdot\chi = y_t^*.$$

Log-output under rational expectations in the rules equilibrium is given by:

$$y_t^* = -\frac{\chi}{\phi_y}\left[1 - \frac{1}{(1+\sigma\phi_y)^{T-t}}\right],$$

and the level-1 equilibrium is given by:

$$y_t^1 = -\frac{\sigma\chi + \log\left((1-\beta)\frac{1-e^{(T-t-1)(\sigma\chi-\rho)}}{1-e^{\sigma\chi-\rho}} + e^{(T-t-1)(\sigma\chi-\rho)}\right)}{1+\sigma\phi_y}.$$

(1) For any t, if the shock is zero then output stays at steady state, i.e., if $\chi = 0$, then using the expressions above it is clear that

$$y_t^* = y_t^1 = 0.$$

(2) Furthermore, the effects of χ on y_t^* and y_t^1 are given by

$$\frac{dy_t^*}{d\chi} = -\frac{1}{\phi_y}\left[1 - \frac{1}{\left(1+\sigma\phi_y\right)^{T-t}}\right] < 0,$$

and since $\frac{1-\left(e^{\sigma\chi-\rho}\right)^{T-t-1}}{1-e^{\sigma\chi-\rho}} = \sum_{s=0}^{T-t-2} e^{s(\sigma\chi-\rho)}$, we can write

$$\frac{dy_t^1}{d\chi} = -\frac{\sigma}{1+\sigma\phi_y}\left[1 + \frac{(1-\beta)\sum_{s=0}^{T-t-2} s e^{s(\sigma\chi-\rho)} + (T-t-1)\left(e^{\sigma\chi-\rho}\right)^{T-t-1}}{(1-\beta)\sum_{s=0}^{T-t-2} e^{s(\sigma\chi-\rho)} + \left(e^{\sigma\chi-\rho}\right)^{T-t-1}}\right] < 0. \quad \text{(A.2)}$$

(3) The rational-expectations equilibrium in this economy is exactly log-linear as a function of the shock, which implies that

$$y_t^* = \left\{\frac{dy_t^*}{d\chi}|_{\chi=0}\right\}\cdot\chi.$$

However, the same is not true under bounded rationality. To show this note that, for $t \leq T-2$,

$$\frac{d^2 y_t^1}{d\chi^2} = -\frac{\sigma^2}{1+\sigma\phi_y}\left[\frac{(1-\beta)\sum_{s=0}^{T-t-2} s^2 e^{s(\sigma\chi-\rho)} + (T-t-1)^2\left(e^{\sigma\chi-\rho}\right)^{T-t-1}}{\overline{\mu}_t}\right]$$
$$+\frac{\sigma}{1+\sigma\phi_y}\frac{\left\{(1-\beta)\sum_{s=0}^{T-t-2} s e^{s(\sigma\chi-\rho)} + (T-t-1)\left(e^{\sigma\chi-\rho}\right)^{T-t-1}\right\}^2}{\overline{\mu}_t^2}$$

where $\overline{\mu}_t \equiv (1-\beta)\sum_{s=0}^{T-t-2} e^{s(\sigma\chi-\rho)} + \left(e^{\sigma\chi-\rho}\right)^{T-t-1} > 0$. Define $\mu_{t,s} \equiv \frac{(1-\beta)e^{s(\sigma\chi-\rho)}}{\overline{\mu}_t}$ if $s < T-t-1$ and $\mu_{t,T-t-1} \equiv \frac{e^{(T-t-1)(\sigma\chi-\rho)}}{\overline{\mu}_t}$, and note that: $\mu_{t,s} > 0$, $\sum_{s=0}^{T-t-1}\mu_{t,s} = 1$. Using these definitions, we can rewrite the derivative as follows:

$$\frac{d^2 y_t^1}{d\chi^2} = -\frac{\sigma^2}{1+\sigma\phi_y}\left\{\sum_{s=0}^{T-t-1}\mu_{t,s}s^2 - \left(\sum_{s=0}^{T-t-1}\mu_{t,s}s^2\right)^2\right\} = -\frac{\sigma^2}{1+\sigma\phi_y}\sum_{s=0}^{T-t-1}\mu_{t,s}\left(s - \sum_{s=0}^{T-t-1}\mu_{t,s}s\right)^2 < 0.$$

This shows that log-output in the level-1 equilibrium is concave in χ.

(4) Evaluating (A.2) at $\chi = 0$ we obtain:

$$\frac{dy_t^1}{d\chi}|_{\chi=0} = -\frac{\sigma}{1+\sigma\phi_y} - \frac{\sigma}{1+\sigma\phi_y}\left[(1-\beta)\sum_{s=0}^{T-t-2}se^{-s\rho} + (T-t-1)\,\beta^{T-t-1}\right].$$

We want to show that $\frac{dy_t^1}{d\chi}|_{\chi=0} < \frac{dy_t^*}{d\chi}|_{\chi=0}$, which is equivalent

$$-\frac{\sigma}{1+\sigma\phi_y}\left[1+(1-\beta)\sum_{s=0}^{T-t-2}se^{-s\rho} + (T-t-1)\,\beta^{T-t-1}\right] \leq -\frac{\sigma}{1+\sigma\phi_y}\left[1+\frac{\sum_{s=0}^{T-t-2}(1+\sigma\phi_y)^{-s}}{1+\sigma\phi_y}\right]$$

$$\Leftrightarrow \left[(1-\beta)\sum_{s=0}^{T-t-2}se^{-s\rho} + (T-t-1)\,\beta^{T-t-1}\right] \geq \frac{\sum_{s=0}^{T-t-2}(1+\sigma\phi_y)^{-s}}{1+\sigma\phi_y}$$

Define

$$\Delta_t \equiv \left[(1-\beta)\sum_{s=0}^{T-t-2}se^{-s\rho} + (T-t-1)\,\beta^{T-t-1}\right] - \frac{\sum_{s=0}^{T-t-2}(1+\sigma\phi_y)^{-s}}{1+\sigma\phi_y}.$$

The desired inequality follows if $\Delta_t \geq 0$. First, let us note that this is true for $t = T-1$ because:

$$\Delta_{T-2} = \beta - \frac{1}{1+\sigma\phi_y} \geq 0,$$

by assumption. Then, for any $t \leq T-2$ note that:

$$\begin{aligned}\Delta_{t-1} - \Delta_t = &\left[(1-\beta)\sum_{s=0}^{T-t-1}se^{-s\rho} + (T-t)\,\beta^{T-t}\right] - \frac{\sum_{s=0}^{T-t-1}(1+\sigma\phi_y)^{-s}}{1+\sigma\phi_y} \\ &- \left[(1-\beta)\sum_{s=0}^{T-t-2}se^{-s\rho} + (T-t-1)\,\beta^{T-t-1}\right] + \frac{\sum_{s=0}^{T-t-2}(1+\sigma\phi_y)^{-s}}{1+\sigma\phi_y}\end{aligned}$$

$$\Leftrightarrow \Delta_{t-1} - \Delta_t = (1-\beta)(T-t-1)\beta^{(T-t-1)} + (T-t)\beta^{T-t} - (T-t-1)\beta^{T-t-1} - \frac{(1+\sigma\phi_y)^{-(T-t-1)}}{1+\sigma\phi_y}$$

$$\Leftrightarrow \Delta_{t-1} - \Delta_t = \beta^{T-t} - (1+\sigma\phi_y)^{-(T-t)}.$$

This implies that, under the same assumption, $\Delta_{t-1} \geq \Delta_t$. Since $\Delta_{T-2} \geq 0$, it follows that $\Delta_t \geq \Delta_{T-2} \geq 0$ for all t and the result follows. In addition, this logic also delivers the fact that $y_t^* - y_t^1$ decreases with t and increases with χ.

A.5 Proof of proposition 4

As described above, for level-1 we find that $y_t^1 \geq y_t^*$. Furthermore,

$$y_t^1 = \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} \leq 0.$$

Since $y_t^{e,k} = y_t^{k-1}$ and

$$y_t^k = \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,k}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\},$$

then,

$$y_t^2 = \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,2}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} \leq \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} = y_t^1$$

with strict inequality if $t \leq T-2$. Also, because $y_t^{e,2} \geq y_t^*$ then

$$y_t^2 = \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,2}}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} \geq \log\left\{\frac{\sum_{s\geq 1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^*}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} = y_t^*.$$

This shows that $y_t^2 \in \left[y_t^*, y_t^1\right]$, and $y_t^2 < y_t^1$ if $y_t^1 \neq y_t^*$,i.e., if $t \leq T-2$.

For each k, suppose that $y_t^{e,k} = y_t^{k-1} \in \left[y_t^*, y_t^{e,k-1}\right]$, with $y_t^{k-1} < y_t^{e,k-1}$ if $y_t^{e.k-1} \neq y_t^*$.

Then,

$$y_t^k = \log\left\{\frac{\sum_{s\geq1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,k}}}{\sum_{s\geq1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^\sigma\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} \leq \log\left\{\frac{\sum_{s\geq1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,k-1}}}{\sum_{s\geq1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^\sigma\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} = y_t^{k-1},$$

with strict inequality if $y_{t+s}^{e,k} \neq y_{t+s}^*$ for some $s \geq 1$. Also,

$$y_t^k = \log\left\{\frac{\sum_{s\geq1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^{e,k}}}{\sum_{s\geq1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^\sigma\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} \geq \log\left\{\frac{\sum_{s\geq1} Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}e^{y_{t+s}^*}}{\sum_{s\geq1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^\sigma\left[Q_{t,t+s}\frac{1+\tau_{t+s}^{c,r}}{1+\tau_t^{c,r}}\right]^{1-\sigma}}\right\} = y_t^*.$$

This shows that y_t^k forms a decreasing sequence in k, $y_t^k \leq y_t^{k-1}$, and $y_t^k \to y_t^*$ as $k \to \infty$.

B Bounded rationality – alternative models

In the benchmark model, we assume that people are standard level-k thinkers. However, our results do not depend crucially on the specific assumptions underlying this model of bounded rationality. In this appendix, we show that the main results of our model continue to hold under alternative models of bounded rationality. We first derive the benchmark model under a *generalized level-k thinking* model based on Camerer et al. (2004). Second, we show that our results are also robust to assuming that people have *reflective expectations* as in García-Schmidt and Woodford (2019). Finally, we also show that our results hold under the *shallow reasoning* model of Angeletos and Sastry (2020). For simplicity, we show this for the benchmark model without inflation, but these same principles hold more generally.

B.1 Generalized level-k thinking

In this section, we show that our results for the standard level-k thinking in the benchmark model go through in the generalized level-k thinking model. We restrict our analysis to the case in which policies are announced as targets, since we already discuss the implications of this model under rules in the main text.

While in standard level-k thinking, an individual with ability k believes that everyone else is level $k-1$, the generalized model allows individuals to conjecture that the population is distributed across all lower cognitive levels. Formally, we assume that individuals with ability k believe that a fraction $f_k(j)$ of the population is level $j = 0, 1, ..., k-1$. The reasoning process is initialized with some equilibrium if the economy is populated by level-0 agents, Y_t^0. For technical reasons, it is useful to define the beliefs $\left\{Y_t^{e,0}\right\}$ which justify $Y_t^0 = \mathcal{Y}_t\left(\left\{Y_{t+s}^{e,0}\right\}_{s\geq 1}\right)$ for all t.

Level-1 agents believe that everyone is level 0, i.e., $f_1(0) = 1$, and so they believe that output is given by:

$$Y_t^{e,1} = Y_t^0.$$

The equilibrium in an economy where all individuals are level-1 is given by

$$Y_t^1 = \mathcal{Y}_t\left(\left\{Y_{t+s}^{e,1}\right\}_{s\geq 1}\right).$$

Level-2 people believe that a fraction $f_2(0)$ and $f_2(1)$ are level 0 and 1, respectively. Under the assumptions discussed in section 2.2, we can write their beliefs as

$$Y_t^{e,2} = \sum_{j=0}^{1} f_2(j)\, Y_t^j.$$

More generally, the level-k beliefs can be constructed recursively

$$Y_t^{e,k} = \sum_{j=0}^{1} f_2(j)\, Y_t^k.$$

We assume that agents of different cognitive levels agree on the relative proportions of lower cognitive levels. Let $\gamma_k \equiv f_k(k-1)$ for all k. Then assumption (2.27) implies that $f_k(j) = (1-\gamma_k) f_{k-1}(j)$ for $j \leq k-2$. We can write the expectation of level-k individuals as follows:

$$Y_t^{e,k} = (1-\gamma_k)\, Y_t^{e,k-1} + \gamma_k Y_t^{k-1}. \tag{B.1}$$

Intuitively, the beliefs of a level-k thinker are given by a weighted average of the beliefs of level $k-1$ agents and the temporary equilibrium that would arise under those beliefs. Standard level-k thinking corresponds to the case of $\gamma_k = 1$. By varying γ_k, we can control the intensity of learning across level-k iterations.

While the standard level-k thinking model assumes that everyone is level k, the generalized level-k thinking model also allows for heterogeneity cognitive abilities. We let $f(k)$ for $k = 0, 1, ...$ denote the share of individuals who are level k in the economy. The observed equilibrium path is thus given by

$$Y_t = \sum_{k=0}^{\infty} f(k)\, Y_t^k. \tag{B.2}$$

B.1.1 Government spending multipliers

We continue to define the level-k multiplier as $\Delta Y_t^k / \Delta G_t$ which is given by

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y_{t+s}^{e,k}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t},$$

where

$$\frac{\Delta Y_{t+s}^{e,k}}{\Delta G_{t+s}} = (1-\gamma_k) \frac{\Delta Y_{t+s}^{e,k-1}}{\Delta G_{t+s}} + \gamma_k \frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}}$$

for $k \geq 2$, and given $\Delta Y_{t+s}^{e,1} / \Delta G_{t+s} = \Delta Y_{t+s}^{0} / \Delta G_{t+s} = 0$. The observed government spending multiplier is given by:

$$\frac{\Delta Y_t}{\Delta G_t} = \sum_{k=0}^{\infty} f(k) \frac{\Delta Y_t^k}{\Delta G_t}.$$

Suppose that $\Delta Y_t^{e,1} / \Delta G_t = 0$, this implies that

$$\frac{\Delta Y_t^1}{\Delta G_t} = 1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \leq 1.$$

As a result, $\Delta Y_t^{e,2} / \Delta G_t \leq 1$. For any k, if $\Delta Y_t^{e,k} / \Delta G_t \leq 1$ then $\Delta Y_t^k / \Delta G_t \leq 1$, which implies that $\Delta Y_t^{e,k+1} / \Delta G_t \leq 1$. As a result, for any $f(k)$,

$$\frac{\Delta Y_t}{\Delta G_t} = \sum_{k=0}^{\infty} f(k) \frac{\Delta Y_t^k}{\Delta G_t} \leq 1.$$

Suppose that $1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} > 0$, then $\Delta Y_t^1 / \Delta G_t > 0$ and $\Delta Y_t^{e,2} / \Delta G_t > \Delta Y_t^{e,1} / \Delta G_t = 0$. This immediately implies that $\Delta Y_t^2 / \Delta G_t > \Delta Y_t^1 / \Delta G_t$.

We now show that $\Delta Y_t^{e,k} / \Delta G_t$ and $\Delta Y_t^k / \Delta G_t$ are increasing in k. To see this, suppose that $\Delta Y_t^j / \Delta G_t > \Delta Y_t^{j-1} / \Delta G_t$ for all $j \leq k$ then this implies that $\Delta Y_t^{e,k+1} / \Delta G_t > \Delta Y_t^{e,k} / \Delta G_t$.

Furthermore,

$$\frac{\Delta Y_t^{k+1}}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y_{t+s}^{e,k+1}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \geq 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y_{t+s}^{e,k}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^k}{\Delta G_t}.$$

This shows that $\Delta Y_t^k / \Delta G_t$ is increasing in individual cognitive ability k. But the equilibrium spending multiplier depends on the full distribution $f(k)$. The analog statement to proposition 1 requires assumptions on the distribution $f(k)$. When comparing to economies, we say that one economy is strictly more sophisticated than another if its distribution of cognitive abilities first-order dominates the distribution of the second one. Formally, consider two economies with distributions $f^A(k)$ and $f^B(k)$. Suppose that $\sum_{s=0}^{k} f^A(s) \leq \sum_{s=0}^{k} f^B(s)$ for all k. Then, the government spending multiplier is higher in economy B than economy A.

Finally, if $\Delta G_t = \zeta^t \Delta G_0$ then, for any k,

$$\frac{\Delta Y_{T-2}^k}{\Delta G_{T-2}} = 1 + \Omega_t \left[\frac{\Delta Y_{T-1}^{e,k}}{\Delta G_{T-1}} - 1 \right] \zeta \Rightarrow \frac{d \frac{\Delta Y_{T-2}^k}{\Delta G_{T-2}}}{d\zeta} = \Omega_t \left[\frac{\Delta Y_{T-1}^{e,k}}{\Delta G_{T-1}} - 1 \right] \leq 0,$$

and for $k = 1$,

$$\frac{d \frac{\Delta Y_t^1}{\Delta G_t}}{d\zeta} \leq 0, \quad \forall t.$$

We now show that $d \frac{\Delta Y_t^k}{\Delta G_t} / d\zeta \leq 0$ for all t. Suppose that $d \frac{\Delta Y_{t+s}^k}{\Delta G_{t+s}} / d\zeta \leq 0$ for all $s \geq 1$ and $d \frac{\Delta Y_t^j}{\Delta G_t} / d\zeta \leq 0$ for all $j \leq k$, then

$$d \frac{\Delta Y_t^{e,k}}{\Delta G_t} / d\zeta \leq 0$$

for all t, which implies that

$$d \frac{\Delta Y_t^k}{\Delta G_t} / d\zeta = \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y_{t+s}^{e,k}}{\Delta G_{t+s}} - 1 \right] s \zeta^{s-1} + \Omega_t \sum_{s=1}^{T-t-1} \frac{d \frac{\Delta Y_{t+s}^{e,k}}{\Delta G_{t+s}}}{d\zeta} \zeta^s \leq 0.$$

Since this property holds for all k, then it holds for the observed spending multiplier for any $f(k)$.

B.1.2 Consumption tax policy

The equilibrium in this economy is given by

$$Y_t = \left(\frac{1+\tau^c}{1+\tau_t^c}\right)^\sigma \frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^c}{1+\tau^c}\right)\sum_{k=0}^{\infty} f(k)\, Y_{t+s}^{e,k} + 1}{(1-\beta)\sum_{s=1}^{T-t-1} e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma} + e^{(T-t)\sigma(\chi-\rho)}}.$$

As before, beliefs about future output $Y_{t+s}^{e,k}$ for any k is only a function of future tax policy, which implies that the analog construction of tax policy τ_t^c implements $Y_t = 1$. Note, however, that this policy may now imply consumption heterogeneity across different cognitive levels, because they may have different beliefs about future output. As it turns out, this is not the case if $Y_t^{e,1} = 1$. We show this next.

Suppose now that $Y_t^{e,1} = 1$. Then, announcing the tax policy $\tau_t^{c,*}$ implies that $Y_t^1 = 1$. It then follows that $Y_t^{e,k} = Y_t^k = 1$ for all k. As a result,

$$Y_t = 1$$

for any $f(k)$. This shows that proposition 2 continues to hold.

B.2 Reflective expectations

García-Schmidt and Woodford (2019) describe a different process of belief formation which they call *reflective expectations.* This process allows cognitive ability to vary continuously but is otherwise similar in spirit to level k. Indexing beliefs by the cognitive ability n, García-Schmidt and Woodford (2019) assume that beliefs evolve according to

$$\frac{dY_t^{e,n}}{dn} = Y_t^n - Y_t^{e,n},$$

for $n \geq 0$ and starting from the initial expectations $Y_t^{e,0}$, where Y_t^n denotes the equilibrium in an economy with level-n people.We use supercript k to denote equilibria and beliefs under level-k thinking and superscript n to denote equilibria and beliefs under reflective expectations.

García-Schmidt and Woodford (2019) show that the beliefs of a level-n individual with reflective expectations are equivalent to a convex combination of standard level-k beliefs determined by a Poisson distribution with mean n, i.e.,

$$Y_t^{e,n} = \sum_{k=1}^{\infty} \frac{n^{k-1}e^{-n}}{(k-1)!} Y_t^{e,k}, \tag{B.3}$$

where $Y_t^{e,k}$ denote the beliefs that standard level-k thinkers have, which we develop in section 2. Equation (B.3) can be used to analyze the relationship between the equilibrium properties of standard level-k thinking and reflective expectations economies.

B.2.1 Government spending multipliers

For the case of the government spending multiplier, the beliefs of a level n individual can be computed from the beliefs under level-k thinking as follows:

$$\frac{\Delta Y_t^{e,n}}{\Delta G_t} = \sum_{k=1}^{\infty} \frac{n^{k-1}e^{-n}}{(k-1)!} \frac{\Delta Y_t^{e,k}}{\Delta G_t}.$$

Since $\Delta Y_t^k / \Delta G_t \leq 1$ for all k, then $\Delta Y_t^{e,n} / \Delta G_t \leq 1$ for all n. Also, since the level-k multiplier increases with k, then so does the level-n belief over the multiplier. In addition, since $\Delta Y_t^k / \Delta G_t$ is decreasing in the persistence parameter γ, then $\Delta Y_t^{e,n} / \Delta G_t$ is decreasing in γ as well.

The equilibrium spending multiplier under reflective expectations is given by:

$$\frac{\Delta Y_t^n}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta Y_{t+s}^{e,n}}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t}.$$

This relationship follows directly from Lemma 1. Again, since $\Delta Y_t^{e,n} / \Delta G_t \leq 1$ for all t, then $\Delta Y_t^n / \Delta G_t \leq 1$ for all t. Also, since the $\Delta Y_t^{e,n} / \Delta G_t$ is increasing with n, so to is $\Delta Y_t^n / \Delta G_t$. In addition, since $\Delta Y_t^{e,n} / \Delta G_t$ is decreasing in the persistence parameter γ for all t, then $\Delta Y_t^n / \Delta G_t$ is decreasing in γ as well.

B.2.2 Consumption tax policy

The temporary equilibrium with reflective expectations is given by:

$$Y_t^n = \left(\frac{1+\tau^c}{1+\tau_t^c} \right)^{\sigma} \frac{(1-\beta) \sum_{s=1}^{T-t-1} \left(\frac{1+\tau_{t+s}^c}{1+\tau^c} \right) Y_{t+s}^{e,n} + 1}{(1-\beta) \sum_{s=1}^{T-t-1} e^{\sigma(\chi-\rho)s} \left[\frac{1+\tau_{t+s}^c}{1+\tau^c} \right]^{1-\sigma} + e^{(T-t)\sigma(\chi-\rho)}},$$

where

$$\frac{dY_t^{e,n}}{dn} = Y_t^n - Y_t^{e,n}.$$

As it turns out, the results of Proposition 2 extend to the model with reflective expectations. We prove this result below.

Set τ_{T-1}^c to the value implied by (A.1). Then, proceed recursively from that date. For each $t \leq T-2$, fix τ_{t+s}^c for $s \geq 1$. These imply a path for $Y_{t+s}^{e,n}$ for $s \geq 1$. Let us choose τ_t^c so that

$$\left(\frac{1+\tau^c}{1+\tau_t^c}\right)^\sigma \frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^c}{1+\tau^c}\right)Y_{t+s}^{e,n}+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}} = 1$$

or, equivalently,

$$1+\tau_t^c = (1+\tau^c)\left(\frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^c}{1+\tau^c}\right)Y_{t+s}^{e,k}+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}\right)^{1/\sigma}.$$

This implies that

$$Y_t^n = 1$$

for all t.

Suppose that $Y_t^{e,0} = 1$ and

$$\tau_t^c = \tau_t^{c,*} = (1+\tau^c)\,e^{-(T-t)(\chi-\rho)} - 1.$$

Then,

$$Y_t^0 = \left(\frac{1+\tau^c}{1+\tau_t^{c,*}}\right)^\sigma \frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^{c,*}}{1+\tau^c}\right)+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}} = 1$$

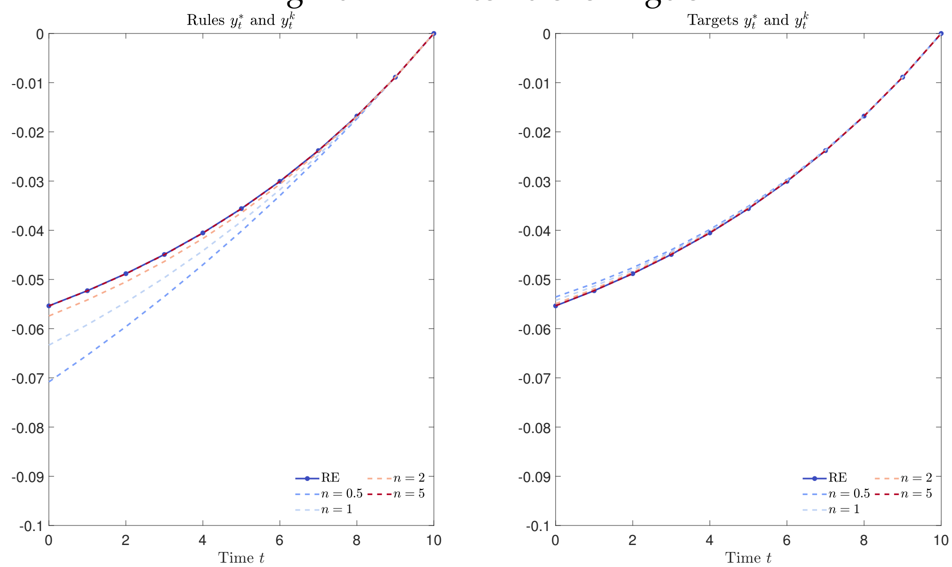
and

$$\frac{dY_t^{e,n}}{dn}|_{n=0} = Y_t^0 - Y_t^{e,0} = 1 - 1 = 0,$$

which implies that $dY_t^n/dn = 0$ for all n and then $Y_t^n = Y_t^0 = 1$ for all n.

Rules versus targets Figure B.1 shows the reflective equilibria for different levels of n both for rules-based communication and targets-communication in the left and right panels, respectively. Consistent with the results for the generalized level-k model, output contracts more sharply for lower levels of cognitive ability. As highlighted by Angeletos and Sastry (2020), the peculiar oscillatory feature that is present under standard level-k thinking does not arise under reflective expectations. We see that as cognitive ability rises, output converges to that under rational expectations. Also in line with the results

Figure B.1: Rules versus targets



in the baseline model, we see that, with targets, output contracts less with lower levels of cognitive sophistication and the level of output also converges to the rational expectations equilibrium as n increases.

This confirms the claim in the paper that all the results in the benchmark model extend to the reflective expectations model.

B.3 Shallow reasoning

Angeletos and Sastry (2020) describe a different process of belief formation which they refer to as *shallow reasoning*. In this model it is assumed that everyone is rational and attentive, knows that everyone else is rational but believe that only a fraction λ are attentive to changes in the economic environment. For simplicity, we work with the linearized equilibrium relation. The consumption of individual i can be written as follows:

$$c_{i,t} = (1-\beta) \sum_{s=0}^{T-1-(t-s)} \beta^s \frac{Y}{C} \left[\mathbb{E}_i y_{t+s} - g_{t+s}\right] - \sigma\beta \sum_{s=0}^{T-1-t} \beta^s \left\{r_{t+s} - \Delta\hat{\tau}^c_{t+s+1} + \chi_{t+s}\right\},$$

where $\mathbb{E}_i\left[y_t\right]$ denotes individual i's expectation of output. Lower-case letters denote log-deviations from steady-state values, except for $g_t = G_t/Y$. Market clearing requires $y_t = \frac{C}{Y}\int c_{i,t}di + g_t$. Individual i fully understands that other individuals have the same policy function, conditional on their beliefs. Using the market clearing condition we can write

$$y_t = g_t + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\overline{\mathbb{E}}y_{t+s} - g_{t+s}\right] - \frac{C}{Y}\sigma\sum_{s=0}^{T-1-t}\beta^s\left\{r_{t+s} - \Delta\hat{\tau}^c_{t+s+1} + \chi_{t+s}\right\},$$

where $\overline{\mathbb{E}}[y_t] \equiv \int_0^1 \mathbb{E}_i[y_t]\,di$ denotes the average expectation in the economy. Let

$$\Psi_t \equiv g_t - (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}g_{t+s} - \frac{C}{Y}\sigma\beta\sum_{s=0}^{T-1-t}\beta^s\left\{r_{t+s} - \Delta\hat{\tau}^c_{t+s+1} + \chi_{t+s}\right\}$$

We can write

$$\boldsymbol{y} = (1-\beta)\boldsymbol{M}\overline{\mathbb{E}}[\boldsymbol{y}] + \boldsymbol{\Psi}$$

where

$$\boldsymbol{y} \equiv \begin{bmatrix} y_0 \\ y_1 \\ \dots \\ y_{T-1} \end{bmatrix}, \quad \boldsymbol{M} \equiv \begin{bmatrix} 0 & 1 & \beta & \dots & \beta^{T-1} \\ 0 & 0 & 1 & \dots & \beta^{T-2} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad \boldsymbol{\Psi} \equiv \begin{bmatrix} \Psi_0 \\ \Psi_1 \\ \dots \\ \Psi_{T-1} \end{bmatrix}.$$

This implies that

$$\overline{\mathbb{E}}[\boldsymbol{y}] = (1-\beta)\boldsymbol{M}\overline{\mathbb{E}}^2[\boldsymbol{y}] + \overline{\mathbb{E}}[\boldsymbol{\Psi}],$$

where $\overline{\mathbb{E}}^h[\cdot] \equiv \overline{\mathbb{E}}\left[\overline{\mathbb{E}}^{h-1}[\cdot]\right]$. Note that the law of iterated expectations does not apply for the average expectation. Then, iterating on this relation and using the fact that $\boldsymbol{M}^h$ converges to a zero matrix as h goes to infinity, we obtain

$$\overline{\mathbb{E}}[\boldsymbol{y}] = \sum_{h=1}^{\infty}\left\{(1-\beta)\boldsymbol{M}\right\}^{h-1}\overline{\mathbb{E}}^h[\boldsymbol{\Psi}].$$

Following Angeletos and Sastry (2020), the behavioral assumptions imply that $\overline{\mathbb{E}}^h[\boldsymbol{\Psi}] = \lambda^h\boldsymbol{\Psi}$, and so

$$\overline{\mathbb{E}}[\boldsymbol{y}] = \lambda\left[\boldsymbol{I} - (1-\beta)\boldsymbol{M}\lambda\right]^{-1}\boldsymbol{\Psi} = \lambda\boldsymbol{y},$$

where the last equality follows from the fact that

$$\begin{aligned}\boldsymbol{y} &= (1-\beta)\boldsymbol{M}\overline{\mathbb{E}}[\boldsymbol{y}] + \boldsymbol{\Psi} = (1-\beta)\boldsymbol{M}\lambda\left[\boldsymbol{I} - (1-\beta)\boldsymbol{M}\lambda\right]^{-1}\boldsymbol{\Psi} + \boldsymbol{\Psi} \\ &= \left[\boldsymbol{I} - (1-\beta)\boldsymbol{M}\lambda\right]^{-1}\boldsymbol{\Psi}\end{aligned}$$

As a result, we can write the equilibrium relation as:

$$y_t = g_t + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\lambda y_{t+s} - g_{t+s}\right] - \frac{C}{Y}\sigma\beta\sum_{s=0}^{T-1-t}\beta^{s}\left\{r_{t+s} - \Delta\hat{\tau}^{c}_{t+s+1} + \chi_{t+s}\right\}. \tag{B.4}$$

B.3.1 Government spending multipliers

Using the equilibrium relation (B.4), we find that the fiscal spending multiplier solves the following recursion:

$$\frac{\Delta Y_t}{\Delta G_t} = 1 + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\lambda\frac{\Delta Y_{t+s}}{\Delta G_{t+s}} - 1\right]\frac{\Delta G_{t+s}}{\Delta G_t}. \tag{B.5}$$

For consistency with earlier results, the multiplier is expressed in terms of levels of Y_t and G_t. As in the benchmark model, the date $T-1$ fiscal multiplier is the same as the rational expectations fiscal multiplier:

$$\frac{\Delta Y_{T-1}}{\Delta G_{T-1}} = 1.$$

This then implies that

$$\frac{\Delta Y_{T-2}}{\Delta G_{T-2}} = 1 - (1-\beta)\left[1-\lambda\right]\frac{\Delta G_{T-1}}{\Delta G_{T-2}}.$$

Since $\lambda < 1$, then $\Delta Y_{T-2}/\Delta G_{T-2} < 1$. As $\lambda \to 1$ then $\Delta Y_{T-2}/\Delta G_{T-2} \to 1$ which coincides with the rational expectations multiplier. We can also see that the fiscal multiplier is monotonically increasing in λ,

$$\frac{d\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}}{d\lambda} = (1-\beta)\frac{\Delta G_{T-1}}{\Delta G_{T-2}} > 0,$$

so as λ increases the multiplier gets closer to the rational expectations multiplier. Via standard inductive arguments these properties extend to all time t multipliers. To see this result, note that for $\lambda < 1$, if $\Delta Y_{t+s}/\Delta G_{t+s} \leq 1$ for all $s \geq 1$ then

$$\frac{\Delta Y_t}{\Delta G_t} = 1 + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\lambda\frac{\Delta Y_{t+s}}{\Delta G_{t+s}} - 1\right]\frac{\Delta G_{t+s}}{\Delta G_t} < 1.$$

Furthermore,

$$\lim_{\lambda\to 1}\frac{\Delta Y_t}{\Delta G_t}=1+(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\lim_{\lambda\to 1}\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}-1\right]\frac{\Delta G_{t+s}}{\Delta G_t}=1$$

as long as $\lim_{\lambda\to 1}\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}=1$. This result shows that all time t spending multipliers converge to the rational expectations multipliers as λ goes to one. Furthermore, under the assumption that

$$1-(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\frac{\Delta G_{t+s}}{\Delta G_t}>0, \tag{B.6}$$

we find that $\Delta Y_t/\Delta G_t>0$ for all t. Differentiating (B.5) with respect to λ, we obtain:

$$\frac{d\frac{\Delta Y_t}{\Delta G_t}}{d\lambda}=(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}+\lambda\frac{d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}}{d\lambda}\right]\frac{\Delta G_{t+s}}{\Delta G_t}.$$

Under assumption (B.6), we know that $\Delta Y_{t+s}/\Delta G_{t+s}>0$. Then, if

$$\frac{d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}}{d\lambda}>0,$$

then $d\frac{\Delta Y_t}{\Delta G_t}/d\lambda>0$. Since we have shown that $d\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}/d\lambda>0$, then it is true that $d\frac{\Delta Y_t}{\Delta G_t}/d\lambda>0$ for all t. This confirms that the shallow reasoning spending multiplier is increasing in the sophistication parameter λ.

Finally, suppose that $\Delta G_t=\zeta^t\Delta G_0$ for $\zeta>0$, then

$$\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}=1-(1-\beta)\left[1-\lambda\right]\zeta\Rightarrow d\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}/d\zeta<0$$

and

$$\frac{\Delta Y_t}{\Delta G_t}=1+(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\lambda\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}-1\right]\zeta^s. \tag{B.7}$$

$$d\frac{\Delta Y_t}{\Delta G_t}/d\zeta=(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda\frac{d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}}{d\zeta}\zeta^s+(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\underbrace{\lambda\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}-1}_{<0}\right]s\zeta^{s-1}<0$$

as long as $d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}/d\zeta<0$. As a result, the spending multiplier is decreasing in the persistence of government spending.

B.3.2 Consumption tax policy

Suppose that $g_t = 0$ for all t and for simplicity suppose that $Y = C$. Interest rates are at the ZLB for $t \leq T-1$, and go back to steady state levels for $t \geq T$:

$$r_t = \log R_t - \rho = \begin{cases} -\rho & \text{if } t \leq T-1 \\ 0 & \text{if } t \geq T. \end{cases}$$

Then, we find that for $t \geq T$ output is back to steady state $y_t = 0$. However, for $t \leq T-1$ output solves the fixed-point system of equations of $\{y_t\}_{t=0}^{T-1}$:

$$y_t = (1-\beta) \sum_{s=1}^{T-1-t} \beta^{s-1} \lambda y_{t+s} - \sigma \sum_{s=0}^{T-1-t} \beta^s \left\{ (\chi - \rho) - \left(\hat{\tau}^c_{t+s+1} - \hat{\tau}^c_{t+s} \right) \right\}. \quad \text{(B.8)}$$

Then, consider the policy that implements full stabilization under rational expectations:

$$1 + \tau^c_t = (1 + \tau^c)\, e^{-(T-t)(\chi-\rho)}$$

which implies that

$$\frac{1+\tau^c_t}{1+\tau^c_{t+1}} = e^{-(\chi-\rho)} \Rightarrow \hat{\tau}^c_{t+1} - \hat{\tau}^c_t = \chi - \rho.$$

Replacing these consumption taxes in the equilibrium relation (B.8), we obtain

$$y_t = (1-\beta) \sum_{s=1}^{T-1-t} \beta^{s-1} \lambda y_{t+s},$$

which implies that $y_t = 0$ for all t is a shallow reasoning equilibrium under this policy. In sum, the same policy that implements the flexible price allocation under rational expectations also implements the flexible price allocation irrespective of the degree of rationality λ.

Rules versus targets Consider now the case in which policy is designed as rules, i.e., such that interest rates and consumption taxes are set so that

$$r_t = \max \left\{ \phi_y y_t, -\rho \right\},$$

and

$$\hat{\tau}^c_{t+1} - \hat{\tau}^c_t = \min \left\{ \phi_y y_t + \rho, 0 \right\}$$

which implies that:

$$r_t + \hat{\tau}^c_{t+1} - \hat{\tau}^c_t = \phi_y y_t.$$

The shallow reasoning equilibrium is a solution to the fixed point system of equations given by:

$$y_t = -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda y_{t+s}.$$

As before, if $\lambda = 1$, then $y_t = -\frac{\chi}{\phi_y}\left[1-\left(1+\sigma\phi_y\right)^{-(T-t)}\right] = y^*_t < 0$ which is the rational expectations equilibrium. Furthermore, note that for $t = T-1$:

$$y_{T-1} = -\frac{\sigma\chi}{1+\sigma\phi_y} = y^*_{T-1} < 0$$

for any λ. Next, we show that, if $\beta > \left(1+\sigma\phi_y\right)^{-1}$, for $\lambda < 1$, $y_t < y^*_t$ for all $t \leq T-2$.

Output at time $t = T-2$ is given by

$$\begin{aligned} y_{T-2} &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\lambda y_{T-1} \\ &< -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right) y^*_{T-1} = y^*_{T-2}, \end{aligned}$$

which shows that $y_{T-2} < y^*_{T-2}$. Furthermore, we also find that $\lambda y_{T-2} > y^*_{T-2}$, which follows from the fact that:

$$\begin{aligned} \lambda y_{T-2} - y^*_{T-2} &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta}(\lambda-1) - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\left(\lambda^2 y_{T-1} - y^*_{T-1}\right) \\ &= (\lambda-1)\left\{-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)(\lambda+1)\, y^*_{T-1}\right\} \\ &> (\lambda-1)\left\{-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right) y^*_{T-1}\right\} = (\lambda-1)\, y^*_{T-2} > 0. \end{aligned}$$

Therefore, we find that $y_{T-2} < y^*_{T-2}$, but $\lambda y_{T-2} > y^*_{T-2}$, i.e., $y_{T-2} \in (\lambda^{-1} y^*_{T-2}, y^*_{T-2})$. For any t, suppose that $y_{t+s} \in (\lambda^{-1} y^*_{t+s}, y^*_{t+s}]$ for all $s = 1, ..., T-t-1$, then

$$\begin{aligned} y_t &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda y_{t+s} \\ &< -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda\lambda^{-1} y^*_{t+s} = y^*_t. \end{aligned}$$

Furthermore, we also find that $\lambda y_t > y_t^*$, which follows from the fact that

$$\begin{aligned}
\lambda y_t - y_t^* &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta}(\lambda-1) - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\left(\lambda^2 y_{t+s} - y_{t+s}^*\right) \\
&> -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta}(\lambda-1) - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\left(\lambda^2 y_{t+s}^* - y_{t+s}^*\right) \\
&= (\lambda-1)\left[-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}(\lambda+1)\, y_{t+s}^*\right] \\
&> (\lambda-1)\left[-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1} y_{t+s}^*\right] \\
&> (\lambda-1)\, y_t^* > 0.
\end{aligned}$$

Then, by induction, we find that $y_t \in (\lambda^{-1}y_t^*, y_t^*]$, which shows that the stabilizing power of fiscal policy under rules becomes weaker.

Suppose now, that the policy is communicated as targets. We show that under targets-based communication $y_t \geq y_t^*$ for all t. First, using (B.8) we find that:

$$\lim_{\lambda\to 0} y_t = -\sigma \sum_{s=0}^{T-1-t}\beta^s \left\{(\chi-\rho) - \left(\hat{\tau}_{t+s+1}^{c,r} - \hat{\tau}_{t+s}^{c,r}\right)\right\} < 0,$$

and

$$\frac{dy_{T-2}}{d\lambda} = (1-\beta)\, y_{T-1}^* < 0 \Rightarrow y_{T-2} < 0,$$

for all λ. Now, note that

$$\frac{dy_t}{d\lambda} = (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1} y_{t+s} + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda\frac{dy_{t+s}}{d\lambda}.$$

So, as long as $y_{t+s} \leq 0$ and $dy_{t+s}/d\lambda \leq 0$ for all $s \geq 1$, with one strict inequality, then we find that $dy_t/d\lambda < 0$ and $y_t < 0$. Furthermore, to show that $y_t > y_t^*$, note that

$$y_t - y_t^* = (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left\{\lambda y_{t+s} - y_{t+s}^*\right\}.$$

As before, this implies that $y_{T-1} = y_{T-1}^*$. Now, evaluating time $t = T-1$, we see that

$$y_{T-2} - y_{T-2}^* = (1-\beta)\{\lambda - 1\}\, y_{T-1}^* > 0 \Rightarrow y_{T-2} > y_{T-2}^*.$$

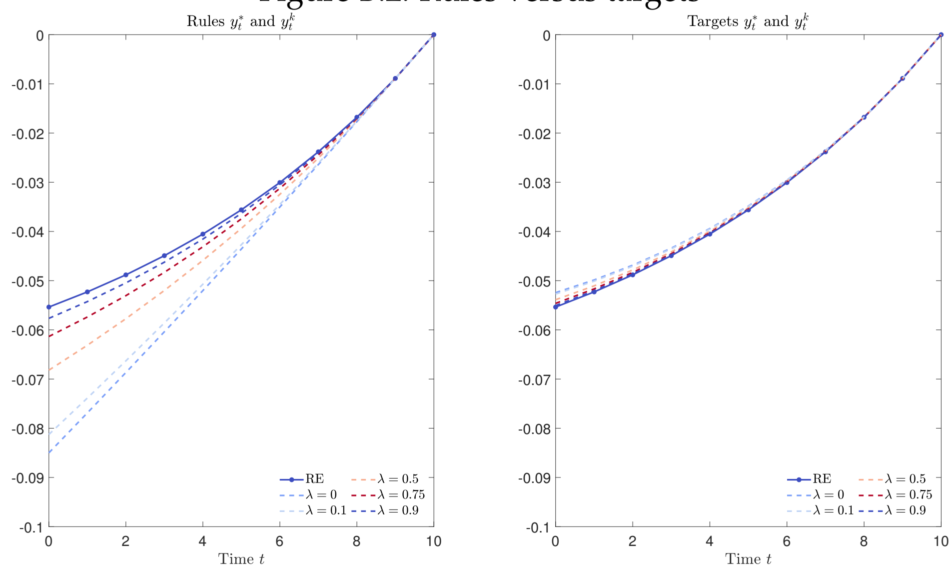
This result serves as the base for the inductive argument. Suppose that $0 > y_{t+2} > y_{t+s}^*$

for all s, then

$$y_t - y_t^* = \sum_{s=1}^{T-1-t} \beta^{s-1} \left\{ \lambda y_{t+s} - y_{t+s}^* \right\} > 0.$$

Figure B.2 shows the equilibrium path for log-output in the economy with shallow reasoning for different levels of λ. As highlighted by Angeletos and Sastry (2020), the peculiar oscillatory feature that is present under simple level-k thinking does not arise under reflective expectations. We see that as cognitive ability rises, output converges to that under rational expectations. Also in line with the results in the baseline model, we see

Figure B.2: Rules versus targets



that, with targets, output contracts less with lower levels of cognitive sophistication and the level of output also converges to the rational expectations equilibrium as λ increases.

This confirms the claim in the paper that all the results in the benchmark model extend to the shallow reasoning model.

C Appendix to section 3

C.1 Consumption function

The household's optimal consumption plan satisfies:

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} \left\{ \left(1 - \tau_{t+s}^n\right) W_{t+s}^e N_{t+s}^e + \Omega_{t+s}^e - T_{t+s}^e \right\} + R_{t-1} B_t}{P_t \left(1 + \tau_t\right) \left[1 + \sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t} \right)^\sigma \left[Q_{t,t+s} \frac{P_{t+s}^e \left(1 + \tau_{t+s}^c\right)}{P_t \left(1 + \tau_t\right)} \right]^{1-\sigma} \right]}.$$

Given their beliefs for output, the household's expectations for lump-sum taxes are given by 3.6. Replacing beliefs for lump-sum taxes, we obtain:

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s}\left\{W^e_{t+s}N^e_{t+s} + \tau^c_{t+s}P^e_{t+s}C^e_{t+s} + \Omega^e_{t+s} - P^e_{t+s}G^e_{t+s}\right\}}{P_t\left(1+\tau^c_t\right)\left[1+\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau^c_t\right)}\right]^{1-\sigma}\right]}.$$

Using the fact that

$$Y^e_{t+s} = \frac{W^e_{t+s}}{P^e_{t+s}}N^e_{t+s} + \frac{\Omega^e_{t+s}}{P^e_{t+s}}$$

and

$$C^e_{t+s} = Y^e_{t+s} - G_{t+s}$$

we can write the consumption function as

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s}P^e_{t+s}\left\{Y^e_{t+s} - G_{t+s} + \tau^c_{t+s}\left(Y^e_{t+s} - G_{t+s}\right)\right\}}{P_t\left(1+\tau^c_t\right)\left[1+\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau^c_t\right)}\right]^{1-\sigma}\right]},$$

or equivalently

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s}\frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau^c_t\right)}\left[Y^e_{t+s} - G_{t+s}\right]}{1+\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau^c_t\right)}\right]^{1-\sigma}},$$

C.2 Unions and wage setting

In this appendix we solve the problem of the union and derive the wage equation 3.8. The problem of a union that gets to reset its wage is

$$\max_{w_{u,t},\{\tilde{n}_{u,t+s}\}} \sum_{s\geq 0}(\beta\lambda)^s\left\{u'\left(C^e_{t+s}\right)\frac{1-\tau^n_{t+s}}{1+\tau^c_{t+s}}\frac{w_{u,t}\tilde{n}_{u,t+s}}{P^e_{t+s}} - v'\left(L^e_{t+s}\right)\tilde{n}_{u,t+s}\right\}$$

subject to the constraint

$$\tilde{n}_{u,t+s} = \left(\frac{w_{u,t}}{W^e_{t+s}}\right)^{-\theta}N^e_{t+s}.$$

Because every union represents an infinitesimal number of workers in each household, the union does not directly affect aggregate consumption, C_t, hours worked by the household, L_t, the composite labor input, N_t, aggregate wages, W_t, and prices, P_t. As discussed in the main text, we assume that the union has rational expectations with respect to the exogenous variables, but is boundedly rational with respect to future endogenous variables.

The optimal reset wage W_t^* solves the following first order condition:

$$\sum_{s\geq 0}(\beta\lambda)^s\left\{-(\theta-1)\,u'\left(C_{t+s}^e\right)\frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}\frac{W_t^*\left(\frac{W_t^*}{W_{t+s}^e}\right)^{-\theta}N_{t+s}^e}{P_{t+s}^e}+\theta v'\left(L_{t+s}^e\right)\left(\frac{W_t^*}{W_{t+s}^e}\right)^{-\theta}N_{t+s}^e\right\}=0$$

which can be equivalently written as follows:

$$\frac{W_t^*}{P_t}=\frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty}(\beta\lambda)^s\,\xi_{t+s}\left(\frac{P_{t+s}^e}{P_t}\right)^{\theta}\left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta}N_{t+s}^e v'\left(L_{t+s}^e\right)}{\sum_{s=0}^{\infty}(\beta\lambda)^s\,\xi_{t+s}\left(\frac{P_{t+s}^e}{P_t}\right)^{\theta-1}\left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta}N_{t+s}^e u'\left(C_{t+s}^e\right)\frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}}.$$

C.3 Sufficient conditions for equilibrium and the linearized system

Given beliefs, a temporary equilibrium denotes a solution to the following system of equations:

1. The consumption function

$$C_t=\frac{\sum_{s=1}^{\infty}Q_{t,t+s}\frac{P_{t+s}^e\left(1+\tau_{t+s}^c\right)}{P_t\left(1+\tau_t^c\right)}\left\{Y_{t+s}^e-G_{t+s}\right\}}{\sum_{s=1}^{\infty}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{P_{t+s}^e\left(1+\tau_{t+s}^c\right)}{P_t\left(1+\tau_t^c\right)}\right]^{1-\sigma}},$$

   where we have imposed market clearing, $C_t=Y_t-G_t$.

2. Unions optimal wage setting

$$\frac{W_t^*}{P_t}=\frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty}(\beta\lambda)^s\,\xi_{t+s}\left(\frac{P_{t+s}^e}{P_t}\right)^{\theta}\left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta}N_{t+s}^e v'\left(L_{t+s}^e\right)}{\sum_{s=0}^{\infty}(\beta\lambda)^s\,\xi_{t+s}\left(\frac{P_{t+s}^e}{P_t}\right)^{\theta-1}\left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta}N_{t+s}^e u'\left(C_{t+s}^e\right)\frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}}$$

   and the aggregate wage is

$$W_t=\left[\lambda W_{t-1}^{1-\theta}+(1-\lambda)\left(W_t^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

3. Real wages are equal to the marginal productivity of labor

$$\frac{W_t}{P_t} = (1-\alpha)\, A \left(\frac{\overline{K}}{N_t} \right)^{\alpha}.$$

4. Output is given by

$$Y_t = A\overline{K}^{\alpha} N_t^{1-\alpha},$$

where

$$L_t = \mu_t N_t$$

$$\mu_t = \int_0^1 \left(\frac{w_{u,t}}{W_t} \right)^{-\theta} du = \lambda \mu_{t-1} \left(\frac{W_{t-1}}{W_t} \right)^{-\theta} + (1-\lambda) \left(\frac{W_t^*}{W_t} \right)^{-\theta}$$

where $\mu_{-1} = 1$.

5. Market clearing

$$C_t + G_t = Y_t.$$

For each quantity and price X_t we denote their log-linear deviation from steady state by $x_t \equiv \log X_t - \log X$, except for $g_t = G_t/Y$. For taxes we denote their log-linear deviation by $\hat{\tau}_t^c = \log(1+\tau_t^c) - \log(1+\tau^c)$ and $\hat{\tau}_t^n = -\{\log(1-\tau_t^n) - \log(1-\tau^n)\}$. Finally, $\log \xi_{t+1}/\xi_t = \chi_t$, where $\chi_t = \chi > 0$ for $t \leq T-1$ and $\chi_t = 0$ for $t \geq T$. The log-linear system can be written as follows.

Consumption is given by

$$c_t = \frac{(1-\beta)}{\beta} \sum_{s=1}^{\infty} \beta^s \frac{Y}{C} \{ y_{t+s}^e - g_{t+s} \} - \sigma \sum_{s=0}^{\infty} \beta^s \left\{ r_{t+s} - \pi_{t+s+1}^e - \left(\hat{\tau}_{t+s+1}^c - \hat{\tau}_{t+s}^c \right) + \chi_{t+s} \right\}. \tag{C.1}$$

Wage inflation $\pi_t^w = w_t - w_{t-1}$ is given by

$$\pi_t^w = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{ \varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c + \alpha n_{t+s}^e \right\} + \frac{1-\lambda}{\lambda} \sum_{s\geq 1}^{\infty} (\beta\lambda)^s \pi_{t+s}^{w,e}. \tag{C.2}$$

Below, we show how to derive these two equations below.

Price inflation $\pi_t = p_t - p_{t-1}$ is given by

$$\pi_t = \pi_t^w + \alpha \Delta n_t. \tag{C.3}$$

Finally, output is given by

$$y_t = (1-\alpha)\, n_t, \tag{C.4}$$

and the market clearing condition is

$$\frac{C}{Y} c_t + g_t = y_t. \tag{C.5}$$

To first order, $n_t = l_t$.

Log-linearized wage inflation Wage setting is given by

$$\frac{W_t^*}{P_t} = \frac{\theta}{\theta-1} \frac{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(\frac{P_{t+s}^e}{P_t}\right)^{\theta} \left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta} N_{t+s}^e v'\left(L_{t+s}^e\right)}{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(\frac{P_{t+s}^e}{P_t}\right)^{\theta-1} \left(\frac{W_{t+s}^e}{P_{t+s}^e}\right)^{\theta} N_{t+s}^e u'\left(C_{t+s}^e\right) \frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}}$$

and the aggregate wage is

$$W_t = \left[\lambda W_{t-1}^{1-\theta} + (1-\lambda)\left(W_t^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

Log-linearizing the wage setting condition we obtain

$$w_t^* - p_t = (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{\varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c\right\} + \sum_{s=1}^{\infty} (\beta\lambda)^s p_{t+s}^e - \sum_{s=0}^{\infty} (\beta\lambda)^{s+1} p_{t+s}^e,$$

$$\begin{aligned}\Leftrightarrow w_t^* - w_t = {} & (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{\varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c\right\} + \sum_{s=1}^{\infty} (\beta\lambda)^s p_{t+s}^e \\ & - \sum_{s=0}^{\infty} (\beta\lambda)^{s+1} p_{t+s}^e + p_t - w_t\end{aligned}$$

or equivalently,

$$w_t^* - w_t = (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{\varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c - (w_{t+s}^e - \hat{p}_{t+s}^e)\right\} + \sum_{s\geq 1}^{\infty} (\beta\lambda)^s \pi_{t+s}^{w,e}$$

since $w_{t+s}^e - \hat{p}_{t+s}^e = -\alpha n_{t+s}^e$ then

$$w_t^* - w_t = (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{\varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c + \alpha n_{t+s}^e\right\} + \sum_{s\geq 1}^{\infty} (\beta\lambda)^s \pi_{t+s}^{w,e}.$$

Log-linearizing the aggregate wage condition we obtain

$$w_t = \lambda w_{t-1} + (1-\lambda)\, w_t^*.$$

Now, define $\pi_t^w = w_t - w_{t-1}$, we can use the equation above to show that

$$\lambda \pi_t^w = (1-\lambda)\left(w_t^* - w_t\right) \Leftrightarrow \pi_t^w = \frac{1-\lambda}{\lambda}\left(w_t^* - w_t\right).$$

Replacing $w_t^* - w_t$ we find that

$$\pi_t^w = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\sum_{s\geq 0}^{\infty}(\beta\lambda)^s\left\{\varphi n_{t+s}^e + \sigma^{-1}c_{t+s}^e + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c + \alpha n_{t+s}^e\right\} + \frac{1-\lambda}{\lambda}\sum_{s\geq 1}^{\infty}(\beta\lambda)^s\,\pi_{t+s}^{w,e}.$$

C.4 Proof of proposition 5

Part 1 The proof strategy is as follows. First, we show that if level-1 people believe that the economy will stay at steady state for $t \geq T$, then all level-k beliefs and corresponding equilibria feature output, consumption, labor and wage inflation remaining at their steady state levels from $t \geq T$, and price inflation is zero for $t \geq T+1$. Second, we note that beliefs about future output, inflation, consumption, and labor are a function only of future tax rates and policies. Finally, for a given level k, we recursively construct a sequence of policies $\left\{\hat{\tau}_t^{c,k}, \hat{\tau}_t^{n,k}\right\}$ which implements the flexible price allocation and always features zero inflation for all t.

(1) Suppose that $y_t^e = c_t^e = n_t^e = 0$ and $\pi_{t+1}^{w,e} = \pi_{t+1}^e = 0$ if $t \geq T$. Then, setting $g_t = \hat{\tau}_t^c = \hat{\tau}^n = r_t = 0$ for all $t \geq T$, implies that consumption, output, and labor for $t \geq T$ are given by

$$c_t = \frac{(1-\beta)}{\beta}\sum_{s=1}^{\infty}\beta^s\frac{Y}{C}y_{t+s}^e = 0,$$

$$y_t = \frac{C}{Y}c_t = 0,$$

and

$$n_t = \frac{y_t}{1-\alpha} = 0,$$

respectively. Then, wage inflation for $t \geq T$ is given by

$$\pi_t^w = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\left\{\varphi n_t + \sigma^{-1}c_t + \alpha n_t\right\} = 0.$$

Finally, this implies that price inflation is

$$\pi_t = \pi_t^w + \alpha \Delta n_t = 0$$

for $t \geq T+1$, and $\pi_T = -\alpha n_{T-1}$. This then shows the initial beliefs $y_t^{e,1} = c_t^{e,1} = n_t^{e,1} = \pi_t^{w,e,1} = \pi_{t+1}^{e,1} = 0$ are consistent with what happens in equilibrium. This result implies that all level-k people believe $y_t^{e,k} = c_t^{e,k} = n_t^{e,k} = \pi_t^{w,e,k} = \pi_{t+1}^{e,k} = 0$ for $t \geq T$.

(2) Recall that the temporary equilibrium for time t solves the system of equations (C.1)-(C.5). This equilibrium does not depend on policies before time t. So, for each t, level-k beliefs are unaffected by past policies, $\left\{\hat{\tau}_s^{c,k}, \hat{\tau}_s^{n,k}\right\}_{s=0}^{t-1}$.

(3) For $t = T-1$, the level-k equilibrium levels of consumption and wage inflation solve

$$c_{T-1}^k = -\sigma\left\{-\pi_T^{e,k} + \hat{\tau}_{T-1}^{c,k} + \chi - \rho\right\},$$

and

$$\pi_{T-1}^{w,k} = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\left\{\varphi n_{T-1}^k + \sigma^{-1} c_{T-1}^k + \hat{\tau}_{T-1}^{n,k} + \hat{\tau}_{T-1}^{c,k} + \alpha n_{T-1}^k\right\}.$$

Note that by setting $\hat{\tau}_{t+s}^{c,k} = \rho + \pi_T^{e,k} - \chi$, then $c_{T-1}^k = 0$. Since consumption remains at its steady-state level, then $y_{T-1}^k = n_{T-1}^k = 0$. Setting $\hat{\tau}_{T-1}^{n,k} = -\hat{\tau}_{T-1}^{c,k}$, implies that $\pi_{T-1}^{w,k} = 0$. Furthermore, since $\pi_T^k = -\alpha n_{T-1}^k$ then this policy also implies that $\pi_T^k = 0$.

We now proceed recursively. At time t, fix the future policies $\left\{\hat{\tau}_{t+s}^{c,k}, \hat{\tau}_{t+s}^{n,k}\right\}_{s\geq 1}$ and the implied beliefs $\left\{y_{t+s}^{e,k}, c_{t+s}^{e,k}, n_{t+s}^{e,k}, \pi_{t+s}^{w,e,k}, \pi_{t+s}^{e,k}\right\}_{s\geq 1}$. Consumption at time t is given by we set $\hat{\tau}_t^{c,k}$ so that

$$c_t^k = \frac{(1-\beta)}{\beta}\sum_{s=1}^{\infty}\beta^s \frac{Y}{C} y_{t+s}^{e,k} - \sigma\sum_{s=0}^{\infty}\beta^s\left\{r_{t+s} - \pi_{t+s+1}^{e,k} - \left(\hat{\tau}_{t+s+1}^{c,k} - \hat{\tau}_{t+s}^{c,k}\right) + \chi_{t+s}\right\}.$$

We set $\hat{\tau}_t^{c,k}$ such that $c_t^k = 0$, which implies

$$\hat{\tau}_t^{c,k} = \frac{(1-\beta)}{\beta\sigma}\sum_{s=1}^{T-t-1}\left[\beta^s \frac{Y}{C} y_{t+s}^{e,k}\right] - \left\{-\pi_{t+1}^{e,k} - \hat{\tau}_{t+1}^{c,k} + \chi - \rho\right\}$$
$$- \sum_{s=1}^{\infty}\beta^s\left\{-\pi_{t+s+1}^{e,k} - \left(\hat{\tau}_{t+s+1}^{c,k} - \hat{\tau}_{t+s}^{c,k}\right) + \chi_{t+s} - \rho\right\}.$$

Since $c_t^k = 0$, it follows from (C.4) and (C.5) that $n_t^k = y_t^k = 0$. Wage inflation is given by

$$\pi_t^{w,k} = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\sum_{s\geq 0}^{\infty}(\beta\lambda)^s\left\{\varphi n_{t+s}^{e,k} + \sigma^{-1} c_{t+s}^{e,k} + \hat{\tau}_{t+s}^{n,k} + \hat{\tau}_{t+s}^{c,k} + \alpha n_{t+s}^{e,k}\right\} + \frac{1-\lambda}{\lambda}\sum_{s\geq 1}^{\infty}(\beta\lambda)^s \pi_{t+s}^{w,e,k}.$$

We set $\hat{\tau}_t^{n,k}$ such that $\pi_t^{w,k} = 0$, which implies

$$\hat{\tau}_t^{n,k} = -\,\hat{\tau}_t^{c,k} - \sum_{s=1}^{\infty} (\beta\lambda)^s \left\{ \varphi n_{t+s}^{e,k} + \sigma^{-1} c_{t+s}^{e,k} + \hat{\tau}_{t+s}^{n,k} + \hat{\tau}_{t+s}^{c,k} + \alpha n_{t+s}^{e,k} \right\} - \frac{1}{1-\beta\lambda} \sum_{s\geq 1}^{\infty} (\beta\lambda)^s \pi_{t+s}^{w,e,k}.$$

These policies implement an allocation in which $n_t^k = 0$ and $\pi_t^{w,k} = 0$ for all t. It follows (C.3) from then $\pi_t^k = 0$ for all t.

Part 2 Suppose that beliefs are anchored at the initial steady state. Consider setting taxes on consumption and labor such that

$$\tau_t^c = (1+\tau^c)\, e^{-(T-t)(\chi-\rho)} - 1.$$

$$\frac{1-\tau_t^n}{1+\tau_t^c} = \frac{1-\tau^n}{1+\tau^c}.$$

Then, consumption is given by

$$C_t = \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c} \{Y-G\}}{\sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c}\right]^{1-\sigma}} = \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c}}{\sum_{s\geq 1} \beta^s \frac{\xi_{t+s}}{\xi_t}} \{Y-G\} = C.$$

This implies that

$$Y_t = C_t + G = C + G = Y,$$

and then

$$N_t = \left(\frac{Y}{A\overline{K}^{\alpha}}\right)^{\frac{1}{1-\alpha}} = N.$$

The reset wage is:

$$\begin{aligned} \frac{W_t^*}{P_t} &= \frac{\theta}{\theta-1} \frac{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(\frac{W}{P}\right)^{\theta} N v'(L)}{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(\frac{W}{P}\right)^{\theta} N u'(C) \frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}} \\ &= \frac{\theta}{\theta-1} \frac{1+\tau^c}{1-\tau^n} \frac{v'(L)}{u'(C)} = \frac{W}{P}. \end{aligned}$$

Then, from the first-order condition of the firm we see that W_t/P_t is constant

$$\frac{W_t}{P_t} = (1-\alpha)\, A \left(\frac{\overline{K}}{N}\right)^{\alpha} = \frac{W}{P}$$

which, combined with

$$\frac{W_t}{P_t} = \left[\lambda \left(\frac{W_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}\right)^{1-\theta} + (1-\lambda)\left(\frac{W_t^*}{P_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

implies that $P_t = P_{t-1}$ for all t. Finally, this implies that $\mu_t = 1$ for all t and $N_t = L_t = N$. Since this result holds for the non-linear model, it trivially extends to the linearized model.

D A model with sticky prices

In this appendix, we present an alternative New Keynesian model with sticky prices instead of sticky wages and show that our main results continue to hold for this alternative specification. We assume that households have the same utility function as the one in our benchmark model, see (2.1).

The final good is produced using a continuum of intermediate inputs $y_{u,t}$ for $u \in [0,1]$ according to the technology:

$$Y_t = \left[\int_0^1 y_{u,t}^{\frac{\theta-1}{\theta}} du\right]^{\frac{\theta}{\theta-1}}.$$

Each variety u is produced by a monopolistic firm using the technology:

$$y_{u,t} = A n_{u,t}^{1-\alpha}.$$

The good market clearing condition is still given by (2.3). We assume that the government has access to the same monetary and fiscal instruments as in section 3.

Final goods firms The representative final goods producer maximizes profits

$$P_t Y_t - \int_0^1 p_{u,t} y_{u,t} du,$$

which implies that demand for the intermediate input is given by

$$y_{u,t} = \left(\frac{p_{u,t}}{P_t}\right)^{-\theta} Y_t.$$

The aggregate price level satisfies:

$$P_t = \left[\int_0^1 p_{u,t}^{1-\theta} du \right]^{\frac{1}{1-\theta}}.$$

Intermediate goods producers Each intermediate good u is produced by a monopolist. Producers set prices subject to Calvo frictions. At time t, a fraction $1-\lambda$ can reset their price. As is standard, it is optimal for producers to choose the same reset price, P_t^*. The optimal reset price is the solution to:

$$\max_{P_t^*} \sum_{s=0}^{\infty} \lambda^s Q_{t,t+s} \frac{P_{t+s}^e}{P_t} \left\{ \left(\frac{P_t^*}{P_{t+s}^e} \right)^{1-\theta} Y_{t+s}^e - \frac{W_{t+s}^e}{P_{t+s}^e A^{\frac{1}{1-\alpha}}} \left(\frac{P_t^*}{P_{t+s}^e} \right)^{-\frac{\theta}{1-\alpha}} \left(Y_{t+s}^e \right)^{\frac{1}{1-\alpha}} \right\}.$$

We assume that the monopolist has rational expectations with respect to exogenous variables, but is boundedly rational with respect to endogenous variables. In particular, we assume that the firm forms beliefs about future aggregate prices, P_t^e, wages, W_t^e,and output Y_t^e using level-k thinking.

The first-order condition implies that:

$$\frac{P_t^*}{P_t} = \left\{ \frac{\theta}{(\theta-1)(1-\alpha)} \frac{\sum_{s=0}^{\infty} \lambda^s Q_{t,t+s} \frac{P_{t+s}^e}{P_t} \frac{W_{t+s}^e}{P_{t+s}^e} \frac{1}{A^{\frac{1}{1-\alpha}}} \left(\frac{P_{t+s}^e}{P_t} \right)^{\frac{\theta}{1-\alpha}} \left(Y_{t+s}^e \right)^{\frac{1}{1-\alpha}}}{\sum_{s=0}^{\infty} \lambda^s Q_{t,t+s} \frac{P_{t+s}^e}{P_t} \left(\frac{P_{t+s}^e}{P_t} \right)^{\theta-1} Y_{t+s}^e} \right\}^{\frac{1-\alpha}{1-\alpha(1-\theta)}}. \tag{D.1}$$

Let lower case letters denote the log-deviation of a variable from its steady-state value, $x_t \equiv \log X_t - \log X$. Using (D.1) we obtain

$$p_t^* - p_t = \zeta (1-\lambda\beta) \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ w_{t+s}^e - p_{t+s}^e + \frac{\alpha}{1-\alpha} y_{t+s}^e \right\} + \sum_{s=1}^{\infty} (\beta\lambda)^s \pi_{t+s}^e, \tag{D.2}$$

where $\zeta \equiv \frac{1-\alpha}{1-\alpha(1-\theta)}$.

The price level is given by

$$P_t = \left[\lambda P_{t-1}^{1-\theta} + (1-\lambda) \left(P_t^* \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

so

$$p_t = \lambda p_{t-1} + (1-\lambda) p_t^* \Leftrightarrow \pi_t = \frac{1-\lambda}{\lambda} (p_t^* - p_t). \tag{D.3}$$

Combining (D.2) and (D.3) we obtain:

$$\pi_t = \kappa \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ w^e_{t+s} - p^e_{t+s} + \frac{\alpha}{1-\alpha} y^e_{t+s} \right\} + \frac{1-\lambda}{\lambda} \sum_{s=1}^{\infty} (\beta\lambda)^s \pi^e_{t+s}, \tag{D.4}$$

where $\kappa \equiv \xi \frac{(1-\lambda)(1-\lambda\beta)}{\lambda}$.

Household The household chooses consumption and labor to maximize:

$$\max \sum_{s=0}^{\infty} \beta^s \xi_{t+s} \left[u\left(\widetilde{C}_{t+s}\right) - v\left(\widetilde{N}_{t+s}\right) \right]$$

$$\sum_{s=0}^{\infty} Q_{t,t+s} P^e_{t+s} \left(1 + \tau^c_{t+s}\right) \widetilde{C}_{t+s} = \sum_{s=0}^{\infty} Q^e_{t,t+s} \left[\left(1 - \tau^n_{t+s}\right) W^e_{t+s} \widetilde{N}_{t+s} + \Omega^e_{t+s} - T^e_{t+s} \right] + R_{t-1} B_t.$$

The solution to this problem implies

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} \left\{ \left(1 - \tau^n_{t+s}\right) W^e_{t+s} \widetilde{N}_{t+s} + \Omega^e_{t+s} - T^e_{t+s} \right\} + R_{t-1} b_{i,t}}{\left(1 + \tau_t\right) P_t \left[1 + \sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t} \right)^{\sigma} \left[Q_{t,t+s} \frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau_t\right)} \right]^{1-\sigma} \right]},$$

where

$$\widetilde{N}^{\varphi}_{t+s} = \frac{\left(1 - \tau^n_{t+s}\right) W^e_{t+s}}{\left(1 + \tau^c_{t+s}\right) P^e_{t+s}} \left(\beta^s \frac{\xi_{t+s}}{\xi_t} \right)^{-1} \frac{Q_{t,t+s} P^e_{t+s} \left(1 + \tau^c_{t+s}\right)}{P_t \left(1 + \tau^c_t\right)} C_t^{-\sigma^{-1}}. \tag{D.5}$$

Using peoples' beliefs about the government budget constraint, (3.6), and the aggregate resource constraint, (2.3), we obtain

$$C_t = \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau^c_t\right)} \left\{ \left(\frac{1-\tau^n_{t+s}}{1-\tau^c_{t+s}} \right) \frac{W^e_{t+s}}{P^e_{t+s}} \left\{ \widetilde{N}_{t+s} - N^e_{t+s} \right\} + Y^e_{t+s} - G_{t+s} \right\}}{\sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t} \right)^{\sigma} \left[Q_{t,t+s} \frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau_t\right)} \right]^{1-\sigma}}. \tag{D.6}$$

Log-linearizing equations D.5and D.6 yields:

$$\begin{aligned} \widetilde{n}_{t+s} = & -\varphi^{-1} \left(\hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} \right) + \varphi^{-1} \left(w^e_{t+s} - p^e_{t+s} \right) \\ & - \varphi^{-1} \sum_{m=0}^{s-1} \left(r_{t+m} - \pi^e_{t+m+1} - \Delta\hat{\tau}^c_{t+m} + \chi_{t+m} \right) - (\varphi\sigma)^{-1} c_t \end{aligned} \tag{D.7}$$

and

$$c_t = \frac{1-\beta}{\beta}\sum_{s\geq 1}\beta^s \frac{Y}{C}\left\{y^e_{t+s} - g_{t+s} - \omega_N n^e_{t+s}\right\} + \frac{1-\beta}{\beta}\sum_{s\geq 1}\beta^s \frac{Y}{C}\omega_N \widetilde{n}_{t+s} \tag{D.8}$$
$$- \sigma \sum_{m=0}^{\infty}\beta^s\left\{r_{t+s} - \pi^e_{t+s+1} - \Delta\hat{\tau}^c_{t+s} + \sigma\chi_{t+s}\right\}$$

where $\omega_N = \left(\frac{1-\tau^n}{1-\tau^c}\right)\frac{W}{P}\frac{N}{Y}$. Replacing (D.7) in (D.8), we obtain:

$$c_t = \psi \sum_{s\geq 1}\beta^s \frac{Y}{C}\left\{y^e_{t+s} - g_{t+s} - \omega_N n^e_{t+s} + \varphi^{-1}\left\{w^e_{t+s} - p^e_{t+s} - (\hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s})\right\}\right\}$$
$$- \sigma \sum_{m=0}^{\infty}\beta^s\left\{r_{t+s} - \pi^e_{t+s+1} - \Delta\hat{\tau}^c_{t+s} + \sigma\chi_{t+s}\right\}$$

where $\psi \equiv \frac{\sigma}{\sigma + \frac{Y}{C}\omega_N\varphi^{-1}}\frac{1-\beta}{\beta}$.

Equilibrium In equilibrium, labor-market clearing, $N_t = \int n_{u,t}du$, implies that:

$$N_t = \int n_{u,t}du = \int\left(\frac{y_{u,t}}{A}\right)^{\frac{1}{1-\alpha}}du = \int\left(\frac{Y_t}{A}\right)^{\frac{1}{1-\alpha}}\left(\frac{p_{u,t}}{P_t}\right)^{-\frac{\theta}{1-\alpha}}du$$

which implies that

$$Y_t = \mu_t^{\alpha-1}AN_t^{1-\alpha} = C_t + G_t,$$

where $\mu_t = \int\left(\frac{p_{u,t}}{P_t}\right)^{-\frac{\theta}{1-\alpha}}$ denotes the standard price distortion. Starting from an non-distorted steady state implies $\mu_{-1} = 1$ and to first order the price distortion is zero.

The temporary equilibrium conditions are as follows.

1. Consumption is given by

$$c_t = \psi \sum_{s\geq 1}\beta^s \frac{Y}{C}\left\{y^e_{t+s} - g_{t+s} - \omega_N n^e_{t+s} + \varphi^{-1}\left\{w^e_{t+s} - p^e_{t+s} - (\hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s})\right\}\right\} \tag{D.9}$$
$$- \sigma \sum_{m=0}^{\infty}\beta^s\left\{r_{t+s} - \pi^e_{t+s+1} - \Delta\hat{\tau}^c_{t+s} + \chi_{t+s}\right\}.$$

2. Inflation is given by

$$\pi_t = \kappa\sum_{s=0}^{\infty}(\beta\lambda)^s\left\{(\hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s}) + \varphi n^e_{t+s} + \sigma^{-1}c^e_{t+s} + \frac{\alpha}{1-\alpha}y^e_{t+s}\right\} + \frac{1-\lambda}{\lambda}\sum_{s=1}^{\infty}(\beta\lambda)^s\pi^e_{t+s}. \tag{D.10}$$

3. Output is given by

$$y_t = (1 - \alpha) n_t. \tag{D.11}$$

4. Market clearing implies

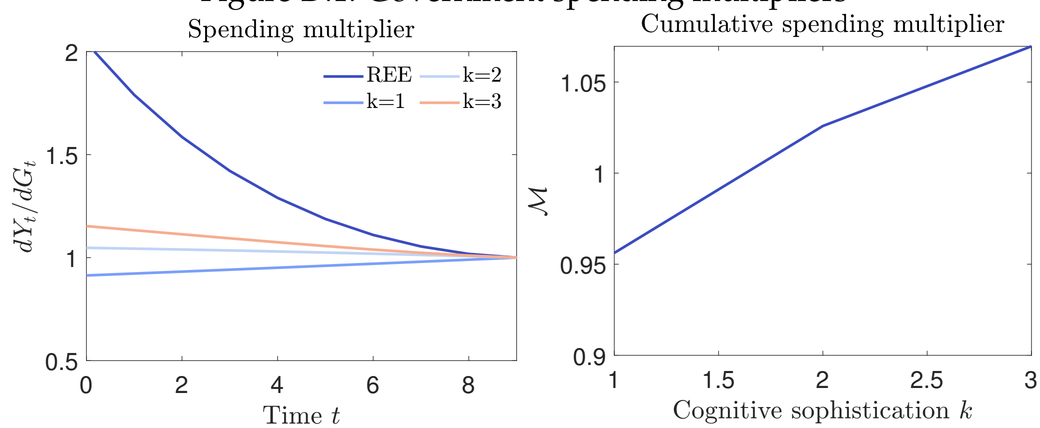
$$y_t = \frac{C}{Y} c_t + g_t. \tag{D.12}$$

Note that we assume that the beliefs that firms have about the real wage are consistent with household labor supply. An equilibrium is a solution to this system along with a specification of belief formation corresponding to level-k thinking.

D.1 Government spending multipliers

In this section we briefly illustrate the analog to Proposition 1 for the case in which tax rates are constant and government spending rises by ΔG during the ZLB period.

Figure D.1: Government spending multipliers



Comparing figures 3.1 and D.1, we see that the implications of level-k thinking for the government multiplier are essentially the same, regardless of whether Calvo frictions apply to wages and prices.

D.2 Consumption tax policy

Proposition 5 continues to hold for the economy in which prices, rather than wages, are subject to Calvo frictions.

Proof. (Part 1) The proof strategy is as follows. Fix a k. First, we show that if the level-1 believe that the economy will stay at steady state for $t \geq T$, then this implies that all level-k beliefs and equilibrium feature output, consumption, labor and wage inflation remaining at their steady state levels from $t \geq T$, and price inflation becoming zero from $t \geq T+1$ on. Second, we note that beliefs about future output, inflation, consumption, and labor are a function only of future tax rates and policies. (3) Finally, we recursively construct a sequence of policies $\left\{\hat{\tau}_t^{c,k}, \hat{\tau}^{n,k}\right\}$ which implements the flexible price allocation and always features zero inflation. □

(1) Suppose that $y_t^e = c_t^e = n_t^e = 0$ and $\pi_t^e = 0$ if $t \geq T$, then the policies $g_t = \hat{\tau}_t^{c,k} = \hat{\tau}^{n,k} = r_t = 0$ for all $t \geq T$ imply that consumption, output, and labor for $t \geq T$ are given by

$$c_t = \psi \sum_{s \geq 1} \beta^s \frac{Y}{C} \left\{ y_{t+s}^e - \omega_N n_{t+s}^e + \varphi^{-1} \left\{ w_{t+s}^e - p_{t+s}^e \right\} \right\} = 0.$$

$$y_t = \frac{C}{Y} c_t = 0,$$

and

$$n_t = \frac{y_t}{1-\alpha} = 0,$$

respectively. Finally, inflation is given by

$$\pi_t = \kappa \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ \varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \frac{\alpha}{1-\alpha} y_{t+s}^e \right\} + \frac{1-\lambda}{\lambda} \sum_{s=1}^{\infty} (\beta\lambda)^s \pi_{t+s}^e = 0.$$

This then shows that starting from the initial beliefs $y_t^{e,1} = c_t^{e,1} = n_t^{e,1} = 0$ and $\pi_t^{e,1} = 0$ implies that the same holds for all k.

(2) Note that the temporary equilibrium for time t, which solves the system of equations (D.9)-(D.12) does not depend on policies before time t. This implies that for each t, $y_t^{e,k}$ is unaffected by policies $\left\{\hat{\tau}_s^{c,k}, \hat{\tau}_s^{n,k}\right\}_{s=0}^{t-1}$.

(3) We now proceed recursively. At time t, given policies $\left\{\hat{\tau}_{t+s}^{c,k}, \hat{\tau}_{t+s}^{n,k}\right\}_{s \geq 1}$ and beliefs $\left\{y_{t+s}^{e,k}, c_{t+s}^{e,k}, n_{t+s}^{e,k}, \pi_{t+s}^{e,k}\right\}_{s \geq 1}$, we set the consumption tax $\hat{\tau}_t^{c,k}$ so that

$$\begin{aligned} \hat{\tau}_t^{c,k} = & \frac{\psi}{\sigma} \sum_{s \geq 1} \beta^s \frac{Y}{C} \left\{ y_{t+s}^{e,k} - \omega_N n_{t+s}^e + \varphi^{-1} \left\{ w_{t+s}^{e,k} - p_{t+s}^{e,k} - \left(\hat{\tau}_{t+s}^{n,k} + \hat{\tau}_{t+s}^{c,k} \right) \right\} \right\} \\ & - \left\{ -\pi_{t+1}^{e,k} - \hat{\tau}_{t+1}^{c,k} + \chi - \rho \right\} - \sum_{s=1}^{\infty} \beta^s \left\{ r_{t+s} - \pi_{t+s+1}^{e,k} - \Delta \hat{\tau}_{t+s}^{c,k} + \chi_{t+s} \right\}, \end{aligned}$$

which implies that $c_t^k = 0$. It then follows that $n_t^k = y_t^k = 0$. Then, setting $\hat{\tau}_{t+s}^{n,k}$ such that

$$\hat{\tau}_t^{n,k} = -\hat{\tau}_t^{c,k} - \sum_{s=1}^{\infty} (\beta\lambda)^s \left\{ (\hat{\tau}_{t+s}^{n} + \hat{\tau}_{t+s}^{c}) + \varphi n_{t+s}^e + \sigma^{-1} c_{t+s}^e + \frac{\alpha}{1-\alpha} y_{t+s}^e \right\} - \frac{1-\lambda}{\lambda\kappa} \sum_{s=1}^{\infty} (\beta\lambda)^s \pi_{t+s}^e$$

which implies that $\pi_t^k = 0$.

Proof. (Part 2) Under this assumption, the consumption function still implies that $C_t^1 = C$, which implies that $N_t^1 = N$ and $Y_t^1 = Y$, i.e., both consumption, labor, and output in the level-1 economy stay at their steady state levels. Using the fact that $(1-\tau_t^n) / (1+\tau_t^c) = (1-\tau^n) / (1+\tau^c)$, this implies that the relative wage W_t^1 / P_t^1 remains at its pre-shock steady state as well. Finally, this implies that $p_t^{*,1} = p_t$ and so inflation is always zero. The same argument then holds for $k > 1$. □

D.2.1 Rules versus targets

Figure D.2 is the analog to Figure 3.2, assuming prices are subject to Calvo-style frictions. We see that the implications of level-k thinking for the efficacy of fiscal policy when prices, rather than wages, are subject to Calvo-style frictions are essentially the same.

Figure D.2: Rules equilibrium